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SV8: Continuous Phase Modulation

Many digital modulation techniques result in a transmitted signal which - depending on the data to be transmitted - might change abruptly. This leads to detrimental spectral characteristics (e.g., poor spectral efficiency). In contrast, continuous phase modulation (CPM) modulates the data bits in a continuous manner and has therefore a high spectral efficiency. This is particularly interesting in wireless communication where bandwidth is expensive. In fact, CPM is most notably used in GSM.

In this experiment you will investigate a communication system that uses CPM modulation. With the aid of SIMULINK, you will simulate the modulation and decoding processes of CPM systems, and you will analyze key figures such as spectral characteristics of the modulated signal or probability of a decoding error.

1 Introduction

Figure 1 illustrates the functional diagram and the basic elements of digital communication systems. The data symbols $\{d_\ell\}_{\ell\in\mathbb{Z}}$ (for short $\{d_\ell\}$) are converted by the *encoder* into a sequence of code symbols $\{x_k\}_{k\in\mathbb{Z}}$ (for short $\{x_k\}$) from a finite alphabet $\mathcal{X}$. The sequence at the output of the encoder is passed to the *modulator*, which serves as the interface to the communication channel. The primary purpose of the modulator is to map the code symbols $\{x_k\}$ into a signal[1] $s(t)$, i.e., an electrical signal that can be transmitted by an antenna or through a wired line.

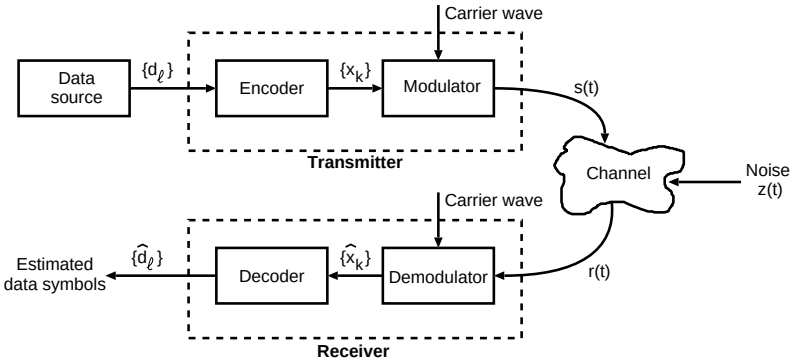

Figure 1: Model of a digital passband transmission system.

This signal $s(t)$ propagates through the *channel*, i.e., the physical medium that is used to send the signal from transmitter to receiver. For example, the channel might be a pair

[1]Here, we write $g(t)$ when we mean a function $g\colon \mathbb{R}\to\mathbb{C};\ t\mapsto g(t)$. The variable t stands for time and, thus, $g(t)$ is a continuous time signal that can be either complex or real.

of wires, a coaxial cable, or the air between the transmit and receive antenna. In this lab exercise, we model the channel by a system with input signal $s(t)$, additive white Gaussian noise $z(t)$, and output signal $r(t) = s(t) + z(t)$.[2]

The receiver wants to guess the sent data symbols $\{d_\ell\}$ based on the received signal $r(t)$. To this end, the *demodulator* processes $r(t)$ and reduces the signal to a sequence of numbers $\{\hat{x}_k\}$ that represent estimates of the transmitted code symbols $\{x_k\}$. This sequence of numbers is passed to the *decoder*, which attempts to reconstruct the original data symbols, i.e., the decoder produces the guesses $\{\hat{d}_\ell\}$ based on the numbers $\{\hat{x}_k\}$.

A performance measure of the system is the relative frequency with which errors occur in the decoded sequence, i.e., the number of wrongly guessed data symbols divided by the total number of sent symbols.

2 Passband Signals and their Representation

Almost all communication systems modulate information onto a sinusoidal carrier waveform. Table 1 lists the carrier frequencies of some popular communication systems. Notice

Type of Transmission	Carrier Frequency of Transmission
FM radio	88 - 108 MHz
GSM900	890 - 915 MHz / 935 - 960 MHz
WLAN (IEEE 802.11b/g)	2.4 - 2.4835 GHz

Table 1: Carrier frequency assignments for different methods of information transmission.

that the carrier frequency of the transmitted signal is not the component which contains the information. The waveform that is modulated onto the carrier is the signal which contains the information.

A general sinusoidal transmit waveform, which we will also call a *passband signal*, can be written in the form

$$s(t) = 2a(t)\cos\big(2\pi f_c t + \phi(t)\big), \quad t \in \mathbb{R}, \tag{1}$$

where $f_c > 0$ is the carrier frequency, $a(t)$ the amplitude, $\phi(t)$ the phase of the signal $s(t)$, and the factor 2 is a normalization factor. The information that is to be transmitted is either contained in the amplitude $a(t)$, in the phase $\phi(t)$, or in both the phase and amplitude.

Applying the identity

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta) \tag{2}$$

to (1), we have

$$s(t) = 2 \cdot a(t) \cdot \cos(2\pi f_c t) \cdot \cos\big(\phi(t)\big) - 2 \cdot a(t) \cdot \sin(2\pi f_c t) \cdot \sin\big(\phi(t)\big). \tag{3}$$

We observe that $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ do not depend on the data symbols. Conversely, the terms that contain the information $a(t) \cdot \cos\big(\phi(t)\big)$ and $a(t) \cdot \sin\big(\phi(t)\big)$ do not depend on the carrier frequency. It would be convenient to have a representation of the passband signal that does not contain the carrier frequency f_c. To this end, we rewrite (3) as

$$s(t) = 2 \cdot v_I(t) \cdot \cos(2\pi f_c t) - 2 \cdot v_Q(t) \cdot \sin(2\pi f_c t), \tag{4}$$

[2] In more general communication scenarios, the transmit signal is not only affected by additive noise but might also be filtered by the channel. See lab exercise SV4 for the equalization of such channel impairments.

