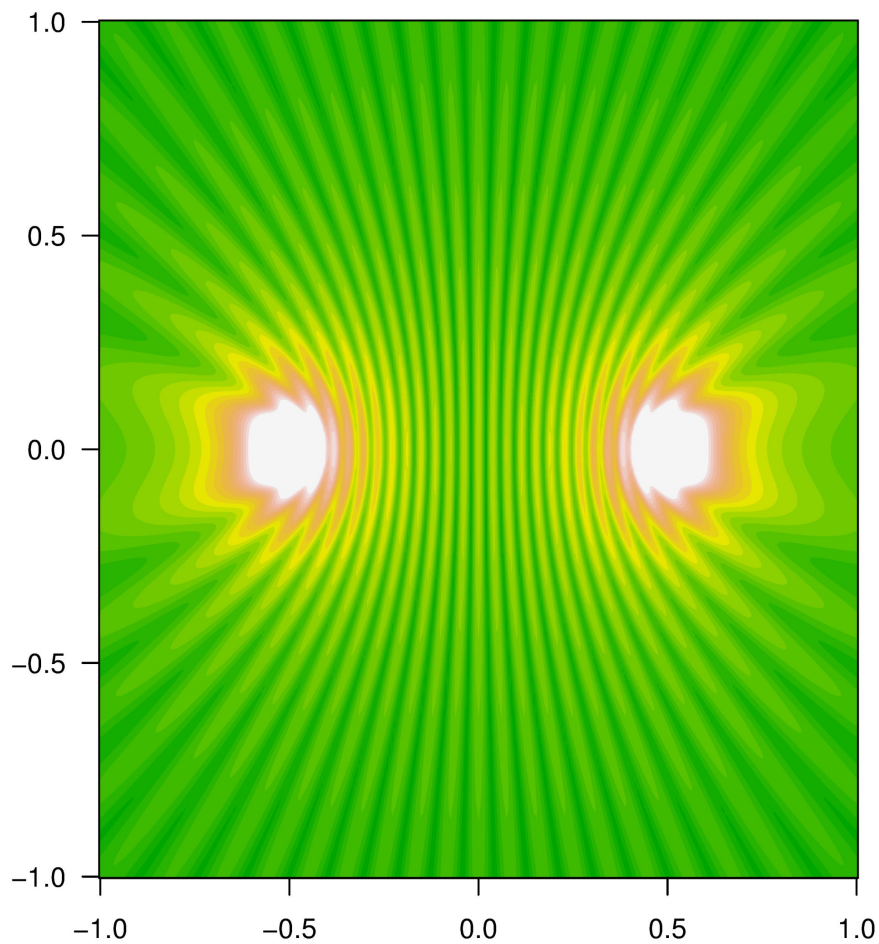


# Lecture Notes on Acoustics I



Kurt Heutschi  
Institute for Signal- and Informationprocessing, ISI  
Swiss Federal Institute of Technology, ETH  
CH-8092 Zurich

2021-09-14

The figure shows the interference pattern of two sound sources located at  $[-0.5, 0.0]$  and  $[0.5, 0.0]$  for a frequency of 4.5 kHz. Bright regions show high sound pressure. The local variation is highest on a line between the source points.



# Contents

<b>1</b>	<b>Acoustic fundamentals</b>	<b>1</b>
1.1	Introduction: Acoustics and sound	1
1.2	Basic sound wave phenomena	2
1.2.1	Geometrical spreading	2
1.2.2	Reflection of sound waves	2
1.2.3	Scattering of sound waves	3
1.2.4	Interference of sound waves	3
1.2.5	Diffraction of sound waves	3
1.3	Fundamental quantities	4
1.3.1	Sound pressure, sound particle displacement and sound velocity	4
1.3.2	Sound intensity and sound power	4
1.3.3	Impedance	5
1.3.4	Volume velocity	5
1.4	Fundamental equations	5
1.4.1	Wave equation	5
1.4.2	Sinusoidal waves	10
1.4.3	Complex representation of sinusoidal quantities	11
1.4.4	Helmholtz equation	11
1.5	Solutions of the wave equation	12
1.5.1	Plane waves	12
1.5.2	Spherical waves	13
1.5.3	Cylindrical waves	15
1.6	Sound pressure and sound power for point sources	15
1.7	Superposition of point sources	15
1.7.1	Superposition of incoherently radiating point sources	16
1.7.2	Superposition of coherently radiating point sources	17
1.8	Reflection of sound waves at acoustically hard surfaces	18
1.8.1	Specular reflection	18
1.8.2	Source directivity for limited radiation angles	19
1.8.3	Diffuse reflection	19
1.9	Doppler effect	19
1.10	Sonic boom	20
1.11	dB - scale	21
1.11.1	Quantities expressed as levels	21
1.11.2	Consequences of the dB scale	22
1.11.3	Subtlety of the dB scale	22
1.11.4	Computations involving dB quantities	22
1.11.5	Typical values of sound pressure levels	22
1.12	Classification of acoustical signals	22
1.13	Standing waves	24
1.13.1	Superposition of waves traveling in opposite directions	24
1.13.2	Quarter wave length resonators	25
1.14	Sound field calculations	26
1.14.1	General problem of reflection	26
1.14.2	Kirchhoff-Helmholtz Integral and Boundary Element Method	27
1.14.3	Applications of the Kirchhoff-Helmholtz Integral	27
1.14.4	Method of Finite Differences	31

1.14.5	Method of finite elements . . . . .	35
1.14.6	Acoustical Holography . . . . .	39
1.14.7	Equivalent sources technique . . . . .	40
1.14.8	Principle of reciprocity . . . . .	40
1.15	Production of sound . . . . .	41
1.15.1	Relaxation of compressed air . . . . .	41
1.15.2	Abrupt gas production (explosion) . . . . .	41
1.15.3	Modulated air flow . . . . .	41
1.15.4	Oscillating air column . . . . .	41
1.15.5	Vibrating bodies . . . . .	43
1.15.6	Thermo-acoustical machines . . . . .	46
<b>2</b>	<b>Acoustical measurements</b>	<b>48</b>
2.1	Introduction . . . . .	48
2.2	Signal attributes . . . . .	48
2.2.1	Overview . . . . .	48
2.2.2	Application of the measurement attributes . . . . .	50
2.2.3	Algorithm to determine the moving square average . . . . .	51
2.3	Filters . . . . .	51
2.3.1	Weighting filters . . . . .	52
2.3.2	Filters for frequency analysis . . . . .	52
2.4	Uncertainty of measurements . . . . .	54
2.4.1	Degrees of freedom of a bandlimited random signal . . . . .	54
2.4.2	Expectation value and variance of various functions of statistically independent samples . . . . .	55
2.4.3	Uncertainty of the calculation of the root mean square . . . . .	56
2.5	Measurement instruments . . . . .	58
2.5.1	Microphones . . . . .	58
2.5.2	Calibrators . . . . .	58
2.5.3	Sound level meter . . . . .	59
2.5.4	Level recorders . . . . .	60
2.5.5	Analyzers for level statistics . . . . .	60
2.5.6	Frequency analyzers . . . . .	60
2.5.7	Sound recorders . . . . .	60
2.6	Special measurement tasks . . . . .	61
2.6.1	Sound intensity measurements . . . . .	61
2.6.2	System identification . . . . .	62
2.6.3	Measurement of reverberation times . . . . .	68
2.7	Pressure zone microphone configuration . . . . .	70
2.8	Uncertainty of acoustical measurements . . . . .	71
<b>3</b>	<b>The human hearing</b>	<b>73</b>
3.1	Structure and principle of operation of the ear . . . . .	73
3.2	Properties of the auditory system for stationary signals . . . . .	74
3.2.1	Loudness . . . . .	74
3.2.2	Frequency discrimination . . . . .	76
3.2.3	Critical bands . . . . .	76
3.2.4	Audibility of level differences . . . . .	76
3.2.5	Masking . . . . .	76
3.2.6	Loudness summation . . . . .	76
3.2.7	Virtual pitch . . . . .	77
3.2.8	Audibility of phase . . . . .	77
3.2.9	Methods to calculate and measure the loudness . . . . .	78
3.2.10	Nonlinear distortions of the ear . . . . .	78
3.3	Properties of the ear for non stationary signals . . . . .	78
3.3.1	Loudness dependency on the signal length . . . . .	78
3.3.2	Temporal masking . . . . .	79
3.4	Binaural hearing: localization of sound sources . . . . .	79
3.4.1	Localization in the horizontal lane . . . . .	80

3.4.2	Localization in the vertical plane (elevation) . . . . .	80
3.4.3	Perception of distances . . . . .	80
3.4.4	Echoes and the precedence effect . . . . .	81
3.5	Hearing damage . . . . .	81
3.5.1	Mechanisms . . . . .	81
3.5.2	Assessment of the danger for a possible hearing damage . . . . .	82
<b>4</b>	<b>Musical Intervals</b>	<b>83</b>
<b>5</b>	<b>Outdoor sound propagation</b>	<b>84</b>
5.1	Basic equation . . . . .	84
5.2	Directivity of the source . . . . .	84
5.3	Attenuation terms . . . . .	85
5.3.1	Geometrical divergence . . . . .	85
5.3.2	Atmospheric absorption . . . . .	85
5.3.3	Ground effect . . . . .	86
5.3.4	Obstacles . . . . .	89
5.4	Reflections . . . . .	91
5.5	Meteorological effects . . . . .	92
5.5.1	Temperature gradients . . . . .	92
5.5.2	Wind . . . . .	93
5.5.3	Favorable and unfavorable sound propagation conditions . . . . .	94
5.5.4	Turbulence . . . . .	95
5.5.5	Calculation of meteorological effects on sound propagation . . . . .	95
<b>6</b>	<b>Absorption and reflection</b>	<b>97</b>
6.1	Characterization . . . . .	97
6.2	Types of absorbers . . . . .	97
6.2.1	Porous absorbers . . . . .	97
6.2.2	Resonance absorbers of type Helmholtz . . . . .	97
6.2.3	Membrane absorbers . . . . .	99
6.3	Measurement of absorption and reflection . . . . .	100
6.3.1	Kundt's tube . . . . .	100
6.3.2	Impedance tube . . . . .	101
6.3.3	Reverberation chamber . . . . .	101
6.3.4	In situ measurement of impulse responses . . . . .	102
6.4	Calculation of absorption and reflection from impedance relations . . . . .	102
6.4.1	Normal incidence . . . . .	102
6.4.2	Oblique incidence . . . . .	103
6.5	Typical values of absorption coefficients . . . . .	103
6.6	Cover for porous absorbers . . . . .	105
<b>7</b>	<b>Room acoustics</b>	<b>107</b>
7.1	Introduction . . . . .	107
7.2	Room acoustics of large rooms . . . . .	107
7.2.1	Statistical room acoustics . . . . .	107
7.2.2	Geometrical room acoustics . . . . .	114
7.2.3	Acoustical design criteria for rooms . . . . .	118
7.2.4	Room acoustical design tools . . . . .	119
7.2.5	Some room acoustical effects that are not considered with statistical or geometrical acoustics . . . . .	122
7.2.6	Reflections at spherical surfaces . . . . .	124
7.3	Room acoustics of small rooms, wave theoretical acoustics . . . . .	129
7.3.1	Wave equation and boundary conditions . . . . .	129
7.3.2	Solution for rectangular rooms with acoustically hard surfaces . . . . .	130
7.3.3	Source - receiver transfer function . . . . .	133
7.3.4	Acoustical design of small rooms . . . . .	133
7.4	Room acoustical measurements . . . . .	136

<b>8</b>	<b>Building acoustics</b>	<b>139</b>
8.1	Introduction	139
8.2	Airborne sound insulation	139
8.2.1	Sound insulation index $R$	139
8.2.2	Sound insulation of single walls	140
8.2.3	Sound insulation of double walls	141
8.2.4	Standard sound pressure level difference	141
8.3	Impact sound insulation	141
8.4	SIA 181	142
8.5	Construction hints for good building acoustical conditions	142
<b>9</b>	<b>Noise abatement</b>	<b>143</b>
9.1	Introduction - definition of noise	143
9.2	Effects of noise	144
9.3	General remarks for the assessment of noise	144
9.4	Influence of the source type	144
9.5	Definition of limiting values	145
9.6	Legal basis in Switzerland	145
9.6.1	Environment protection law USG	145
9.6.2	Noise Abatement Ordinance LSV	145
9.7	Soundscape concept	148
<b>A</b>	<b>Acoustic physical constants</b>	<b>150</b>
A.1	speed of sound in air	150
A.2	density of air at sea level	150
A.3	acoustic impedance	150

# Chapter 1

## Acoustic fundamentals

### 1.1 Introduction: Acoustics and sound

Acoustics is the science of sound. Typical questions deal with the generation of sound, the propagation and interaction with matter and the perception by humans. The term sound stands for mechanical oscillations with wave-like propagation. Sound waves can propagate in air, in liquids or in solid bodies. Figure 1.1 shows plane wave propagation in an open ended tube as a movie sequence.

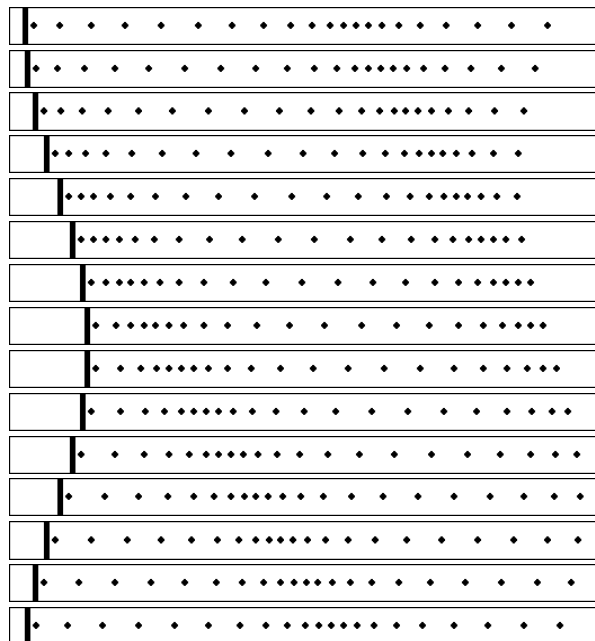


Figure 1.1: Movie pictures of the sound propagation in a long and open tube. The sound waves are generated by the moving piston shown on the left. The dots depict air particles. Of special interest is the local density of the particles which corresponds to sound pressure and the speed of the particles which corresponds to sound particle velocity. It should be noted that on average there is no net movement of the particles.

Corresponding to the perceptual capabilities of the human ear, three different frequency ranges are distinguished. The range of hearing stretches from about 16 Hz to 16 kHz. Lower frequencies are called infra-sound, higher frequencies are called ultra-sound.

The field of acoustics can be subdivided into several special topics such as:

**Theoretical acoustics** analytical and numerical methods for sound field calculations.

**Nonlinear acoustics** nonlinear effects that occur at events of extremely high sound pressure such as explosions or sonic booms of objects that move faster than the speed of sound.

**Underwater acoustics** sound propagation in water, sonar systems, seismic explorations.

**Ultrasound** non destructive test procedures for materials, medical applications.

**Vibrations** vibrational behavior of bodies, sound radiation of vibrating structures.

**Noise control** description and modeling of noise sources, investigations on noise protection measures.

**Room acoustics** assessment, planing and prediction of sound fields in rooms.

**Building acoustics** noise control in buildings, transmission loss of building structures.

**Electroacoustics** transducers (microphones, loudspeakers), recording devices, public address systems, signal processing in acoustics.

**Acoustics of the ear** structure of the ear, characteristics of the ear, perception and subjective evaluation of noise.

## 1.2 Basic sound wave phenomena

### 1.2.1 Geometrical spreading

Sound wave fronts that origin from a source with finite extension spread with growing distance over an increasing surface. Correspondingly the amplitude of the sound wave decreases (Figure 1.2).

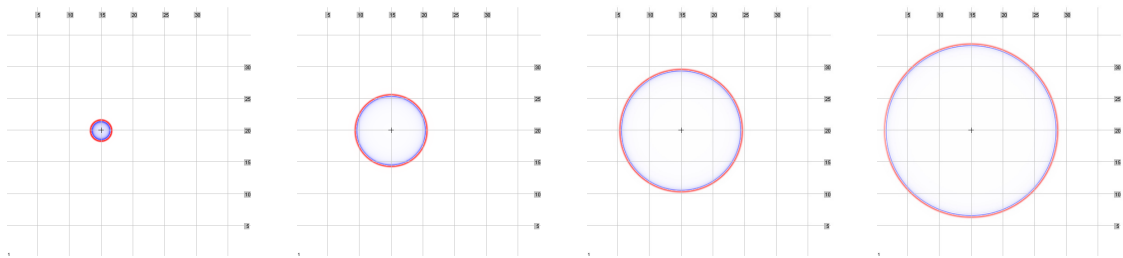


Figure 1.2: Geometrical spreading of a pulse shaped sound wave (time progresses from left to right). The location dependent sound pressure is color coded where intense red corresponds to high positive values and intense blue stands for high negative values.

### 1.2.2 Reflection of sound waves

If a sound wave hits an object, the free propagation is disturbed. At least a portion of the incident wave will be thrown back that i.e. will be reflected. If the reflecting object is large and flat, a specular reflection occurs. In this case the billiard rule *angle of incidence = angle of reflection* holds. The reflected wave has a distinct orientation and has the same temporal characteristics as the incident wave (Figure 1.3).

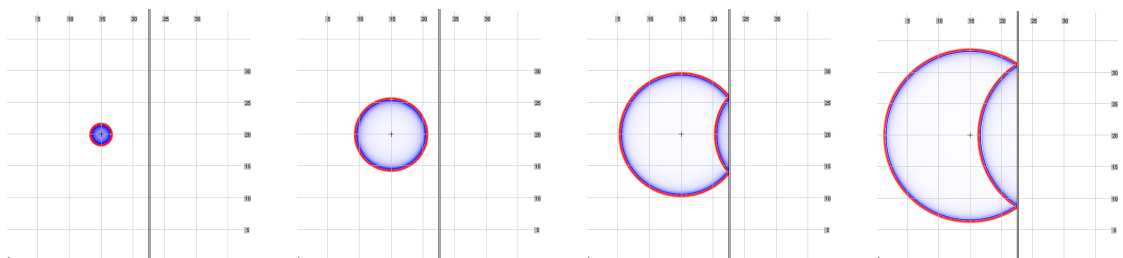


Figure 1.3: Reflection of a pulse shaped sound wave at a large smooth surface.



### 1.2.3 Scattering of sound waves

If the size of the reflecting object is small or the reflecting surface is significantly structured in depth (compared to the wave length), the reflection is no longer specular but scattering or diffuse. Reflected waves represent no distinct direction and are smeared over time (Figure 1.4).

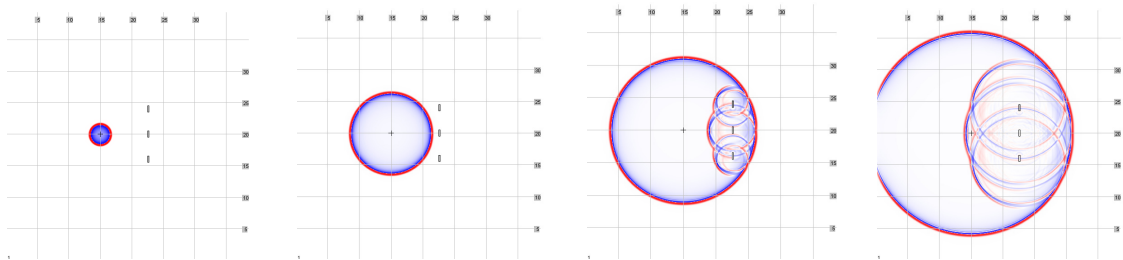


Figure 1.4: Reflection or scattering of a pulse shaped sound wave at several small objects.

### 1.2.4 Interference of sound waves

If two or more sound waves superpose, the resulting wave has sound pressure and sound velocity corresponding to the sum of the individual pressures and velocities. This summation has to be understood for each point in space and time. If the individual waves have identical frequency, their relative phase decides whether they amplify or attenuate each other. If the phase difference between two waves is small, an amplification occurs and the interference is called constructive. If the phase difference tends to  $180^\circ$  the waves attenuate each other and the interference is called destructive (Figure 1.5).

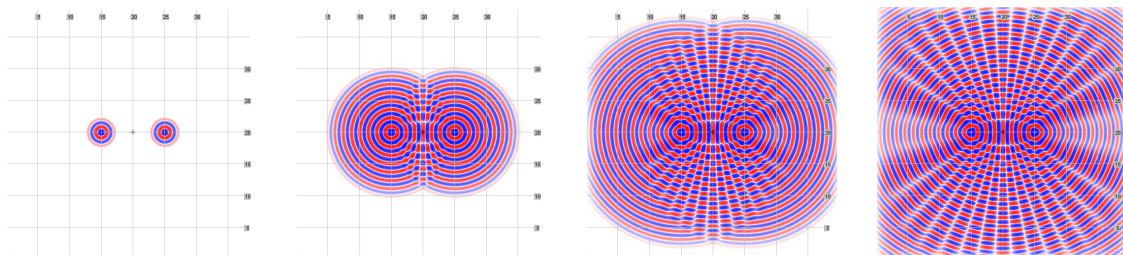


Figure 1.5: Superposition of two sound sources emitting sinusoidal waves resulting in location specific constructive or destructive interference.

### 1.2.5 Diffraction of sound waves

Diffraction describes the phenomenon that waves are bent around obstacles (Figure 1.6). The deflection into the geometrical shadow is stronger for lower frequencies.

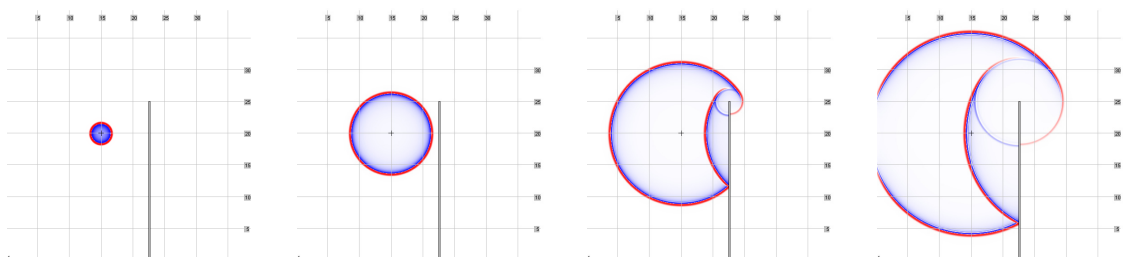


Figure 1.6: Diffraction of a pulse shaped sound wave at an edge (time progresses from left to right). The edge of the barrier is the origin of a secondary wave.

## 1.3 Fundamental quantities

### 1.3.1 Sound pressure, sound particle displacement and sound velocity

One of the consequences of the hull of air surrounding the earth is a static pressure. This atmospheric pressure is highest at sea level and decreases with height. On average the atmospheric pressure is about 100'000 Pa. A variation in altitude of 1 m results in a change of about 12 Pa. The atmospheric pressure is superimposed by small fluctuations as a consequence of sound waves. The human ear is only sensitive to these variations. Consequently these fast fluctuations relative to the atmospheric pressure got a special name. The corresponding quantity is called *sound pressure* (dt: Schalldruck) and is defined as (Eq. 1.1):

$$p(t) = P(t) - P_{\text{atm}} \quad (1.1)$$

where

$p(t)$ : sound pressure

$P(t)$ : momentary air pressure

$P_{\text{atm}}$ : atmospheric pressure

The production of local pressure variations leads to waves that travel with the speed of sound. Sound waves transport energy by the interaction of adjacent elements. Therefore they require matter with a mass and spring characteristics. In air (airborne sound) sound waves are always longitudinal waves which means that the gas particles move back and forth in the propagation direction. The movement of the gas particles is described by the *sound particle displacement*  $\zeta$  (dt: Schallausschlag) and by the *sound particle velocity*  $\vec{v}$  (dt: Schallschnelle). The sound particle velocity  $\vec{v}$  is a vector which points in the propagation direction. The displacement and the velocity are related by Eq. 1.2.

$$v(t) = \frac{d\zeta}{dt} \quad (1.2)$$

Sound pressure and sound particle velocity represent the two fundamental quantities to describe acoustical processes.

A sound field describes the acoustical conditions in a region in space. A complete description of a sound field requires in principle knowledge of sound pressure and sound particle velocity at every point in space. However as sound particle velocity can be related to the sound pressure gradient (see below), the velocity field can be calculated from complete information about sound pressure alone.

#### Typical numerical values

**sound pressure** normal speech produces in 1 m distance typical root mean squared sound pressure values  $p_{\text{typ,rms}}$  of about 0.1 Pa. At frequencies around 1 kHz sound pressure values  $p_{\text{min,rms}}$  of  $2 \times 10^{-5}$  Pa are just audible. The threshold of pain of the human auditory system is at  $p_{\text{max,rms}} \approx 100$  Pa.

**sound particle displacement** at a frequency of 1 kHz the above indicated sound pressure values correspond to sound particle displacements of  $\zeta_{\text{typ,rms}} \approx 4 \times 10^{-8}$  m,  $\zeta_{\text{min,rms}} \approx 8 \times 10^{-12}$  m and  $\zeta_{\text{max,rms}} \approx 4 \times 10^{-5}$  m.

**sound particle velocity** in a plane wave the sound pressure values from above correspond to the following particle velocities:  $v_{\text{typ,rms}} \approx 2.5 \times 10^{-4}$  m/s,  $v_{\text{min,rms}} \approx 5 \times 10^{-8}$  m/s and  $v_{\text{max,rms}} \approx 0.25$  m/s.

### 1.3.2 Sound intensity and sound power

The energy transport related to a sound wave can be described by the sound intensity (dt: Schallintensität)  $\vec{I}$ . The intensity indicates the amount of sound energy per unit time or sound power that passes through an orthogonal unit area. The sound intensity is a vector and points in the same direction as the sound particle velocity. The absolute value equals the product of sound pressure and sound particle velocity (taking into account a possible phase shift).

$$I = \overline{p\vec{v}} \quad [W/m^2] \quad (1.3)$$

The bar in Eq. 1.3 indicates averaging in time. In the vicinity of sound sources or reflectors there is usually a phase shift between  $p$  and  $v$ . In extreme cases this can lead to low intensity values although sound pressure and sound particle velocity have both high amplitudes. The physical interpretation is that air moves back and forth without significant compression. In other words there is a lot of reactive power but only little effective power.

If the sound intensity is known, the sound power  $W$  passing through an area  $S$  is given by the integral in Eq. 1.4.

$$W = \int_S \vec{I} d\vec{S} \quad [W] \quad (1.4)$$

The multiplication denotes the scalar product of the intensity vector  $\vec{I}$  and the orthogonal vector of the area element  $d\vec{S}$ . The sound power  $W$  corresponds to the total radiated power of the source if the area  $S$  encloses the source completely.

The sound power of typical sources is very small as shown in Table 1.1.

	sound power [W]
human voice, normal	$7 \times 10^{-6}$
human voice, max.	$2 \times 10^{-3}$
violin, fortissimo	$1 \times 10^{-3}$
Hi-Fi loudspeaker (10 W el.)	0.1
jackhammer	1
organ, fortissimo	10
orchestra (75 persons)	70
airplane Boeing 747	6'000
airplane FA-18	200'000

Table 1.1: Examples of sound sources and their emitted sound power.

### 1.3.3 Impedance

The ratio of sound pressure and sound particle velocity is defined as acoustical impedance  $Z$  (dt. Impedanz).

$$Z = \frac{\check{p}}{\check{v}} \quad (1.5)$$

The symbol  $\check{\phantom{x}}$  stands for the complex amplitude that contains an amplitude and a phase information. In general the impedance  $Z$  is a complex quantity.

### 1.3.4 Volume velocity

In the discussion of sound radiation the quantity volume velocity  $Q$  (dt: Schallfluss) plays an important role. It indicates the amount of sound that passes through a certain area (Eq. 1.6). The multiplication stands for the scalar product of the sound particle velocity and orthogonal vector of the area element  $d\vec{S}$ .

$$Q = \int_S \vec{v} d\vec{S} \quad (1.6)$$

## 1.4 Fundamental equations

### 1.4.1 Wave equation

The wave equation is the fundamental differential equation that describes in a compact form the physics of sound fields. For their derivation the interactions between sound pressure and sound particle velocity will be formulated.

### Interaction between sound pressure and sound particle velocity

The effect of sound pressure on sound particle velocity is investigated in a small cube with dimensions  $\Delta l \cdot \Delta l \cdot \Delta l$ . It is assumed that the sound pressure  $p$  on all six faces of the cube is known. Given this we are looking for the behavior of the sound particle velocity  $\vec{v}$  in the cube (Fig. 1.7).

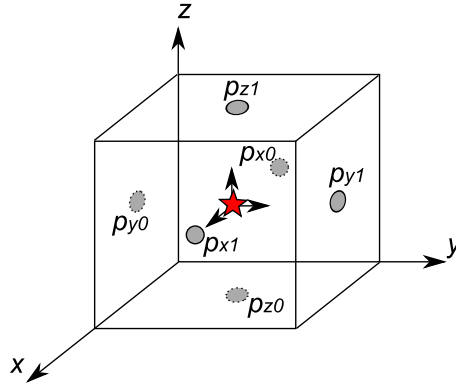


Figure 1.7: Situation to investigate the interaction between sound pressure  $p$  and sound particle velocity  $\vec{v}$  in a small cube.

The consequence of pressure differences on opposite sides of the cube is an acceleration  $a$  of the air with mass  $m$  in between. Once the acceleration is known, the sound particle velocity can be deduced easily. The relevant physical equation is Newton's law (1.7).

$$F_{\text{res}} = m \cdot a \quad (1.7)$$

The resulting force  $F_{\text{res}}$  corresponds to the pressure difference multiplied by the area. The acceleration equals the time derivative of the sound particle velocity in the corresponding direction. Here this is shown for the  $x$  coordinate direction (1.8).

$$\Delta l^2(p_{x0} - p_{x1}) = m \frac{\Delta v_x}{\Delta t} \quad (1.8)$$

The mass of the cube is related to density  $\rho$  as:

$$m = \Delta l^3 \cdot \rho \quad (1.9)$$

Eq. 1.8 becomes

$$\Delta l^2(p_{x0} - p_{x1}) = \Delta l^3 \cdot \rho \frac{\Delta v_x}{\Delta t} \quad (1.10)$$

Finally with division by the volume of the cube  $\Delta l^3$  it follows from Eq. 1.10

$$\frac{p_{x0} - p_{x1}}{\Delta l} = \rho \frac{\Delta v_x}{\Delta t} \quad (1.11)$$

Eq. 1.11 can be written as separated differential equations for the three directions in space:

$$\begin{aligned} \frac{\partial p}{\partial x} &= -\rho \frac{\partial v_x}{\partial t} \\ \frac{\partial p}{\partial y} &= -\rho \frac{\partial v_y}{\partial t} \\ \frac{\partial p}{\partial z} &= -\rho \frac{\partial v_z}{\partial t} \end{aligned} \quad (1.12)$$

or in vector equation form:

$$\text{grad}(p) = -\rho \frac{\partial \vec{v}}{\partial t} \quad (1.13)$$

### Interaction between sound particle velocity and sound pressure

The effect of sound particle velocity on sound pressure is again investigated in a small cube with dimensions  $\Delta l \cdot \Delta l \cdot \Delta l$ . It is assumed that the sound particle velocity  $\vec{v}$  is given on all six faces of the cube. We are looking for the behavior of the sound pressure  $p$  in the cube (Fig. 1.8).

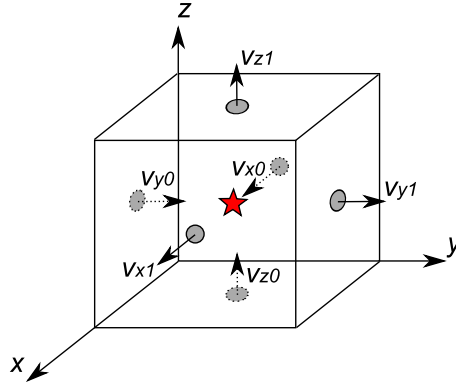


Figure 1.8: Situation to investigate the interaction between sound particle velocity  $\vec{v}$  and sound pressure  $p$  in a small cube.

A difference in sound particle velocity on two opposite sides of the cube results in a change of the cube volume  $\Delta V$ . This volume change is connected to a change in pressure  $\Delta P$ . Assuming an adiabatic process, the relation between  $\Delta V$  and  $\Delta P$  is described by the Poisson law (1.14). The assumption of an adiabatic process is usually fulfilled for sound in air. This implies that there is no heat exchange between the sound wave and the surrounding. However in special cases such as a loudspeaker box filled with porous material the process is no longer adiabatic but isothermal. For adiabatic processes an expansion of the gas leads to a pressure decrease and a cooling of the gas.

$$PV^\kappa = \text{constant} \quad (1.14)$$

with

$P$ : pressure of the gas

$V$ : volume

$\kappa$ : adiabatic exponent, for air  $\kappa = 1.4$

For small variations the Poisson law in Eq. 1.14 can be linearized. A small pressure variation  $\Delta P$  is related to a small volume change  $\Delta V$ :

$$(P_0 + \Delta P)(V_0 + \Delta V)^\kappa = P_0 V_0^\kappa \quad (1.15)$$

The first term in Eq. 1.15 can be rewritten as

$$P_0 + \Delta P = P_0 \left( 1 + \frac{\Delta P}{P_0} \right) \quad (1.16)$$

For small changes  $\Delta V$  compared to  $V$  the expression  $(V_0 + \Delta V)^\kappa$  can be expanded into a series. Ignoring the higher order elements of the series one gets:

$$(V_0 + \Delta V)^\kappa \approx V_0^\kappa + \Delta V \kappa V_0^{\kappa-1} = V_0^\kappa \left( 1 + \kappa \frac{\Delta V}{V_0} \right) \quad (1.17)$$

(1.15) and (1.17) in (1.14) yields:

$$P_0 \left( 1 + \frac{\Delta P}{P_0} \right) V_0^\kappa \left( 1 + \kappa \frac{\Delta V}{V_0} \right) \approx P_0 V_0^\kappa \quad (1.18)$$

$$\left( 1 + \frac{\Delta P}{P_0} \right) \left( 1 + \kappa \frac{\Delta V}{V_0} \right) \approx 1 \quad (1.19)$$

$$\frac{\Delta P}{P_0} \approx -\kappa \frac{\Delta V}{V_0} - \kappa \frac{\Delta P}{P_0} \frac{\Delta V}{V_0} \quad (1.20)$$

The product  $\Delta P \Delta V$  can be ignored under the assumption of small quantities. So finally we get:

$$\frac{\Delta P}{P_0} \approx -\kappa \frac{\Delta V}{V_0} \quad (1.21)$$

The linearized form of the Poisson Equation (1.21) connects in a simple way the pressure variation and the volume variation due to the sound particle velocity differences on all sides of the cube. Let the volume of the cube at time  $t$  be

$$V(t) = V_0 = \Delta l^3 \quad (1.22)$$

Short time later the volume is

$$V(t + \Delta t) = [\Delta l + \Delta t(v_{x1} - v_{x0})] \cdot [\Delta l + \Delta t(v_{y1} - v_{y0})] \cdot [\Delta l + \Delta t(v_{z1} - v_{z0})] \quad (1.23)$$

The products of two and three sound particle velocity differences become very small and can be neglected:

$$V(t + \Delta t) \approx \Delta l^3 + \Delta l^2 \Delta t(v_{x1} - v_{x0}) + \Delta l^2 \Delta t(v_{y1} - v_{y0}) + \Delta l^2 \Delta t(v_{z1} - v_{z0}) \quad (1.24)$$

The volume change  $\Delta V$  during the time step  $\Delta t$  is

$$\Delta V = V(t + \Delta t) - V(t) \approx \Delta l^2 \Delta t(v_{x1} - v_{x0}) + \Delta l^2 \Delta t(v_{y1} - v_{y0}) + \Delta l^2 \Delta t(v_{z1} - v_{z0}) \quad (1.25)$$

Insertion of Eq. 1.25 in Eq. 1.21 gives

$$\Delta P = \frac{-\kappa P_0}{\Delta l^3} [\Delta l^2 \Delta t(v_{x1} - v_{x0}) + \Delta l^2 \Delta t(v_{y1} - v_{y0}) + \Delta l^2 \Delta t(v_{z1} - v_{z0})] \quad (1.26)$$

or

$$\frac{\Delta P}{\Delta t} = -\kappa P_0 \left( \frac{v_{x1} - v_{x0}}{\Delta l} + \frac{v_{y1} - v_{y0}}{\Delta l} + \frac{v_{z1} - v_{z0}}{\Delta l} \right) \quad (1.27)$$

It should be noticed that the variation of the pressure  $\Delta P$  equals the sound pressure change  $\Delta p$ . Translated into a differential equation, Eq. 1.27 results in

$$\frac{\partial p}{\partial t} = -\kappa P_0 \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \quad (1.28)$$

or abbreviated

$$\frac{\partial p}{\partial t} = -\kappa P_0 \text{div}(\vec{v}) \quad (1.29)$$

The two equations 1.12 and 1.28 represent the fundamental physical relations for acoustical processes. The wave equation combines these two relations into one single differential equation. For its derivation, the equations 1.12 are differentiated with respect to the three coordinates  $x$ ,  $y$  and  $z$ , Eq. 1.28 is differentiated regarding to  $t$ . One gets:

$$\begin{aligned} \frac{\partial^2 p}{\partial x^2} &= -\rho \frac{\partial^2 v_x}{\partial t \partial x} \stackrel{*)}{=} -\rho \frac{\partial^2 v_x}{\partial x \partial t} \\ \frac{\partial^2 p}{\partial y^2} &= -\rho \frac{\partial^2 v_y}{\partial t \partial y} \stackrel{*)}{=} -\rho \frac{\partial^2 v_y}{\partial y \partial t} \\ \frac{\partial^2 p}{\partial z^2} &= -\rho \frac{\partial^2 v_z}{\partial t \partial z} \stackrel{*)}{=} -\rho \frac{\partial^2 v_z}{\partial z \partial t} \end{aligned} \quad (1.30)$$

\*) theorem of Schwarz  
and

$$\frac{\partial^2 p}{\partial t^2} = -\kappa P_0 \left( \frac{\partial^2 v_x}{\partial x \partial t} + \frac{\partial^2 v_y}{\partial y \partial t} + \frac{\partial^2 v_z}{\partial z \partial t} \right) \quad (1.31)$$

insertion of 1.30 in 1.31 results in the wave equation:

$$\frac{\partial^2 p}{\partial t^2} = \frac{\kappa P_0}{\rho} \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right) \quad (1.32)$$

or

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{\rho}{\kappa P_0} \frac{\partial^2 p}{\partial t^2} \quad (1.33)$$

The wave equation (1.33) represents a relation between the derivatives of sound pressure with respect to space and time. From experience follows that a local sound field disturbance propagates as a sound wave. It is postulated that the disturbance propagates with the speed of sound  $c$ . The one-dimensional sound field can be written as an arbitrary function with argument of form  $(x - ct)$  where  $x$  is the space coordinate and  $t$  is time. Insertion into the wave equation (1.33) yields for the speed of sound  $c$ :

$$1 = \frac{\rho}{\kappa P_0} c^2 \quad (1.34)$$

or

$$c = \sqrt{\kappa \frac{P_0}{\rho}} \quad (1.35)$$

It turns out that  $c$  is almost independent of pressure and density as these two quantities compensate each other to large extent in the term  $P_0/\rho$ . The speed of sound is almost identical on top of the Himalaya and at sea level. The impedance on the other hand is considerably lower at high altitudes which means that the sound pressure produced by a vibrating body is smaller.

With the speed of sound  $c$  the wave equation for the sound pressure  $p$  can be written as

$$\Delta p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (1.36)$$

where

$\Delta p$ : three dimensional Laplace operator.

For a cartesian coordinate system the Laplace operator is

$$\Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \quad (1.37)$$

For cylindrical coordinates the Laplace operator is given by

$$\Delta p = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \phi^2} + \frac{\partial^2 p}{\partial z^2} \quad (1.38)$$

The wave equation is the basis for the description of sound fields. The equation has to be fulfilled for each point in space. The specification of a concrete problem makes it necessary to indicate boundary conditions such as the velocity distribution of a vibrating surface or the acoustical impedance of field limiting areas. The solution for the sound field is found as the function that fulfills both the wave equation and the boundary conditions at the same time.

For the application of the wave equation one has to bear in mind that the equations used for its derivation were found by linearization of the fundamental physical equations. Strictly speaking the wave equation is no longer valid for high pressure or velocity values. An explicit application of the non linear behavior of air is the usage of modulated high frequency sound for public address systems. The high frequency waves that can be emitted focused to a narrow angle in space demodulate in the air and produce in this way the hearable audio signal<sup>1</sup>.

The speed of sound  $c$  and the density  $\rho_0$  of the air depend on temperature. As good approximation one can write

<sup>1</sup>F. Joseph Pompei, The Use of Airborne Ultrasonics for Generating Audible Sound Beams, Journal of the Audio Engineering Society, vol. 47, p. 726-731 (1999).

$$c \approx 343.2 \sqrt{\frac{T}{293}} \quad (1.39)$$

and

$$\rho_0 \approx \rho_{\text{ref}} \frac{P_a T_0}{P_0 T} \quad (1.40)$$

where

$T$ : temperature in Kelvin

$P_a$ : air pressure in Pascal [Pa]

$T_0$ : 293 K

$P_0$ : 101325 Pa

$\rho_{\text{ref}}$ : 1.186 kg/m<sup>3</sup>

A more accurate expression (1.42) for the speed of sound can be found by taking into account the parameters temperature, pressure, humidity and CO<sub>2</sub> concentration<sup>2</sup>. Besides the temperature influence there is a weak dependency on humidity (Fig. 1.9).

$$c(t, P, x_w, x_c) = a_0 + a_1 t + a_2 t^2 + (a_3 + a_4 t + a_5 t^2) x_w + (a_6 + a_7 t + a_8 t^2) P \\ + (a_9 + a_{10} t + a_{11} t^2) x_c + a_{12} x_w^2 + a_{13} P^2 + a_{14} x_c^2 + a_{15} x_w P x_c \quad (1.41)$$

where

$t$ : temperature in degrees Celsius

$P$ : air pressure in Pascal

$x_w$ : water vapour mole fraction, where  $x_w \approx (h/P)(1.00062 + 3.14 \times 10^{-8} P + 5.6 \times 10^{-7} t^2) \exp(1.2811805 \times 10^{-5} T^2 - 1.9509874 \times 10^{-2} T + 34.04926034 - 6.3536311 \times 10^3 T^{-1})$

$h$ : relative humidity as a fraction ( $0 < h < 1$ )

$T$ : temperature in Kelvin =  $t + 273.15$

$x_c$ : CO<sub>2</sub> mole fraction, typical value: = 0.000314

$a_0 = 331.5024, a_1 = 0.603055, a_2 = -0.000528, a_3 = 51.471935, a_4 = 0.1495874, a_5 = -0.000782, a_6 = -1.82 \times 10^{-7}, a_7 = 3.73 \times 10^{-8}, a_8 = -2.93 \times 10^{-10}, a_9 = -85.20931, a_{10} = -0.228525, a_{11} = 5.91 \times 10^{-5}, a_{12} = -2.835149, a_{13} = -2.15 \times 10^{-13}, a_{14} = 29.179762, a_{15} = 0.000486$

The formula is valid for  $t$  between 0 and 30° C, for  $P$  between 75'000 and 102'000 Pa and for  $x_w$  between 0 and 0.06.

## 1.4.2 Sinusoidal waves

Waves with sinusoidal time dependency play an important role for theoretical considerations. Such waves are characterized by their frequency  $f$  or their angular frequency  $\omega$  or their period length  $T$ .

$$f = \frac{1}{T} \quad (1.42)$$

$$\omega = 2\pi f \quad (1.43)$$

A fixed point on a sinusoidal wave train travels one wave length  $\lambda$  within the time  $T$  (Fig. 1.10). Therefore

$$\lambda = cT = \frac{c}{f} \quad (1.44)$$

Often usage of wave number  $k$  it is helpful where

$$k = \frac{2\pi}{\lambda} \quad (1.45)$$

<sup>2</sup>Owen Cramer, The variation of the specific heat ratio and the speed of sound in air with temperature, pressure, humidity, and CO<sub>2</sub> concentration, Journal of the Acoustical Society of America, vol. 93, p.2510-2516 (1993).



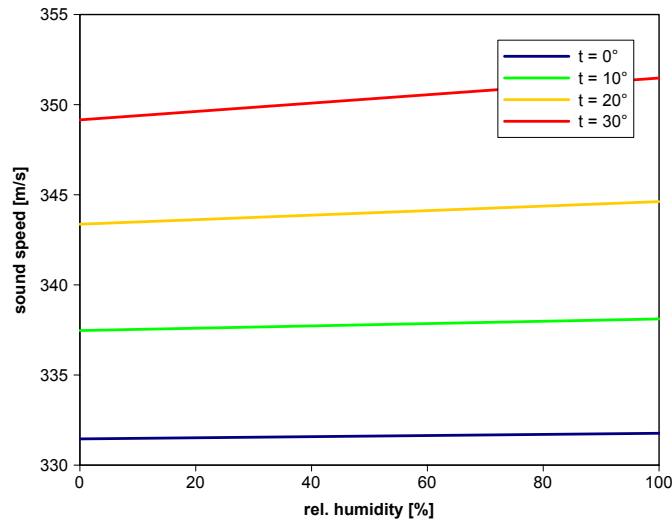


Figure 1.9: Speed of sound  $c$  for an air pressure of 1013 hPa as a function of air humidity with temperature as parameter.

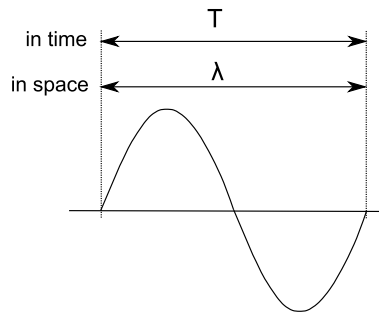


Figure 1.10: Sinusoidal wave with period length  $T$  in time and wave length  $\lambda$  in space.

### 1.4.3 Complex representation of sinusoidal quantities

Quantities with sinusoidal behavior may be represented as pointers in the complex plane. The pointer has a certain length - corresponding to the amplitude - and rotates according to angular frequency with constant angular velocity counter clockwise. The angle of the pointer at  $t = 0$  corresponds to the initial phase  $\phi$ . The pointer marks a complex number with an imaginary part that describes the sine function of the quantity (Fig. 1.11). The real part describes the corresponding cosine function. The quantity  $p$  with sinusoidal variation:

$$p(t) = \hat{p} \sin(\omega t + \phi) \quad (1.46)$$

is represented by the pointer  $\underline{p}$ :

$$\underline{p}(t) = \hat{p} e^{j(\omega t + \phi)} \quad (1.47)$$

Calculations with complex pointers are often easier to perform than dealing with sine and cosine functions.

### 1.4.4 Helmholtz equation

With the restriction to sinusoidal time dependencies, the wave equation simplifies to the Helmholtz equation. The sinusoidal excitation of a sound field (assumed to be linear) yields sinusoidal time dependencies for all field variables. It is therefore sufficient to indicate the amplitudes and phase relations in each field point.

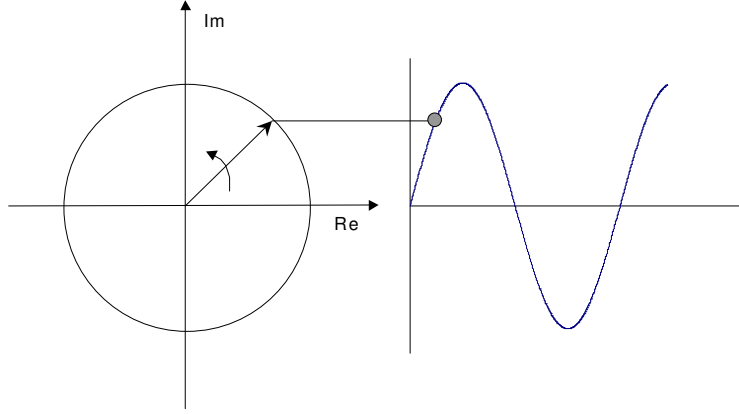


Figure 1.11: Representation of a quantity with sinusoidal dependency as imaginary part of a rotating pointer in the complex plane.

In complex writing the sound pressure  $\underline{p}$  can be written as product of a complex, location-dependent amplitude function  $\check{p}(\text{location})$  and an oscillation term  $e^{j\omega t}$  (Eq. 1.48).

$$\underline{p}(\text{location}, t) = \check{p}(\text{location})e^{j\omega t} \quad (1.48)$$

For the Laplace operator can be written

$$\Delta \underline{p} = \Delta \check{p}e^{j\omega t} \quad (1.49)$$

and

$$\frac{\partial^2 \underline{p}}{\partial t^2} = -\omega^2 \check{p}(\text{location})e^{j\omega t} \quad (1.50)$$

Insertion of (1.49) and (1.50) in (1.36) yields the Helmholtz equation (1.51):

$$\Delta \check{p} + \frac{\omega^2}{c^2} \check{p} = 0 \quad (1.51)$$

The complex amplitude function  $\check{p}$  is only a function of the position in space.

## 1.5 Solutions of the wave equation

### 1.5.1 Plane waves

A plane wave is the simplest wave type. The sound field variables  $p$  and  $\vec{v}$  are both in phase and depend only on one space coordinate. For propagation in the  $x$ -direction, all points in the  $y, z$  plane have identical values of  $p$  and  $\vec{v}$ . Most relevant for plane waves is the fact that there is no geometrical divergence. Plane waves occur e.g. in tubes with a diameter that is much smaller than the wave length. Far away from sources of limited size, the waves can usually be approximated as plane waves with good accuracy.

The solutions of the one-dimensional wave equation

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \quad (1.52)$$

represent the set of possible sound pressure dependencies. All functions  $p(x, t)$  that fulfill Eq. 1.52 have the form

$$p(x, t) = f(ct \pm x) \quad (1.53)$$

In the above equation  $f$  stands for an arbitrary function. The one dimensional wave equation is thus fulfilled if the argument of  $f$  has the form  $ct \pm x$ . A certain value of the argument can be obtained by

adapting the time or the space variable - time and space are thus exchangeable. The minus sign in the argument stands for a wave propagating in positive  $x$  direction (to the right), a plus sign is a wave in the negative  $x$  direction (to the left).

It is often convenient to describe the arbitrary function  $f$  as the superposition of sine waves according to the theorem of Fourier. It is usually sufficient to solve a certain problem by investigating the behavior for a sine wave of arbitrary frequency. In complex representation according to 1.48 we can write for the sound pressure  $p$ :

$$\underline{p}(x, t) = \hat{p}e^{j(-kx+\phi)}e^{j\omega t} \quad (1.54)$$

where

$\hat{p}$ : amplitude of the sine oscillation

$\phi$ : constant phase term

The sound particle velocity can be determined from sound pressure with Eq. (1.12). The plane wave in the  $x$  direction causes the air particles to move back and forth in the  $x$  direction. There is no movement in the  $y$  and the  $z$  direction. The sound particle velocity in  $x$  is in complex notation

$$\underline{v}_x(x, t) = \check{v}_xe^{j\omega t} \quad (1.55)$$

where

$\check{v}_x$ : complex, location dependent amplitude function.

Inserting (1.54) and (1.55) in (1.12) yields

$$\hat{p}jk e^{j(-kx+\phi)}e^{j\omega t} = \check{v}_x\rho j\omega e^{j\omega t} \quad (1.56)$$

With  $\omega = kc$  one gets

$$\check{v}_xe^{j\omega t} = \frac{1}{\rho c}\hat{p}e^{j(-kx+\phi)}e^{j\omega t} \quad (1.57)$$

or

$$\underline{v}_x(x, t) = \frac{1}{\rho c}\underline{p}(x, t) \quad (1.58)$$

In a plane wave sound pressure and sound particle velocity are in phase and the ratio of their amplitudes (corresponding to the impedance  $Z_0$ ) is

$$Z_0 = \rho c \quad (1.59)$$

## 1.5.2 Spherical waves

Spherical waves can be thought of emitted by a point source. They propagate spherically in all directions. The two dimensional analogue are water waves that occur as the results of a local distortion, for example a stone falling into the water. Due to symmetry reasons the sound pressure and the amplitude of the sound particle velocity have to be constant on a spherical surface with arbitrary radius and a center that coincides with the source point. The vector of the sound particle velocity points in radial direction outwards.

As a guess for the solution of the sound pressure in spherical waves the approach for plane waves is assumed and complemented with a  $1/r$  ( $r$ : Radius) amplitude dependency (1.60).

$$\underline{p}(r, t) = \frac{1}{r}\hat{p}e^{j(-kr+\phi)}e^{j\omega t} \quad (1.60)$$

The validity of Eq. 1.60 can be proved with help of the Helmholtz equation. In spherical coordinates the equation for a sound field variable that depends on the radius only is

$$\frac{\partial^2 \check{p}}{\partial r^2} + \frac{2}{r} \frac{\partial \check{p}}{\partial r} + k^2 \check{p} = 0 \quad (1.61)$$

In analogy to plane waves the sound particle velocity can be deduced from sound pressure with help of Eq. 1.12. The radial component is found as

$$v_r(r, t) = \underline{p}(r, t) \left( \frac{1}{\rho c} + \frac{1}{j\omega\rho r} \right) \quad (1.62)$$

For the impedance  $Z_K$  of spherical waves follows

$$Z_K = \rho c \frac{jkr}{1 + jkr} \quad (1.63)$$

$Z_K$  depends on frequency and distance. For large distances (compared to the wave number  $k$ )  $Z_K$  approaches the value of plane waves. If the distance gets small and smaller both real and imaginary part of  $Z_K$  drop off (Fig. 1.12).

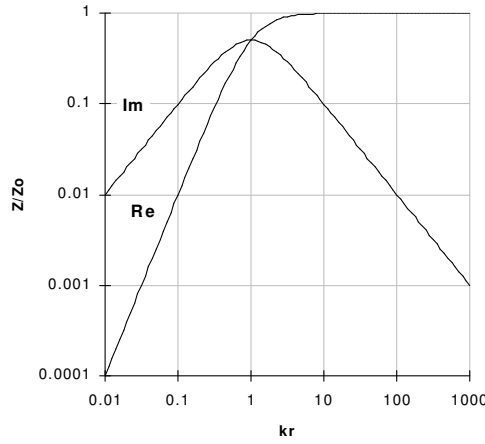


Figure 1.12: Real and imaginary part of the impedance for spherical waves. The abscissa is scaled as product  $kr$  with  $k$ : wave number  $= 2\pi/\lambda$  and  $r$ : distance. The ordinate shows the impedance relative to the value for plane waves  $Z_0 = \rho c$ .

The model for an ideal source that emits spherical waves is a small (relative to the wave length) pulsating sphere. A sphere with radius  $r_0$  and surface velocity  $v_r$  in radial direction produces a volume velocity  $Q$  of

$$Q = 4\pi r_0^2 v_r \quad (1.64)$$

With Eq. 1.63 and under the assumption  $kr_0 \ll 1$ , the sound pressure on the surface of the sphere is found as

$$\check{p}(r_0) = \check{v}_r(r_0) \rho c jkr_0 \quad (1.65)$$

Following Eq. 1.60 the spherical wave approach can be rewritten as follows with the stipulation that the phase is now referred to the surface of the sphere:

$$\underline{p}(r, t) = \frac{1}{r} \hat{p} e^{j(-k(r-r_0)+\phi)} e^{j\omega t} \quad (1.66)$$

Comparison of Eq. 1.65 with 1.66 yields

$$\frac{1}{r_0} \hat{p} e^{j(\phi)} = \check{v}_r(r_0) \rho c jkr_0 \quad (1.67)$$

or

$$\hat{p} e^{j(\phi)} = \frac{Q}{4\pi} \rho c jk \quad (1.68)$$

Amplitude and phase can be found as:

$$\begin{aligned}\hat{p} &= \frac{Q}{4\pi} \rho c k \\ \phi &= \frac{\pi}{2}\end{aligned}\tag{1.69}$$

Considering the fact that  $r_0$  is very small compared to the wave length, the difference  $r - r_0$  in Eq. 1.66 can be approximated as  $r$ . With this the sound pressure  $\check{p}(r)$  at distance  $r$  from a point source with volume velocity  $Q$  is found as

$$\check{p}(r) = \frac{jk\rho c Q e^{-jkr}}{4\pi r}\tag{1.70}$$

### 1.5.3 Cylindrical waves

Sound pressure, sound particle velocity and impedance for cylindrical waves can be determined analogously to the case of spherical waves. The sound pressure dependency with distance  $r$  results as

$$p_{\text{cyl.}} \sim \frac{1}{\sqrt{r}}\tag{1.71}$$

Similarly to spherical waves, cylindrical waves show a near field and a far field. However, the transition is at  $kr \approx 1$  in contrast to  $kr \approx 2$  for spherical waves.

## 1.6 Sound pressure and sound power for point sources

By definition, ideal point sources radiate sound equally in all directions. If the sound pressure  $p(r)$  at distance  $r$  in the far field (not too close to the source) is known, the sound power  $W$  of the source can be found as follows:

The impedance  $Z$  in the far field is

$$Z = \frac{p(r)}{v(r)} = \rho c\tag{1.72}$$

The intensity  $I_{\text{rms}}(r)$  at distance  $r$  from the source is

$$I_{\text{rms}}(r) = p_{\text{rms}}(r)v_{\text{rms}}(r) = \frac{p_{\text{rms}}^2(r)}{\rho c}\tag{1.73}$$

The totally emitted sound power can be found by integration of the intensity (1.73) over a closed surface  $S$  that encloses the source.

$$W = \int_S \vec{I} d\vec{S}\tag{1.74}$$

Most naturally,  $S$  is chosen as surface of a sphere with center at the position of the source. In this case the intensity is constant over  $S$  and the integration yields:

$$W = I_{\text{rms}}(r)4\pi r^2 = \frac{p_{\text{rms}}^2(r)}{\rho c}4\pi r^2\tag{1.75}$$

## 1.7 Superposition of point sources

Here the sound pressure at a receiver position is investigated in the case of several active sound sources. As long as the amplitudes of the sound field variables are not too large (linear case) the superposition principle holds for sound pressure and sound particle velocity. This implies that the sound pressure at a receiver corresponds to the sum of the sound pressures resulting for each single source.

