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SV1: Active RC Filters

1 Introduction

In this experiment, an active filter is constructed and measured. Consisting of resistors, capacitors, and an amplifier, it belongs to the class of active RC filters. Properties such as the frequency and time domains' behavior are investigated using a second-order low-pass filter. The experimental circuits are built on a plug-in board with discrete components.

2 Linear filters

In most cases, electrical filters are used to attenuate specific frequency components of a signal. These frequency-selective filters include, for example, low-pass, high-pass, band-pass, and band-stop. All these filters also change the phase response of the signal. However, this is less relevant for many applications than the frequency-dependent change in amplitude. On the other hand, some filters (e.g., all-pass filters) only influence the phase of the output signal.

2.1 Transfer function

The continuous-time transfer function of a linear filter describes the relationship between input and output signals:

$$T(s) = \frac{V_{OUT}}{V_{IN}} \tag{1}$$

$T(s)$ is a rational function of the complex frequency $s = \sigma + j\omega$. The numerator $N(s)$ and denominator $D(s)$ of $T(s)$ are complex polynomials (2). If they are decomposed into linear factors, the poles p_i and zeros z_i of the transfer function (3) are obtained.

$$T(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} \tag{2}$$

$$T(s) = k \cdot \frac{(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)} \tag{3}$$

Evaluating $T(s)$ along the imaginary axis ($s = j\omega$), we obtain the frequency response $T(j\omega)$, which can be decomposed into the amplitude response $|T(j\omega)|$ and the phase response $\phi(\omega)$:

$$T(j\omega) = |T(j\omega)| \cdot e^{j\phi(\omega)}. \tag{4}$$

The group delay $\tau_g(\omega)$ is a measure of the change in phase response:

$$\tau_g(\omega) = -\frac{d\phi(\omega)}{d\omega}. \tag{5}$$

Using the transfer function of a second-order low-pass filter, let's take a closer look at the meaning of the poles. Suppose $T(s)$ has a complex conjugates poles pair $p_1 = p = -\sigma_p + j\tilde{\omega}_p$ and $p_2 = p^* = -\sigma_p - j\tilde{\omega}_p$. $T(s)$ is then

$$T(s) = k \cdot \frac{\omega_p^2}{s^2 + 2\sigma_p s + \omega_p^2} = k \cdot \frac{\omega_p^2}{s^2 + \frac{\omega_p}{q_p} s + \omega_p^2}. \tag{6}$$

The values σ_p and $\tilde{\omega}_p$ are needed for the representation of the poles in the complex s-plane. However, for filter design, the values k (gain at $\omega = 0$), ω_p (pole frequency), and q_p (pole quality) are more useful since these quantities are physically measurable. The phase shift at $\omega = \omega_p$ is exactly 90°; the amplitude response has a value of kq_p at this point. Thus, the pole quality q_p measures the exaggeration of the amplitude response. These relationships are illustrated in Figure 1.

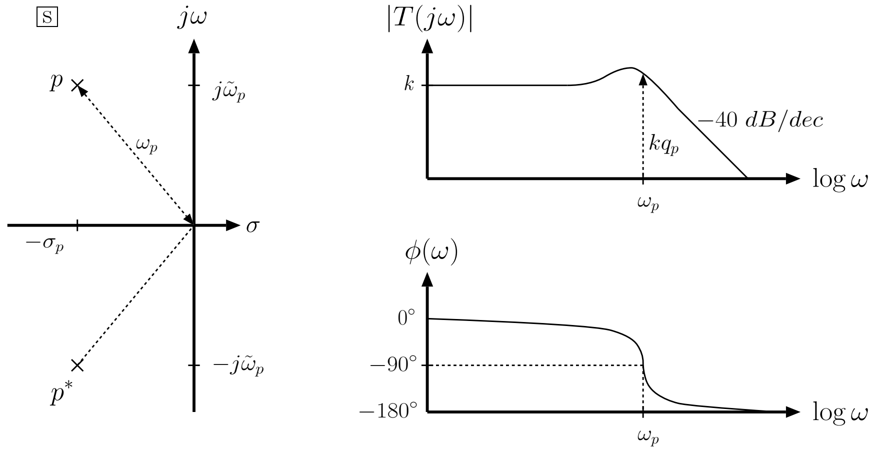

Figure 1: Pole/zero diagram of a low-pass filter with one complex conjugates pole pair with the corresponding amplitude and phase response.

