Exercise 8 - Turbocompressors

A *turbocompressor* (TC) or turbocharger is a mechanical device used in internal combustion engines to enhance their power output. The basic idea of a TC is to force additional air ("boost") into the ignition chamber, allowing for more fuel to be combusted and thus increasing the torque produced by the motor.

TC systems are well described with the coupling of a compressor and a turbine, as depicted in Figure 1. The processes are usually modeled as follows:

- $1 \rightarrow 2$: compression of fresh air in the compressor.
- $2 \rightarrow 3$: isobaric fuel and air combustion in the ignition chamber.
- $3 \rightarrow 4$: expansion of the exhaust gases in the turbine.
- $4 \rightarrow 1$: isobaric heat rejection into atmosphere.

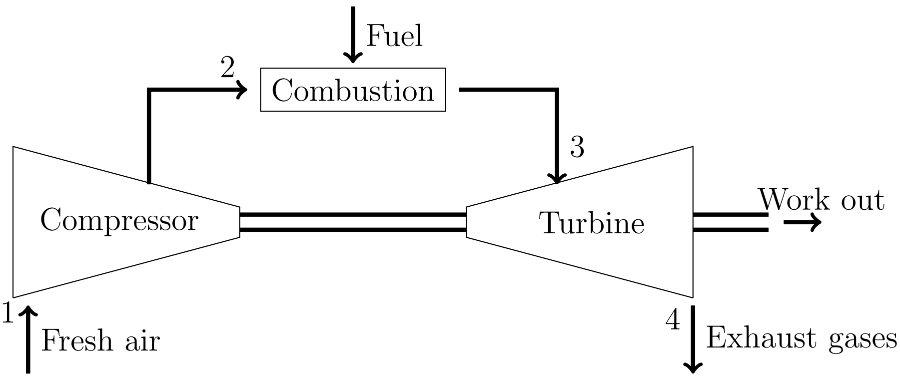

Figure 1: Schematic turbine/compressor system

8.1 Gas Turbines

We model gas turbines as static elements with the causality diagram shown in Figure 2.

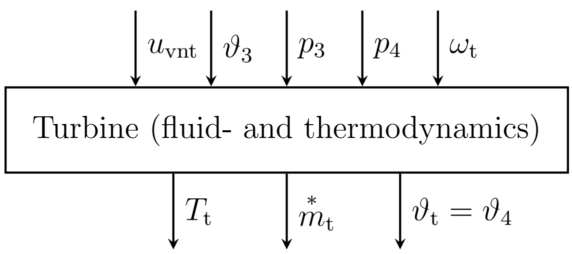

Figure 2: Causality diagram of a gas turbine.

Inputs:

- u_{vnt}: variable nozzle geometry control input $[-]$; determines the inlet area of the turbine A_{vnt}.

This material is based on the HS17 teaching assistance taught by Nicolas Lanzetti and Gioele Zardini.
This document can be downloaded at `https://n.ethz.ch/~lnicolas`

- ϑ_3: temperature before the turbine [K].
- p_3: pressure before the turbine [Pa].
- p_4: pressure after the turbine [Pa].
- ω_t: turbine speed [rad/s].

Outputs:

- T_t: turbine torque [Nm].
- $\overset{*}{m}_\text{t}$: mass flow through the turbine [kg/s].
- ϑ_4 temperature after the turbine [K].

As usual, to model the system, we seek a relationship between the outputs and the inputs.

Outputs Derivation

The starting point is the energy balance for an open system

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \overset{*}{H}_\text{in} - \overset{*}{H}_\text{out} - \overset{*}{W}_\text{t} + \overset{*}{Q},$$

together with two simplifying assumptions, namely:

- the turbine does not store energy over time $\rightarrow \frac{\mathrm{d}E}{\mathrm{d}t} = 0$;
- the turbine is assumed to be **adiabatic** (no heat transfer)$\rightarrow \overset{*}{Q} = 0$.

