Control Systems 2

Lecture 1: Closed-loop feedback control

Roy Smith

Feedback control

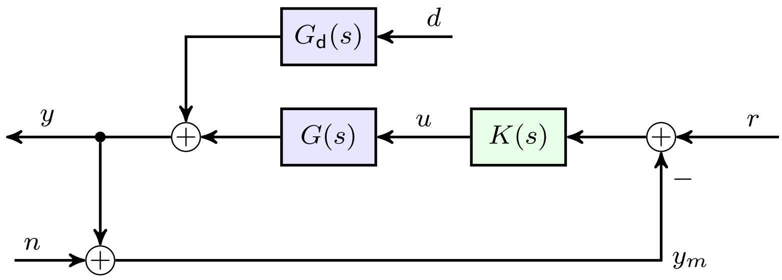
How well can you control a given system or plant?

Limitations arise from:

- Fundamental system/plant properties;
- Limits in knowledge about the plant;
- Variability in the plant.

Can you tell, in advance, whether a plant will be easy or hard to control?

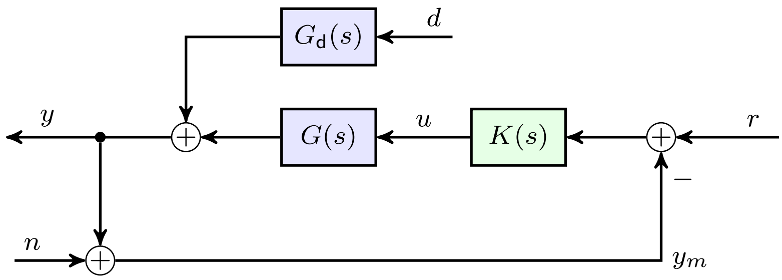
Feedback control



Objectives:

- Closed-loop stability

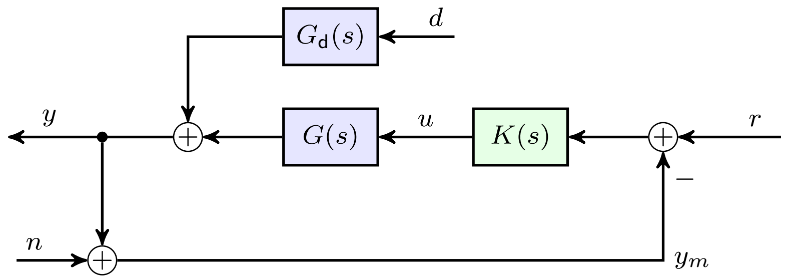
Feedback control



Objectives:

- Closed-loop stability
- Reference tracking

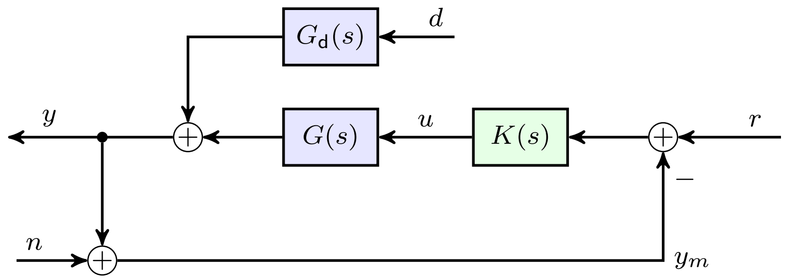
Feedback control



Objectives:

- Closed-loop stability
- Reference tracking
- Disturbance rejection

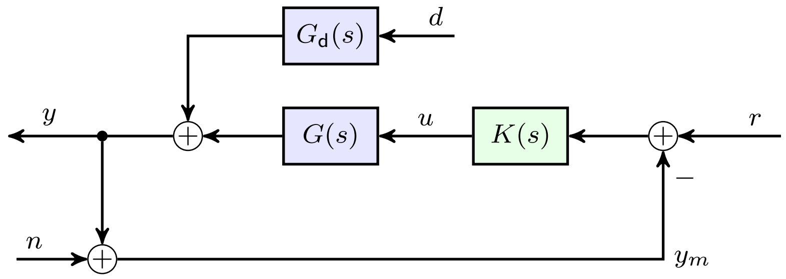
Feedback control



Objectives:

- Closed-loop stability
- Reference tracking
- Disturbance rejection
- Noise response

Feedback control



Difficulties:

