System Identification

Lecture 1: Introduction to System Identification

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Identification objectives

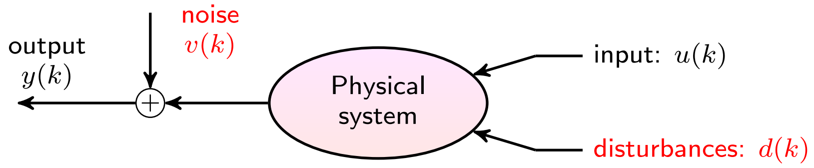
- Simple approximation to the system.
- Model for prediction purposes.
- Clearer understanding of system behaviour.
- Model for use in control design.

Fundamental issues: identifying dynamical systems

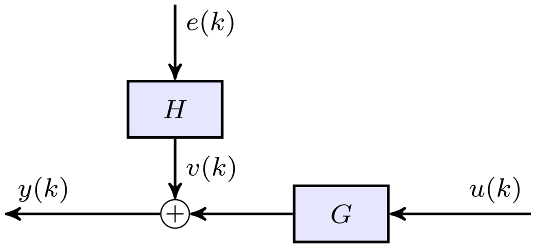
- Purpose of the model.
- Open- or closed-loop experiments.
- Linearity of the system. Linear model assumptions.
- Choice of sample period, T.
- Finite data records.
- Constraints on the input excitation.
- Noise on the measurements.
- Experiment time constraints.
- Data efficiency (how big is the data?)

Identification framework: open-loop configuration

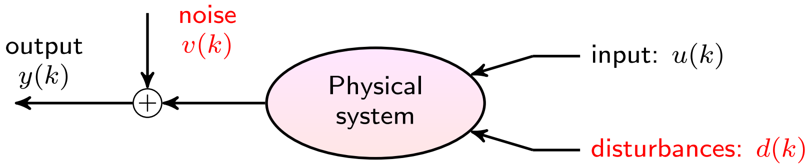
Experimental system



Identified model



Fundamental concepts



An Identification Procedure: from data to a model

1. Choose an input of length K: $u(k),\ k = 0, \ldots, K-1$.
2. Apply $u(k)$ to the plant, G.
3. Measure $y(k),\ k = 0, \ldots, K-1$.
4. Use $\{u(k), y(k)\},\ k = 0, \ldots, K-1$ to get a model, $\hat{G}$, for the plant G.

Basic idea:

$$y = G\,u, \qquad \text{so} \quad \hat{G} \approx y/u.$$

Models

First principles (from the physics)

Derived from physical principles.

$$f = m\ddot{x} \quad \Longrightarrow \quad G(s) = \frac{1/m}{s^2}, \text{ where } y = x, \text{ and } u = f.$$

"Black-box" models

Response estimated from experimental data (measurements of $f(t)$ and $x(t)$).

"Grey-box" models

A combination of first principles and experimentally derived parameters.

$$G(s) = \frac{\theta}{s^2}, \quad \text{where } \theta \text{ is to be estimated from data.}$$

Modeling philosophy

Applicability

"All models are wrong, but some are useful." George Box, 1978

George Box (1919–2013)

Parsimony

Make the model as simple as possible.

Occam's razor - remove extraneous assumptions.

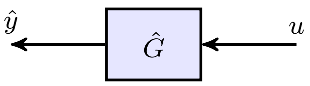
Simplier models are more easily falsified.

Fewer parameters require less data for identification.

William of Ockham (1285–1347)