With these simplifications the power output of the turbine reads

$$P_\text{t} = \overset{*}{W}_\text{t} = \overset{*}{H}_\text{in} - \overset{*}{H}_\text{out} = \overset{*}{m}_\text{t} \cdot c_\text{p} \cdot (\vartheta_3 - \vartheta_4).$$

The temperature at the turbine outlet can be found by introducing the **turbine pressure ratio**

$$\Pi_\text{t} = \frac{p_3}{p_4} > 1,$$

and combinining the **isentropic relation** and **isentropic efficiency** of a turbine

$$\frac{\vartheta_3}{\vartheta_{4,\text{is}}} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} \qquad \eta_\text{t} = \frac{\vartheta_3 - \vartheta_4}{\vartheta_3 - \vartheta_{4,\text{is}}}.$$

The result is

$$\vartheta_4 = \vartheta_3 \cdot \left(1 - \eta_\text{t} \cdot \left(1 - \Pi_\text{t}^{\frac{(1-\gamma)}{\gamma}}\right)\right). \tag{8.1}$$

The torque produced at the shaft can be obtained by inserting ϑ_4 in the expression for the turbine power and dividing by ω_t

$$T_\text{t} = \frac{\eta_\text{t} \cdot \overset{*}{m}_\text{t} \cdot c_\text{p} \cdot \vartheta_3}{\omega_\text{t}} \cdot \left(1 - \Pi_\text{t}^{\frac{1-\gamma}{\gamma}}\right). \tag{8.2}$$

Finally, the mass flow through the turbine can be computed as

$$\overset{*}{m}_\text{t} = \frac{p_3}{p_\text{ref,0}} \cdot \sqrt{\frac{\vartheta_\text{ref,0}}{\vartheta_3}} \cdot \overset{*}{\mu}_\text{t}. \tag{8.3}$$

where ($p_\text{ref,0}$, $\vartheta_\text{ref,0}$) is a reference state and $\overset{*}{\mu}_\text{t}$ is the normalized mass flow obtained from the so-called *mass flow maps* (see next section).

Turbine Efficiency and Mass Flow Maps

In practice, η_t is obtained from turbine efficiency maps as

$$\eta_t = \mathcal{M}(\text{various parameters}).$$

These maps are represented in diagrams as depicted by Figure 3.

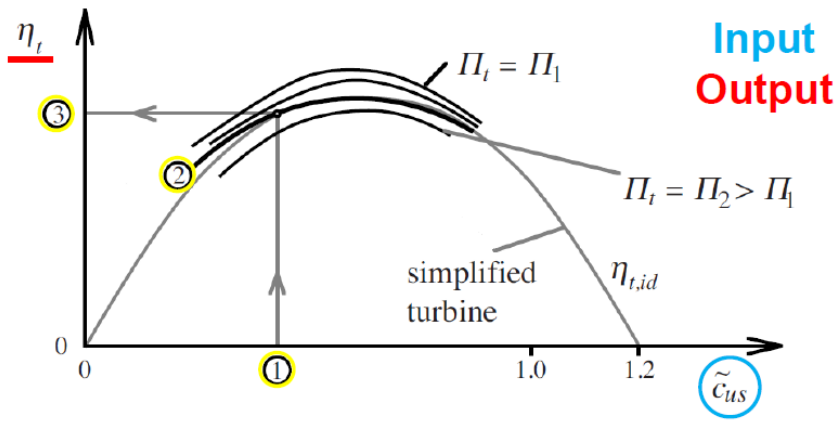

Figure 3: Turbine efficiency map.

where

$$c_{us} = \sqrt{2 \cdot c_p \cdot \vartheta_3 \cdot \left(1 - \Pi_t^{\frac{(1-\gamma)}{\gamma}}\right)}, \qquad \tilde{c}_{us} = \frac{r_t \cdot \omega_t}{c_{us}}$$

are the **isentropic velocity** and the **turbine blade speed ratio**, respectively.

Similar maps exist for $\overset{*}{\mu}_t$, such that

$$\overset{*}{\mu}_t = \mathcal{M}(\text{various parameters}).$$

These maps are usually represented by diagrams such as the one depicted by Figure 4[1].