Two cases have to be distinguished:

In the first case the sources radiate coherently, that is to say there is a fixed phase relation between the sources. Here the resulting pressure equals the phase sensitive addition of the pressure contributions of each source. It is most beneficial to perform this addition using the complex representation of sound pressure.

In the second case the sources radiate incoherent signals, that is to say it is impossible to conclude from the time signal of one source to the time signal of any other source. In this case the superposition simplifies in the sense that intensities can be summed up. The resulting mean squared pressure equals the sum of the squared pressure contributions of each source.

### 1.7.1 Superposition of incoherently radiating point sources

#### Incoherently radiating point sources distributed along a straight line

An infinite row of equally distributed point sources along a straight line is considered (Fig. 1.13).

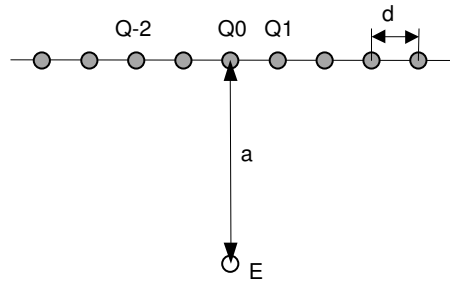


Figure 1.13: Situation of an infinite row of incoherently radiating point sources  $Q_{-\infty} \dots Q_{+\infty}$ . The receiver  $E$  is located in distance  $a$  from the line.

The mean squared pressure  $p_{rms,n}^2$  at the receiver  $E$  caused by source  $n$  is

$$p_{rms,n}^2 = \frac{K}{a^2 + (nd)^2} \quad (1.76)$$

where

$K$ : constant to describe the source strength

The superposition of all sources yields

$$p_{rms,tot}^2 = \sum_{n=-\infty}^{+\infty} p_{rms,n}^2 = K \sum_{n=-\infty}^{+\infty} \frac{1}{a^2 + (nd)^2} = K \frac{1}{d^2} \sum_{n=-\infty}^{+\infty} \frac{1}{\frac{a^2}{d^2} + n^2} \quad (1.77)$$

From symmetry follows that the sum from  $-\infty$  to  $+\infty$  in Eq. 1.77 can be written as two times the sum from 1 to  $+\infty$  and a correction for the term for  $n = 0$ .

A good formulary tells us that

$$\coth x = \frac{1}{x} + \frac{2x}{\pi^2} \sum_{n=1}^{+\infty} \frac{1}{\frac{x^2}{\pi^2} + n^2} \quad (1.78)$$

With substitution of  $\frac{x}{\pi} = \frac{a}{d}$ , Eq. 1.78 can be rewritten as

$$\sum_{n=1}^{+\infty} \frac{1}{\frac{a^2}{d^2} + n^2} = \frac{\pi d}{2a} \coth\left(\frac{\pi a}{d}\right) - \frac{d^2}{2a^2} \quad (1.79)$$

Finally Eq. 1.77 can be written as

$$p_{rms,tot}^2 = \frac{K}{d^2} \frac{\pi d}{a} \coth\left(\frac{\pi a}{d}\right) = \frac{K\pi}{ad} \coth\left(\frac{\pi a}{d}\right) \quad (1.80)$$

For the discussion of Eq. 1.80, two cases have to be distinguished:

$\frac{\pi a}{d}$  **small (small distances)** in this case the approximation holds:  $\coth\left(\frac{\pi a}{d}\right) \approx \frac{d}{\pi a}$ . It follows for  $p_{\text{rms,tot}}^2 \approx \frac{K}{a^2}$  or  $p_{\text{rms,tot}} \approx \frac{\sqrt{K}}{a}$ . The dependency of  $p(a)$  with distance corresponds to  $1/a$ , just as for a point source. In the proximity of the row of point sources the pressure at the receiver is dominated just by the source that is nearest.

$\frac{\pi a}{d}$  **large (large distances)** in this case the approximation holds:  $\coth\left(\frac{\pi a}{d}\right) \approx 1$ . It follows for  $p_{\text{rms,tot}}^2 \approx \frac{K\pi}{ad}$  or  $p_{\text{rms,tot}} \approx \sqrt{\frac{K\pi}{d}} \frac{1}{\sqrt{a}}$ . The dependency of  $p(a)$  with distance corresponds to  $1/\sqrt{a}$ , just as for a line source.

The transition between the two distance regimes can be localized where the two approximations yield the same result:

$$a = \frac{d}{\pi} \quad (1.81)$$

### Incoherent radiating point sources distributed along a line of finite length

If the row of point sources has limited length, there is for large distances a transition from the line source behavior to a point source behavior. The mathematical proof is easiest if the separation between the point sources tends to 0. The summation corresponds then to an integration over a distinct range. The final result is a transition distance  $a = L/\pi$  if  $L$  is the length of the point source row.

### Incoherent radiating point sources distributed over an area of finite size

The distance dependency of an incoherent radiating rectangular area of length  $L$  and width  $B$  ( $L > B$ ) can be described by three regions according to Table 1.2.

$a < B/\pi$	plane wave behavior (sound pressure independent of distance)
$B/\pi < a < L/\pi$	line source behavior
$L/\pi < a$	point source behavior

Table 1.2: Distance dependency of sound pressure for incoherent radiating rectangular areas.  $a$  depicts the distance,  $L$  is the length and  $B$  is the width of the radiating area.

## 1.7.2 Superposition of coherently radiating point sources

### Dipole radiator

The dipole radiator consists of two coherently radiating point sources of equal amplitudes but opposite phase. The sound pressure at a receiver point is given as the phase sensitive addition of the contributions of the two point sources (Fig. 1.14).

With (1.60) the sound pressure in E can be written as:

$$\underline{p}(r, t) = \hat{p} \left( \frac{1}{r_1} e^{-jkr_1} - \frac{1}{r_2} e^{-jkr_2} \right) e^{j\omega t} \quad (1.82)$$

At low frequencies and in the far field, that is to say for  $\Delta r \ll r$  and  $k\Delta r \ll 1$ ,  $r_1$  and  $r_2$  can be approximated as:

$$r_1 \approx r - \frac{\Delta r}{2} \quad (1.83)$$

$$r_2 \approx r + \frac{\Delta r}{2} \quad (1.84)$$

With this follows

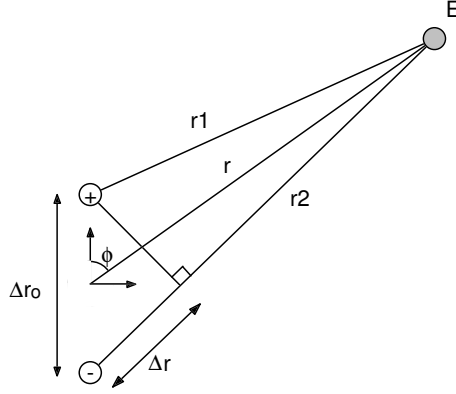


Figure 1.14: Geometry of a dipole with the receiver E.

$$\begin{aligned} \underline{p}(r, t) &\approx \hat{p} \frac{e^{-jkr}}{r} \left( \frac{e^{jk\Delta r/2}}{1 - \frac{\Delta r}{2r}} - \frac{e^{-jk\Delta r/2}}{1 + \frac{\Delta r}{2r}} \right) e^{j\omega t} \quad (1.85) \\ &= \hat{p} \frac{e^{-jkr}}{r} \left( \frac{\cos(k\Delta r/2) + j \sin(k\Delta r/2)}{1 - \frac{\Delta r}{2r}} - \frac{\cos(-k\Delta r/2) + j \sin(-k\Delta r/2)}{1 + \frac{\Delta r}{2r}} \right) e^{j\omega t} \end{aligned}$$

Making use of approximations for small arguments ( $\cos \epsilon \approx 1$  and  $\sin \epsilon \approx \epsilon$ ) yields

$$\begin{aligned} \underline{p}(r, t) &\approx \hat{p} \frac{e^{-jkr}}{r} \left( \frac{1 + jk\Delta r/2}{1 - \frac{\Delta r}{2r}} - \frac{1 - jk\Delta r/2}{1 + \frac{\Delta r}{2r}} \right) e^{j\omega t} \quad (1.86) \\ &= \hat{p} \frac{e^{-jkr}}{r} \left( \frac{(1 + jk\frac{\Delta r}{2})(1 + \frac{\Delta r}{2r}) - (1 - jk\frac{\Delta r}{2})(1 - \frac{\Delta r}{2r})}{1 - (\frac{\Delta r}{2r})^2} \right) e^{j\omega t} \end{aligned}$$

Under the far field assumption, the denominator in the brackets on the right hand side can be approximated as 1:

$$\underline{p}(r, t) \approx \hat{p} \frac{e^{-jkr}}{r} \left( \frac{\Delta r}{r} (r + jkr) \right) e^{j\omega t} \quad (1.87)$$

With  $\Delta r \approx \Delta r_0 \cos \phi$  follows:

$$\underline{p}(r, t) \approx \hat{p} \frac{\Delta r_0 \cos \phi}{r^2} (r + jkr) e^{-jkr} e^{j\omega t} \quad (1.88)$$

For  $k \gg 1$  that is to say  $f \gg 50$  Hz Eq. 1.88 simplifies to

$$\underline{p}(r, t) \approx \hat{p} \frac{jk\Delta r_0 \cos \phi}{r} e^{-jkr} e^{j\omega t} \quad (1.89)$$

It should be noted that the amplitude term in (1.89) is proportional to  $k$  and therefore to frequency. The dipole radiation is very inefficient at low frequencies.

## 1.8 Reflection of sound waves at acoustically hard surfaces

### 1.8.1 Specular reflection

The presence of an acoustically hard surface implements a boundary condition for the normal component of the sound particle velocity with

$$\underline{v}_n = 0 \quad (1.90)$$



An elegant concept to deal with such a boundary condition is the introduction of one or more additional equivalent sources. These sources are set and adjusted in order for their superposition with the original source to satisfy the boundary condition. With this in mind the reflection of sound waves at a large, acoustically hard surface can be treated with the concept of a mirror source. The corresponding additional source is placed at the mirrored position of the original source. The effect of the reflector can then be replaced by the contribution of this additional source. The mirror source emits the same signal as the original source (Fig. 1.15).

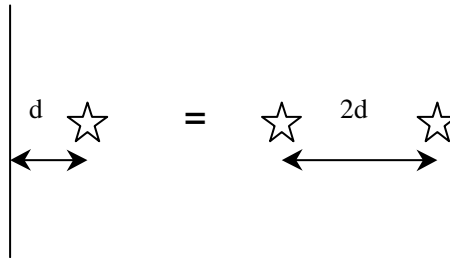


Figure 1.15: Replacement of a reflecting surface by a mirror source.

### 1.8.2 Source directivity for limited radiation angles

Sources located close to acoustically hard surfaces no longer radiate in all directions. For a broadband source the limitation of radiation to a solid angle  $\Phi$  results in an amplification corresponding to the ratio  $4\pi/\Phi$ . For example, a source next to a corner appears with an sound power amplified by a factor 8.

### 1.8.3 Diffuse reflection

The directivity of diffuse reflections of sound waves is often described by Lambert's law, originally developed in Optics. It assumes that the reflection intensity  $I(\phi)$  in direction  $\phi$  is independent of the incident direction and proportional to the cosine of  $\phi$  (1.91) where  $\phi$  is understood relative to the normal direction (Fig. 1.16).

$$I(\phi) = I_0 \cos \phi \quad (1.91)$$

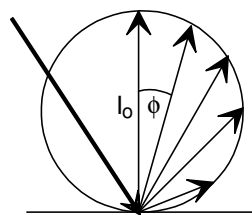


Figure 1.16:  $\cos \phi$  dependency of the intensity of a diffuse reflection assuming Lambert's law.

## 1.9 Doppler effect

In case of moving sources or moving receivers (relative to each other) a frequency shift occurs. This effect is named after Ch. Doppler (1803-1852, Vienna) who discovered and explained the phenomenon. The effect is omnipresent in daily life, for example in connection with passing cars. The Doppler effect plays an important role in sound radiation by loudspeakers that consists of only one membrane.

The mathematical discussion shall be based on the situation in Fig. 1.17. A point source  $Q$  is in  $x = 0$  at time  $t = 0$ .  $Q$  moves in positive  $x$  direction with speed  $v_Q$ . It is assumed that  $Q$  emits a pure tone

of frequency  $f_0$ . We are looking for the frequency  $f$  that is registered at a receiver point  $E$  at distance  $d$  under an angle  $\phi$  relative to the  $x$  direction.

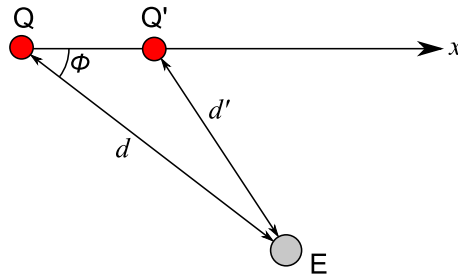


Figure 1.17: Situation to investigate the Doppler frequency shift.

The frequency  $f$  is determined by evaluation of the time interval  $T$  between two sound pressure maxima. A sound pressure maximum emitted at position  $Q$  reaches the receiver at time  $t = d/c$ . The next sound pressure maximum is emitted at position  $Q'$  at time  $t = T_0 = 1/f_0$ . Consequently this maximum reaches the receiver at time  $t = T_0 + d'/c$ . With this the time interval between two maxima at the receiver is

$$T = T_0 + \frac{d'}{c} - \frac{d}{c} \quad (1.92)$$

For the frequency  $f$  at the receiver position follows:

$$f = \frac{1}{\frac{1}{f_0} - \frac{d-d'}{c}} \quad (1.93)$$

where  $d'$  is found as

$$d' = \sqrt{d^2 - 2dv_Q T_0 \cos \phi + v_Q^2 T_0^2} \quad (1.94)$$

If the receiver is located on the  $x$ -axis ( $\phi = 0$ ), Eq. 1.94 simplifies to

$$d' = d - v_Q T_0 \quad (1.95)$$

and Eq. 1.93 becomes

$$f = f_0 \frac{c}{c - v_Q} \quad (1.96)$$

## 1.10 Sonic boom

Sources that move faster than the speed of sound produce a sonic boom. Typical examples are air planes, projectiles or the end of a whipcord in action. Fig. 1.18 shows the development of such a boom (Mach's cone). At time 0 the source is in position  $Q_0$ . After time  $t$  the source has reached position  $Q_3$ . After time  $t$  the wave front emitted in  $Q_0$  corresponds to a sphere of radius  $ct$ . The wave fronts emitted from source positions between  $Q_0$  and  $Q_3$  are correspondingly smaller spheres. The envelope of all wave fronts forms a cone with very high sound pressure. The opening angle  $\alpha$  of the cone is

$$\sin \alpha = \frac{c}{v} \quad (1.97)$$

The tip of the cone moves with the source. At the moment where the cone reaches the receiver, a sharp bang is heard.

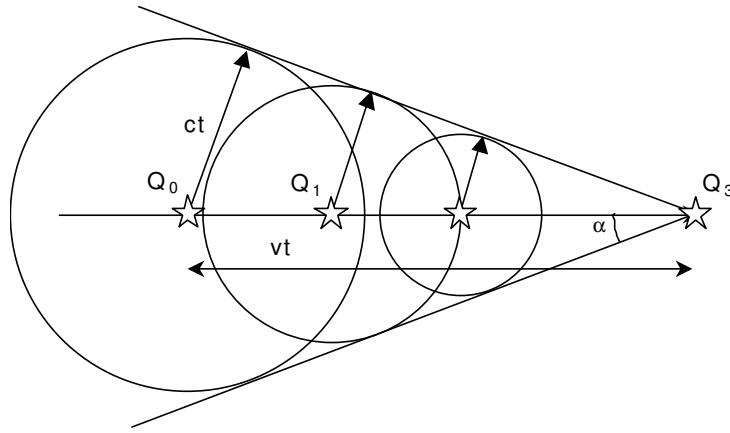


Figure 1.18: Development of Mach's cone for a source moving faster than the speed of sound.

## 1.11 dB - scale

### 1.11.1 Quantities expressed as levels

An important characterization of the behavior of a system is the ratio of the power at the output  $y$  and the power at the input  $x$ . Instead of indicating this ratio linearly, often the logarithm of base 10 is used. The corresponding unit is [Bel].

$$\log_{10} \left( \frac{\text{power}_Y}{\text{power}_X} \right) \quad [\text{Bel}] \quad (1.98)$$

The Bel scale is very coarse. It is often more appropriate to introduce a factor of 10 yielding tenth of a Bel or decibel [dB].

$$10 \log_{10} \left( \frac{\text{power}_Y}{\text{power}_X} \right) \quad [\text{dB}] \quad (1.99)$$

It is very common to express the acoustic quantities such as sound pressure, sound intensity and sound power in the decibel scale. If doing so the quantities get the name *level* as an appendix (sound pressure level, ...). One of the reasons to use the dB scale is the fact that the sensation of the human ear follows basically a logarithmic law. To express sound field variables as levels, they have to be converted to power proportional quantities (if necessary) and related to reference values as follows:

#### Sound pressure level $L_p$

$p_0 = 2 \times 10^{-5} \text{Pa}$  is chosen as sound pressure reference value. This corresponds to the threshold of hearing at 1 kHz.

$$L_p = 10 \log_{10} \left( \left( \frac{p_{\text{rms}}}{p_0} \right)^2 \right) \quad [\text{dB}] \quad (1.100)$$

#### Sound intensity level $L_I$

The reference value for sound intensity is  $I_0 = 10^{-12} \text{W/m}^2$ .

$$L_I = 10 \log_{10} \left( \frac{I}{I_0} \right) \quad [\text{dB}] \quad (1.101)$$

#### Sound power level $L_W$

The reference value for sound power is  $W_0 = 10^{-12} \text{W}$ .

$$L_W = 10 \log_{10} \left( \frac{W}{W_0} \right) \quad [\text{dB}] \quad (1.102)$$

The above reference values are chosen in such a way that for a plane wave the sound pressure level and the sound intensity level match within 0.1 dB. For a point source follows from (1.75), (1.100) and (1.102) that sound power level and sound pressure level are identical in a distance of approximately 0.3 m

### 1.11.2 Consequences of the dB scale

Using the dB scale signifies that the range of hearing is transformed to sound pressure levels between 0 and 120 dB. A constant dB step corresponds to a constant variation in sensation. Furthermore, a multiplication of physical quantities becomes a simple summation in the dB domain.

### 1.11.3 Subtlety of the dB scale

The question of the relevance of a change of x dB can be answered for example with help of the human auditory sensation according to Table 1.3.

change in sound pressure level	sensation
< 2 dB	not audible
2...4 dB	just audible
5...10 dB	clearly audible
> 10 dB	very convincing

Table 1.3: Sensation for changes in sound pressure level for the human hearing.

An other hint regarding the relevance of level differences can be derived from tolerances of modern sound level meters. The overall uncertainty of such devices in the order of 1 dB.

### 1.11.4 Computations involving dB quantities

Special care is needed when calculations involve dB quantities. The addition of dB values corresponds to a multiplication of the physical quantities. Very often a summation of physical quantities is needed. In this case the dB values firstly have to be converted back to linear quantities before the operation can be applied. It has to be considered, what quantities add up. In case of coherent signals the summation goes for sound pressure, in case of incoherent contributions the corresponding sound pressure square values have to be summed up. Often the result is then again converted and expressed as a level.

### 1.11.5 Typical values of sound pressure levels

Table 1.4 indicates typical sound pressure levels at a certain distance for different sound sources.

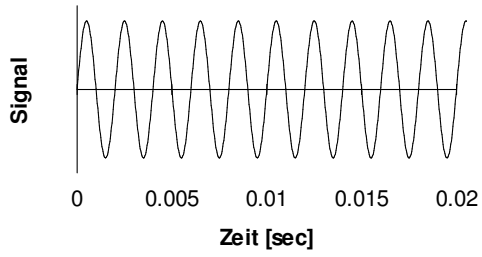
sound source	sound pressure level
tick of an alarm clock in 0.5 m	30 dB
human voice in 2 m	60 dB
road traffic in 10 m (1000 vehicles/h, 80 km/h)	70 dB
jet air plane in 100 m	120 dB

Table 1.4: Some typical sound pressure level values.

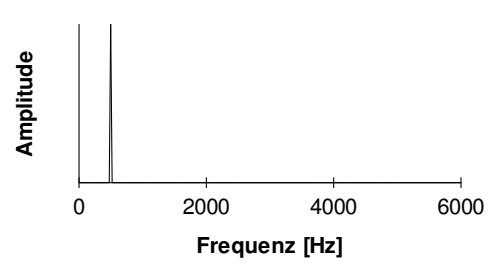
## 1.12 Classification of acoustical signals

Acoustical signals can be subdivided into few fundamental types. In daily life they almost never occur in pure form, but often one or the other fundamental type can be identified as predominant. The following figures show the basic signal types (on the left: time dependency, on the right: spectrum).

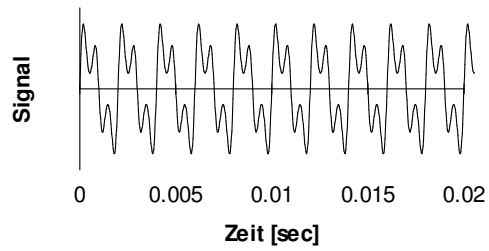
*pure tone (Reinton): time dependency*



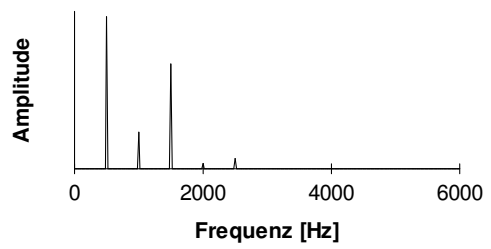
*pure tone: spectrum*



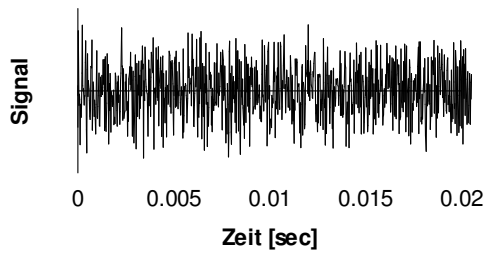
*complex tonal sound (Klang): time dependency*



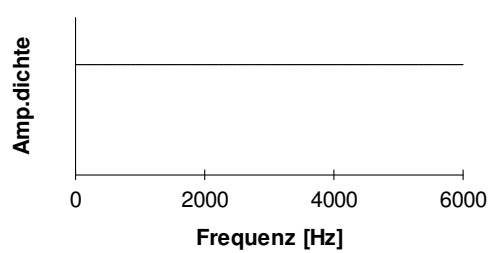
*complex tonal sound: spectrum*



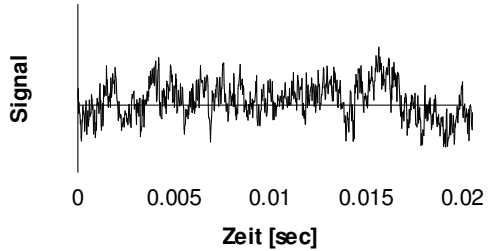
*white noise (weisses Rauschen): time dependency*



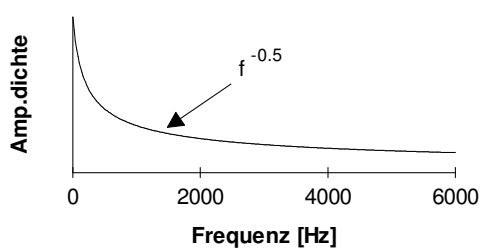
*white noise: spectrum*



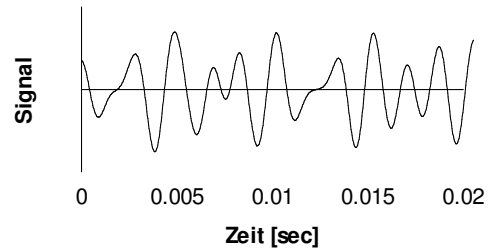
*pink noise (rosa Rauschen): time dependency*



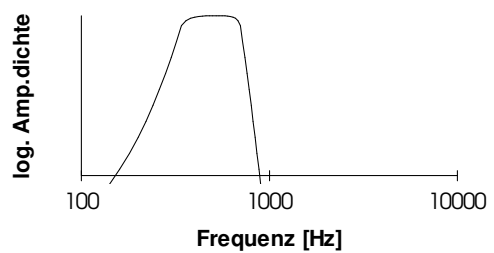
*pink noise: spectrum*



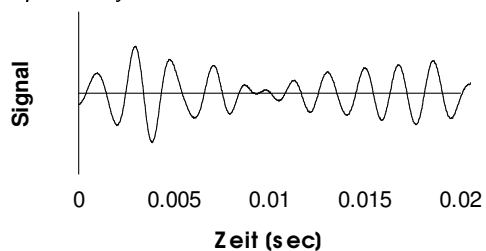
*500 Hz octave band filtered noise: time dependency*



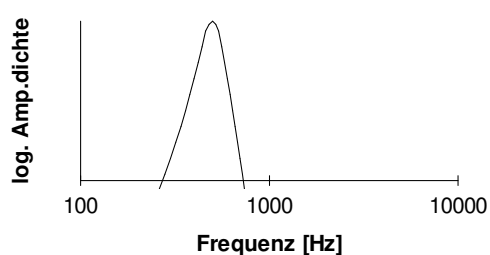
*500 Hz octave band filtered noise: spectrum*



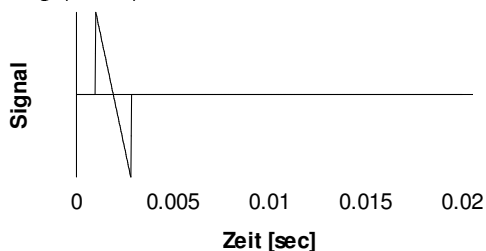
500 Hz third octave band filtered noise: time dependency



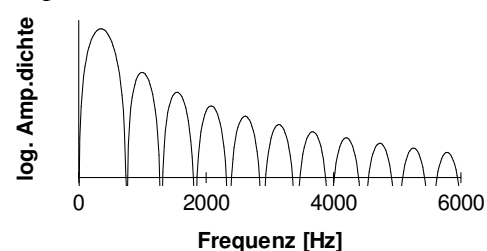
500 Hz third octave band filtered noise: spectrum



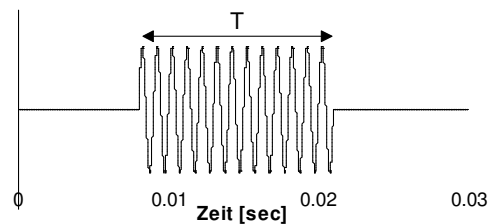
bang (Knall): time dependency



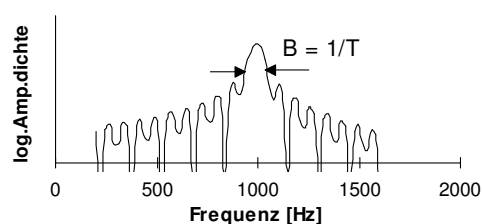
bang: spectrum



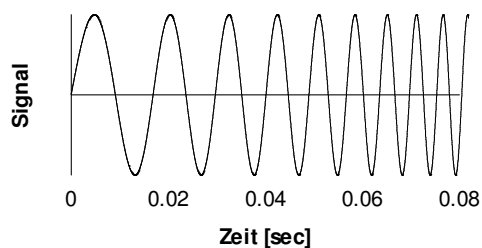
tone burst: time dependency



tone burst: spectrum



sweep: time dependency



sweep: The spectrum depends on the time dependency of the frequency variation.

## 1.13 Standing waves

### 1.13.1 Superposition of waves traveling in opposite directions

The superposition of two sine waves of equal frequency and amplitude but opposite directions results in a standing wave. For a mathematical investigation the two waves are introduced in complex writing:

$$\underline{p}_1(x, t) = \hat{p} e^{j(\omega t - kx)} \quad (1.103)$$

$$\underline{p}_2(x, t) = \hat{p}e^{j(\omega t + kx)} \quad (1.104)$$

The sum yields:

$$\begin{aligned} \underline{p}_{\text{tot}}(x, t) &= \underline{p}_1(x, t) + \underline{p}_2(x, t) = \hat{p}e^{j\omega t} (e^{-jkx} + e^{jkx}) \\ &= \hat{p}e^{j\omega t} (\cos(-kx) + j \sin(-kx) + \cos(kx) + j \sin(kx)) = \hat{p}e^{j\omega t} 2 \cos(kx) \end{aligned} \quad (1.105)$$

The superposition of the two waves is no longer a propagating wave but a harmonic oscillation that is modulated in space with  $\cos(kx)$ . As a consequence at certain locations maxima and at other locations minima arise.

### 1.13.2 Quarter wave length resonators

An example for the application of standing waves is the quarter wave length ( $\lambda/4$ ) resonator. It consists of a tube with an acoustically hard termination at one end. At the end with the hard termination the sound waves are perfectly reflected. If the sound wave length equals four times the length of the tube, a sound pressure minimum results at the open end of the tube. This is in conflict with sound pressure of the excitation outside the tube. The tube has to react with a high amplification resulting in high pressures at the terminated end. If a pressure microphone is placed at this position its sensitivity can easily be increased by more than 20 dB for the resonance frequencies  $f_{\text{res},i}$  according to Eq. 1.106.

$$f_{\text{res},i} = \frac{2i - 1}{4} \frac{c}{L} \quad (1.106)$$

where

$i$ : 1,2,3,...

$c$ : speed of sound

$L$ : length of the tube

Fig. 1.19 shows an example of a measured frequency response for a microphone placed at the end of such a tube.

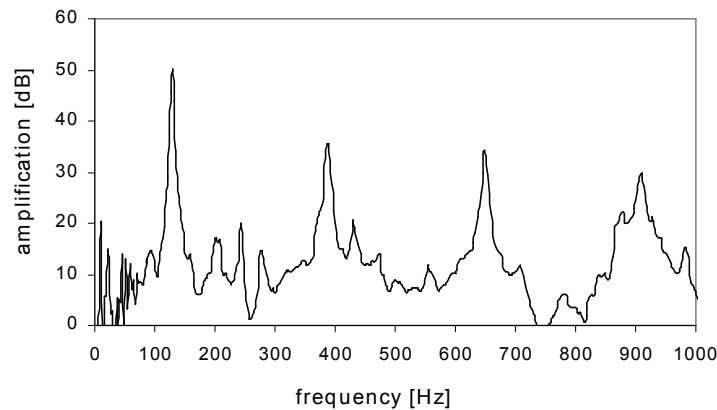


Figure 1.19: Measured frequency response at the closed end of a tube of length 66 cm relative to a microphone placement in free field. In the experiment the distance between the source and the microphone was 130 cm. At the resonance frequencies (129 Hz, 387 Hz, ...) the tube produces very high amplifications. Besides the resonances a small amplification of typically 6 dB can be observed due to the fact that the tube acts as a sonde microphone. Indeed the sound field is observed at the location of the open end of the tube which was approximately in half the distance compared to the free field reference measurement.

## 1.14 Sound field calculations

Generally sound field calculations are seeking for space-time dependencies of the sound field variables of interest. These solutions have to fulfill the wave equation and the situation specific boundary conditions. The boundary conditions are defined by the source and the presence of possible surfaces with their corresponding impedance. Often the solutions are not searched in the time domain. In many cases it is easier to handle the problem in the frequency domain. To do so the task is formulated for an arbitrary frequency. In this case the Helmholtz equation (1.51) can be used instead of the wave equation. Analytical solutions for sound fields can only be found for special situations. In the general case, approximations or numerical solutions based on strategies such as Finite Elements or Boundary Elements have to be applied.

### 1.14.1 General problem of reflection

The general problem of reflection cares about the resulting sound field if a sound wave is reflected at an object. If the sound wave length is larger than the dimensions of the object the reflection process is usually called scattering.

In the mathematical description the reflecting or scattering object introduces a boundary condition on the object surface. This is easily done for so called locally reacting surfaces which can be described by a surface impedance ( $p/v_n$ ). Many materials can be handled as a locally reacting surfaces. On the other hand there are structures that behave as extended reacting surfaces with a relevant amount of sound propagation in the material itself. An example for a medium with extended reaction is ballast that is used in the superstructure of railway lines. Here we will restrict the discussion to locally reacting surfaces.

In the most simple case of an acoustically hard surface the boundary condition simplifies to

$$v_n = 0 \text{ or } \frac{\partial p}{\partial n} = 0 \quad (1.107)$$

where

$v_n$ : sound particle velocity component perpendicular to surface

$\partial p/\partial n$ : partial derivative of the sound pressure in direction perpendicular to the surface

Outside the reflecting object the resulting sound field has to fulfill the wave equation. In the view of the frequency domain, the corresponding condition is the Helmholtz equation. It is a good idea to split up the resulting sound pressure field  $\check{p}$  in an *incident*  $\check{p}_e$  and a *reflected* (or *scattered*)  $\check{p}_s$  component:

$$\check{p} = \check{p}_e + \check{p}_s \quad (1.108)$$

Usually the incident wave  $\check{p}_e$  is known and the problem lies in the determination of the reflected component  $\check{p}_s$ . The Helmholtz equation has to be fulfilled for the total sound field  $\check{p}$ . With the two parts  $\check{p}_e$  and  $\check{p}_s$  the Helmholtz equation becomes:

$$\Delta(\check{p}_e + \check{p}_s) + k^2(\check{p}_e + \check{p}_s) = 0 \quad (1.109)$$

Eq. 1.109 can be rewritten as

$$\Delta\check{p}_e + k^2\check{p}_e + \Delta\check{p}_s + k^2\check{p}_s = 0 \quad (1.110)$$

The Helmholtz equation has also to be fulfilled for the incident wave  $\check{p}_e$  alone. Therefore

$$\Delta\check{p}_s + k^2\check{p}_s = 0 \quad (1.111)$$

It follows that the reflected wave has to fulfill the Helmholtz equation as well. In addition, the reflected wave has to ensure that the boundary condition at the surface of the object is fulfilled. In case of an acoustically hard object this implies that  $\check{v}_n = 0$ . With  $\check{v}_{n,e}$  as the normal component of the sound particle velocity of the incident wave and  $\check{v}_{n,s}$  as the normal component of the sound particle velocity of the reflected wave follows  $\check{v}_{n,s} = -\check{v}_{n,e}$ . This condition can be formulated for the sound pressure with help of Eq. 1.12:

$$\frac{\partial \check{p}_s}{\partial n} = j\omega\rho\check{v}_{n,e} \quad (1.112)$$



### 1.14.2 Kirchhoff-Helmholtz Integral and Boundary Element Method

With help of Green's theorem the Helmholtz equation (1.51) can be transformed into an integral equation (1.113), called the Kirchhoff - Helmholtz integral.

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_S \left( j\omega\rho\check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS \quad (1.113)$$

What Eq. 1.113 says is that the complex amplitude function  $\check{p}$  at any point  $P$  in space can be calculated if the normal component of the sound particle velocity  $\check{v}_S$  and the sound pressure  $\check{p}_S$  is known on a closed surface  $S$ . The point  $P$  can be located inside or outside of  $S$ .  $r$  is the distance between  $P$  and the surface element  $dS$ ,  $\partial/\partial n$  stands for the derivative in the direction perpendicular to the surface<sup>3</sup>. The surface  $S$  may lie partially in the infinity. Furthermore it can be shown that the Kirchhoff Helmholtz integral holds even for points  $P$  on the surface  $S$  itself. Though in this case Eq. 1.113 yields  $\check{p}/2$ .

With help of the Kirchhoff Helmholtz integral (KHI), typical radiation problems can be solved for vibrating bodies that are acoustically hard. It is usually assumed that the mechanical vibration is known on the surface of the body. With this knowledge the problem is completely specified.

The sound particle velocity on the surface corresponds to the normal component of the speed of the mechanical vibration. With this information the first term of the KHI is known. The sound pressure on the body surface as the second field variable necessary to evaluate the KHI is unknown at this point. However it is possible to express the sound pressure by the KHI itself. By discretization of the body surface, a finite number of pressure variables can be introduced. This discretization has to be fine compared to the shortest wave length of interest. Typically 6 to 10 points per wave length have to be chosen. With the pressure variables and the KHI, a system of equations can be set up and solved. Once the pressure is known on the surface, the KHI allows for the calculation of the sound pressure at any point in space outside the vibrating body. The numerical implementation of this procedure is called *Boundary Element Method* or short *BEM*<sup>4</sup>.

### 1.14.3 Applications of the Kirchhoff-Helmholtz Integral

#### Rayleigh Integral for the sound radiation of a piston in an infinitely extended wall

In the following it is assumed that an infinitely extended and acoustically hard wall at rest contains a limited region  $S$  with given normal component of the velocity  $\check{v}_n(x, y)$ . As the wall acts as a reflector the problem can be transformed into an equivalent one with eliminated wall but an additional mirror source. The mirror source makes sure that the resulting normal component of the sound particle velocity component vanishes on the wall outside  $S$ . The translational back and forth movement of the region  $S$  has thus to be replaced by a pulsating movement where the front and back side both move in phase outwards and inwards.  $S$  can be interpreted as a body with variable thickness mounted in free space (Fig. 1.20).

The sound pressure on the surface of the piston is unknown but due to symmetry, the values are identical on both sides. In the evaluation of the Kirchhoff-Helmholtz integral the contribution of the sound pressure is multiplied with the derivative of a distance function in the outward direction. Therefore the pressure contributions add up to 0 and can thus be omitted. On the other hand, the contribution of the velocity is identical for both sides of the piston  $S$ . It is sufficient to perform the integration over one side only and multiply the result by 2. The remaining relation is called Rayleigh integral:

$$\check{p}(x, y, z, \omega) = \frac{j\omega\rho}{2\pi} \int_S \check{v}_n(x, y, \omega) \frac{e^{-jkr}}{r} dS \quad (1.114)$$

<sup>3</sup>The derivative of the function  $f$  in a point  $\vec{a}$  in direction  $\vec{n}$  is given by:  $\lim_{\beta \rightarrow 0} \frac{f(\vec{a} + \beta\vec{n}) - f(\vec{a})}{\beta}$  where  $\vec{n}$  is a vector of length 1.

<sup>4</sup>Ochmann M., Wellner F. Berechnung der Schallabstrahlung dreidimensionaler schwingender Körper mit Hilfe eines Randelemente-Mehrgitterverfahrens. *Acustica* 73 (1991) pp 177 - 190.

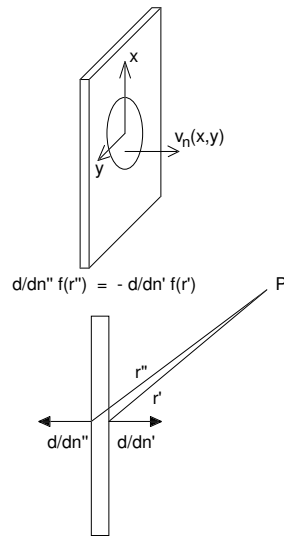


Figure 1.20: Situation of sound radiation of a piston in an infinitely extended wall.

### Kirchhoff's approximation to handle diffraction problems

The situation to discuss here is a infinitely extended screen with an opening  $S$  and an incident plane wave (Fig. 1.21). If the normal component of the sound particle velocity  $\tilde{v}_n(x, y)$  over  $S$  is known, the sound field behind the screen can be calculated by applying the Rayleigh integral. Kirchhoff's approximation assumes that the sound particle velocity in the opening is identical to the situation without screen. This simplification becomes more and more critical as the opening gets smaller compared to the wave length. Figure 1.22 shows the calculated sound pressure field behind an opening using Kirchhoff's approximation.

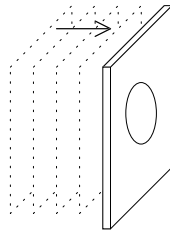


Figure 1.21: Situation of a plane wave incident on a screen with an opening.

### Kirchhoff's approximation to handle problems of reflections at small screens

The wave that is reflected at an acoustically hard screen can be determined with help of Kirchhoff's approximation as well. The screen introduces the boundary condition of vanishing resulting normal component of the sound particle velocity. Kirchhoff's approximation for the sound pressure lies in the assumption that the pressure doubles in front of the screen and vanishes at the back side. The reflected sound wave has to ensure that these conditions are fulfilled.

This is accomplished by a reflected sound particle velocity distribution on the screen that has equal amplitude but opposite direction compared to the incident wave. Kirchhoff's approximation ignores boundary effects. It is further assumed that the sound particle velocity is homogeneous over the screen. The sound pressure of the reflection contribution has to be identical in amplitude and phase to the incident sound pressure on the front side of the screen. On the rear side, the reflected sound pressure has equal amplitude but opposite phase.

Knowing sound particle velocity and sound pressure of the reflection on the surface of the screen, the reflected sound field can be calculated at any point by evaluating the Kirchhoff-Helmholtz integral over the front and back side of the screen.

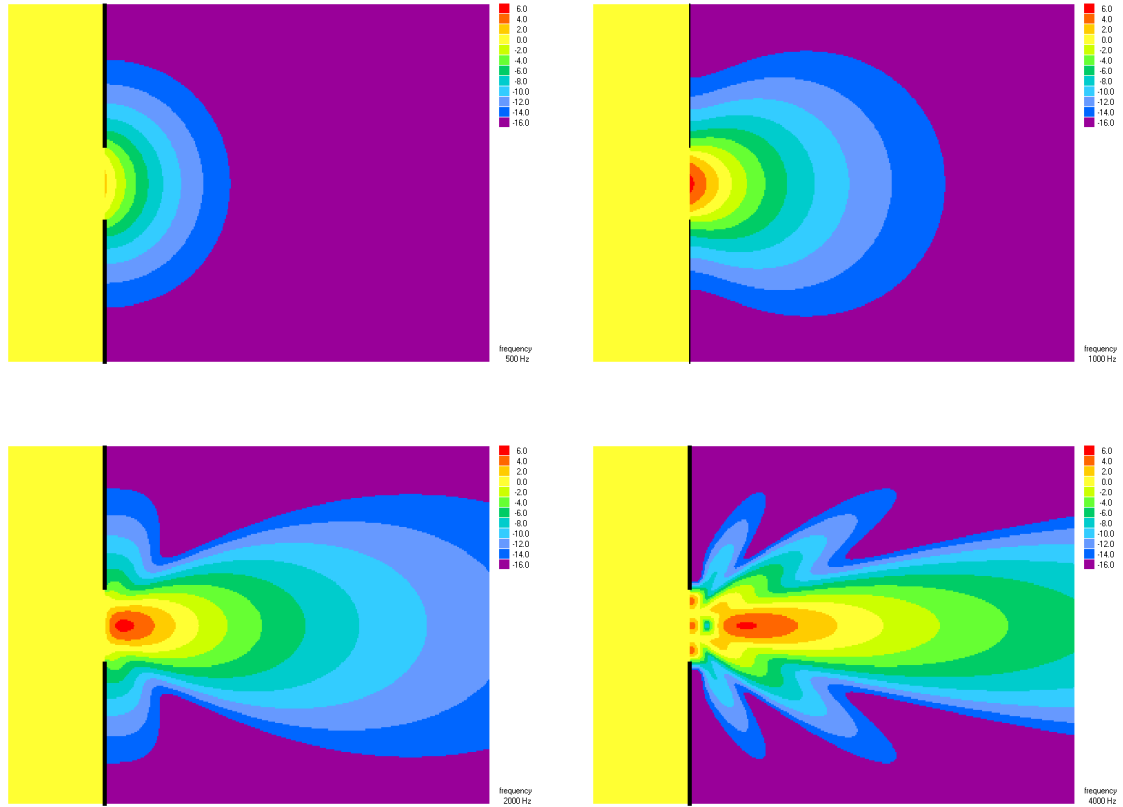


Figure 1.22: Sound pressure field behind a screen with an opening of diameter 25 cm. The calculation assumes an incident plane wave from left to right and Kirchhoff's approximation. Shown are the frequencies 500, 1000, 2000 and 4000 Hz. The sound pressure is color coded. Compared to the sound pressure of the incident wave the amplification due to focusing effects in *hot spots* can reach +6 dB.

Figure 1.23 shows the situation for an incident plane wave with sound pressure  $p$  and sound particle velocity  $v$ . With  $v_{SSv}$  and  $v_{SSr}$  and  $p_{SSv}$  and  $p_{SSr}$  as sound particle velocities and sound pressures of the reflected wave on the front ( $v$ ) and the rear ( $r$ ) side of the screen it can be written:

$$\begin{aligned}
 v_{SSv} &= v \\
 v_{SSr} &= -v \\
 p_{SSv} &= p \\
 p_{SSr} &= -p
 \end{aligned} \tag{1.115}$$

With the Kirchhoff-Helmholtz integral the sound pressure of the reflected wave is given as

$$\check{p}_{\text{streu}}(x, y, z, \omega) = \frac{1}{4\pi} \int_S \left( j\omega\rho\check{v}_{SS}(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_{SS}(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS \tag{1.116}$$

As  $v_{SSv}$  and  $v_{SSr}$  have identical amplitude and opposite sign, their contributions cancel each other during integration over the front and rear side of the screen. Sound pressure behaves differently. The derivative of the distance function yields opposite signs for the front and rear side. As  $p_{SSv}$  and  $p_{SSr}$  have opposite signs as well they add up constructively. Instead of integrating over the front and the rear side, the integration can be restricted to the front side and the result is multiplied with a factor of 2. Figure 1.24 shows the result of such a calculation.

### Huygens elementary sources and the construction of Fresnel zones

As shown above the calculation of the reflection at a screen or the diffraction at an opening requires the evaluation of the Kirchhoff-Helmholtz or the Rayleigh integral. This integration corresponds to

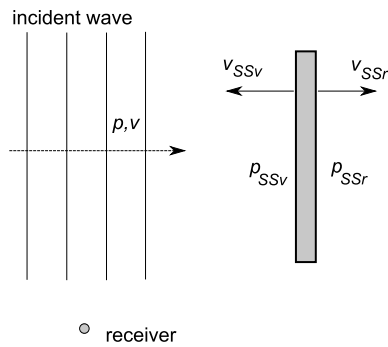


Figure 1.23: Reflection of a sound wave at a screen. Note that the sound particle velocity normal components are oriented in the outward direction.

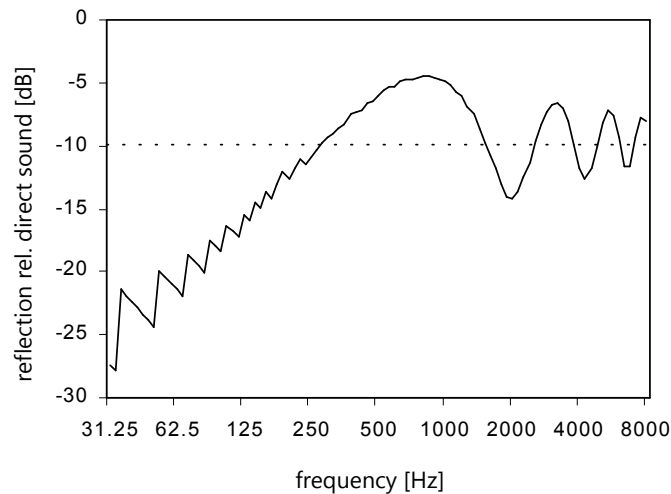


Figure 1.24: Kirchhoff-Helmholtz integral calculation of the frequency response of the normal incident reflection at a screen of dimensions  $2 \times 2$  m. The source is at 5 m, the receiver at 10 m distance to the reflector. The discretization used in the calculation was set to 1/10 of the wave length under consideration. At low frequencies the reflection is weak. Actually it is more likely a scattering in all directions. At high frequencies the sound pressure of the reflected wave tends to -10 dB as expected for the infinitely extended reflector.

a summation of the contributions of monopole and dipole sources on the surface  $S$ . In a qualitative sense this concept was already proposed by Huygens. He introduced elementary sources on the front of a wave to extrapolate the wave front at a later time. By this concept, a fundamental understanding of wave phenomena such as diffraction can be obtained.

Fresnel developed this concept in a more quantitative manner. In many cases the amplitude changes only slowly during the integration over the surface  $S$ . As a first approximation it is therefore sufficient to take care of the phase change alone. A further simplification is the classification of the phase in just two categories  $+1$  ( $0 \dots 180^\circ$ ) and  $-1$  ( $180 \dots 360^\circ$ ). With this the integration reduces to additions and subtractions of amplitudes. With the smallest phase assumed as 0, the phase classes can be enumerated. The phase of the  $n$ th class lies in the interval  $(n - 1) \times 180^\circ \dots n \times 180^\circ$ .  $+1$ -classes (positive contributions) have odd  $n$ , even  $n$  stand for  $-1$ -classes (negative contributions).

A region on the surface  $S$  for which the contributions belong to the  $n$ th class, is called the  $n$ th Fresnel zone<sup>5</sup>. The dimension of a Fresnel zone is frequency dependent. For low frequencies the Fresnel zones are large, for high frequencies the zones are small. On a plane surface the Fresnel

<sup>5</sup>Cremer L., Fresnels Methoden zur Berechnung von Beugungsfeldern, Acustica, vol. 72, p.1-6 (1990).

zones are elliptic rings. The sound pressure at a receiver point is given as sum and differences of contributions that are proportional to the area of the Fresnel zones and inversely proportional to the distance. Thereby most of the contributions do cancel each other. Finally what remains is the contribution of half of the first zone. Consequently for a plane sound wave that passes a screen with an opening that corresponds to the first Fresnel zone, an amplification by a factor of 2 or +6 dB is expected.

The first Fresnel zone can be regarded as the relevant region for a reflection. If the reflector is smaller than half of the first Fresnel zone, the amplitude of the reflected sound pressure scales with the corresponding area ratio. Sometimes the first Fresnel zone is defined as the region for a maximum phase shift of only a quarter of a wave length. This automatically accounts for the fact that only half of the area of the first  $\lambda/2$ -Fresnel zone remains as net contribution for a total reflection.

### Interpretation of the Kirchhoff-Helmholtz integral with monopole and dipole sources

The Kirchhoff-Helmholtz integral

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_S \left( j\omega \rho \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS \quad (1.117)$$

can be rewritten with the following considerations:

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \Rightarrow e^{-j\omega r/c} = e^{-jkr} \quad (1.118)$$

$$\begin{aligned} \frac{\partial}{\partial n} \left( \frac{e^{-jkr}}{r} \right) &= \frac{1}{r^2} \left( r \frac{\partial}{\partial n} (e^{-jkr}) - e^{-jkr} \frac{\partial r}{\partial n} \right) \\ &= \frac{1}{r^2} \left( -jkr \frac{\partial r}{\partial n} e^{-jkr} - e^{-jkr} \frac{\partial r}{\partial n} \right) = \frac{1}{r^2} e^{-jkr} (-jkr - 1) \frac{\partial r}{\partial n} \end{aligned} \quad (1.119)$$

In (1.119)  $\partial r/\partial n$  corresponds to the projection of  $r$  on the normal direction  $\vec{n}$ . So we get  $\partial r/\partial n = -\cos \phi$  where  $\phi$  is the angle between the normal direction  $\vec{n}$  and the direction to the receiver point  $(x, y, z)$ .

Therewith the Kirchhoff-Helmholtz integral becomes:

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_S \left( j\omega \rho \check{v}_S(\omega) \frac{e^{-jkr}}{r} + \check{p}_S(\omega) \frac{1 + jkr}{r^2} \cos \phi e^{-jkr} \right) dS \quad (1.120)$$

The integrand in (1.120) is composed of two parts. The first term corresponds to the sound pressure produced by an omnidirectional monopole source. The second term is the contribution of a dipole source with the dipole axis pointing in the surface normal direction. The Kirchhoff-Helmholtz integral can thus be interpreted as summation of monopole and dipole contributions distributed over the surface  $S$ . The strength of the monopole sources is given by the normal component of the surface sound particle velocity, the strength of the dipole sources is determined by the sound pressure on  $S$ . Eq. 1.120 is the mathematical basis to synthesize a three dimensional sound field by controlling sound pressure and sound particle velocity on a closed surface<sup>6, 7</sup>.

#### 1.14.4 Method of Finite Differences

The method of finite differences is a widely used approach to numerically solve differential equations. For sound field calculations the region of interest has to be discretized sufficiently fine and represented by a finite number of grid points. The relevant differential equations are then approximated by linear equations for the field variables in the grid points. Thereby derivatives translate into differences.

<sup>6</sup>A. J. Berkhout, A Holographic Approach to Acoustical Control, Journal of the Audio Engineering Society, vol. 36, n.12, p.977-995 (1988).

<sup>7</sup>Diemer de Vries, Sound Reinforcement by Wavefield Synthesis: Adaption of the Synthesis Operator to the Loudspeaker Directivity Characteristics, Journal of the Audio Engineering Society, vol. 44, n.12, p.1120-1131 (1996).

## Finite differences in the frequency domain

The sound field calculation in the frequency domain is usually based on the Helmholtz equation <sup>8</sup>. The system of equations that has to be established uses the unknown amplitudes and phase values of the sound field variable (usually sound pressure) in each grid point. The parameters of the equations are determined by application of the Helmholtz equation and the boundary conditions. In most cases the information about the boundary is given in form of impedances (ratio of sound pressure and sound particle velocity). As sound pressure and sound particle velocity are related to each other by a differential equation, it is possible to get rid of one variable in order to describe the boundary condition with one field variable alone.

In complex writing sound pressure and sound particle velocity read as

$$\underline{p} = \check{p}e^{j\omega t} \quad (1.121)$$

$$\underline{v} = \check{v}e^{j\omega t} \quad (1.122)$$

where  $\check{p}$  and  $\check{v}$  represent complex amplitude functions. The impedance  $Z$  is

$$Z = \frac{\check{p}}{\check{v}} \quad (1.123)$$

With (1.12) one gets for one direction (here:  $x$ )

$$\frac{\partial \check{p}}{\partial x} e^{j\omega t} = -\rho j\omega \check{v} e^{j\omega t} \quad (1.124)$$

$$\frac{\partial \check{p}}{\partial x} = -\rho j\omega \frac{\check{p}}{Z} \quad (1.125)$$

The angular frequency  $\omega$  can be expressed with the wave number  $k$  as:

$$\omega = 2\pi \frac{c}{\lambda} = kc \quad (1.126)$$

By inserting Eq. (1.126) in (1.125) one finally gets

$$\frac{\partial \check{p}}{\partial x} = -\rho c j k \frac{\check{p}}{Z} \quad (1.127)$$

Eq. (1.127) represents an impedance boundary condition with only sound pressure as variable. With this the system of linear equations for the  $n$  complex pressure values (amplitude and phase) can be established. In general this makes it necessary to invert an  $n \times n$  matrix for each frequency. Taking into account that the grid spacing has to be in the order of 1/6 of the shortest wavelength of interest, it becomes clear that the method is restricted to small volumes or low frequencies. The method is not very flexible as only homogeneous and equidistant grids can be applied.

## Finite differences in the time domain

It is possible and often beneficial to apply the finite differences concept in the time domain <sup>9,10</sup>. The result of such a simulation is an impulse response that contains information about all frequencies. An other advantage is that fact that an iterative, time-step wise updating scheme can be used without the necessity of solving a system of equations. However a difficulty with the time domain formulation is the specification of boundary conditions. Typically these are defined as impedances in the frequency domain. An exact transformation to the time domain would require a convolution operation which is very expensive in the sense of computational effort. Therefore appropriate approximation are usually used <sup>11,12</sup>.

<sup>8</sup> Alfredson R.J., A Note on the Use of the Finite Difference Method for Predicting Steady State Sound Fields. *Acustica* 28 (1973) pp 296 - 301.

<sup>9</sup> D. Botteldooren, Finite-difference time-domain simulation of low-frequency room acoustic problems, *Journal of the Acoustical Society of America*, vol. 98, p.3302-3308 (1995).

<sup>10</sup> S. Sakamoto, H. Tachibana, Numerical study on sound propagation from depressed/semi-underground roads, *Proceedings inter-noise 2001*.

<sup>11</sup> B. Van den Nieuwenhof, J.-P. Coyette, Treatment of frequency-dependent admittance boundary conditions in transient acoustic finite/infinite-element models, *Journal of the Acoustical Society of America*, vol. 110, p.1743-1751 (2001)

<sup>12</sup> Benoit Van den Nieuwenhof, Jean-Pierre Coyette, Treatment of frequency-dependent admittance boundary conditions in transient acoustic finite-infinite-element models, *Journal of the Acoustical Society of America*, vol. 111, p.1743-1751 (2001).

The method of *Finite Differences in the Time Domain* (FDTD) is based on Eq. 1.12 and 1.29. In cartesian coordinates these equations read as:

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial v_x}{\partial t} \quad (1.128)$$

$$\frac{\partial p}{\partial y} = -\rho \frac{\partial v_y}{\partial t} \quad (1.129)$$

$$\frac{\partial p}{\partial z} = -\rho \frac{\partial v_z}{\partial t} \quad (1.130)$$

$$-\frac{\partial p}{\partial t} = \kappa P_0 \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) \quad (1.131)$$

The region of interest is covered with a regular grid. The sound pressure is evaluated at grid positions  $\langle i\Delta x, j\Delta y, k\Delta z \rangle$  where  $i, j, k$  are whole-numbered indices and  $\Delta x, \Delta y, \Delta z$  are the discretization widths in the three coordinate directions. The sound particle velocity component in the  $x$ -direction is evaluated at grid points  $\langle (i \pm 0.5)\Delta x, j\Delta y, k\Delta z \rangle$ , the  $y$ -component at  $\langle i\Delta x, (j \pm 0.5)\Delta y, k\Delta z \rangle$  and the  $z$ -component at  $\langle i\Delta x, j\Delta y, (k \pm 0.5)\Delta z \rangle$ . The Figure 1.25 shows the grid in two dimensions.

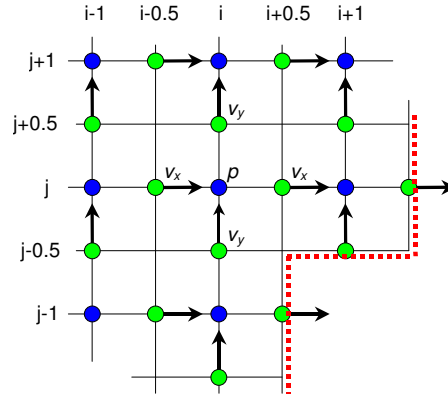


Figure 1.25: FDTD discretization of the region of simulation in the two dimensional case. Sound pressure is investigated at  $\langle i\Delta x, j\Delta y \rangle$ , the sound particle velocity components in the  $x$  and the  $y$ -direction at  $\langle (i \pm 0.5)\Delta x, j\Delta y \rangle$  and  $\langle i\Delta x, (j \pm 0.5)\Delta y \rangle$ . The dashed line marks the border of the region where boundary conditions have to be defined.