where

$$v_I(t) \triangleq a(t)\cos\big(\phi(t)\big) \tag{5}$$

and

$$v_Q(t) \triangleq a(t)\sin\big(\phi(t)\big). \tag{6}$$

The signals $v_I(t)$ and $v_Q(t)$ are known as the *in-phase* (I) and *quadrature* (Q) components, respectively. They are the only terms that contain the information. We define the complex *baseband representation* $v(t)$ as

$$v(t) \triangleq v_I(t) + \mathsf{i} \cdot v_Q(t), \tag{7}$$

where $\mathsf{i} = \sqrt{-1}$. Note that $v(t)$ is a complex signal and hence not physically realizable. It is just a representation of the real transmit signal $s(t)$ that still contains all the information but is independent of the carrier frequency. In fact, the passband signal $s(t)$ can easily be reconstructed from its baseband representation $v(t)$ and the carrier frequency f_c with

$$s(t) = 2\mathrm{Re}\left(v(t) \cdot e^{\mathsf{i}2\pi f_c t}\right). \tag{8}$$

Therefore, we will look at the signals in the baseband representation. In order to generate a real passband signal $s(t)$ that can be transmitted, the modulator can generate a baseband representation $v(t)$ and up-convert it to the passband signal $s(t)$ prior to the transmission. To this end, Formula (8) can be applied.

Exercise 1. *Verify that* (8) *is indeed equivalent to* (1) *by applying Euler's formula:*

$$e^{\mathsf{i}\varphi} = \cos(\varphi) + \mathsf{i} \cdot \sin(\varphi).$$

3 Continuous Phase Modulation

Some of the modulation techniques used in mobile radio systems are special cases of the broad class of phase modulation techniques. In such systems, the information is only contained in the phase $\phi(t)$ of the transmit signal, while the amplitude $a(t) = A$ remains constant, i.e.,

$$s(t) = 2 \cdot A \cdot \cos\big(2\pi f_c t + \phi(t)\big). \tag{9}$$

In this lab exercise we consider the special case of *continuous phase modulation* (CPM), where the carrier phase varies in a continuous manner. This is attractive because such signals have excellent spectral characteristics.

As a first step, the encoder maps the binary data symbols $\{d_\ell\}$, $d_\ell \in \{0, 1\}$, to code symbols $\{x_k\}$. In CPM, the code symbols $\{x_k\}$ are from the finite alphabet $\mathcal{X} = \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$ with M denoting the alphabet size.

Baseband Representation With the stream of code symbols $\{x_k\}$, the modulator constructs the *baseband representation* of CPM signals of the form

$$v(t) = A \cdot \exp\left(\mathsf{i}\phi(t)\right) = A \cdot \exp\left(\mathsf{i}2\pi k_f \int_{-\infty}^{t} \sum_k x_k h_f(\tau - kT_b) d\tau\right), \tag{10}$$

where

A : is the amplitude,
k_f : the peak frequency deviation,
$h_f(t)$: the frequency shaping pulse, and
T_b : the symbol duration.

We call $\phi(t) = 2\pi k_f \int_{-\infty}^{t} \sum_k x_k h_f(\tau - kT_b) d\tau$ the *excess phase*. Note that the excess phase is continuous whenever the shaping pulse $h_f(t)$ does not contain impulses, thus the name *continuous phase modulation*.

The shaping pulse $h_f(t)$ can have a variety of forms. A *full response* CPM scheme uses shaping pulses that have a duration equal to the symbol duration T_b. Some common pulse forms are:

$$h_f(t) = u_{T_b}(t) \triangleq \begin{cases} 1, & \text{if } 0 \le t \le T_b \\ 0, & \text{else} \end{cases} \qquad \text{non-return-to-zero (NRZ)} \tag{11}$$

$$h_f(t) = \sin\left(\frac{\pi}{T_b} t\right) \cdot u_{T_b}(t) \qquad \text{half sinusoid (HS)} \tag{12}$$

$$h_f(t) = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi}{T_b} t\right)\right] \cdot u_{T_b}(t) \qquad \text{raised cosine (RC)} \tag{13}$$

$$h_f(t) = \left[1 - \frac{\left|t - \frac{T_b}{2}\right|}{\frac{T_b}{2}}\right] \cdot u_{T_b}(t) \qquad \text{triangular (TRI).} \tag{14}$$

Exercise 2. *Consider the bit sequence $\{d_k\} = \{1, 1, 0, 1, 0, 0, 1\}$. Sketch the curve of the excess phase for this sequence if an NRZ shaping pulse $u_{T_b}(t)$ is used and each data symbol d_k is mapped into a code symbol from the binary antipodal symbol alphabet $\mathcal{X} = \{-1, +1\}$, i.e.,*

$$x_k = \begin{cases} -1, & \text{if } d_k = 1 \\ +1, & \text{if } d_k = 0, \end{cases} \qquad k = 0, 1, \ldots, 6.$$

These symbols are also called BPSK symbols.

Phase and Frequency The *instantaneous frequency* $f_{dev}(t)$ of the baseband representation $v(t)$, defined as

$$f_{dev}(t) \triangleq \frac{1}{2\pi} \cdot \frac{d\phi(t)}{dt}, \tag{15}$$

is given by

$$f_{dev}(t) = k_f \sum_k x_k h_f(t - kT_b). \tag{16}$$

The peak frequency deviation k_f strongly influences the bandwidth of the signal. It is therefore an important parameter in the design of CPM systems in order to fit the signal to spectral requirements. The *average frequency deviation* of a full response CPM signal is defined as

$$\overline{k_f} \triangleq k_f \frac{1}{T_b} \int_0^{T_b} h_f(t) dt, \tag{17}$$

and the *modulation index* is defined as

$$h \triangleq 2\overline{k_f} T_b. \tag{18}$$

Exercise 3. *Compute the peak frequency deviation k_f as a function of the modulation index h with respect to the shaping pulses given in* (11), (12), (13), *and* (14).

CPM signals can be conveniently described by sketching the phase for all possible code symbol sequences $\{x_k\}$. Such a plot is called a *phase tree*. For a typical phase tree with an NRZ shaping pulse $h_f(t) = u_{T_b}(t)$ and BPSK code symbol alphabet $\mathcal{X} = \{-1, +1\}$ see Figure 2.