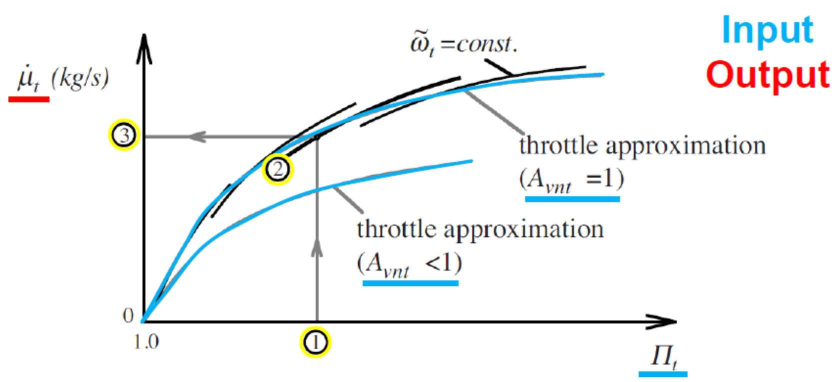

Figure 4: Turbine mass flow maps.

Remark. For control purposes, the mass flow behaviour of fluid-dynamic turbines can be modeled quite well as a compressible flow through a valve.

[1]Here, A_{vnt} and $\tilde{\omega}_t$ represent normalized quantities.

8.2 Compressors

We model compressors as static elements with the causality diagram shown in Figure 5.

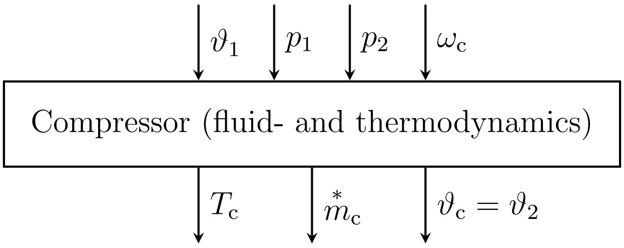

Figure 5: Causality diagram of a gas compressor.

Inputs:

- ϑ_1: temperature before the compressor [K].
- p_1: pressure before the compressor [Pa].
- p_2: pressure after the compressor [Pa].
- ω_c: compressor speed [rad/s].

Outputs:

- T_c: compressor torque [Nm].
- $\overset{*}{m}_\mathrm{c}$: mass flow through the compressor [kg/s].
- ϑ_2 temperature after the compressor [K].

Outputs Derivation

The starting point is, again, the energy balance for an open system

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \overset{*}{H}_\mathrm{in} - \overset{*}{H}_\mathrm{out} - \overset{*}{W}_\mathrm{c} + \overset{*}{Q},$$

together with two simplifying assumptions, namely:

- the compressor does not store energy over time $\rightarrow \frac{\mathrm{d}E}{\mathrm{d}t} = 0$;
- the compressor is assumed to be **adiabatic** (no heat transfer)$\rightarrow \overset{*}{Q} = 0$.

With these simplifications the power input of a compressor reads

$$P_\mathrm{c} = -\overset{*}{W}_\mathrm{c} = \overset{*}{H}_\mathrm{out} - \overset{*}{H}_\mathrm{in} = \overset{*}{m}_\mathrm{c} \cdot c_\mathrm{p} \cdot (\vartheta_2 - \vartheta_1).$$

Introducing the compressor pressure ratio

$$\Pi_\mathrm{c} = \frac{p_2}{p_1} > 1,$$

and combining the **isentropic relation** and the **isentropic efficiency** for a compressor

$$\frac{\vartheta_{2,\text{is}}}{\vartheta_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \eta_\text{c} = \frac{\vartheta_{2,\text{is}} - \vartheta_1}{\vartheta_2 - \vartheta_1},$$

one can compute the temperature at the outlet of the compressor as

$$\vartheta_2 = \vartheta_1 \cdot \left[1 + \frac{1}{\eta_\text{c}} \cdot \left(\Pi_\text{c}^{\frac{\gamma-1}{\gamma}} - 1\right)\right]. \tag{8.4}$$

The torque absorbed by the compressor then reads

$$T_\text{c} = \frac{\overset{*}{m}_\text{c} \cdot c_\text{p} \cdot \vartheta_1}{\eta_\text{c} \cdot \omega_\text{c}} \cdot \left(\Pi_\text{c}^{\frac{\gamma-1}{\gamma}} - 1\right). \tag{8.5}$$

