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Eidgenössische Technische Hochschule Zürich
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Acoustics II: electrical-mechanical-acoustical analogies

Reto Pieren
2024

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Motivation

- microphones and loudspeakers consist of coupled mechanical, acoustical and electrical subsystems
- → analysis has to consider all subsystems in an integral way
- the fundamental differential equations have identical form in all systems
- → introduction of analogies and conversion of mechanical and acoustical systems into electrical ones
- excellent tools available for analysis of electrical networks

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Describing quantities

At a specific point of a mechanical system, the two quantities of interest are:

- velocity $u = \frac{dx}{dt}$
- force F

From u, further quantities are derived to describe the movement:

- displacement $x = \int u \mathrm{d}t$
- acceleration $a = \frac{du}{dt}$

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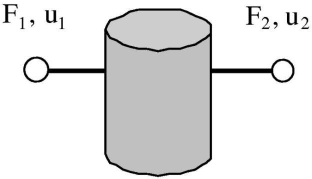
mechanical elements

Mechanical mass

ideal mass:

- incompressible → each point of the mass has identical velocity
- physical description: $F_{res} = m \cdot a$ (Newton)



ports 1 and 2:

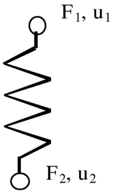
$$u_1 = u_2 = u$$

$$F_1 - F_2 = m\frac{du}{dt}$$

Spring

ideal spring:

- stiffness s independent of excursion x
- physical description: $F = s \cdot x$ (Hook's law)
- no force difference along the spring



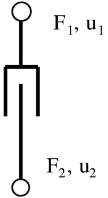
$$F_1 = F_2 = F$$

$$F = s \int (u_1 - u_2)\,\mathrm{d}t$$

Friction

ideal friction:

- proportionality between friction force and velocity
- physical description: $F = R \cdot u$
- no force difference along the friction element



$$F_1 = F_2 = F$$
$$F = R(u_1 - u_2)$$

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Links

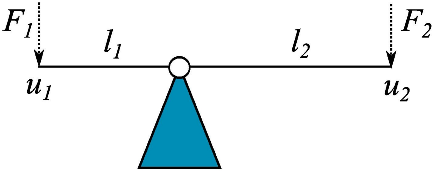
ideal link:

- connecting element for mechanical building blocks
- no mass and incompressible

Lever

ideal lever:

- frictionless and massless
- mechanical transformer



$$F_1 l_1 - F_2 l_2 = 0$$

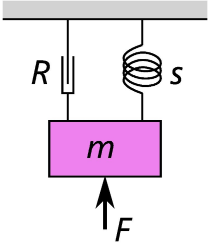
$$u_1 l_2 + u_2 l_1 = 0$$

Mechanical sources

Mechanical sources:

- force source
  - example: conductor in a magnetic field → force $\sim B \times I$ but independent of velocity
- velocity source
  - example: motor with a flywheel → velocity independent of load (force)

resonance system: spring pendulum



- given: external force: $F(t) = \hat{F}\sin(\omega t)$
- unknown: movement of mass
  - excursion $x(t)$
  - velocity $u(t) = dx/dt$
  - acceleration $a(t) = d^2x/dt^2$

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resonance system: spring pendulum: solution?

equilibrium of forces:

$$F_{acceleration} + F_{friction} + F_{spring} = F$$

differential equation for x in complex writing for harmonic excitation:

$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + sx = \hat{F}e^{j\omega t}$$

resonance system: spring pendulum: solution?

general solution of the differential equation:

$$x(t) = \frac{\hat{F} e^{j\omega t}}{(j\omega)^2 m + j\omega R + s} = \frac{\hat{F} e^{j\omega t}}{j\omega \left(j\omega m + R + \frac{s}{j\omega} \right)}$$

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resonance system: spring pendulum: solution?

with

$$Z_m = j\omega m + R + \frac{s}{j\omega}$$

we obtain:

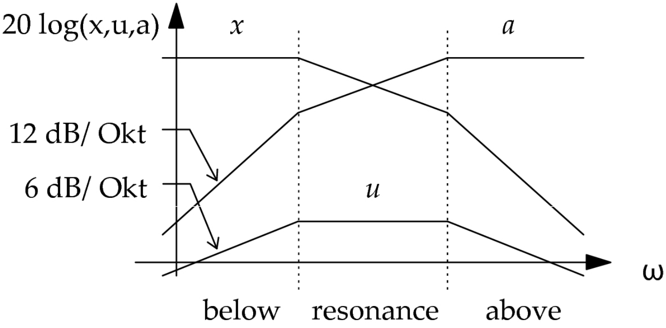
$$x(t) = \frac{\hat{F} e^{j\omega t}}{j\omega Z_m}$$

$$u(t) = \frac{\hat{F} e^{j\omega t}}{Z_m}$$

$$a(t) = \frac{j\omega \hat{F} e^{j\omega t}}{Z_m}$$

resonance system: spring pendulum: solution?

amplitude responses:



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resonance system: spring pendulum

consequences for microphones:

- majority of mics use a membrane (mechanical spring-mass resonance system)
- condition for flat amplitude response:
    - operation below resonance if electrical output is proportional to membrane excursion
    - operation at resonance if electrical output is proportional to membrane velocity

resonance system: spring pendulum

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describing quantities

fundamental acoustical quantities:

- sound pressure p
- sound particle velocity v or volume flow $Q = \int_S v \mathrm{d}S$

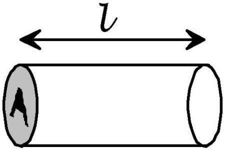
movie: rohr-open5-4

Acoustical elements

Acoustical mass

Acoustical mass:

- accelerated but not compressed air
- realization: tube of length l, diameter d where $\lambda \gg l$ and $\lambda \gg d$



$$\Delta F = \rho A l \frac{dv}{dt} \quad \text{Newton}$$

where

ρ: density of air

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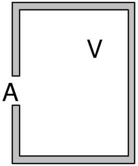
Acoustical compliance

Acoustical compliance:

- compressed but not accelerated air
- realization: cavity V with opening area A (largest dimension $l \ll \lambda$)



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Acoustical compliance

- experiment: virtual piston in opening A with applied force F
- piston sinks in by $\Delta l = \frac{F}{s}$ (s: stiffness of the compliance)

with the assumption of adiabatic behavior (no temperature exchange):

$$s = c^2 \rho \frac{A^2}{V}$$

where

c: speed of sound

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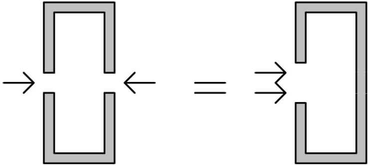
Acoustical compliance

- position of the opening is irrelevant



Acoustical resistance

Acoustical resistance:

- element introduces losses (conversion of sound energy into heat)
- realization: porous material, small tube

resistance of a small tube:

$$\Delta p = v\frac{8l\eta}{r^2}$$

where
Δp: sound pressure difference on both sides
v: sound particle velocity
l: length of the tube
r: radius of the tube ($r \ll l$)
η: dynamic viscosity of air: $1.82 \times 10^{-5}\mathrm{Nsm}^{-2}$

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Acoustical resistance

- acoustical resistance of a tube is always accompanied by an acoustical mass
- mass behavior can be neglected for small diameters and low frequencies

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analogies

Analog quantities

	potential quantity	flow quantity
electrical system:	voltage	current
mechanical system:	velocity u	force F
acoustical system:	sound pressure p	volume flow Q

→ dual analogy would be possible as well

Analog quantities

Amplitude scaling:

- conversion of acoustical and mechanical quantities into electrical ones requires amplitude and unit conversion factors
- arbitrary amplitude conversion factors are allowed
- here: $= 1$