Model purpose

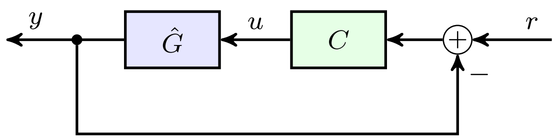
Open-loop prediction



$$\hat{y} = \hat{G}u.$$

Criterion: $\|y - \hat{y}\|$

Closed-loop design



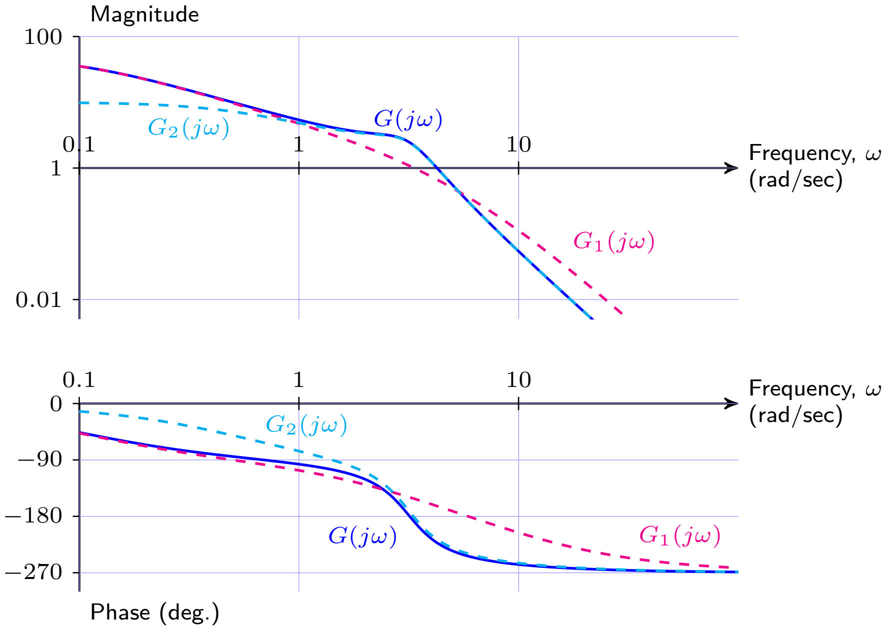
$$\frac{y}{r} = \frac{\hat{G}C}{1 + \hat{G}C}$$

Criteria: $\dfrac{GC}{I + GC}$ stable

$$\left\| \frac{GC}{1 + GC} - \frac{\hat{G}C}{1 + \hat{G}C} \right\|$$

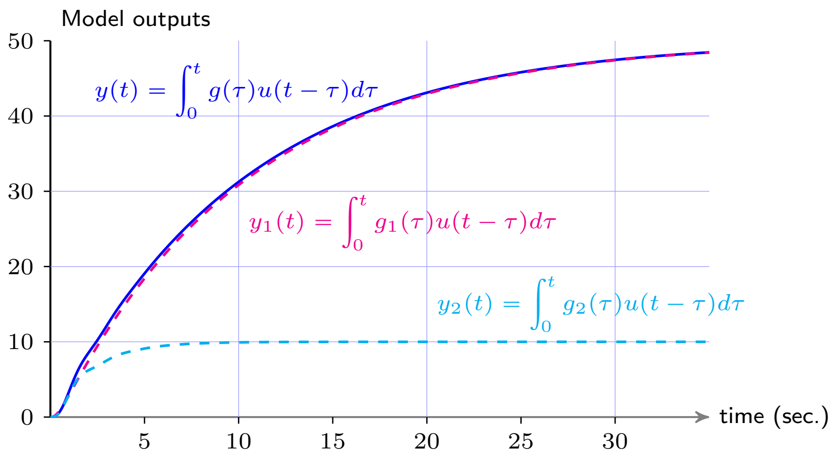
Model purpose

Model comparison: frequency domain



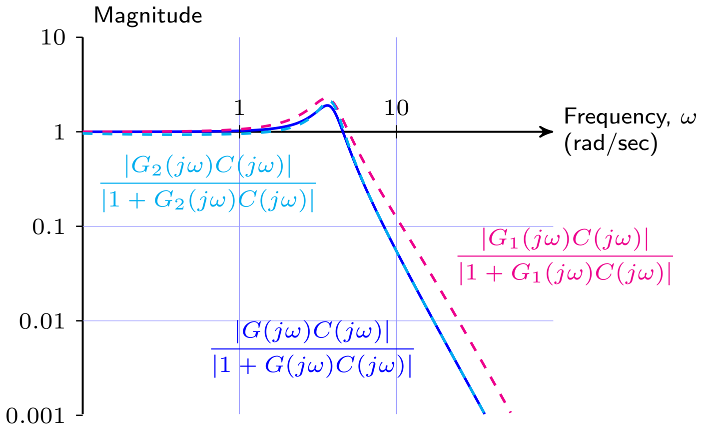
Model purpose

Step responses



Model purpose

Closed-loop responses



Controller:

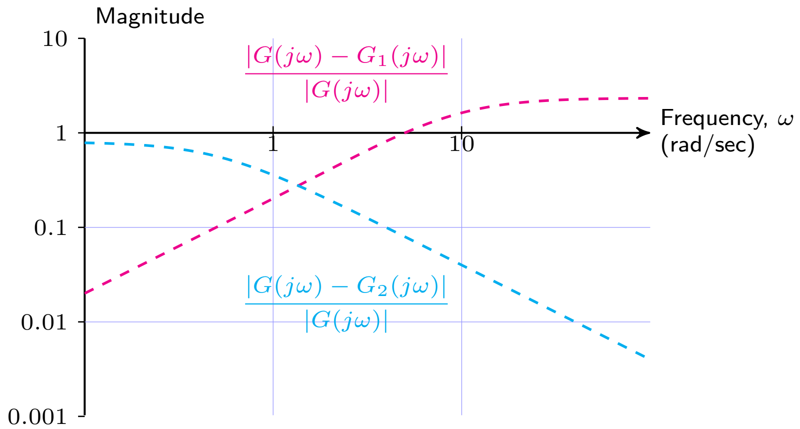
$$C(s) = \frac{0.01(10s+1)}{0.1s}$$

Closed-loop stability:

G_1	stable
G_2	unstable
G	unstable

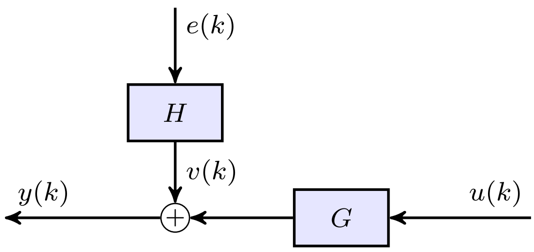
Model purpose

Relative errors



Open-loop identification

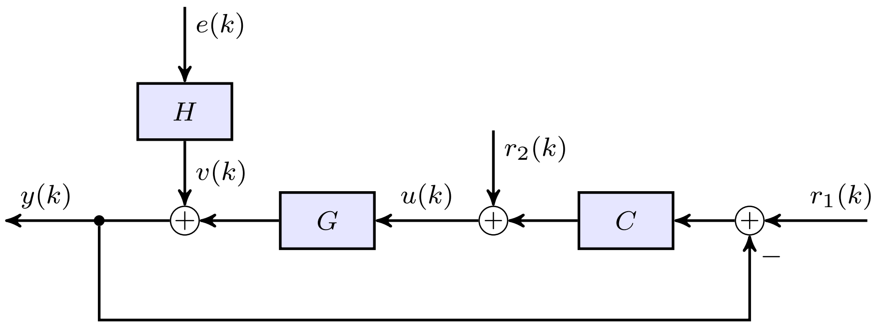
Discrete-time model:



Main assumptions

- G is linear.
- $e(k)$ is zero-mean white noise (with known variance).
- H may be known approximately.

Closed-loop identification



Key features

- Closed-loop operation can set the operating point of the plant.
- For many plants closed-loop operation is required.
- For unstable plants, closed-loop experiments are essential.
- A well designed controller masks variations in the plant.
- We can now measure $y(k)$, $u(k)$, $r_1(k)$ and $r_2(k)$.
- All of these signals are now correlated.