- Model errors
- Fundamental limits on controllability of $G(s)$
- Actuation constraints

Transfer functions

Loop transfer function

$$L(s) = G(s)K(s)$$

Sensitivity function

$$S(s) = \frac{1}{1 + G(s)K(s)} = \frac{1}{1 + L(s)}$$

Complementary sensitivity

$$T(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} = \frac{L(s)}{1 + L(s)}$$

Output response

$$y = T(s)\,r \;+\; S(s)G_d(s)\,d \;-\; T(s)n$$

Error response

$$\begin{aligned} e &= r - y \\ &= S(s)\,r \;-\; S(s)G_d(s)\,d \;+\; T(s)n \end{aligned}$$

Conflicting objectives

Performance requirements

Reference tracking	$T(s) \approx 1$
Noise rejection	$T(s) \ll 1$
Disturbance rejection	$S(s)G_{\mathsf{d}}(s) \ll 1$
Low closed-loop plant sensitivity	$S(s) \ll 1$

Constraint

$$S(s) + T(s) = 1 \quad \text{for all } s$$

Sensitivity

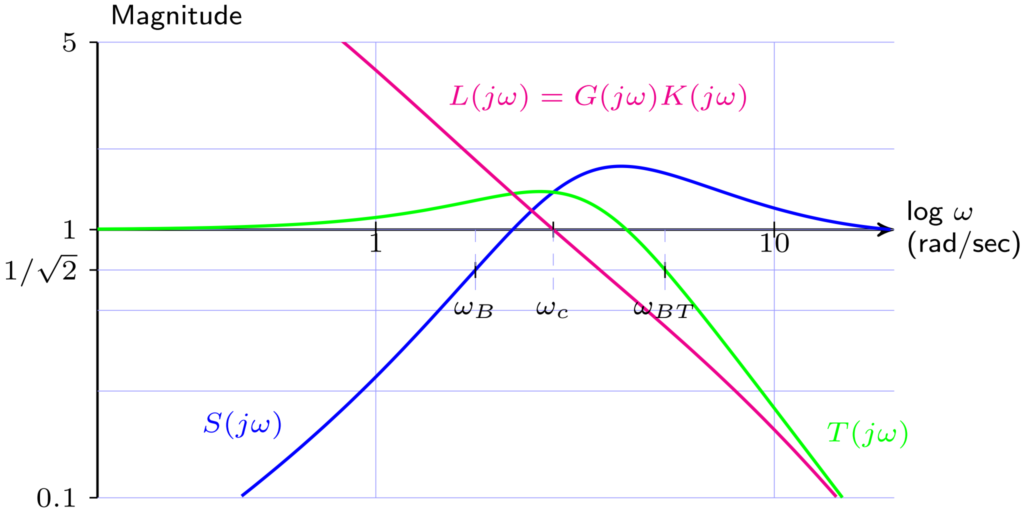
Sensitivity to plant gain changes

$$S(s) = \frac{\text{relative closed-loop response change}}{\text{relative open-loop response change}}$$

$$= \frac{dT(s)/T(s)}{dG(s)/G(s)}$$

A change of α percent in the open-loop plant DC gain gives a magnitude change of $|\alpha S(0)|$ percent in the closed-loop DC gain.

Closed-loop performance



$$G(s) = \frac{5\,e^{-0.1s}}{(s+1)(0.1s+1)} \qquad K(s) = \frac{0.5s+1}{s}$$

Closed-loop performance

(Closed-loop) bandwidth ω_B: Frequency at which $|S(j\omega)| = -3\text{dB} = 1/\sqrt{2}$.

Crossover frequency ω_c: Frequency at which $|L(j\omega)| = 1$.

If PM $< 90^\circ$ then, $\omega_B < \omega_c < \omega_{BT}$

Maximum control frequency: Frequency where $|K(j\omega)|$ is still significant.