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Analog quantities

mechanical → electrical:

$$U = G_1 u \quad \text{with} \quad G_1 = 1\frac{\mathrm{Vs}}{\mathrm{m}}$$

$$I = \frac{1}{G_2} F \quad \text{with} \quad G_2 = 1\frac{\mathrm{N}}{\mathrm{A}}$$

Analog quantities

acoustical → electrical:

$$U = G_3 p \quad \text{with} \quad G_3 = 1 \frac{\mathrm{Vm}^2}{\mathrm{N}}$$

$$I = \frac{1}{G_4} Q \quad \text{with} \quad G_4 = 1 \frac{\mathrm{m}^3}{\mathrm{As}}$$

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Impedances: general

Conversion of mechanical and acoustical elements into analog electrical ones is performed using impedance definition:

$$\text{impedance} = \frac{\text{potentialQuantity}}{\text{flowQuantity}}$$

Impedances: mechanical mass

impedance Z of the mechanical mass:

$$Z = \frac{u}{F} = \frac{u}{m\frac{du}{dt}}$$

with $u = u_0 e^{j\omega t}$ follows $\frac{du}{dt} = u_0 j\omega e^{j\omega t}$ and finally

$$Z = \frac{1}{j\omega m}$$

- mechanical mass $\triangleq$ electrical capacitance
- inertia of the mass is understood relative to reference system at rest → capacitors always at ground potential

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Impedances: mechanical spring

impedance Z of the mechanical spring:

$$Z = \frac{u}{F} = \frac{u}{s\int u\mathrm{d}t}$$

with $u = u_0 e^{j\omega t}$ follows $\int u\mathrm{d}t = u_0 \frac{1}{j\omega} e^{j\omega t}$ and finally

$$Z = j\omega \frac{1}{s}$$

- mechanical spring $\triangleq$ electrical inductance

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Impedances: mechanical friction

impedance Z of a mechanical friction element:

$$Z = \frac{u}{F} = \frac{u}{Ru} = \frac{1}{R}$$

- mechanical friction element $\triangleq$ electrical resistance

Impedances: acoustical mass

impedance Z of the acoustical mass:

$$Z = \frac{p}{Q} = \frac{\frac{\Delta F}{A}}{Av} = \frac{\rho A l \frac{dv}{dt}}{AAv}$$

with $v = v_0 e^{j\omega t}$ follows $\frac{dv}{dt} = v_0 j\omega e^{j\omega t}$ and finally

$$Z = j\omega \frac{\rho l}{A}$$

- acoustical mass $\triangleq$ electrical inductance

Impedance: acoustical compliance

impedance Z of the acoustical compliance:

$$Z = \frac{p}{Q} = \frac{\frac{\Delta F}{A}}{Av} = \frac{c^2 \rho \frac{A^2}{V} \Delta l}{AAv} = \frac{c^2 \rho \frac{A^2}{V} \int v \mathrm{d}t}{AAv}$$

with $v = v_0 e^{j\omega t}$ follows $\int v \mathrm{d}t = v_0 \frac{1}{j\omega} e^{j\omega t}$ and finally

$$Z = \frac{c^2 \rho}{j\omega V}$$

- acoustical compliance $\triangleq$ electrical capacitance

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Impedance: acoustical resistance

impedance Z of the acoustical resistance:

$$Z = \frac{p}{Q} = \frac{8l\eta}{\pi r^4}$$

where
l: length of the tube
r: radius of the tube ($r \ll l$)
η: dynamic viscosity in air $= 1.82 \times 10^{-5}$ Nsm^{-2}

- acoustical resistance $\triangleq$ electrical resistance

Impedances: overview

	electrical	mechanical	acoustical
$Z = R$	resistance	resistance	resistance
$Z = \frac{1}{j\omega C}$	capacitor	mass	compliance
$Z = j\omega L$	inductance	spring	mass

Impedances: overview

	electrical	mechanical	acoustical
$Z = R$	R	$R = \frac{1}{R_m} G_1 G_2$	$R = R_m G_3 G_4$
$Z = \frac{1}{j\omega C}$	C	$C = m \frac{1}{G_1 G_2}$	$C = \frac{V}{\rho c^2} \frac{1}{G_3 G_4}$
$Z = j\omega L$	L	$L = \frac{1}{s} G_1 G_2$	$L = \frac{\rho l}{A} G_3 G_4$