The modulation index h is often restricted to be a rational number. This constraint ensures that the number of *phase states* $\Phi_k \triangleq \phi(kT_b)$, $k \in \mathbb{Z}$, is finite which might simplify

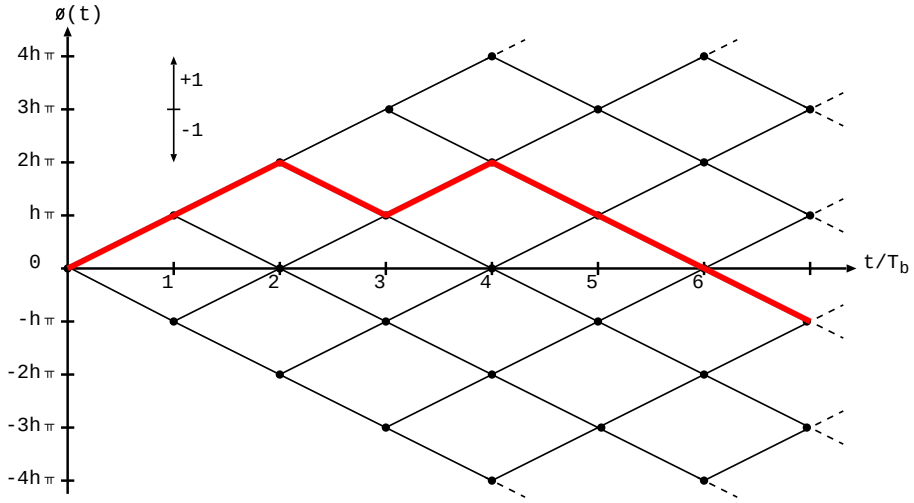

Figure 2: Phase tree of binary CPM with NRZ shaping pulse and arbitrary modulation index. The excess phase derived from the data symbols $\{0, 0, 1, 0, 1, 1, 1\}$ is shown as a bold line.

the receiver design, as we will see in the next section. For example, if $h = \frac{1}{2}$, then the CPM signal has phase states $\Phi_k \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$. This means that two consecutive phase states are always $h\pi = \frac{\pi}{2}$ apart.

3.1 Minimum Shift Keying

Minimum shift keying (MSK) is a special case of CPM resulting when choosing the NRZ shaping pulse $h_f(t) = u_{T_b}(t)$, the BPSK code symbol alphabet $\mathcal{X} = \{-1, +1\}$, and modulation index $h = \frac{1}{2}$. A slightly modified[3] form of this modulation scheme is used in the Global System for Mobile communications (GSM).

In the following we consider MSK. Then, the excess phase $\phi(t)$ in the interval $nT_b \leq t \leq (n+1)T_b$, $n \in \mathbb{Z}$, can be simplified to

$$\begin{aligned}
\phi(t) &= 2\pi k_f \int_{-\infty}^{t} \sum_{k\in\mathbb{Z}} x_k h_f(\tau - kT_b) d\tau \\
&= 2\pi \frac{h}{2T_b} \int_{-\infty}^{t} \sum_{k\in\mathbb{Z}} x_k u_{T_b}(\tau - kT_b) d\tau \\
&\overset{(a)}{=} \pi \frac{1/2}{T_b} \sum_{k=-\infty}^{n-1} x_k \int_{kT_b}^{(k+1)T_b} 1 d\tau + \pi \frac{1/2}{T_b} x_n \int_{nT_b}^{t} 1 d\tau \\
&= \frac{\pi}{2T_b} \sum_{k=-\infty}^{n-1} x_k \cdot T_b + \frac{\pi}{2T_b} x_n \cdot (t - nT_b) \\
&\overset{(b)}{=} \Phi_n \pm \frac{\pi}{2T_b} (t - nT_b), \qquad\qquad nT_b \leq t \leq (n+1)T_b,
\end{aligned} \tag{19}$$

where (a) follows from interchanging the integral with the sum and separating the summand which contains the pulse that lies in the considered time interval, and (b) follows because $\Phi_n = \phi(nT_b)$ is the phase state at time $t = nT_b$. The plus sign corresponds to sending code symbol $x_n = +1$, and the minus sign corresponds to sending code symbol $x_n = -1$.

[3] In GSM, the NRZ pulses are first filtered with a Gaussian low-pass filter prior to the integration. This leads to a more compact spectrum. The scheme is then called *Gaussian MSK* (GMSK).

We observe that $\phi(t)$ depends on the past history of the modulation process. We further observe that the total phase change in the interval $nT_b \le t \le (n+1)T_b$ is exactly $\pm\frac{\pi}{2}$. Therefore, all even phase states Φ_{2n} are either 0 or π, and all odd phase states Φ_{2n+1} are either $\frac{\pi}{2}$ or $-\frac{\pi}{2}$.

The passband signal of an MSK system is

$$s(t) = 2A \cdot \cos\big(2\pi f_c t + \phi(t)\big), \tag{20}$$

or expressed in its in-phase and quadrature components

$$s(t) = 2 \cdot A\cos\big(\phi(t)\big) \cdot \cos(2\pi f_c t) - 2 \cdot A\sin\big(\phi(t)\big) \cdot \sin(2\pi f_c t). \tag{21}$$

From this we can determine the in-phase and quadrature component of $s(t)$. The in-phase component will be described in the interval $(2n-1)T_b \le t \le (2n+1)T_b$, i.e., the interval of length $2T_b$ where the center $2nT_b$ is an *even* multiple of T_b; whereas the quadrature component will be described in the interval $2nT_b \le t \le (2n+2)T_b$, i.e., the interval of length $2T_b$ where the center $(2n+1)T_b$ is an *odd* multiple of T_b.