The original differential equations of the sound field are approximated by finite differences. Besides a spacial discretization, a temporal discretization has to be introduced for that purpose. Sound particle velocity is evaluated at times  $t = (l + 0.5)\Delta t$ , sound pressure is evaluated at times  $t = l\Delta t$  ( $l$  being a running index). The corresponding difference equations become:

$$v_x^{[l+0.5]}(i + 0.5, j, k) = v_x^{[l-0.5]}(i + 0.5, j, k) - \frac{\Delta t}{\rho \Delta x} \left( p^{[l]}(i + 1, j, k) - p^{[l]}(i, j, k) \right) \quad (1.132)$$

$$v_y^{[l+0.5]}(i, j + 0.5, k) = v_y^{[l-0.5]}(i, j + 0.5, k) - \frac{\Delta t}{\rho \Delta y} \left( p^{[l]}(i, j + 1, k) - p^{[l]}(i, j, k) \right) \quad (1.133)$$

$$v_z^{[l+0.5]}(i, j, k + 0.5) = v_z^{[l-0.5]}(i, j, k + 0.5) - \frac{\Delta t}{\rho \Delta z} \left( p^{[l]}(i, j, k + 1) - p^{[l]}(i, j, k) \right) \quad (1.134)$$

$$\begin{aligned} p^{[l+1]}(i, j, k) = & p^{[l]}(i, j, k) - \frac{\rho c^2 \Delta t}{\Delta x} \left( v_x^{[l+0.5]}(i + 0.5, j, k) - v_x^{[l+0.5]}(i - 0.5, j, k) \right) \\ & - \frac{\rho c^2 \Delta t}{\Delta y} \left( v_y^{[l+0.5]}(i, j + 0.5, k) - v_y^{[l+0.5]}(i, j - 0.5, k) \right) \\ & - \frac{\rho c^2 \Delta t}{\Delta z} \left( v_z^{[l+0.5]}(i, j, k + 0.5) - v_z^{[l+0.5]}(i, j, k - 0.5) \right) \end{aligned} \quad (1.135)$$

For points at the border of the simulation region, boundary conditions have to be defined. A difficulty is the handling of an open space. At the border of the region of calculation, total reflection occurs. To avoid unwanted artifacts, a zone with damped propagation has to be introduced. A very efficient method is the perfectly matched layer, originally proposed for electro-magnetic field calculations<sup>13</sup>. Without significant restrictions it can be assumed that the boundary conditions need only be formulated at grid points where the sound particle velocity is evaluated. A local reaction condition is usually assumed which means that the boundary conditions makes a statement about the ratio of sound pressure and the normal component of the sound particle velocity. This corresponds to an impedance that is usually frequency dependent.

If the possible frequency dependency is restricted, the formulation of the boundary conditions simplifies dramatically. Here it is assumed that the impedance can be described with Eq. 1.136. In<sup>14</sup> a more subtle second order extension is discussed.

$$Z(\omega) = a_{-1} \frac{1}{j\omega} + a_0 + a_1 j\omega \quad (1.136)$$

where

$a_{-1}, a_0, a_1$ : positive real numbers.

For the Fourier transform in the frequency domain it can be written:

$$P(\omega) = Z(\omega)V(\omega) = V(\omega)a_{-1} \frac{1}{j\omega} + V(\omega)a_0 + V(\omega)a_1 j\omega \quad (1.137)$$

where

$P(\omega)$ : Fourier transform of the sound pressure time history

$V(\omega)$ : Fourier transform of the sound particle velocity time history

Equation 1.137 translates into the time domain as:

$$p(t) = \int_{-\infty}^t a_{-1} v_n(\tau) d\tau + a_0 v_n(t) + a_1 \frac{dv_n(t)}{dt} \quad (1.138)$$

As already mentioned it is assumed that the boundary condition is defined at a grid point where the sound particle velocity is evaluated. In these points Eq. 1.132 to 1.134 have to be replaced accordingly.

Exemplarily this is demonstrated here for the sound particle velocity component in the  $x$ -direction with the assumption, that the border runs through the grid point  $\langle (i + 0.5)\Delta x, j\Delta y, k\Delta z \rangle$  and the simulation region lies on the left (at lower  $x$  values).

As for any point in space, Eq. 1.139 has to be fulfilled for the boundary point  $\langle (i + 0.5)\Delta x, j\Delta y, k\Delta z \rangle$  as well.

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial v_x}{\partial t} \quad (1.139)$$

In contrast to the symmetrical approximation from above (1.132), a onesided approximation for (1.139) is used here:

$$v_x^{[l+0.5]}(i + 0.5, j, k) = v_x^{[l-0.5]}(i + 0.5, j, k) - \frac{2\Delta t}{\rho\Delta x} \left( p^{[l]}(i + 0.5, j, k) - p^{[l]}(i, j, k) \right) \quad (1.140)$$

In Eq. 1.140 the sound pressure at the point  $\langle (i + 0.5)\Delta x, j\Delta y, k\Delta z \rangle$  is unknown. However with knowledge of the boundary condition (1.138) this unknown sound pressure can be expressed with the sound particle velocity:

<sup>13</sup>J. P. Berenger, A perfectly matched layer for the absorption of electro magnetic waves, Journal of Computational Physics, vol. 114, p.185-200 (1994)

<sup>14</sup>K. Heutschi, M. Horvath, J. Hofmann, Simulation of Ground Impedance in Finite Difference Time Domain Calculations of Outdoor Sound Propagation, Acta Acustica united with Acustica, vol. 91, 35-40 (2005).



$$p^{[l]}(i + 0.5, j, k) = a_{-1}\Delta t \left( \sum_{m=-\infty}^l v_x^{[m-0.5]}(i + 0.5, j, k) \right) + a_0 v_x^{[l]}(i + 0.5, j, k) + a_1 \frac{v_x^{[l+0.5]}(i + 0.5, j, k) - v_x^{[l-0.5]}(i + 0.5, j, k)}{\Delta t} \quad (1.141)$$

In Eq. 1.141 a Term with the sound particle velocity at time  $l$  occurs. As sound particle velocity values are evaluated at times  $\cdot + 0.5$ , this value has to be approximated as average of the two temporal neighbors:

$$v_x^{[l]} = \frac{v_x^{[l+0.5]} + v_x^{[l-0.5]}}{2} \quad (1.142)$$

Finally Eq. 1.140 can be dissolved for the wanted value  $v_x^{[l+0.5]}(i + 0.5, j, k)$  by insertion of Eq. 1.141 and 1.142. Eq. 1.143 is the replacement of Eq. 1.132 as updating equation.

$$v_x^{[l+0.5]}(i + 0.5, j, k) = v_x^{[l-0.5]}(i + 0.5, j, k) \frac{\rho\Delta x - a_0\Delta t + 2a_1}{\rho\Delta x + a_0\Delta t + 2a_1} + p^{[l]}(i, j, k) \frac{2\Delta t}{\rho\Delta x + a_0\Delta t + 2a_1} - \sum_{m=-\infty}^l v_x^{[m-0.5]}(i + 0.5, j, k) \frac{2a_{-1}(\Delta t)^2}{\rho\Delta x + a_0\Delta t + 2a_1} \quad (1.143)$$

The infinite sum in Eq. 1.143 makes it necessary to introduce an additional register to accumulate the corresponding contributions over time.

If the region of simulation lies on the right side of the boundary the updating equation for the sound particle velocity is found as:

$$v_x^{[l+0.5]}(i + 0.5, j, k) = v_x^{[l-0.5]}(i + 0.5, j, k) \frac{\rho\Delta x - a_0\Delta t + 2a_1}{\rho\Delta x + a_0\Delta t + 2a_1} - p^{[l]}(i + 1, j, k) \frac{2\Delta t}{\rho\Delta x + a_0\Delta t + 2a_1} - \sum_{m=-\infty}^l v_x^{[m-0.5]}(i + 0.5, j, k) \frac{2a_{-1}(\Delta t)^2}{\rho\Delta x + a_0\Delta t + 2a_1} \quad (1.144)$$

The equations for the two other coordinate directions are found by adapting the corresponding indices.

A possible initial condition to investigate the impulse response is a smooth and continuous sound pressure distribution at and around the source position as given in Eq. 1.145. It has to be ensured that no aliasing occurs, neither in space nor in time.

$$p^{[0]}(i, j, k) = e^{-((0.3(i-i_S))^2 + (0.3(j-j_S))^2 + (0.3(k-k_S))^2)} \quad (1.145)$$

with

$i_S, j_S, k_S$ : indices of the grid point of the source position.

Finally the set of difference equations represents an updating scheme to determine new sound particle velocity and sound pressure values from the corresponding old ones. Observing the reaction on an impulse excitation, the temporal evolution of the sound field at each grid point is obtained. These impulse responses represent the complete information about the system. By application of a Fourier transformation the frequency responses can easily be calculated.

Figure 1.26 shows an example of a FDTD simulation.

### 1.14.5 Method of finite elements

As in many disciplines Finite Elements can successfully be applied for sound field calculations<sup>15,16</sup>. The finite element method is especially well suited for bounded domains, however it is

<sup>15</sup>W. J. Anderson, Numerical Acoustics, Multimedia study guide (CD-ROM), Ann Arbor, Automated Analysis Corporation (1996).

<sup>16</sup>G. Dhatt, G. Touzot, The Finite Element Method Displayed, John Wiley & Sons (1984).

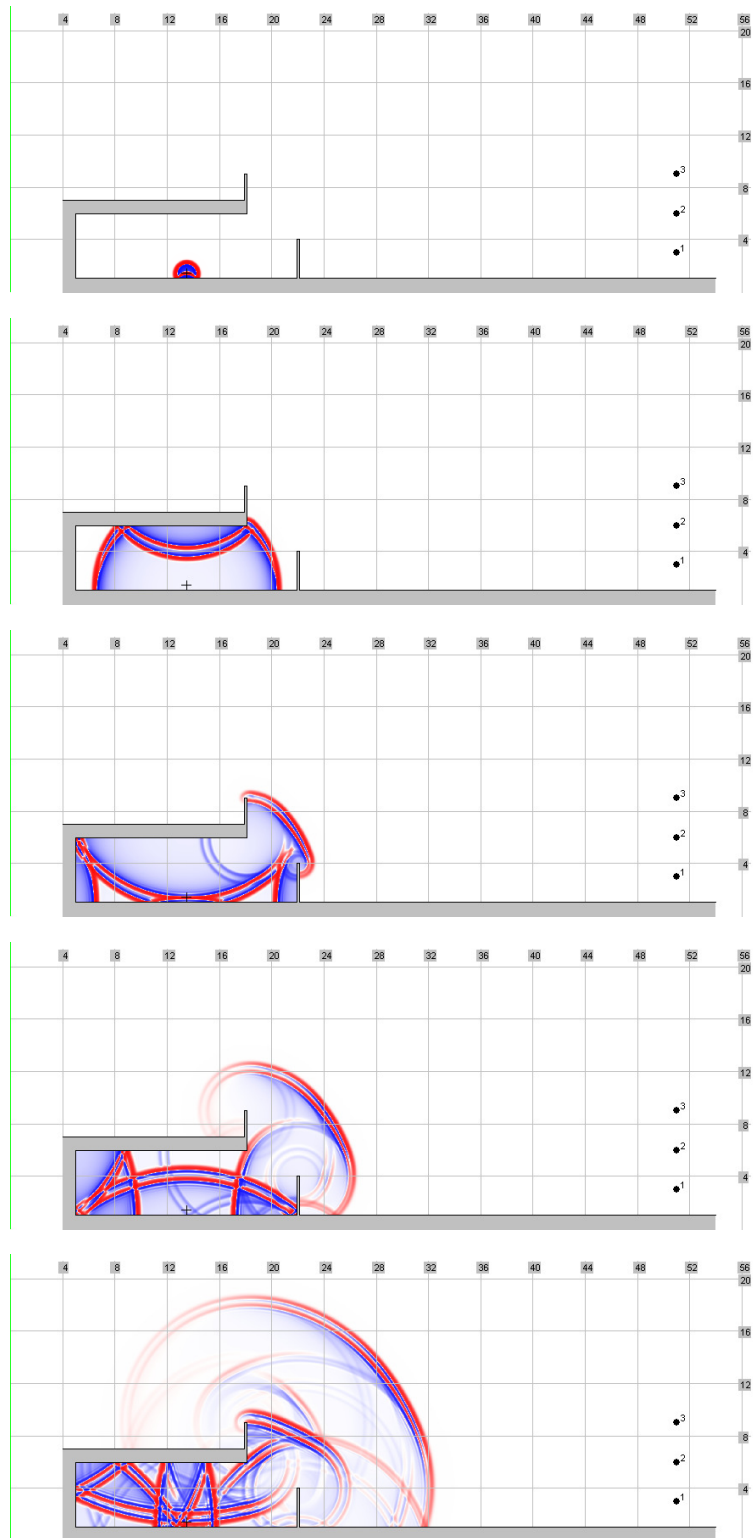


Figure 1.26: 2D FDTD simulation of the temporal evolution of the sound field in a road gallery (cross sectional view) after excitation with a pressure pulse. Dark red corresponds to high positive, dark blue to high negative sound pressure.

possible to handle infinite domains as well with help of so called infinite elements. The underlying equations are usually formulated in the frequency domain, but time domain approaches are also possible.

In the following the principles of the finite element method are introduced for a general 3-dimensional

bounded domain. The sound field variable of interest is usually sound pressure. It is assumed that the time dependency is sinusoidal with angular frequency  $\omega$ . Consequently the search of the sound field reduces to the determination of the the complex amplitude  $\check{p}$  as a function of the location.

The general problem can be formulated by the Helmholtz equation and three possible boundary conditions as follows:

$$\nabla^2 \check{p} + k^2 \check{p} = 0 \quad \text{in the considered volume } V \quad (1.146)$$

$$\check{p} = \bar{p} \quad \text{on the surface } S_1 \quad (1.147)$$

$$\check{v}_n = \bar{v}_n \rightarrow \frac{\partial \check{p}}{\partial n} = -j\rho\omega\bar{v}_n \quad \text{on the surface } S_2 \quad (1.148)$$

$$\frac{\check{v}_n}{\check{p}} = A_n = \frac{1}{Z_n} \rightarrow \frac{\partial \check{p}}{\partial n} = -j\rho\omega A_n \check{p} \quad \text{on the surface } S_3 \quad (1.149)$$

where

$k$ : wave number =  $\frac{\omega}{c}$

$\bar{p}$ : predefined sound pressure

$\bar{v}_n$ : predefined normal component of the sound particle velocity

$A_n$ : predefined admittance

$Z_n$ : predefined impedance

$S_1$ ,  $S_2$  and  $S_3$  form the total surface  $S$  that encloses the field volume completely. With the finite element procedure an approximate solution  $\check{p}'$  for the true pressure  $\check{p}$  is searched. The quality of the approximation is measured with help of the residues that correspond to the differences between the actual and the nominal values:

$$R_V = \nabla^2 \check{p}' + k^2 \check{p}' \quad (1.150)$$

$$R_{S_1} = \bar{p} - \check{p}' \quad (1.151)$$

$$R_{S_2} = -\frac{\partial \check{p}'}{\partial n} - j\rho\omega\bar{v}_n \quad (1.152)$$

$$R_{S_3} = -\frac{\partial \check{p}'}{\partial n} - j\rho\omega A_n \check{p}' \quad (1.153)$$

where

$R_V$ : residuum for the considered volume  $V$

$R_{S_1}$ : residuum for the surface  $S_1$  with predefined sound pressure  $\bar{p}$

$R_{S_2}$ : residuum for the surface  $S_2$  with predefined normal component of the sound particle velocity  $\bar{v}_n$

$R_{S_3}$ : residuum for the surface  $S_3$  with predefined admittance  $A_n$

The approximate solution  $\check{p}'$  is searched for the condition of a vanishing weighted average sum of the residues:

$$\int_V W R_V dV + \int_{S_1} W R_{S_1} dS + \int_{S_2} W R_{S_2} dS + \int_{S_3} W R_{S_3} dS = 0 \quad (1.154)$$

with

$W$ : weighting function

The weighting function  $W$  in Eq. 1.154 can be chosen arbitrarily. However the solution  $\check{p}'$ , that fulfills Eq. 1.154 depends on  $W$ . On the surface  $S_1$  the boundary condition specifies the sound pressure  $\bar{p}$ . It is most plausible to choose there  $\check{p}'$  identical to  $\bar{p}$ . Consequently on  $S_1$  the residuum  $R_{S_1}$  vanishes independently of  $W$ . As will be seen later it is beneficial to chose  $W$  in such a way that it takes the value 0 on  $S_1$ . Inserting the residues in Eq. 1.154 yields:

$$\int_V W \nabla^2 \check{p}' dV + \int_V W k^2 \check{p}' dV - \int_{S_2} W \left( \frac{\partial \check{p}'}{\partial n} + j\rho\omega\bar{v}_n \right) dS - \int_{S_3} W \left( \frac{\partial \check{p}'}{\partial n} + j\rho\omega A_n \check{p}' \right) dS = 0 \quad (1.155)$$

The first summand in Eq. 1.155 can be rewritten with the first Green's formula:

$$\int_V W \nabla^2 \check{p}' dV = - \int_V \text{grad} W \bullet \text{grad} \check{p}' dV + \oint_S W \frac{\partial \check{p}'}{\partial n} dS \quad (1.156)$$

where

•: scalar product

In Eq. 1.156 the integration over the surface  $S$  can be written as sum of the integrals over the partial surfaces  $S_1$ ,  $S_2$  and  $S_3$ . Finally Eq. 1.155 becomes:

$$- \int_V \text{grad} W \bullet \text{grad} \check{p}' dV + \int_V W k^2 \check{p}' dV + \int_{S_1} W \frac{\partial \check{p}'}{\partial n} dS - \int_{S_2} W j \rho \omega \check{v}_n dS - \int_{S_3} W j \rho \omega A_n \check{p}' dS = 0 \quad (1.157)$$

In Eq. 1.157 the integration over  $S_1$  vanishes as the weighting function  $W$  was chosen to 0 on  $S_1$ .

The next step is the discretization. That fore the whole region of interest is subdivided into small elements. These elements may vary in size and may have different shapes (Fig. 1.27). By suitable element selection, an optimal adaption to the geometry of interest is possible. This flexibility is an essential advantage compared to the equidistant discretisation in the finite differences method.

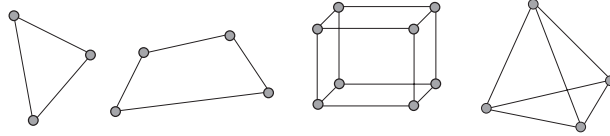


Figure 1.27: Examples of 2D and 3D finite elements.

An element describes a small part of the field region of interest. In three dimensions these can be cubes, tetrahedrons and so on. Suitable shapes in two dimensions are triangles and four-sided forms. An element is defined by nodes that are typically located at the corners. The elements have to cover the whole simulation region. Some elements share a common boundary and some have the same nodes. For each element  $M$ , so called interpolation functions or shape functions  $N_i$  are determined where  $M$  corresponds to the number of the nodes of the element. The interpolation functions  $N_i$  depend on location and describe the field variable  $\check{p}'$  within the element from the values at the nodes (1.158). Outside of the element the functions  $N_i$  vanish.

$$\check{p}'(x, y, z) = \sum_{i=1}^M \check{p}'_i N_i(x, y, z) \quad (1.158)$$

with

$\check{p}'_i$ : sound pressure in node  $i$

$N_i(x, y, z)$ : interpolation function  $i$

The finite element algorithms differ in the choice of the weighting functions  $W$ . A common approach is the so called Galerkin method. Thereby the weighting functions are identical to the interpolation functions. The formula 1.157 represents one equation for each element and node. These equations contain information about each isolated element only. In a so called assembling procedure the equations are put together under consideration of the fact that some elements have common nodes. This process introduces the situation geometry. In the last step the resulting system of equations has to be solved for the field variable sound pressure in each node.

As already mentioned above the finite element method is very well suited for bounded domains. Open domains can be treated with the idea of infinite elements<sup>17, 18</sup>. An alternative approach for unbounded

<sup>17</sup>D. S. Burnett, A three-dimensional acoustic infinite element based on a prolate spheroidal multipole expansion. Journal of the Acoustical Society of America, vol. 96, p.2798-2816 (1994).

<sup>18</sup>D. S. Burnett, R. L. Holford, An ellipsoidal acoustic infinite element. Comput. Methods Appl. Mech. Eng. vol. 164, p.49-76 (1998).

domains (which means nothing is reflected back) is the introduction of an arbitrary boundary where the boundary conditions corresponds to the free field impedance  $Z = \rho c$ . For plane waves this works fine, however in the general case a certain impedance discontinuity will occur, resulting in some reflected sound energy.

Within the concept of finite elements it is possible to account for locally varying medium properties and thus propagation conditions. Furthermore coupled structure fluid systems can be treated, taking into account e.g. the force of the sound wave that is acting on a structure.

### 1.14.6 Acoustical Holography

As already discussed, the Helmholtz equation can be transformed into the Kirchhoff-Helmholtz integral by use of Greens theorem. For that purpose the free field Green's function is applied. The Kirchhoff-Helmholtz integral expresses the sound pressure in an arbitrary point in three dimensional space by the integral evaluated on a closed surface  $S$ . For certain geometries of  $S$ , other Green's functions may be applied that deliver simpler field descriptions. A case of such a specially chosen surface is a plane that is closed in infinity in form of a hemisphere (Figure 1.28).

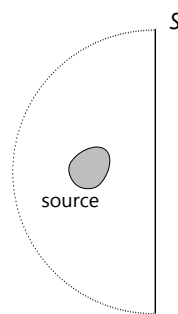


Figure 1.28: The surface  $S$  encloses the source completely.  $S$  consists of a plane and a hemisphere with infinite radius.

Using Sommerfeld's radiation condition, the contribution of the integral over the hemisphere of  $S$  can be neglected, meaning that the integral has to be evaluated over the plane only. An adapted Green's function that takes the mirror source into account yields an integral formulation with sound pressure alone, the contribution of the sound particle velocity vanishes. The sound pressure at any point in space on the right hand side of the plane (in the half space not occupied by the source) is then given as <sup>19</sup>:

$$\check{p}(x, y, z, \omega) = j \int_S \check{p}_S(\omega) \cos \phi \left( 1 - \frac{j}{kr} \right) \frac{e^{-jkr}}{\lambda r} dS \quad (1.159)$$

where

$\check{p}_S(\omega)$ : sound pressure (amplitude and phase) on the plane  $S$

$\lambda$ : wavelength

$\omega$ : angular frequency

$k$ : wave number =  $2\pi/\lambda$

$r$ : distance of the point of interest  $(x, y, z)$  to the point on the plane

$\phi$ : angle between the direction from the point on the plane to the point of interest and the normal direction of the plane

Most remarkable in Eq. 1.159 is the fact that a 3D sound pressure field is determined by the sound pressure distribution over a 2D plane. This is the essential property of holography where an interference pattern in a photography can store information about a 3D object.

In a practical applications of acoustical holography sound pressure (with respect to phase and amplitude) is determined in a plane at discrete grid points. The sampling region has to be large enough so that the sound pressure outside can be neglected. The sampling can be performed simultaneously

<sup>19</sup>Jorgen Hald, STSF - a unique technique for scan-based Near-field Acoustic Holography without restrictions on coherence, Br'uel + Kjaer Technical Review, no. 1, (1989).

with an array of microphones or sequentially with one microphone that is moved from one sampling position to the other. In this case a reference is needed to determine the phase.

In some cases one is interested in the conversion of the values measured in one plane to the sound pressure in an other plane. This operation can be performed very efficiently by a spacial Fourier transformation<sup>20</sup>. This calculation can be performed for target planes that are close to the source. By this procedure the near field of an extended source can be investigated. In such a plane partial sources can be detected easily. Information about sound particle velocity can be deduced by using the corresponding relation of the gradient of the sound pressure (Eq. 1.12).

### 1.14.7 Equivalent sources technique

In some cases the method of equivalent sources can be a very efficient strategy to find approximate solutions for sound fields defined by boundary conditions and a driving source. Cases with rigid boundaries are especially well suited. The basic idea is to introduce auxiliary sources in order to satisfy the boundary conditions. To adjust the position and strength of the auxiliary sources an optimization procedure is needed. The quality of a solution is measured as the sum of the squared error at discrete points on the boundary. In general the error can not be made zero because the number of auxiliary sources is usually much lower than the number of test points on the boundary. The art in the application of the method is to find reasonably good solutions with a low number of auxiliary sources<sup>21</sup>.

### 1.14.8 Principle of reciprocity

In a homogeneous medium at rest the so called principle of reciprocity holds for acoustical quantities such as sound pressure or sound particle velocity<sup>22,23</sup>. The principle states that the effect at a receiver point that is produced by a source is identical if source and receiver are exchanged. In free field situations the validity of the principle is obvious. However the interchangeability is maintained even if arbitrary boundaries such as walls and reflectors are introduced. In general the principle of reciprocity is violated for sound propagation outdoors due to the fact that the medium is not at rest and not homogeneous.

A remarkable consequence of this principle is the so called time-reversed acoustics.<sup>24,25</sup> In a typical experiment firstly the sound emitted by a source is registered at several receivers in the vicinity. Then the recorded signals at each receiver are emitted time-inverted (backwards) at these former receiver positions. In accordance with the principle of reciprocity the emitted signals will focus perfectly in the original source position. This focussing effect is especially pronounced if sources and receivers are omnidirectional.

Although experiments with time-reversed acoustics are usually performed with several microphones and consequently several loudspeakers, the principle can also be applied with a single microphone and loudspeaker. However in this case reflections are needed to produce relevant focusing amplifications. It is assumed that the source emits a short pulse. The receiver will then record the impulse response of the system. The principle of reciprocity states that this impulse response from the source to the receiver is identical to the impulse response from the receiver to the source. If the time-inverted impulse response signal is emitted at the original receiver position, the signal that results at the original source position corresponds to the convolution of the time-inverted impulse response with the impulse response. This operation yields the autocorrelation function of the impulse response with a distinct peak at the corresponding point in time.

Time-reversed acoustics can be found e.g. in medical applications for diagnosis purposes and in mechanical treatments such as destroying of kidney stones.

<sup>20</sup>Maynard, J. D. et al., Nearfield acoustic holography. I: Theory of generalized holography and the development of NAH, J. Acoustical Society of America, vol. 78, p. 1395- (1985).

<sup>21</sup>M. E. Johnson, An equivalent source technique for calculating the sound field inside an enclosure containing scattering objects. Journal of the Acoustical Society of America, vol. 104, p.1221-1231 (1998).

<sup>22</sup>Allan D. Pierce, Acoustics, published by the Acoustical Society of America (1989).

<sup>23</sup>M. Heckl, H.A. Müller, Taschenbuch der Technischen Akustik, Springer-Verlag (1994).

<sup>24</sup>M. Fink, Zeitumkehr-Akustik, Spektrum der Wissenschaft, p.68-74, März (2000).

<sup>25</sup>M. Fink, Time-Reversed Acoustics, Physics Today, vol.50, p.34-40 (1997).

## 1.15 Production of sound

Audible sound pressure can be understood as AC component of the absolute air pressure. The generation of sound makes thus in one form or the other a time varying excitation necessary. Possible sound generation mechanisms are:

- abrupt relaxation of compressed air (bursting balloon)
- abrupt gas production (explosion)
- modulated air flow (siren)
- oscillating air column (organ pipe, acoustical laser <sup>26</sup>...)
- vibrating body (loudspeaker membrane, tuning fork)
- abrupt local heating of air (lightening and thunder)

### 1.15.1 Relaxation of compressed air

A possible source to produce an impulse-like sound is a bursting balloon. The balloon filled with air represents a volume of higher pressure. At the moment of bursting this over-pressure can propagate in all directions. Thereby peak levels may exceed 125 dB in a distance of 1 m.

### 1.15.2 Abrupt gas production (explosion)

The muzzle blast of a fire arm is the result of an abrupt gas production. An other example is the air bag widely applied in cars. In case of an accident a small explosion is ignited that inflates a bag to mechanically protect the passenger. On the other hand the inflating bag leads to very high sound pressure peaks that may damage the ear <sup>27</sup>. The linear peak ranges up to 167 dB, the linear event or exposure level is about 139 dB. These values surpass the SUVA limiting values for impulsive noise by 6 to 8 dB.

### 1.15.3 Modulated air flow

A modulated air flow can produce very high sound pressure values. Probably the most common application of this principle is a siren. In its simplest form a siren consists of a perforated rotating disk that controls the passage of an air flow. The speed of revolution and the geometry of the holes in the disk define the frequency of the generated sound.

An other application is the so called airflow speaker. This speaker consists of a unit that contains air under high pressure. By a valve that is controlled by the audio signal an airflow producing very high sound pressure can be established. A major challenge with airflow speakers is the suppression of unwanted flow noise that appears at the nozzle.

### 1.15.4 Oscillating air column

The air column in a tube represents a system of resonances that can be used to generate tones. Here the system of an organ pipe shall be discussed in some detail.

The organ pipe is excited at one end to maintain the oscillation at the resonance frequency while the other end is terminated by a certain impedance  $Z_L$ . It is assumed that the tube has the length  $L$ . The region of interest ranges thus from  $x = 0$  to  $x = L$  where the excitation is at  $x = 0$  (Figure 1.29). In a first step the impedance seen at the input ( $x = 0$ ) will be determined.

It is assumed that the wave length is much larger than the diameter of the tube. With this in mind the sound propagation can be described as an incident and a reflected plane wave running in  $x$ -direction. Sound pressure and sound particle velocity are in phase everywhere, their ratio corresponds to  $\rho c$ .

<sup>26</sup>Echos, The newsletter of the Acoustical Society of America, no. 3, vol 10 (2000).

<sup>27</sup>Beat W. Hohmann, Geh'orsch'aden durch Airbags, Fortschritte der Akustik DAGA 98, p.722-723 (1998).

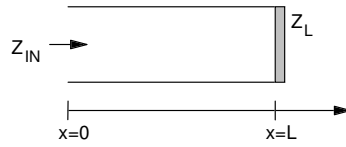


Figure 1.29: Situation for the discussion of the impedances at arbitrary position in the organ tube.

Thermal and viscous losses at the circumference are ignored.

Assuming a harmonic oscillation with angular frequency  $\omega$ , sound pressure and sound particle velocity of the wave running to the right are given as:

$$p_r(x, t) = Ae^{-jkx} e^{j\omega t} \quad (1.160)$$

$$v_r(x, t) = \frac{A}{\rho c} e^{-jkx} e^{j\omega t} \quad (1.161)$$

where

$A$ : amplitude of sound pressure

$k$ : wave number =  $2\pi/\lambda$

Sound pressure and sound particle velocity of the wave running to the left are given as:

$$p_l(x, t) = Be^{jkx} e^{j\omega t} \quad (1.162)$$

$$v_l(x, t) = -\frac{B}{\rho c} e^{jkx} e^{j\omega t} \quad (1.163)$$

where

$B$ : amplitude of sound pressure

It should be noted that the sound particle velocity of the wave running to the left has a negative sign. The superposition of both waves yields the total sound field:

$$p(x, t) = (Ae^{-jkx} + Be^{jkx}) e^{j\omega t} \quad (1.164)$$

$$v(x, t) = \left( \frac{A}{\rho c} e^{-jkx} - \frac{B}{\rho c} e^{jkx} \right) e^{j\omega t} \quad (1.165)$$

At the position  $x = L$  the impedance is known, namely  $Z_L$ :

$$\frac{p(L, t)}{v(L, t)} = \frac{Ae^{-jkL} + Be^{jkL}}{\frac{A}{\rho c} e^{-jkL} - \frac{B}{\rho c} e^{jkL}} = Z_L \quad (1.166)$$

From this the reflection factor (the ratio of the constants  $B$  to  $A$ ) can be determined as

$$\frac{B}{A} = e^{-2jkL} \frac{Z_L - \rho c}{Z_L + \rho c} \quad (1.167)$$

With knowledge of this ratio (Eq. 1.167) the input impedance at the position of excitation  $x = 0$  can be found as

$$Z_{IN} = \frac{p(0, t)}{v(0, t)} = \rho c \frac{1 + \frac{B}{A}}{1 - \frac{B}{A}} = \rho c \frac{Z_L \cos(kL) + j\rho c \sin(kL)}{jZ_L \sin(kL) + \rho c \cos(kL)} \quad (1.168)$$

Regarding the termination of the tube  $Z_L$ , two important cases can be distinguished:

- closed end:  $Z_L = \infty$
- open end:  $Z_L =$  radiation impedance of the opening



For the closed end the input impedance is

$$Z_{\text{IN}} = -j\rho c \cot(kL) \quad (1.169)$$

For the open end and at low frequencies  $k \times \text{tube diameter} \ll 1$  the radiation impedance  $Z_L$  is much smaller than  $\rho c$ . With this follows

$$Z_{\text{IN}} = j\rho c \tan(kL) \quad (1.170)$$

At higher frequencies the radiation impedance can no longer be neglected. This effect can be modeled by an end correction.

The excitation process is air that is blown across a cutting edge. This results in high sound particle velocity and low sound pressure. The organ pipe is in resonance, if this excitation condition is supported by the tube, that is to say  $Z_{\text{IN}} = 0$ . From the relations above the resonance frequencies can be calculated as:

*closed end:*

$$\begin{aligned} \cot(kL) = 0 \rightarrow kL = (2n-1)\frac{\pi}{2} \rightarrow \omega = (2n-1)\frac{\pi c}{2L}, \quad n = 1, 2, \dots \\ \text{the fundamental mode } n = 1 \text{ corresponds to } L = \frac{\lambda}{4} \end{aligned} \quad (1.171)$$

*open end:*

$$\begin{aligned} \tan(kL) = 0 \rightarrow kL = n\pi \rightarrow \omega = n\frac{\pi c}{L}, \quad n = 1, 2, \dots \\ \text{the fundamental mode } n = 1 \text{ corresponds to } L = \frac{\lambda}{2} \end{aligned} \quad (1.172)$$

### 1.15.5 Vibrating bodies

Many sound sources are based on vibrating bodies, such as loudspeaker membranes, string instruments, motors, wheels, and so forth. If the normal component of the surface velocity is known, the sound pressure can be calculated at any point in space by application of the Boundary Element method. The required surface velocity can be measured e.g. with laser vibrometers.

#### Strings

Vibrating strings played an important role in early history of acoustics. Experiments with strings allowed for the discovery of musical intervals and made it possible to establish a relation between the pitch of a musical tone and the number of oscillations per second.

Many instruments contain strings as excitation element. Due to the small cross sectional dimensions, a vibrating string is a very inefficient sound radiator. For improved radiation, the vibrations of the strings are usually coupled to larger areas and bodies. In the following paragraph the wave equation for the transverse motion of a string will be deduced.

Figure 1.30 shows a short segment of a string with the force vectors  $\vec{T}$ . The amplitudes of the force vectors on both sides of the segment have to be equal. If the string is not in its neutral position they do not point exactly in opposite directions. The resulting force component in  $y$  direction acts as a restoring force.

The resulting force component  $F_{\text{res},y}$  in  $y$ -direction is

$$F_{\text{res},y} = T \sin(\theta(x+dx)) - T \sin(\theta(x)) \quad (1.173)$$

where

$\theta(x)$ : angle of the string force at position  $x$

$\theta(x+dx)$ : angle of the string force at position  $x+dx$

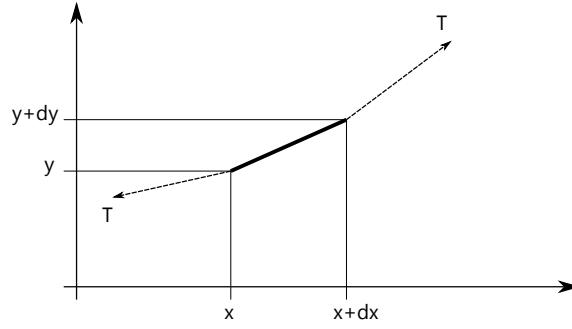


Figure 1.30: Short segment of a string with the force vectors  $T$ .

The function  $\sin(\theta(x + dx))$  can be developed as a Taylor series according to

$$f(x + dx) = f(x) + \frac{\partial f(x)}{\partial x} dx + \dots \quad (1.174)$$

Ignoring the higher order terms, Eq. 1.173 can be written as

$$F_{\text{res},y} = T \sin(\theta(x)) + T \frac{\partial(\sin(\theta(x)))}{\partial x} dx - T \sin(\theta(x)) = T \frac{\partial(\sin(\theta(x)))}{\partial x} dx \quad (1.175)$$

Under the assumption that the displacement of the string is small, the angle  $\theta$  remains small as well. With this we get

$$\sin \theta \approx \tan \theta \approx \frac{\partial y}{\partial x} \text{ for } \theta \rightarrow 0 \quad (1.176)$$

insertion of Eq. 1.176 in Eq. 1.175 yields:

$$F_{\text{res},y} = T \frac{\partial \left( \frac{\partial y}{\partial x} \right)}{\partial x} dx = T \frac{\partial^2 y}{\partial x^2} dx \quad (1.177)$$

With Newton's law the force  $F_{\text{res},y}$  in Eq. 1.177 can be expressed with mass and acceleration:

$$T \frac{\partial^2 y}{\partial x^2} dx = \mu dx a_y = \mu dx \frac{\partial^2 y}{\partial t^2} \quad (1.178)$$

where

$T$ : tension of the string

$\mu$ : density of the string per unit length

$dx$ : length of considered string section

$a_y$ : acceleration in  $y$ -direction

Rearranging Eq. 1.178 finally yields the differential equation of the transverse motion of the string:

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} \quad (1.179)$$

Eq. 1.179 has the same structure as the one dimensional wave equation for sound. Consequently the general solution is given by:

$$y = f_1(ct - x) + f_2(ct + x) \quad (1.180)$$

where

$c$ : propagation velocity =  $\sqrt{\frac{T}{\mu}}$

In Eq. 1.180  $f_1$  and  $f_2$  denote two arbitrary functions. The arguments  $(ct - x)$  and  $(ct + x)$  express that a certain value for  $y$  can be obtained by an adjustment of time or position. This corresponds to two waves running to the left and right. The configurations at both ends of the string define the boundary conditions.

To find harmonic solutions of the wave equation (1.180), the following function in space is put on for  $y$ :

$$y = A \sin(\omega t - kx) + B \cos(\omega t - kx) + C \sin(\omega t + kx) + D \cos(\omega t + kx) \quad (1.181)$$

where

$$k: \text{wave number} = \frac{\omega}{c}$$

For the string of length  $L$  that is clamped on both ends, the boundary conditions are:

$$y(0, t) = 0 \text{ und } y(L, t) = 0 \quad (1.182)$$

From  $y(0, t) = 0$  follows for the parameters in Eq. 1.181:  $C = -A$  and  $D = -B$ .

Using the sum and difference formulas for  $\sin(x)$  and  $\cos(x)$ :

$$\begin{aligned} \sin(a \pm b) &= \sin(a) \cos(b) \pm \cos(a) \sin(b) \\ \cos(a \mp b) &= \cos(a) \cos(b) \pm \sin(a) \sin(b) \end{aligned} \quad (1.183)$$

Eq. 1.181 can be simplified to

$$y(x, t) = -2A \cos(\omega t) \sin(kx) + 2B \sin(\omega t) \sin(kx) = 2 \sin(kx) (B \sin(\omega t) - A \cos(\omega t)) \quad (1.184)$$

The second condition in Eq. 1.182 calls for  $\sin(kL) = 0$ , which means

$$kL = n\pi \text{ for } n = 1, 2, \dots \quad (1.185)$$

From that follows the condition for the angular frequency

$$\omega = n \frac{c\pi}{L} = n \sqrt{\frac{T}{\mu}} \frac{\pi}{L} \quad (1.186)$$

The string clamped on both ends can only vibrate at discrete frequencies. Associated with each frequency is a distribution of oscillation (mode) with regions of maximum and regions with minimum oscillations. However more than one mode is possible simultaneously. The occurrence of the modes depends on the external excitation. The most general solution of the vibrating string is the superposition of all modes.

$$y(x, t) = \sum_{n=1}^{\infty} (A_n \cos(\omega_n t) + B_n \sin(\omega_n t)) \sin(k_n x) \quad (1.187)$$

where

$$\omega_n = n \sqrt{\frac{T}{\mu}} \frac{\pi}{L}$$

$$k_n = n \frac{\pi}{L}$$

$A_n, B_n$ : amplitude factor of the  $n$ -th mode, depending on the excitation

A possible excitation is the plucking of the string. Thereby the string is pulled away from its neutral position at a certain point. As a first approximation the string forms a triangle. After the release the string will oscillate in those modes that were excited by this triangular shape. The corresponding modes can be found by development of the function  $y(x, 0)$  in a Fourier series. If the string is plucked in a distance  $L/m$  from one end, the  $m$ -th mode is missing.

## Rods

In rods different types of vibration can occur:

- longitudinal (in direction of the rod)
- transversal (perpendicular to the rod)
- twisting (torsion)

With exception of the longitudinal vibration the mathematical description is expensive, see e.g. <sup>28</sup>.

<sup>28</sup>Thomas D. Rossing, Neville H. Fletcher, Principles of Vibration and Sound, Springer, 1995.

## Membranes

Membranes are foils that are clamped at the circumference. They represent so to say a two dimensional extension of the one dimensional string. It is assumed that plane of the membrane coincides with the  $xy$ -plane. The deflection from the neutral position is described by the  $z$ -coordinate. Similarly to the case of the string, a wave equation can be formulated for the membranes, describing the transversal vibration. For rectangular membranes the wave equation in cartesian coordinates reads as:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\sigma}{T} \frac{\partial^2 z}{\partial t^2} \quad (1.188)$$

where

$z$ : deflection of the membrane

$x, y$ : coordinates of the membrane point

$\sigma$ : density of membrane as mass per unit area

$T$ : tension of the membrane

The above equation ignores the stiffness of the membrane and the influence of the surrounding air. Analogous to the string, there exist only solutions for discrete frequencies. These modes have to be described by a pair of nonnegative integers  $m, n$ . Figure 1.31 shows a couple of modes for the rectangular membrane.

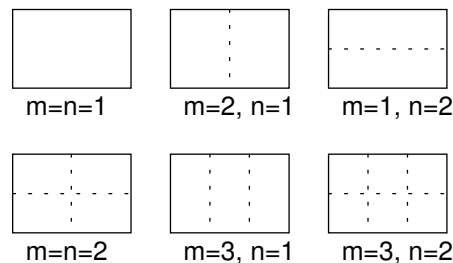


Figure 1.31: Some modes for the rectangular membrane. The sides are clamped, resulting in a boundary condition of vanishing movement. The node lines (dashed) represent regions without movement.

For a discussion of circular membranes see the book by Rossing <sup>29</sup>.

### 1.15.6 Thermo-acoustical machines

Glassblowers know the phenomenon that - under certain circumstances - glass tubes can suddenly produce a loud pure tone when exposed to heat.

As already pointed out above, sound in air is an adiabatic process. This means that a passing sound wave is accompanied by a temperature variation, connected to the momentary pressure. High pressure creates a temperature increase while low pressure leads to a temperature decrease. Of special interest is the case of a standing wave. Thereby air packages move back and forth. The movement in one direction is connected to compression and thus increases temperature. In the other direction the air is relaxing and thus cooling down. By external installation of an appropriate local temperature gradient the oscillation of the standing wave can be excited from outside.

An oscillator of this type can be realized quite easily <sup>30</sup>. Thereby a glass tube with one open and one closed end is used. The tube can thus act as quarter-wave-length resonator. In the fundamental resonance the standing wave in the tube produces a pressure maximum at the closed end and a pressure minimum at the open end. Figure 1.32 shows the movement of the air particles at progressing moments in time.

To stimulate the resonance situation shown in Figure 1.32 an appropriate temperature gradient has to be established. Appropriate means that the implemented temperature gradient supports the

<sup>29</sup>Thomas D. Rossing, Neville H. Fletcher, Principles of Vibration and Sound, Springer, 1995.

<sup>30</sup>Steven L. Garrett, Scott Backhaus, The Power of Sound, American Scientist, vol. 88, no. 6, p.516-525 (2000)

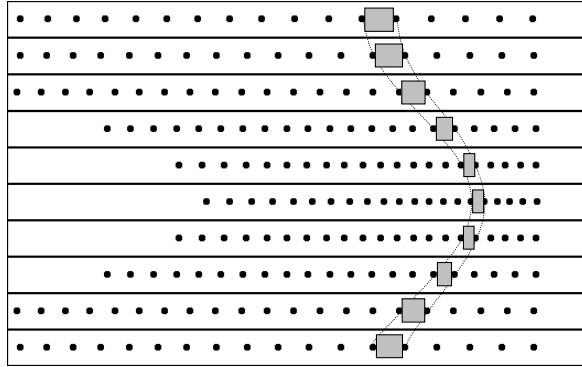


Figure 1.32: Movie representation with progressing time from top to bottom of the movement of air particles in resonance in a tube closed on the right hand end and open at the other end. While moving to the right the air is compressed and heated up while the movement to the left corresponds to a relaxation with associated temperature decrease.

temperature gradient of the standing wave.

Instead of exciting a sound wave by an external temperature gradient, the effect can be reversed. If the standing wave is excited by a vibrating membrane, a temperature gradient is created by the sound wave that be used for heating or cooling.

# Chapter 2

## Acoustical measurements

### 2.1 Introduction

Acoustical measurements can typically be classified as shown in Table 2.1:

task	aim	example
emission measurements (passive)	description of the strength of a source	sound radiation of a lawn mover
measurement at a receiver position (passive)	description of the strength of a source including the propagation to the receiver	road traffic noise measurement in the living room of a resident
measurements of a transmission system (active)	description of a transmission system	measurement of the frequency response of a loudspeaker

Table 2.1: Categories of typical tasks in acoustical measurements.

The complete description of an acoustical process encompasses the specification of the time history of sound pressure and sound particle velocity at each point. Usually for practical questions one can restrict to a few attributes of the sound field. Most often the sound field variable sound pressure is investigated. Indeed sound pressure is significantly easier to measure than sound particle velocity. For sound pressure excellent and accurate transducers (microphones) are available to convert the acoustical signal into an electrical one.

### 2.2 Signal attributes

#### 2.2.1 Overview

As mentioned above it is usually not necessary to represent the complete time history of the variable of interest. Thus the question arises what are meaningful signal attributes that can be extracted from a time signal. Figure 2.1 gives an example of a typical noise-like sound pressure signal  $p(t)$ . In addition sound pressure squared  $p^2(t)$  is shown.

From the time history of a signal as shown in Fig. 2.1, various attributes can be evaluated such as:

- peak value of sound pressure or sound pressure square
- linearly or exponentially time-weighted integrations of sound pressure square
- statistical quantities, e.g. the portion of the signal duration with sound pressure exceeding a certain limit

The most common quantities used in acoustical measurements are integrations over time of sound pressure square. Peak values and statistical quantities play only a minor role. It should be noted that the

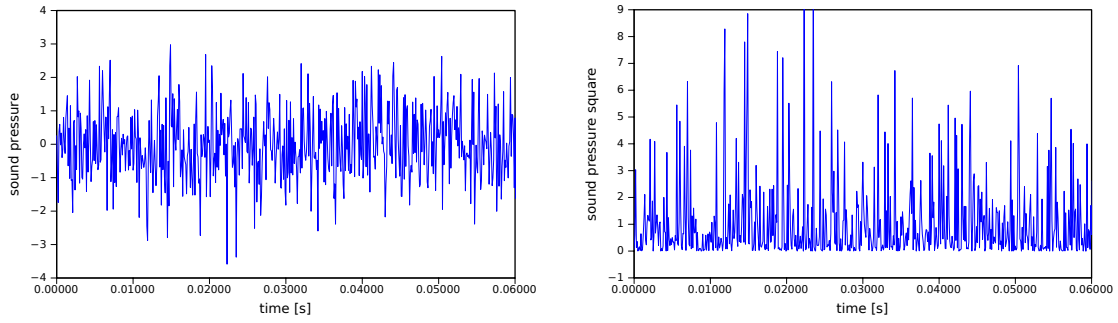


Figure 2.1: Typical time history of a noise-like sound pressure signal (left) and the signal squared (right).

integration over time of sound pressure makes no sense, as this quantity yields 0 in the average. The integration of sound pressure square can be interpreted as a measure for the energy or power of the signal.

Three different integration quantities are used:

*Momentary sound pressure level  $L(t)$ :*

→ logarithmic form of the mean sound pressure square moving average (exponential time weighting)

$$L(t) = 10 \log \left( \frac{1}{RC} \int_{-\infty}^t \frac{p^2(\tau)}{p_0^2} e^{\frac{\tau-t}{RC}} d\tau \right) \quad [\text{dB}] \quad (2.1)$$

where

$RC$ : time constant

$p(\tau)$ : instantaneous sound pressure

$p_0$ : reference sound pressure =  $2 \times 10^{-5}$  Pa

*Equivalent continuous sound pressure level  $Leq$ :*

→ logarithmic form of the mean sound pressure square taken over a certain time frame

$$Leq = 10 \log \left( \frac{1}{T} \int_0^T \frac{p^2(\tau)}{p_0^2} d\tau \right) \quad [\text{dB}] \quad (2.2)$$

where

$T$ : measurement time interval

$p(\tau)$ : instantaneous sound pressure

$p_0$ : reference sound pressure =  $2 \times 10^{-5}$  Pa

*Sound exposure level  $L_E$  or  $SEL$  (former designation):*

→ logarithmic form of the integral of the sound pressure square over a certain time frame and normalized to 1 s.

$$L_E = 10 \log \left( \frac{1}{1 \text{ sec}} \int_0^T \frac{p^2(\tau)}{p_0^2} d\tau \right) \quad [\text{dB}] \quad (2.3)$$

where

$T$ : measurement time interval

$p(\tau)$ : instantaneous sound pressure

$p_0$ : reference sound pressure =  $2 \times 10^{-5}$  Pa

The momentary, moving average sound pressure level exists at any moment in time. It follows the original signal with a more or less pronounced averaging effect depending on the selected time constant. Short peaks are underestimated in their amplitude and overestimated in their pulse width. Typical time constants that are FAST (125 ms) and SLOW (1 s)<sup>1</sup>. The time history of the FAST- or SLOW time weighted momentary sound pressure level is typically evaluated for certain single number attributes such as the maximum value.

The integrations over time windows of arbitrary length become possible when the analyzers got digital microprocessors.  $L_{eq}$  or  $L_E$  both describe as a single value the signal power or signal energy of the selected time interval.

Figure 2.2 shows the different integrations discussed above.

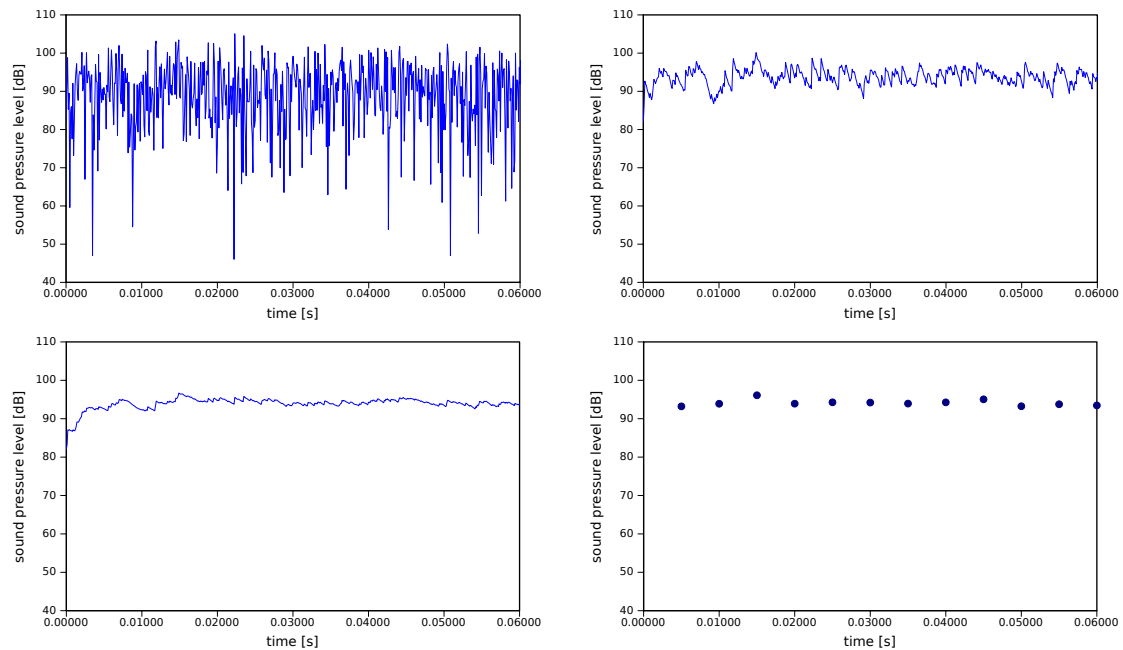


Figure 2.2: Momentary squared sound pressure in dB (top left), momentary sound pressure level with small time constant (top right), momentary sound pressure level with large time constant (bottom left) and equivalent continuous sound pressure (evaluated every 5 ms) (bottom right).

## 2.2.2 Application of the measurement attributes

Depending on the measurement task, different measurement attributes are used. The following list gives some typical examples:

*Momentary sound pressure level  $L$ :*

- maximum level with time constant FAST:  $L_{max, Fast}$  → attribute to describe shooting noise or the passage of road vehicles
- minimum level:  $L_{min}$  → estimation of a stationary signal with occurrence of transient unwanted noise

*Equivalent continuous sound pressure level  $L_{eq}$ :*

- characterization of non-stationary sources and signals

*Sound exposure level  $L_E$ :*

- measurement of single events such as e.g. train passages

<sup>1</sup>IEC Standard 61672, Electroacoustics - Sound level meters, 2002-05.



### 2.2.3 Algorithm to determine the moving square average

In the analogue world the moving average according to Eq. 2.1 can be realized by an RC low-pass filter. The following derivation will end up with the formula for a digital implementation.

Starting point is a time signal (e.g. sound pressure)  $x(t)$ . Then the moving average  $x_{\text{rms}}^2(t)$  of  $x^2(t)$  can be determined as follows:

At time  $t + \Delta t$  the attribute  $x_{\text{rms}}^2$  is given as

$$\begin{aligned} x_{\text{rms}}^2(t + \Delta t) &= \frac{1}{RC} \int_{-\infty}^{t+\Delta t} x^2(\tau) e^{-\frac{t+\Delta t-\tau}{RC}} d\tau \\ &= \frac{1}{RC} \int_{-\infty}^t x^2(\tau) e^{-\frac{t-\tau}{RC}} e^{-\frac{\Delta t}{RC}} d\tau + \frac{1}{RC} \int_t^{t+\Delta t} x^2(\tau) e^{-\frac{t+\Delta t-\tau}{RC}} d\tau \\ &= e^{-\frac{\Delta t}{RC}} x_{\text{rms}}^2(t) + \frac{1}{RC} \int_t^{t+\Delta t} x^2(\tau) e^{-\frac{t+\Delta t-\tau}{RC}} d\tau \end{aligned} \quad (2.4)$$

For  $RC \gg \Delta t$  and  $t < \tau < t + \Delta t$ ,  $e^{-\frac{t+\Delta t-\tau}{RC}}$  can be approximated as 1. Then follows

$$x_{\text{rms}}^2(t + \Delta t) \approx e^{-\frac{\Delta t}{RC}} x_{\text{rms}}^2(t) + \frac{1}{RC} \int_t^{t+\Delta t} x^2(\tau) d\tau \quad (2.5)$$

The integral  $\int_t^{t+\Delta t} x^2(\tau) d\tau$  can be approximated by the area of the rectangle  $\Delta t x^2(t + \Delta t)$ :

$$x_{\text{rms}}^2(t + \Delta t) \approx e^{-\frac{\Delta t}{RC}} x_{\text{rms}}^2(t) + \frac{1}{RC} \Delta t x^2(t + \Delta t) \quad (2.6)$$

The exponential-function  $e^{-\frac{\Delta t}{RC}}$  can be developed into a series. Ignoring the higher order terms we get:

$$e^{-\frac{\Delta t}{RC}} \approx 1 - \frac{\Delta t}{RC} \quad (2.7)$$

And finally

$$\begin{aligned} x_{\text{rms}}^2(t + \Delta t) &\approx \left(1 - \frac{\Delta t}{RC}\right) x_{\text{rms}}^2(t) + \frac{1}{RC} \Delta t x^2(t + \Delta t) \\ &= x_{\text{rms}}^2(t) + \frac{x^2(t + \Delta t) - x_{\text{rms}}^2(t)}{\frac{RC}{\Delta t}} \end{aligned} \quad (2.8)$$

With Eq. 2.8 the moving average at time  $t + \Delta t$  is expressed as the former value at time  $t$  and a correction term.  $\Delta t$  can be understood as sampling interval of the digital representation of the signal. By evaluating Eq. 2.8 the moving average can easily be updated for every new signal sample. It may be beneficial to chose  $\Delta t$  in such a way that  $RC/\Delta t$  corresponds to a power of 2. In this case the division reduces to a simple shift operation.

## 2.3 Filters

Up to now it was assumed that the signal attributes are evaluated for the sound pressure time history. However it is often of interest to take into account the frequency composition of the signal. For that reason acoustical measurements often use filters to apply an appropriate frequency weighting or select a limited frequency range for the analysis. The signal attributes introduced above can then be applied in the same way for filtered signals.

### 2.3.1 Weighting filters

The sensitivity of the human hearing depends strongly on frequency. For that reason frequency weighting filters have been defined to simulate the frequency response of the ear. However a serious difficulty is the fact, that the frequency response of the ear depends on sound pressure level. At lower levels the frequency dependency is more pronounced than at higher levels. For that reason several filters were originally defined. They got the names *A*, *B* and *C*.<sup>2,3</sup> The *A*-filter was designed for low levels, the *B*-filter for medium levels and the *C*-filter for high levels. The *B*-filter has disappeared completely. Most often used today is the *A*-filter, the *C*-filter is applied in special cases only. Evaluations performed with the *A*-filter are labeled with the unit dB(A).

