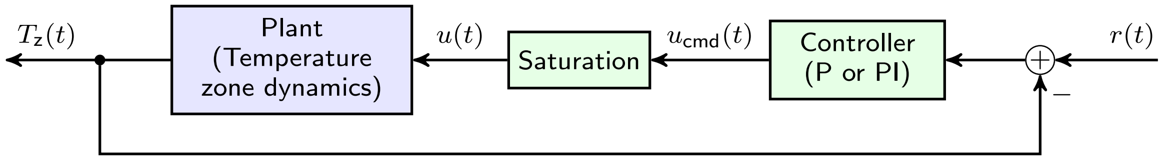
Building control

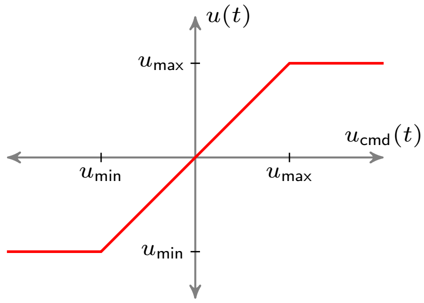
Lecture 5: Saturation, relay control and discretization

Roy Smith

Saturation

Plant actuation limitations





$$u(t) = \begin{cases} u_{max} & \text{if } u_{cmd}(t) > u_{max} \\ u_{cmd}(t) & \text{if } u_{min} \leq u_{cmd}(t) \leq u_{max} \\ u_{min} & \text{if } u_{cmd}(t) < u_{min} \end{cases}$$

Saturation

Plant actuation effects

When the input is saturated (e.g. $u_{\mathsf{cmd}}(t) > u_{\mathsf{max}}$) then the interconnection is of the plant and controller is non-linear.

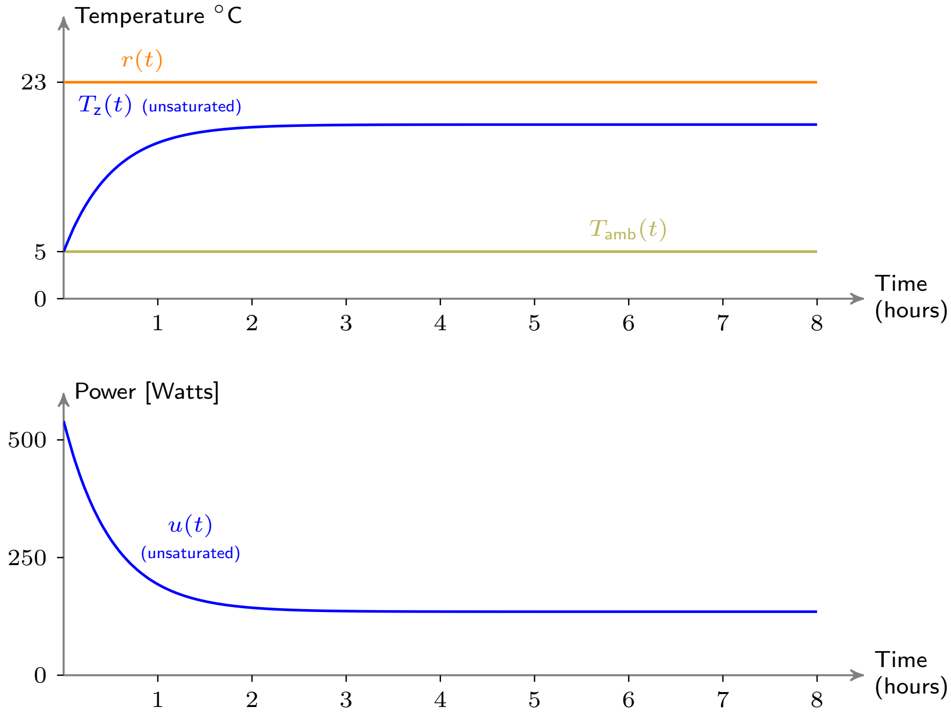
For small changes in the disturbance or the reference, the input to the plant will not change.

Cost efficient actuation

Many building systems operate with their actuators in saturation due to capital equipment cost constraints.