In the interval $(2n-1)T_b \le t \le (2n+1)T_b$, the in-phase component is with (19)

$$v_I(t) = A\cos\big(\phi(t)\big) \tag{22}$$

$$= A\cos\left(\Phi_{2n} \pm \frac{\pi}{2T_b}(t - 2nT_b)\right), \qquad (2n-1)T_b \le t \le (2n+1)T_b. \tag{23}$$

Substituting $\tilde{t} \triangleq t - 2nT_b$ and applying the trigonometric identity (2) to (23) yields

$$\begin{aligned} v_I(\tilde{t}) &= A\cos\left(\Phi_{2n} \pm \frac{\pi}{2T_b}\tilde{t}\right) \\ &= A\cos(\Phi_{2n})\cos\left(\frac{\pi}{2T_b}\tilde{t}\right) \mp A\sin(\Phi_{2n})\sin\left(\frac{\pi}{2T_b}\tilde{t}\right) \\ &= \pm A \cdot \cos\left(\frac{\pi}{2T_b}\tilde{t}\right), \qquad\qquad -T_b \le \tilde{t} \le T_b, \end{aligned}$$

which follows because $\cos(\Phi_{2n}) = \pm 1$ and $\sin(\Phi_{2n}) = 0$, since Φ_{2n} is either 0, or π. Therefore, in the interval $(2n-1)T_b \le t \le (2n+1)T_b$ we have

$$v_I(t) = \pm A \cdot \cos\left(\frac{\pi}{2T_b}(t - 2nT_b)\right), \qquad (2n-1)T_b \le t \le (2n+1)T_b, \tag{24}$$

where the plus sign corresponds to $\Phi_{2n} = 0$ and the minus sign corresponds to $\Phi_{2n} = \pi$. Thus, the in-phase component $v_I(t)$ consists of a *half-cycle cosine pulse* whose polarity depends on the even phase state Φ_{2n}.

Similarly, one can show that the quadrature component $v_Q(t)$ consists in the interval $2nT_b \le t \le (2n+2)T_b$ of a *half-cycle sine pulse*, whose polarity depends on the odd phase state Φ_{2n+1}.

$$\begin{aligned} v_Q(t) &= A\sin\big(\phi(t)\big) \\ &= A\sin\left(\Phi_{2n+1} \pm \frac{\pi}{2T_b}\big(t - (2n+1)T_b\big)\right), \qquad 2nT_b \le t \le (2n+2)T_b. \end{aligned}$$

Substituting $\tilde{t} \triangleq t - 2nT_b$ yields

$$\begin{aligned} v_Q(\tilde{t}) &= A\sin\left(\Phi_{2n+1} \pm \frac{\pi}{2T_b}\big(\tilde{t} - T_b\big)\right) \\ &= A\sin\left(\Phi_{2n+1} \mp \frac{\pi}{2} \pm \frac{\pi}{2T_b}\tilde{t}\right), \qquad\qquad 0 \le \tilde{t} \le 2T_b. \end{aligned}$$

Applying the identity $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$ leads to

$$\begin{aligned} v_Q(\tilde{t}) &= A \sin\left(\Phi_{2n+1} \mp \frac{\pi}{2}\right) \cos\left(\frac{\pi}{2T_b}\tilde{t}\right) \pm A \cos\left(\Phi_{2n+1} \mp \frac{\pi}{2}\right) \sin\left(\frac{\pi}{2T_b}\tilde{t}\right) \\ &= \pm A \cdot \sin\left(\frac{\pi}{2T_b}\tilde{t}\right), \qquad\qquad 0 \le \tilde{t} \le 2T_b, \end{aligned}$$

because $\Phi_{2n+1} = \pm\frac{\pi}{2}$ and hence adding $\mp\frac{\pi}{2}$ to Φ_{2n+1} leads to 0 or π. Therefore,

$$v_Q(t) = \pm A \sin\left(\frac{\pi}{2T_b}(t - 2nT_b)\right), \qquad 2nT_b \le t \le (2n+2)T_b. \tag{25}$$

The plus sign corresponds to $\Phi_{2n+1} = \frac{\pi}{2}$ and the minus sign corresponds to $\Phi_{2n+1} = -\frac{\pi}{2}$.

Looking at the interval $2nT_b \le t \le (2n+1)T_b$, the passband signal $s(t)$ is determined by the two phase states Φ_{2n} and Φ_{2n+1}. Since Φ_{2n} and Φ_{2n+1} can each take on one of two possible values, one of four possibilities can arise:

- $\Phi_{2n} = 0$ and $\Phi_{2n+1} = \frac{\pi}{2}$, corresponding to the transmission of $x_{2n} = +1$,
- $\Phi_{2n} = \pi$ and $\Phi_{2n+1} = \frac{\pi}{2}$, corresponding to the transmission of $x_{2n} = -1$,
- $\Phi_{2n} = \pi$ and $\Phi_{2n+1} = -\frac{\pi}{2}$, corresponding to the transmission of $x_{2n} = +1$,
- $\Phi_{2n} = 0$ and $\Phi_{2n+1} = -\frac{\pi}{2}$, corresponding to the transmission of $x_{2n} = -1$.

This, in turn, means that in each interval $nT_b \le t \le (n+1)T_b$, the MSK signal $s(t)$ can take on any one of four possible forms, depending on the values of the two consecutive phase states Φ_n and Φ_{n+1}. If n is even, the phase states Φ_n and Φ_{n+1} are related to x_n as described above. If n is odd:

- $x_n = -1$ corresponds to $\Phi_n = \frac{\pi}{2}$ and $\Phi_{n+1} = 0$, or $\Phi_n = -\frac{\pi}{2}$ and $\Phi_{n+1} = \pi$,
- $x_n = +1$ corresponds to $\Phi_n = \frac{\pi}{2}$ and $\Phi_{n+1} = \pi$, or $\Phi_n = -\frac{\pi}{2}$ and $\Phi_{n+1} = 0$.

In summary, the transmission of a code symbol $x_n = -1$ corresponds to a *decreasing* excess phase, the transmission of a code symbol $x_n = +1$ corresponds to an *increasing* excess phase in the time interval $nT_b \le t \le (n+1)T_b$.