Finally, the mass flow through the compressor is

$$\overset{*}{m}_\text{c} = \frac{p_1}{p_{\text{ref},0}} \cdot \sqrt{\frac{\vartheta_{\text{ref},0}}{\vartheta_1}} \cdot \overset{*}{\mu}_\text{c}. \tag{8.6}$$

Compressor Efficiency and Mass Flow Maps

Similarly to turbines, the compressor efficiency and mass flow are usually described by parameter dependent maps

$$\overset{*}{\mu}_\text{c} = \mathcal{M}(\text{various parameters}), \qquad \eta_\text{c} = \mathcal{M}(\text{various parameters}).$$

One such example is depicted in Figure 6.

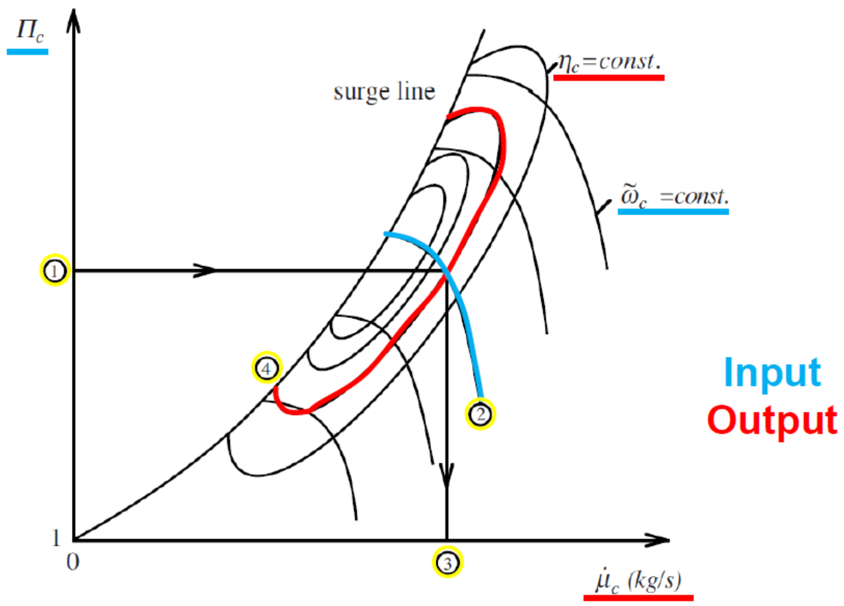

Figure 6: Compressor map for mass flow and efficiency.

In this map, both the efficiency and the mass flow can be obtained by known pressure ratio and speed.

8.3 Tips

Exercise (a): No tips.

Exercise (b): No tips

Exercise (c): No tips.

Exercise (d): No tips.

Exercise (e): No tips.

8.4 Example

Inspired by Mr. Balerna's lecture, you aim to reduce the consumptions and increase the power of the car of SpaghETH. Among others, the power unit of your vehicles should include an turbocharger. However, this makes the operation of your vehicle more challenging. In order not to overwhelm your drivers with additional tasks, you decide to design a control system to take over the task of managing the energy in the system.
The system consists of an internal combustion engine, a compressor, and a turbine, mechanically connected through a shaft of inertia Θ, as depicted in Figure 7. Air flows from the ambient to the intake manifold (volume V_{IM} and constant temperature ϑ_{IM}) through the compressor and the intercooler which ensures a constant temperature of the manifold. Then, air flows to the engine through a throttle of opening surface $A(t)$ to the engine. After the combustion, the warm mixture flows to the exhaust manifold (volume V_{EM} and isolated) through the turbine back to the ambient. The following assumptions are made:

- The mass-flow and efficiency of the compressor and the turbine are to be found in maps:
$$\overset{*}{m}_{\mathrm{c}} = \mathcal{M}_{c,1}(\Pi_{\mathrm{c}}, \omega_{\mathrm{c}}), \qquad \overset{*}{m}_{\mathrm{t}} = \mathcal{M}_{t,1}(\Pi_{\mathrm{t}}, \omega_{\mathrm{t}}),$$
$$\eta_{\mathrm{c}} = \mathcal{M}_{c,2}(\Pi_{\mathrm{c}}, \omega_{\mathrm{c}}), \qquad \eta_{\mathrm{t}} = \mathcal{M}_{t,2}(\Pi_{\mathrm{t}}, \omega_{\mathrm{t}}).$$
- The temperature after the engine is a known function of the fuel injected in the engine. That is, $\vartheta_{\mathrm{e}} = \Phi(\overset{*}{m}_{\mathrm{f}})$.
- The throttle has discharge coefficient c_{d} and, as the pressure before the engine a known function of $p_{\mathrm{IM}}(t)$, the function $\Psi(\cdot)$ is known and reads $\Psi(p_{\mathrm{IM}}(t))$.
- Assume that air does not accumulate before the engine, i.e., $\overset{*}{m}_{\mathrm{cyl}} = \overset{*}{m}_{\mathrm{th}}$. Moreover, the fuel burns stoichiometrically, i.e., $\overset{*}{m}_{\mathrm{f}} = \overset{*}{m}_{\mathrm{cyl}}/\sigma$.
- Both air the and air-fuel mixture have constant c_p, c_v, R, and γ.
- The engine power is given by $P_{\mathrm{e}} = K_1 \cdot \overset{*}{m}_{\mathrm{f}} + K_2 \cdot (p_{\mathrm{IM}} - p_{\mathrm{EM}})$, where $K_1, K_2 > 0$.
- The intercooler keeps the intake manifold at constant temperature.

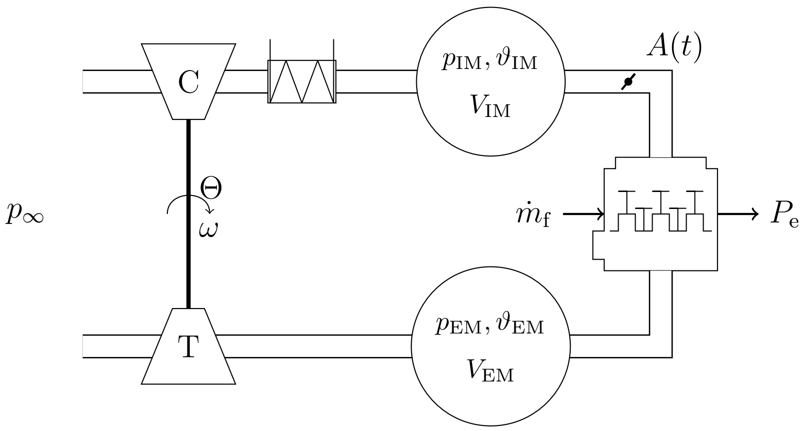

Figure 7: Sketch of the system.

1. What are the input(s) and the output(s)?
2. What are the reservoirs and the corresponding level variables?
3. Draw a causality diagram of the system.
4. Formulate the algebraic and differential equations describing the system.
5. Use your model to state the two main advantages and the main disadvantage of turbochargers.

Solution.

1. The input is the throttle position/area. The output is the engine power.
2. The reservoirs are
   - Mass in the intake manifold with level variable $p_{\mathrm{IM}}(t)$;
   - Mass in the exhaust manifold with level variable $m_{\mathrm{EM}}(t)$;
   - (Thermal) Energy in the exhaust manifold with level variable $\vartheta_{\mathrm{EM}}(t)$;
   - (Kinetic) Energy of the turbocharger with level variable $\omega(t)$.
3. The causality diagram is shown in Figure 8.
4. The dynamics of the intake manifold are described by

$$\frac{\mathrm{d}}{\mathrm{d}t}p_{\mathrm{IM}}(t) = \frac{1}{V_{\mathrm{IM}}}(\overset{*}{m}_{\mathrm{c}} - \overset{*}{m}_{\mathrm{th}}) \cdot R \cdot \vartheta_{\mathrm{IM}},$$

where

$$\begin{aligned}\overset{*}{m}_{\mathrm{c}} &= \mathcal{M}_{\mathrm{c},1}(p_{\mathrm{IM}}(t)/p_\infty, \omega(t)),\\ \overset{*}{m}_{\mathrm{th}} &= c_{\mathrm{d}} \cdot A(t) \cdot \frac{p_{\mathrm{IM}}(t)}{\sqrt{R \cdot \vartheta_{\mathrm{IM}}}} \cdot \Psi(p_{\mathrm{IM}}(t)).\end{aligned}$$