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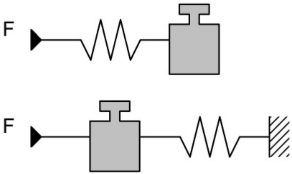
Examples of mechanical networks

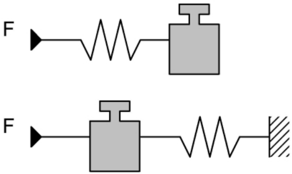
Example: mechanical resonance circuit

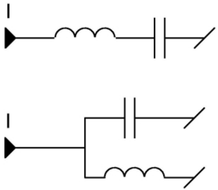
combinations of mechanical mass and spring:



Example: mechanical resonance circuit

combinations of mechanical mass and spring:





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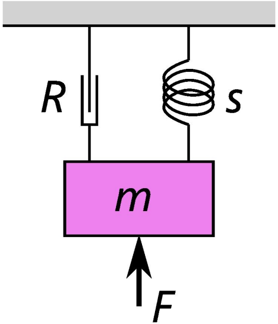
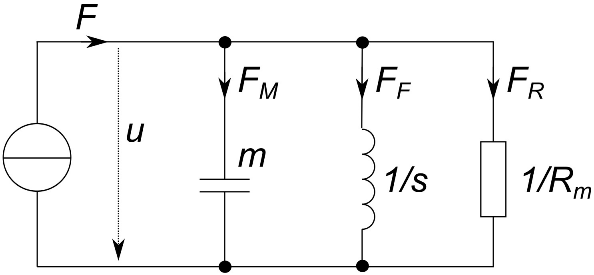
Example: mechanical resonance circuit



- sinusoidal force excitation
- analysis of the mechanical spring pendulum with help of an equivalent electrical circuit.

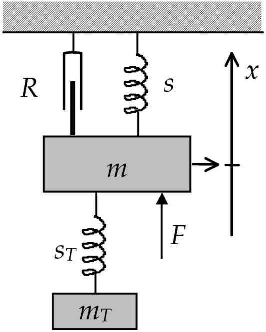
Example: mechanical spring pendulum



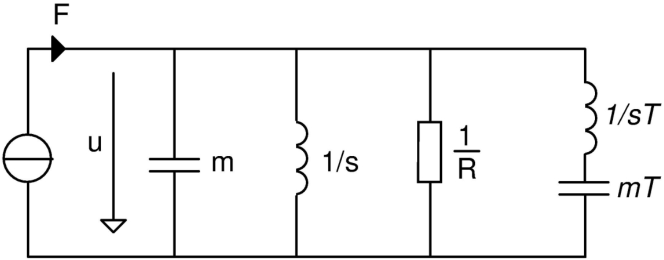
- $x = F_F / s$
- $u = F_R / R$
- $a = F_M / m$

Example: mechanical vibration damper

- strategy in case of sinusoidal vibration → implementation of an additional resonance system



Example: mechanical vibration damper



- additional series resonance circuit
- at resonance: impedance → 0
- short-circuit for the excitation force

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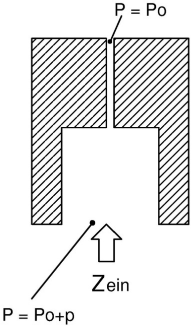
Example: acoustical resonance circuit

combination of acoustical mass and compliance:



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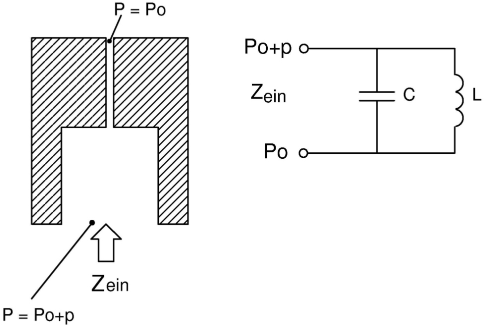
Example: acoustical resonance circuit