3.2 MSK Receiver

Before the receiver structure is presented, we will have a closer look at the MSK signal. Based on expanding (21) with (24) and (25), we define the *basis functions*,

$$s_I(t) \triangleq \cos\left(\frac{\pi}{2T_b}(t - 2nT_b)\right)\cos(2\pi f_c t), \qquad (2n-1)T_b \le t \le (2n+1)T_b, \tag{26}$$

$$s_Q(t) \triangleq \sin\left(\frac{\pi}{2T_b}(t - 2nT_b)\right)\sin(2\pi f_c t), \qquad 2nT_b \le t \le (2n+2)T_b. \tag{27}$$

Then, in the interval $2nT_b \le t \le (2n+1)T_b$, the transmitted passband signal can be expressed as

$$s(t) = b_{I,2n} \cdot s_I(t) + b_{Q,2n+1} \cdot s_Q(t), \qquad 2nT_b \le t \le (2n+1)T_b, \tag{28}$$

where the coefficients $b_{I,2n}$ and $b_{Q,2n+1}$ are related to the phase states Φ_{2n} and Φ_{2n+1}. To evaluate $b_{I,2n}$, one can integrate the product $s(t) \cdot s_I(t)$ between the limits $(2n-1)T_b$ and $(2n+1)T_b$

$$\begin{aligned} b_{I,2n} &= \int_{(2n-1)T_b}^{(2n+1)T_b} s(t) \cdot s_I(t) dt \\ &= A\cos(\Phi_{2n}), \qquad\qquad (2n-1)T_b \le t \le (2n+1)T_b. \end{aligned} \tag{29}$$

The verification of (29) is left as an exercise.

Similarly, to evaluate $b_{Q,2n+1}$ one can integrate the product $s(t) \cdot s_Q(t)$ between the limits $2nT_b$ and $(2n+2)T_b$

$$\begin{aligned} b_{Q,2n+1} &= \int_{2nT_b}^{(2n+2)T_b} s(t) \cdot s_Q(t) dt \\ &= -A\sin(\Phi_{2n+1}), \qquad\qquad 2nT_b \le t \le (2n+2)T_b. \end{aligned} \tag{30}$$

When the coefficients $b_{I,2n}$ and $b_{Q,2n+1}$ are known, the phase states can easily be recovered with (29) and (30):

- if $b_{I,2n} > 0 \Rightarrow \Phi_{2n}$ must be 0,
  if $b_{I,2n} < 0 \Rightarrow \Phi_{2n}$ must be π,
- if $b_{Q,2n+1} > 0 \Rightarrow \Phi_{2n+1}$ must be $-\frac{\pi}{2}$,
  if $b_{Q,2n+1} < 0 \Rightarrow \Phi_{2n+1}$ must be $\frac{\pi}{2}$.

With this, we can draw the *signal-space* diagram for the MSK signal (Figure 3). With this diagram, the phase states can be deduced from the values of $b_{I,2n}$ and $b_{Q,2n+1}$, which lie on the s_I and s_Q axis, respectively. For example, $b_{I,2n} > 0$, $b_{Q,2n+1} > 0$ corresponds to $\Phi_{2n} = 0$ and $\Phi_{2n+1} = -\frac{\pi}{2}$ and the transmitted code symbol must be $x_{2n} = -1$.

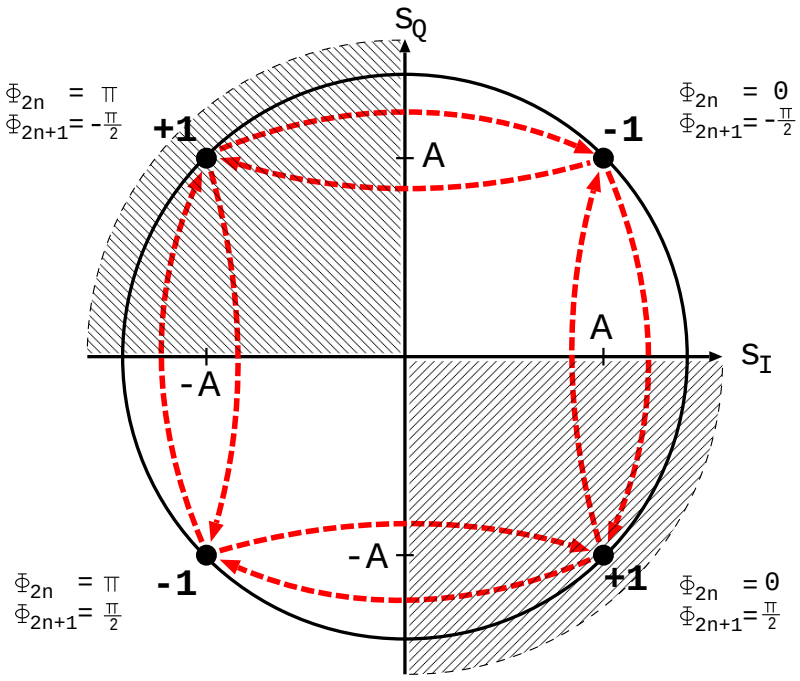

Figure 3: Signal-space diagram for MSK.

In the case of an *additive white Gaussian noise* (AWGN) channel, the received signal is given by

$$r(t) = s(t) + z(t) \tag{31}$$

where $s(t)$ is the transmitted MSK signal, and $z(t)$ is a white Gaussian noise process of zero mean and power spectral density $N_0/2$. From $r(t)$, we wish to guess $\{x_n\}$, i.e., to decide whether $x_n = +1$ or $x_n = -1$. To this end, the phase states $\{\Phi_n\}$ are guessed.