3 Filter types

Ideally, the amplitude response of a filter is rectangular or step-shaped. In addition, a linear phase response is often desired. In practice, however, these two characteristics can only be realized approximately. The characteristics of five popular filter types are described below. Figure 2 shows the amplitude responses of various second-order low-pass filters.

- **Bessel filter:** Smooth amplitude response with low slope (flatter than Butterworth filter). Hardly any overshoot of the step response. The phase response is closest to the linear ideal; the group delay in the passband is largely constant. Also called *linear phase filter*.

- **Butterworth filter:** Linear amplitude response in the passband ("maximally flat"), smooth amplitude response in the entire frequency band. Represents a compromise between the steepness of the amplitude response in the stopband and linear phase response.

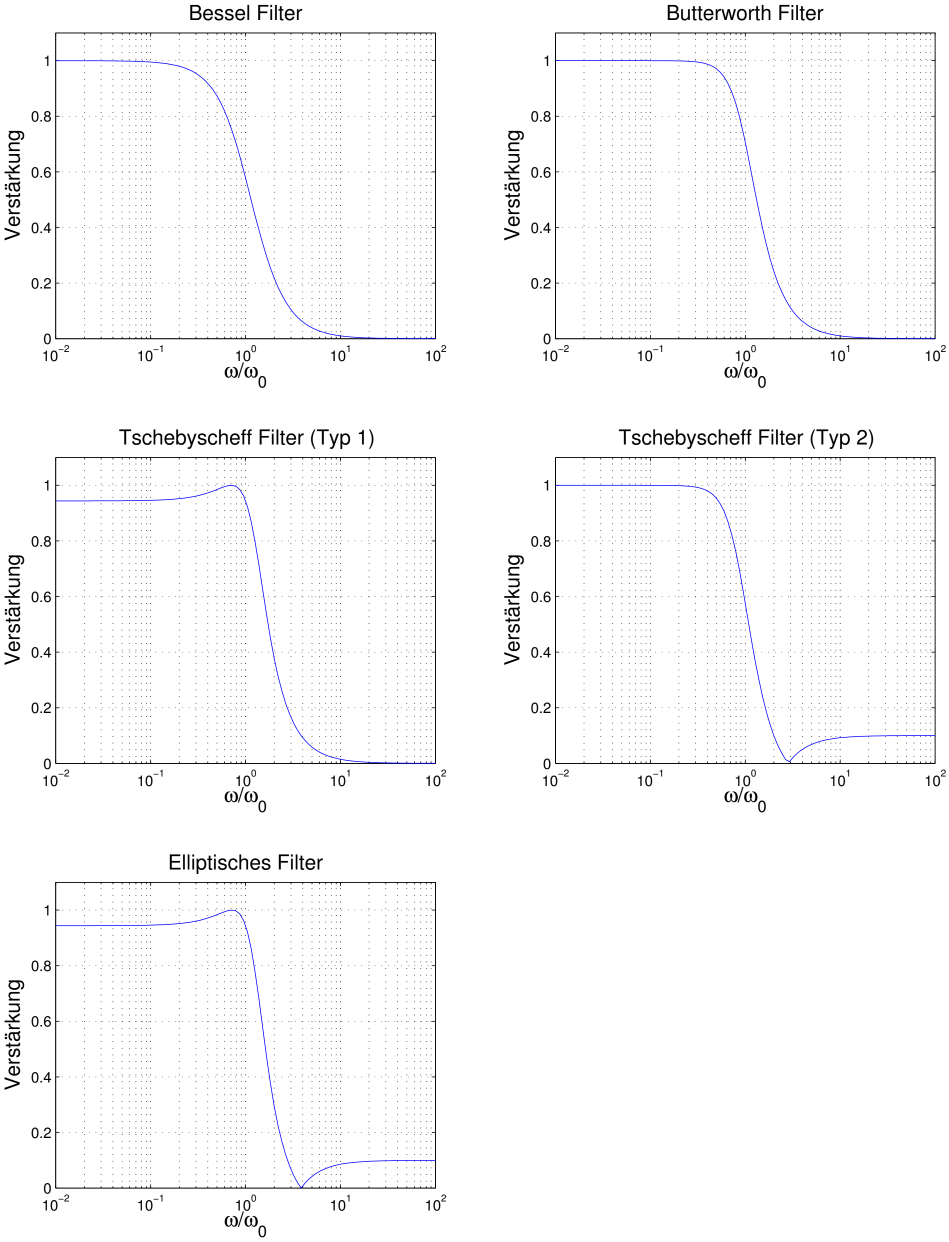

Figure 2: Amplitude responses of five different filter types. All are second-order low-pass filters

- **Chebyshev filter (type 1):** Wavy amplitude response in the passband (passband ripple), steep bend at the cutoff frequency. Wavy phase response. Also called *equiripple filter.*

- **Chebyshev filter (type 2):** Wavy amplitude response in stopband ripple, steep bend at the cutoff frequency. Wavy phase response. Also called *inverse Chebyshev filter, inverse equiripple filter.*

- **Elliptic filters:** Highest slope of amplitude response for a given filter order. Wavy amplitude response in passband and stopband. Strongly wavy phase response. Also called *Chebyshev-Cauer filter.*

4 Active RC filters