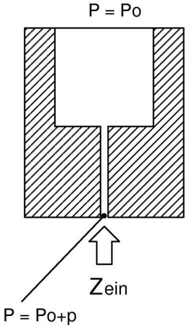
combination of acoustical mass and compliance:



→ parallel resonance circuit

Example: acoustical resonance circuit

combination of acoustical mass and compliance:



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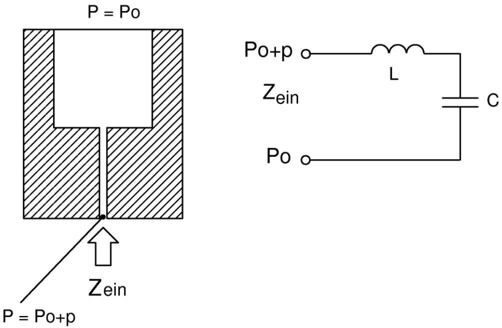
Example: acoustical resonance circuit

combination of acoustical mass and compliance:

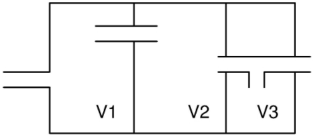


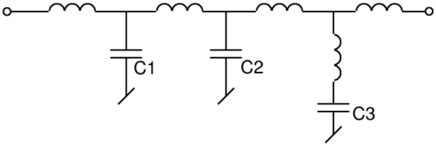
→ series resonance circuit

Example: muffler

muffler:

- no damping for low frequency air flow
- high damping for high frequency sound
- → acoustical low pass filter





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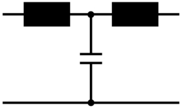
Example: acoustical transmission line

tubes (cross sect. area $= A$, $d \ll \lambda$) with length $> \lambda$:

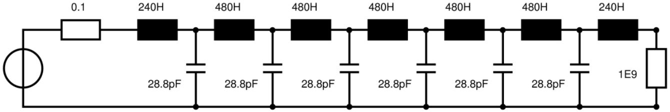
- subdivision of the tube into small sections of length l ($l \ll \lambda$)
- representation of each section by mass and compliance properties
- equivalent network: L, C, L T-section with
  - $L = \frac{\rho l}{2A}$
  - $C = \frac{Al}{\rho c^2}$



Example: acoustical transmission line

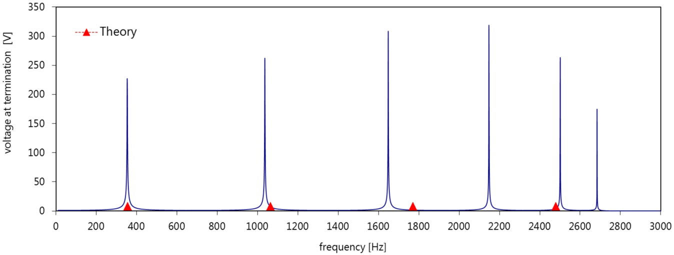
numerical example:

- hard terminated tube of length 24 cm
- cross sectional area $A = 10^{-4}$ m^2
- discretization: $l = 0.04$ m



Example: acoustical transmission line

Spice simulation of the voltage at the hard/open termination:



resonance frequencies expected from theory:

$$\frac{1\lambda}{4} = 0.24m \rightarrow 354\text{ Hz}, \ \frac{3\lambda}{4} = 0.24m \rightarrow 1063\text{ Hz}$$

$$\frac{5\lambda}{4} = 0.24m \rightarrow 1771\text{ Hz}, \ \frac{7\lambda}{4} = 0.24m \rightarrow 2479\text{ Hz}$$