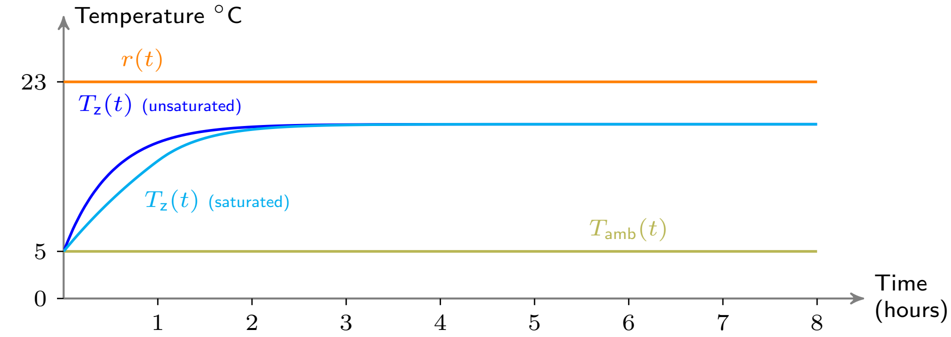
P-control, unsaturated input

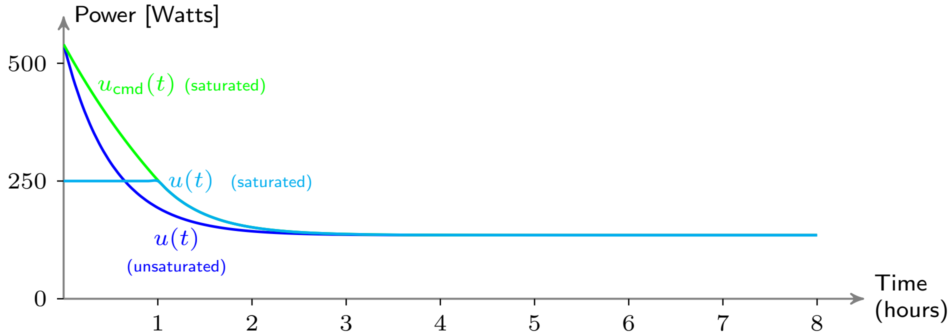
$K_{\mathsf{P}} = 30$



P-control, saturated input

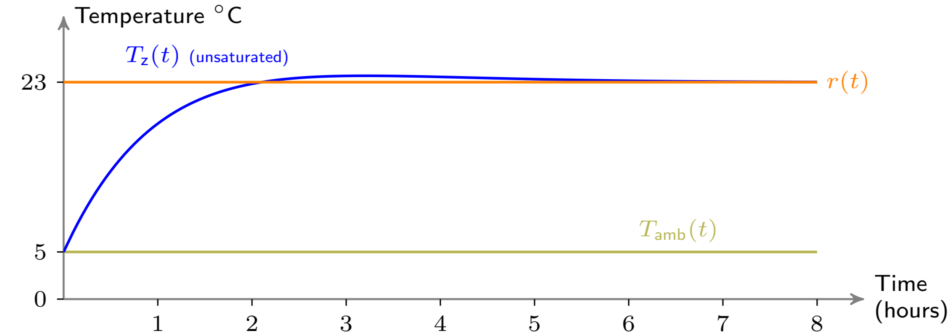
$K_P = 30, \quad u_{min} = 0W, \quad u_{max} = 250\text{W}$