The transfer function for the *C*-filter is given by<sup>4</sup>:

$$T_{C\text{-Filter}}(s) = \frac{Ks^2}{(s + \omega_1)^2(s + \omega_2)^2} \quad (2.9)$$

where

$$\begin{aligned} \omega_1 &= 1.29 \times 10^2 \text{ [rad/sec]} \\ \omega_2 &= 7.67 \times 10^4 \text{ [rad/sec]} \end{aligned}$$

With  $f$  as frequency in Hz, the amplitude in dB of the transfer function of the *C*-filter is

$$\text{C-weighting} = 20 \log \left( \frac{1.498 \times 10^8 f^2}{(f^2 + 20.6^2)(f^2 + 12200^2)} \right) \quad (2.10)$$

The transfer function of the *A*-filter corresponds to the one of the *C*-filter but complemented by two zeros at the origin and two simple poles:

$$T_{A\text{-filter}}(s) = \frac{Ks^4}{(s + \omega_1)^2(s + \omega_2)^2(s + \omega_3)(s + \omega_4)} \quad (2.11)$$

where

$$\begin{aligned} \omega_1 &= 1.29 \times 10^2 \text{ [rad/sec]} \\ \omega_2 &= 7.67 \times 10^4 \text{ [rad/sec]} \\ \omega_3 &= 6.77 \times 10^2 \text{ [rad/sec]} \\ \omega_4 &= 4.64 \times 10^3 \text{ [rad/sec]} \end{aligned}$$

The amplitude in dB of the transfer function results in

$$\text{A-weighting} = 20 \log \left( \frac{1.873 \times 10^8 f^4}{(f^2 + 20.6^2)(f^2 + 12200^2) \sqrt{f^2 + 107.7^2} \sqrt{f^2 + 737.9^2}} \right) \quad (2.12)$$

Figure 2.3 shows the amplitude responses of the *A*- and *C*-filter.

In Figure 2.4 a possible *RC* realization of an *A*-filter is depicted. The attenuation of this filter at 1 kHz is 3.2 dB, which means an additional amplification of 3.2 dB is needed, preferably at the output to guarantee a high-resistance filter load.

### 2.3.2 Filters for frequency analysis

The frequency analysis process evaluates signal contributions that lie within a certain frequency band. For a complete analysis the whole frequency range of interest is divided into a series of bands that follow each other seamlessly. The signal attributes discussed above are then evaluated for each band individually. The frequency axis can be divided in different ways. For acoustical applications linear and logarithmic partitioning are very common. A linear partitioning results in filters of constant absolute bandwidth, the logarithmic partitioning corresponds to filters of constant relative bandwidth.

<sup>2</sup>ISO Norm 10845 Acoustics - Frequency weighting A for noise measurements. Draft 1995.

<sup>3</sup>ISO Norm 14938 Acoustics - Revision of B- and C-weightings and Lin-response for noise measurements. 1998.

<sup>4</sup>IEC Standard 61672, Electroacoustics - Sound level meters, 2002-05.

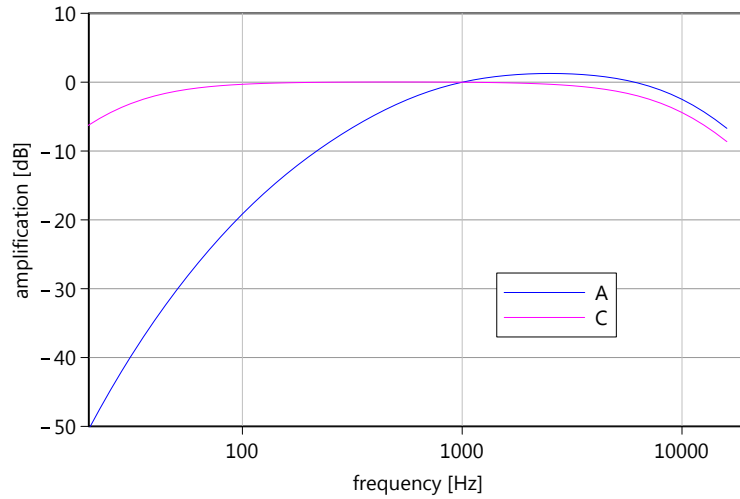


Figure 2.3: Frequency response of the A-(blue) and C-(purple) filter. The A-weighting shows a small amplification between 1 and 6 kHz.

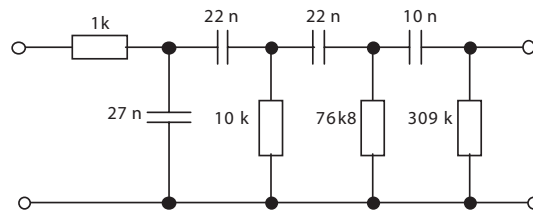


Figure 2.4: Possible realization of the A-weighting as a passive  $RC$  filter fulfilling the requirements of class 1 according to IEC 61672. An additional amplification of 3.2 dB is needed.

### Filters of constant relative bandwidth

Filters of constant relative bandwidth have a width  $B$  that is proportional to the center frequency  $f_m$  of the filter. As a standardized basis a center frequency of 1 kHz was defined. With this the complete series can be developed:

$$B = f_m g \quad (2.13)$$

where

$B$ : bandwidth, evaluated at the -3 dB points

$f_m$ : center frequency of the filter

$g$ : constant

The bandwidth is distributed logarithmically around the center frequency:

$$\begin{aligned} f_o &= f_m h \\ f_u &= f_m \frac{1}{h} \\ f_o - f_u &= B \\ g &= h - \frac{1}{h} \end{aligned} \quad (2.14)$$

where

$f_o$ : upper limiting frequency

$f_u$ : lower limiting frequency

$h$ : constant

With the condition that the filters follow each other seamlessly, the  $n$ -th and the  $n + 1$ -th filter are specified as:

$$f_{o,n} = f_{u,n+1} \text{ or } \frac{f_{m,n+1}}{f_{m,n}} = h^2 \quad (2.15)$$

For the center frequency of the  $n$ -th filter follows:

$$f_{m,n} = 1000(h^2)^n \text{ for } n = \dots - 3, -2, -1, 0, 1, 2, \dots \quad (2.16)$$

The most important filters of this type are octave and third-octave filters<sup>5</sup>. Third octave bands are of special interest as this partitioning of the frequency axis is related to human perception (critical bands). The constants  $g$  and  $h$  have to be chosen according to Table 2.2.

	$g$	$h$	$h^2$
octave filter	0.705	$10^{\frac{3}{20}}$	$10^{\frac{3}{10}}$
third-octave filter	0.231	$10^{\frac{1}{20}}$	$10^{\frac{1}{10}}$

Table 2.2: Values of the constants  $g$  and  $h$  for octave and third-octave filters.

Octave filters have a bandwidth of about 70% of the center frequency. The bandwidth of a third-octave filter is about 23% of the center frequency. Table 2.3 shows the standardized octave and third octave filter series for the audio range from 16 Hz to 16 kHz. It should be noted that the steepness of the filters is finite, meaning that several filters show a response even in case of narrow band signals. Nowadays frequency analyzers are available that can evaluate different signal attributes in third octave bands simultaneously in real-time.

### Filters of constant absolute bandwidth

Filters with constant absolute bandwidth have a fixed bandwidth independent of the center frequency. Narrow band filters with typical bandwidths of a few Hz belong to this category, as well as FFT analyzers. This sort of analysis is typical for technical tasks such as the investigation of the frequency of a pure tone signal component.

## 2.4 Uncertainty of measurements

Acoustical signals are often noise-like and thus have random character. If only a limited time is available, the exact determination of the signal power or the RMS (root mean square) is impossible. Starting point for the further discussion is an analog, noise-like signal. It is then assumed that a finite number of samples are taken from the signal and based on these samples a RMS value is calculated. This evaluation shows a fundamental uncertainty (Figure 2.5) that depends on the time window and the signal or analysis bandwidth as will be demonstrated below.

### 2.4.1 Degrees of freedom of a bandlimited random signal

A consequence of the frequency limitation of a random signal is the fact that two samples lying close to each other on the time axis are no longer statistically independent. The narrower the frequency band, the more the time between the samples has to be increased to guarantee statistical independency. A sample that is not statistically independent relative to the preceding one doesn't yield relevant information and can thus be omitted.

From a random signal  $u(t)$  of bandwidth  $B$ ,  $n$  statistically independent samples can be taken within a time frame  $T$ <sup>6</sup>:

$$n = 2BT \quad (2.17)$$

<sup>5</sup>IEC 61260: Electroacoustics - Octave-band and fractional-octave-band filter, 2014.

<sup>6</sup>Jens Trampe Broch, Principles of Analog and Digital Frequency Analysis, Tapir, 1981.

octave band			third octave band		
$f_u$	$f_m$	$f_o$	$f_u$	$f_m$	$f_o$
11.3	16	22.6	11.1	12.5	14.0
			14.3	16	18.0
			17.8	20	22.4
22.3	31.5	44.5	22.3	25	28.1
			28.1	31.5	35.5
			35.7	40	44.9
44.5	63	89.1	44.6	50	56.1
			56.1	63	70.7
			71.3	80	89.8
88.4	125	177	89.0	100	112
			111	125	140
			143	160	180
177	250	354	178	200	224
			223	250	281
			281	315	353
354	500	707	357	400	449
			446	500	561
			561	630	707
707	1000	1410	713	800	898
			890	1000	1120
			1110	1250	1400
1410	2000	2830	1430	1600	1800
			1780	2000	2240
			2230	2500	2810
2830	4000	5660	2810	3150	3530
			3570	4000	4490
			4460	5000	5610
5660	8000	11300	5610	6300	7070
			7130	8000	8980
			8900	10000	11200
11300	16000	22600	11100	12500	14000
			14300	16000	18000
			17800	20000	22400

Table 2.3: Standard octave and third octave filters with their nominal center and their lower and upper limiting frequencies.

The variable  $n$  denotes the degrees of freedom of the signal  $u(t)$  in the time window  $T$ . Taking into account that a bandlimited signal can be interpreted as an amplitude modulated carrier and that the information is contained in the modulation, Eq. 2.17 follows directly from the sampling theorem.

## 2.4.2 Expectation value and variance of various functions of statistically independent samples

Here a gaussian random signal  $u(t)$  is assumed with expectation value = 0 and variance = 1 ( $u_{\text{rms}}^2 = 1$ ). From this signal a certain number of statistically independent samples  $u_i$  are taken. The set of the samples corresponds to the random variable  $U$ .

### Amplification

An amplification of the signal  $u(t)$  by a factor  $\alpha$  results in an random variable  $U'$  where

$$\begin{aligned}
 u'_i &= \alpha u_i \\
 \text{expectation value}(U') &= 0 \\
 \text{variance}(U') &= \alpha^2
 \end{aligned}
 \tag{2.18}$$

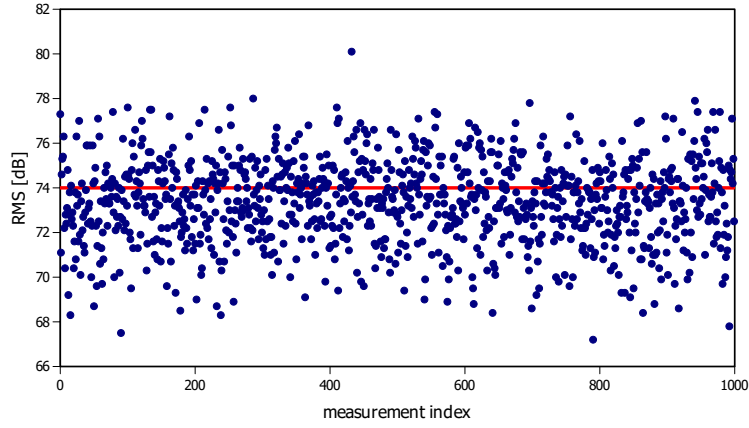


Figure 2.5: Spread of Leq-measurements of pink noise for an analysis bandwidth of 10 Hz and an integration time of 0.5 s. The values lie asymmetrical relative to the *true* average value (red).

#### Summation over $u_i$

Based on the random variable  $U$ , a new quantity  $U''$  is generated by summation over  $n$  samples of  $U$ . For  $U''$  follows:

$$\begin{aligned}
 u_i'' &= \sum_{i=1}^m u_i \\
 \text{expectation value}(U'') &= 0 \\
 \text{variance}(U'') &= m
 \end{aligned} \tag{2.19}$$

#### Summation over $u_i^2$

Based on  $U$  a new quantity  $U'''$  is determined by summation over  $m$  squared values of the samples.  $U'''$  is  $\chi^2$  distributed with:

$$\begin{aligned}
 u_i''' &= \sum_{i=1}^m u_i^2 \\
 \text{expectation value}(U''') &= m \\
 \text{variance}(U''') &= 2m
 \end{aligned} \tag{2.20}$$

Fig. 2.6 shows the density function  $f(y)$  of the  $\chi^2$  distribution for different values of the parameter  $m$  (degrees of freedom) <sup>7</sup>.

### 2.4.3 Uncertainty of the calculation of the root mean square

The first step in the calculation of the RMS (root mean square) of a signal  $u(t)$  is the determination of the available number of independent samples. For a fixed time frame  $T$  and an analysis bandwidth  $B$  this number corresponds to degrees of freedom  $n = 2BT$  according to Eq. 2.17. The square of the RMS value is found as summation of the  $n$  squared samples and division by the number  $n$ .

It is assumed that the variance of the signal under investigation  $u(t)$  equals 1. The uncertainty of the sum  $S$  of the  $n$  samples can then be estimated by the quantiles of the corresponding  $\chi^2$  distribution. The quantile  $\chi_{n,1-\alpha}^2$  is the value for  $S$  that is exceeded with a probability  $\alpha$ . Table 2.4 gives some quantiles of the  $\chi^2$  distribution.

From Table 2.4 follows finally the uncertainty of the RMS value in the decibel scale. For that purpose an upper and lower bound  $\Delta+$ ,  $\Delta-$  are determined that cover the measurement value with the probability

<sup>7</sup>Hubert Weber, Einführung in die Wahrscheinlichkeitsrechnung und Statistik für Ingenieure, Teuber, 1992.

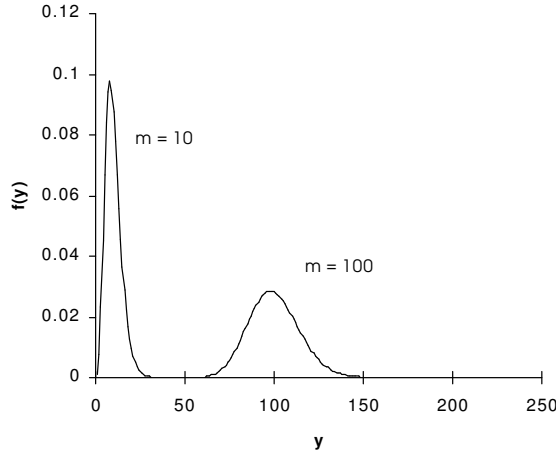


Figure 2.6: Density function  $f(y)$  of the  $\chi^2$  distribution for two values of  $m$  (degrees of freedom). The area under the curve evaluated up to a certain threshold  $y$  corresponds to the probability that the  $\chi^2$  distributed random variable is smaller or equal to  $y$ .

$n$	$\chi_{n,0.005}^2$	$\chi_{n,0.010}^2$	$\chi_{n,0.050}^2$	$\chi_{n,0.100}^2$	$\chi_{n,0.900}^2$	$\chi_{n,0.950}^2$	$\chi_{n,0.990}^2$	$\chi_{n,0.995}^2$
10	2.156	2.558	3.940	4.865	15.99	18.31	23.21	25.19
100	67.33	70.07	77.93	82.36	118.5	124.3	135.8	140.2
1000	888.5	898.9	927.6	943.1	1058	1075	1107	1119

Table 2.4: Quantiles of the  $\chi^2$  distribution where  $n$  corresponds to the degrees of freedom.

$p$ . It is assumed that the measurement value lies with  $p/2$  below the lower bound and with  $p/2$  above the upper bound. The bounds are then found as

$$\begin{aligned}\Delta- &= 10 \log \left( \frac{\chi_{n,[(1-p)/2]}^2}{n} \right) \\ \Delta+ &= 10 \log \left( \frac{\chi_{n,[(1+p)/2]}^2}{n} \right)\end{aligned}\quad (2.21)$$

Table 2.5 shows a few corresponding bounds, calculated with Eq. 2.21.

$n$	$p = 0.90$	$p = 0.99$
10	-4.0 ... + 2.6 dB	-6.6 ... + 4.0 dB
100	-1.1 ... + 0.9 dB	-1.7 ... + 1.5 dB
1000	-0.3 ... + 0.3 dB	-0.5 ... + 0.5 dB

Table 2.5: Ranges of uncertainty in the RMS calculation of noise-like signal as a function of the degrees of freedom  $n$  for the probabilities  $p$  of 90 and 99%.

The derivation above is based on the RMS determination over a fixed time frame  $T$ . It can be shown<sup>8</sup> that for a moving average RMS calculation with time constant  $RC$  the same uncertainty is obtained for

$$2RC = T \quad (2.22)$$

<sup>8</sup>C. G. Wahrmann, J. T. Broch, On the Averaging Time of RMS Measurements, B&K Technical Review, No. 2 (1975).

## 2.5 Measurement instruments

### 2.5.1 Microphones

Microphones are transducers that transform an acoustical signal into an electrical one. For measurement purposes only omnidirectional pressure sensitive condenser microphones are used. However towards high frequencies, pressure microphones lose their omnidirectionality if the sound wave length is in the same order of magnitude as the microphone diameter. This defines an upper frequency limit. High frequency sound waves hitting the microphone in direction of the membrane normal produce a pressure pile-up which corresponds to an increase in sensitivity. This deviation from a flat frequency response can range up to 10 dB. Such a microphone can be used without further measures only for sound incident direction parallel to the membrane or in small cavities where no wave propagation takes place. Consequently these microphones are called *pressure response types*.

Microphones can be designed for usage under normal incident direction by a compensation of the above mentioned effect by appropriate frequency dependent attenuation. These microphones are called *free field response types*. They are more common than pressure response microphones. Fig. 2.7 shows the above mentioned pressure pile-up in form of the frequency response of a pressure response microphone for different sound incident directions.

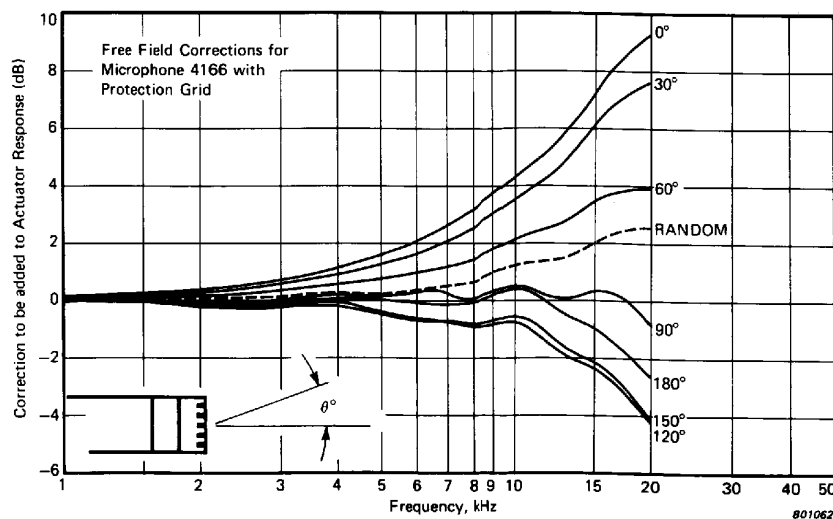


Figure 2.7: Frequency response of a 1/2" pressure response microphone for different sound incident directions (B&K 4166).

Some measurement instruments allow for a selection of the incident direction dependent frequency correction by the user. So it becomes e.g. possible to measure with a free field microphone in a diffuse field with sound incidence equally distributed over all directions.

The two most important properties of a measuring microphone are:

- dynamic range (lower limit defined by self noise, upper limit given by a specific level of distortion)
- frequency range

Regarding these two properties no ideal microphone exists. The optimization of one parameter results in a deterioration of the other. Table 2.6 shows specifications of typical measuring microphones.

### 2.5.2 Calibrators

Calibrators are devices that can be mounted on microphones and produce a highly stable and reproducible sound pressure. Calibrators are used prior to a measurement to calibrate the microphone and the instrument. There are two common types:

- pistonphone



microphone diameter in " (1 " = 2.5 cm)	dynamic range	frequency range
1"	10 dB(A)..146 dB(A)	2 Hz ...18 kHz
1/2"	15 dB(A)..146 dB(A)	2 Hz ...20 kHz
1/4"	29 dB(A)..164 dB(A)	2 Hz ...100 kHz

Table 2.6: Specifications of typical measuring microphones of varying diameter.

- acoustical calibrator

The pistonphone generates the reference sound pressure by the movement of two small pistons with extremely precise lift. It operates at a frequency of 250 Hz and produces a nominal sound pressure level of 124 dB (+/- 0.15 dB). As the produced sound pressure depends on the density of the air a correction for the ambient air pressure is necessary.

The acoustical calibrator generates the reference sound pressure by aid of a small loudspeaker. Usually a frequency of 1 kHz is used, the sound pressure level is typically 94 dB and possibly 114 dB with a reproducibility of +/- 0.3 dB. The excitation frequency of 1 kHz has the advantage that it doesn't matter if the A-filter is involved as the A-filter is transparent at 1 kHz.

### 2.5.3 Sound level meter

The sound level meter is the standard measuring instrument of the acoustician. Today's instruments operate digitally. They measure sound pressure and allow for the evaluation of a variety of signal attributes such as maximum and minimum levels, equivalent energy levels and event levels. Figure 2.8 shows the block diagram of a sound level meter.

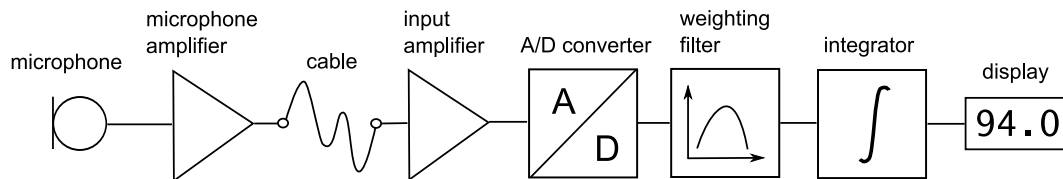


Figure 2.8: Block diagram of a sound level meter.

*Functional units of a sound level meter:*

**microphone and amplifier** the microphones used are omnidirectional condenser microphones, usually prepolarized.

**cable** the microphone cable represents a significant load for the microphone capsule. To drive such a load, a microphone amplifier is absolutely necessary. Long cables can lead to nonlinear distortions at high levels and high frequencies.

**input amplifier** the input amplifier allows for a stepwise adaptation of the measuring range to the signal. The dynamic range of sound level meters is typically in the order of 80 dB.

**weighting filter** A- or C-weighting can be applied to account for the frequency response of the human hearing. Some instruments allow to insert external filters.

**integrator** different signal attributes are evaluated simultaneously and stored for the presentation in the display.

**display** indication of the selected signal attribute.

The International Electrotechnical Commission (IEC) has specified requirements for class 1 (precision) and class 2 sound level meters<sup>9</sup>. Measurements in connection with the Swiss noise legislation<sup>10</sup> have to

<sup>9</sup>IEC Norm 61672 Electroacoustics - Sound level meters, 2002-05.

<sup>10</sup>LSV: Schweizerische L<sup>1</sup>armschutzverordnung, [http://www.admin.ch/ch/d/sr/c814\\_41.html](http://www.admin.ch/ch/d/sr/c814_41.html)

fulfill the requirements according to class 1. Furthermore the instruments need approval from METAS, the Federal Office of Metrology. All instruments have to be initially calibrated by METAS. Every two years the instruments need a recalibration by METAS or by a certified body.

#### 2.5.4 Level recorders

Level recorders can register the level time history of an acoustical signal. Today's level recorders operate digitally. They write the information in a memory for further data processing and evaluation.

#### 2.5.5 Analyzers for level statistics

Analyzers for level statistics allow for the evaluation of statistical quantities such as  $L1$  or  $L50$ . They indicate the levels that are exceeded during 1% ( $L1$ ) or 50% ( $L50$ ) of the measurement time. In today's practice, statistical levels play a minor role. One reason is the fact that due to nonlinear behavior it is not possible to perform calculations based on these quantities.

#### 2.5.6 Frequency analyzers

With help of frequency analyzers it is possible to investigate the spectral contents of a signal. In many areas of signal processing FFT analyzers are common. For acoustical applications on the other hand, often a frequency resolution that is constant relative to frequency is preferred (e.g. octave and third-octave bands). For special purposes even smaller bandwidths (one sixth or one twelfth of an octave) are available. Frequency analyzers are capable to perform the bandpass filtering in real-time simultaneously in a range from 20 Hz to 20 kHz. Figure 2.9 shows a hand-held two-channel analyzer with a maximal temporal resolution of 5 ms.



Figure 2.9: Example of a hand-held third-octave band analyzer.

#### 2.5.7 Sound recorders

It is often useful to record the microphone signal with a sound recorder for possible additional subsequent analysis. Today's state of the art are portable digital recorders. They offer a frequency range up to at least 20 kHz and a dynamic range of 90 dB or more. To establish a relation to an absolute signal level, the calibration tone is recorded at the beginning of a measurement. A repetition at the end of the recording allows for a control that the properties of the measurement chain haven't changed.

## 2.6 Special measurement tasks

### 2.6.1 Sound intensity measurements

Sound intensity meters can capture and evaluate sound intensity. Sound intensity is a vector quantity and has thus an orientation. The intensity can be calculated as product of sound pressure and sound particle velocity (Eq. 2.23).

$$|\vec{I}| = \overline{p(t) |\vec{v}(t)|} \quad (2.23)$$

While the measurement of sound pressure is relatively simple, the sound particle velocity is much more difficult to capture. A interesting development in this context is the microflown transducer<sup>11,12,13</sup>. The principle behind the microflown transducer is a of hot-wire anemometer reacting directly on the sound particle velocity. The transducer can be built with dimensions much smaller than the wave lengths of interest in the audio range. However the frequency range is limited towards high frequencies by the fact that the heating and cooling of the wires needs some time due to their thermal capacity.

Still a common method to evaluate sound particle velocities is the two microphone technique. It uses the relationship between the temporal derivative of the sound particle velocity and the local derivative of the sound pressure:

$$\rho \frac{\partial v_x}{\partial t} = - \frac{\partial p}{\partial x} \quad (2.24)$$

where  $v_x$  is the sound particle component in the  $x$ -direction. Integration yields

$$v_x = - \frac{1}{\rho} \int \frac{\partial p}{\partial x} dt \quad (2.25)$$

The partial derivative of sound pressure relative to the  $x$  component can be approximated by a finite difference:

$$\frac{\partial p}{\partial x} = \frac{p(x + \Delta x) - p(x)}{\Delta x} \quad (2.26)$$

where  $p(x)$  and  $p(x + \Delta x)$  correspond to the sound pressure at positions  $x$  and  $x + \Delta x$ . The approximation by a difference is valid only if  $\Delta x$  is much smaller than the projection of the wave length onto the  $x$  axis.

With Eq. 2.26 inserted in Eq. 2.25 and Eq. 2.25 in Eq. 2.23 the  $x$ -component of the intensity finally becomes - expressed in  $p(x)$  and  $p(x + \Delta x)$

$$I_x = \frac{1}{T} \int_0^T \left( - \frac{1}{2} \frac{1}{\rho \Delta x} (p(x) + p(x + \Delta x)) \int p(x) - p(x + \Delta x) dt \right) d\tau \quad (2.27)$$

where

$T$ : time of integration (averaging)

The availability of sound intensity allows for an elegant measurement of the sound power of a source<sup>14</sup>. To do so the sound source is surrounded by a closed virtual surface. At representative points on this surface the normal component of sound intensity is measured. By multiplication with the corresponding areas and summation the total emitted sound power of the source is determined.

<sup>11</sup>Jörg Sennheiser, MICRO-MINIATURIZED MICROPHONE FOR COMMUNICATION APPLICATIONS, 2nd Convention of the EAA, Berlin, 1999.

<sup>12</sup>W.F. Druyvesteyn, H.E. de Bree, A Novel Sound Intensity Probe Comparison with the Pair of Pressure Microphones Intensity, Journal of the Audio Engineering Society, vol. 48, p.49-56 (2000).

<sup>13</sup>R. Raangs et al., A Low-Cost Intensity Probe, Journal of the Audio Engineering Society, vol. 51, p.344-357 (2003).

<sup>14</sup>ISO Norm 9614-1,2 Acoustics - Determination of sound power levels of noise sources using sound intensity; Measurement at discrete points and measurement by scanning. 1993, 1996.

## Additional sound power measurement strategies

If no intensity measurement is available, the sound power of a source can be estimated by pure sound pressure measurements alone:

A first method is to install the source in a reverberant room and to measure the sound pressure in the diffuse field. From this sound pressure and with knowledge of the sound absorption in the reverberant room, the sound power can be determined<sup>15, 16</sup>.

In the second arrangement the source is installed in an absorbing environment above a reflecting ground. This can be in an anechoic room or outdoors. At several positions in defined distance from the source the sound pressure is evaluated. If the distance from the microphones to the source is large enough so that near field effects can be neglected, the sound particle velocity can be deduced from sound pressure. The sound power of the source is then evaluated analogously to the case where intensity is measured directly<sup>17, 18, 19, 20</sup>.

The third method is based on the comparison of the source under consideration with a reference source of known sound power. The sound pressures produced by the two sources are measured in the diffuse field of an environment with not too much absorption. The ratio of the square of the two sound pressure values corresponds to the ratio of the sound power of the two sources<sup>21</sup>.

## 2.6.2 System identification

### General

A common task in the field of acoustics is the description of the transmission properties of systems with an input  $x$  and an output  $y$ . In many systems the input and output are different physical quantities, as e.g. in case of a loudspeaker with an electrical input and an acoustical output. Here it is assumed that the systems are linear and time invariant which means that they don't change their properties over time.

There are two fundamental possibilities for the description of such a system. In the time domain it is the impulse response  $h(t)$ , in the frequency domain the frequency response  $H(\omega)$ . Both representations describe the system completely. By help of the Fourier transformation they can be converted one into the other.

$$H(\omega) = \int_{-\infty}^{+\infty} h(t)e^{-j\omega t} dt \quad (2.28)$$

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\omega)e^{j\omega t} d\omega \quad (2.29)$$

In the time domain the output  $y(t)$  of the system is given as the convolution of the input  $x(t)$  with the impulse response  $h(t)$ :

$$y(t) = \int_{-\infty}^t x(\tau)h(t - \tau)d\tau \quad (2.30)$$

<sup>15</sup>ISO Norm 3741 Acoustics - Determination of sound power levels of noise sources using sound pressure. Precision methods for reverberation rooms. Draft 1998.

<sup>16</sup>ISO Norm 3742 Acoustics - Determination of sound power levels of noise sources. Precision methods for discrete-frequency and narrow-band sources in reverberation rooms. 1988.

<sup>17</sup>ISO Norm 3744 Acoustics - Determination of sound power levels of noise sources using sound pressure. Engineering method in an essentially free field over a reflecting plane. 1994.

<sup>18</sup>ISO Norm 3745 Acoustics - Determination of sound power levels of noise sources. Precision methods for anechoic and semi-anechoic rooms. 1977.

<sup>19</sup>ISO Norm 3746 Acoustics - Determination of sound power levels of noise sources using sound pressure. Survey method using an enveloping measurement surface over a reflecting plane. 1995.

<sup>20</sup>ISO Norm 3748 Acoustics - Determination of sound power levels of noise sources. Engineering method for small, nearly omnidirectional sources under free-field conditions over a reflecting plane. 1983.

<sup>21</sup>ISO Norm 3747 Akustik - Bestimmung der Schallleistungspegel von Geräuschausgleichsquellen aus Schalldruckmessungen - Vergleichsverfahren zur Verwendung unter Einsatzbedingungen. Entwurf 1998.

In the frequency domain the system output  $Y(\omega)$  corresponds to the product of the input  $X(\omega)$  and the frequency response  $H(\omega)$ :

$$Y(\omega) = X(\omega)H(\omega) \quad (2.31)$$

A serious difficulty with the practical measurement of the properties of a system is the presence of unwanted noise. Usually the noise adds at the output of the system as depicted in Fig. 2.10. Only the signal  $v(t)$  can be measured as superposition of the system output  $y(t)$  and the noise  $n(t)$ .

The question arises, in which way a statement about the system output  $y(t)$  is possible. If it is sufficient to determine the signal power of  $y$ , the noise power  $n_{rms}^2$  can be estimated (with input  $x(t) = 0$ ) and subtracted from  $v_{rms}^2$ , yielding an estimate for  $y_{rms}^2$ . This works under the assumption that no correlation exists between the unwanted noise  $n(t)$  and the system input  $x(t)$ . Furthermore it is necessary that  $n(t)$  is stationary, meaning it doesn't change its properties over time.

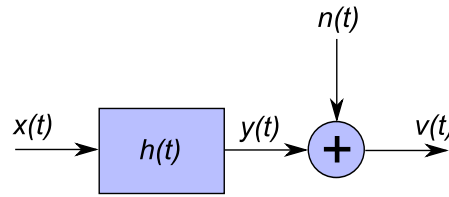


Figure 2.10: Identification of the system  $h$  with additional superposition of unwanted noise at the output.

A more sophisticated approach is the usage of correlation functions. Hereby auto correlation functions  $R_{xx}(\tau)$  and cross correlation functions  $R_{xy}(\tau)$  are needed according to the following definitions:

$$R_{xx}(\tau) = \frac{1}{2T} \int_{-T}^{+T} x(t-\tau)x(t)dt \quad (T \rightarrow \infty) \quad (2.32)$$

$$R_{xy}(\tau) = \frac{1}{2T} \int_{-T}^{+T} x(t-\tau)y(t)dt \quad (T \rightarrow \infty) \quad (2.33)$$

The cross correlation between the input and output of a linear system with impulse response  $h(t)$  can be written as (with  $T \rightarrow \infty$ ):

$$\begin{aligned} R_{xy}(\tau) &= \frac{1}{2T} \int_{-T}^{+T} x(t-\tau) \int_0^{\infty} x(t-u)h(u)du dt \\ &= \int_0^{\infty} h(u) \frac{1}{2T} \int_{-T}^{+T} x(t-\tau)x(t-u)dt du \\ &= \int_0^{\infty} h(u) \frac{1}{2T} \int_{-T}^{+T} x(t-(\tau-u))x(t)dt du \\ &= \int_0^{\infty} h(u)R_{xx}(\tau-u)du \\ &= h(t) * R_{xx}(\tau) \end{aligned} \quad (2.34)$$

where

\*: convolution

The relation (2.34) is called Wiener-Hopf equation. With known auto correlation function of the stimulus  $x(t)$  the system impulse response can be determined from the measured cross correlation function

between the input and output. The big advantage of evaluation of the cross correlation function is the fact that uncorrelated noise cancels out perfectly in the limiting case of infinite measuring time. Applied to the system identification task from Fig. 2.10 it can be concluded that  $R_{xy} = R_{xv}$  and therefore

$$R_{xx}(\tau) * h(t) = R_{xv}(\tau) \quad (2.35)$$

In the limiting case of an infinitely long measurement, the impulse response  $h(t)$  can thus be determined perfectly with help of Eq. 2.35.

The Wiener-Khinchine theorem states that the auto correlation function  $R_{xx}(\tau)$  and the power spectrum  $G_{xx}(\omega)$  as well as the cross correlation function  $R_{xy}(\tau)$  and the cross power spectrum  $G_{xy}(\omega)$  are related by the Fourier transformation. Consequently Eq. 2.34 can be translated into the frequency domain as

$$H(\omega) = \frac{G_{xy}(\omega)}{G_{xx}(\omega)} \quad (2.36)$$

where

$G_{xx}(\omega)$  : power spectrum of the input signal  $x(t)$

$G_{xy}(\omega)$  : cross power spectrum of the input signal  $x(t)$  and output signal  $y(t)$

The power spectrum  $G_{xx}(\omega)$  and the cross power spectrum  $G_{xy}(\omega)$  are given as

$$G_{xx}(\omega) = E[X^*(\omega)X(\omega)] \quad (2.37)$$

$$G_{xy}(\omega) = E[X^*(\omega)Y(\omega)] \quad (2.38)$$

where

$E$  : expectation value

$X(\omega)$  : Fourier transform of the input signal  $x(t)$

$Y(\omega)$  : Fourier transform of the output signal  $y(t)$

\* : complex conjugate

In the context of system identification, the coherence  $\gamma_{xy}^2(\omega)$  function is often evaluated to describe the quality of the measurement. The coherence is defined as

$$\gamma_{xy}^2(\omega) = \frac{|G_{xy}(\omega)|^2}{G_{xx}(\omega)G_{yy}(\omega)} \quad (2.39)$$

If there is a strict linear relationship between the input and output of a system, the coherence  $\gamma_{xy}^2(\omega)$  equals 1 everywhere. If there is no correlation at all between the input and output the coherence becomes 0. In practical applications the coherence is usually a little below 1, meaning that

- the measurement is distorted by noise and/or
- input and output are related not only linearly and/or
- the output depends on the input but is influenced by further quantities

### Correlation measurement in the time domain

The correlation measurement in the time domain evaluates the impulse response of a system by evaluation of the cross correlation function between the input and output signal according to Eq. 2.34. If a stimulus with a dirac-like auto correlation function is chosen, Eq. 2.34 simplifies to

$$h(t) = R_{xy}(\tau) \quad (2.40)$$

where

$h(t)$ : impulse response of the system

$R_{xy}(\tau)$ : cross correlation function between input and output

Different stimulus signals with dirac-like auto correlation functions are worth to be considered. White noise for example is one possibility. An other interesting signal class are two-valued pseudo random sequences or maximum length sequences (MLS). Such sequences  $s(k)$  can be found for lengths  $L$  with

$$L = 2^n - 1 \quad n: \text{integer} > 0 \quad (2.41)$$

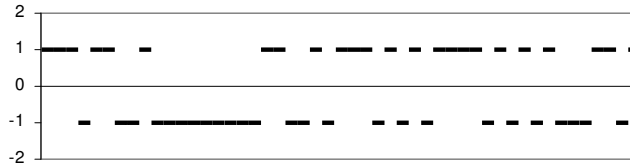


Figure 2.11: Part of a two valued pseudo random sequence. The sequence values 0 are mapped to +1, sequence values 1 are mapped to -1.

Pseudo random sequences can be generated with help of shift registers. The secret lies in a suitable exclusive-or operation and feed-back of the correct digits. Table 2.7 shows for different orders  $n$  examples of primitive polynomials. Fig. 2.12 shows exemplarily the translation of a primitive polynomial into a feed-back structure of the shift register. More details about primitive polynomials can be found in the book by Weldon <sup>22</sup>.

order $n$	primitive polynomial	order $n$	primitive polynomial
1	$x + 1$	16	$x^{16} + x^5 + x^3 + x^2 + 1$
2	$x^2 + x + 1$	17	$x^{17} + x^3 + 1$
3	$x^3 + x + 1$	18	$x^{18} + x^7 + 1$
4	$x^4 + x + 1$	19	$x^{19} + x^6 + x^5 + x + 1$
5	$x^5 + x^2 + 1$	20	$x^{20} + x^3 + 1$
6	$x^6 + x + 1$	21	$x^{21} + x^2 + 1$
7	$x^7 + x + 1$	22	$x^{22} + x + 1$
8	$x^8 + x^6 + x^5 + x + 1$	23	$x^{23} + x^5 + 1$
9	$x^9 + x^4 + 1$	24	$x^{24} + x^4 + x^3 + x + 1$
10	$x^{10} + x^3 + 1$	25	$x^{25} + x^3 + 1$
11	$x^{11} + x^2 + 1$	26	$x^{26} + x^8 + x^7 + x + 1$
12	$x^{12} + x^7 + x^4 + x^3 + 1$	27	$x^{27} + x^8 + x^7 + x + 1$
13	$x^{13} + x^4 + x^3 + x + 1$	28	$x^{28} + x^3 + 1$
14	$x^{14} + x^{12} + x^{11} + x + 1$	29	$x^{29} + x^2 + 1$
15	$x^{15} + x + 1$	30	$x^{30} + x^{16} + x^{15} + x + 1$

Table 2.7: Examples of primitive polynomials of order  $n$  1...30.

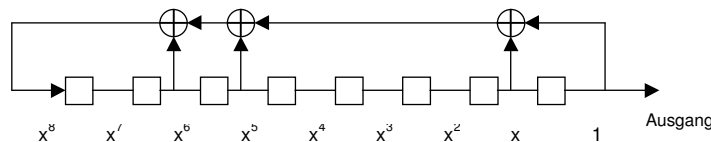


Figure 2.12: Feed-back structure for the primitive polynomial  $x^8 + x^6 + x^5 + x + 1$ .

If maximum length sequences  $s(k)$  are repeated periodically, the autocorrelation function  $R_{ss}(k)$  becomes:

$$R_{ss}(k) = \begin{cases} 1 & : k = iL, i: \text{integer} \geq 0 \\ -1/L & : \text{else} \end{cases} \quad (2.42)$$

For large  $L$  the function  $R_{ss}$  is a good approximation of the Dirac pulse. Indeed this holds only within one period of length  $L$ . It has to be assured that the system impulse response drops off to small enough

<sup>22</sup>Error-Correcting Codes, W. Wesley Peterson, E. J. Weldon, MIT-Press 1972.

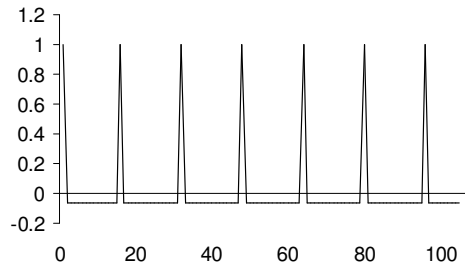


Figure 2.13: Autocorrelation function  $R_{ss}(k)$  of a periodically repeated maximum length sequence of length  $L = 15$ .

values to avoid temporal aliasing. Due to their periodicity the spectrum of maximum length sequences is a line spectrum. The separation between two frequency lines  $\Delta f$  is

$$\Delta f = \frac{1}{L\Delta t} \quad (2.43)$$

where

$L$ : sequence length

$\Delta t$ : 1/clock frequency

The envelope  $E(f)$  is given by

$$E(f) = \frac{\sin^2\left(\frac{\pi f}{f_c}\right)}{\left(\frac{\pi f}{f_c}\right)} \quad (2.44)$$

where

$f_c$ : clock frequency

Up to about half of the clock frequency, the spectrum of a maximum length sequence is flat. A big advantage compared to white noise is the significantly lower crest factor<sup>23</sup>.

Compared to single impulse measurements, the correlation technique shows a tremendously improved signal-to-noise ratio. The correlation process actually performs an average over  $L$  single impulse measurements where  $L$  is the sequence length. During the averaging, the signal of interest adds linearly with correct phase while the noise adds on a square basis only. Thus for each doubling of the sequence length the signal/noise ratio improves by 3 dB. Relative to a single impulse the signal-to-noise ratio improvement  $G$  can be written as:

$$G = 3 \log_2(L) \quad [dB] \quad (2.45)$$

In practical applications, sequence lengths in the order of 100'000 are used, which corresponds to a S/N improvement of about 50 dB. Impulse response measurements based on maximum length sequences *MLS* are widely used in room acoustics<sup>24, 25, 26</sup>.

MLS measurements may also be interesting in situations where the focus lies not on the exact course of the impulse response but on the total energy that is transferred by a system. This energy can be found by integration of the squared impulse response. An interesting property of the MLS technique is the fact that any disturbing noise during the measurement is mapped onto a stationary noise-like signal that is

<sup>23</sup>The crest factor describes the ratio of the peak value to the root mean square value of the signal.

<sup>24</sup>Kenneth W. Goff, Application of correlation techniques to some acoustic measurements, Journal of the Acoustical Society of America, 1955, v.27, p.236.

<sup>25</sup>M. R. Schroeder, Integrated impulse method measuring sound decay without using impulses, Journal of the Acoustical Society of America, 1979, v.66, p.497-500.

<sup>26</sup>R. Bütikofer, K. Baschnagel, Bauakustische Messungen mit MLS; Konsequenzen für das Beschallungssystem, Fortschritte der Akustik, DAGA 98 (1998), p.652-653.



equally smeared over the impulse response. The power of this unwanted noise can be estimated in a region of the impulse where no signal is present (e.g. during the time until the arrival of the direct sound).

The application of maximum length sequences as stimulus allows to use the Hadamard transformation for a very efficient calculation of the cross correlation function <sup>27</sup>.

As already mentioned, the resulting system impulse response is periodic with a period length that corresponds to the stimulus sequence length. In order to avoid temporal aliasing (overlapping) it has to be assured that this sequence length is larger than the length of the impulse response. In room acoustical applications the impulse response length can be assumed as the reverberation time.

The usage of the correlation measurement technique is only possible, if the system under investigation is linear and time invariant. If these conditions are not fulfilled an additional noise component occurs in the resulting impulse response <sup>28, 29, 30</sup>. Typical cases where MLS doesn't work are measurements with moving loudspeakers and/or microphones. Strong turbulent air flows are problematic as well, limiting the applicability outdoors. A serious source of non-linearity are loudspeakers that are driven with high amplitudes. For non-linear systems, measurements with sweeps as stimulus are favorable <sup>31</sup>.

### Swept sine measurements

Another commonly employed type of excitation signal for impulse response measurements is the sine sweep. There are two types of sine sweeps, the linear and the exponential sine sweep. While a linear sine sweep linearly increases or decreases in (instantaneous) frequency at a constant rate over time, an exponential sine sweep, also known as a chirp, does so at an exponential rate. The exponential sine sweep is defined as<sup>32</sup>:

$$x(t) = \sin \left[ \frac{\omega_1 T}{\ln \left( \frac{\omega_N}{\omega_1} \right)} \left( e^{\frac{t}{T} \ln \left( \frac{\omega_N}{\omega_1} \right)} - 1 \right) \right] \quad (2.46)$$

where

$\omega_1$ : starting angular frequency

$\omega_N$ : end angular frequency

$T$ : sweep duration

This measurement technique employs a deconvolution technique based on time-domain convolution with the time-reversal mirror of the sine sweep signal, which can be derived analytically. This method enables a separation of the harmonic distortion products.

### Time - bandwidth uncertainty principle

In many cases one is interested in an bandpass filtered impulse response. However, any filtering produces a temporal smearing. The time-bandwidth uncertainty principle states that the product of *temporal uncertainty* and *analysis bandwidth* can not drop below a certain limit. The more narrow the analysis bandwidth, the larger is the temporal uncertainty which can be described by a minimal pulse width <sup>33</sup>. If the temporal uncertainty is defined as the -3 dB width of the bandlimited impulse, the uncertainty principle says

$$\Delta t \Delta f \geq 0.5 \quad (2.47)$$

<sup>27</sup>J. Borish, J. B. Angell, An efficient algorithm for measuring the impulse response using pseudo random noise, Journal of the Audio Engineering Society, 1983, v.31, p.478-487.

<sup>28</sup>J. Vanderkooy, Aspects of MLS Measuring Systems, Journal of the Audio Engineering Society, vol. 42, p.219-231 (1994).

<sup>29</sup>C. Dunn, M. O. Hawksford, Distortion Immunity of MLS-Derived Impulse Response Measurements, Journal of the Audio Engineering Society, vol. 41, p.314-335 (1993).

<sup>30</sup>U. P. Svensson, J. L. Nielsen, Errors in MLS Measurements Caused by Time Variance in Acoustic Systems, Journal of the Audio Engineering Society, vol. 47, p.907-927 (1999).

<sup>31</sup>G. Stan, J. Embrechts, D. Archambeau Comparison of Different Impulse Response Measurement Techniques Journal of the Audio Engineering Society, vol. 50, no. 4, p.249-262 (2002).

<sup>32</sup>A. Farina, Advancements in impulse response measurements by sine sweeps, 122nd AES convention, Vienna (2007).

<sup>33</sup>J. S. Suh, P. A. Nelson, Measurement of transient response of rooms and comparison with geometrical acoustic models, J. Acoustical Society of America, vol. 105, p. 2304-2317 (1999).

with

$\Delta t$ : temporal width of the impulse in seconds, evaluated at the -3 dB points

$\Delta f$ : frequency bandwidth in Hz

The interesting question is, what kind of bandpass filter function of given width produces a minimal pulse width enlargement so that the *equal* sign holds in Eq. 2.47. A brick wall band pass filter, for example, leads to a  $\Delta t \Delta f$  product of 1. This is a factor 2 away from the optimum. It can be shown that the optimal band filter has a frequency response according to Eq. 2.48.

$$G(\omega) = 0.5 \frac{\sqrt{\pi}}{\alpha} \left( e^{-(\omega+\omega_0)^2/4\alpha^2} + e^{-(\omega-\omega_0)^2/4\alpha^2} \right) \quad (2.48)$$

where

$\omega_0$ : center frequency of the bandpass filter in rad/s

$$\alpha = \frac{\Delta\omega}{\sqrt{2\pi}}$$

$\Delta\omega$ : filter bandwidth in rad/s

In the time domain the frequency response of Eq. (2.48) corresponds to the so called Gabor pulse<sup>34</sup>:

$$g(t) = e^{-\alpha^2 t^2} \cos(\omega_0 t) \quad (2.49)$$

The width of the Gabor pulse (2.49) evaluated at the -3 dB points is found to:

$$\Delta t = \sqrt{\frac{\pi}{2}} \frac{1}{\alpha} \quad (2.50)$$

Figure 2.14 shows an example of such an optimal bandpass filter frequency response and the corresponding time response.

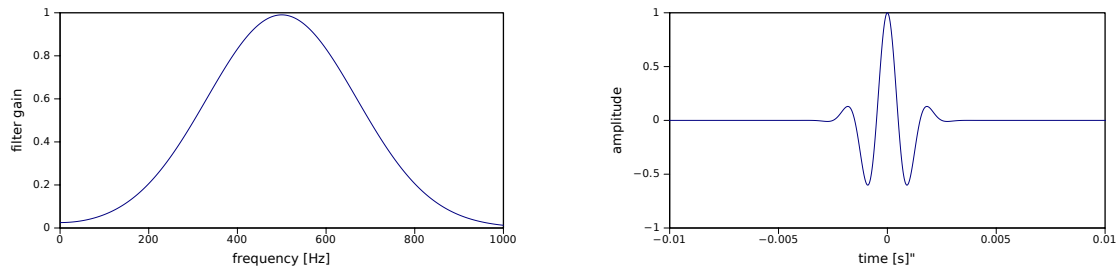


Figure 2.14: Filter frequency response of an optimal bandpass filter according to Eq. 2.48 and the corresponding time response of a filtered impulse (2.49) for a center frequency of 500 Hz ( $\omega_0 = 3142$  rad/s) and a bandwidth of 300 Hz ( $\Delta\omega = 1885$  rad/s).

## 2.6.3 Measurement of reverberation times

### Introduction

The reverberation time is an important quantity to describe the acoustical property of rooms. Reverberation stands for the delayed reaction of a room to temporally varying excitation. If a source of constant level is switched on, the sound travels as direct sound to a receiver position, followed by reflections with increasing temporal density. A few tenths of a second after the switch-on moment, a stationary condition is accomplished with constant sound energy density in the room. This condition represents the equilibrium state where the sound power fed by the source equals the sound power that is absorbed in the air and at the boundary.

The reverberation process itself manifests after switching off the source. After the traveling time from the source to the receiver, the contribution of the direct sound disappears. The great number of reflections however still make their way to the receiver. With time these reflections become weaker and weaker due to absorption in the air and at the boundaries. The sound pressure drops more or

<sup>34</sup>D. Gabor, Theory of Communication, J. IEEE, London, vol. 93(III), p.429-457 (1946).

less exponentially, which means that the sound pressure level follows a straight line. The time span measured from the moment when the source is switched off until the level drops for 60 dB is called reverberation time  $T60$ . Typical values for reverberation times lie between a few tenth of a second (living rooms) and several seconds (large churches).

As the absorption properties of the room boundaries are frequency dependent, the reverberation times are frequency dependent. Consequently the reverberation times are evaluated in third octave or octave bands.

### Schroeder reverse integration

The classical method to determine reverberation times is to use a loudspeaker that emits a random signal which is switched off after a certain time. The observation of the sound pressure level as a function of time will show random variations that differ from measurement to measurement. The reason for this is the random phase of the room modes at the switch-off moment (Fig. 2.15).

To eliminate these variations and to smoothen the level - time curves, the measurements have to be performed several times and averaged. Schroeder<sup>35</sup> has shown that the average of  $n$  measurements with  $n \rightarrow \infty$  can be found by one measurement alone. To do so one has to determine the squared impulse response  $r^2(t)$  of the room for the source and receiver position under consideration. By a so called reverse integration it is then found how the squared sound pressure dies away on average (2.51).

$$\langle s^2(t) \rangle \sim \int_t^{\infty} r^2(\tau) d\tau \quad (2.51)$$

where

$\langle s^2(t) \rangle$ : average of all possible decays of the squared time response

$r^2(t)$ : squared impulse response of the room for the selected source and microphone positions

### Measurement of short reverberation times at small filter bandwidths

Often the reverberation is measured in third octave bands. At the lowest third octave bands the bandwidth is so small that the filter applied to the impulse response may dominate the decay process (Fig. 2.16).

As a rule of thumb it can be concluded that the following condition has to be fulfilled in order to guarantee a valid reverberation time measurement<sup>36</sup>:

$$B \times T60 > 16 \quad (2.52)$$

where

$B$ : bandwidth of the filter

$T60$ : reverberation time.

The impulse response of a bandpass filter is asymmetrical (Fig. 2.16). It is therefore beneficial to reverse the time axis<sup>37,38,39</sup>. This can be done either by playing a recorded signal backwards or by using a filter with time reversed impulse response. In both cases the frequency content remains the same. Compared to the condition in Eq. 2.52, a factor 4 can be gained, meaning that only the following condition has to be fulfilled:

$$B \times T60 > 4 \quad (2.53)$$

where

$B$ : bandwidth of the filter

<sup>35</sup>M. R. Schroeder, New Method of Measuring Reverberation Time, Journal of the Acoustical Society of America, 1965, p.409-412.

<sup>36</sup>F. Jacobsen, A note on acoustic decay measurements, Journal of Sound and Vibration, v.115, 1987.

<sup>37</sup>F. Jacobsen, J. H. Rindel, Time reversed decay measurements, Journal of Sound and Vibration, v.117, p.187-190, 1987.

<sup>38</sup>B. Rasmussen, J. H. Rindel, H. Henriksen, Design and Measurement of Short Reverberation Times at Low Frequencies in Talk Studios, Journal of the Audio Engineering Society, v. 39, n. 1/2, p.47-57, 1991.

<sup>39</sup>M. A. Sobreira-Seoane, D. Perez Cabo, F. Jacobsen, The influence of the group delay of digital filters on acoustic decay measurements, Applied Acoustics, v. 73, p. 877-883, 2012.

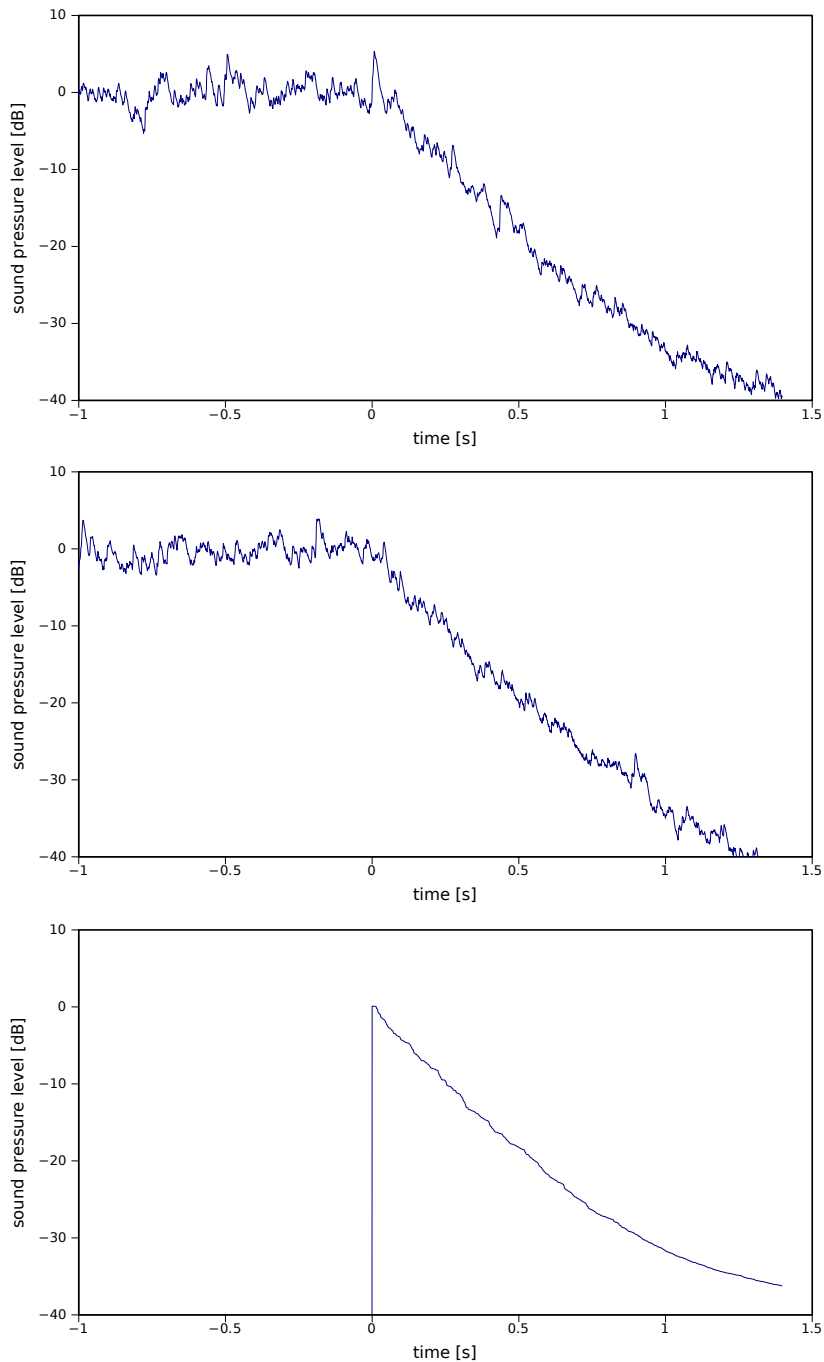


Figure 2.15: Classical measurement of the decay in a room after switching off the source (top and middle). The bottom curve is found with the Schroeder reverse integration of the squared impulse response.