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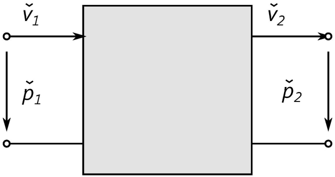
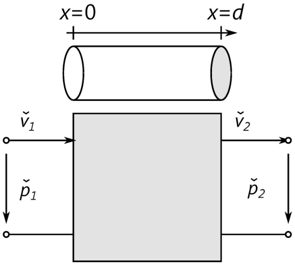
distributed acoustical elements

distributed acoustical elements

- modeling methods of the acoustical behavior of long tubes:
  1. sequence of short sections, each represented by lumped elements
  2. introduction of a distributed acoustical element → four-pole



distributed acoustical elements



- assumption: two plane waves traveling in opposite directions

$$\check{p}(x) = Ae^{-jkx} + Be^{jkx}$$

$$\check{v}(x) = \frac{A}{\rho c}e^{-jkx} - \frac{B}{\rho c}e^{jkx}$$

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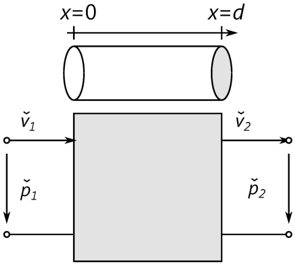

- definition:
  - $\check{p}(x = 0) \equiv \check{p}_1$ and $\check{p}(x = d) \equiv \check{p}_2$
  - $\check{v}(x = 0) \equiv \check{v}_1$ and $\check{v}(x = d) \equiv \check{v}_2$

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distributed acoustical elements

insert definition:

- $\check{p}(x=0)=\check{p}_1$ and $\check{p}(x=d)=\check{p}_2$
- $\check{v}(x=0)=\check{v}_1$ and $\check{v}(x=d)=\check{v}_2$

in:

$$\check{p}(x)=Ae^{-jkx}+Be^{jkx}$$

$$\check{v}(x)=\frac{A}{\rho c}e^{-jkx}-\frac{B}{\rho c}e^{jkx}$$

resolving for $\check{p}_2$ and $\check{v}_2$

distributed acoustical elements

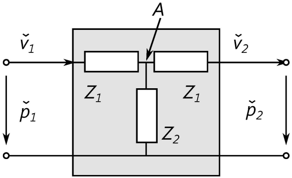
yields the four-pole equations:

$$\begin{aligned}\check{p}_2 &= \frac{\check{p}_1}{2}e^{-jkd} + \frac{\check{p}_1}{2}e^{jkd} + \frac{\check{v}_1\rho c}{2}e^{-jkd} - \frac{\check{v}_1\rho c}{2}e^{jkd} \\ &= \check{p}_1\cosh(jkd) - \check{v}_1\rho c\sinh(jkd)\end{aligned}$$

$$\begin{aligned}\check{v}_2 &= \frac{1}{\rho c}\frac{\check{p}_1}{2}e^{-jkd} - \frac{1}{\rho c}\frac{\check{p}_1}{2}e^{jkd} + \frac{\check{v}_1}{2}e^{-jkd} + \frac{\check{v}_1}{2}e^{jkd} \\ &= -\check{p}_1\frac{1}{\rho c}\sinh(jkd) + \check{v}_1\cosh(jkd)\end{aligned}$$

distributed acoustical elements

representation of the above four-pole relations by T-type circuit:



with:

$$Z_1 = j\rho c \frac{1 - \cos(kd)}{\sin(kd)}$$

$$Z_2 = \frac{-j\rho c}{\sin(kd)}$$

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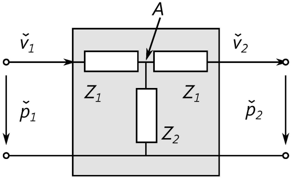
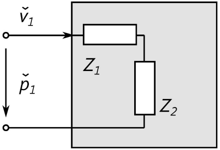
distributed acoustical elements

application example: impedance at the entrance of tube of length d with hard termination:





$$\frac{\check{p}_1}{\check{v}_1} = Z_1 + Z_2$$

$$\frac{\check{p}_1}{\check{v}_1} = \frac{j\rho c - j\rho c\cos(kd) - j\rho c}{sin(kd)} = -j\rho c\cot(kd)$$