Originally, electrical filters were constructed exclusively from coils and capacitors (LC filters). Theoretically, filters with arbitrarily high qualities can be realized since these elements ideally work without loss. Filters can also be built from resistors and capacitors. However, the quality is limited due to the losses in the resistors. Adding an active element, such as an operational amplifier, can compensate for these losses. In this way, filters with high qualities can be realized.

4.1 Second-order RC filters

There are many ways to build filters from resistors, capacitors, and op-amps. The circuits used in this experiment are called Sallen-Key filters or *VCVS*[1] filters. They have a simple positive feedback path (Figure 3). This arrangement can be used to implement low-pass and high-pass filters. The circuit of the high-pass filter results from the low-pass filter shown in figure 3, if capacitors are used instead of resistors and vice versa (for all elements except R_5 and R_6). Figure 4 shows the high-pass filter derived in this way.

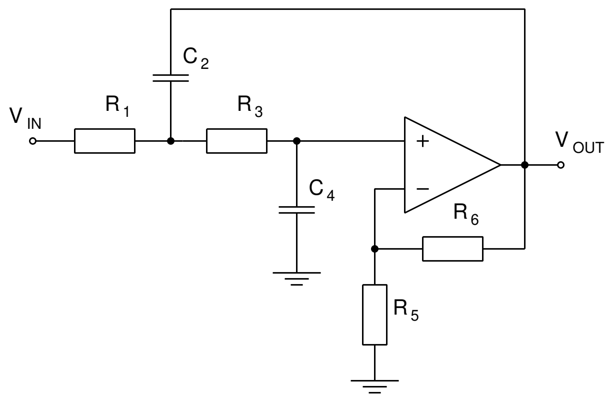

Figure 3: Active second-order low-pass filter (*VCVS* filter) with simple positive feedback.

4.2 Filter design

The circuits shown can be used to build filters with Bessel, Butterworth, or Chebyshev characteristics, among others; inverse Chebyshev and elliptic filters cannot be realized with these circuits because they need additional zeros. The following equations (*design equations*) are used to dimension the circuit elements in Figure 3 and 4:

[1] The operational amplifier forms with the resistors R_5 and R_6 a *Voltage Controlled Voltage Source*

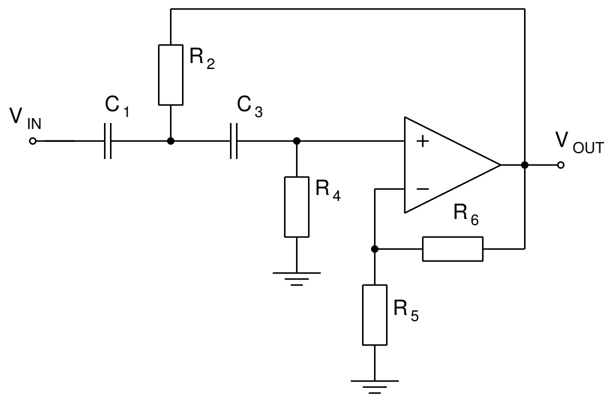

Figure 4: Second order active RC high-pass filter. Note the duality to the low-pass filter in Figure 3.