(The $\omega_c < \omega_{BT}$ statement is true for "reasonable" control designs)

Closed-loop performance

Maximum peak criteria

$$M_S = \max_{\omega} |S(j\omega)| = \|S(s)\|_{\mathcal{H}_\infty} \quad \text{and} \quad M_T = \max_{\omega} |T(j\omega)| = \|T(s)\|_{\mathcal{H}_\infty}$$

Typical specifications:

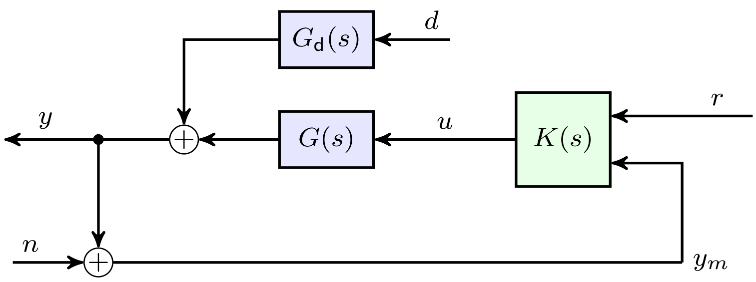
$M_S \leq 2$ and $M_T < 1.25$

Gain and phase margins:

$$\text{GM} \geq \frac{M_S}{M_S - 1}, \quad \text{PM} \geq 2 \arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S} \text{(rad)}$$

Alternative structures

2 degrees-of-freedom structure



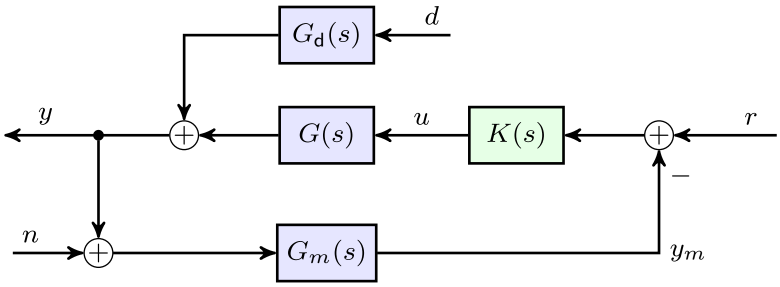
$$u = K(s)\begin{bmatrix} r \\ y_m \end{bmatrix} = \begin{bmatrix} K_r(s) & K_y(s) \end{bmatrix}\begin{bmatrix} r \\ y_m \end{bmatrix}$$

$$L(s) = -G(s)K_y(s)$$

$$y = S(s)G_d(s)\, d \;+\; S(s)G(s)K_r(s)\, r \;+\; S(s)G(s)K_y(s)\, n$$

Alternative structures

Additional loop dynamics

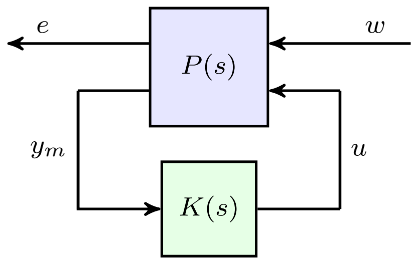


$$L(s) = G(s)K(s)G_m(s)$$

$$y = S(s)G_d(s)\,d \;+\; S(s)G(s)K(s)\,r \;-\; T(s)\,n$$

Alternative structures

A general structure

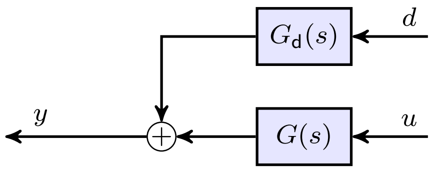


e = performance outputs $\quad$ w = exogenous inputs

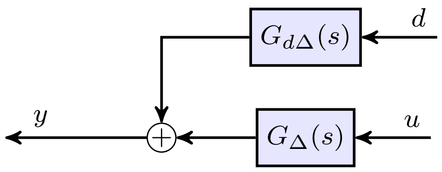
y_m = measured signals $\quad$ u = control actuation

Perturbed systems

Nominal case:



Perturbed case:



$$G_\Delta(s) = \{\, G(s) + \Delta(s) \mid \Delta(s) \in \text{ Set } \,\}$$
$$G_{d\Delta}(s) = \{\, G_d(s) + \Delta_d(s) \mid \Delta_d(s) \in \text{ Set } \,\}$$

Uncertainty

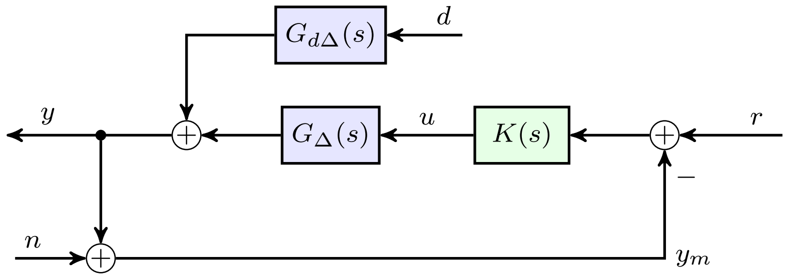
Sources of uncertainty:

- Nonlinear dynamics.
- Operating point variation.
- Neglected dynamics in the model.
- Non-repeatable dynamics.