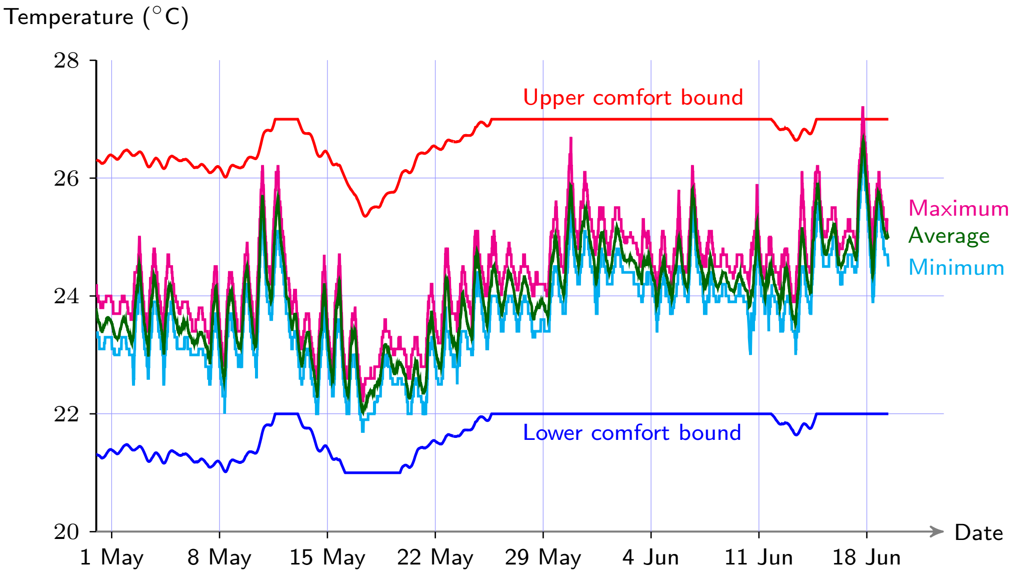
Example: energy flows in buildings

Inputs: Air supply temperatures
In-floor water system temperatures
Blind settings
Solar radiation
Outside temperature

Outputs: Room temperatures

Example: energy flows in buildings

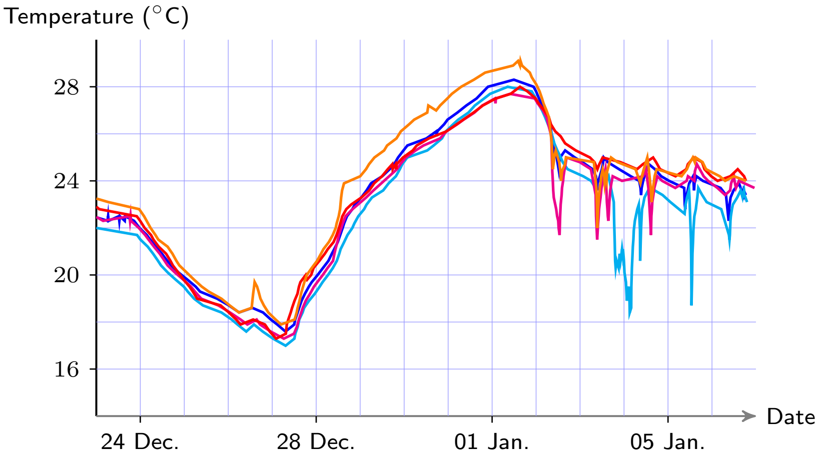
Room temperatures (controlled operation)



Step (or doublet) excitation

Step changes in TABS heating setpoint

Excitation plan: 16 °C for 5 days
30 °C for 5 days

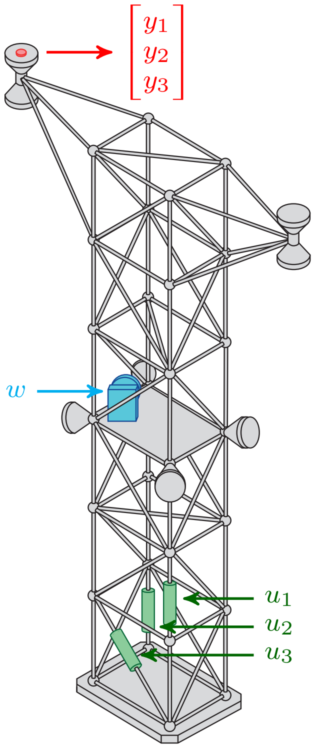


Example: energy flows in buildings

Things to consider:

- The building is occupied and (usually) controlled.
- The dynamics can be (very) slow.
- It takes a year to see a reasonably complete range of weather conditions.