For optimum detection of the even phase states Φ_{2n}, the projection of the received signal $r(t)$ onto the basis function $s_I(t)$ is determined over the interval $(2n-1)T_b \leq t \leq (2n+1)T_b$:

$$\begin{aligned}
\hat{b}_{I,2n} &= \int_{(2n-1)T_b}^{(2n+1)T_b} r(t) \cdot s_I(t) dt \\
&= \int_{(2n-1)T_b}^{(2n+1)T_b} s(t) \cdot s_I(t) dt + \underbrace{\int_{(2n-1)T_b}^{(2n+1)T_b} z(t) \cdot s_I(t) dt}_{\triangleq \tilde{z}_{I,2n}} \\
&= b_{I,2n} + \tilde{z}_{I,2n}.
\end{aligned} \tag{32}$$

From the signal-space diagram in Figure 3, we observe that if $\hat{b}_{I,2n} > 0$, the receiver chooses the estimate $\hat{\Phi}_{2n} = 0$. On the other hand, if $\hat{b}_{I,2n} < 0$, it chooses the estimate $\hat{\Phi}_{2n} = \pi$. This procedure is optimal in the sense that it minimizes the probability of a detection error.[4]

Similarly, for the optimum detection of the odd phase states Φ_{2n+1}, the projection of the received signal $r(t)$ onto the second basis function $s_Q(t)$ over the interval $2nT_b \leq t \leq (2n+2)T_b$ is computed:

$$\begin{aligned}
\hat{b}_{Q,2n+1} &= \int_{2nT_b}^{(2n+2)T_b} r(t) \cdot s_Q(t) dt \\
&= \int_{2nT_b}^{(2n+2)T_b} s(t) \cdot s_Q(t) dt + \underbrace{\int_{2nT_b}^{(2n+2)T_b} z(t) \cdot s_2(t) dt}_{\triangleq \tilde{z}_{Q,2n+1}} \\
&= b_{Q,2n+1} + \tilde{z}_{Q,2n+1}.
\end{aligned} \tag{33}$$

Referring again to the signal-space diagram in Figure 3, we observe that if $\hat{b}_{Q,2n+1} > 0$, the receiver chooses the estimate $\hat{\Phi}_{2n+1} = -\pi/2$. If, on the other hand, $\hat{b}_{Q,2n+1} < 0$, it chooses the estimate $\hat{\Phi}_{2n+1} = \pi/2$.

The coefficients $\hat{b}_{I,2n}$ give a phase estimate for every even multiple of T_b, i.e., $\hat{\Phi}_{2n}$. The coefficients $\hat{b}_{Q,2n+1}$ give a phase estimate for every odd multiple of T_b, i.e., $\hat{\Phi}_{2n+1}$. To reconstruct the original data symbol sequence, the receiver computes at every time-step one of these two estimates, depending on whether the actual time-step is an even or an odd multiple of T_b. Then, the two sets of phase estimates have to be interleaved as follows:

from $\hat{b}_I$:	$\ldots$	$\hat{\Phi}_0$		$\hat{\Phi}_2$		$\hat{\Phi}_4$		$\ldots$	$\hat{\Phi}_{2n}$		$\ldots$
from $\hat{b}_Q$:	$\ldots$		$\hat{\Phi}_1$		$\hat{\Phi}_3$		$\hat{\Phi}_5$	$\ldots$		$\hat{\Phi}_{2n+1}$	$\ldots$

From this stream of interleaved phase estimates, the transmit sequence can be reconstructed. The receiver looks at the phase estimates $\hat{\Phi}_n$ and $\hat{\Phi}_{n+1}$ and chooses $\hat{x}_n$ according to Table 2. From $\hat{x}_n$, we can determine $\hat{d}_n$.

[4] To learn about optimal guessing strategies, visit the lecture *Information Transfer*.

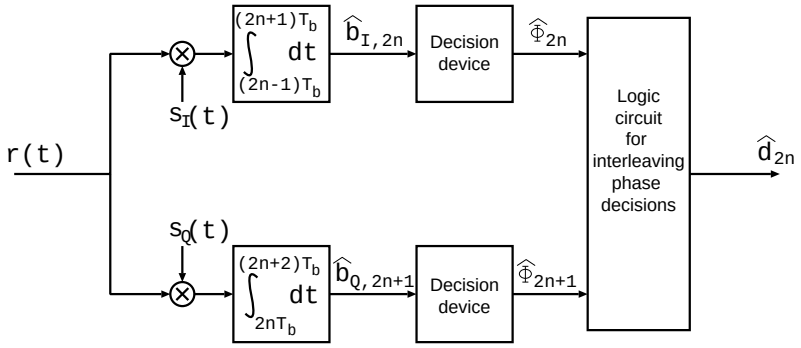

Figure 4: Block diagram of an MSK receiver.

n even					n odd			
$\hat{\Phi}_n$	$\hat{\Phi}_{n+1}$	$\hat{x}_n$	$\hat{d}_n$		$\hat{\Phi}_n$	$\hat{\Phi}_{n+1}$	$\hat{x}_n$	$\hat{d}_n$
0	$-\frac{\pi}{2}$	-1	1		$-\frac{\pi}{2}$	0	$+1$	0
π	$\frac{\pi}{2}$	-1	1		$\frac{\pi}{2}$	π	$+1$	0
π	$-\frac{\pi}{2}$	$+1$	0		$-\frac{\pi}{2}$	π	-1	1
0	$\frac{\pi}{2}$	$+1$	0		$\frac{\pi}{2}$	0	-1	1

Table 2: Detection of MSK.

Literature

A. Lapidoth, *Information Transfer*, Lecture notes.

G. L. Stüber, *Principles of Mobile Communication*, Chapter 4, Kluwer Academic Publishers, 1996.

S. Haykin, *Communication Systems*, 3rd edition, Chapter 8, Wiley, 1994.

J. G. Proakis, *Digital Communications*, 4th edition, Mc Graw Hill, 2001.

4 Experiments

1. Copy `/home/isistaff/glf/fachprak_isi/SV8` to your home directory: (`cp -irL /home/isistaff/glf/fachprak_isi/SV8 ./`) and open MATLAB.

2. Open the SIMULINK model `CPM.mdl` in the directory `SV8/simulink`.

You can see the complete CPM system. It consists of a `Data Source`, a `Transmitter`, an additive white Gaussian noise channel (`AWGN Channel`), and a `Receiver`. In this lab exercise, the red boxes have to be modified.