$T_{60}$ : reverberation time.

Fig. 2.17 shows the significantly steeper decay of the reverse integrated impulse response of the time reversed filter compared to the normal filter.

## 2.7 Pressure zone microphone configuration

Often an acoustical measurement should provide information about the direct sound or the sound power of a source. In these cases the sound reflection at the ground is particularly disturbing, as

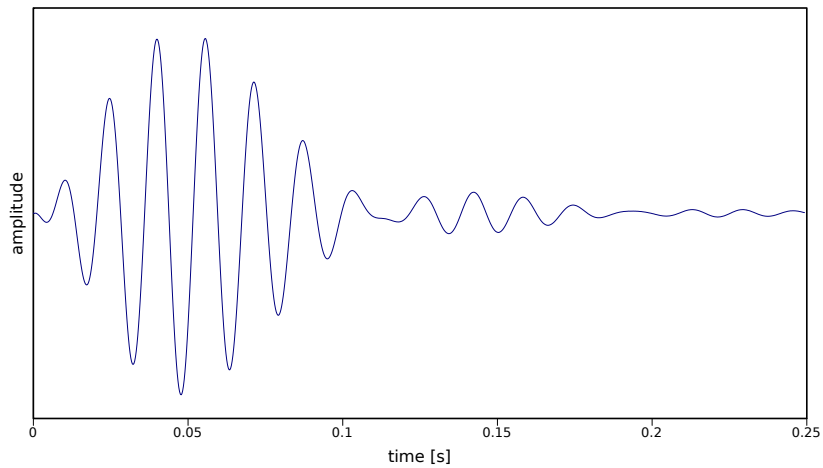


Figure 2.16: Impulse response of a third octave band filter at 63 Hz.

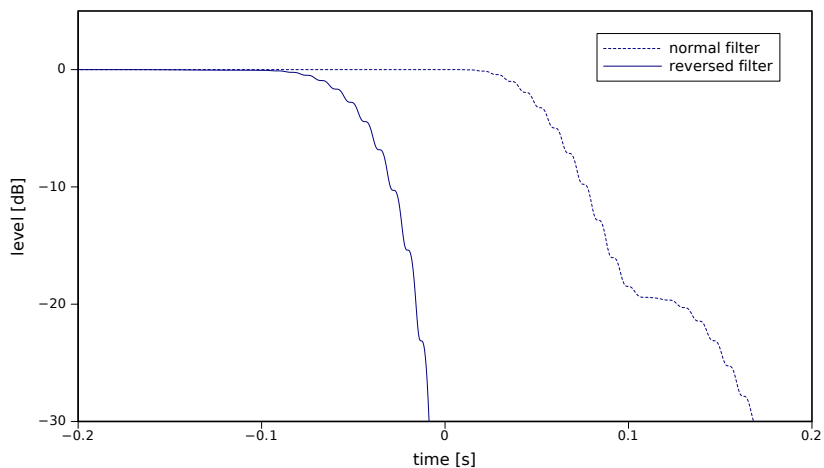


Figure 2.17: Decay (Schroeder reverse integration) of the normal and the time reversed third octave filter at 63 Hz.

interference occurs in combination with the direct sound. If the ground surface is acoustically hard it is possible to put the omnidirectional, pressure sensitive microphone directly on the ground. This set-up is called pressure zone configuration. Independently of the angle of incidence the sound pressure of the incidence wave doubles on the hard surface. In the dB scale this corresponds to a 6 dB increase relative to the direct sound in the free field. A prerequisite is that the reflecting surface is large enough. The condition *large enough* can not easily be converted into specific dimensions.

Here a measurement is shown for a reflecting plate of  $1.50 \times 1.40$  m. In the center of the plate a  $1/2''$  microphone was installed with the membrane parallel to the plate surface in a distance of 2 mm. A loudspeaker in a distance of 2.70 m was used as source and emitted pink noise. The angle of incidence  $\phi$  relative to the plate normal direction was varied between  $0$  and  $90^\circ$ . As the microphone pointed to the plate, the  $0^\circ$  direction corresponded to an angle of  $180^\circ$  for the microphone. Figure 2.18 shows the measured third octave band sound pressure levels relative to free field as a function of  $\phi$ . In the mid-frequency range for not grazing incidence the configuration produces the expected 6 dB amplification. For low frequencies and/or grazing incidence the amplification is significantly reduced due to insufficient size of the reflecting surface. At higher frequencies the amplification drops due to the decreasing sensitivity of the microphone itself for off-axis incident.

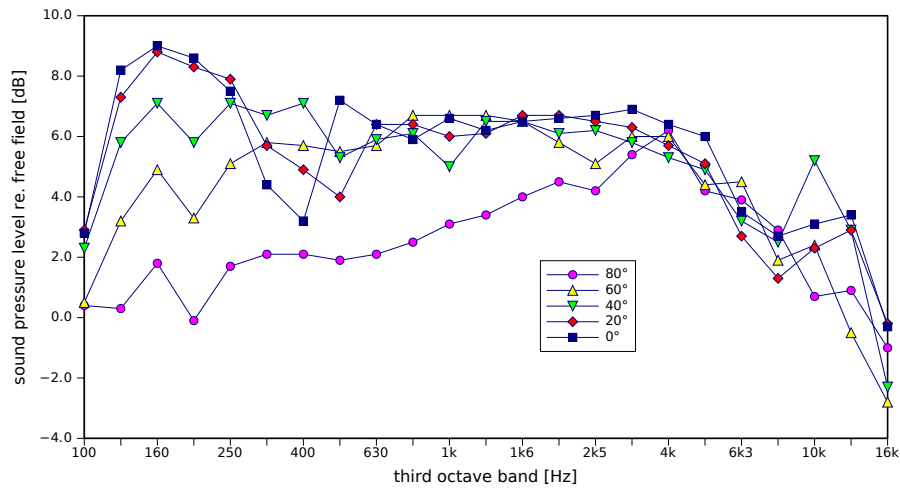


Figure 2.18: Third octave band levels relative to free field for a 1/2" microphone in pressure zone configuration on a plate with dimensions 1.50×1.40 m for different angles of incidence.

## 2.8 Uncertainty of acoustical measurements

In almost all cases acoustical measurements include unwanted effects. If the uncertainty due to these effects is too large, the results may become worthless. For the case of determining the sound pressure at a certain location, the following aspects have to be considered:

- the source may be in a not representative condition
- the propagation medium may be in a not representative condition (e.g. upwind conditions and a negative vertical temperature gradient)
- the surrounding of the microphone may influence the measurement in a non representative way
- the uncertainty of the calibration and tolerances of the measurement instrument
- possible unwanted disturbing noise (often this is the main difficulty and appropriate strategies have to be found to remove or exclude this noise. If this noise is uncorrelated and stationary, its contribution can be estimated and subtracted on a power basis)
- Uncertainty in the determination of the power of random signals

For each measurement the total uncertainty has to be specified. Typical values are in the range of  $\pm 1..3$  dB in the sense of a standard deviation.

## Chapter 3

# The human hearing

### 3.1 Structure and principle of operation of the ear

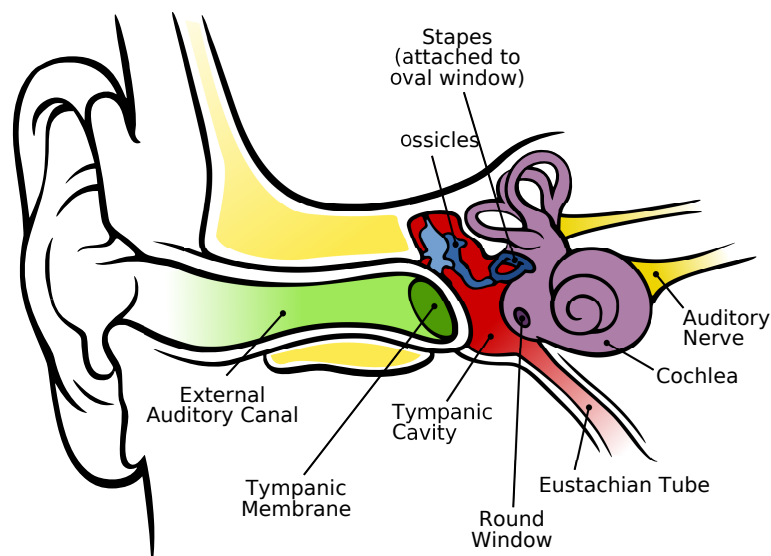


Figure 3.1: Section through the human ear. *source: Chittka L, Brockmann*

The human ear can be separated into three main parts, the outer ear, the middle ear that is filled with air and the inner ear or cochlea, filled with a fluid. The outer ear comprises the auricle and the outer ear canal. It is separated from the middle ear by the tympanic membrane or ear drum. The middle ear is usually closed airtight. However the Eustachian tube provides a connection to the throat and allows for pressure equalization. This can be provoked by swallowing. This configuration with a membrane on top of a closed cavity - and thus exposed to the sound field on one side only - corresponds to a sound pressure receiver.

The vibrations of the tympanic membrane are transmitted to the inner ear by tiny bones (ossicles). These bones convert the relative large excursions of the tympanic membrane into small excursions at the input of the inner ear. The benefit of this transformation is an amplification of the force which is necessary to excite the fluid. The configuration performs an impedance adjustment between air and fluid.

The ossicles in the middle ear are connected to muscles that can influence the transmission properties. If very loud sound signals are perceived, these muscles are contracted by reflex and lower the sensitivity of the ear and thus provide a certain protection of the inner ear. The inner ear is formed by the cochlea. The cochlea is separated into two channels by the basilar membrane. At the far end these two channels are connected to each other. The fluid in the two channels in the inner ear is excited by mechanical vibration of a membrane that is put into motion by

the ossicles. As a consequence of this excitation a traveling wave is formed that runs along the basilar membrane. The amplitudes of the traveling waves are very small. For stimuli that are just audible they are in the order of the diameter of an atom. The location of highest amplitude depends on the frequency of the stimulus. Thus in the inner ear a transformation takes place that maps frequency to location. This mechanism is fundamental for the frequency discrimination of the ear.

The location on the basilar membrane for maximal amplitude can be described by Eq. 3.1<sup>1</sup>.

$$f = 165.4(10^{0.06x} - 1)$$

$$x = \frac{1}{0.06} \log\left(\frac{f + 165.4}{165.4}\right) \quad (3.1)$$

where

$f$ : frequency in [Hz]

$x$ : position of maximum excursion of the basilar membrane in [mm]

The movement of the basilar membrane is detected by hair cells that sit on top of the membrane. The stimulated hair cells emit electrical impulses that are transported to the brain by the auditory nerve. The ear has an excellent frequency discrimination which can not be explained by the frequency dependent amplitudes of the traveling waves alone. Recent investigations have demonstrated that feed-back effects play an important role. There is experimental evidence that the outer hair cells are put into motion actively and by this influence the movement of the basilar membrane. This activity leads on her part to an excitation of the ossicles and the tympanic membrane and can be detected by a microphone in the ear canal. This phenomenon is called otoacoustic emission. Sometimes these emissions occur spontaneously. More relevant is the fact that such an emission results always as a reaction of the ear to an acoustical stimulus, however only if the ear functions properly. These tests are performed most easily with a short click as stimulus. The reaction of the ear can then be detected with a delay of a few milliseconds. This is an excellent possibility to investigate the proper working of the ear in an objective manner without the need of a response of the human being. Many hospitals use this method to detect possible malfunctioning of the auditory system of newborns.

An excellent overview of physiological and psychological aspects of the human ear can be found in the book by Fastl and Zwicker<sup>2</sup>

## 3.2 Properties of the auditory system for stationary signals

### 3.2.1 Loudness

The intensity of the sensation of a sound is described by its loudness. There are two scales in use. Loudness can be expressed on a linear scale, called *sone*, or on a logarithmic scale as loudness level  $L_N$  in phon.

The loudness of a specific sound is investigated by subjective comparison with a reference signal, usually a 1 kHz tone or 1 kHz narrow band noise. The reference signal is adjusted in such a way that the two sounds are perceived as equally loud. The sound pressure level in dB of the reference signal corresponds then to the phon value of the signal under investigation. Figure 3.2 shows curves of equal loudness for pure tones<sup>3</sup>.

A curve of special interest in Figure 3.2 is the auditory threshold. The curve denotes for a given frequency the sound pressure that is necessary to make the tone just audible. The standard ISO 389-7<sup>4</sup> describes the threshold of hearing for binaural hearing of pure tones under free field conditions. The polynomial approximation in Eq. 3.2 reproduces the tabulated values for frequencies between 20 and 16000 Hz with an accuracy better than 0.5 dB.

<sup>1</sup>D. D. Greenwood, A Cochlear Frequency-Position Function for Several Species - 29 Years Later, J. of the Acoustical Society of America, vol. 87, p. 2592-2605 (1990).

<sup>2</sup>Hugo Fastl, Eberhard Zwicker, Psycho-Acoustics, Springer, 2007.

<sup>3</sup>ISO Norm 226: Acoustics - Normal equal-loudness level contours. Second edition 2003.

<sup>4</sup>ISO Norm 389-7, Acoustics - Reference zero for the calibration of audiometric equipment - Part 7: Reference threshold of hearing under free-field and diffuse-field listening conditions (1996).



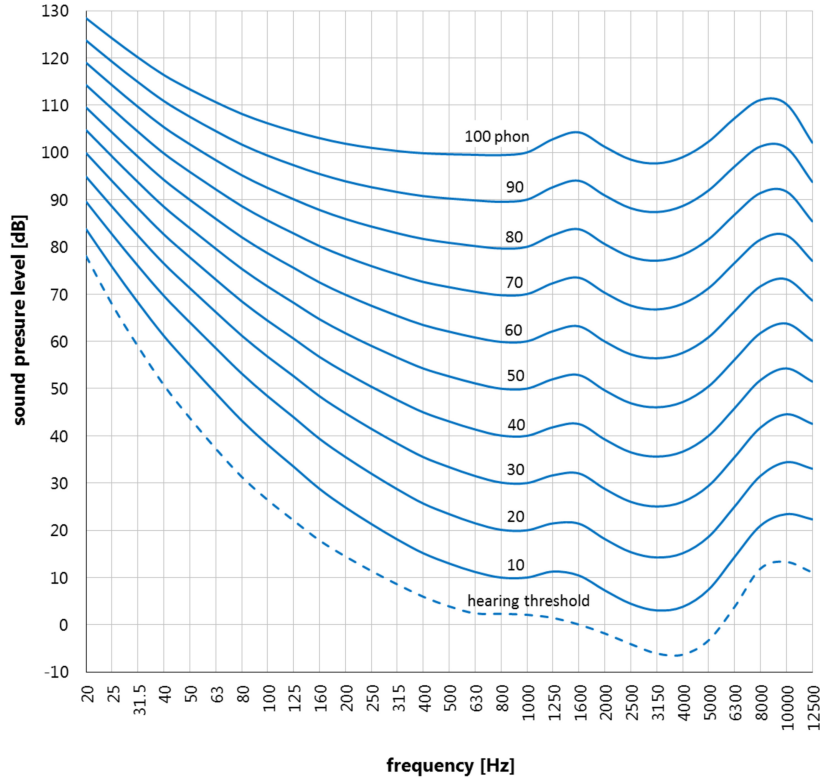


Figure 3.2: Curves of equal loudness, labeled in phon. All combinations of frequency and sound pressure level that lie on one curve result in equal loudness sensations.

$$T(f) \approx \begin{cases} 2.262 \times 10^5 f^{-3} - 3.035 \times 10^4 f^{-2} + 2.357 \times 10^3 f^{-1} + 8.3 - 2.912 \times 10^{-2} f + 2.2066 \times 10^{-5} f^2 & : 20 < f \leq 660 \\ -1.7 + 1.18247 \times 10^{-2} f - 1.0653 \times 10^{-5} f^2 + 2.98811 \times 10^{-9} f^3 - 3.5279 \times 10^{-13} f^4 + 1.86485 \times 10^{-17} f^5 - 3.6299 \times 10^{-22} f^6 & : 660 < f < 16000 \end{cases} \quad (3.2)$$

where

$T(f)$ : sound pressure level of a pure tone of frequency  $f$  that makes the tone for binaural hearing and under free field conditions just audible. With increasing age the hearing capabilities usually decrease and thus the threshold of hearing increases <sup>5</sup>.

The *phon* scale corresponds to a dB scale and is thus not proportional to the sensation. The *sone* scale on the other hand describes directly the sensation. Each doubling of the sone value corresponds to a doubling of the loudness sensation. For levels not too low there is a simple conversion between phon and sone figures. Each doubling of the sone value corresponds to an increase of 10 phon. With the definition of 1 sone  $\equiv$  40 phon as a point of reference the conversion can be written as:

$$N = 2^{\frac{L_N - 40}{10}} \quad (3.3)$$

$$L_N \approx 40 + 33 \log(N) \quad (3.4)$$

For loudness values below 40 phon, the relation from above is no longer valid. A bisection of the sone value is found for a phon step smaller than 10 phon.

<sup>5</sup>ISO Norm 7029: Acoustics: Statistical distribution of hearing thresholds as a function of age (2000).

### 3.2.2 Frequency discrimination

The human hearing can distinguish a little more than 600 frequency steps. For frequencies below 500 Hz the just audible frequency difference  $\Delta f$  is about 3.5 Hz. Above 500 Hz the necessary difference increases as

$$\Delta f = 0.007f \quad (3.5)$$

### 3.2.3 Critical bands

As mentioned above the stimulation of the ear leads to a traveling wave in the inner ear with the consequence of a local excitation of hair cells on the basilar membrane. Even in case of a pure tone stimulation, the region of excitation has a certain width. If the stimulus consists of two tones of frequencies  $f_1$  and  $f_2$ , three different mechanisms of perception can be distinguished. If the difference  $f_2 - f_1$  is below 10 Hz, the beat can be heard. If the difference is increased above 10 Hz, the amplitude modulations are no longer audible, however the beat is perceived as roughness of the sound. For further increasing of the frequency difference this roughness disappears more and more. This point is reached if both regions of excitation on the basilar membrane do no longer overlap. This frequency difference is called critical band. The width of a critical band is almost constant below 500 Hz and amounts to about 100 Hz. Above 500 Hz the width of the critical bands corresponds to about 20% of the signal frequency. This is very close to the bandwidth of third octave band filters. A more accurate description of the width  $\Delta f_{\text{crit}}$  of the critical bands is found in Eq. 3.6<sup>6</sup>:

$$\Delta f_{\text{crit}}[\text{Hz}] \approx 25 + 75 (1 + 1.4(f_s[\text{kHz}])^2)^{0.69} \quad (3.6)$$

where

$f_s$ : signal frequency [Hz]

### 3.2.4 Audibility of level differences

In a direct A/B comparison the smallest level differences that are just audible are in the order of 1 dB. Table 3.1 shows typical level variations and the corresponding differences in sensation. If the two signals are presented with a certain time span in between, the audible differences are significantly higher.

level variation	sensation
0..1 dB	not audible
2..4 dB	just audible
5..10 dB	clearly audible
> 10 dB	very convincing

Table 3.1: Sensation of level differences in a direct A/B comparison.

### 3.2.5 Masking

As a consequence of stimulation by a tone, the basilar membrane is activated in a certain region. A second tone played simultaneously will only be audible if the corresponding activation surmounts the one of the first tone. In other words, the presence of a tone leads to an upwards shift of the auditory threshold. This shift is more pronounced for frequencies higher than the tone frequency. This shift of the auditory threshold due to the presence of a tone is called masking, the tone responsible for the masking effect is called *masker*.

### 3.2.6 Loudness summation

The auditory sensation in case of the superposition of two signals distinguishes between two summation mechanisms:

<sup>6</sup>E. Zwicker, R. Feldkeller, Das Ohr als Nachrichtenempfänger, Hirzel Verlag, 1967.

In the first case the two signals cover the same critical band. Here the intensities add up according to Eq. 3.7).

$$L_{N\text{total}} = 10 \log (10^{0.1L_{N1}} + 10^{0.1L_{N2}}) \quad [\text{Phon}] \quad (3.7)$$

where

$L_{N1}$ : phon figure of the first component

$L_{N2}$ : phon figure of the second component

If for example the two signals have a strength of 50 phon each, the superposition yields a loudness level of 53 phon.

In the second case the two signals are clearly separated in frequency, which means they lie in different critical bands. Here the loudness of the signals adds up. For the example from above the two loudness levels are converted into the corresponding sone figures (50 phon  $\rightarrow$  2 sone). The sum of the sone values equals 4 sone which in turn corresponds to 60 phon.

### 3.2.7 Virtual pitch

Complex tonal sounds consist of a series of sinusoidal signals according to Eq. 3.8).

$$s(t) = \sum_{i=1}^{\infty} A_i \sin(i\omega t) \quad (3.8)$$

where  $\omega$  represents the angular frequency of the fundamental, the components  $i\omega$  are the harmonics.

The fundamental is responsible for the pitch, the harmonics constitute the tone color. It happens that the fundamental is only weak or totally missing. An example are string instruments playing tones at low frequencies. Nonetheless the ear can perceive the pitch of the fundamental. The auditory system complements the missing fundamental from the pattern of the harmonics. This phenomenon is called *virtual pitch*. A speciality of this virtual fundamental component is that it can not be masked by other sounds.

Virtual pitch is responsible for the fact that small loudspeakers appear to radiate sound even at low frequencies although this is not possible for physical reasons. It has been proposed to manipulate audio signals specifically in order to make usage of this effect <sup>7, 8, 9</sup>.

### 3.2.8 Audibility of phase

Helmholtz and Ohm stated that the perceived color of a tone of a complex sound depends only on the amplitude ratios but is independent of the phase spectrum. Indeed for most signals the phase shift of the reproducing system doesn't influence the aural impression at all. However there are some exceptions that show that humans are not strictly *phase deaf*. For instance the masking effect of low frequency tones depends not only on the amplitude spectrum but on the time function as well <sup>10</sup>. An other phase sensitive example signal is a complex sound with many harmonics with specific phase relations:

$$s(t) = \sum_{i=1}^{\infty} \frac{g_i}{i} \cos(2\pi \cdot i \cdot f_0 \cdot t + \phi_i) \quad (3.9)$$

The comparison between an *in-phase* version of  $s$  and one with *random phase* reveals clearly audible differences. For the *in-phase* version the following parameter setting is used:  $f_0 = 100$  Hz,  $g_i = 1$  and  $\phi_i = (i - 1)\pi/2$ . For the *random phase* version  $f_0$  and  $g_i$  are identical while for each  $i$   $\phi_i$  is set to a random number between 0 and  $2\pi$  <sup>11</sup>.

<sup>7</sup>Erik Larsen, Ronald M. Aarts, Reproducing Low-Pitched Signals through Small Loudspeakers, Journal of the Audio Engineering Society, vol. 50, no. 3, p.147-164 (2002).

<sup>8</sup>Nay OO, Woon-Seng Gan, Malcolm Hawksford, Journal of the Audio Engineering Society, vol. 59, no. 11, p.804-824 (2011).

<sup>9</sup>Hao Mu, Woon-Seng Gan, Perceptual Quality Improvement and Assessment for Virtual Bass Systems, Journal of the Audio Engineering Society, vol. 63, no. 11, p.900-913 (2015).

<sup>10</sup>E. Zwicker, M. Zollner, Elektroakustik, Springer-Verlag, 1987, p. 250.

<sup>11</sup>M. Laitinen, S. Disch, V. Pulkki, Sensitivity of Human Hearing to Changes in Phase Spectrum, Journal of the Audio Engineering Society, vol. 61, p. 860-877 (2013)

### 3.2.9 Methods to calculate and measure the loudness

For the determination of the loudness of stationary signals a standardized method - originally developed by Zwicker - exists<sup>12</sup>. The calculation needs the third octave band spectrum of the signal as input. Recently, loudness level meters have been developed that can even handle time varying sounds.

### 3.2.10 Nonlinear distortions of the ear

The transmission of the movement of the tympanic membrane to the inner ear is not a perfectly linear process. Thereby nonlinear distortions occur. They manifest as sum- and difference tones if two tones of different frequencies are presented. The difference tones are especially disadvantageous as they are not masked by the original tones. The strength of the most important difference tone at the frequency  $f = f_2 - f_1$  (where  $f_2$  is the frequency of the higher stimulus tone and  $f_1$  corresponds to the frequency of the lower stimulus tone) can be estimated by<sup>13</sup>:

$$L(f_2 - f_1) = L(f_1) + L(f_2) - 130 \text{ dB} \quad (3.10)$$

where

$L(f_2 - f_1)$ : level of a tone at frequency  $f_2 - f_1$ , that leads to the same sensation as the difference tone produced by the nonlinearity.

$L(f_1)$ : level of the lower frequency stimulus tone

$L(f_2)$ : level of the upper frequency stimulus tone

The summation in Eq. 3.10 has to be understood arithmetically. The stimulation of the ear with two tones of  $L(f_1) = L(f_2) = 90 \text{ dB}$  produces a level of the difference tone of 50 dB.

## 3.3 Properties of the ear for non stationary signals

### 3.3.1 Loudness dependency on the signal length

The hearing process shows a certain delay. Very short events are not perceived with full loudness. The maximal loudness is perceived just after a few tenths of a second. For signals shorter than 100 ms the perceived loudness is proportional to the signal length or signal energy. Figure 3.3 shows the relation between signal duration and loudness<sup>14</sup>.

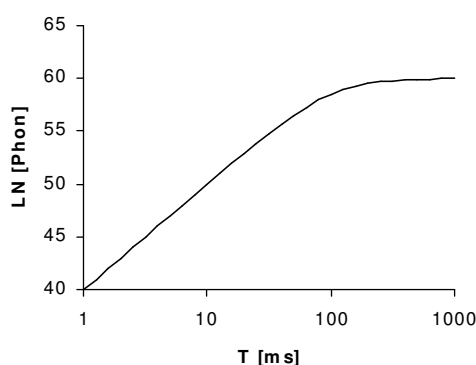


Figure 3.3: Relation of the perceived loudness level  $L_N$  and the signal duration  $T$  for a 2 kHz tone burst of 57 dB.

<sup>12</sup>ISO Norm 532 Acoustics - Method for calculating loudness level. 1975.

<sup>13</sup>E. Zwicker, Psychoakustik, Springer, 1982.

<sup>14</sup>E. Zwicker, Psychoakustik, Springer, 1982.

### 3.3.2 Temporal masking

Similarly as stationary signals can mask other frequency components, strong transient signals can mask weaker signals in the temporal vicinity of the masker. As shown in Figure 3.4, the hearing threshold is shifted upwards just a few milliseconds before and some hundredths of a second after the masker appeared or disappeared.

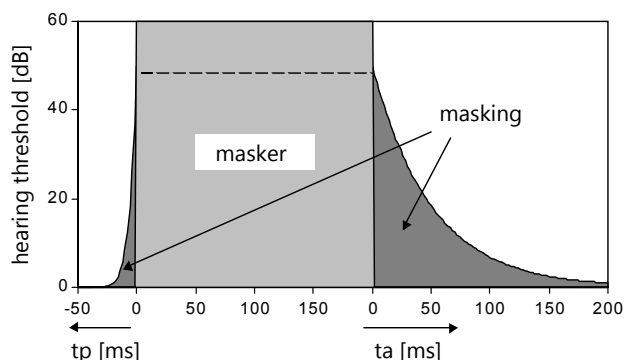


Figure 3.4: Temporal masking effect for a masker of 200 ms duration. The hearing threshold is shifted upwards already shortly before the masker is detected. After the discontinuation of the masker the hearing threshold returns to its normal level after some tenths of a second.

## 3.4 Binaural hearing: localization of sound sources

Within certain limits the auditory system is capable to localize sound sources according to their direction and their distances. For example in a noisy environment a listener can concentrate on a specific speaker and suppress the unwanted sound to a certain extent (cocktail party effect).

The localization of sound sources is usually described with help of a spherical coordinate system with its origin at the head's location. The localization in the vertical plane (elevation of the source) is based on monaural attributes. The localization in the horizontal plane on the other hand (azimuth of the source) uses inter-aural attributes which means differences between the signals at the two ears. To improve the localization, humans perform permanently little rotational movements with their heads and evaluate the resulting small variations. These movements help to discriminate between sources that lie in front of and sources that are behind the listener. This information is not available in the presentation of recordings over headphones.

The information that is available to the auditory system is composed of the signals at the two ear drums. The excitation of the eardrums depends on frequency and the sound incident direction. As a first approximation the problem can be formulated as diffraction pattern on a sphere. The transmission system free-field  $\rightarrow$  ear drum is usually described by the head related transfer function HRTF. This transfer function depends on the direction of incidence and varies to some extent from person to person<sup>15,16,17</sup>. Knowledge of the head related transfer function is essential for the equalization of headphones or in the context of virtual reality applications (auralisation<sup>18</sup>).

<sup>15</sup>H. Moeller, M. F. Soerensen, D. Hammershoei, C. B. Jensen, Head-Related Transfer Functions of Human Subjects, Journal of the Audio Engineering Society, May, n.5, vol 43, p.300-321 (1995).

<sup>16</sup>A. Schmitz, M. Vorländer, Messung von Aussenohrstossantworten mit Maximalfolgen-Hadamard-Transformation und deren Anwendung bei Inversionsversuchen, Acustica, vol. 71, p.257-268 (1990).

<sup>17</sup>Corey I. Cheng, Gregory H. Wakefield, Introduction to Head-Related Transfer Functions (HRTFs): Representations of HRTFs in Time, Frequency, and Space, Journal of the Audio Engineering Society, vol. 49, p.231-249 (2001).

<sup>18</sup>L. Savioja, et. al. Creating Interactive Virtual Acoustic Environments, Journal of the Audio Engineering Society, vol. 47, p.675-705 (1999).

### 3.4.1 Localization in the horizontal lane

The localization in the horizontal plane is based on two attributes. If sound is incident from a lateral direction as shown in Figure 3.5, the two ear signals differ in amplitude and time of arrival. Maximal directional resolution is achieved for frontal sound incidence. In this case azimuth changes in the order of  $1^\circ$  can be discriminated. Figure 3.6 and Figure 3.7 show how time and level differences at the two ears are mapped onto directional information. Completely lateral direction is perceived for time differences of  $630 \mu\text{s}$  and level differences of 10 dB.

At lower frequencies (below about 800 Hz but above about 80 Hz), the auditory system uses mainly time differences, for higher frequencies (above about 1600 Hz), mainly level differences are evaluated<sup>19</sup>. For frequencies in between, both attributes play a role.

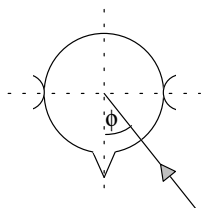


Figure 3.5: Direction  $\phi$  of sound incidence for the hearing in the horizontal plane.

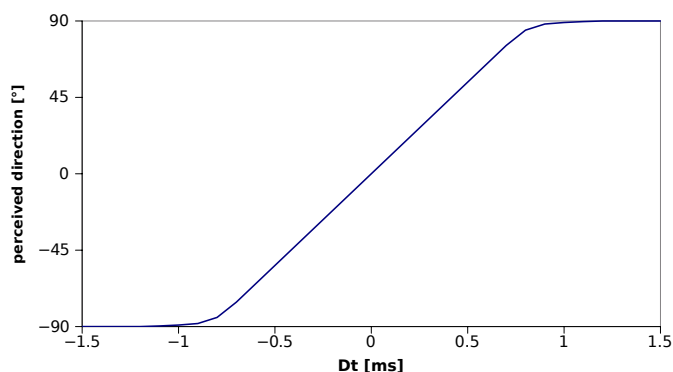


Figure 3.6: Detection of the angle of incidence  $\phi$  in the horizontal plane in relation to the inter-aural time difference  $\Delta t$ .

### 3.4.2 Localization in the vertical plane (elevation)

For a sound source located in front of the head but at different elevation angles, the two ear signals don't differ at all. In this case, no binaural attributes can be evaluated. The only information available is the change of the amplitude response of the HRTF in relation to the elevation angle. The elevational resolution that can be achieved depends strongly in the signal type and lies in the order of  $10^\circ \dots 45^\circ$ .

### 3.4.3 Perception of distances

Up to a certain degree the auditory system can estimate the distance of a sound source. The most important attribute that is evaluated is the strength of the signal. The louder a signal is, the shorter is the perceived distance to the source. In rooms the amount of reverberant sound in relation to the direct sound can be evaluated additionally.

<sup>19</sup>Actually level differences are evaluated over the whole auditory frequency range. However in typical situations no significant level differences occur at low frequencies due to diffraction around the head. In near field applications with small distances to the source level differences at the two ears can occur due to different distance ratios.

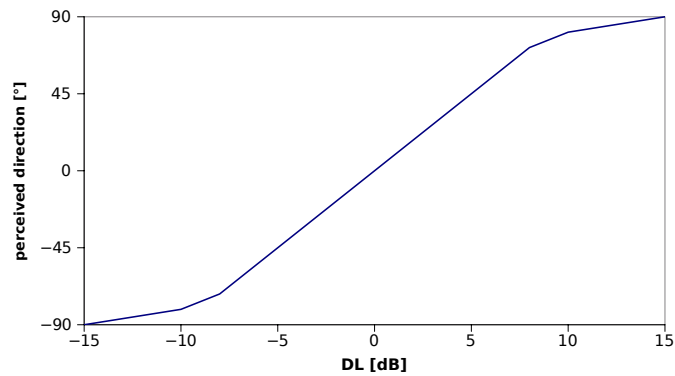


Figure 3.7: Detection of the angle of incidence  $\phi$  in the horizontal plane in relation to the inter-aural level difference  $\Delta L$ .

### 3.4.4 Echoes and the precedence effect

In a situation with direct sound and a shortly delayed copy of it, the auditory systems tends to merge the two signals to one impression and to localize on the signal that arrives first. This property is denoted as *precedence effect* <sup>20,21</sup>. There are two limitations associated with the precedence effect (Figure 3.8). Firstly, the localization on the first arriving signal takes place only if the sound pressure level of the delayed signal is not more than 10 dB higher than the direct sound. Secondly, the delay must be smaller than 30 to 50 ms, depending on the level differences. If the delay is larger than 50 ms, the second signal is perceived as a separate component, as an echo. Echoes are unfavorable in the sense that they disturb communication and thus lower speech intelligibility.

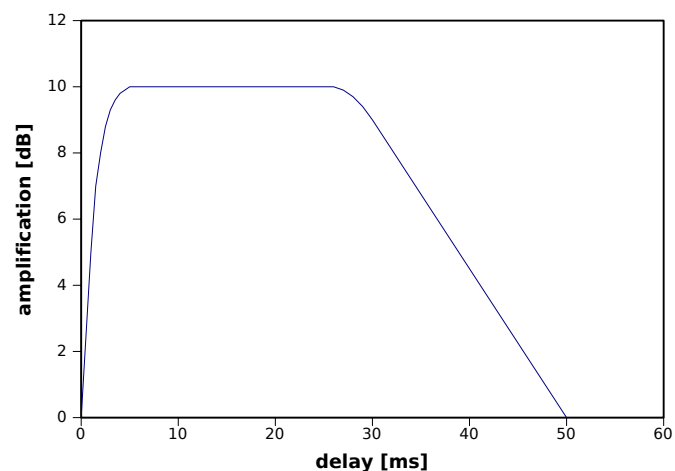


Figure 3.8: The precedence effect occurs for delay and amplification combinations that lie below the curve.

## 3.5 Hearing damage

### 3.5.1 Mechanisms

A hearing damage can have two causes. A first cause is a possible mechanical damage of the inner ear by an intense boom event. A second reason is a permanent long term overload of the auditory system by exuberant sound. In this case the metabolism of the inner ear can be overstrained with the

<sup>20</sup>Helmut Haas, 'Über den Einfluss eines Einfahechos auf die Hörbarkeit von Sprache, *Acustica*, vol. 1, no. 2 (1951).

<sup>21</sup>The precedence effect, Ruth Y. Litovsky et al., *Journal of the Acoustical Society of America*, vol. 106, p.1633-1654 (1999).

consequence that the hair cells are not supplied properly and are dying off over time. As the sensitivity of the ear is biggest around 4 kHz, hearing losses develop often in this frequency range first. Later the affected region enlarges and will cover the important range for communication. This is the moment where the damage will become obvious.

A serious disease of the ear is the tinnitus. Hereby the patient perceives tones and noises that do not exist. In fatal cases tinnitus can seriously disturb the ability to concentrate and to relax. Tinnitus can have different causes. However, in many cases a noise induced hearing loss stands at the beginning. Up to now there is no real treatment available.

### 3.5.2 Assessment of the danger for a possible hearing damage

The evaluation of a possible danger for a hearing damage is based on a dose measure. The relevant factors are sound pressure and time. The dose corresponds to the product of the two factors. An increase of one factor can be compensated by a reduction of the other.

The assessment of impulsive sound is based on the sound exposure level  $SEL$  or event level  $L_E$ , measured over a period of 1 hour. The SUVA defines as a limit an  $SEL = L_E = 120$  dB(A). The single occurrence of a higher level may lead to a permanent damage of the ear. The firing of one shot with an assault rifle for example produces an  $L_E$  of 129 dB(A). In addition to the A-level  $L_E$  criterion, a maximum for the C-weighted peak level of 135 dB(C) has to be met.

For stationary noise SUVA has established the following limiting value: for permanent noise exposure during 8 hours a day and 5 days a week the  $Leq$  must not exceed 85 dB(A). In a year the assumed working time sums up to 2000 hours. If the exposure occurs only during a portion of this time, higher levels are tolerable (Table 3.2).

yearly time of exposure	allowable $Leq$
2000 h	85 dB(A)
1000 h	88 dB(A)
500 h	91 dB(A)
250 h	94 dB(A)

Table 3.2: Allowable  $Leq$  values in dependency of the yearly time of exposure according to the SUVA limiting values.

According to today's knowledge, ultrasonic sound (20 kHz..100 kHz) doesn't cause harm if the unweighted maximum level is below 140 dB and the sound exposure level integrated over a period of 8 hours doesn't exceed 110 dB. For infrasound (2 Hz..20 Hz) the corresponding limits are 150 dB for the maximum level and 135 dB for the exposure level.



# Chapter 4

## Musical Intervals

The octave as a frequency ratio 2:1 is the most fundamental musical interval in western music. The *equally tempered scale* in use today divides each octave on a logarithmic basis in 12 half tones. Each half tone corresponds thus to a frequency ratio of  $2^{1/12} \approx 1.059 \approx 6\%$ . The advantage of the *equally tempered scale* lies in the fact that on a piano all intervals can be played starting from any half tone and a certain interval always corresponds to the same frequency ratio. The disadvantage on the other hand is that besides the octave no other perfect whole-numbered interval can be played. A fifth for example which represents a ratio of 3:2 in *just scale* has to be played as 7 half tones in the *equally tempered scale*, corresponding to a ratio of 1.498. The pure fourth stands for a frequency ratio of 4:3. This has to be approximated by 5 half tones resulting in a ratio of 1.3348. The deviation of the frequency intervals for the *equally tempered scale* compared to the *just scale* are so small, that the pleasure of music is not disturbed. As an overview Table 4.1 shows the intervals and the frequency ratios for the *equally tempered scale*.

interval	tone	number of half tones	frequency ratio	<i>just scale</i>
perfect unison	c	0	1.0000	
minor second	des	1	1.0595	
major second	d	2	1.1225	
minor third	es	3	1.1892	6:5 = 1.2000
major third	e	4	1.2599	5:4 = 1.2500
perfect fourth	f	5	1.3348	4:3 ≈ 1.3333
augmented fourth	fis	6	1.4142	
diminished fifth	ges	6	1.4142	
perfect fifth	g	7	1.4983	3:2 = 1.5000
minor sixth	as	8	1.5874	
major sixth	a	9	1.6818	
minor seventh	b	10	1.7818	
major seventh	h	11	1.8878	
perfect octave	c'	12	2.0000	2:1 = 2.0000

Table 4.1: Musical intervals for the *equally tempered scale*, starting with the tone c.

Alexander John Ellis proposed in 1875 a much finer partition than just half tones, labeled as *cent*. Cent stands for "hundred" and signifies a logarithmic partitioning of a half tone interval into 100 steps. An octave has 12 half tones and corresponds therefore to 1200 cents. A cent stands for a frequency ratio of  $\sqrt[1200]{2} \approx 1.00057779$ . In other words one cent corresponds to a frequency change of 0,057779 %. In general a frequency ratio  $f_2$  to  $f_1$  corresponds  $C$  cent where

$$C = 1200 \log_2 \left( \frac{f_2}{f_1} \right) = 1200 \frac{\ln \left( \frac{f_2}{f_1} \right)}{\ln(2)} \quad [\text{Cent}] \quad (4.1)$$

The other way round,  $C$  cent correspond to a frequency ratio  $f_2/f_1$  of

$$\frac{f_2}{f_1} = 2^{\frac{C}{1200}} \quad (4.2)$$

## Chapter 5

# Outdoor sound propagation

The simplest case of a sound propagation situation is given by a point source radiating in all directions with equal strength in an unbounded homogeneous medium at rest. The sound pressure at an arbitrary receiver position can be determined by taking into account the geometrical spreading and the frequency dependent air absorption. However, in real situations usually further influence factors have to be considered. Firstly the medium is never unbounded. In many cases the source and/or the receiver are in the vicinity of the ground. This ground surface leads to a reflection of the sound waves and in the interaction with the direct sound to interference effects. Besides the reflection at the ground, additional reflections at other objects such as walls or building facades may occur. Secondly, the medium is usually not at rest and not homogeneous. This leads to a refraction of sound waves and in consequence to curved propagation. Thirdly the sound propagation between the source and the receiver may be interrupted by obstacles such as trees or walls. In this case, damping and diffraction effects have to be taken into account.

### 5.1 Basic equation

The calculation of an outdoor sound propagation problem is usually based on an equation in form of Eq. (5.1). The relevant variables are the source strength - specified as a sound power level, a possible correction for the directivity and a sum of attenuation terms <sup>1</sup>.

$$L_p(\text{receiver}) = L_W + D - \sum A \quad (5.1)$$

where

$L_p(\text{receiver})$ : sound pressure level at the receiver

$L_W$ : sound power level of the source

$D$ : directivity of the source

$A$ : attenuation during propagation

As most attenuations  $A$  are frequency dependent, the calculation according to Eq. 5.1 has to be performed for different frequency bands. Therefore the sound power is split into third-octave or octave bands, then the propagation attenuation is calculated for each band and finally the sound pressure values at the receiver for each band are summed up to a total level. For distinct classes of noise sources with a defined spectrum, approximations for the A-weighted may be applied.

### 5.2 Directivity of the source

The simplest model of a source assumes equal radiation in all directions. Such a characteristic is denoted as *omnidirectional* or *spherical*. If such an omnidirectional source is located close to a reflecting surface, the radiation is restricted to a limited solid angle, leading to an amplification in these directions. Table 5.1 lists the corresponding directivity values  $D$  from Eq. 5.1 for different configurations of the source.

In some cases the source itself can show a directivity with stronger radiation in some directions.

---

<sup>1</sup>ISO Standard 9613: Acoustics - Attenuation of sound during propagation outdoors, Part 2: General method of calculation.

source configuration	solid angle	D[dB]
open space	$4\pi$	0
in front of a surface	$2\pi$	+3
in front of two orthogonal surfaces	$\pi$	+6
in front of three orthogonal surfaces (corner)	$\frac{\pi}{2}$	+9

Table 5.1: Directivity corrections  $D$  in Eq. 5.1 for a point source in front of reflecting surfaces.

## 5.3 Attenuation terms

### 5.3.1 Geometrical divergence

The geometrical divergence is independent of frequency and describes the reduction of intensity or sound pressure with distance due to the distribution of the sound power on an area that increases with distance. For an omnidirectional point source, the intensity on a spherical surface around the source is given by Eq.5.2.

$$I = \frac{W}{4\pi d^2} \quad (5.2)$$

where

$I$ : intensity at distance  $d$  from the source

$W$ : sound power

For distances larger than a few wavelengths, the ratio of sound pressure and sound particle velocity equals the free field impedance and therefore

$$I = \frac{p_{rms}^2}{\rho_0 c} \quad (5.3)$$

and

$$p_{rms}^2 = \frac{W \rho_0 c}{4\pi d^2} \quad (5.4)$$

In the dB scale the geometrical divergence  $A_{div}$  is given as (with the conversion constant from sound power level to sound pressure level in 1 m distance)

$$A_{div} = 20 \log \left( \frac{d}{d_0} \right) + 11 \quad [\text{dB}] \quad (5.5)$$

where

$d$ : distance source - receiver

$d_0$ : reference distance = 1 m

### 5.3.2 Atmospheric absorption

During sound propagation, a certain fraction of the sound energy is converted into heat. Per unit distance the fraction of absorbed energy is constant. Translated into the dB scale this corresponds to (Eq. 5.6).

$$A_{atm} = \alpha d \quad [\text{dB}] \quad (5.6)$$

Atmospheric absorption is influenced by air temperature and humidity and depends strongly on frequency. For that reason the calculation of air absorption should preferably be done in third octave bands. Table 5.2 shows the atmospheric absorption in dB/km for some temperature / humidity combinations. The values correspond to the parameter  $\alpha$  in Eq. 5.6 if the distance  $d$  is inserted in  $\text{km}^2$ .

<sup>2</sup>ISO Norm 9613-1: Acoustics - Attenuation of sound during propagation outdoors.

temp [° C]	rel. humidity[%]	63	125	250	500	1k	2k	4k	8k
10	70	0.1	0.4	1.0	1.9	3.7	9.7	32.8	117
20	70	0.1	0.3	1.1	2.8	5.0	9.0	22.9	76.6
30	70	0.1	0.3	1.0	3.1	7.4	12.7	23.1	59.3
15	20	0.3	0.6	1.2	2.7	8.2	28.2	88.8	202
15	50	0.1	0.5	1.2	2.2	4.2	10.8	36.2	129
15	80	0.1	0.3	1.1	2.4	4.1	8.3	23.7	82.8

Table 5.2: Coefficient  $\alpha$  of atmospheric absorption in dB/km as a function of pure tone frequency for different combinations of temperature and humidity.

Table 5.2 shows a very strong increase of the atmospheric absorption towards higher frequencies. Further away from a source, only the low frequency components are audible.

The coefficients  $\alpha$  of atmospheric absorption can be calculated with the following set of formulas:

$$\alpha = 8.686f^2 \left( \left[ 1.84 \times 10^{-11} \left( \frac{p_a}{p_r} \right)^{-1} \left( \frac{T}{T_0} \right)^{1/2} \right] + \left( \frac{T}{T_0} \right)^{-5/2} \times \right. \\ \left. \times \left\{ 0.01275 \left[ \exp \left( \frac{-2239.1}{T} \right) \right] \left[ f_{rO} + \left( \frac{f^2}{f_{rO}} \right) \right]^{-1} + \right. \right. \\ \left. \left. + 0.1068 \left[ \exp \left( \frac{-3352.0}{T} \right) \right] \left[ f_{rN} + \left( \frac{f^2}{f_{rN}} \right) \right]^{-1} \right\} \right) \quad (5.7)$$

$$f_{rO} = \frac{p_a}{p_r} \left( 24 + 4.04 \times 10^4 h \frac{0.02 + h}{0.391 + h} \right) \quad (5.8)$$

$$f_{rN} = \frac{p_a}{p_r} \left( \frac{T}{T_0} \right)^{-1/2} \left( 9 + 280h \exp \left\{ -4.170 \left[ \left( \frac{T}{T_0} \right)^{-1/3} - 1 \right] \right\} \right) \quad (5.9)$$

$$h = h_r \frac{p_{\text{sat}}/p_r}{p_a/p_r} \quad (5.10)$$

$$p_{\text{sat}}/p_r = 10^{-6.8346(273.16/T)^{1.261} + 4.6151} \quad (5.11)$$

where

$\alpha$ : coefficient of air absorption in [dB/m]

$f$ : frequency in [Hz]

$p_a$ : air pressure in [kPa]

$p_r$ : reference air pressure = 101.325 kPa

$T$ : air temperature in Kelvin

$T_0$ : reference air temperature = 293.15 K

$h_r$ : relative humidity of the air as percentage ( $0 < h_r < 100$ )

### 5.3.3 Ground effect

In many practical cases sound propagates close to the ground. For larger distances and small source and receiver heights the angles of incident relative to the ground are almost grazing. In this situation the direct sound interferes with a significant reflection from the ground. The frequencies that are amplified or attenuated depend on the sound path length differences and the phase shift at the reflection. The modification of the sound pressure at a receiver due to the presence of the ground is called *ground effect*. An exact analytical solution can be given for simple geometries only (see below). However for a broad band signal the A-weighted ground effect can be estimated according to Eq. 5.12.

$$A_{\text{ground}} = 4.8 - \frac{2h_m}{d} \left( 17 + \frac{300}{d} \right) \geq 0 \quad [\text{dB(A)}] \quad (5.12)$$

where

$h_m$ : average height above ground of the direct sound propagation path [m]

$d$ : distance source - receiver [m]

If the ground effect calculation is performed based on Eq. 5.12, an additional directivity correction  $D_\Omega$  that increases the receiver level has to be applied as:

$$D_\Omega = 10 \log \left( 1 + \frac{d_p^2 + (h_s - h_r)^2}{d_p^2 + (h_s + h_r)^2} \right) \quad (5.13)$$

where

$h_s$ : height of the source above ground [m]

$h_r$ : height of the receiver above ground [m]

$d_p$ : source-receiver distance projected onto the ground plane [m]

For flat terrain the ISO standard 9613-2<sup>3</sup> describes a more subtle algorithm that yields the ground effect in octave bands.

In case of a point source above flat homogeneous ground, the ground effect can be calculated exactly in the sense of a numerical approximation to the wave theory<sup>4</sup>. Thereby *locally reacting ground* is assumed which means the boundary condition at the ground is defined as the frequency dependent ratio of sound pressure and the normal component of the sound particle velocity (ground impedance). The calculation is based on the following variables (see Fig. 5.1):

$d$ : horizontal distance source - receiver

$h_s$ : source height above ground

$h_r$ : receiver height above ground

$Z$ : impedance of the ground, normalized to  $\rho c$

$R_1$ : distance source - receiver

$R_2$ : distance source - point of reflection - receiver

$\lambda$ : wave length

$k$ : wave number =  $\frac{2\pi}{\lambda}$

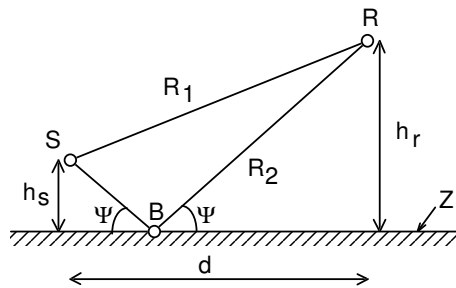


Figure 5.1: Situation of a point source  $S$  above homogeneous ground with impedance  $Z$ ,  $B$  is the reflection point,  $R$  is the receiver.

As already insinuated in Fig. 5.1, the sound pressure  $p(R)$  at the receiver is composed of two contributions: the direct sound and the ground reflection. In complex writing  $p(R)$  can be stated as:

$$p(R) = \frac{1}{R_1} e^{jkR_1} + Q \frac{1}{R_2} e^{jkR_2} \quad (5.14)$$

where

$Q$ : spherical wave reflection coefficient

The spherical wave reflection coefficient  $Q$  can be deduced from the plane wave reflection coefficient  $r_p$  as

<sup>3</sup>ISO Standard 9613: Acoustics - Attenuation of sound during propagation outdoors, Part 2: General method of calculation.

<sup>4</sup>C. I. Chessel, Propagation of noise along a finite impedance boundary. J. Acoustical Society of America, 62, p.825-834 (1977).

$$Q = r_p + (1 - r_p)F(w) \quad (5.15)$$

where

$$r_p = \frac{\sin(\psi) - \frac{1}{Z}}{\sin(\psi) + \frac{1}{Z}}$$

$$w = \frac{1+j}{2} \sqrt{kR_2} \left( \sin(\psi) + \frac{1}{Z} \right)$$

The factor  $F(w)$  in Eq. 5.15 can be approximated as <sup>5</sup>

$$F(w) = 1 + j\sqrt{\pi}we^{-w^2}\operatorname{erfc}(-jw) = 1 + j\sqrt{\pi}w\operatorname{wofz}(w) \quad (5.16)$$

The function  $\operatorname{erfc}(-jw)$  in Eq. 5.16 denotes the complex error function <sup>6</sup>. For the evaluation of the function  $\operatorname{wofz}(w) = e^{-w^2}\operatorname{erfc}(-jw)$ , a very efficient algorithm is available <sup>7,8</sup>.

The impedance  $Z$  of the ground is frequency dependent. Very often, the characterization is based on a one parameter model with the flow resistivity  $\sigma$  as variable. With help of the empirical model by Delany and Bazley <sup>9</sup> (5.17) the impedance normalized to  $\rho c$  can be calculated for all frequencies  $f$ . It should be noted that the sign of the imaginary part of the impedance in Eq. 5.17 depends on the convention of the time dependency in the complex representation. A positive imaginary part as shown here, assumes  $e^{-j\omega t}$  <sup>10</sup>.

$$Z = 1 + 9.08 \left( \frac{f}{\sigma} \right)^{-0.75} + j11.9 \left( \frac{f}{\sigma} \right)^{-0.73} \quad (5.17)$$

where

$Z$ : impedance normalized to  $\rho c$

$f$ : frequency [Hz]

$\sigma$ : flow resistivity [ $\text{kPa}\cdot\text{s}/\text{m}^2$ ].

Table 5.3 shows corresponding flow resistivities for different ground types. Figure 5.2 demonstrates exemplarily the frequency response of the impedance for lawn ( $\sigma = 300 \text{ kPa}\cdot\text{s}/\text{m}^2$ ).

Fig. 5.3 shows the frequency responses of the ground effect, calculated with Eq. 5.14 for different situations. For that purpose the resulting sound pressure at the receiver is referenced to the direct sound pressure. For grassy ground an amplification at very low frequencies and an attenuation in the mid frequency range is very typical.

ground type	flow resistivity $\sigma$ [ $\text{kPa}\cdot\text{s}/\text{m}^2$ ]
asphalt, water	20'000
hard natural ground	5'000
plow soil, gravel	500
lawn	300
grass land	150
hard snow	40
powder snow	10

Table 5.3: Flow resistivity for different ground types.

<sup>5</sup>C. F. Chien, W. W. Soroka, A note on the calculation of sound propagation along an impedance surface. J. Sound and Vib. vol. 69, 340-343 (1980).

<sup>6</sup>M. Abramowitz, J. A. Stegun, Handbook of Mathematical Functions.

<sup>7</sup>W. Gautschi, Efficient Computation of the Complex Error Function. SIAM J. Numer. Anal. vol. 7, 187-198 (1970).

<sup>8</sup>Collected Algorithms from CACM. Algorithm 363.

<sup>9</sup>M. E. Delany, E. N. Bazley, Acoustical properties of fibrous absorbent materials. Applied Acoustics vol. 3, 105-116 (1970).

<sup>10</sup>G. A. Daigle et al. Some comments on the literature of propagation near boundaries of finite acoustical impedance, Journal of the Acoustical Society of America, vol. 66, p.918-919 (1979).

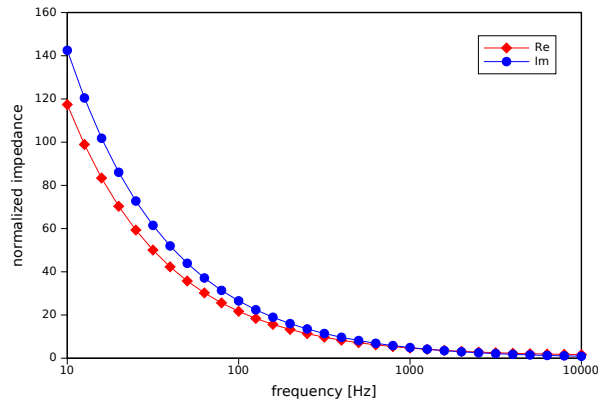


Figure 5.2: Real- and imaginary part of the impedance of lawn, normalized with  $\rho c$

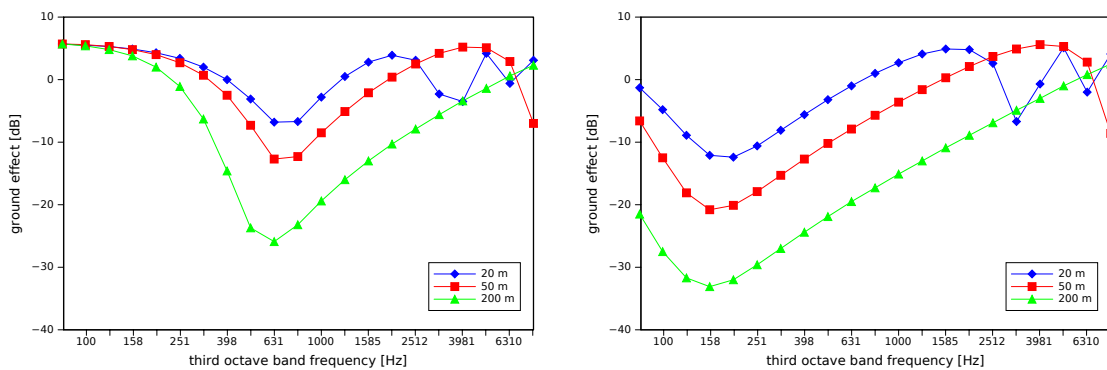


Figure 5.3: Ground effect for lawn (left) and snow covered ground (right) with source and receiver 1 m above ground and distances of 20, 50 and 200 m. The wide dip in the low frequencies in case of snow explains the experience that many transportation noise sources are quieter during periods of snow.

### 5.3.4 Obstacles

#### Vegetation

Sound is attenuated while passing dense vegetation. This is mainly due to scattering effects at trunks and branches. However, significant attenuation is found only for extensions of more than about 20 meters. One row of trees or bushes has no direct effect. Though a second order effect is the fact that vegetation loosens the ground and by this reduces the flow resistivity which in turn influences the ground effect. An additional effect of vegetation is the interruption of view which may be beneficial from a psychological point of view in noise abatement applications.

Table 5.4 shows the average attenuation  $A_{\text{foliage}}$  in octave bands associated with dense vegetation. The effective distance is the sound path that passes through the vegetation.

effective distance	63	125	250	500	1k	2k	4k
10...20m	0dB	0dB	1dB	1dB	1dB	1dB	2dB
20...200m	0.02dB/m	0.03dB/m	0.04dB/m	0.05dB/m	0.06dB/m	0.08dB/m	0.09dB/m
> 200m	4 dB	6 dB	8 dB	10 dB	12 dB	16 dB	18 dB

Table 5.4: Attenuation due to vegetation  $A_{\text{foliage}}$  as a function of frequency.