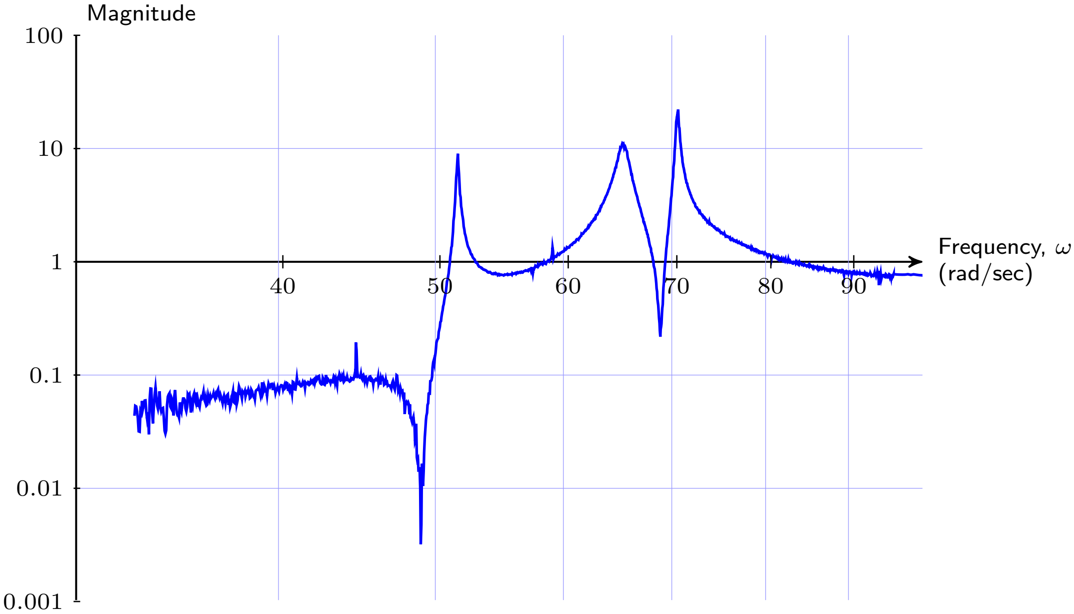
Example: vibration in flexible structures



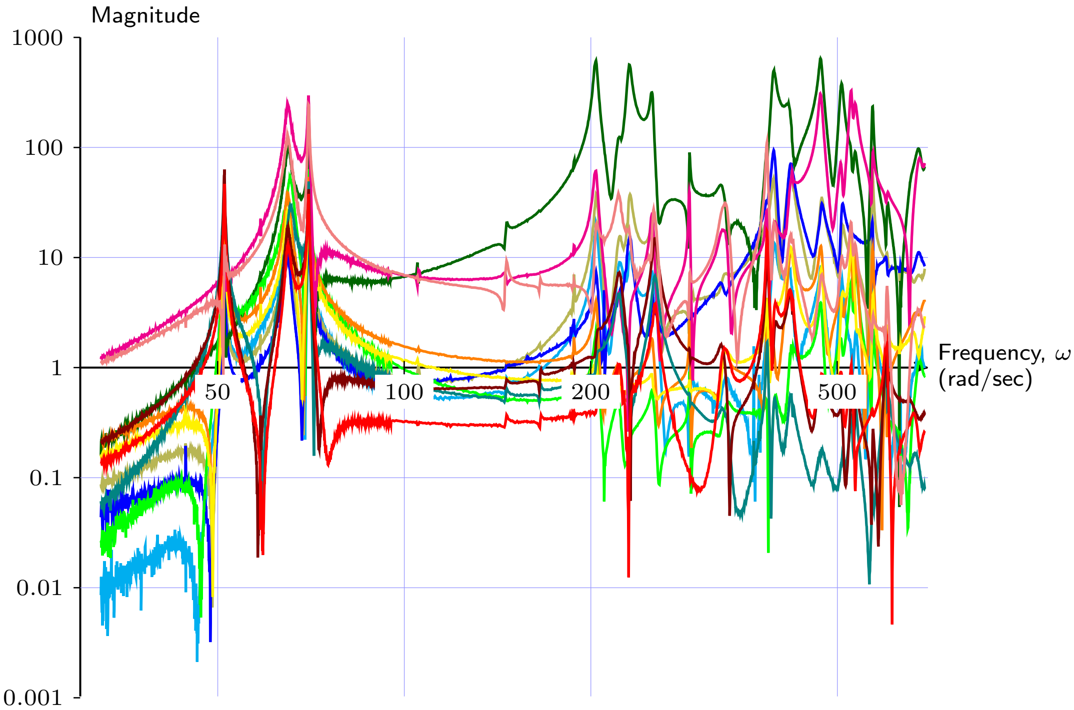
Inputs: Forces in structural members. (u_i)
Shaker disturbance (w)

Outputs: Accelerations in 3 dimensions (y_i)