The dynamics of the exhaust manifold are described by

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}m_{\mathrm{EM}}(t) &= \overset{*}{m}_{\mathrm{th}} + \overset{*}{m}_{\mathrm{f}} - \overset{*}{m}_{\mathrm{t}}\\ &= \left(1 + \frac{1}{\sigma}\right) \cdot \overset{*}{m}_{\mathrm{th}} - \overset{*}{m}_{\mathrm{t}}.\\ \frac{\mathrm{d}}{\mathrm{d}t}(m_{\mathrm{EM}}(t) \cdot c_v \cdot \vartheta_{\mathrm{EM}}(t)) &= (\overset{*}{m}_{\mathrm{th}} + \overset{*}{m}_{\mathrm{f}}) \cdot c_p \cdot \vartheta_{\mathrm{e}} - \overset{*}{m}_{\mathrm{t}} \cdot c_p \cdot \vartheta_{\mathrm{EM}}\\ &= \left(1 + \frac{1}{\sigma}\right) \cdot \overset{*}{m}_{\mathrm{th}} \cdot c_p \cdot \vartheta_{\mathrm{e}} - \overset{*}{m}_{\mathrm{t}} \cdot c_p \cdot \vartheta_{\mathrm{EM}}\end{aligned}$$

where $\vartheta_{\mathrm{e}} = \Phi(\overset{*}{m}_{\mathrm{f}})$. For the turbocharger, conservation of mechanical energy gives

$$\Theta \cdot \frac{\mathrm{d}}{\mathrm{d}t}\omega(t) = T_{\mathrm{t}} - T_{\mathrm{c}},$$

where

$$\begin{aligned}T_{\mathrm{t}} &= \frac{\overset{*}{m}_{\mathrm{t}}(t) \cdot c_p \cdot \vartheta_{\mathrm{EM}}(t)}{\omega(t)} \cdot \left(1 - \left(\frac{p_{\mathrm{EM}}(t)}{p_\infty}\right)^{\frac{1-\gamma}{\gamma}}\right) \cdot \eta_{\mathrm{t}},\\ T_{\mathrm{c}} &= \frac{\overset{*}{m}_{\mathrm{c}}(t) \cdot c_p \cdot \vartheta_\infty}{\omega(t)} \cdot \left(\left(\frac{p_{\mathrm{IM}}(t)}{p_\infty}\right)^{\frac{\gamma-1}{\gamma}} - 1\right) \cdot \frac{1}{\eta_{\mathrm{c}}},\end{aligned}$$

with $\eta_{\mathrm{c}} = \mathcal{M}_{c,2}(p_{\mathrm{IM}}(t)/p_\infty, \omega(t))$ and $\eta_{\mathrm{t}} = \mathcal{M}_{t,2}(p_{\mathrm{EM}}(t)/p_\infty, \omega(t))$. The exhaust manifold pressure can be computed using the ideal gas relation

$$p_{\mathrm{EM}} = \frac{1}{V_{\mathrm{EM}}} \cdot m_{\mathrm{EM}} \cdot R \cdot \vartheta_{\mathrm{EM}}.$$

Finally, the engine power is

$$P_{\mathrm{e}} = K_1 \cdot \overset{*}{m}_{\mathrm{f}} + K_2 \cdot (p_{\mathrm{IM}} - p_{\mathrm{EM}}).$$