## Noise barriers

Massive obstacles that interrupt the line of sight between source and receiver lead to a significant attenuation. In the context of road and railway noise, barriers are indeed a common approach to reduce the noise level at receivers. Due to diffraction effects, still a relevant portion of the sound wave can reach the geometrical shadow zone behind an obstacle. This is due to the fact that sound wave lengths relevant for many noise sources are in the same order of magnitude as typical geometrical dimensions. In the calculation of the attenuation due to obstacles, the portion of sound energy that goes through the obstacle is usually ignored. This is allowable in most cases if the area specific mass of the obstacle is larger than  $10 \text{ kg/m}^2$ .

The calculation of the sound field behind an infinitely extended half plane is a classical task in theoretical acoustics. Maekawa was the first that deduced an empirical formula for the barrier attenuation based on theoretical considerations and measurements in the laboratory. Hereby the attenuation is expressed as a function of one single parameter - the so called Fresnel number  $N$ .  $N$  is defined as the ratio  $N = 2z/\lambda$  where  $\lambda$  is the wave length and  $z$  is the difference of the path lengths around the obstacle and through the obstacle.

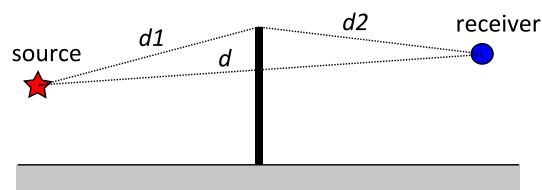


Figure 5.4: Situation of a single obstacle between source and receiver with the sound path around the obstacle edge ( $d_1, d_2$ ) and the path through the the obstacle ( $d$ ).

The ISO standard 9613 calculates the attenuation  $A_{\text{screen}}$  due to a barrier as follows:

$$A_{\text{screen}} = 10 \log \left( 3 + \frac{C_2}{\lambda} C_3 z K_w \right) \quad [\text{dB}] \quad (5.18)$$

where

$C_2 = 20$

$C_3 = 1$  for a single barrier

$\lambda$ : wave length [m]

$z$ : difference of the path lengths around the obstacle and through the obstacle  $z = d_1 + d_2 - d$  (Fig. 5.4) [m]

$K_w$ : correction factor  $\leq 1$  to account for a reduced attenuation effect in case of favorable propagation conditions due to special meteorological conditions (see below).

*Remarks:*

- If the obstacle just touches the line of sight between source and receiver, the path length difference  $z$  yields 0. The barrier attenuation according to Eq. 5.18 becomes 5 dB, independently of the frequency. If the obstacle height is lowered further, still a path length difference can be evaluated. If the corresponding value is used with negative sign, the formula yields a smooth transition to the case where the barrier attenuation vanishes.
- As expected, Eq. 5.18 yields a barrier attenuation in the shadow zone that increases with frequency.
- If a barrier attenuation is present, the attenuation by the ground  $A_{\text{ground}}$  (ground effect) should be ignored.

A more accurate solution of the sound field behind a barrier is given by Pierce<sup>11</sup>. The insertion loss  $IL$ , that means the difference between the receiver level with obstacle and the level without obstacle can be calculated as:

<sup>11</sup>Allan D. Pierce, Acoustics, published by the Acoustical Society of America (1991).



$$IL = -10 \log \left( \left| H(X) - \frac{e^{j\frac{\pi}{4}}}{\sqrt{2}} A_D(X) e^{j\frac{\pi}{2} X^2} \right|^2 \right) \quad [\text{dB}] \quad (5.19)$$

where

$H(X)$ : Heaviside function, = 0 if the receiver is in the geometrical shadow, = 1 in all other cases.

$$X = \sqrt{\frac{2k}{\pi}} (L - R)$$

$k$ : wave number =  $2\pi \frac{f}{c}$

$f$ : frequency

$c$ : speed of sound

$L$ : path length from the source to the receiver around the obstacle

$R$ : path length from the source to the receiver through the obstacle

$A_D(X)$ : diffraction integral =  $f(X) - jg(X)$

$f(X), g(X)$ : auxiliary Fresnel functions, for which the following approximation exist:

$$f(X) \approx \frac{1+0.926X}{2+1.792X+3.104X^2}$$

$$g(X) \approx \frac{1}{2+4.142X+3.492X^2+6.67X^3}$$

In typical outdoor noise control applications - e.g. in the context of road traffic noise - barrier attenuations in the order of 5 to 15 dB can be achieved. A barrier is most effective, if it is positioned close to the source or close to the receiver. As a consequence of turbulence and inhomogeneities of the air the maximum barrier attenuation is limited to 20...25 dB. During the installation of noise barriers it has to be assured that no gaps occur as they would lower the attenuation effect considerably.

In some cases it is important that noise barriers are equipped with an absorbing surface to avoid reflections in the opposite direction. Methods to determine the characteristics of noise barriers in situ are described in the ISO standard *ISO 10847: In-situ determination of insertion loss of outdoor noise barriers of all types*. An excellent overview of possible modifications of the top section of noise barriers to improve the attenuation effect can be found in the paper by Ulrich <sup>12</sup>.

## 5.4 Reflections

Besides the ground, additional surfaces and objects can reflect sound. They introduce additional sound propagation paths and thus rise the sound pressure at the receiver. As the path lengths usually differ significantly from the direct sound, the different contributions can be summed up energetically. If the reflecting object is a flat surface, the reflection can be dealt with the concept of mirror sources. The criteria for the occurrence of specular reflections are

- the point of reflection lies on the reflecting surface
- the reflecting surface is large enough in relation to the sound wave length.

The test of a sufficient reflector size at the frequency  $f_c$  can be performed by checking if Eq. 5.20 is fulfilled.

$$f_c > \frac{2c}{(l_{\min} \cos(\beta))^2} \frac{d_{s,o} d_{o,r}}{d_{s,o} + d_{o,r}} \quad (5.20)$$

where

$c$ : speed of sound

$d_{s,o}$ : distance source - point of reflection

$d_{o,r}$ : distance point of reflection - receiver

$\beta$ : angle of incidence relative to the surface normal direction

$l_{\min}$ : smallest dimension of the reflector

If the reflecting surface has absorbing properties, a corresponding attenuation has to be accounted for.

<sup>12</sup>S. Ullrich, *Vorschläge und Versuche zur Steigerung der Minderungswirkung einfacher L'armschutzwände*, Strasse + Autobahn 7, p.347-354 (1998).

If the reflecting object is not sufficiently flat, the mirror source concept can no longer be applied. The handling of diffuse reflections is usually more difficult. As an example, Fig. 5.5 shows the reflection at a forest rim. Each tree scatters a certain amount of sound energy. There is no sharp reflection as in case of flat surfaces but a sort of reverberation with a distinct temporal smearing.

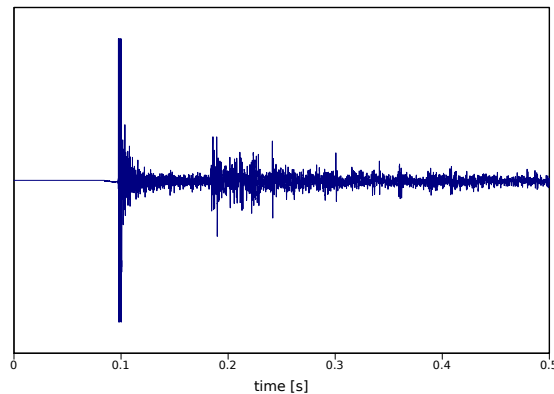


Figure 5.5: Level time curve of a gun shot reflected at a forest rim. The direct sound is followed by a reflection that is strongly smeared over time.

## 5.5 Meteorological effects

Up to now the medium air was assumed to be homogeneous, in rest and time invariant. All three conditions are usually not fulfilled. Of importance regarding possible sound propagation attenuation variations are vertical temperature and wind speed gradients and the temporal and local inhomogeneities in the air layer close to the ground. Temperature and wind speed gradients lead to a curvature of the propagation paths. Local inhomogeneities of the air produce scattering effects.

### 5.5.1 Temperature gradients

The mass of the atmosphere generates an average pressure of 1013 hPa on sea level. With increasing height above ground, the pressure drops by about 12 Pa per meter. As a consequence of this pressure decrease a packet of air that moves upwards cools down with about 1° per one hundred meters. A temperature stratification with a gradient of  $-1^\circ/100\text{m}$  is called adiabatic stratification.

The adiabatic stratification corresponds to the basic state of the atmosphere without additional exterior influences. However, during day time with strong incoming sound radiation the ground and with a certain delay the air layer above is heated up. This leads to a strong negative temperature gradient corresponding to decreasing temperature with increasing height. This is called an unstable stratification. On the other hand during nights with clear sky, the ground loses energy due to outgoing radiation. This leads to a strong cooling of the ground and the adjoining layers of air. In the following, a positive temperature gradient develops in the lowest few meters. This condition is called stable stratification or temperature inversion. It should be noted that a stable stratification can only develop if there are no strong winds.

In both regimes with unstable and stable stratification the temperature gradients are largest close to the ground and become smaller with increasing height. The temperature as a function of height above ground can be described with an approach as shown in Eq. 5.21.

$$T(z) = T(0) + kz^{0.2} \quad (5.21)$$

where

$T(z)$ : temperature [°C] at height  $z$  [m] above the ground

$k$ : constant depending of the stability condition with values  $k = -1.9$  in the very unstable case and  $k = 2.6$  for very stable conditions

## Consequences of temperature stratification for the sound propagation

As the speed of sound depends on temperature, a temperature gradient leads to a gradient of the effective propagation speed. A direct consequence of this is a curvature of the sound paths. In case of unstable stratification during day time the curvature points away from the ground (Figure 5.6). In larger distances shadow zones evolve with a corresponding strong attenuation. On the other hand during clear nights with stable stratification the sound speed increases with height, leading to a curvature towards the ground (Figure 5.7). This results in a lowering of the attenuation compared to day time. It is even possible that obstacles loose their effect as they are surmounted by the propagation path.

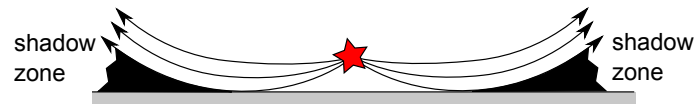


Figure 5.6: Curvature of sound rays due to a negative temperature gradient. In larger distances a shadow zone develops where the sound pressure is strongly attenuated.



Figure 5.7: Curvature of sound rays due to a positive temperature gradient.

### 5.5.2 Wind

Due to friction in the vicinity of the ground, wind speed shows always a vertical gradient. The wind speed profile  $u(z)$  can be described with help of Eq. 5.22<sup>13</sup>.

$$\frac{u(z)}{u_{\text{ref}}} = \left( \frac{z - d_0}{z_{\text{ref}} - d_0} \right)^\alpha \quad (5.22)$$

where

$u(z)$ : average wind speed [m/s] at the height  $z$  [m] above ground

$u_{\text{ref}}$ : average wind speed at the reference height  $z_{\text{ref}}$  [m] above ground (typ. 10 m)

$d_0$ : offset height [m], situation dependent according to Table 5.5

$\alpha$ : profile exponent, situation dependent according to Table 5.5

site	$d_0$ [m]	$\alpha$
water, ice or snow	0	0.08...0.12
gras land	0	0.12...0.18
parks, agglomeration	$0.75h$	0.18...0.24
forests, urban areas	$0.75h$	0.24...0.40

Table 5.5: Offset heights and profile exponents for Eq. (5.22). The parameter  $h$  corresponds to the average height of buildings and / or vegetation [m].

### Consequences of wind regarding sound propagation

The sound propagation in a moving medium has to consider the sound speed vector (normal to the wave front) and the velocity vector of the medium. The wave front at time  $t + \Delta t$  can be found from the front at time  $t$  by vector addition of the sound speed vector and the medium velocity vector (Fig. 5.8).

<sup>13</sup>VDI-Richtlinie 3782, Blatt 12: Umweltmeteorologie, Physikalische Modellierung von Strömungs- und Ausbreitungsvorgängen in der atmosphärischen Grenzschicht (1999).

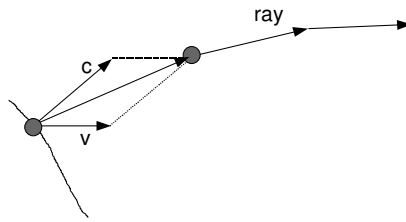


Figure 5.8: The momentary propagation speed of a point on a wave front is given by addition of the sound speed vector  $\vec{c}$  (normal to the wave front) and the medium velocity vector  $\vec{v}$ .

The important influence of wind on sound propagation is a result of the vertical wind speed gradient. In downwind direction sound propagates faster with increasing height. Similarly as in case of stable temperature stratifications, sound propagates no longer along straight lines but becomes a curvature towards the ground. In the upwind direction the curvature points upwards (Figure 5.9).

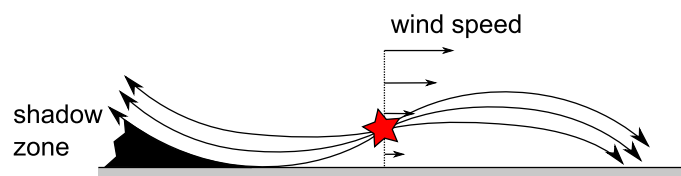


Figure 5.9: Curvature of sound rays due to a wind speed gradient. In the upwind direction a shadow zone develops where the sound pressure is strongly attenuated.

### 5.5.3 Favorable and unfavorable sound propagation conditions

The influence of wind and temperature gradients on sound propagation can be divided roughly into the two categories *favorable* and *unfavorable* sound propagation conditions. Favorable conditions are given if the propagation curvature is oriented towards the ground, unfavorable conditions are encountered in case of a bending upwards.

For engineering applications, the propagation conditions are usually specified in four classes <sup>14</sup>:

- M1: unfavorable sound propagation conditions
- M2: neutral conditions (no bending)
- M3: favorable sound propagation conditions
- M4: very favorable sound propagation conditions

The propagation classes are determined by the temperature stratification and the component of the wind speed  $v$  in propagation direction. A specific meteorological situation can be mapped onto the corresponding propagation class as shown in Table 5.6.

	$v < -1$	$-1 < v < 1$	$1 < v < 3$	$3 < v < 6$	$v > 6$
overcast sky	M1	M2	M3	M3	M4
clear sky during day	M1	M1	M2	M3	M4
clear sky during night	M1	M4	M4	M4	M4

Table 5.6: Mapping of a meteorological situation onto the propagation classes M1 to M4.  $v$  is the wind speed component projected onto the propagation direction from source to receiver in m/s.

<sup>14</sup>ISO 1996-2rev, Acoustics - Description, measurement and assessment of environmental noise - Part 2: Determination of environmental noise levels

For the distinction between overcast/clear sky, the following criteria can be used:

primary criteria

- clear during night: if temperature difference measured at 2.0 m and 0.05 m above ground is larger than 1.5°C
- clear during day: if global radiation  $> 200 \text{ W/m}^2$

alternative criteria

- clear during day/night: if daily (24h) temperature variation at 2.0 m above ground is larger than 10°C
- clear during day/night: if cloud coverage  $< 4/8$

#### 5.5.4 Turbulence

Wind flow over non-flat terrain or locally varying heating of the ground surface lead to inhomogeneities of the air in the surface layer. These inhomogeneities are called turbulence. Turbulence is responsible for arbitrary variations of the propagation attenuation between source and receiver. However more important are scattering effects that can reflect sound energy into geometrical shadow zones and the effect of decorrelation between direct and ground reflected sound. The incorporation of turbulence into calculation schemes can be done in different ways as e.g. described here <sup>15</sup>.

#### 5.5.5 Calculation of meteorological effects on sound propagation

The influence of meteorological effects on sound propagation can be considered in different ways.

**Empirical corrections of barrier attenuation** The possible variation of the propagation attenuation due to meteorological effects is especially large in case of an obstacle between source and receiver. For downwind conditions or for stable stratification the barrier attenuation can be significantly reduced. There are barrier attenuation formulas such as ISO 9613-2 with empirical corrections for favorable propagation conditions.

**Analytical solutions of sound ray paths** Under the assumption of linear vertical profiles of the effective sound speed (constant gradient), the curvature of the sound rays can be described analytically. The resulting rays are circles. They can be constructed for arbitrary source and receiver positions and the consequences for a barrier attenuation or the ground effect can be calculated easily <sup>16</sup>.

**Ray tracing** With ray tracing calculation schemes <sup>17</sup>, the propagation of sound rays can be determined for arbitrary effective sound speed profiles (Fig. 5.10). Sound pressure levels at a receiver point can be determined by evaluating the density of the rays.

**Numerical solutions of the wave equation** Several strategies are known to find approximate numerical solutions of the wave equation. As the distances between source and receiver are usually large, classical methods such as Finite Elements are out of question due to the exploding calculation effort. More suitable are approximations such as the Parabolic Equation (PE) that assume pure forward propagating waves and yield a numerical solution of the wave equation. The benefit of the constraint of forward propagation is that fact that a stepwise solution of small systems of equations is possible <sup>18</sup>.

---

<sup>15</sup>P. Chevret et al. A numerical model for sound propagation through a turbulent atmosphere near the ground. J. Acoustical Society of America, vol. 100, p.3587-3599 (1996).

<sup>16</sup>A. L'Esperance et al., Heuristic Model for Outdoor Sound Propagation Based on an Extension of the Geometrical Ray Theory in the Case of a Linear Sound Speed Profile, Applied Acoustics, vol. 37, p. 111-139 (1992).

<sup>17</sup>Robert J. Thompson, Ray-acoustic intensity in a moving medium, Journal of the Acoustical Society of America, vol. 55, p. 729-737 (1974).

<sup>18</sup>Erik M. Salomons, computational atmospheric acoustics, Kluwer Academic Publishers, 2001.

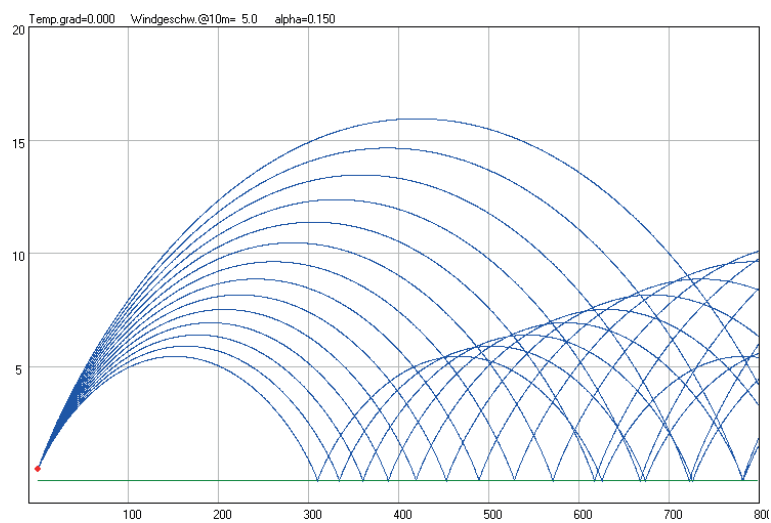


Figure 5.10: Example of a ray tracing simulation for downwind of 5 m/s at a height of 10 m above ground. The horizontal axis is the coordinate in propagation direction, the vertical axis is the height above ground (note the different scaling of the axis). The rays start at the source on the left. They are bent downwards and can thus surmount obstacles. At certain points rays intersect. In these so called caustics the energy density becomes infinitely high which can obviously not be true. Within the ray tracing model, no statement about the sound pressure in these points is possible.

# Chapter 6

## Absorption and reflection

If a sound wave hits a boundary surface, only a portion of the incoming energy is reflected<sup>1</sup>. The energy that is not reflected splits into a portion that is absorbed and a portion that is transmitted. The absorbed energy is converted into heat. The transmission is the result of excitation of the boundary to vibrations and then as a consequence sound is radiated on the rear side. Often the transmitted portion is not addressed explicitly which means that this contribution is added to the absorbed portion.

### 6.1 Characterization

The quantitative description of the property of a surface to absorb or reflect sound uses the absorption coefficient or the reflection coefficient. The absorption coefficient  $\alpha$  is defined as the ratio of the energies of absorbed and incident sound:

$$\alpha = \frac{\text{absorbed energy}}{\text{incident energy}} \quad (6.1)$$

The reflection coefficient  $R$  on the other hand is the ratio of the sound pressures of reflecting and incoming sound:

$$R = \frac{\text{sound pressure of reflected wave}}{\text{sound pressure of incident sound wave}} \quad (6.2)$$

The absorption coefficient is a real number in the range  $0 \dots 1$ . The reflection coefficient is a complex number and describes the amplitude ratio and the phase shift during reflection. Under the assumption that the whole incident energy splits into absorption and reflection, a relation between  $\alpha$  and  $R$  can be established:

$$\alpha = 1 - |R|^2 \quad (6.3)$$

### 6.2 Types of absorbers

#### 6.2.1 Porous absorbers

Porous absorbers are usually made from glass fibers or organic fibers or open foam. They function as absorbers due to friction losses when the air moves back and forth in the pores. The relevant sound field variable is thus the sound particle velocity. Consequently the optimal positioning of porous absorbers is at locations with high sound particle velocity. It is therefore beneficial to install a porous absorber with a certain distance to an acoustically hard boundary.

#### 6.2.2 Resonance absorbers of type Helmholtz

Helmholtz resonance absorbers are formed by an acoustical spring and an acoustical mass. The spring is realized by a compressible volume of air, while the mass corresponds to a column of air that can be accelerated (Fig. 6.1).

---

<sup>1</sup>T. J. Cox, P. D'Antonio, Acoustic Absorbers and Diffusers, Taylor and Francis, 2009.

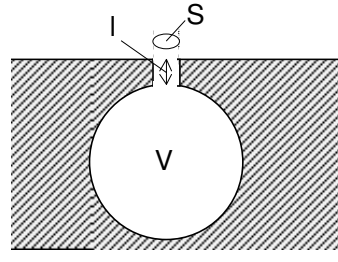


Figure 6.1: Resonance absorber of type Helmholtz. The mass and spring are realized by a column of air (cross section  $S$  and length  $l$ ) and a volume  $V$ .

The resonance frequency of a mass/spring system with mass  $m$  and stiffness  $s$  is

$$f_0 = \frac{\sqrt{\frac{s}{m}}}{2\pi} \quad (6.4)$$

The mass  $m$  is given by the mass in the cylinder and a portion of vibrating air at the end of the cylinder. This additional mass is introduced in the calculation as a mouth correction. With  $\rho$  as density of air, the moving mass is:

$$m = \rho(l + l_{\text{corr}})S \quad (6.5)$$

The mouth correction can be approximated as  $l_{\text{corr}} \approx 0.8R$  where  $R$  corresponds to the radius of the cylinder <sup>2</sup>.

The stiffness  $s$  of the spring can be determined with help of the Poisson law (Eq. 1.14) for adiabatic processes:

$$s = c^2 \rho \frac{S^2}{V} \quad (6.6)$$

where  $c$  is the speed of sound. Finally the resonance frequency  $f_0$  is found as

$$f_0 = \frac{c}{2\pi} \sqrt{\frac{S}{V(l + l_{\text{corr}})}} \quad (6.7)$$

Without further measures the frequency curve of absorption shows a large peak in a narrow band only. The absorption effect can be enlarged over a wider frequency range by introducing damping (an acoustical resistance such as porous material) at the position of the neck where the sound particle velocity is highest.

There are different realizations of Helmholtz resonators. A first possibility is a structure that consists of a layer of damping material and a plate with holes or slits on top of it. The air in the holes or slits acts as an acoustical mass, the air in the damping material is the acoustical spring.

An other version uses a sheet of metal that is installed in a certain distance to the wall or ceiling. In this case the acoustical mass is dominated by the neck correction. The damping is usually realized with a thin tissue mounted on the rear side of the metal sheet.

The extra damping material can be omitted if the holes have a very small diameter ( $< 1$  mm) <sup>3,4,5</sup>. Such Helmholtz resonators are called microperforated absorbers. The friction loss in the small holes is large enough to realize sufficient damping. It is thus possible to construct absorbers from one material only. If this material is glass or acrylic glass, transparent absorbers are possible which opens very

<sup>2</sup>The mouth correction  $l$  yields a non vanishing mass even if the length of the cylinder tends to 0.

<sup>3</sup>Dah-You Maa, Microperforated-Panel Wideband Absorbers, Noise Control Engineering Journal, no. 3, vol. 29 (1987).

<sup>4</sup>Einsatz mikroperforierter Platten als Schallabsorber mit inhärenter Dämpfung, H. V. Fuchs, X. Zha, Acustica, no. 2, vol. 81 (1995).

<sup>5</sup>I. Falsafi, A. Ohadi, Design guide of single layer micro perforated panel absorber with uniform air gap, Applied Acoustics, vol. 126, p.48-57 (2017).



interesting design possibilities. Alternative constructions use slits instead of holes<sup>6</sup> or thin layers of air between two adjacent plates<sup>7</sup> or multi-layer arrangements<sup>8</sup>.

Figure 6.2 shows the performance of absorption for two geometries of a perforated absorber. The smaller the holes, the higher the damping and thus the broader the frequency range with high absorption.

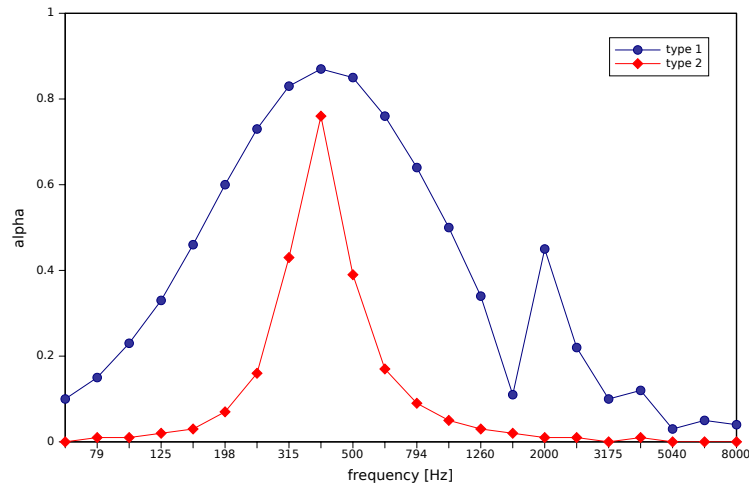


Figure 6.2: Calculated absorption as a function of frequency for normal sound incidence. Absorber type 1: plate thickness = 3 mm, hole diameter = 0.4 mm, spacing between holes = 2 mm, distance to wall = 100 mm; absorber type 2: plate thickness = 3 mm, hole diameter = 2 mm, spacing between holes = 15 mm, distance to wall = 50 mm.

### 6.2.3 Membrane absorbers

Membrane absorbers or panel absorbers are another realization of a spring-mass resonance absorber. In contrast to Helmholtz absorbers the mass is realized by a thin plate or foil<sup>9</sup>. The spring is determined by the stiffness of the layer of air between the plate and the rigid wall. If foils are used as mass, their stiffness has to be added to the stiffness of the air. Diaphragmatic absorbers have to be constructed as boxes to avoid that air can escape at the edges.

As a resonance effect is responsible for the absorption, membrane absorbers are frequency selective. They are mainly used for low frequency absorption. The resonance frequency  $f_0$  for which highest absorption is obtained is given as

$$f_0 = \frac{\sqrt{\frac{s''}{m''}}}{2\pi} \quad (6.8)$$

where

$s''$ : stiffness per unit area

$m''$ : mass per unit area

Similarly to the case of Helmholtz absorbers the stiffness is found as

$$s'' = \frac{\rho c^2}{l_w} \quad (6.9)$$

where

$l_w$ : distance of the panel to the rigid wall

<sup>6</sup>R. T. Randeberg, Adjustable Slitted Panel Absorber, Acta Acustica united with Acustica, vol. 88, p.507-512 (2002).

<sup>7</sup>R. T. Randeberg, A Helmholtz Resonator with a Lateral Elongated Orifice, acta acustica, vol. 86, p.77-82 (2000).

<sup>8</sup>Y.J. Qian et al., Pilot study on wideband sound absorber obtained by adopting a serial-parallel coupling manner, Applied Acoustics, vol. 124, p.48-51 (2017).

<sup>9</sup>K. Sakagami et al., Sound Absorption of a Cavity-Backed Membrane: A Step Towards Design Method for Membrane-Type Absorbers, Applied Acoustics, vol. 49, no. 3, pp. 237-247 (1996).

and finally

$$f_0 = \frac{c\sqrt{\frac{\rho}{m'l_w}}}{2\pi} \quad (6.10)$$

By filling the volume of air between the panel and the rigid back wall with a porous material, the absorption can be increased and extended to a broader frequency range.

For practical applications, certain conditions should be fulfilled. In general, best results are obtained for large values of the distance  $l_w$ . However  $l_w$  needs to be small compared to the wave length  $\lambda_0$  at the resonance frequency. Usually one tries to fulfill the condition  $l_w < \lambda_0/12$ . Further the panel shouldn't be too small, a minimum area of 0.4 m<sup>2</sup> is stipulated. In addition the proportions of the panel shouldn't be too extreme, the minimum length of each panel side is 0.5 m.

Panel absorbers can be combined with porous absorbers that are put on top. At low frequencies where the panel absorber is active, the porous absorber is transparent. However the additional mass of the porous layer has to be considered.

## 6.3 Measurement of absorption and reflection

### 6.3.1 Kundt's tube

The measurement in Kundt's tube allows for the determination of the absorption coefficient under normal incidence for relative small material probes<sup>10</sup>.

Kundt's tube serves to create a one-dimensional plane wave sound field at discrete frequencies (Figure 6.3). For that purpose a loudspeaker located at one end of the tube generates a sine wave. This wave propagates in the tube to the other end and will be reflected at the hard termination. Thereby the incident and reflected sound wave form an interference pattern with pressure maxima and minima. By introducing absorbing material in front of the hard termination, the reflection is reduced and as a consequence the sound pressure maxima decrease and the minima increase. As will be shown below, the absorption coefficient can be determined from the ratio of sound pressure in the maxima and minima alone.

To guarantee that only plane waves along the tube axis occur, the frequency has to be limited to a value such that the corresponding wave length is larger than the diameter of the tube.

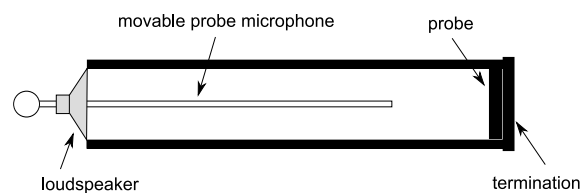


Figure 6.3: Kundt's tube with the loudspeaker at one end and the material probe in front of the hard termination at the other end. In the center of the tube diameter a probe microphone can be moved along the tube axis to detect sound pressure maxima and minima.

With  $p_r$  as sound pressure of the wave reflected at the end of the tube and  $p_e$  as sound pressure of the incident wave one can write:

$$\frac{p_r}{p_e} = \sqrt{1 - \alpha} \quad (6.11)$$

The sound pressure maxima are formed by constructive interference of incident and reflected wave:

$$p_{\max} = p_e + p_r = p_e(1 + \sqrt{1 - \alpha}) \quad (6.12)$$

<sup>10</sup>ISO-Norm 10534: Acoustics - Determination of sound absorption coefficient and impedance in impedance tubes, Part 1: Method using standing wave ratio.

The sound pressure minima on the other hand result as destructive interference between incident and reflected wave:

$$p_{\min} = p_e - p_r = p_e(1 - \sqrt{1 - \alpha}) \quad (6.13)$$

With the ratio

$$n = \frac{p_{\max}}{p_{\min}} \quad (6.14)$$

the absorption coefficient can be calculated as

$$\alpha = 1 - \left( \frac{n - 1}{n + 1} \right)^2 \quad (6.15)$$

### 6.3.2 Impedance tube

The measurement in the tube of Kundt is time consuming, as for each frequency the maxima and minima have to be searched and evaluated. In this respect the impedance tube is a more elegant method <sup>11</sup>. The geometry of loudspeaker, tube and probe is similar to Kundt's tube. However the sound pressure is not observed along the tube axis but at two fix positions. The excitation is wide band noise, allowing to extract spectral information with one single measurement. For a given geometry (distanced between the two microphones and distances to the probe) the ratio between incoming and reflected wave can be evaluated by measuring the complex transfer function between the microphones <sup>12</sup>. From the complex pressure reflection factor the impedance and the absorption coefficient can be calculated.

### 6.3.3 Reverberation chamber

The measurement of sound absorption in the reverberation chamber <sup>13</sup> is based on the influence of absorption on the reverberation process. After switching off a sound source in a room with hard surfaces, the sound pressure doesn't drop to zero immediately. The sound waves are still reflected back and forth between the walls, floor and ceiling. As they loose energy only slowly, the observable reverberation process can last for several seconds. The reverberation is described by the reverberation time  $T$ . The parameter measures the time for a decrease of the sound energy density to 1/1'000'000 of its initial value.

If sound absorbing material is introduced, the reverberation time decreases. The relation between reverberation time  $T$ , room volume  $V$  and Absorption  $A$  can be expressed by the formula of Sabine:

$$T = \frac{0.16V}{A} \quad (6.16)$$

From two measurements of  $T$  in the empty room and in the the room with absorbing material, the increase of absorption  $\Delta A$  by the material can be determined. With knowledge of the area  $S$  of the introduced material the absorption coefficient is found as  $\alpha_s = \Delta A/S$ .

For maximum accuracy it is beneficial to aim at large differences between the empty room measurement and the measurement with the material installed. For that reason reverberation chambers are constructed with as less initial absorption as possible. The walls, the floor and the ceiling are thus made from acoustically hard materials. To reduce the tendency of low frequency resonances, the walls are usually oriented in such a way that opposite walls are not in parallel. In addition, reflectors and diffusers may be installed in the room to improve the diffusivity of the sound field. The area of the material probe lies usually between 10 and 12m <sup>2</sup>.

The absorption coefficients  $\alpha_s$  determined in the reverberation chamber do not match exactly with the values found in Kundt's tube of the impedance tube. One reason is the difference in the exciting sound

<sup>11</sup>ISO-Norm 10534: Acoustics - Determination of sound absorption coefficient and impedance in impedance tubes, Part 2: Transfer-function method.

<sup>12</sup>J. Y. Chung, D. A. Blaser, Transfer function method of measuring in-duct acoustic properties. Journal of the Acoustical Society of America, vol. 68, p. 907-921, 1980.

<sup>13</sup>ISO Norm 354 Acoustics - Measurement of sound absorption in a reverberation room. 1985.

field. In the tubes only perpendicular incidence is investigated while in the reverberation chamber the angles vary between  $0^\circ$  and  $90^\circ$ . In some cases  $\alpha_s$  values  $> 1$  occur, which doesn't make sense from a physical point of view. The reason for this is that important assumptions for the Sabine formula are violated.

### 6.3.4 In situ measurement of impulse responses

In some cases it is not possible to put the structure or material of interest in the impedance tube or bring it to the reverberation chamber. Here in situ impulse response measurements in an appropriate geometrical configuration may yield useful information. The loudspeaker - microphone - absorber geometry has to be chosen in such way that the direct sound, the reflection from the absorber and other unwanted reflections can be separated on the time axis. Two main difficulties are linked to the problem of evaluating an absorption coefficient. To account for the direct and reflected sound path length ratio, a normalization step is necessary. This is easily done for flat absorbers but can cause major difficulties if the surface of interest is significantly structured in depth. The second problem arises from the requirements at low frequencies. The evaluation of the low frequency range makes large dimensions of the absorber necessary (see Fresnel zones).

For a recent review of *in situ* absorption measurement techniques see <sup>14</sup>.

## 6.4 Calculation of absorption and reflection from impedance relations

### 6.4.1 Normal incidence

A plane wave is considered that propagates in a medium with impedance  $Z_0$ . The medium is bounded by a medium with impedance  $Z_1$ . It is assumed that the wave hits the impedance discontinuity  $Z_0 \rightarrow Z_1$  perpendicularly.

The incident sound wave has sound pressure  $p_I$  and sound particle velocity  $v_I$  with

$$\frac{p_I}{v_I} = Z_0 \quad (6.17)$$

The reflected wave has sound pressure  $p_{II}$  and sound particle velocity  $v_{II}$  where

$$\frac{p_{II}}{v_{II}} = Z_0 \quad (6.18)$$

At the surface of the medium  $Z_1$ , sound pressure and sound particle velocity add up to <sup>15</sup>

$$\begin{aligned} p &= p_I + p_{II} \\ v &= v_I - v_{II} \end{aligned} \quad (6.19)$$

with the condition:

$$\frac{p}{v} = Z_1 \quad (6.20)$$

From

$$p_I + p_{II} = Z_1 \left( \frac{p_I}{Z_0} - \frac{p_{II}}{Z_0} \right) \quad (6.21)$$

follows finally

$$\frac{p_{II}}{p_I} = R = \frac{Z_1 - Z_0}{Z_1 + Z_0} \quad (6.22)$$

<sup>14</sup>E. Brandao, A. Lenzi, S. Paul, A Review of the In Situ Impedance and Sound Absorption Measurement Techniques, Acta Acustica united with Acustica, vol. 101, p. 443-463, 2015.

<sup>15</sup>The scalar quantity sound pressure adds up with a positive sign, while the vectors sound particle velocity adds up with negative sign due to reversed orientation.

Eq. 6.22 demonstrates that the reflection factor  $R$  approaches 1 for increasing difference of  $Z_1$  and  $Z_0$ . On the other hand, maximum absorption will show for  $Z_1 = Z_0$ . An absorber is characterized by the property that it doesn't introduce a significant resistance to the incoming wave.

If a layer of porous absorption is placed in front of a hard wall, the resulting impedance is increased compared to the impedance of the absorber itself. As a rule of thumb the thickness of the absorber should be larger than a quarter of the wave length of the lowest frequency that should be absorbed.

### 6.4.2 Oblique incidence

For many materials it can be assumed (as a first order approximation) that the propagation in the material itself is perpendicular to the surface due to refraction at the entry of the oblique incident wave. In this case the reaction of the material at any point is independent of the reaction at any other point, which is called *local reaction*. With this assumption one finds

$$\frac{p_{II}}{p_I} = R = \frac{Z_1 - \frac{Z_0}{\cos(\phi)}}{Z_1 + \frac{Z_0}{\cos(\phi)}} \quad (6.23)$$

with

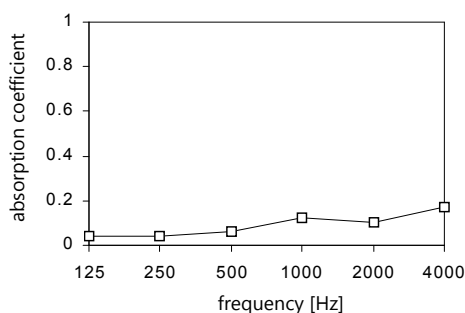
$\phi$ : angle of incident and outgoing wave relative to the surface normal direction

The nominator in Eq. 6.23 can become 0 also for  $Z_1 > Z_0$  by adjustment of  $\phi$ . This means that for any impedance discontinuity  $Z_0 \rightarrow Z_1$  perfect absorption is achieved for a certain angle of incidence. In the extreme case of  $\phi \rightarrow 90^\circ$  the reflection factor  $R$  approaches -1, independently of  $Z_1$ . This corresponds to total reflection with a phase shift of  $180^\circ$ .

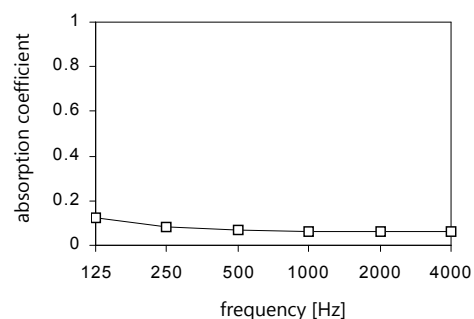
## 6.5 Typical values of absorption coefficients

There exist collections of data of absorption coefficients for different materials<sup>16</sup>. Usually octave band values of  $\alpha_s$  measurements in the reverberation chamber are shown. The following figures give a little overview.

*parquet floor*

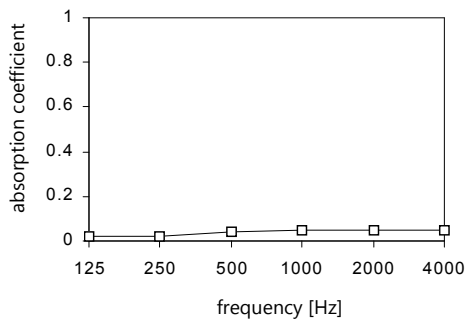


*wooden stage*

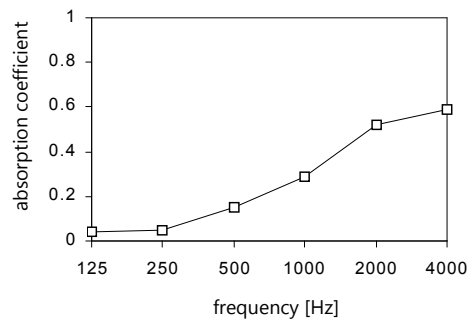


<sup>16</sup>Fasold, Sonntag, Winkler, Bau- und Raumakustik, Verlagsgesellschaft Rudolf M'uller, K'oln-Braunsfeld, 1987.

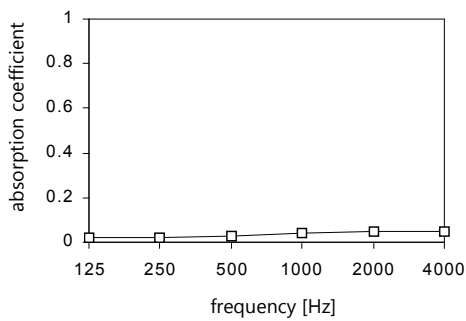
stone floor



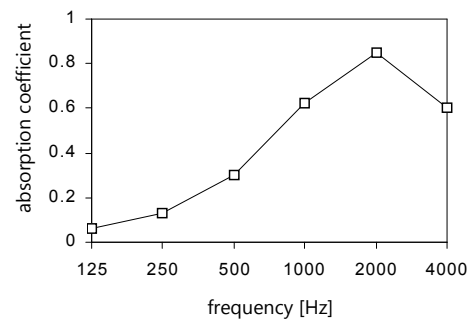
carpet, thickness 5 mm



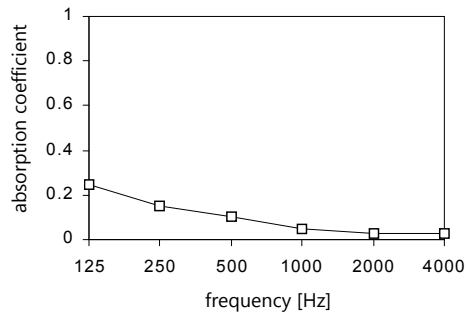
plaster



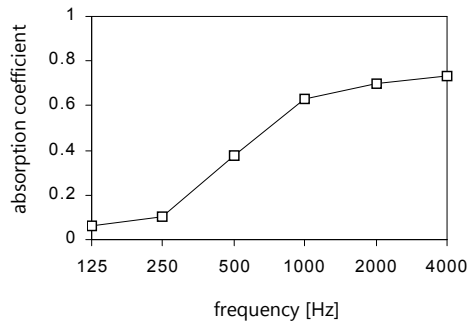
acoustically optimized plaster, thickness 20 mm



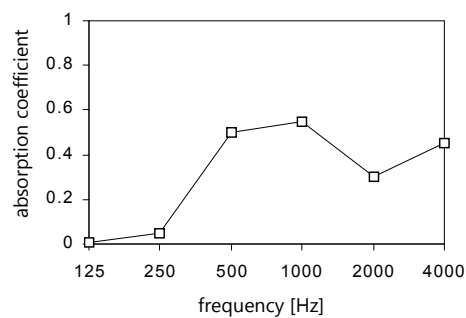
window



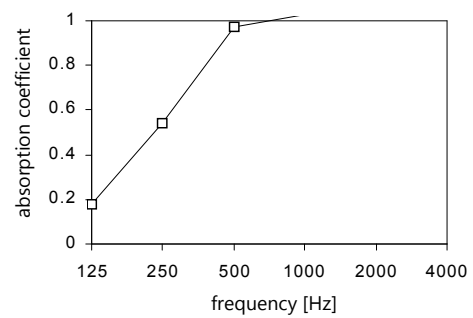
heavy curtain



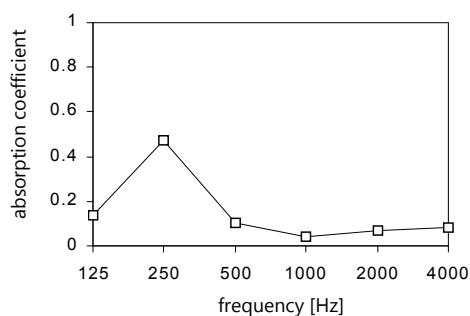
egg carton



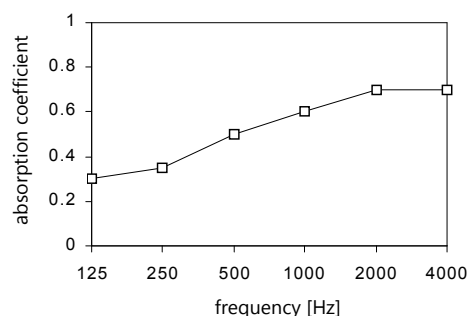
glass fiber panel, thickness 50 mm



panel resonator, 4 mm wood, 120 mm air layer



audience on upholstered chair



## 6.6 Cover for porous absorbers

In most cases porous absorbers need a cover for mechanical protection. Often used are panels with slits or holes. The openings have to be designed in such a way that the degree of transmission is close to 1 in the frequency range of interest. The problem lies in the high frequencies<sup>17</sup>. The sound wave can pass the panel only by an oscillation of air columns in the holes. Due to the inertia this gets more and more difficult for increasing frequency. Figure 6.4 shows the fundamental frequency dependency of the degree of transmission.

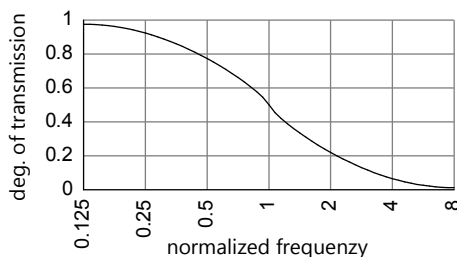


Figure 6.4: Normalized frequency dependency of the degree of transmission for perforated panels.

The relevant parameters of the panel are the ratio  $\epsilon$  of the area of the holes relative to the area of the panel, the diameter  $r$  of the holes and the thickness  $l$  of the panel. The length of the oscillating air columns does not exactly correspond to the thickness of the panel but is a little larger. This fact is accounted for by introducing a correction  $2\Delta l$ , resulting in an effective panel thickness of  $l^*$  with

$$l^* = l + 2\Delta l \quad (6.24)$$

The frequency  $f_{0.5}$  where the degree of transmission has dropped to 0.5 can be estimated as

$$f_{0.5} \approx 1500 \frac{\epsilon}{l^*} \quad (6.25)$$

where

$\epsilon$ : ratio of the area of the holes relative to the area of the panel in %

$l^*$ : effective panel thickness in mm

Table 6.1 shows some parameter combinations for  $f_{0.5} = 6300$  Hz.

In some cases it may be interesting to explicitly limit the absorption of porous materials at high frequencies due to the fact that there is often plenty of high frequency absorption existent. This can be done by a proper adjustment of the perforated panel parameters.

<sup>17</sup>Fasold, Sonntag, Winkler, Bau- und Raumakustik, Verlagsgesellschaft Rudolf M'uller, K'oln-Braunsfeld, 1987.

panel thickness $l$	1 mm	1 mm	4 mm	4 mm
$\epsilon$	5 %	10 %	17 %	20%
hole diameter $r$	0.5 mm	3 mm	0.5 mm	3 mm

Table 6.1: Parameters of a perforated panel for  $f_{0.5} = 6300$  Hz.



# Chapter 7

## Room acoustics

### 7.1 Introduction

What makes sound fields in rooms special is the superposition of direct sound and many first and higher order reflections. As a consequence of the sound energy that is stored in these reflections, there is so called reverberation. After switching off a source in a room, the sound pressure is only slowly fading away.

This reverberation effect is objectively described by the parameter *reverberation time*  $T$ . For a thorough discussion of reverberation, see e.g. Blesser <sup>1</sup>.

From an acoustical point of view the limiting surfaces (walls, floor and ceiling) are the relevant elements of a room. The sound field is influenced by their geometry, their absorption properties and their diffusivity. For the investigation of the sound field three methods are in use

- *Statistical room acoustics* assumes a diffuse sound field as a central simplification. The analysis focuses on the ratio of direct and diffuse sound and deals with the reverberation. Walls, floor and ceiling are described by the statistical absorption coefficient  $\alpha_s$ .
- *Geometrical room acoustics* models the sound propagation as energy that propagates along straight sound rays. This is a high frequency approximation that holds for wave lengths that are much smaller than the dimensions of the elements of the room. The reflection properties are defined by an absorption coefficient and a diffusivity to describe the scattering behavior.
- *Wave based room acoustics* is seeking solutions of the wave equation. The sound propagation is modeled physically correct and considers wave phenomena such as resonance, interference and diffraction. However analytical solutions are available for a few simple geometries only. In general, specific solutions have to be found with numerical approximations such as the Boundary Element method (BEM) or Finite Element method (FEM). The corresponding computational efforts restricts the application to small geometries or low frequencies. The boundary surfaces have to be described with their proper impedances. A difficulty arises as in practice this information is usually not available.

### 7.2 Room acoustics of large rooms

Sound fields in large rooms are characterized by a high density of room resonances already at relative low frequencies. As a consequence the fluctuations in the transfer functions from a source to a receiver have arbitrary character. Under these conditions statistical and geometrical room acoustic methods can be applied.

#### 7.2.1 Statistical room acoustics

Statistical room acoustics is based on the concept of a diffuse sound field, which means that

---

<sup>1</sup>Barry Blesser, An Interdisciplinary Synthesis of Reverberation Viewpoints, Journal of the Audio Engineering Society, vol. 49, p.867-903 (2001).

1. the sound energy density in the whole room is constant.
2. there is no predominant sound incident direction.

These two conditions are never totally fulfilled in real situations. However for practical applications a diffuse sound field can be assumed if there is not too much absorption in a room and if this absorption is more or less evenly distributed over the surface of the room <sup>2</sup>.

### Intensity on a wall

For a given sound energy density  $w$  in a room, the sound intensity on a wall shall be determined. The intensity corresponds to the incoming power per unit area. The power is given by the energy that hits the considered surface element  $dS$  within one second (Figure 7.1).

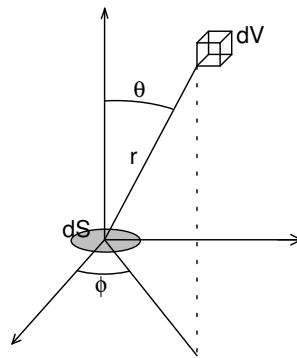


Figure 7.1: Situation to determine the energy contribution of a volume element  $dV$  to the surface element  $dS$  in a diffuse sound field.

The energy  $E$  that stems from the volume element  $dV$  and hits the surface element  $dS$  is

$$E = \frac{dS \cos \theta}{4\pi r^2} w dV \quad (7.1)$$

In spherical coordinates the volume  $dV$  is

$$dV = r^2 dr d\theta \sin(\theta) d\phi \quad (7.2)$$

The sound power  $W$ , that hits  $dS$  within one second corresponds to the energy contribution stemming from a half sphere with radius  $R = c \times 1 \text{ sec}$ :

$$W = I dS = \frac{w dS}{4\pi} \int_0^{c \times 1 \text{ sec}} \int_0^{2\pi} \int_0^{\pi/2} \cos(\theta) \sin(\theta) d\phi d\theta dr = \frac{w c}{4} dS \quad (7.3)$$

With this the intensity on a wall in a diffuse field with energy density  $w$  is found as

$$I = \frac{w c}{4} \quad (7.4)$$

### Total absorption and power balance in the diffuse field

If a sound source in a room is switched on, the sound energy density steadily increases until a final state of sound power balance is reached. This state is characterized by the condition that sound power that is absorbed is just as large as the sound power that is fed to the room by the source. The absorption of the room is described by the *total absorption*  $A$ , defined as

$$A = \sum_{i=1}^n S_i \alpha_i \quad \text{where} \quad \sum_{i=1}^n S_i = \text{area of the surface of the room} \quad (7.5)$$

<sup>2</sup>Murray Hodgson, When is Diffuse-Field Theory Applicable? Applied Acoustics, vol.49, n.3, p.197-207 (1996).

where  $\alpha_i$  is the absorption coefficient and  $S_i$  the area of the room surface element with index  $i$ ,  $n$  is the total number of surface elements.

The total sound power that is absorbed by the room surface is

$$W_{\text{absor}} = I_{\text{wall}} A = \frac{wc}{4} A \quad (7.6)$$

The balance condition is

$$W_{\text{absor}} = W_{\text{source}} \quad (7.7)$$

$$w = \frac{4W_{\text{source}}}{Ac} \quad (7.8)$$

The diffuse sound field can be understood as superposition of many plane waves that arrive from all possible directions. In case of a plane wave the energy that flows through an area of  $1 \text{ m}^2$  in  $1 \text{ sec}$  corresponds to the energy contained in a cylinder of base  $1 \text{ m}^2$  and height  $c \times 1 \text{ sec}$ . With this follows

$$I = wc = \frac{p^2}{\rho c} \quad (7.9)$$

Finally the sound pressure  $p_{\text{diffuse}}$  in a diffuse field can be expressed in dependency of the source power  $W_{\text{source}}$  and the total absorption  $A$  as

$$p_{\text{diffuse}}^2 = \frac{4W_{\text{source}}\rho c}{A} \quad (7.10)$$

Relation 7.10 is valid only under the idealized assumption that the diffuse field is constant throughout the room. However there are empirical formulas to consider a distance dependency of the sound pressure<sup>3</sup>:

$$p_{\text{diffuse}}^2 = \frac{4W_{\text{source}}\rho c}{A} e^{-\left(\frac{2\partial r}{c}\right)} \quad (7.11)$$

where

$r$ : source - receiver distance

$\partial$ : decay constant =  $3 \ln(10)/T$

$T$ : reverberation time

$c$ : speed of sound

### Direct sound and diffuse field contribution, critical distance

Up to now only the diffuse field was considered. Of course a diffuse field can't exist without a direct sound field. Under the assumption of an omnidirectional source that excites the sound fields, the pressure square  $p_{\text{direct}}$  of the direct sound is given as:

$$p_{\text{direct}}^2 = \frac{W_{\text{source}}\rho c}{4\pi r^2} \quad (7.12)$$

and hence the total sound pressure square  $p^2$  sums up to

$$p^2 = p_{\text{direct}}^2 + p_{\text{diffuse}}^2 = W_{\text{source}}\rho c \left( \frac{1}{4\pi r^2} + \frac{4}{A} \right) \quad (7.13)$$

For small distances  $r$ , the first term in the brackets dominates. This indicates that the direct sound with its  $1/r^2$  distance dependency is larger than the diffuse sound. For increasing distances the significance of the direct sound decreases and the location independent diffuse sound field determines more and more the total sound pressure (Figure 7.2). The distance where direct and diffuse sound have equal strengths is called *critical distance* and is usually labeled as  $r_c$ .

$$\text{with } \frac{1}{4\pi r_c^2} = \frac{4}{A} \quad \text{follows } r_c = \sqrt{\frac{A}{16\pi}} \quad (7.14)$$

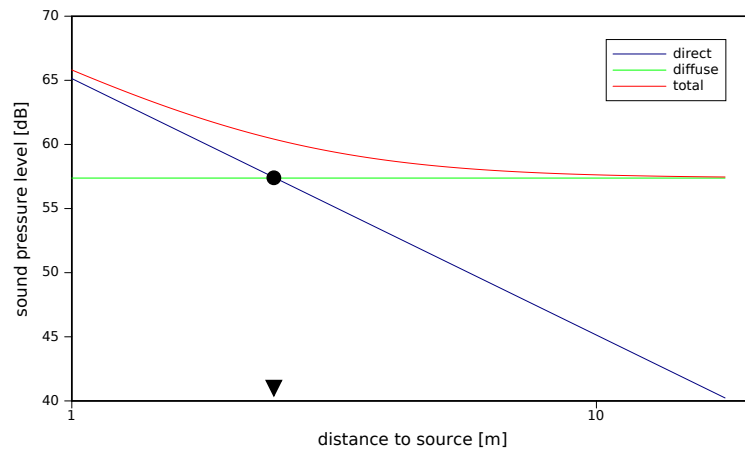


Figure 7.2: Distance dependency of sound pressure in a room with direct sound and an ideal diffuse sound field. The arrow marks the critical distance where direct and diffuse sound have equal strength.

If the source shows enhanced radiation in one direction, the critical distance in this direction increases accordingly.

In reality the distance dependency of sound pressure in a room doesn't follow exactly the relation shown in Fig. 7.2. A more subtle description is based on Eq. 7.11 and yields <sup>4</sup>:

$$L(r) = 10 \log \left( \frac{100}{r^2} + \frac{31200T e^{-0.04r/T}}{V} \right) \quad [\text{dB}] \quad (7.15)$$

where

$L(r)$ : sound pressure level at distance  $r$  relative to the value in 10 m

$T$ : reverberation time [sec]

$V$ : room volume [ $\text{m}^3$ ]

Figure 7.3 shows the corresponding distance dependency of sound pressure for a room with volume  $V = 20'000 \text{ m}^3$  and a reverberation time  $T = 2 \text{ sec}$ .