Example: vibration in flexible structures



Example: vibration in flexible structures

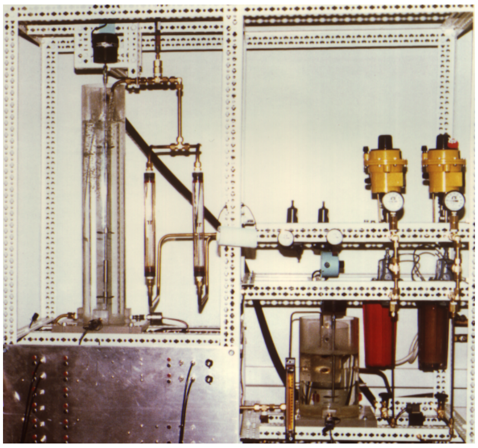


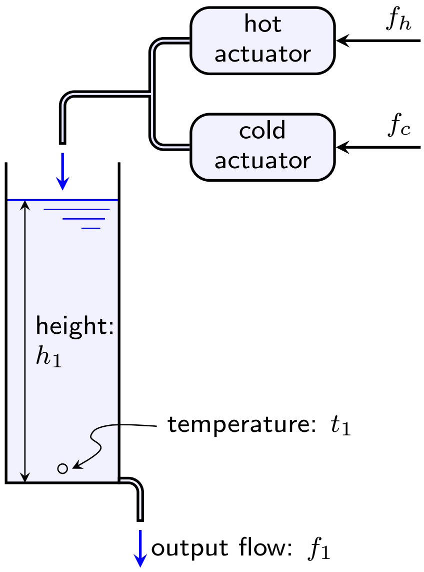
Example: vibration in flexible structures

Things to consider:

- Extremely fine frequency resolution is needed to resolve the modes.
- Long data records are required.
- There is noise and drift on the accelerometer measurements.
- Experiments are very sensitive to environmental disturbances.

Example: process control



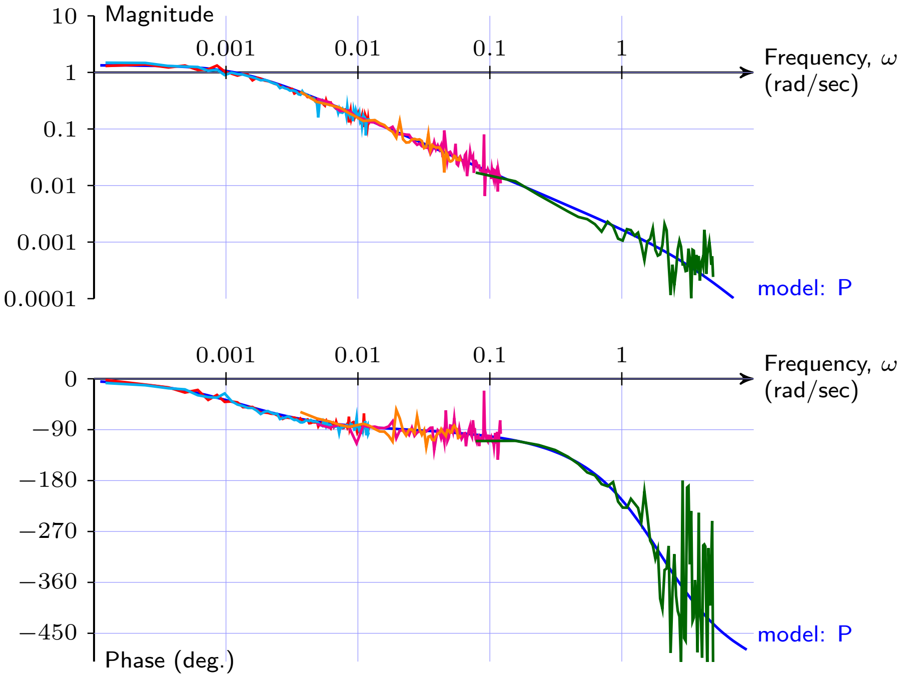


Inputs: Hot and cold water flows (f_h and f_c)

Outputs: Tank height and temperature (h_1 and t_1)