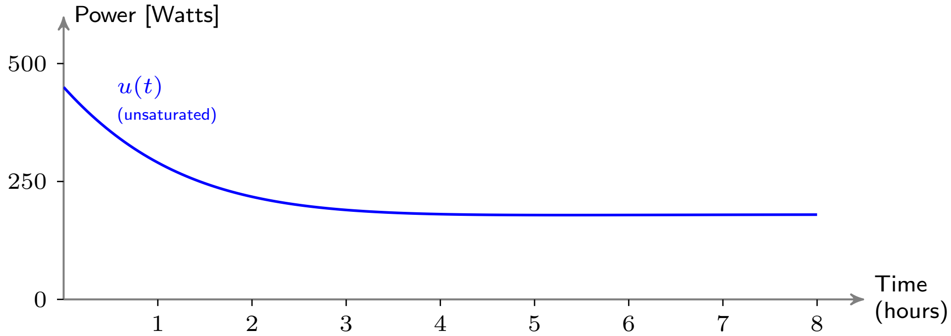




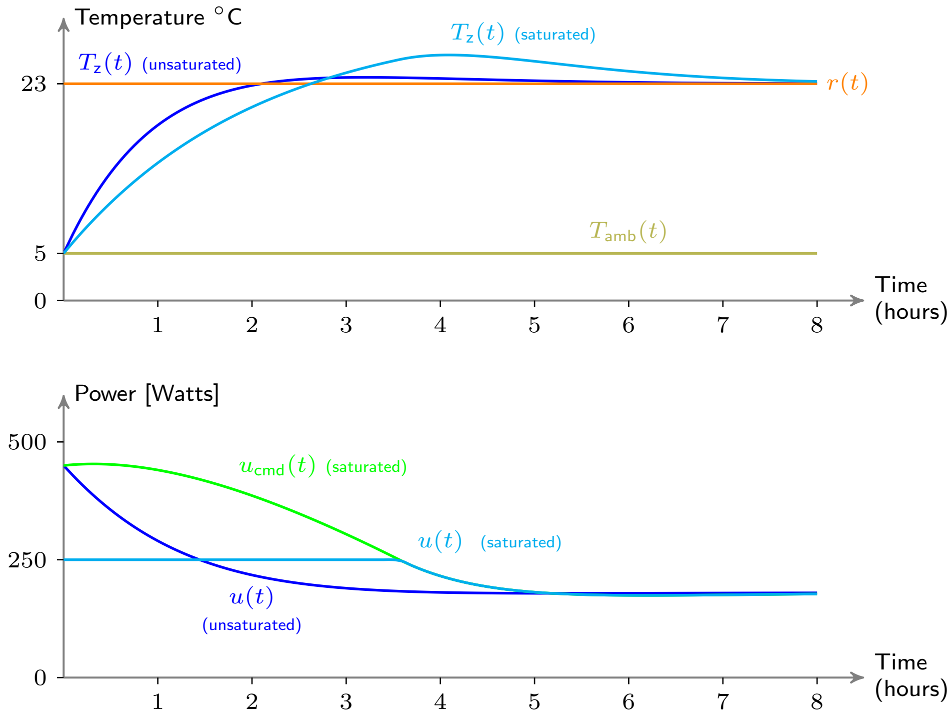
PI-control, unsaturated input

$K_P = 25, \quad K_I = 0.005$





PI-control, saturated input



Saturation

Integrator windup

PI controller:

$$u(t) \;=\; K_P\, e(t) + K_I \int_0^t e(t)\, dt.$$

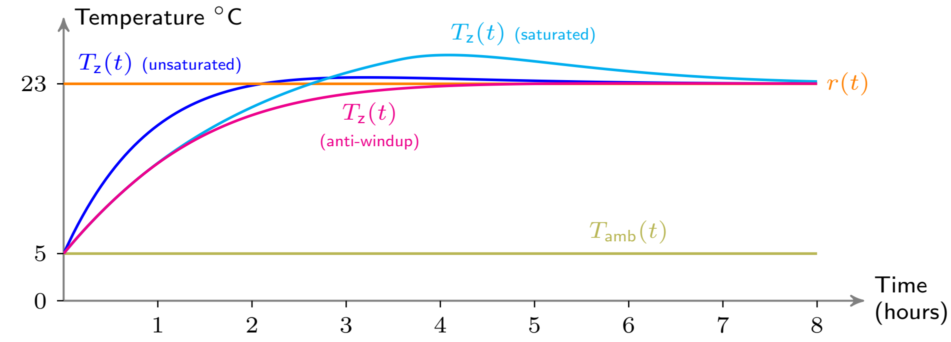
Saturation has the effect of keeping $e(t)$ larger than it would have otherwise been.

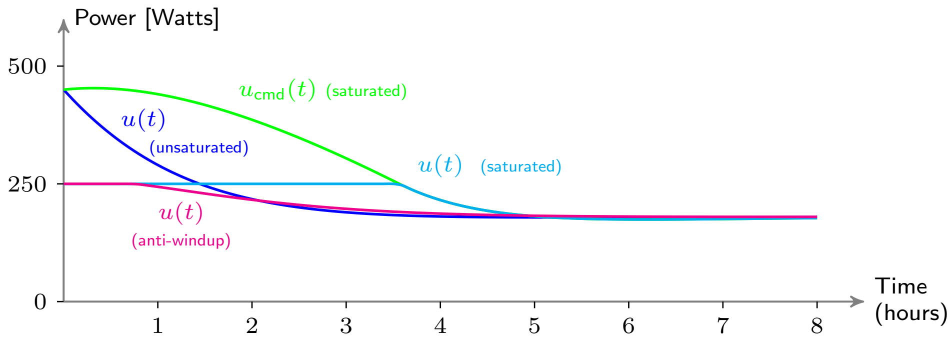
The integrator continues to integrate an error that the controller cannot easily reduce ("windup").