Parameters:

- Symbol duration: $T_b = 1$ sec,
- Sampling frequency of the SIMULINK model: $F_s = 1000$ Hz,
- Simulation duration: 20 sec.

Exercises:

3. Each $T_b = 1$ sec, the `Data Source` generates a bit `d`. Open `Scope1 Data Signal` with a double click to see a sequence of bits. Choose `Start` in the menu `Simulation` of the SIMULINK model or press `<CTRL>+<T>` in order to run the simulation.

4. With this bit stream, we modulate a CPM baseband representation. To this end, open the block `Transmitter` with a double click. Here you see the block `Pulse Shape` which maps each bit `d` to a frequency shaping pulse. Open this box (double click). The MATLAB editor appears with a MATLAB script that has to be completed. The function `pulseshape` has two inputs, the time `t` and the bit `d`.

   Implement the mapping from the bits `d` to the BPSK symbols `x` from the alphabet $\mathcal{X} = \{-1, +1\}$:

   $$\texttt{x} = \begin{cases} -1, & \text{if } \texttt{d} = 1 \\ +1, & \text{if } \texttt{d} = 0. \end{cases}$$

   Find a way to implement this mapping as efficient as possible (without using `if` and `else`). Write your code into the script `pulseshape`. Note that the output variable must be `x`. Save the script and close the editor. Open `Scope2 Pulse Shape` and run the simulation (`<CTRL>+<T>`). The signal you see corresponds to the NRZ shaping pulse.

5. Now try the half sinusoid HS frequency shaping pulse:

   $$h_f(t) = \sin\left(\frac{\pi}{T_b}t\right), \qquad 0 \le t \le T_b.$$

   Remark: Note that $h_f(t)$ is defined in the interval $[0, T_b]$ only. Use $t \pmod{T_b}$ instead of t to extend $h_f(t)$ for all times (MATLAB instruction `mod(t,Tb)`instead of `t`).

   Open again the MATLAB script `pulseshape` and multiply the symbol `x` with the HS pulse. Use again `x` as the output variable. Save, close the editor, run the simulation, and watch the signal in `Scope2`.

6. Try other shaping pulses:

- Raised Cosine (RC): $h_f(t) = \frac{1}{2}\left(1 - \cos(\frac{2\pi t}{T_b})\right), \qquad 0 \le t \le T_b$
- Triangular (TRI): $h_f(t) = \left(1 - \frac{|t - \frac{T_b}{2}|}{\frac{T_b}{2}}\right), \qquad 0 \le t \le T_b.$

Use again `mod(t,Tb)` instead of `t`.

What is the difference between the pulse forms? How do the frequency deviations with different $h_f(t)$ look like?

7. To compute the excess phase of the CPM signal, we need to integrate the sequence `x*hf(t)`. To this end, delete the `Terminator` and the `Ground` in the `Transmitter` and replace them with an `Integrator`:

   i) Type `simulink3` in the MATLAB command line. This opens the SIMULINK library.

   ii) Open `Continuous` with a double click and look for the integrator. Drag and drop this block to the place where the `Terminator` and the `Ground` have been.

   iii) Connect the input and the output, respectively, of the integrator with the lines.

   Run the simulation and watch the integrated signal in `Scope3 Integral` for the different shaping pulses. How can you back-reference from the integral curve to the sent bits?

8. We have computed

$$\int_{-\infty}^{t} \sum_k x_k \cdot h_f(\tau - kT_b) d\tau. \tag{34}$$

   From (34), the baseband representation (10) is generated as follows:

$$v(t) = A \cdot \exp\left\{\mathrm{i}2\pi k_f \int_{-\infty}^{t} \sum_k x_k \cdot h_f(\tau - kT_b) d\tau\right\}. \tag{35}$$

   Open the MATLAB script `baseband` (with a double click) and define the following parameters:

   - Amplitude: $A = 1$;
   - Peak Frequency Deviation: $k_f = 10$.

   Implement (35), where $\int_{-\infty}^{t} \sum_k x_k \cdot h_f(\tau - kT_b) d\tau$ is available as the input variable `u` and the output variable must be `v`. In SIMULINK, you have to write the imaginary number `i` as `1i` or `1j`. Save and close the editor.

9. Open now the block `passband` to up-convert the baseband representation $v(t)$ to the passband signal:

$$s(t) = 2\mathrm{Re}\left(v(t) \cdot e^{\mathrm{i}2\pi f_c t}\right).$$

   To do so, choose first a carrier frequency $f_c > 0$, i.e., the center frequency of the transmit signal. Note that f_c should not be greater than 500 Hz since the SIMULINK model is sampled with 1000 Hz (Sampling Theorem!).

10. Look at the spectrum of the transmit signal (`Vector Scope Spectrum`) for different f_c (change in `passband`) and for different k_f (change in `baseband`). Try for example $f_c = 50, 200$ and $k_f = 1/4, 5, 30$. Try also different pulse forms $h_f(t)$ (in `pulseshape`). Can you see differences in the spectrum? Can you explain them? What are the advantages and disadvantages of the different shaping pulses? How can you decide whether a 1 or a 0 has been transmitted by observing the spectrum?

11. Choose $f_c = 4$ (in `passband`) and $k_f = 2$ (in `baseband`) and compare the signal $s(t)$ (`Scope4 Transmit Signal`) of all different shaping pulses. Can you guess the sent bits?

Based on the general CPM, we consider now the special case of MSK. Recall that MSK is a special case of CPM with:

$$h_f(t) = u_{T_b}(t), \qquad h = \frac{1}{2}, \qquad x_k \in \{-1, +1\}.$$

12. Compute the peak frequency deviation k_f for $h = 1/2$ and $h_f(t) = u_{T_b}(t)$ (NRZ pulse) by using

$$\overline{k_f} = k_f \frac{1}{T_b} \int_0^{T_b} h_f(t)dt$$
$$h = 2\overline{k_f} T_b.$$

13. Change the parameters in the `Transmitter` so that it generates MSK signals:

    - $h_f(t)$ in `pulseshape`
    - k_f in `baseband`.