Which dynamics should we model accurately?

Robustness

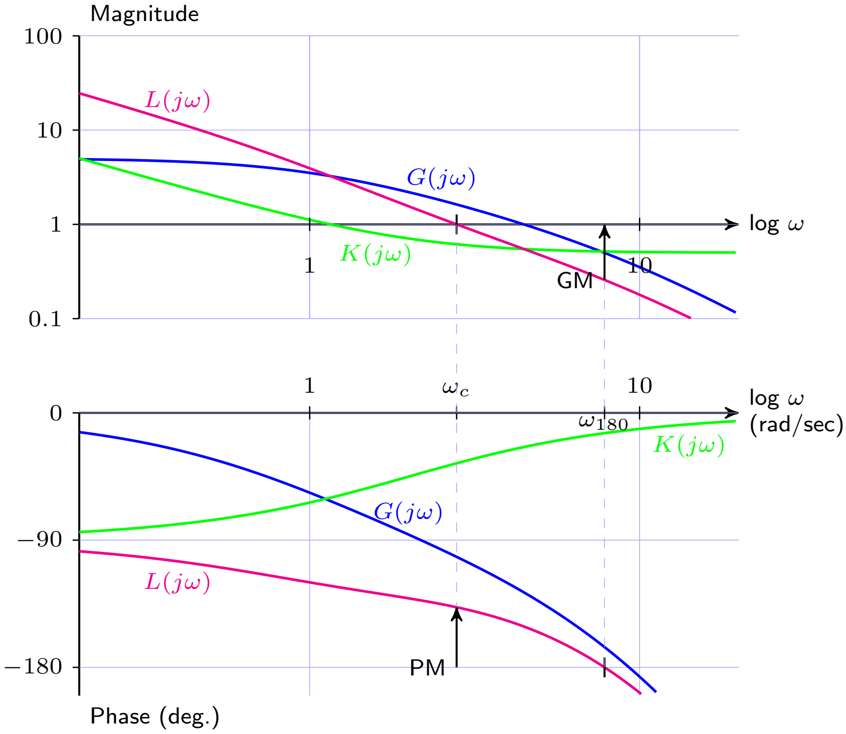
Closed-loop configuration:



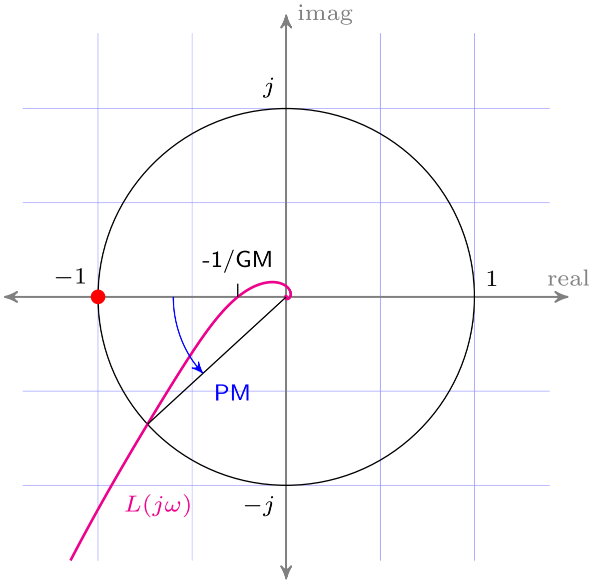
$$G_\Delta(s) = \{\ G(s) + \Delta(s) \mid \Delta(s) \in \text{ Set }\ \}$$
$$G_{\mathsf{d}\Delta}(s) = \{\ G_{\mathsf{d}}(s) + \Delta_{\mathsf{d}}(s) \mid \Delta_{\mathsf{d}}(s) \in \text{ Set }\ \}$$

What happens for the different Δ that may occur in practice?