So $u(t)$ becomes larger than it should,

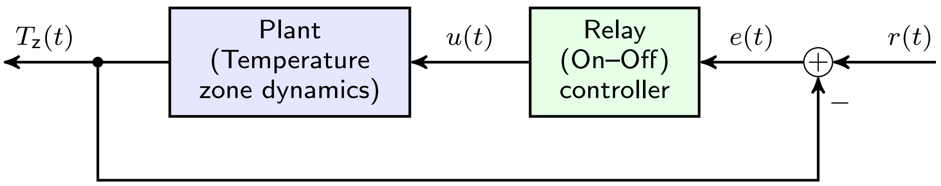
and we have to have error in the opposite direction (overshoot) to bring the integrator back down to it's correct level.

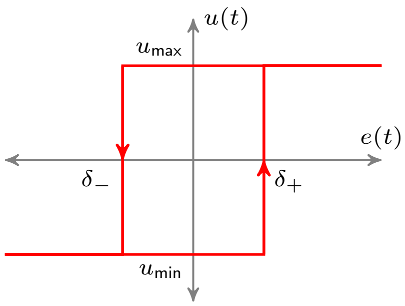
PI-control with anti-windup compensation





Relay control (with hysteresis)

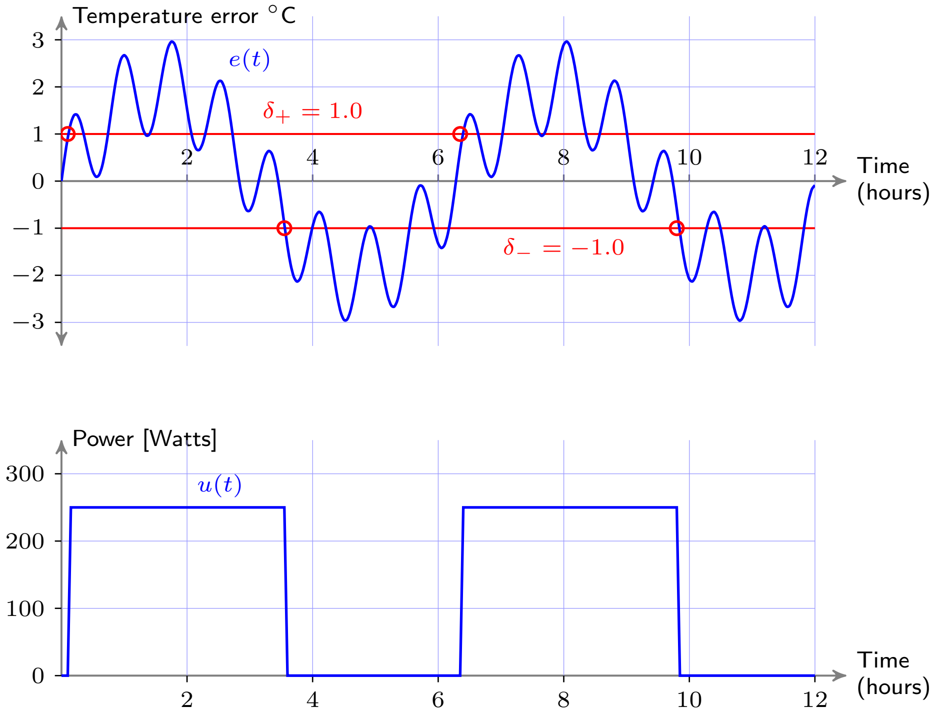




Switching rules

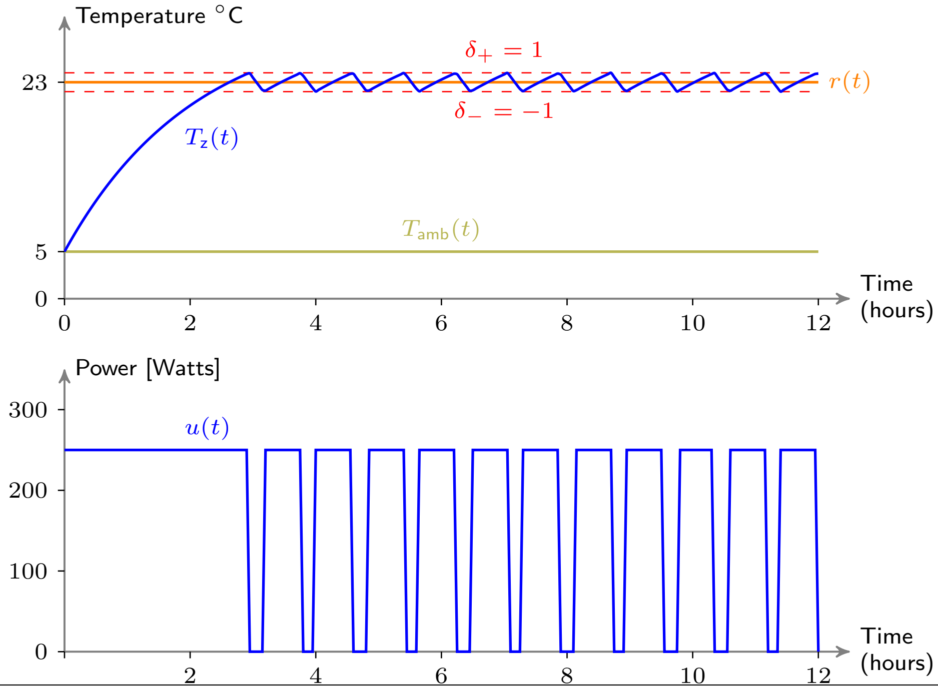
- If $u(t) = u_{min}$ and $e(t) \geq \delta_+$,