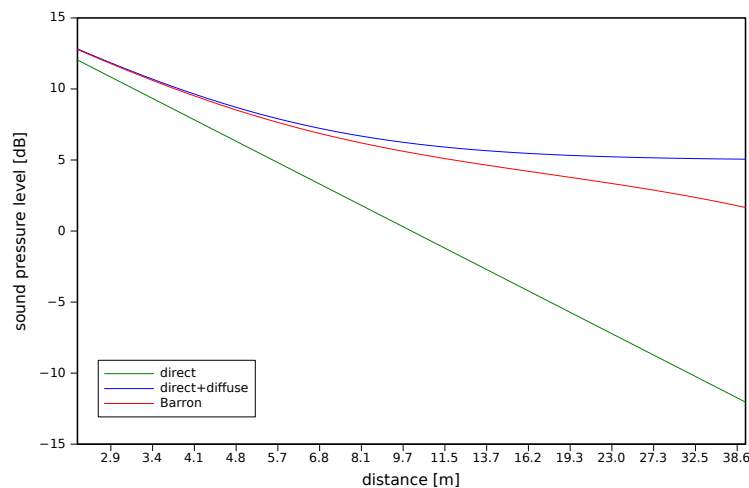


Figure 7.3: Sound pressure level as a function of distance in a room with  $V = 20'000 \text{ m}^3$  and a reverberation time  $T = 2 \text{ sec}$ . Direct sound, diffuse field theory and the formula of Barron are shown.

<sup>3</sup>M. Barron, L. J. Lee, Energy relations in concert auditoriums, Journal of the Acoustical Society of America, vol 84, p.618-628 (1988).

<sup>4</sup>M. Barron, Loudness in Concert Halls, Acustica - acta acustica, vol.82, suppl. 1 (1996).

## Reverberation, reverberation time

Above, the sound power relations for the stationary condition have been discussed. In the following, the situation of a sound source that is switched off shall be investigated. Due to the energy that is stored in the reflections, the sound energy density in the room decreases only slowly, depending on the room volume and the absorption of the room surfaces. This process is called reverberation and described quantitatively by the so called reverberation time. For the power balance can be written

$$W_{\text{source}} = W_{\text{absor}} + V \frac{dw}{dt} \quad (7.16)$$

where

$W_{\text{source}}$ : sound power emitted by the source

$W_{\text{absor}}$ : sound power that is absorbed by the room surfaces

$V$ : room volume

$w$ : energy density

From

$$W_{\text{absor}} = \frac{wc}{4}A \quad (7.17)$$

follows

$$W_{\text{source}} = \frac{wc}{4}A + V \frac{dw}{dt} \quad (7.18)$$

Eq. 7.18 represents a differential equation for the energy density  $w$ . If the source is switched off, the reverberation process manifests. The solution of  $w(t)$  that fulfills the equation

$$0 = \frac{wc}{4}A + V \frac{dw}{dt} \quad (7.19)$$

has the form

$$w(t) = w_0 e^{bt} \quad (7.20)$$

where

$$b = -\frac{cA}{4V}$$

Eq. 7.20 describes the reverberation process as an exponentially decaying time history. This corresponds to a straight line in the level-time representation as shown in 7.4.

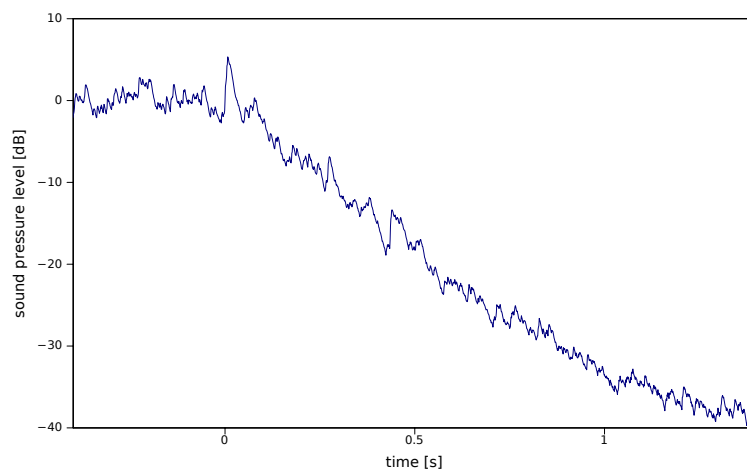


Figure 7.4: Example of sound pressure decay in a room after switching off the source at time  $t = 0$ .

The reverberation time  $T$  is defined as the time that passes until the energy density has decreased to  $1E-6$  of its initial value. In the dB scale this corresponds to -60 dB. From

$$e^{-\frac{cA}{4V}T} = 10^{-6} \quad (7.21)$$

follows

$$T = \frac{-\ln(10^{-6})4V}{cA} = \frac{0.16V}{A} \quad (7.22)$$

The relation between  $T$ , the room volume  $V$  and total absorption  $A$  was found experimentally by W. C. Sabine in 1900. To his honor, Eq. 7.22 is usually called Sabine equation.

An other derivation of the reverberation time was given by Eyring. His conception was that sound propagates in form of energy packets along straight lines. Whenever such a packet hits a room surface, a certain amount of energy is absorbed while the remaining energy is reflected. Besides the average absorption coefficient  $\alpha$  of the room surfaces, the mean free path length between two reflections  $\ell$  is the second relevant parameter. For a rectangular room  $\ell$  can be calculated from the volume  $V$  and the room surface area  $S$ :

$$\ell = \frac{4V}{S} \quad (7.23)$$

The reverberation process can now be observed for one single energy packet. It is assumed that the average absorption coefficient over the whole room surface is  $\alpha$  with ( $\alpha = 1/S \sum S_i \alpha_i$ ). At each reflection the energy is reduced by  $\alpha \times 100$  %. Thus after  $N$  reflections the remaining energy  $E$  is

$$E(N) = E_0(1 - \alpha)^N \quad (7.24)$$

The decay to  $1E-6$  of the initial energy is reached after  $M$  reflections where

$$M = \frac{-13.8}{\ln(1 - \alpha)} \quad (7.25)$$

$M$  reflections correspond to a path length  $L = M \cdot \ell$ , or a time  $T$

$$T = \frac{M\ell}{c} = \frac{-13.8 \times 4V}{\ln(1 - \alpha)cS} = \frac{0.16V}{-\ln(1 - \alpha)S} \quad (7.26)$$

For little absorption ( $\alpha \rightarrow 0$ ) the reverberation formula of Eyring (Eq. 7.26) approximates the formula of Sabine (Eq. 7.22). For highly damped rooms ( $\alpha \rightarrow 1$ ) the formula of Eyring takes on the reasonable value  $T = 0$ , while Sabines formula predicts a value  $T > 0$ . Eyring predicts in any case a lower reverberation time than Sabine.

At high frequencies, air absorption may become a relevant factor that influences reverberation. This can be considered by introducing an additional factor in the Eyring reverberation formula:

$$E(N) = E_0(1 - \alpha)^N e^{-mNl'} \quad (7.27)$$

where

$m$  : intensity damping constant for air according to Table 7.1

The above derivation (Eq. 7.26) for the reverberation time  $T$  is accordingly modified with Eq. 7.27 as

$$T = \frac{0.16V}{-\ln(1 - \alpha)S + 4mV} \quad (7.28)$$

For outdoor sound propagation applications, comprehensive tables of air absorption coefficients are available (ISO 9613-1). The air absorption is specified by a coefficient  $\alpha$  that describes the level reduction in dB per meter. The damping constant  $m$  used here can be expressed in  $\alpha$  as

$$m = \ln(10^{0.1 \cdot \alpha}) \quad (7.29)$$

relative humidity [%]	500 Hz	1000 Hz	2000 Hz	4000 Hz	8000 Hz
30	0.00058	0.00115	0.00325	0.01125	0.03874
40	0.00060	0.00107	0.00258	0.00838	0.02992
50	0.00063	0.00107	0.00228	0.00683	0.02423
60	0.00064	0.00111	0.00214	0.00590	0.02047
70	0.00064	0.00115	0.00208	0.00531	0.01787
80	0.00064	0.00119	0.00207	0.00493	0.01599

Table 7.1: Intensity damping constant  $m$  of air as a function of frequency and relative humidity at a temperature of 20°.

### Rooms with non-diffuse behavior

Besides the above mentioned cases where a diffuse field establishes and thus the energy density shows an exponential decay, there are room situations with a deviating decay curve. This is the case for rooms with very inhomogeneous distribution of the absorption or coupled rooms where two rooms with different damping are arranged that they can communicate with each other.

Table 7.2 shows the calculated reverberation times for a rectangular room with different absorber configurations and varying degree of diffusivity of the surfaces. In any case the total absorption was kept constant. The calculations were performed with a ray tracing model (see next section).

	surface diffusivity	$T$
calculation according to Sabine		1.33 s
ray tracing, absorption concentrated on one surface of 20x15 m	30%	2.10 s
ray tracing, absorption distributed on the whole surface	30%	1.32 s
ray tracing, absorption concentrated on one surface of 20x15 m	90%	1.07 s

Table 7.2: Comparison of calculated reverberation times  $T$  in a rectangular room with dimensions 20 x 15 x 6.7 m = 2000 m<sup>3</sup> and total absorption of 240 m<sup>2</sup> for different distributions of the absorbing surfaces. The column *surface diffusivity* describes the assumed diffusivity of the reflecting surfaces.

For equally distributed (homogeneous) absorption the ray tracing calculation is very close to the Sabine result. However for concentrated absorption and low diffusivity the reverberation times can increase considerably.

A typical example of coupled rooms is a hall with a foyer that gets sound energy from the hall by doors or other small openings. Further examples are churches with adjacent chapels. If the source is located in the room with less absorption, a decay curve as shown in Fig. 7.5 will occur.

### Absorption of audience

In many rooms, especially in concert halls, the audience contributes significantly or even dominates the absorption. It is therefore of great importance to know the corresponding absorption characteristics precisely. However the exact absorption coefficient depends on different factors such as density and arrangement of the seating, the upholstery of the seats or the type of clothes people are wearing. Typical  $\alpha$  values are given in Table 7.3 <sup>5</sup>.

	125 Hz	250 Hz	500 Hz	1000 Hz	2000 Hz	4000 Hz
upholstered seat, row spacing 1.15 m	0.30	0.35	0.50	0.60	0.70	0.70

Table 7.3: Typical absorption coefficients  $\alpha_s$  for audience areas.

<sup>5</sup>Fasold, Sonntag, Winkler, Bau- und Raumakustik, Rudolf M<sup>u</sup>ller Verlag, 1987.

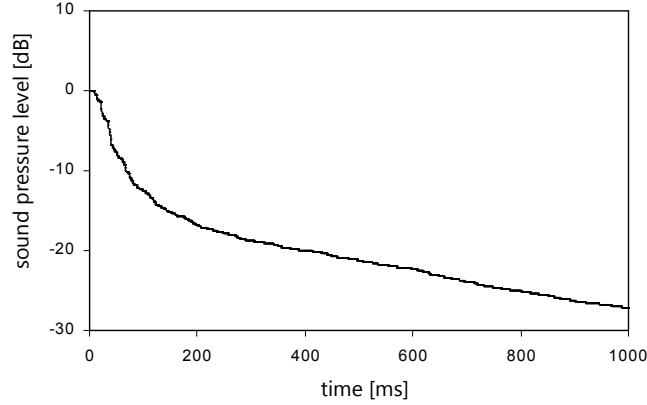


Figure 7.5: Sound decay for two coupled rooms where the source is in the room with lower absorption and the receiver in the room with higher absorption.

### Statistical impulse responses

For general, non-specific room acoustical investigations statistical impulse response models may be of interest. In the context of statistical room acoustics such a model has to define the direct sound and the diffuse field contribution. Thus the necessary specifying parameters are source directivity, distance between source and receiver, room volume and absorption. The direct sound is represented as a Dirac pulse with appropriate amplitude and delay. The diffuse field contribution is simulated by an exponentially decaying noise signal. With this a statistical, time discrete impulse response  $h(i)$  can be written as <sup>6</sup>:

$$h(i) = \frac{\sqrt{\Gamma}}{r} \Delta \left( i - \text{trunc} \left( \frac{f_c r}{c} \right) \right) + \sqrt{\frac{4\pi c}{V f_c}} e^{-\frac{i\partial}{f_c}} \xi(i) \theta \left( i - \text{trunc} \left( \frac{f_c r}{c} \right) \right) \quad (7.30)$$

where

$i$ : sample number

$\Gamma$ : directivity factor as ratio of the intensity in direction of the receiver and the intensity averaged over all directions

$r$ : source - receiver distance

$\Delta(i)$ : impulse function, = 1 for  $i = 1$ , elsewhere 0

$\text{trunc}()$ : truncate-function, round off to the next lower whole number

$f_c$ : clock frequency

$c$ : speed of sound

$V$ : room volume

$\partial$ : decay constant of the room,  $\partial = 3 \ln(10)/T$  ( $T$ : reverberation time)

$\xi(i)$ : sequence of samples of white noise,  $\xi(i)$  and  $\xi(i+1)$  are independent samples of a normally distributed random variable with mean = 0 and standard deviation = 1 <sup>7</sup>

$\theta(i)$ : step function, = 1 for  $i \geq 1$ , elsewhere 0

Figure 7.6 shows a statistical impulse response that was created with the above procedure.

## 7.2.2 Geometrical room acoustics

Geometrical acoustics assumes that sound propagates in form of rays along straight lines. This geometrical approach is a high frequency approximation and ignores wave phenomena such as diffraction

<sup>6</sup>U. P. Svensson, Energy-time relations in a room with an electroacoustic system, Journal of the Acoustical Society of America, vol. 104, p.1483-1490 (1998).

<sup>7</sup>Normally distributed random numbers can be generated from equally distributed random numbers as follows: get two random numbers  $R_A$  and  $R_B$  that are equally distributed in the interval (0,1), then convert them to two normally distributed random numbers  $S_A$  and  $S_B$  with standard deviation  $\sigma$  according to:

$$S_A = \sigma \sqrt{-2 \ln(R_A)} \cos(2\pi R_B)$$

$$S_B = \sigma \sqrt{-2 \ln(R_A)} \sin(2\pi R_B)$$



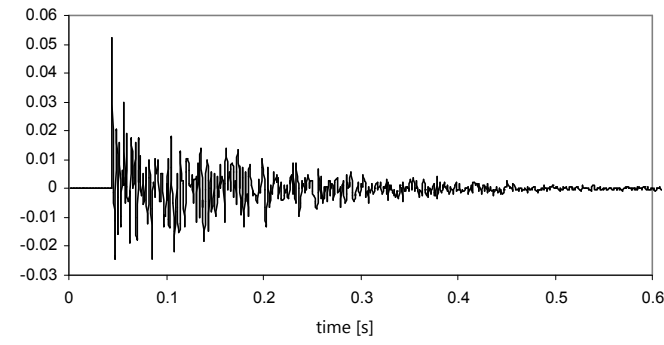


Figure 7.6: Example of an artificially generated room impulse response with a decay constant  $\partial = 6.9$  (corresponding to a reverberation time  $T = 1$  sec), a room volume  $V = 10'000 \text{ m}^3$  and a source - receiver distance  $r = 15$  m.

or interference.

### Reflection at plane surfaces, specular sources

If a sound ray hits a surface, it loses a certain amount of its energy depending on the absorption coefficient of the corresponding surface. The remaining energy is reflected according to the law of reflection (angle of incidence = angle of reflection). A certain sound path can be determined by construction of mirror sources (see Fig. 7.7).

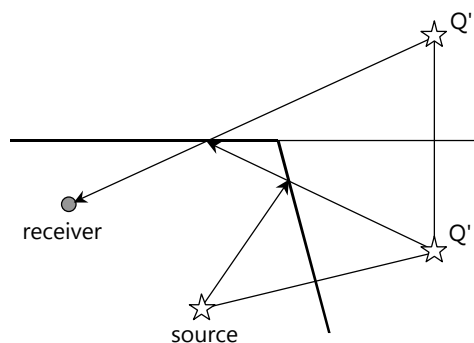


Figure 7.7: Construction of the reflection of sound rays by introduction of mirror sources.

### Reflection at structured surfaces, diffuse reflection and scattering

A reflection at a surface with significant depth structuring is no longer specular but rather diffuse like. The degree of diffusivity depends on the ratio of the structure dimension and the wave length. Diffuse reflections usually occur at higher frequencies while low frequencies show specular behavior. More specifically, three cases can be distinguished as shown in Fig. 7.8.

A diffuse reflection returns sound energy into a large solid angle. Often the idealized Lambert reflection characteristics is assumed<sup>8</sup>. It states that the intensity of the reflection in direction  $\phi$  relative to the surface normal is proportional to the cosine of  $\phi$ .

### Energy impulse response

Within the concept of geometrical room acoustics, sound propagation is modeled by aid of energy packages that travel along straight lines (sound rays). After emission at the source the packages that

<sup>8</sup>Max Born, Emil Wolf, Principles of Optics, Pergamon Press, 1980.

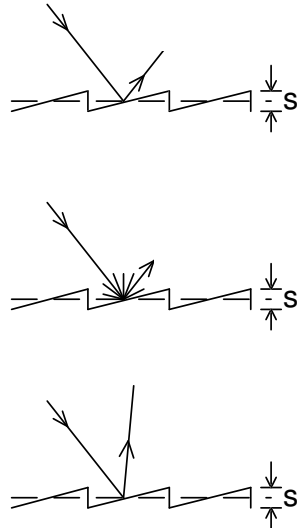


Figure 7.8: Reflection at a structured surface. Top: For  $\lambda \gg$  structure dimension  $s \rightarrow$  the structure has no effect  $\rightarrow$  specular reflection at an 'average' plane. Middle: For  $\lambda \approx$  structure dimension  $s \rightarrow$  the structure acts as a whole  $\rightarrow$  diffuse reflection. Bottom:  $\lambda \ll$  structure dimension  $s \rightarrow$  the single structure elements act as reflectors  $\rightarrow$  specular reflection at the structure details.

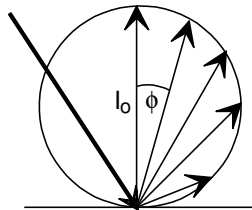


Figure 7.9: Ideal diffuse reflection according to Lambert. Independent of the sound incidence direction the intensity of the reflection in direction  $\phi$  is proportional to  $\cos(\phi)$ .

arrive at a receiver can be collected and registered with regard to the energy they represent and their travel time. This collection corresponds to an energy impulse response (Fig. 7.10) for the chosen source and receiver position.

### Objective room acoustical criteria

For the considered source and receiver position the energy impulse response represents the finger print of the room. In the past, many different features of such impulse responses have been proposed to relate the subjective quality of a room to objective criteria. From the large catalogue, a small set of these criteria has proven to be sufficient and relevant to describe the acoustical quality of rooms<sup>9</sup>. These criteria are usually evaluated for the octave bands from 125 Hz to 4 kHz. In the following, the origin of the time axis  $t = 0$  is understood as the moment of arrival of the direct sound.

- Reverberation time  $T$  [s]  
The reverberation time is the most fundamental feature to describe the room acoustical properties. It has global character, which means that the value is not changing a lot for different positions. The reverberation time is usually measured with backward integration of the squared impulse response. The decay curve is then evaluated between -5 and -35 dB. This time is doubled to get the reverberation decay of 60 dB.

<sup>9</sup>ISO Norm 3382 Measurement of the reverberation time of rooms with reference to other acoustical parameters. 1997.

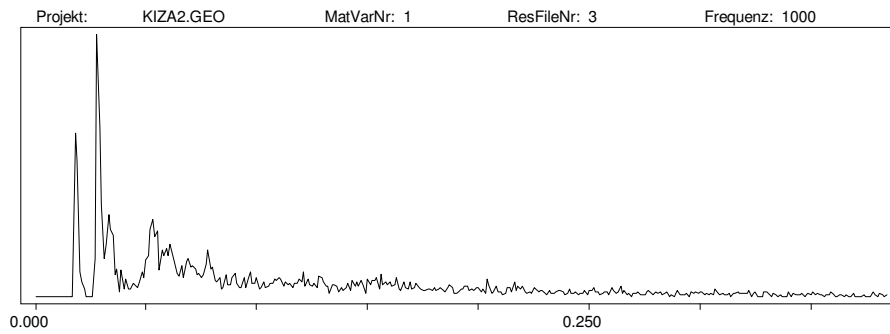


Figure 7.10: Example of an energy impulse response. The earliest contribution corresponds to the direct sound. Then first and higher order reflections follow with increasing density. Note the unusual strong reflection due to focusing effects of a concave room surface.

- Early Decay Time  $EDT$  [s]

The Early Decay Time  $EDT$  is defined similarly to the reverberation time, but is based on the decay over the top 10 dB. This time is then multiplied by 6 to extrapolate for a decay over 60 dB. From a subjective point of view the  $EDT$  is more relevant for a listener, as the dynamic range for music performances is typically in the order of 10...20 dB. The  $EDT$  may depend strongly on the listening position. The just audible difference of a variation of  $EDT$  is in the order of 5 % in an A/B comparison <sup>10</sup>.

- Clarity  $C80$  [dB]

Clarity measures the ratio of early arriving energy relative to the late energy in the impulse response.  $C80$  describes the transparency of music. With the energy impulse response  $h^2(t)$ , clarity is calculated as follows:

$$C80 = 10 \log \left( \frac{\int_0^{80ms} h^2(t) dt}{\int_{80ms}^{\infty} h^2(t) dt} \right) \quad (7.31)$$

A typical value for  $C80$  is 0 dB, an increase of the value means higher clarity. The just audible difference is in the order of 0.5 dB in the direct A/B comparison.

- Strength  $G$  [dB]

The strength  $G$  is a measure that describes the level at the receiver position relative to the level under free field conditions at a distance of 10 m. If the source receiver distance is 10 m,  $G$  specifies directly the amplification by the room. The strength is found by integration over the energy impulse response  $h^2(t)$ :

$$G = 10 \log \left( \frac{\int_0^{\infty} h^2(t) dt}{\int_0^{\infty} h_{f,10m}^2(t) dt} \right) \quad (7.32)$$

where

$h_{f,10m}$ : energy impulse response under free field conditions at 10 m distance.

The just audible difference is about 1 dB in a direct A/B comparison.

- Deutlichkeit  $D50$  [%]

Similarly to clarity  $C80$ , Deutlichkeit  $D50$  describes the clearness of a room acoustical situation.  $D50$  is defined as the energy ratio of useful early energy up to 50 ms after the direct sound relative to the total energy in the impulse response.  $D50$  is mainly used to investigate the clearness of speech signals. With the energy impulse response  $h^2(t)$   $D50$  is found as

<sup>10</sup>M. Vorländer, International Round Robin Test on Room Acoustical Computer Simulation, ICA 1994 Bergen.

$$D50 = \frac{\int_0^{50ms} h^2(t)dt}{\int_0^{\infty} h^2(t)dt} \times 100\% \quad (7.33)$$

A  $D50$  value of 40 % corresponds to an intelligibility of syllables of about 87 %, a  $D50$  of 60 % means an intelligibility of syllables of about 93 %. The just audible difference is about 5 % in the direct A/B comparison.

- Center time  $TS$  [ms]  
The center time describes similarly to  $C80$  and  $D50$  the temporal distribution of incoming energy. However  $TS$  avoids strict separations to distinguish between beneficial and detrimental energy.  $TS$  corresponds to the center of gravity of the energy impulse response  $h^2(t)$ :

$$TS = \frac{\int_0^{\infty} th^2(t)dt}{\int_0^{\infty} h^2(t)dt} \quad (7.34)$$

The just audible difference is about 10 ms in the direct A/B comparison.

- Lateral energy fraction  $LF$  [%]  
The lateral energy fraction measures the ratio of early lateral energy relative to early omnidirectional energy. The  $LF$  describes spaciousness which is a result of inter-aural signal differences. To evaluate  $LF$  the energy impulse response has to be determined once with an omnidirectional microphone ( $\rightarrow h^2(t)$ ) and once with a figure of eight microphone ( $\rightarrow h_{\infty}^2(t)$ ) where the orientation has been chosen in such a way that the sensitivity in frontal direction is zero.

$$LF = \frac{\int_0^{80ms} h_{\infty}^2(t)dt}{\int_0^{80ms} h^2(t)dt} \times 100 \quad (7.35)$$

The just audible difference is about 5 % in the direct A/B comparison.

For reverberation times  $T$  there is consensus about optimal values as a function of room volume for a wide variety of different applications. Fig. 7.11 shows optimal values in the mid frequency range for music and speech performance. In general one aims at reverberation times that are more or less independent of frequency. In concert halls however a slight increase at lower frequencies is usually perceived as beneficial ("warmer sound").

For the other objective criteria, only preliminary optimal values exist due to lack of sufficient experience. For concert halls the values in Table 7.4<sup>11</sup> may be applied.

parameter	EDT	C80 (500...2 kHz)	G (500...2 kHz)	LF (125...1 kHz)
optimal range	1.8...2.2 s	-2...+2 dB	> 0 dB	0.1...0.35

Table 7.4: Values of further room acoustical criteria considered as optimal in concert halls.

### 7.2.3 Acoustical design criteria for rooms

The design of a room for good room acoustics has to consider different aspects that vary in their relevance depending on the function and the usage. The most important criteria are:

- Silence** Any audible noise that has nothing to do with the performance on stage has to be avoided. Possible unwanted noise in auditoriums may stem from external traffic or from adjacent rooms. An other possible noise source is the air conditioning system of the auditorium.

<sup>11</sup>M. Barron, Auditorium Acoustics and Architectural Design. 1993

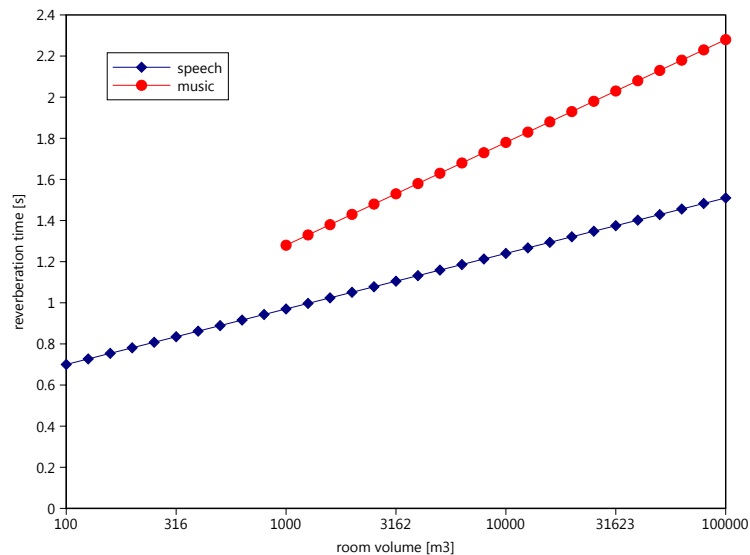


Figure 7.11: Optimal values of the reverberation time at mid frequencies in dependency of the room volume for speech and music performance.

**Direct sound** The whole audience area should receive sufficient direct sound from the source. Early reflections (within 50 ms) can support the direct sound supply.

**Reverberation** Depending on usage, room volume and room type, an appropriate reverberation time has to be adjusted.

**Lateral reflections** The feeling of spaciousness is triggered by uncorrelated signals at the two ears of a listener. This makes strong lateral reflections necessary.

**Diffusivity** With the exception of early lateral reflections, the reflections should typically be diffuse and not specular. This spreads reflected sound energy over time and reduces the danger of focusing effects.

**Balance** Different sections of extended sources such as orchestras should be heard in the audience with equal strength.

**Audibility on stage** To guarantee an optimal performance, the musicians in an orchestra should hear each other reasonably well.

For certain room types or usages, specific recommendations exist regarding the acoustical design:

- rooms for speech communication up to a room volume of about 5'000 m<sup>3</sup> such as conference rooms, schools or restaurants <sup>12</sup>.
- recording studios <sup>13</sup>.

## 7.2.4 Room acoustical design tools

The optimal acoustical design of a room requires appropriate analysis tools. They help to proof the efficacy of planned measures. Depending on the questions asked, a variety of design tools are available.

### Construction of sound rays

A preliminary estimate of the sound distribution in a room can be achieved by the construction of sound rays by hand. Thereby one usually restricts to a horizontal or vertical section through the room. Assuming an omnidirectional source some ten or twenty sound rays are drawn in all directions. At

<sup>12</sup>Hörbarkeit in kleinen bis mittelgrossen Räumen, DIN 18041.

<sup>13</sup>DIN 15996, Elektronische Laufbild- und Tonbearbeitung in Film-, Video- und Rundfunkbetrieben (1996).

the intersections with boundary surfaces the rays are reflected. The resulting density of the rays at a specific receiver locations determines the sound pressure level at that point. The manual construction of sound rays is suitable for example to investigate fundamental ceiling shapes or the optimal orientation of reflectors. The effort to construct higher order reflections grows quickly, one will then use ray tracing computer models.

### Calculation of reverberation times

As mentioned above, the reverberation time is the most fundamental room acoustical parameter. If the materialization is known, the reverberation time of a room can be calculated by application of the Sabine or Eyring formula. In concert halls, the audience is usually the dominating absorber. In these cases it is therefore possible to estimate the reverberation time  $T$  with the area of the audience  $S_P$ , the room volume  $V$  as:

$$T \approx \frac{0.15V}{S_P} \quad (7.36)$$

### Scale models

Sound propagation in rooms can be simulated with help of scale models<sup>14,15,16,17</sup>. If all dimensions are scaled by a factor  $1/s$  and at the same time the frequency is scaled by  $s$  (preservation of the ratio of wavelength and dimension) the sound propagation phenomena remain unaltered. A difficulty is to find materials for the scale models that have similar absorption characteristics in the transformed frequency domain as the original material in the original frequency domain. In addition, strategies are necessary to overcome the strong air absorption in the scale model frequency range (up to 50 kHz). One solution is to dry the air down to a relative humidity of a few percent- Under these conditions the air shows low absorption up to high frequencies. An other approach is to compensate for the absorption by way of a calculation. As travel times have to be known this can only be done on basis of the impulse response. Typical values for the scale factor  $s$  are between 10 and 50.

### Computer simulations

Nowadays it becomes more and more common to use computer software to simulate sound propagation in rooms. The first attempt in this direction was most probably made by Schroeder<sup>18</sup>, however the first who actually wrote a computer program were Krokstad and his colleagues<sup>19</sup>.

Room acoustical computer simulations can be divided roughly into two categories. The first category comprises numerical methods that find solutions to the wave equation. The second category contains methods that simulate sound propagation based on geometrical acoustics.

All numerical methods that solve the wave equation have in common that the room volume and/or the room surface have to be discretized. The corresponding mesh has to be significantly finer than the shortest wave length of interest. The computational effort becomes extremely high for large rooms and high frequencies.

The methods based on geometrical acoustics assume sound propagation along straight lines. Wave phenomena such as interference or resonance can not be considered. Computer models based on geometrical acoustics can be split into two groups: ray tracing and mirror sources.

<sup>14</sup>F. Spand'ock, Akustische Modellversuche, Annalen der Physik, vol. 20, 1934, p.345.

<sup>15</sup>A. F. B. Nickson, R. W. Muncey, Some experiments in a room and its acoustic model; *Acustica*, 1956, vol. 6, p.295-302

<sup>16</sup>D. Brebeck, R. Buecklein, E. Krauth, F. Spand'ock, Akustisch 'ahnliche Modelle als Hilfsmittel f'ur die Raumakustik, *Acustica*, 1967, v.18, p.213-226.

<sup>17</sup>J. D. Polack, A. H. Marshall, G. Dodd, Digital evaluation of the acoustics of small models: The MIDAS package, *Journal of the Acoustical Society of America*, 1989, v.85, p.185-193.

<sup>18</sup>M. R. Schroeder, B. S. Atal, C. Bird, Digital Computers in room acoustics, *Proc. 4th Intern. Congr. of Acoustics*, 1962, Paper M21.

<sup>19</sup>A. Krokstad, S. Strom, S. Sorsdal, Calculating the acoustical room response by the use of a ray tracing technique, *Journal of Sound and Vibration*, 1986, p.118-124.

Ray tracing methods<sup>20</sup> simulate sound propagation by emitting many sound particles at the source position (Fig. 7.12). The particles propagate along straight lines. If a particle hits a boundary surface, the energy is reduced corresponding to the absorption coefficient of the surface. The particle with adjusted energy is then reflected based on a certain reflection characteristics that is described by a diffusivity factor. If a particle is reflected diffusely, the outgoing direction  $\phi$  is determined randomly where the probability of a certain angle  $\phi$  is proportional to cosine of  $\phi$ . At each receiver position a sphere of small diameter is constructed. Each time a sound particle passes such a receiver volume, the corresponding travel time and energy of the particle is noted in a table.

With the mirror source method, all possible sound paths between a source and a receiver are determined by constructing all visible mirror sources up to a certain order<sup>21</sup>. All room surfaces are assumed to reflect specularly. The attenuation of a certain sound path is given by the product of the absorption coefficients of all surfaces involved and a factor  $1/d^2$  with  $d$  the travel distance.

The ray tracing or mirror source method deliver finally an energy impulse response for the room and the chosen source and receiver points. From this the above mentioned room acoustical criteria such as *EDT* or *C80* can be evaluated. Furthermore sound pressure impulse responses can be derived for auralization purposes. For a recent overview of geometrical room acoustic modeling see the tutorial paper by Savioja<sup>22</sup>.

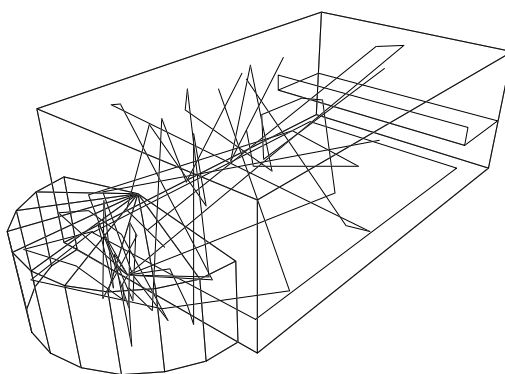


Figure 7.12: Example of the beginning of a ray tracing simulation.

## Auralization

As seen above there are different parameters to evaluate and describe the acoustical quality of a room. These parameters can be calculated in advance during the planning phase of a project. However the ultimate criterion is the listening experience in the room. The process of simulating the audible impression of a room is called auralization. First attempts of auralization with help of scale models go back to Spandöck<sup>23,24</sup>. Thereby the signal of interest was up-shifted in frequency by an appropriate scale factor and emitted in the scale model. At the listener position the signal was recorded, down-shifted in frequency and played back through headphones.

With the introduction of room acoustical computer simulations, a new auralization approach was introduced<sup>25,26,27</sup>. With help of the computer simulation it is determined, when how much energy from

<sup>20</sup>M. Vorlaender, Ein Strahlverfolgungsverfahren zur Berechnung von Schallfeldern in Räumen, *Acustica*, 1988, v.65, p.138-148.

<sup>21</sup>J. B. Allen, D. A. Berkley, Image method for efficiently small-room acoustics, *Journal of the Acoustical Society of America*, 1979, v.65, p.943-950.

<sup>22</sup>L. Savioja, U. P. Svensson, Overview of geometrical room acoustic modeling techniques, *J. Acoust. Soc. Am.* 2015, vol. 138, 708-730.

<sup>23</sup>F. Spandöck, *Annalen der Physik V*, vol. 20, 1934, p.345

<sup>24</sup>F. Spandöck Das Raumakustische Modellverfahren mit maßstabsgerechter Frequenztransponierung und die Möglichkeiten seiner Verwirklichung. *Third International Congress of Acoustics*, 1959, pp. 925-928.

<sup>25</sup>Hilmar Lehnert, Jens Blauert, *Principles of Binaural Room Simulation*, *Applied Acoustics*, 1992, v.36, p.259-291.

<sup>26</sup>Mendel Kleiner, Bengt-Inge Dalenbaeck, Peter Svensson, Auralization - An Overview; *Journal of the Audio Engineering Society*, 1993, v.41, p.861-875.

<sup>27</sup>L. Savioja, et. al. Creating Interactive Virtual Acoustic Environments, *Journal of the Audio Engineering Society*, vol.

which direction hits the receiver. According to this distribution, the signal of interest is then delayed accordingly and played back over a cloud of loudspeakers installed in an anechoic chamber <sup>28</sup> (Figure 7.13).

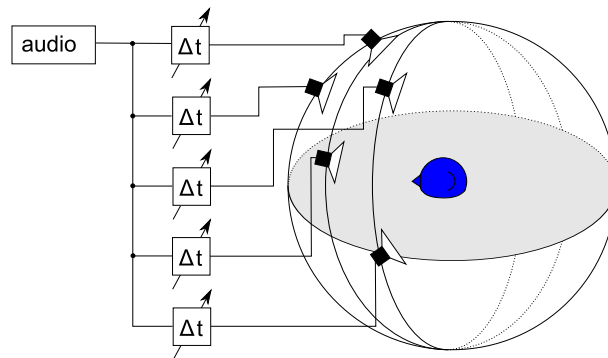


Figure 7.13: Schematic representation of a cloud of loudspeakers distributed around a listener position to auralize the acoustics of an auditorium. The loudspeakers are fed with appropriate delayed and weighted copies of the reverberation free source signal.

A serious drawback of the loudspeaker cloud is the space requirements and the need for an anechoic room. Indeed all that has to be done with auralization is to produce appropriate signals at the two eardrums of the listener. It should therefore be possible to realize an auralization playback system with help of headphones <sup>29</sup>. To do so, additional information about the head related transfer functions (HRTF) is necessary. As discussed above, the room acoustical simulation delivers impulse responses for different categories of incidence angles. The room impulse responses between source and the two eardrums are obtained by convolution with the corresponding HRTFs. Finally the headphone auralization signals are generated as convolution of the dry source signal with the two room impulse responses to the eardrums.

Compared to the loudspeaker cloud solution two problems are associated with the auralization by headphones. The first difficulty is the fact that the head related transfer functions differ from person to person. For optimal results these HRTFs should be determined individually. The second problem is that the headphone representation can not map head movements <sup>30</sup>.

Most of today's software packages for room acoustical simulations allow for auralization by headphones.

### 7.2.5 Some room acoustical effects that are not considered with statistical or geometrical acoustics

The modeling of sound propagation in rooms by means of statistical or geometrical acoustics ignores the wave nature of sound and is therefore only a coarse approximation to reality. In the following a few aspects are discussed that may have relevance in rooms but are usually not considered.

#### Sound propagation at grazing incidence over audience areas

If sound propagates at grazing incidence over audience areas, additional damping can be observed. This is firstly due to destructive interference between direct sound and sound that is reflected and/or scattered at heads and shoulders of the audience and secondly due to energy that is lost as a consequence

47, p.675-705 (1999)

<sup>28</sup>Y. Korenaga, Y. Ando, A Sound-Field Simulation System and Its Application to a Seat-Selection System, J. Audio Eng. Soc., vol. 41, 1993, pp. 920-930.

<sup>29</sup>K. H. Kuttruff, Auralization of Impulse Responses Modeled on the Basis of Ray-Tracing Results, J. Audio Eng. Soc., vol. 41, 1993, pp. 876-880.

<sup>30</sup>A solution to overcome this difficulty is the implementation of head tracking systems that capture the orientation of the head and adjust the headphone signals accordingly



of diffraction. This additional damping is called *seat dip effect* in the literature<sup>31, 32, 33, 34</sup>. Figure 7.14 shows measurements of Mommertz<sup>35</sup> demonstrating the order of magnitude of the seat dip effect.

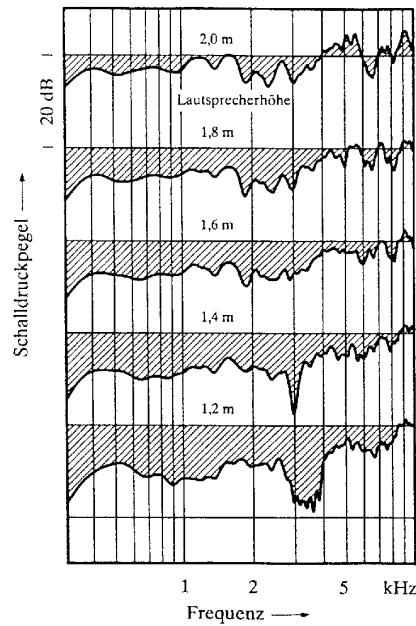


Figure 7.14: Frequency response of the additional damping for sound propagating at grazing incidence over an audience area. The measurement position was in the 12th row at a height of 1.2 m. The height of the source varied between 1.2 and 2.0 m.

### Reflection at finite surfaces

The reflection of sound waves at hard surfaces of infinite extension can be handled with the mirror source concept. This is a fundamental assumption behind geometrical room acoustical tools. However this concept is no longer fully correct for small reflectors, low frequencies and grazing sound incidence. In these cases where the extension of the reflector has to be taken into account, the concept of Fresnel zones may help to identify the frequency dependent dimension that is necessary for a full reflection.

For a given reflector geometry (Fig. 7.15), the lower limiting frequency  $f_u$  for full reflection can be estimated with Eq. 7.37<sup>36</sup>

$$f_u = \frac{2c}{(l \cos \beta)^2} \frac{d_{QR}d_{RE}}{(d_{QR} + d_{RE})} \quad (7.37)$$

where

$c$ : speed of sound [m/s]

$d_{QR}$ : distance source  $\rightarrow$  point of reflection [m]

$d_{RE}$ : distance point of reflection  $\rightarrow$  receiver [m]

$l$ : dimension of the reflector [m]

$\beta$ : angle of incidence relative to the reflector normal direction

<sup>31</sup>E. Meyer, H. Kuttruff, F. Schulte. Versuche zur Schallausbreitung  $\hat{f}$  über Publikum. *Acustica*, vol.15, 1965, p.175-182.

<sup>32</sup>S. Bradley. Some further investigations of the seat dip effect. *J. Acoustical Society of America*, vol. 90, 1991, p.324-333.

<sup>33</sup>R. Hecht, E. Mommertz. Ein Schallteilchenverfahren zur Simulation der streifenden Schallausbreitung  $\hat{f}$  über Publikum. *DAGA 94*, 1994, p.229-232.

<sup>34</sup>D. Takahashi, Seat dip effect: the phenomena and the mechanism, *J. Acoustical Society of America*, vol. 102, 1997, p.1326-1334.

<sup>35</sup>E. Mommertz. Einige Messungen zur streifenden Schallausbreitung  $\hat{f}$  über Publikum und Gest $\hat{f}$ uhl. *Acustica*, vol. 79, 1993, p.42-52.

<sup>36</sup>ISO 9613-2, Acoustics - Attenuation of sound during propagation outdoors - Part 2.

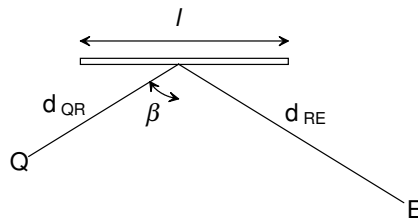


Figure 7.15: Situation for the estimation of the lower limiting frequency for a full reflection at a reflector of limited size.

## 7.2.6 Reflections at spherical surfaces

Curved structures and concave room shapes need special attention<sup>37,38</sup>. Convex curvatures are unproblematic under normal conditions as they increase scattering of reflected sound energy. Concave curvatures on the other hand show often unwanted focusing effects with highly inhomogeneous sound field distributions. Spectacular examples are whispering galleries that allow for communication between distant points with unnatural low damping. There exist quite a few historical buildings that contain sound focusing elements. From a today's perspective it is not clear whether these amplifying effects have been implemented deliberately or whether they are a product of accident<sup>39</sup>.

In many cases domed structures can be approximated by parts of a sphere. In two dimensions, this leads to the discussion of reflection of rays at a small arc of a circle (Fig. 7.16).

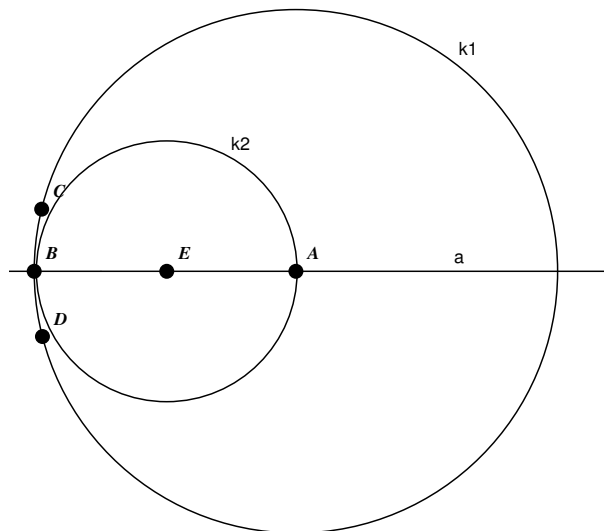


Figure 7.16: Situation of the reflection of sound rays at a small arc of a circle. On the circle  $k_1$  with center  $A$  the arc between the points  $C$  and  $D$  is considered. The source position is assumed on the line  $a$  or on the circle  $k_2$  where the points  $B$  and  $A$  define the diameter of  $k_2$ .

### Reflection at circles: source position on axis $a$

If the source point location is on the axis  $a$  (see Fig. 7.16), emitted sound rays are reflected as shown in Fig. 7.17.

<sup>37</sup>M. Vercammen, Sound Reflections from Concave Spherical Surfaces. Part I: Wave Field Approximation, Acta Acustica united with Acustica, vol. 96, 82-91 (2010).

<sup>38</sup>M. Vercammen, Sound Reflections from Concave Spherical Surfaces. Part II: Geometrical Acoustics and Engineering Approach, Acta Acustica united with Acustica, vol. 96, 92-101 (2010).

<sup>39</sup>K. Heutschi, Akustik der Evangelischen Filialkirche in Guarda-Giarsun, Schweizer Ingenieur und Architekt SI+A Nr. 41 (2000).

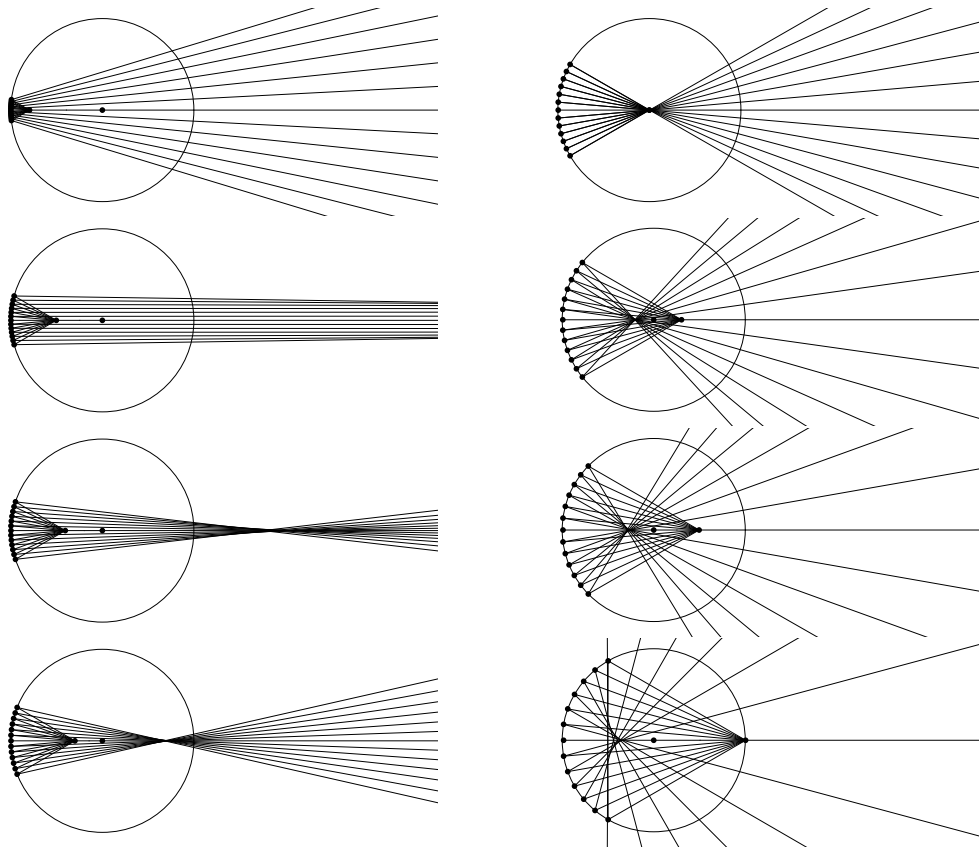


Figure 7.17: Reflection of sound rays at a circle for different source positions.

According to Figure 7.17 the reflection at a circle can show hyperbolic, parabolic or elliptic behavior, depending on the source position in relation to the center of the circle. For a mathematical discussion, a unity circle is assumed with center at  $x_Z = 1.0$ ,  $y_Z = 0.0$ . The circle is then described by Eq. 7.38 or 7.39.

$$(x - 1)^2 + y^2 = 1 \quad (7.38)$$

or

$$y^2 = 2x - x^2 \quad (7.39)$$

In the following it is assumed that the source position is on the  $x$ -axis and that sound rays are emitted into a small angular segment in  $-x$ -direction. For the reflection only a small region of the circumference ( $x$  small) is of interest. Eq. 7.39 can then be approximated by

$$y^2 \approx 2x \quad (7.40)$$

It can be shown that Eq. 7.40 approximates a small portion of an ellipse, a parabola or a hyperbola. The behavior of the reflection can easily be discussed if the source point is interpreted as the focal point of the corresponding conic section.

**Ellipse** The equation for an ellipse as shown in Fig. 7.18 is given by:

$$\frac{(x - a)^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (7.41)$$

where:

$a$ : semi-major axis

$b$ : semi-minor axis

$d = a - \sqrt{a^2 - b^2}$ :  $x$ -coordinate of the first focal point

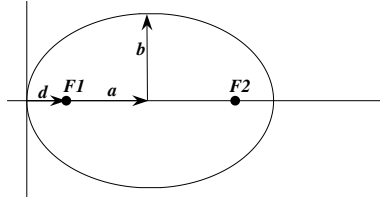


Figure 7.18: Ellipse with extreme point at the origin and the two focal points  $F1$  and  $F2$ .

Equation 7.41 can be rewritten as:

$$y^2 = 2x \frac{b^2}{a} - x^2 \frac{b^2}{a^2} \quad (7.42)$$

If only small values for  $x$  are of interest (see above), the second term in Eq. 7.42 can be ignored so that the equation simplifies to:

$$y^2 \approx 2x \frac{b^2}{a} \quad (7.43)$$

If the parameters  $a$  and  $b$  are chosen in such a way that  $b^2 = a$ , the simplified equation for the ellipse (7.43) corresponds to the simplified equation for the circle (7.40).

The  $x$ -coordinate  $d$  of the first focal point becomes

$$d = a - \sqrt{a^2 - b^2} = a - \sqrt{a^2 - a} \quad (7.44)$$

If - the other way round - the  $x$ -coordinate  $d$  of the first focal point is given, the semi-major axis  $a$  is found as

$$a = \frac{d^2}{2d - 1} \quad (7.45)$$

Eq. 7.45 reveals for  $a$  only positive (valid) solutions, if  $d > 0.5$ . For a source position with  $x$ -coordinate  $x_Q > 0.5$ , the reflection at the circular arc can thus be approximated as reflection at an ellipse where the source point corresponds to the first focal point and the second focal point is given as:

$$x_{F2} = 2 \frac{x_Q^2}{2x_Q - 1} - x_Q = \frac{x_Q}{2x_Q - 1} \quad (7.46)$$

where:

$x_{F2}$ :  $x$ -coordinate of the second focal point

The reflection at the elliptically shaped boundary manifests in such a way that rays emitted at the first focal point all meet in the second focal point.

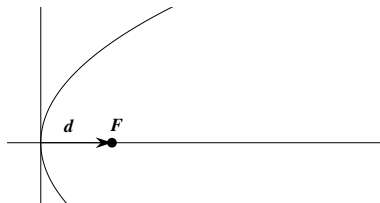


Figure 7.19: Parabola with vertex at the origin and focal point  $F$ .

**Parabola** The equation that describes the parabola in Fig. 7.19 is given by:

$$y^2 = 2px \quad (7.47)$$

where:

$p$ : parameter

$d = \frac{p}{2}$ :  $x$ -coordinate of the focal point

The equation for the parabola (7.47) with  $p = 1$  corresponds directly to the equation for the circle in the approximation (7.40) for small  $x$ . Consequently for a source point with  $x_Q = 0.5$ , the reflection at the arc of a circle can be approximated by the reflection at a parabola with focal point at  $x_Q = 0.5$ . All rays emitted at the focal point of a parabola are reflected back in parallel to the  $x$  axis.

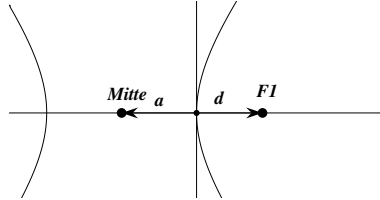


Figure 7.20: Hyperbola with vertex at the origin and the first focal point  $F1$ .

**Hyperbola** The hyperbola in Fig. 7.20 is described by:

$$\frac{(x + a)^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (7.48)$$

where:

$a$ :  $x$ -axis parameter

$b$ :  $y$ -axis parameter

$d = -a + \sqrt{a^2 + b^2}$ :  $x$ -coordinate of the first focal point

The equation for the hyperbola 7.48 can be rewritten as:

$$y^2 = 2x \frac{b^2}{a} + x^2 \frac{b^2}{a^2} \quad (7.49)$$

Under the assumption of small  $x$  values, the second term in Eq. 7.49 can be ignored:

$$y^2 \approx 2x \frac{b^2}{a} \quad (7.50)$$

If the parameters  $a$  and  $b$  are chosen in such a way that  $b^2 = a$ , the approximated equation of the hyperbola (7.50) corresponds to the approximated equation of the circle (7.40). The  $x$ -coordinate  $d$  of the first focal point becomes

$$d = -a + \sqrt{a^2 + b^2} = -a + \sqrt{a^2 + a} \quad (7.51)$$

If - the other way round - the  $x$ -coordinate  $d$  of the first focal point is given, the axis parameter  $a$  is found as

$$a = \frac{d^2}{1 - 2d} \quad (7.52)$$

In Eq. 7.52 positive (valid) solutions for  $a$  result only if  $d < 0.5$ . For a source point with  $x_Q < 0.5$ , the reflection at an arc of a circle can be approximated by the reflection at a hyperbola with the first focal point corresponding to the source position and the second focal point at:

$$x_{F2} = -2 \frac{x_Q^2}{1 - 2x_Q} - x_Q = \frac{x_Q}{2x_Q - 1} \quad (7.53)$$

where:

$x_{F2}$ :  $x$ -coordinate of the second focal point

Sound rays that are emitted at the first focal point are reflected in such a way that they seem to originate from the second focal point. According to Eq. 7.53 the  $x$ -coordinate of the second focal

point is always smaller than  $-x_Q$  which implies that the divergence of the reflection is weaker than a reflection at a plane surface.

It should be noted that the equations for the second focal point are identical for the ellipse and the hyperbola. Indeed the equation holds even for the parabola in the limiting condition of  $x_{F2} \rightarrow \infty$ .

Table 7.5 summarizes the the above findings for the geometrical reflection at a circular arc.

	hyperbolic	parabolic	elliptical
source position:	$x_Q < 0.5$	$x_Q = 0.5$	$x_Q > 0.5$
reflection:	<b>divergent</b>	<b>parallel</b>	<b>focusing</b>
second focal point:	$x_{F2} = \frac{x_Q}{2x_Q - 1}$	$\infty$	$x_{F2} = \frac{x_Q}{2x_Q - 1}$

Table 7.5: Reflection at a circular arc (bold) with radius  $r = 1$  for different source positions.

### Reflection at circles: source on the circle $k_2$

If the source is located on the circle  $k_2$  (see Fig. 7.16) the reflecting arc of the large circle corresponds approximately to a segment of a vertically orientated ellipse with the first focal point at the source position. Thus the reflected rays all meet at the second focal point. The second focal lies symmetrically to the first focal point relative to the line  $a$  (see Fig. 7.21).

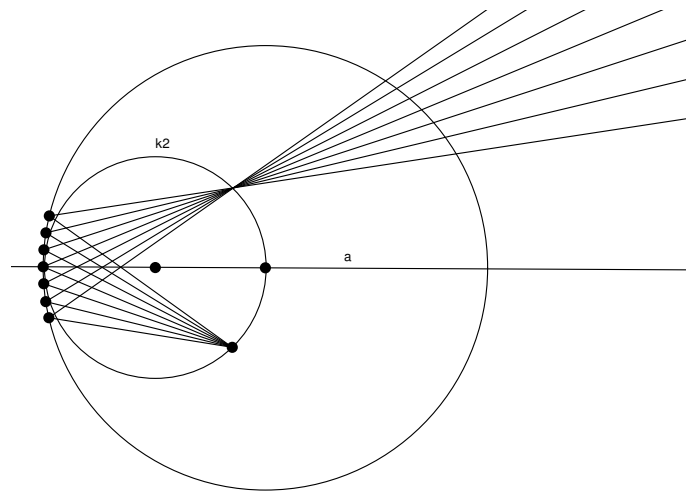


Figure 7.21: A source point on circle  $k_2$  produces reflections that focus in a point symmetrical to the source position relative to  $a$ .

The analytical investigation follows the considerations from above. Again the reflecting circular arc can be approximated for small  $x$  values by

$$y^2 \approx 2x \quad (7.54)$$

A vertically orientated ellipse through the origin can be described by Eq. 7.55.

$$\frac{(x - b)^2}{b^2} + \frac{y^2}{a^2} = 1 \quad (7.55)$$

where:

$a$ : major half axis

$b$ : minor half axis

$x = b, y = +\sqrt{a^2 - b^2}$ : coordinate of the first focal point

$x = b, y = -\sqrt{a^2 - b^2}$ : coordinate of the second focal point

For small values of  $x$ , Eq. 7.55 can be approximated by Eq. 7.56.

$$y^2 \approx 2x \frac{a^2}{b} \quad (7.56)$$

With the condition  $a^2 = b$ , Eq. 7.56 corresponds to the equation of the circle (7.54). This implies that the arc of the circle looks like a segment of an ellipse. The focal points of this ellipse are given by

$$y^2 = x - x^2 \quad (7.57)$$

Eq. 7.57 describes a circle with center at  $x_Z = 0.5, y_Z = 0$  and radius = 0.5 (7.58).

$$(x - 0.5)^2 + y^2 = 0.25 \quad \Leftrightarrow \quad y^2 = x - x^2 \quad (7.58)$$

## 7.3 Room acoustics of small rooms, wave theoretical acoustics

The sound field in small rooms at low frequencies is dominated by discrete resonances (Eigenfrequencies) with low spectral density. In these situations the methods of statistical and geometrical acoustics are not applicable. The wave nature of sound has to be considered explicitly with help of wave theoretical room acoustics.