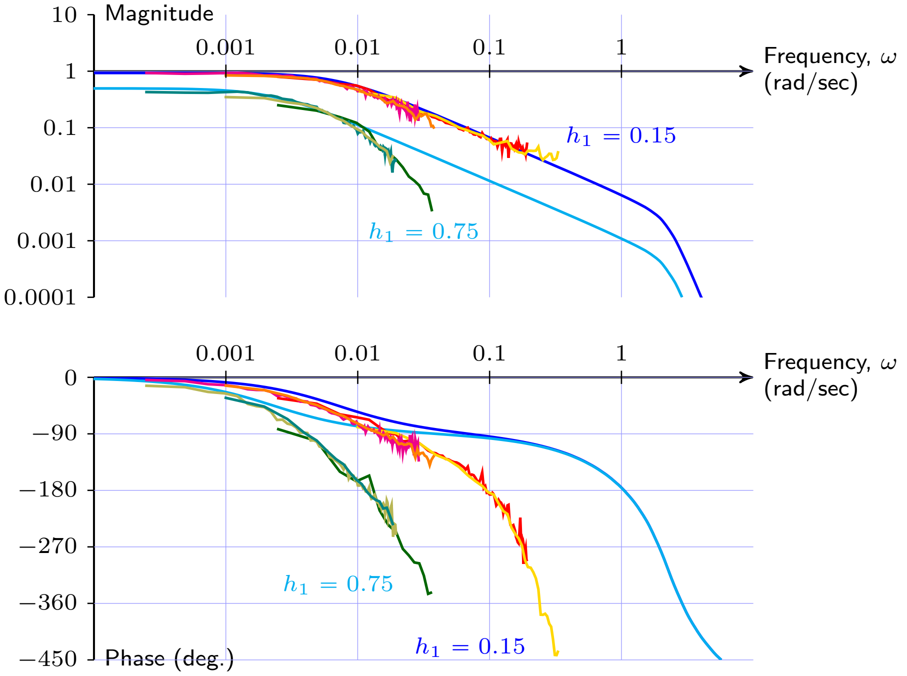
Example: process control

Height response: $f_h + f_c$ to h_1



Example: process control

Temperature response: $f_h - f_c$ to t_1



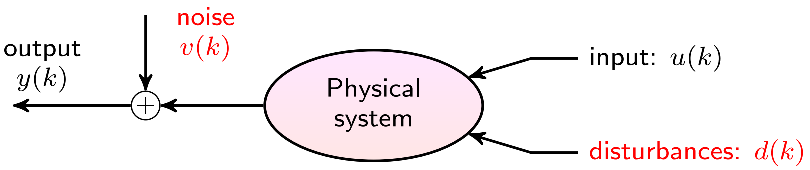
Example: process control

Things to consider:

- Temperature and pressure (height) measurements are noisy.
- A wide range of frequencies were identified (multiple experiments).
- Nonlinear mixing dynamics are significant in the temperature response.
- The temperature response is a strong function of the height.
- Closed-loop control was used for one variable (h_1 or t_1), while the other was excited in open-loop.

Identification: static open-loop case

Experimental system



Experiment:

Length K data record: $\mathcal{Z} \;=\; \{\,(y(k), u(k))\,\} \quad k = 0, \ldots, K-1.$

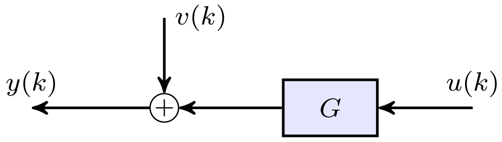
We may have multiple experiments, $\mathcal{Z}_r, \quad r = 1, \ldots, R.$

Identification: static open-loop case

Experiment:

Length K data record: $\mathcal{Z} \;=\; \{\,(y(k), u(k))\,\} \quad k = 0, \ldots, K-1.$

Model framework



Model set

- Static, scalar map: $G \in \mathcal{R}$.
- Linear system: $y(k) \;=\; G\,u(k) \;+\; v(k), \quad k = 0, \ldots, K-1.$
- No disturbances: $d(k) = 0, \quad k = 0, \ldots, K-1.$
- All of the uncertainty is to be described by $v(k)$.

Fitting the data

Identification procedure

Given $\mathcal{Z}$, find an estimate for G (denoted by $\hat{G}$).

We would like to pick the "best" estimate $\hat{G}$.

Fitting the experimental data

Our estimate would give a predicted output:

$$\hat{y}(k) \;=\; \hat{G}\,u(k), \quad k = 0, \ldots, K-1.$$

Define

$$\hat{y} = \begin{bmatrix} \hat{y}(0) \\ \vdots \\ \hat{y}(K-1) \end{bmatrix}, \quad u = \begin{bmatrix} u(0) \\ \vdots \\ u(K-1) \end{bmatrix}, \quad \text{so} \quad \hat{y} = \hat{G}\,u.$$

$$\underset{\hat{G} \in \mathcal{R}}{\text{minimise}} \left\| y - \hat{G}u \right\|_2^2$$

Fitting criteria

More than just fitting the data ...