  then $u(t) = u_{max}$.
- If $u(t) = u_{max}$ and $e(t) \leq \delta_-$,
  then $u(t) = u_{min}$.

Relay actuator



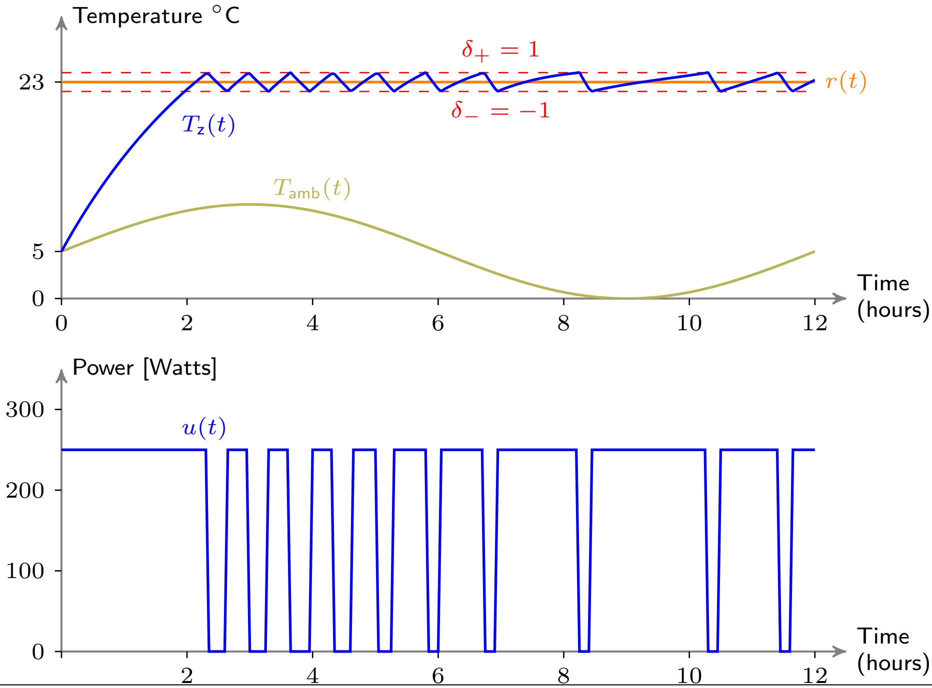
Relay control

Reference tracking



Relay control

Disturbance rejection



Relay control

Common building applications

- Thermostatic control:
  - Room temperature (heating, air-conditioning);
  - Refrigeration;
  - Water heaters.
- Fans (chillers, ventilation systems).
- Simple gas burners.