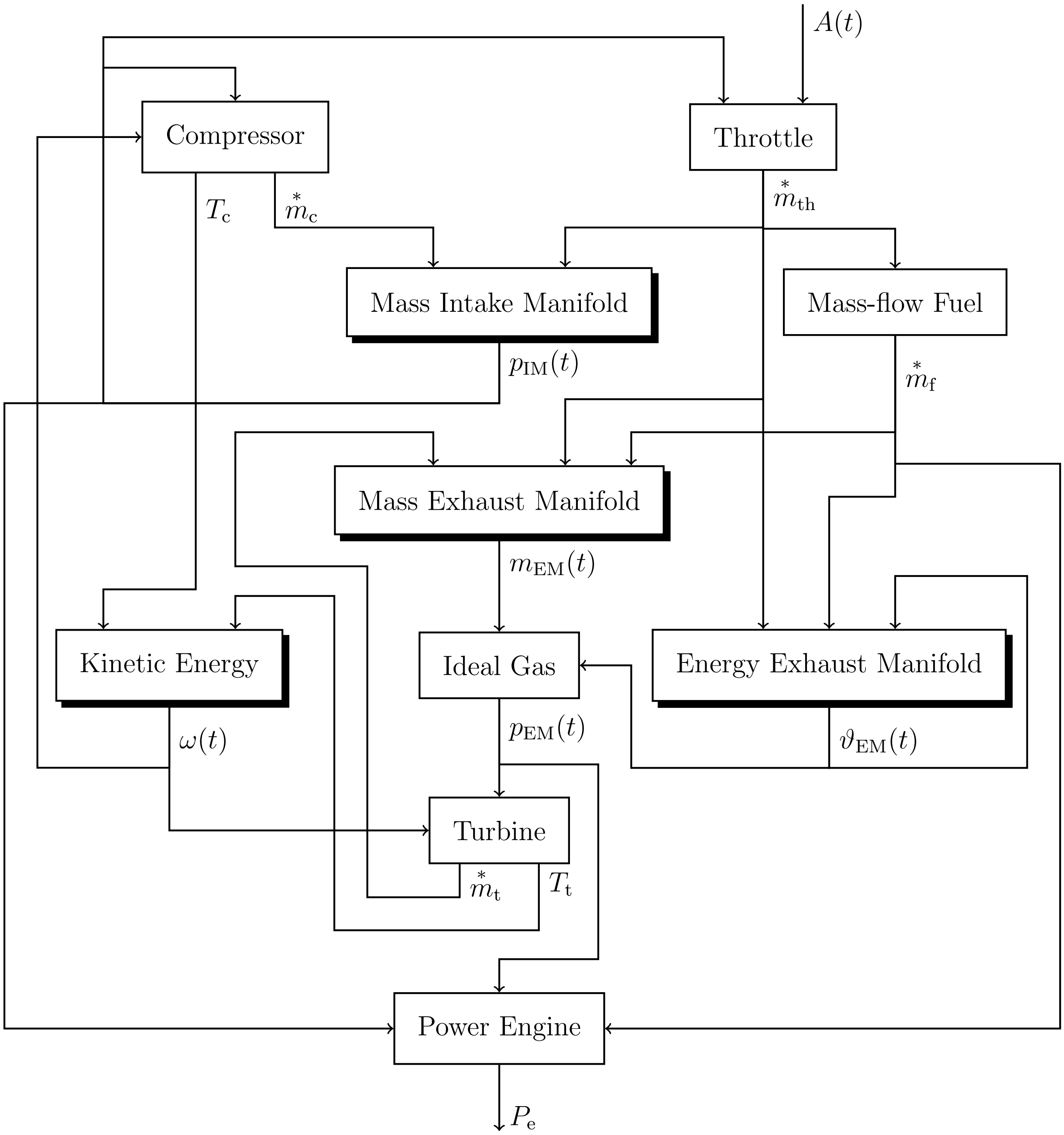

Figure 8: Causality diagram.

5. Advantages:

   - Given the same motor, the larger mass of air in the cylinder allows for more fuel to be combusted during the single engine stroke, generating more power.
   - With no turbocharger one has $p_\infty = p_{EM} = p_{IM}$. The addition of a turbocharger, creates a pressure difference $\frac{p_{IM}}{p_{EM}} > 1$ increasing the power output of the engine *with a set fuel mass flow* (see equation for engine power).

   Disadvantages:

   - The turbocompressor has a mass and, therefore, an inertia. During the transients (i.e. when there is a change in power requested from the engine) the engine has to accelerate and the turbocharger, initially idle, also has to spin up to operating speed. Thus, the engine is slower to reach the maximum speed than if there were no turbocharger.