Minimising the error in the fit addresses the fitting of the past data.

What about the predictive capabilities of the model (future data).

Would $\hat{G}$ still be a good model?

An additional assumption ...

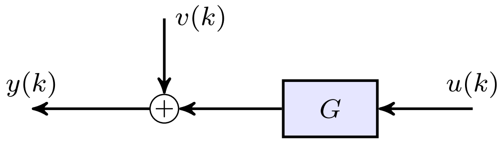
The "true plant" is an element of the model set.

This implies that there exists an optimal model: G_0

So all input-output data (future and unmeasured past) are described by G_0.

Model class

Static, linear model with zero-mean noise



$$y(k) \;=\; G_0\, u(k) \;+\; v(k), \quad \text{for all } k, \qquad \text{Assumptions:} \begin{cases} G_0 \in \mathcal{R}, \\ \mathcal{E}\{v(k)\} = 0 \\ \mathcal{E}\{|v(k)|^2\} = \sigma_v^2 \end{cases}$$

Statistical statements about the model quality

- Quantify (in expectation) to ability of an estimate $\hat{G}$ to predict future outputs.
- Quantify (in expectation) the accuracy of the model with respect to the experiment length.

Identification objectives

Mean square error criterion

$$\underset{\hat{G} \in \mathcal{R}}{\text{minimise}}\, \mathcal{E}\left\{ \|y - \hat{G}\, u\|_2^2 \right\} \qquad \text{MSE}$$

Note that the expectation allows us to make statements about the expected square-error when a different (future) input is applied.

Identification objectives

Statistical criteria

Every identification procedure is a mapping: $\mathcal{Z} \longrightarrow \hat{G}$.

The data record $\mathcal{Z}$ is a random variable $\Longrightarrow$ $\hat{G}$ is a random variable.

Statistical criteria on $\hat{G}$:

- Bias: $\mathcal{E}\left\{\hat{G}\right\} - G$.
- Asymptotic bias:

$$\lim_{K \longrightarrow \infty} \mathcal{E}\left\{\hat{G}\right\} - G.$$

- Variance: $\mathcal{E}\left\{\left|\hat{G} - \mathcal{E}\left\{\hat{G}\right\}\right|^2\right\}$.
- Consistency:

$$\lim_{K \longrightarrow \infty} \hat{G}_K \xrightarrow{p} G, \text{ or equivalently, } \lim_{K \longrightarrow \infty} \mathrm{Prob}\left\{\left|\hat{G}_K - G\right| > \epsilon\right\} = 0.$$

Bias-variance trade-offs

Bias-variance relationship

Relationship: MSE $=$ bias^2 $+$ variance.
This is easily shown by multiplying out:

$$\begin{aligned}
\mathrm{MSE}(\hat{G}) &= \mathcal{E}\left\{|G - \hat{G}|^2\right\} \\
&= \mathcal{E}\left\{\left|(G - \mathcal{E}\left\{\hat{G}\right\}) - (\hat{G} - \mathcal{E}\left\{\hat{G}\right\})\right|^2\right\} \\
&= \mathcal{E}\left\{|G - \mathcal{E}\left\{\hat{G}\right\}|^2\right\} + \mathcal{E}\left\{|\hat{G} - \mathcal{E}\left\{\hat{G}\right\}|^2\right\} \\
&\quad - \mathcal{E}\left\{2\,\mathrm{real}\left((G - \mathcal{E}\left\{\hat{G}\right\})(\hat{G} - \mathcal{E}\left\{\hat{G}\right\})^*\right)\right\} \\
&= \mathrm{bias}^2\left(\hat{G}\right) + \mathrm{var}\left(\hat{G}\right) \\
&\quad - \mathcal{E}\left\{2\,\mathrm{real}\left((G - \mathcal{E}\left\{\hat{G}\right\})(\hat{G} - \mathcal{E}\left\{\hat{G}\right\})^*\right)\right\}
\end{aligned}$$

The final step involves showing that the last term is zero.