Digital controllers

Why use digital controllers?

- Easily reprogrammed or modified.
- Complex algorithms (or optimisations) can be implemented.
- Integration with remote systems (via internet).

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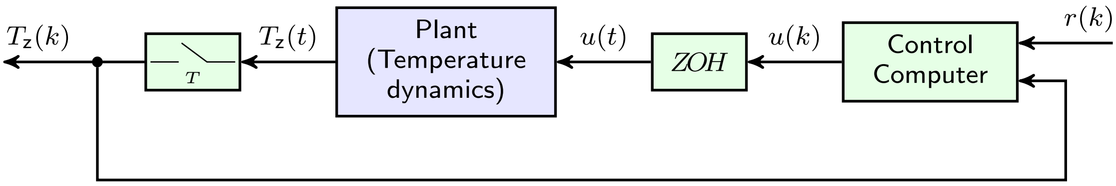
Why use analogue controllers?

- Simple and low cost in mass production.
- Highly reliable.

Digital controllers in a feedback loop

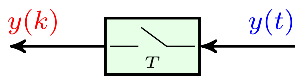


Components:

- Plant: continuous-time thermal dynamics;
- Controller: discrete-time control calculations
- Sampler (A/D converter). Measures $T_z(t)$ every T seconds;
- Zero-order hold (D/A converter): "holds" the value of $u(k)$ for T seconds.

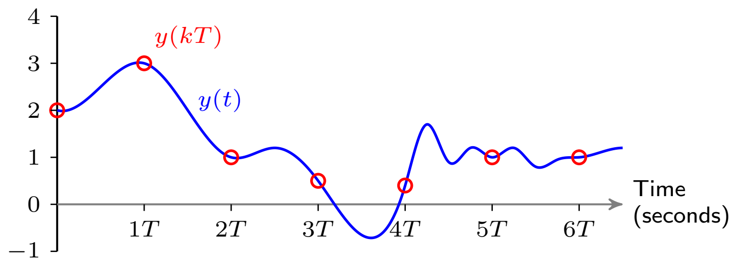
Components: sampler

$y(k) = y(t)\mid_{t=kT}, k = 0, 1, 2, \ldots$

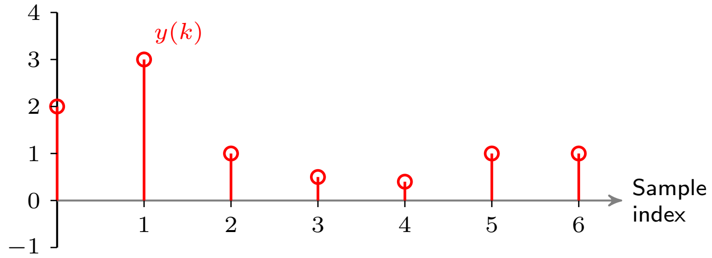


T is the sampling period.

Continuous signal:



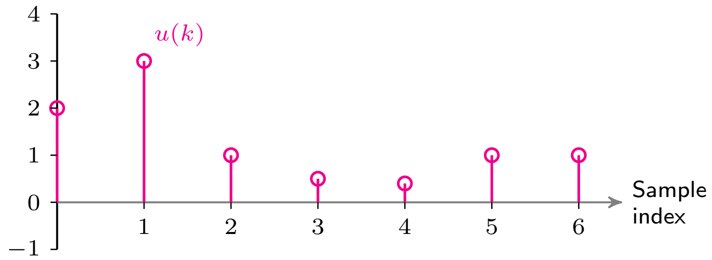
Discrete sequence:



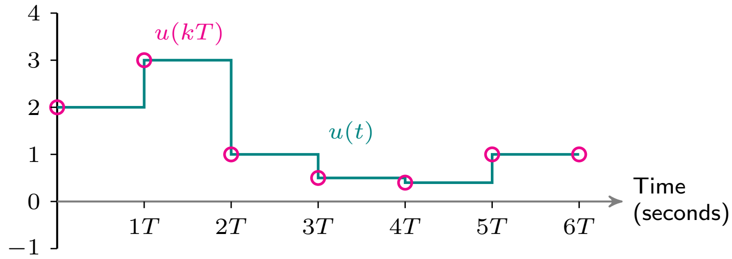
Components: zero-order hold

$u(t) = u(k)$, for $kT \le t < kT + T$.