    Watch the transmit signal (`Scope4 Transmit Signal`) and compare it with the noisy version after the AWGN Channel (`Scope5 Noisy Received Signal`). Can you still guess the sent bits?

14. Now we implement the MSK receiver. The basis functions (26) and (27) are

$$s_I(t) \triangleq \cos\left(\frac{\pi}{2T_b}(t - 2nT_b)\right)\cos(2\pi f_c t), \quad (2n-1)T_b \le t \le (2n+1)T_b,$$
$$s_Q(t) \triangleq \sin\left(\frac{\pi}{2T_b}(t - 2nT_b)\right)\sin(2\pi f_c t), \qquad 2nT_b \le t \le (2n+2)T_b.$$

    Notice that by periodicity of the cosine and because $\cos\left(\frac{\pi}{2T_b}(t - 2nT_b)\right) \ge 0$ in the interval $(2n-1)T_b \le t \le (2n+1)T_b$,

$$\cos\left(\frac{\pi}{2T_b}(t - 2nT_b)\right) = \left|\cos\left(\frac{\pi}{2T_b}t\right)\right|,$$

    and similarly because $\sin\left(\frac{\pi}{2T_b}(t - 2nT_b)\right) \ge 0$ in the interval $2nT_b \le t \le (2n+2)T_b$

$$\sin\left(\frac{\pi}{2T_b}(t - 2nT_b)\right) = \left|\sin\left(\frac{\pi}{2T_b}t\right)\right|.$$

    Therefore, the basis functions $s_I(t)$ and $s_Q(t)$ are given by

$$s_I(t) = \left|\cos\left(\frac{\pi}{2T_b}t\right)\right| \cdot \cos(2\pi f_c t), \qquad t \in \mathbb{R},$$
$$s_Q(t) = \left|\sin\left(\frac{\pi}{2T_b}t\right)\right| \cdot \sin(2\pi f_c t), \qquad t \in \mathbb{R}.$$

    Implement these functions in the blocks `basisI` and `basisQ`, respectively, in the `Receiver`. Note that f_c must be chosen as in the `Transmitter`! The output variables have to be called `sI` and `sQ`.

In the in-phase and the quadrature channel, the received signal $r(t)$ is multiplied with $s_I(t)$ and and $s_Q(t)$, respectively, and integrated:

when n is even

$$\hat{b}_{I,n} = \int_{(n-1)T_b}^{(n+1)T_b} r(t) \cdot s_I(t)dt, \quad n = 0, 2, 4, \ldots$$

when n is odd

$$\hat{b}_{Q,n} = \int_{(n-1)T_b}^{(n+1)T_b} r(t) \cdot s_Q(t)dt, \quad n = 1, 3, 5, \ldots$$

After the integration, the `Inphase Decision Device` and the `Quadature Decision Device` decide whether $\hat{b}_{I,n}$ and $\hat{b}_{Q,n}$ are smaller or larger than 0.

15. Open `Scope6 Inphase` and `Scope7 Quadrature`, and run the simulation. The scopes plot the in-phase and the quadrature components as well as a sequence of rectangular functions which are $+1$ if the coefficients $\hat{b}_{I,n}, \hat{b}_{Q,n} > 0$ and -1 if the coefficients $\hat{b}_{I,n}, \hat{b}_{q,n} < 0$. Note that the coefficients $\hat{b}_{I,n}$ and $\hat{b}_{Q,n}$ are delayed by T_b with respect to $r(t) \cdot s_I(t)$ and $r(t) \cdot s_Q(t)$.

16. The block `Interleaver` combines the two streams of $\text{sign}(\hat{b}_{I,n})$ and $\text{sign}(\hat{b}_{Q,n})$ such that for every symbol time nT_b the two values `signb1` and `signb2` are available in the block `estimator`:
If n even

$$\texttt{signb1} = \text{sign}(\hat{b}_{I,n})$$
$$\texttt{signb2} = \text{sign}(\hat{b}_{Q,n+1}), \qquad n = 0, 2, 4, \ldots,$$

if n odd

$$\texttt{signb1} = \text{sign}(\hat{b}_{Q,n})$$
$$\texttt{signb2} = \text{sign}(\hat{b}_{I,n+1}), \qquad n = 1, 3, 5, \ldots,$$

where

$$\text{sign}(a) = \begin{cases} +1, & \text{if } a \geq 0 \\ -1, & \text{if } a < 0 \end{cases}, \qquad a \in \mathbb{R}.$$

Implement a function that makes a decision on the bits based on the input values `signb1` and `signb2`. Use

$$\hat{\Phi}_n = \begin{cases} 0, & \text{if } \hat{b}_{I,n} \geq 0 \\ \pi, & \text{if } \hat{b}_{I,n} < 0 \end{cases}, \text{ if } n \text{ even}, \qquad \hat{\Phi}_n = \begin{cases} -\frac{\pi}{2}, & \text{if } \hat{b}_{Q,n} \geq 0 \\ \frac{\pi}{2}, & \text{if } \hat{b}_{Q,n} < 0 \end{cases}, \text{ if } n \text{ odd},$$

and Table 2 to develop your function. Call the output variable `dhat`. Can you find a function that makes the decision without using $\hat{\Phi}_n$ and $\hat{\Phi}_{n+1}$, i.e., based on `signb1` and `signb2` only?

Run the simulation and see in `Scope8` if the decoded bits are equal to the sent bits. Note that the received and decoded bits are delayed by $2T_b$ due to the integration and estimation. Therefore, the first decoded bit is at time instance $2T_b$; the values in the first two time slots are meaningless. In `scope8`, the transmitted bits are also delayed by $2T_b$, such that the sent and the correctly decoded bits lie perfectly on top of each other.

17. Open `AWGN Channel` and change the value of the noise `Variance`. At which value can you observe errors in the decoded bit stream?