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Input impedances of cylindrical and conical tubes

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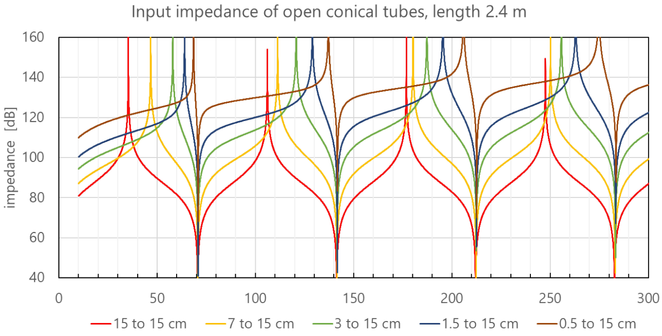
Input impedances of cylindrical and conical tubes

- brass instruments: playable at frequencies for which input impedance Z is maximal
- simulation of Z by Spice analysis of the equivalent electrical network
- termination at open end: short circuit

Input impedances of cylindrical and conical tubes



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Measurement of acoustical impedances

- investigation of unknown acoustical impedances
    - e.g. quality control of wind instruments
    - e.g. audiometric tests → input impedance of the middle ear

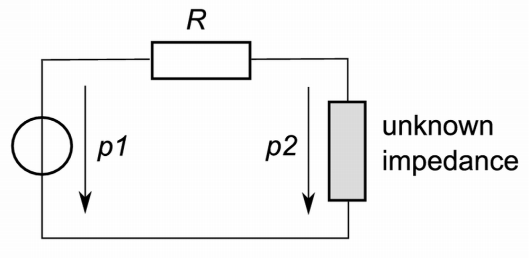
how to do this?

Measurement of acoustical impedances

- measurement methods
  1. combination of sound pressure and sound particle velocity sensors, e.g. μFlown
  2. two pressure sensors and known acoustical resistance

Measurement of acoustical impedances

method with two pressure sensors and known acoustical resistance



possible evaluation strategies:

- loudspeaker signal controlled for constant difference $p_1 - p_2 \rightarrow p_2 \sim Z$
- calculation from measured spectra

coupling of mechanical, acoustical and electrical systems

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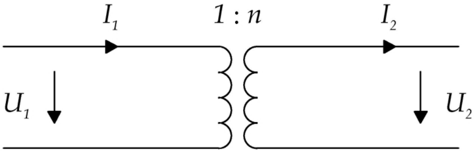
interfaces

linking of different subsystems:

- transformation of different physical quantities with help of interfaces:
  - type 1: conversion of potential quantity into potential quantity and flow quantity into flow quantity
  - type 2: conversion of potential quantity into flow quantity and vice versa

interfaces: transformer

interface type 1: transformer



relations for the transformer:

$$U_2 = nU_1 \tag{1}$$

$$I_2 = \frac{1}{n} I_1 \tag{2}$$

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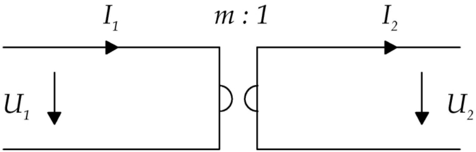
interface: gyrator

interface type 2: gyrator



relations for the gyrator:

$$U_2 = m I_1 \tag{3}$$

$$I_2 = \frac{1}{m} U_1 \tag{4}$$

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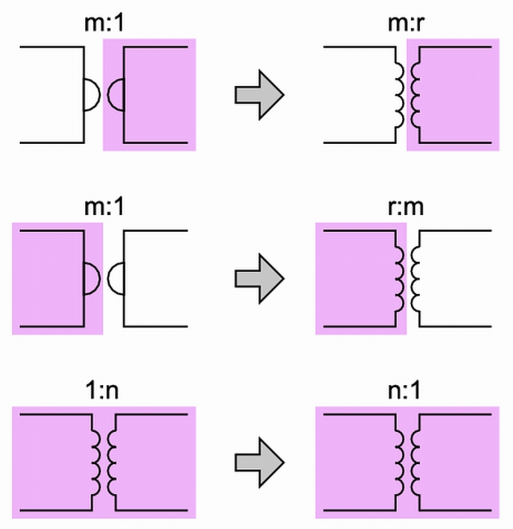
Dual conversion