Discrete sequence:

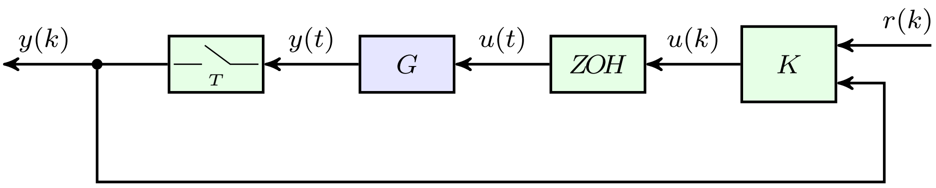


Continuous signal:

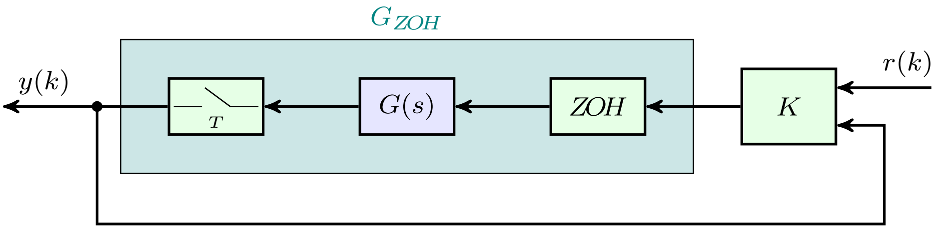


Digital control system design

Digital controller and continuous plant: closed-loop

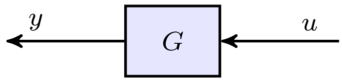


Discrete-time equivalence



Discrete-time state-space models

Plant model:

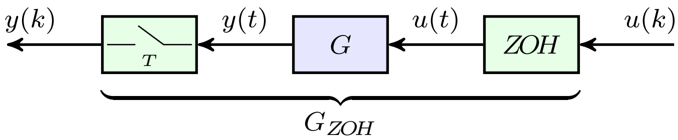


$$x(k+1) \;=\; A\,x(k) \;+\; B\,u(k)$$
$$y(k) \;=\; C\,x(k) \;+\; D\,u(k)$$

The discrete-time state-space system is stable if and only if

$$|\mathrm{eig}(A)| \;<\; 1.$$

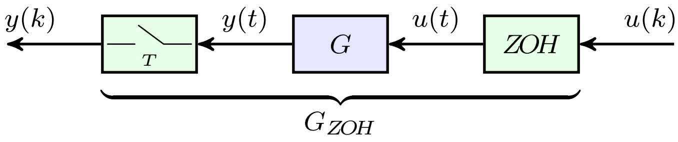
Zero-order hold equivalence — state space



Integrating $\Phi(t)$ over a single sample period (kT to $kT+T$):

$$x(kT+T) \;=\; \mathrm{e}^{AT}\,x(kT) \;+\; \int_{kT}^{kT+T} \mathrm{e}^{A(kT+T-\tau)}\,Bu(\tau)\,d\tau,$$

Zero-order hold equivalence — state space



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$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \quad \overset{ZOH\text{-equivalence}}{\Longrightarrow} \quad \left[\begin{array}{c|c} \mathrm{e}^{AT} & \displaystyle\int_0^T \mathrm{e}^{A\eta}\, B\, d\eta \\ \hline C & D \end{array}\right]$$