### 7.3.1 Wave equation and boundary conditions

The possible sound fields in a room are given by functions of sound pressure that fulfill the wave equation as well as the boundary conditions. If one restricts to sinusoidal time dependencies, the wave equation can be replaced by the Helmholtz equation (1.51) with the complex, location dependent amplitude function  $\check{p}$ :

$$\Delta \check{p} + k^2 \check{p} = 0 \quad (7.59)$$

where

$$k = \frac{\omega}{c} \text{ (wave number)}$$

The boundary conditions are defined by the room limiting surfaces. It is assumed that the surfaces are locally reacting which means that they can be specified by an impedance  $Z$  given as the ratio of sound pressure and normal component of the sound particle velocity on the surface.

With Eq. 1.12 it can be written for a point on the surface:

$$\frac{\partial p}{\partial n} = -\rho \frac{\partial v_n}{\partial t} \quad (7.60)$$

Inserting the impedance  $Z$  of the surface, the sound particle velocity in Eq. 7.60 can be eliminated:

$$\frac{1}{\rho} \frac{\partial p}{\partial n} = -\frac{1}{Z} \frac{\partial p}{\partial t} \quad (7.61)$$

Introducing complex writing for the sinusoidal sound pressure  $p = \check{p} e^{j\omega t}$  yields

$$\frac{\partial p}{\partial t} = \check{p} j\omega e^{j\omega t} \quad (7.62)$$

Insertion of (7.62) in (7.61) gives

$$\frac{1}{\rho} \frac{\partial \check{p}}{\partial n} = -\frac{1}{Z} \check{p} j\omega \quad (7.63)$$

or

$$Z \frac{\partial \check{p}}{\partial n} + j\omega\rho\check{p} = 0 \quad (7.64)$$

### 7.3.2 Solution for rectangular rooms with acoustically hard surfaces

Solutions of the wave equations that fulfill the boundary conditions can be found analytically for a few special geometries only. One important example is the rectangular room. Rooms with such a fundamental shape are often encountered in real life.

In the following, a rectangular room with dimensions  $L_x, L_y, L_z$  according to Fig. 7.22 is considered.

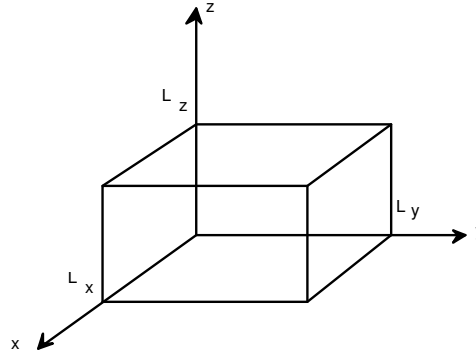


Figure 7.22: Coordinate system to be used for the discussion of the sound field in a rectangular room with dimensions  $L_x, L_y, L_z$ .

As a simplification it is assumed that all surfaces are acoustically hard ( $Z \rightarrow \infty$ ). With Eq. 7.64 the boundary conditions read as:

$$\begin{aligned} \frac{\partial \check{p}}{\partial x} &= 0 & \text{for } x = 0, x = L_x \\ \frac{\partial \check{p}}{\partial y} &= 0 & \text{for } y = 0, y = L_y \\ \frac{\partial \check{p}}{\partial z} &= 0 & \text{for } z = 0, z = L_z \end{aligned} \quad (7.65)$$

All possible sound fields in the rectangular room are given by sound pressure functions  $\check{p}(x, y, z)$  that fulfill the Helmholtz equation (7.59) and the boundary conditions (7.65). In cartesian coordinates the Helmholtz equation reads as

$$\frac{\partial^2 \check{p}}{\partial x^2} + \frac{\partial^2 \check{p}}{\partial y^2} + \frac{\partial^2 \check{p}}{\partial z^2} + k^2 \check{p} = 0 \quad (7.66)$$

As a guess for the solution, the following approach will be tested:

$$\check{p}(x, y, z) = C \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) \quad (7.67)$$

where

$n_x, n_y, n_z$ : arbitrary whole number  $\geq 0$

$C$ : arbitrary constant

The approach (7.67) describes a field of standing waves with maxima and minima, depending on location. As a proof, the approach is inserted into the Helmholtz equation and in the boundary



condition equations.

#### Verification of the boundary conditions

For that purpose the Eq. (7.67) is differentiated regarding the coordinates  $x, y$  and  $z$ . For the  $x$ -coordinate this yields:

$$\begin{aligned} \frac{\partial \check{p}}{\partial x} &= -C \frac{n_x \pi}{L_x} \sin\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) \\ &\rightarrow \frac{\partial \check{p}}{\partial x} = 0 \text{ for } n_x \text{ integer} \end{aligned} \quad (7.68)$$

#### Verification of the Helmholtz equation

Eq. (7.67) is differentiated two times regarding the coordinates  $x, y$  and  $z$ :

$$\begin{aligned} \frac{\partial^2 \check{p}}{\partial x^2} &= -C \frac{n_x^2 \pi^2}{L_x^2} \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) \\ \frac{\partial^2 \check{p}}{\partial y^2} &= -C \frac{n_y^2 \pi^2}{L_y^2} \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) \\ \frac{\partial^2 \check{p}}{\partial z^2} &= -C \frac{n_z^2 \pi^2}{L_z^2} \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) \end{aligned} \quad (7.69)$$

Inserted in (7.66) yields:

$$\begin{aligned} C \left[ -\frac{n_x^2 \pi^2}{L_x^2} - \frac{n_y^2 \pi^2}{L_y^2} - \frac{n_z^2 \pi^2}{L_z^2} \right] \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) + \\ k^2 C \cos\left(\frac{n_x \pi x}{L_x}\right) \cos\left(\frac{n_y \pi y}{L_y}\right) \cos\left(\frac{n_z \pi z}{L_z}\right) = 0 \end{aligned} \quad (7.70)$$

The above equation is satisfied if

$$k^2 = \frac{n_x^2 \pi^2}{L_x^2} + \frac{n_y^2 \pi^2}{L_y^2} + \frac{n_z^2 \pi^2}{L_z^2} \quad (7.71)$$

In the rectangular room with acoustically hard surfaces the Helmholtz equation is only fulfilled for discrete values of the wave number  $k$  (so called Eigenvalues). Each positive, whole numbered triple  $n_x, n_y, n_z$  determines with Eq. 7.71 an Eigenvalue. The corresponding function  $\check{p}(x, y, z)$  is called mode.

With

$$k = \frac{2\pi}{\lambda} = 2\pi \frac{f}{c} \quad (7.72)$$

relation (7.71) can be expressed in frequency  $f$ :

$$f = \frac{c}{2} \sqrt{\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}} \quad (7.73)$$

Figure 7.23 shows some examples of sound pressure distributions (modes) in a rectangular room. All modes have a sound pressure maximum in the corners of the room. Modes with *one*  $n_i = 0$  have a maximum at the edges while modes with *two*  $n_i = 0$  show a maximum on the corresponding planes. This is of relevance for the placement of low frequency absorbers that react on sound pressure (plate or membrane absorbers).

Table 7.6 shows exemplarily the lowest ten Eigenfrequencies for a small rectangular room with dimensions  $4.7 \times 4.1 \times 3.1$  m.

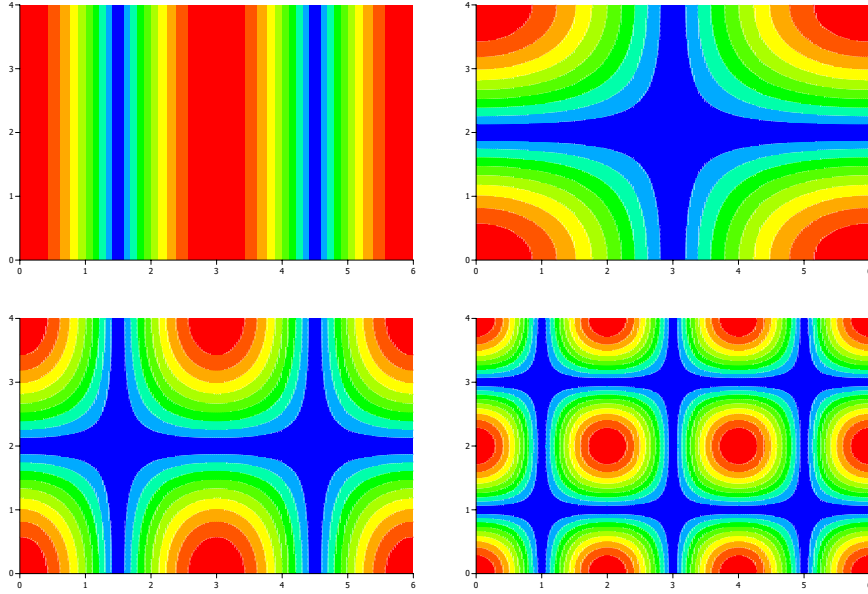


Figure 7.23: Sound pressure amplitude distribution in a rectangular room for a few modes. The amplitude is colour coded where red stands for maximum and blue for minimum amplitudes. From left to right and top to bottom: mode (2,0,0), mode (1,1,0), mode (2,1,0), mode (3,2,0).

Eigenfrequency [Hz]	$n_x$	$n_y$	$n_z$
36.2	1	0	0
41.5	0	1	0
54.8	0	0	1
55.0	1	1	0
65.7	1	0	1
68.6	0	1	1
72.3	2	0	0
77.7	1	1	1
82.9	0	2	0
83.4	2	1	0

Table 7.6: The ten lowest Eigenfrequencies and the corresponding modes for a rectangular room with dimensions  $4.7 \times 4.1 \times 3.1$  m.

The frequency differences between the adjacent Eigenfrequencies are quite large at the low frequency end. For increasing frequency these differences become smaller. In [40] the number  $N_f$  of Eigenfrequencies between 0 and the frequency  $f$  [Hz] in a rectangular room of volume  $V$  [m<sup>3</sup>] is estimated as

$$N_f \approx \frac{4\pi}{3} V \left( \frac{f}{c} \right)^3 \quad (7.74)$$

The density  $dN_f/df$  (number of Eigenfrequencies per Hz) at frequency  $f$  is then

$$\frac{dN_f}{df} \approx 4\pi V \left( \frac{f^2}{c^3} \right) \quad (7.75)$$

If the resonances overlap, the room modes are no longer isolated and lose their relevance. For practical applications, a resonance width of about 1 Hz can be assumed. Evaluation of Eq. 7.75 yields a corresponding frequency  $f_S$  for a density of 1 mode per Hz:

$$f_S \approx \frac{1800}{\sqrt{V}} \quad (7.76)$$

<sup>40</sup>Philip M. Morse, *Vibration and Sound* (1936).

$f_S$  can be interpreted as lower limiting frequency, above which the investigation of the sound field with statistical or geometrical acoustics is valid.

### 7.3.3 Source - receiver transfer function

The above discussed modes in a rectangular room represent the sound fields that are allowed by the room. In a concrete situation the question arises whether a certain mode can be excited. This leads to the source - receiver transfer function. The mathematical treatment makes the introduction of a source term in the wave equation necessary <sup>41</sup>. Here only the solution is given. The sound pressure amplitude  $\check{p}(E, \omega)$  at a receiver point  $E$ , specified by the coordinates  $e_x, e_y, e_z$ , with volume excitation at a source point  $Q$  given by  $(q_x, q_y, q_z)$  and angular frequency  $\omega$  is

$$\check{p}(E, \omega) \sim \omega \sum_n \frac{\check{p}_n(E)\check{p}_n(Q)}{(\omega^2 - \omega_n^2)K_n} \quad (7.77)$$

where

$\sum_n$ : sum over all modes

$\check{p}_n(E)$ : complex sound pressure amplitude for the mode  $n$  at point  $E$

$\check{p}_n(Q)$ : complex sound pressure amplitude for the mode  $n$  at point  $Q$

$\omega_n$ : Eigenfrequency for the mode  $n$

$K_n$ : constant

From Eq. 7.77 follows that a certain mode  $n$  produces relevant sound pressure at the receiver  $E$  only if both  $Q$  and  $E$  are in the vicinity of a pressure maximum. As already mentioned, all modes have a pressure maximum in the corners of a rectangular room. Thus if a loudspeaker is expected to excite all possible modes, it should be placed in a corner.

Up to now perfectly hard surfaces were assumed. In reality all rooms show at least little absorption. The consequences compared to the above derived results are

- at the resonance frequencies only quasi standing waves establish with finite maxima and not vanishing minima
- the quality of the resonances in the transfer function is finite (lowering and widening of the peaks).

The bandwidth of a resonance in the transfer function is a measure for the damping of the corresponding mode. In a well damped room this bandwidth is typically in the order of 5 Hz. The dying away of a mode can be characterized by a sort of reverberation time which can be estimated according to 7.78 <sup>42</sup>

$$RT = \frac{2.2}{B} \quad (7.78)$$

where

$RT$ : reverberation time in seconds

$B$ : bandwidth (at the -3 dB points)

### 7.3.4 Acoustical design of small rooms

#### Introduction

In small undamped rooms the following acoustical difficulties are typical:

- At low frequencies the transfer function is very uneven due to the low density of resonances. Figure 7.24 shows an example.
- At mid and high frequencies strong reflections lead to comb filter distortions and errors in the stereo image. These effects are irrelevant if there is no other contribution stronger than -15 dB relative to the direct sound within 20 ms after the direct sound <sup>43</sup>.

<sup>41</sup>H. Kuttruff, Room Acoustics, Elsevier, 1991.

<sup>42</sup>F. Alton Everest, Master Handbook of Acoustics, McGraw Hill, 2001

<sup>43</sup>James A. S. Angus, Controlling Early Reflections Using Diffusion, AES Convention 102nd (1997).

- At all frequencies the reverberation is too large which leads to low transparency of the acoustical image.

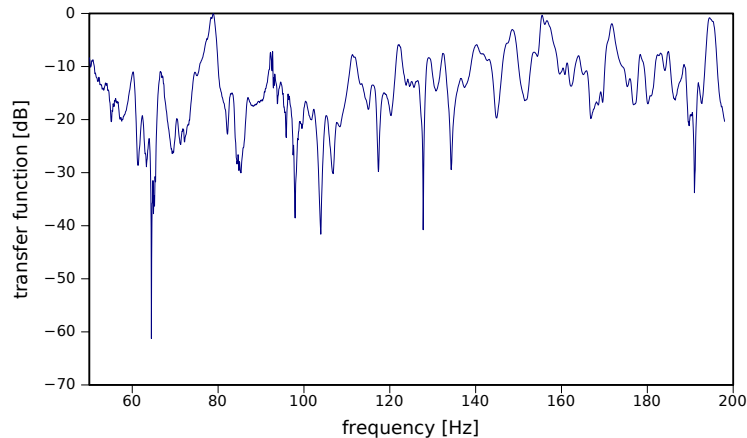


Figure 7.24: Example of a transfer function between a loudspeaker and a microphone in an undamped studio room.

The acoustical design of a small room has to ensure that the above mentioned problems are avoided. There are two fundamental strategies:

- installation of absorbers
- installation of diffusers

### Absorbers

Low frequency absorbers for the low frequency range are typically realized as plate or membrane absorbers. To obtain a broad frequency band of absorption, different modules are necessary with adjusted resonance frequency. In the mid and high frequency range porous absorbers can be used.

### Diffusers

The use of diffusers aims at replacing reflections by scattering<sup>44</sup>. In the best case the scattered sound energy is equally distributed in all directions. In small rooms, scattering may help to avoid room resonances. In order to create diffuse reflections a surface has to introduce locally inhomogeneous reflection conditions. This inhomogeneity can be realized by phase or amplitude variation. An important category of diffusers are Schroeder diffusers that are based on thorough mathematical investigations<sup>45,46</sup>. Schroeder diffusers are built from a series of narrow channels of varying depth (Fig. 7.25). An incident sound wave that hits the diffuser runs down in each channel, is reflected and re-emitted at the channel entrance. The varying channel depth introduces a random phase shift that yields a more or less uniform radiation.

A serious drawback of this configuration is the relative high absorption associated with the reflection. This is due to partial sound pressure compensation of adjacent channels during radiation. The channel concept can be extended to fractal structures where the primary channel with and depth is designed for low frequencies and the high frequency scattering is realized by a smaller structure at the bottom of each channel (Fig. 7.26).

A difficulty arises if identical panels are put in line. Due to the introduced periodicity certain frequencies will be reflected predominantly in certain directions. This unwanted artefact can be overcome with the

<sup>44</sup>T. J. Cox, P. D'Antonio, *Acoustic Absorbers and Diffusers*, Taylor and Francis, 2009.

<sup>45</sup>M. R. Schroeder, Diffuse Sound Reflection by Maximum Length Sequences, *Journal of the Acoustical Society of America*, vol. 57, p. 149-150 (1975).

<sup>46</sup>M. R. Schroeder, Binaural Dissimilarity and Optimum Ceilings for Concert Halls: More Lateral Sound Diffusion, *Journal of the Acoustical Society of America*, vol. 65, p.958-963 (1979).

usage of different panel types. An excellent overview of diffusers can be found in <sup>47</sup>.

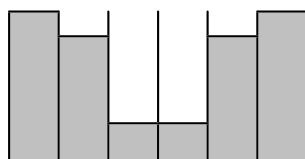


Figure 7.25: Section through a Schroeder diffuser with channels of varying depth according to a distinct number sequence.

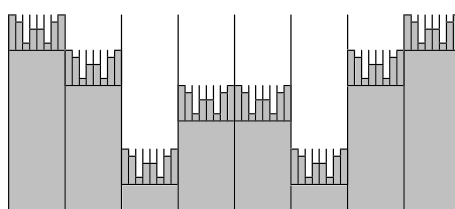


Figure 7.26: Example of a fractal Schroeder diffuser.

Depending on the structure depth, the frequency range of a diffuser is limited to low frequencies. However recent developments show that it is possible to further lower this limit with help of active strategies <sup>48</sup>.

The determination of the diffusivity of a structure by measurements can be performed according to the ISO standard 17497-1 <sup>49</sup>. The method yields a frequency independent single figure in form of a so called scattering coefficient. The measurement is based on several impulse response measurements in the reverberation chamber while the structure is rotating. By phase sensitive averaging of the responses, the specular reflection (coherent contribution) separates from the diffuse reflection (incoherent contribution).

### Design of listening rooms

The design of listening rooms can be based on the standard DIN 15996 <sup>50</sup>. The standard specifies the maximum allowable noise level, the reverberation time and the sound insulation between different facilities. Listening rooms should be larger than 40 m<sup>3</sup> and symmetrical relative to the listening axes.

The maximum allowable noise levels are given by limiting curves in form of third octave band spectra. The noise may not be higher than the limiting values in none of the third octave bands. An advanced listening room should comply with the limit GK10 (Fig. 7.27).

The sound insulation between two different listening rooms should be so high that the mutual disturbance lies below the GK10 curve. For this evaluation a listening spectrum according to Fig. 7.28 is assumed.

Depending on the room volume, the reverberation time in the 500 Hz third octave band should lie between 0.3 (50 m<sup>3</sup>) and 0.5 (1000 m<sup>3</sup>) seconds. The reverberation time should be constant over frequency ( $\pm 10\%$  in the range from 125 to 2000 Hz).

<sup>47</sup>Peter D'Antonio, Trevor Cox, Two Decades of Sound Diffuser Design and Development, Part 1: Applications and Design, Journal of the Audio Engineering Society, vol. 46, no. 11, p.955-976 (1998).

<sup>48</sup>Trevor Cox, et al., Maximum length sequences and Bessel diffusers using active technologies, Journal of Sound and Vibration, vol. 289, p.807-829 (2006).

<sup>49</sup>ISO 17497-1, Acoustics - Sound-scattering properties of surfaces - Measurement of the random-incidence scattering coefficient in a reverberation room (2004).

<sup>50</sup>DIN 15996, Elektronische Laufbild- und Tonbearbeitung in Film-, Video- und Rundfunkbetrieben (1996).

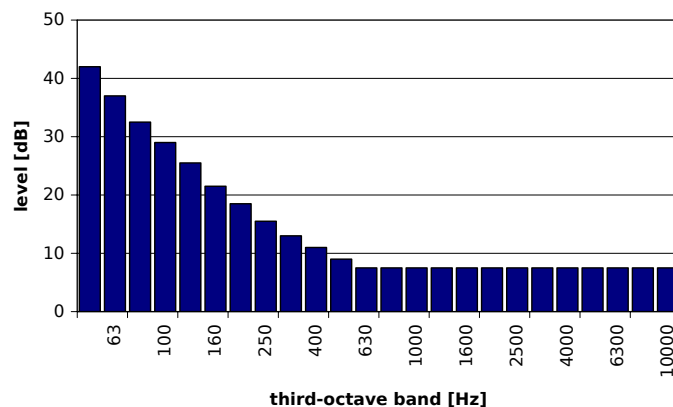


Figure 7.27: Limiting curve GK10 to specify the maximum allowable noise level in third octave bands.

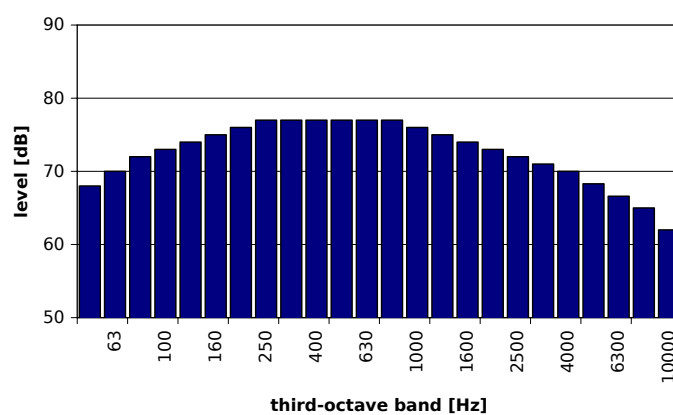


Figure 7.28: Assumed sound pressure spectrum in a typical listening situation.

## 7.4 Room acoustical measurements

The traditional measurement quantity in room acoustics is reverberation time. There are different ways to measure the reverberation time as e.g. with noise that is switched-off or by reverse integration of the squared impulse response. The reverberation time represents a global attribute, in the frame of diffuse field theory the reverberation time does not depend on source and receiver positions. However in practical measurements there occur differences for varying positions. Therefore the reverberation time of a room has to be determined as the average over typically two source and five receiver positions. Along with the measurements, the air temperature and humidity have to be logged to estimate and normalize the effect of air absorption. Further information regarding room acoustical measurements can be found in the standard ISO-3382.

In recent years room impulse response measurements become more and more popular. For given source and receiver positions the impulse response contains the complete information of the room (Fig. 7.29). The main advantage of room impulse responses lies in the possibility to investigate the strength of single reflections and to evaluate further objective criteria such as *clarity*, *EDT*, etc.

The impulse response and the derived objective criteria are very sensitive to the source directivity. To get results of general validity an omnidirectional source is used. Possible sources to excite a room are pistol shots or balloon bursts<sup>51</sup> or loudspeakers. However the practical realization of a wide-band, omnidirectional loudspeaker is difficult. One strategy is to place several speaker chassis on a sphere-like surface such as a dodecahedron (Fig. 7.30).

If an impulse response measurement is performed with a line array of microphones, additional information

<sup>51</sup>J. P'atynen et al., Investigations on the balloon as an impulse source, J. Acoust. Soc. Am., EL27-EL33, vol. 129 (2011).

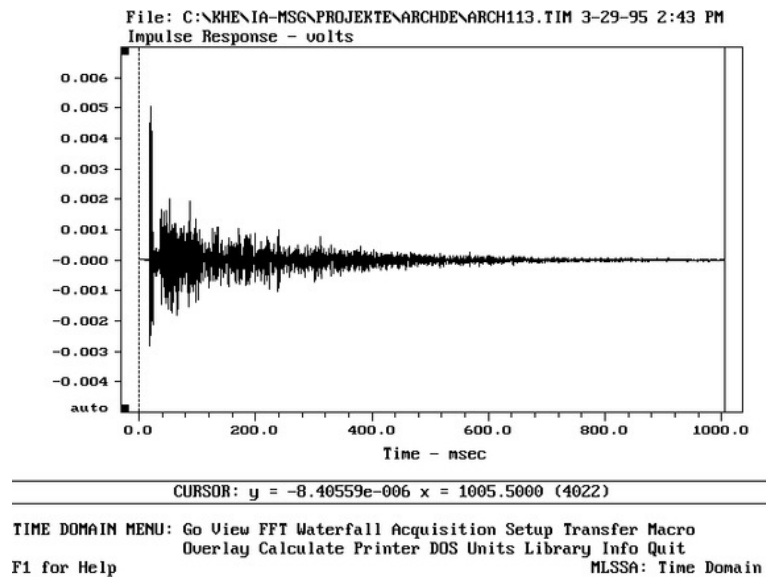


Figure 7.29: Example of a measured impulse response in a multi-purpose hall. The first peak corresponds to the direct sound, followed by weak reflections at the ground and at nearby objects. Later, more pronounced reflections from the walls and the ceiling arrive and finally the reverberation tail can be observed. From the section before the arrival of the direct sound the unwanted noise and thus the quality of the measurement can be estimated.

about the sound incidence direction can be obtained <sup>52</sup>. This allows for a more reliable identification of single reflections.

<sup>52</sup>A. J. Berkhout, D. de Vries, J. J. Sonke, Array technology for acoustic wave field analysis in enclosures, J. of the Acoustical Society of America, vol. 102, no. 5 (1997), p.2757-2770.



Figure 7.30: Dodecahedron loudspeaker with 12 chassis for omnidirectional sound radiation.



# Chapter 8

## Building acoustics

### 8.1 Introduction

Building acoustics deals with noise control in buildings. The fundamental aim is the avoidance or sufficient reduction of noise from neighbors. Usually there is no connection by air between two adjacent rooms. However air borne or structure borne sound in one room finds its way to the other room by vibration of the structure. Finally this vibration is emitted in form of air borne sound in the receiver room. The capability of a wall to suppress this transmission is called sound insulation. Two forms of excitation are possible. The first type of excitation is air borne sound such as a talking person or a loudspeaker. The sound insulation in this context is called airborne sound insulation. The second type is structure borne sound which means the structure is excited directly by a mechanical force. The most important source of this type is impact sound that occurs while walking. In this case the sound insulation is called impact sound insulation.

### 8.2 Airborne sound insulation

#### 8.2.1 Sound insulation index $R$

The airborne sound insulation of a structure that separates two rooms (Figure 8.1) is described by the transmission loss or airborne sound insulation index  $R$  according to Eq. 8.1.

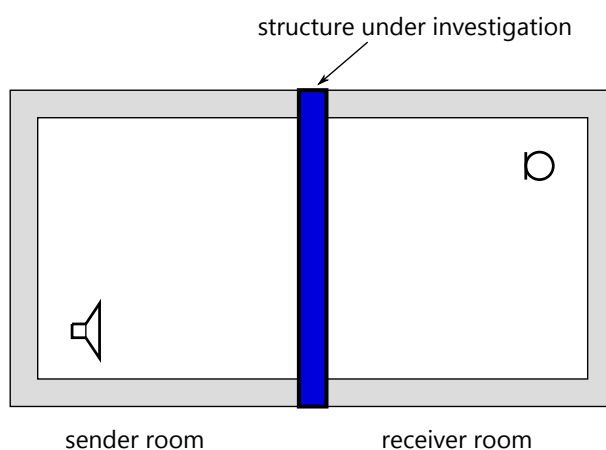


Figure 8.1: Configuration of a sender and a receiving room with the separating structure to be investigated.

$$R = 10 \log \left( \frac{P_1}{P_2} \right) \quad [\text{dB}] \quad (8.1)$$

where

$P_1$ : incident sound power on the sender side

$P_2$ : sound power that is radiated on the rear side of the structure

The sound insulation index  $R$  is independent of the area of the structure.

The measurement of  $R$  is based on a sound pressure level difference  $L_1 - L_2$  in third octaves between the sender and receiving room. However two corrections have to be applied:

- The level  $L_1$  describes the sound pressure square in the sender room. Under the assumption that the sound field can be thought of as composed of plane waves arriving from all directions, the incident sound power  $P_1$  can be determined by integration over a half sphere and taking the cosine of the incident angle into account. With  $S$  as area of the structure the sound power results as  $P_1 = Sp_1^2/4\rho c$ .
- The sound pressure square  $p_2^2$  in the receiving room is inverse proportional to the total absorption  $A_2$  in the receiving room. Consequently the power  $P_2$  is given as  $P_2 = A_2p_2^2/4\rho c$ .  $A_2$  is determined with the reverberation time  $T_2$  and the room volume  $V_2$  as  $A_2 = 0.16V_2/T_2$ .

Finally for the sound insulation index can be written

$$R = L_1 - L_2 + 10 \log \left( \frac{S}{A_2} \right) \quad [\text{dB}] \quad (8.2)$$

Details about the measurement of sound insulation of building elements can be found in the series of standards ISO 140-3. For easier handling the third octave spectrum of  $R$  is converted to a single figure  $R_w$  (rated sound insulation index) by application of a reference spectrum.

## 8.2.2 Sound insulation of single walls

Sound insulation of homogeneous and dense plates depends on frequency and the plate parameters:

- thickness
- density
- modulus of elasticity

The frequency dependency of  $R$  follows essentially the curve shown in Figure 8.2. Hereby three regions A, B and C can be distinguished.

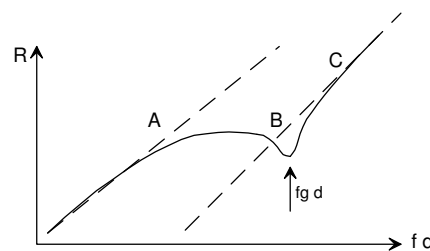


Figure 8.2: General frequency dependency of the sound insulation index  $R$  for a single wall. The abscissa shows the product frequency  $\times$  thickness of the element ( $= fd$ ). Region A: mass law, region B: coincidence, region C: above coincidence.

*Region A:*

For low frequencies the sound insulation follows the mass law that can be written for random incident sound waves as <sup>1</sup>

$$R = 20 \log \left( \frac{\pi f m''}{\rho c} \right) - 5 \quad [\text{dB}] \quad (8.3)$$

<sup>1</sup>Fasold, Sonntag, Winkler, Bau- und Raumakustik, Verlag R. Müller (1987).

where

$f$ : frequency

$m''$ : area specific mass

For a given structure the sound insulation increases by 6 dB for a doubling of frequency. In the same manner for a given frequency the sound insulation increases by 6 dB for a doubling of the mass.

*Region B:*

The excitation of the wall by a sound leads to the formation of bending waves. These waves propagate along the surface with a velocity that depends on the modulus of elasticity and the thickness and density of the structure. If the wave length of the airborne sound excitation on the wall (projection of the wave) coincides with the wave length of the bending wave, the sound insulation collapses. This condition is called coincidence. Exact coincidence occurs for a certain frequency and a certain sound incidence direction. Due to the random distribution of the angle of incidences the coincidence collapse is not that strong in the diffuse field and smeared over a wider frequency region.

*Region C:*

For frequencies above the coincidence the sound insulation increases again with frequency. The steepness is around 25 dB/decade.

### 8.2.3 Sound insulation of double walls

An improvement of the sound insulation can be achieved by adding a second wall. The space between the walls is usually air. The two walls together with the air space in between form a resonance system with two masses coupled by a spring. At the resonance frequency the sound insulation breaks down and is lower than in the case of a corresponding single wall. Above resonance the sound insulation increases strongly with frequency up to the point where again coincidence kicks in.

### 8.2.4 Standard sound pressure level difference

In a given situation the disturbance of neighbors does not depend primarily on the sound insulation index of the structural elements, but rather on the sound pressure level difference  $D_{nT}$  between the rooms. This level difference is given by the sound insulation index  $R$  and the shared area  $F$ . As the sound pressure level in the receiving room is influenced by the total absorption  $A$ , an agreement has to be achieved to get representative results. This is done by normalizing the results to a reverberation time in the receiver room of 0.5 s. For a receiver room volume  $V$ , the standard sound pressure level difference can be written as

$$D_{nT} = R + 10 \log \left( \frac{V}{F} \right) - 4.9 \quad (8.4)$$

If the rated sound insulation index  $R_w$  is inserted in Eq. 8.4, the corresponding value is called rated standard sound pressure level difference with the symbol  $D_{nT,w}$ .

## 8.3 Impact sound insulation

The measurement of the impact sound insulation is usually based on excitation by a standardized tapping machine. The machine uses hammers of defined mass and form that fall on the floor from defined height. In the receiving room the resulting sound pressure level is measured at different positions. From the average sound pressure level  $L_i$  the standard impact sound level  $L_n$  is determined by normalization for a total absorption of  $10 \text{ m}^2$ . With the receiving room volume  $V$  this can be expressed with help of the reverberation time  $T$  in the receiving room as

$$L_n = L_i - 10 \log \left( \frac{10T}{0.163V} \right) \quad (8.5)$$

The spectral values  $L_n$  can be translated into a single value  $L_{n,w}$  by comparison with a reference curve.

## 8.4 SIA 181

The Swiss standard SIA 181 represents the state of the art in building technology regarding building acoustical requirements. The standard defines the necessary noise protection on two levels. The minimal requirements have to be fulfilled in any case. Apart from the minimal requirements elevated requirements are specified that can be agreed by contract. In some cases such as single family houses that are built together, the elevated requirements are compulsory.

The SIA 181 defines minimal values of sound pressure level differences of the building structure for exterior airborne sound and interior airborne sound. In addition, limiting values are given for impact sound. The limiting values differentiate regarding the intensity of the source and the degree of sensitivity of the inhabitants for a certain usage of the room.

## 8.5 Construction hints for good building acoustical conditions

**Arrangement of rooms** Often building acoustical problems can be avoided by suitable arrangement of rooms. It should be avoided that rooms with different usage (e.g. a bed room and a kitchen) are located next to each other (horizontally and vertically).

**Doors and windows** Doors and windows have typically a maximum sound insulation of 35 to 40 dB. Higher values can only be obtained with special constructions. Compared to doors and windows the sound transmission through the surrounding walls can usually be neglected.

**Leakage** The sound insulation between adjacent rooms is drastically reduced if there is leakage in form of cracks. Similarly lead-throughs for cables or ventilation ducts are critical.

**Floating floors** Usually walls are put directly on the concrete floors. To avoid significant structure borne sound transmission through the floor, floating floors can be installed. Hereby a layer of low stiffness is put in between the concrete floor and the top cover. It is absolutely crucial that any connection between the floating floor and other parts of the building construction is avoided.

# Chapter 9

## Noise abatement

### 9.1 Introduction - definition of noise

Noise is sound but sound is not necessarily noise. The assessment of an acoustical situation regarding possible annoyance for a human being depends strongly on the individual. Noise is very subjective and as such can't be measured. Each person has his own noise scale. Furthermore annoyance depends on the momentary condition of the individual (psychological situation, weariness, etc.). A short definition of noise is:

#### **Noise is unwanted sound**

Noise has to be assessed, there is no objective scale. For well defined noise sources such as road traffic or railways a relation between an objective acoustical measure (exposure) and the annoyance can be established. However such a relation is only valid for an average person, the individual reporting can deviate significantly. The outcome of studies about annoyance follows typically a curve as shown in Fig. 9.1. The sigmoid curve expresses the fact that even for very low exposure always a certain portion of people reports high annoyance. At the other end, there are very insensitive people that are not significantly annoyed even at very high exposure.

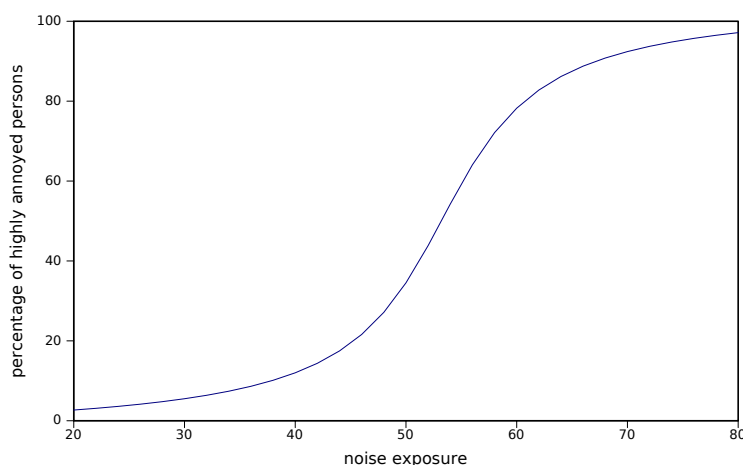


Figure 9.1: Typical relation between noise exposure and annoyance, shown as percentage of people that are highly annoyed.

In the meantime it is widely accepted that excess noise may cause health problems. The corresponding relations are difficult to establish due to the complexity and number of factors that play a role. However it can be assumed that risk of health impairment due to noise increases for average sound pressure levels higher than 65 to 70 dB(A) during the day. At night the sound pressure level at the ear of the sleeping person should not exceed 30 dB(A) in order not to affect sleep quality.

## 9.2 Effects of noise

The effects of noise can be categorized as follows:

**physiological effects** such as headache, cardio-vascular diseases, increased blood pressure, extensive pouring out of stress hormones, sleep disturbances and hearing defects in extreme cases

**psychological effects** such as stress and nervousness, reduction of productivity

**social effects** such as obstruction of communication, social segregation (those who can afford live in quieter areas)

In addition to the above mentioned effects noise has economical consequences as well. The noise burden is a factor that has significant influence on the prices of real estates. In many situations measures have to be taken against noise (such as e.g. noise barriers). In case of public noise sources (roads, railway lines, etc.) the costs are payed by the public. Finally noise induced health problems cause health costs and loss of productivity.

## 9.3 General remarks for the assessment of noise

The assessment of noise is usually based on the exposure principle. Besides the intensity of the noise events the number of events in a certain time interval is taken into account. This leads to the consideration of average values such as the  $L_{eq}$  (energy equivalent sound pressure level). The averaging period is often a year.

The sensitivity to noise is highest during nighttime, somewhat lower at the evening period and lowest during the day. Switzerland has chosen the approach to define separate limiting values for day and night. In Europe and the U.S. the so called *day-evening-night* level  $L_{den}$  is used. The  $L_{den}$  maps the noise exposure to a single number whereby the level for the night period is increased by 10 dB and the evening level is increased by 5 dB. These "malus" values reflect the increased sensitivity during night and evening periods.

$$L_{den} = 10 \log \left( \frac{1}{24} \left[ 12 \cdot 10^{0.1(L_d)} + 4 \cdot 10^{0.1(L_e+5)} + 8 \cdot 10^{0.1(L_n+10)} \right] \right) \quad (9.1)$$

where

$L_d$ : average sound pressure level  $L_{eq}$  during daytime (12 hours)

$L_e$ : average sound pressure level  $L_{eq}$  during the evening period (4 hours)

$L_n$ : average sound pressure level  $L_{eq}$  at night (8 hours)

In some cases the *day-night* level  $L_{dn}$  is used. It is defined analogously to the  $L_{den}$ , however without consideration of the evening period.

$$L_{dn} = 10 \log \left( \frac{1}{24} \left[ 15 \cdot 10^{0.1(L_d)} + 9 \cdot 10^{0.1(L_n+10)} \right] \right) \quad (9.2)$$

where

$L_d$ : average sound pressure level  $L_{eq}$  during daytime (7:00 till 22:00)

$L_n$ : average sound pressure level  $L_{eq}$  during nighttime (22:00 till 7:00)

The assessment of a noise situation is finally based on a comparison of the exposure at a receiver location with a limiting value. This yields a simple "yes/no" decision. In addition there exist more sophisticated assessment schemes that evaluate a continuous relationship between exposure and annoyance - an example is the *Zürcher FluglärmindeX*,  $ZFI$ .

## 9.4 Influence of the source type

At equal exposure people report different annoyance for different noise sources. Railway noise for example is significantly less annoying compared to road traffic noise or noise from aircrafts (Fig. 9.2)<sup>1</sup>.

<sup>1</sup>M. E. Miedema, H. Vos, Exposure-response relationships for transportation noise, Journal of the Acoustical Society of America, vol. 104, p.3432-3445 (1998).

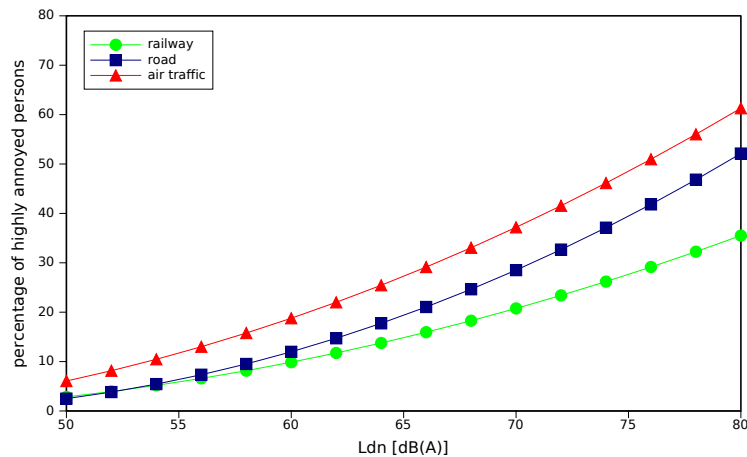


Figure 9.2: Exposure - annoyance relation for different noise sources. The annoyance is expressed as percentage of people that are highly annoyed, the exposure is described as  $L_{dn}$ .

The curves in Fig. 9.2 correspond to the functions in Eq. 9.3, where  $\%HA$  is the percentage of highly annoyed people.

$$\begin{aligned}
 \text{railwaynoise :} & \quad \%HA = 0.01(L_{dn} - 42) + 0.0193(L_{dn} - 42)^2 \\
 \text{roadtrafficnoise :} & \quad \%HA = 0.03(L_{dn} - 42) + 0.0353(L_{dn} - 42)^2 \\
 \text{aircraftnoise :} & \quad \%HA = 0.53(L_{dn} - 42) + 0.0285(L_{dn} - 42)^2 \quad (9.3)
 \end{aligned}$$

There are several reasons for a source type dependent annoyance sensitivity. An important influence factor is the personal attitude towards the noise polluter. Furthermore spectral or temporal differences in the noise signal may play a role. Consequently in practice each kind of noise is investigated and assessed separately.

## 9.5 Definition of limiting values

As discussed above the noise burden is investigated by evaluating a suitable exposure measure and subsequent comparison with limiting values. The definition of these limiting values is based on exposure - annoyance relationships as shown in Fig. 9.1. Usually the annoyance is reported on a scale from 0 to 10. The percentage of highly annoyed people is then determined by counting the answers 8..10. The limiting value is typically set to the exposure that creates between 15 and 25% highly annoyed people. In other words if the limiting value is reached, almost one quarter of the people is highly annoyed.

## 9.6 Legal basis in Switzerland

### 9.6.1 Environment protection law USG

The environment protection law was implemented in 1985. It specifies the fundamental principles for the protection of humans, animals and plants against harmful and annoying impacts. As a central instruction the principle of precaution was established. It says that potential impacts should be detected in advance and limited accordingly. All emissions should be limited at the source according to the possibilities given. The exposure at residents has to be assessed by comparison with impact thresholds. These limits have to be fixed in such a way that - according to best knowledge - exposures below the limits guarantee that the population is not sincerely annoyed. The law is further detailed in the Noise Abatement Ordinance LSV.

### 9.6.2 Noise Abatement Ordinance LSV

The Noise Abatement Ordinance (LSV) specifies the execution of the environment protection law in the domain of noise. The LSV has been put into force in 1987 and has experienced different extensions

and adaptations since. The LSV gives declarations regarding construction, operation and rehabilitation of facilities and regularizes the construction of new buildings with noise sensitive usage.

### **Scheme of limiting values**

The LSV specifies not only *impact thresholds*, but *planning values* and *alarm values* as well. The planning values are typically 5 dB lower than the impact thresholds. They come into play for new buildings and new facilities and implement the principle of precaution. The alarm values on the other hand (typically 5 dB higher than the impact thresholds) help to identify severe situations with urgent need for the realization of noise abatement measures. All limiting values are specified separately for day and night periods. Further they are differentiated according to four sensitivity levels. Sensitivity level I corresponds to special zones for recreation, sensitivity level II qualifies zones for living, sensitivity level III is assigned to zones for living and industry. Sensitivity level IV finally corresponds to zones with industry only.

### **Construction, operation and sanitation of facilities**

As a fundamental principle the LSV claims that any noise source has to reduce its emissions as much as possible at least to a degree that is affordable.

A *new or heavily altered installation* has to reduce its emissions, so that the planning values in the neighborhood are respected. For private installations relaxations can be granted if the installation is of general interest or if the effort to fulfill the planning values would be disproportional. Public installations can get relaxations as well, even if the impact threshold is violated. However in these cases protection measures have to be taken at the receivers in form of sound-proof windows.

*Existing installations* have to respect the impact thresholds in the neighborhood. If a private installation exceeds these values, the installation has to be improved. Relaxations are possible between the impact threshold and the alarm value. Public installations can get relaxations even above the alarm value if protection measures are taken at the receivers.

If a private installation is *significantly altered* and the impact thresholds were violated so far, measures have to be taken to respect the impact thresholds.

### **Construction permits**

An important aim of the LSV is the prevention that new buildings with noise sensitive usage are built in areas with high noise burden. Therefore the allowance for new buildings is coupled to certain conditions regarding noise that is already present. The authorities can install new zones for buildings only if the planning values can be respected. Similarly, areas that are already defined as zones for buildings but are not developed yet have to respect the planning values. Houses are allowed in zones for buildings that are already developed if the impact thresholds are kept. Exceptions are possible if the construction is of public interest, e.g. if a gap in row of houses is closed to create a quiet backyard.

Relevant for the verification of the limiting values is the center of the most exposed open window of a room with noise sensitive usage such as living rooms or bed rooms. In the vicinity of line noise sources such as roads or railway lines it may be possible to construct new houses even in short distance if the orientation of the sensitive rooms is optimized. For windows that can not be opened the noise limits do not apply.

### **Assessment of road traffic noise**

To evaluate the road traffic noise burden two rating levels  $L_r$  are determined separately for day (6-22) and night (22-6) as follows:

$$L_r = L_{eq} + K_1 \quad (9.4)$$

$L_{eq}$  corresponds to the yearly average A-weighted sound pressure level, evaluated for day and night. The correction  $K_1$  depends on traffic volume. For less than 32 vehicles per hour  $K_1$  is -5 dB, for more than 100 vehicles per hour  $K_1$  equals 0 dB.



The two rating levels evaluated with Eq. 9.4 are finally compared with the scheme of limiting values in Table 9.1.

Sens.level	PW-day	PW-night	IGW-day	IGW-night	AW-day	AW-night
I	50	40	55	45	65	60
II	55	45	60	50	70	65
III	60	50	65	55	70	65
IV	65	55	70	60	75	70

Table 9.1: Scheme of limiting values for road traffic noise for the sensitivity levels I to IV. PW is the planning value, IGW is the impact threshold and AW is the alarm value.

### Assessment of railway noise

To evaluate the railway noise burden two rating levels  $Lr$  are determined separately for day (6-22) and night (22-6) as follows:

$$Lr = Leq + K1 \quad (9.5)$$

$Leq$  corresponds to the yearly average A-weighted sound pressure level, evaluated for day and night. The correction  $K1$  depends on traffic volume. For less than 8 train passages per hour  $K1$  is -15 dB, for more than 80 passages per hour  $K1$  equals -5 dB. This bonus reflects the lower sensitivity against railway noise compared to road traffic noise.

### Assessment of industry noise

Noise from industries shows larger variation in character compared with road traffic or railway noise. Usually industrial noise sources vary over time significantly. For that reason the assessment is based on different phases of equal noise character. The rating level is defined for day (7-19) and night (19-7) as follows

$$Lr = 10 \log \left( \sum 10^{(0.1Lr_i)} \right) \quad (9.6)$$

where the  $Lr_i$  correspond to partial rating levels of the individual phases of equal noise character. The partial rating levels are determined as

$$Lr_i = Leq_i + K1_i + K2_i + K3_i + 10 \log \left( \frac{t_i}{t_o} \right) \quad (9.7)$$

where:

$Leq_i$ : energy equivalent A-weighted sound pressure level during phase  $i$

$K1_i$ : source type dependent correction for phase  $i$

$K2_i$ : tone correction for phase  $i$

$K3_i$ : impulse correction for phase  $i$

$t_i$ : average daily duration of phase  $i$  in minutes, where  $t_i = \frac{T_i}{B}$

$T_i$ : yearly duration of phase  $i$  in minutes

$B$ : number of days per year the plant is in service <sup>2</sup>

$t_o = 720$  minutes

The correction for the source type  $K1$  lies between 5 and 10 dB (10 dB are applied for heating, ventilation and air condition installations).

The correction for tonal sound is set according to the listening impression. If there is no tone (with a distinct pitch) audible,  $K2$  equals = 0, for weakly audible tones  $K2$  is set to 2, for clearly audible tones  $K2$  is 4 and finally if the signal contains tones that are strongly audible,  $K2$  is set to 6.

<sup>2</sup>In some cases the definition of  $B$  is tricky.

The correction for impulsive sound is determined subjectively as well.  $K3 = 0$  stands for no audible impulsiveness,  $K3 = 2$  signifies weakly audible impulses,  $K3 = 4$  is for clearly audible impulses and  $K3 = 6$  is for strongly audible impulsiveness.

The day and night rating levels according to Eq. 9.6 are compared to the limiting values for road traffic noise (Table 9.1).

### Assessment of noise from shooting ranges

The assessment of noise from 50 m and 300 m shooting ranges is based on a rating level  $Lr$  as follows:

$$Lr = L + K \quad (9.8)$$

where  $L$  corresponds to the average maximum level (A-Fast) of a single shot. The correction  $K$  for the number of shots is determined as:

$$K = 10 \log(Dw + 3 \cdot Ds) + 3 \log(M) - 44 \quad (9.9)$$

where:

$Dw$ : number of half-days with activity during the week per year

$Ds$ : number of half-days with activity at Sundays per year

$M$ : number of shots fired in one year

Finally the rating levels are compared with the limiting values scheme according to Table 9.2. As shooting ranges operate only during daytime, there are no limiting values for the night period.

Sens.level	PW	IGW	AW
I	50	55	65
II	55	60	75
III	60	65	75
IV	65	70	80

Table 9.2: Scheme of limiting values for noise from shooting ranges for the sensitivity levels I to IV. PW is the planning value, IGW is the impact threshold and AW is the alarm value.

### Assessment of aircraft noise

The assessment of aircraft noise in the surroundings of the airports Zurich, Basel and Geneva is based on separate rating levels for the day period (6-22), the first hour of the night (22-23), the second hour of the night (23-24) and the last hour of the night (5-6). The level for the day period  $Lr_t$  is determined as follows:

$$Lr_t = 10 \log(10^{0.1Lr_k} + 10^{0.1Lr_g}) \quad (9.10)$$

$Lr_k$  corresponds to the rating level for small aviation. The level is determined as the A-weighted average sound pressure level for a day with average peak service and a correction based on the number of flight operations.  $Lr_g$  is the A-weighted, yearly average sound pressure level stemming from large aviation in the period between 6 and 22.

The rating levels for the night hours correspond directly to the A-weighted average sound pressure levels produced by large aviation. The rating levels are finally compared to the scheme given in Table 9.3. The impact thresholds for the second and last night hour are identical to the nighttime values for road traffic noise. However the separate evaluation of hourly values in case of aircraft noise is stricter compared to road traffic noise where higher values in one hour are smeared over the whole night period.

## 9.7 Soundscape concept

Standard noise abatement strategies try to lower the A-level at the residents locations. However the potential for attenuation measures in urban environments is usually rather small as classical solutions

Sens.level	PW <sub>d</sub>	IGW <sub>d</sub>	AW <sub>d</sub>	PW <sub>n1</sub>	IGW <sub>n1</sub>	AW <sub>n1</sub>	PW <sub>n2l</sub>	IGW <sub>n2l</sub>	AW <sub>n2l</sub>
I	53	55	60	43	45	55	43	45	55
II	57	60	65	50	55	65	47	50	60
III	60	65	70	50	55	65	50	55	65
IV	65	70	75	55	60	70	55	60	70

Table 9.3: Scheme of limits for aircraft noise for the sensitivity levels I to IV. PW is the planning value, IGW is the impact threshold and AW is the alarm value. The index *d* denotes the day period (6-22), *n1* indicates the first night hour (22-23), *n2l* means the second and last night hour (23-24, 5-6).

such as noise barriers are not applicable. Therefore acousticians and authorities start to reconsider the fundamental noise abatement goal. The noise situation of residents can usually be improved by lowering the A-level but this is not necessarily the only path to go. Indeed people assess noise annoyance by taking into account many more factors. It seems therefore promising to consider additional aspects when it comes to future noise abatement policies. All relevant aspects that affect noise perception are usually summarized and described by the *Soundscape*.

A mighty factor in this context is the fact that subjective annoyance depends on the type of noise source. At identical A-levels, we are usually more annoyed by man-made sounds compared to natural sounds. This offers the possibility to mask unwanted sound by more favored sounds such as water sounds<sup>3</sup>.

---

<sup>3</sup>L. Galbrun, T. T. Ali, Acoustical and perceptual assesment of water sounds and their use over road traffic noise, J. Acoustical Society America, vol. 133, p. 227-237 (2013)

# Appendix A

## Acoustic physical constants

### A.1 speed of sound in air

temperature [°C]	speed of sound $c$ [m/s]
0	331.3
10	337.3
20	343.2

### A.2 density of air at sea level

temperature [°C]	density of air $\rho$ [kg/m <sup>3</sup> ]
0	1.292
10	1.247
20	1.204

### A.3 acoustic impedance

temperature [°C]	$\rho c$ [Ns/m <sup>3</sup> ]
0	428.0
10	420.5
20	413.3

# Index

- A-filter, 52
- absorber, 97
- absorption, 97
- absorption coefficient, 97
- acoustical calibrator, 59
- acoustical holography, 39
- adiabatic stratification, 92
- airborne sound, 4
- airborne sound insulation, 139
- airflow speaker, 41
- angular frequency, 10
- atmospheric absorption, 85
- auralization, 121
- auto correlation function, 63
  
- B-filter, 52
- bang, 24
- basilar membrane, 73
- boundary conditions, 9
  
- C-filter, 52
- C80, 117
- calibrators, 58
- cent, 83
- center time, 118
- clarity, 117
- cochlea, 73
- cocktail party effect, 79
- coherent sources, 16
- complex tonal sound, 23
- critical band, 76
- critical distance, 109
- cross correlation functions, 63
- cylindrical waves, 15
  
- day-evening-night-level, 144
- day-night-Pegel, 144
- dB - scale, 21
- degrees of freedom, 55
- diffraction, 3
- diffuse reflection, 19
- diffuse sound field, 107
- diffusers, 134
- dipole radiator, 17
- dodecahedron loudspeaker, 136
- Doppler effect, 19
  
- ear, 73
- Early Decay Time, 117
  
- Eigenfrequencies, 129
- emission measurements, 48
- energy impulse response, 116
- environment protection law, 145
- equally tempered scale, 83
- Equivalent continuous sound pressure level, 49
- equivalent source technique, 40
- Eyring, 112
  
- FAST time constant, 50
- filters, 51
- finite element method, 35
- free field response microphone, 58
- frequency, 10
- frequency analysis, 52
- frequency analyzers, 60
- frequency response, 62
- Fresnel number, 90
- Fresnel zone, 30
- Fresnel zones, 123
  
- Gabor pulse, 68
- geometrical spreading, 2
- ground effect, 86
  
- head related transfer functions, 122
- Helmholtz equation, 11
- HRTF, 79, 122
- Huygens elementary sources, 30
  
- impedance, 5
- impedance tube, 101
- impulse correction, 148
- impulse response, 62
- incoherent sources, 16
- interference, 3
  
- Kirchhoff - Helmholtz integral, 27
- Kundt's tube, 100
  
- L1, 60
- L50, 60
- Lambert's reflection characteristics, 115
- lateral energy fraction, 118
- level recorders, 60
- levels, 21
- loudness, 74
- loudness summation, 76
  
- masking, 76

maximum length sequences, 64  
 measurement uncertainty, 54  
 membrane absorbers, 99  
 membranes, 46  
 Microflown, 61  
 microperforated absorbers, 98  
 microphones, 58  
 mirror source, 19  
 mirror sources (room acoustics), 120  
 MLS, 64  
 mode, 131  
 momentary level, 49  
  
 noise, 143  
 Noise Abatement Ordinance, 145  
 noise: evaluation of railway noise, 147  
 noise: evaluation of road traffic noise, 146  
 noise: industrial installations, 147  
 noise: shooting ranges, 148  
  
 octave filters, 54  
 organ pipe, 41  
 otoacoustic emission, 74  
  
 period length, 10  
 phon, 74  
 pink noise, 23  
 pistonphone, 58  
 plane waves, 12  
 point sources, 15  
 precedence effect, 81  
 pressure response microphone, 58  
 pressure zone configuration, 71  
 principle of reciprocity, 40  
 pure tone, 23  
  
 quarter wave length resonator, 25  
  
 ray tracing (room acoustics), 120  
 Rayleigh integral, 27  
 reflection, 2, 19, 97  
 reflection coefficient, 97  
 reverberation time, 69, 111, 116, 136  
 reverberation time measurement, 68  
 rods, 45  
 room impulse response measurement, 136  
  
 Sabine, 112  
 scale models, 120  
 scattering, 3, 26  
 Schroeder reverse integration, 69  
 Schroeder-diffusers, 134  
 seat dip effect, 123  
  
 SIA 181, 142  
 siren, 41  
 SLOW time constant, 50  
 sone, 74  
 sonic boom, 20  
 sound exposure level, 49  
 sound field, 4  
 sound insulation, 139  
 sound insulation index, 139  
 sound intensity, 4  
 sound intensity level, 21  
 sound intensity meters, 61  
 sound level meter, 59  
 sound particle displacement, 4  
 sound particle velocity, 4  
 sound power, 4  
 sound power level, 21  
 sound pressure, 4  
 sound pressure level, 21  
 sound recorders, 60  
 soundscape, 149  
 source directivity, 19  
 specular reflection, 18  
 spherical waves, 13  
 standing wave, 24  
 strength, 117  
 strings, 43  
 superposition of point sources, 15  
 superposition principle, 15  
 sweep, 24  
 system identification, 62  
  
 temporal masking, 79  
 thermo-acoustical machines, 46  
 third-octave filters, 54  
 time-bandwidth uncertainty principle, 67  
 time-reversed acoustics, 40  
 tone burst, 24  
 tone correction, 147  
 total absorption, 108  
 transmission, 97  
  
 uncertainty of measurements, 54  
  
 virtual pitch, 77  
 volume velocity, 5  
  
 wave equation, 5, 8  
 wave equation of a string, 43  
 wave length, 10  
 wave number, 10  
 white noise, 23