Dual conversion

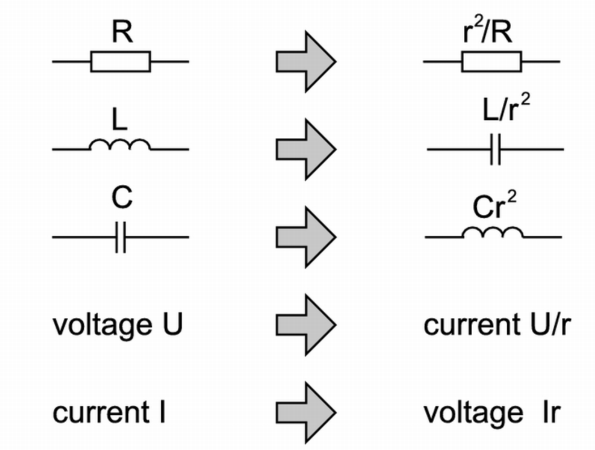
motivation: elimination of gyrators

- selection of an arbitrary conversion factor r
- dual conversion of a suitable region of the network (cut in half all gyrators)
- gyrators → transformers
- series arrangement → parallel arrangement
- parallel arrangement → series arrangement
- $L \rightarrow C$, $C \rightarrow L$, $R \rightarrow 1/R$

Dual conversion



Dual conversion



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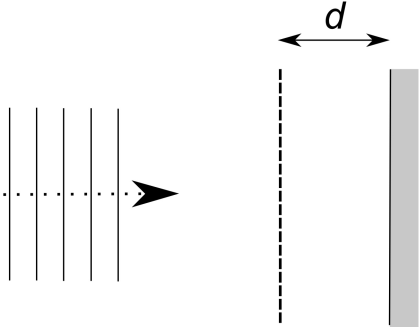
application:
thin absorber
in front of a hard wall

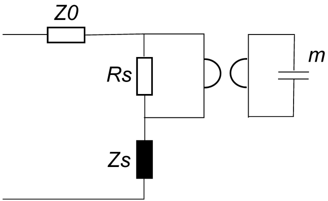
Thin absorber in front of a hard wall

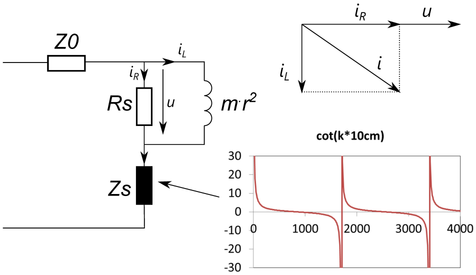
discussion of the behavior of a thin porous layer in front of a hard wall with help of an equivalent network for normal incidence



→ equivalent electrical network?

Thin absorber in front of a hard wall





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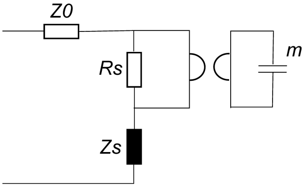
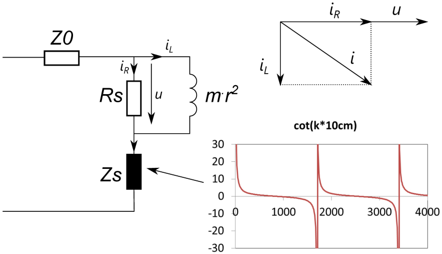
Thin absorber in front of a hard wall

how to maximise absorption?





Thin absorber in front of a hard wall

- absorption: dissipated power according to $i_R \cdot u$
- max. absorption for
    - $Z_S \to 0$
    - $R_S = Z_0$
    - $m \to \infty$

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