MATLAB calculation: `c2d`

Discrete-time controllers

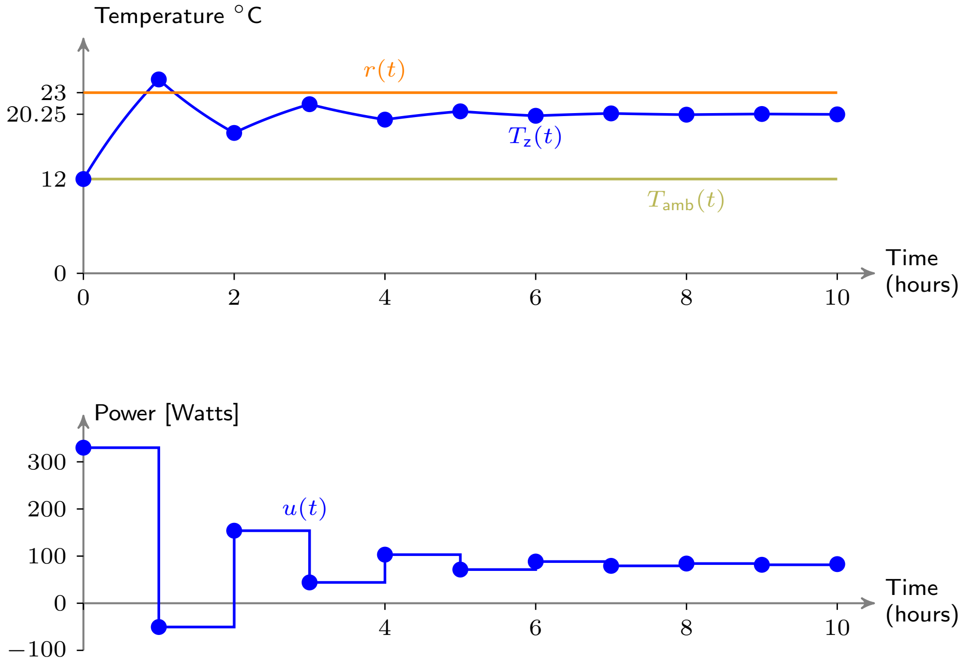
Proportional controller

$$u(k) \;=\; K_{\mathsf{P}}\, e(k)$$

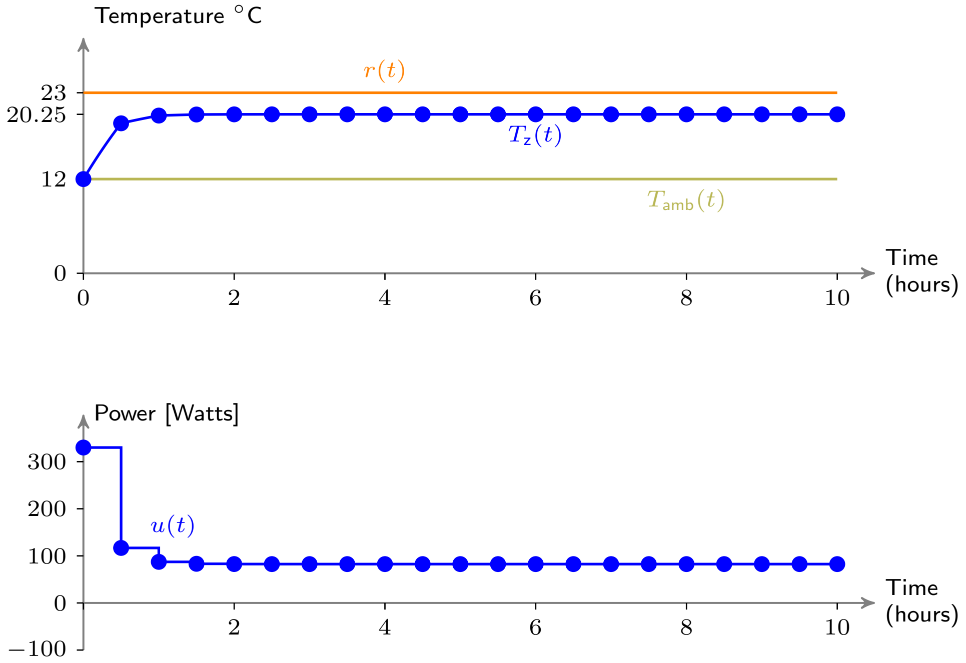
PI controller

$$x_{\mathsf{K}}(k+1) \;=\; 1\, x_{\mathsf{K}}(k) \;+\; T\, e(k)$$
$$u(k) \;=\; K_{\mathsf{I}}\, x_{\mathsf{K}}(k) \;+\; K_{\mathsf{P}}\, e(k)$$

P-control



P-control, faster sampling



PI-control, discrete-time