$\boxed{\text{low-pass}}$

$$T(s) = k \cdot \frac{\omega_p^2}{s^2 + (\omega_p/q_p)s + \omega_p^2}$$
$$k = 1 + \frac{R_6}{R_5}$$
$$\omega_p^2 = \frac{1}{R_1 C_2 R_3 C_4}$$
$$q_p = \frac{\sqrt{R_3 C_2/(R_1 C_4)}}{1 + R_3/R_1 - R_6 C_2/(R_5 C_4)}$$

$\boxed{\text{high-pass}}$

$$T(s) = k \cdot \frac{s^2}{s^2 + (\omega_p/q_p)s + \omega_p^2}$$
$$k = 1 + \frac{R_6}{R_5}$$
$$\omega_p^2 = \frac{1}{C_1 R_2 C_3 R_4}$$
$$q_p = \frac{\sqrt{R_4 C_1/(R_2 C_3)}}{1 + C_1/C_3 - R_4 R_6/(R_2 R_5)}$$

4.3 Filter parameters tuning

The component values are subject to considerable tolerances in discretely constructed and integrated circuits. Therefore it is often necessary to adjust the filter parameters after production (*tuning*). In the circuits shown, the values of ω_p and q_p can be adjusted by changing resistor values using the following table:

Parameter	Low-pass	High-pass
ω_p	R_3	R_2 or R_4
q_p	R_6	R_6

In each case, ω_p is set first and then q_p is set.

5 Experiments

1. Set up your workstation, connect the equipment and familiarize yourself with the instruments. Please contact the assistant if you have any questions. (Hint: Beware of floating grounds!)

2. We build a second-order active Butterworth low-pass filter (see figure 3) with DC gain of 6 dB and cutoff frequency $f_g = 1000\,\text{Hz}$. This gives the following values for the filter parameters: $k = 2$, $\omega_p = 2\pi \cdot 1000\ s^{-1}$ and $q_p = \sqrt{2}/2$. Now the values of the resistors and capacitors must be determined. With $R_6 = 10\,\text{k}\Omega$, $C_2 = 10\,\text{nF}$ and $C_4 = 15\,\text{nF}$,

3. Build the filter circuit on the breadboard. Practical advice: use the schematic (figure 3) as a guide for the placement of the components, this increases the clarity. The pinout of the operational amplifier TL072 is shown in figure 5. Use a supply voltage of $\pm 15\,\mathrm{V}$.

4. Measure the frequency response. Try to determine f_g and q_p as accurately as possible. Note: for $f = f_g = f_p$, the phase rotation is exactly $90°$. Do the measurement results agree with the expected frequency response?

5. Insert potentiometers instead of resistors R_3 and R_6 so that you can adjust the values of ω_p and q_p. Observe the frequency response in different settings.

6. Investigate how the circuit behaves in the time domain: Look at the step response at different values of q_p (R_6). Apply a square wave signal to the input.

7. The circuit can be easily transformed into a high-pass filter by replacing the resistors with capacitors and vice versa (R_5 and R_6 are excluded). Figure 4 shows the circuit. Measure the frequency response of the high-pass filter and vary ω_p and q_p.

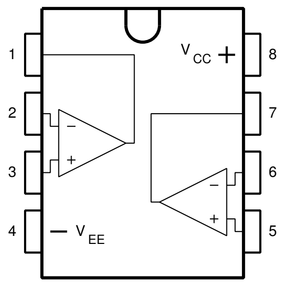

Figure 5: Pin assignment of the TL072 operational amplifier (top view).

6 Material

- Function generator, oscilloscope, multimeter.
- Accessories: BNC cable, probes, lab cable, tool, wire reel.
- Plug-in board, components: resistors $7.5\,\mathrm{k\Omega}$, 2x $10\,\mathrm{k\Omega}$, $22\,\mathrm{k\Omega}$; trimmers 2x $20\,\mathrm{k\Omega}$; capacitors $10\,\mathrm{nF}$, $15\,\mathrm{nF}$; operational amplifier TL072.