Bode plot: margins

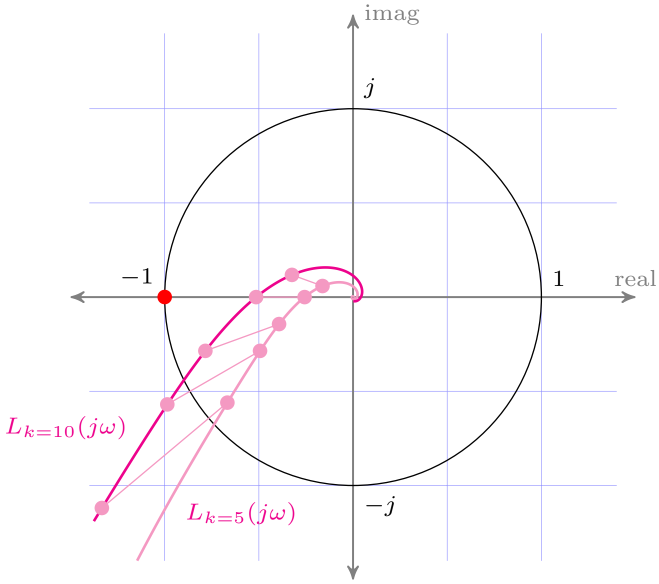


Nyquist plot: margins



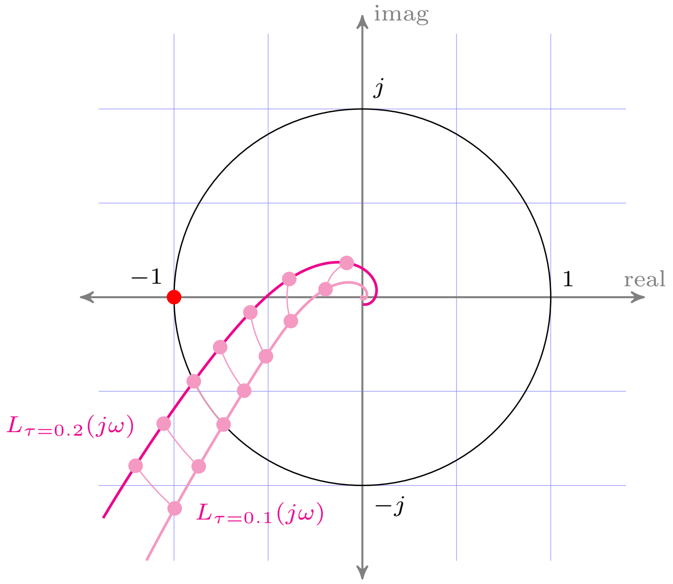
$$G(s) = \frac{5\,\mathrm{e}^{-0.1s}}{(s+1)(0.1s+1)} \qquad K(s) = \frac{0.5s+1}{s}$$

Nyquist plot: gain perturbation



$$G(s) = \frac{k\,\mathrm{e}^{-0.1s}}{(s+1)(0.1s+1)}, \quad k \in [5, 10] \qquad K(s) = \frac{0.5s+1}{s}$$

Nyquist plot: delay perturbation



$$G(s) = \frac{5\,\mathrm{e}^{-\tau s}}{(s+1)(0.1s+1)}, \quad \tau \in [0.1, 0.2] \qquad K(s) = \frac{0.5s+1}{s}$$

Robustness objectives

Nominal Stability (NS)
Closed-loop system stable with no model uncertainty.

Nominal Performance (NP)
Closed-loop system satisfies the performance requirements with no model uncertainty.

Robust Stability (RS)
Closed-loop system is stable for all models in a prescribed set.

Robust Performance (RP)
Closed-loop system satisfies the performance requirements for all models in a prescribed set.

Control design

Loop shaping
Design $K(s)$ so that the loop, $L(s)$, has the required properties (classical approach).

Signal-based optimal control
Design $K(s)$ to satisfy certain closed-loop system or signal objectives.
For example: LQG methods.

Numerical optimisation-based
Use multi-objective optimisation with closed-loop and robustness objectives.

Notes and references

Skogestad & Postlethwaite (2nd Ed.)

Introduction: Sections 1.1 – 1.6, & 2.1

Feedback control: Sections 2.2.

Closed-loop performance: Sections 2.3 & 2.4

Controller design: Section 2.5