Eidgenössische Technische Hochschule Zürich
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Acoustics I: sound field calculations

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introduction

introduction

sound field calculations:

- calculation of a situation specific location- and time dependent sound field (often p)
- conditions for a valid solution:
  - fulfillment of the wave equation or Helmholtz equation
  - fulfillment of the boundary conditions:
    - sources
    - boundaries (borders of space)
- analytical solutions for special geometries only
- numerical solutions in the general case:
  - finite elements
  - boundary elements
  - time domain methods such as FDTD

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Kirchhoff - Helmholtz integral

Kirchhoff - Helmholtz integral

Green's theorem:
Helmholtz equation $\curvearrowright$ Kirchhoff - Helmholtz integral:

$$\check{p}(x,y,z,\omega) = \frac{1}{4\pi}\int_S \left(j\omega\rho_0 \check{v}_S(\omega)\frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega)\frac{\partial}{\partial n}\frac{e^{-j\omega r/c}}{r}\right)\mathrm{d}S$$

S: closed surface
$\check{v}_S$: sound particle velocity on and normal to S
$\check{p}_S$: sound pressure on S
r: distance of the surface point to the receiver point (x, y, z)

- Kirchhoff-Helmholtz integral $\rightarrow$ wave field synthesis

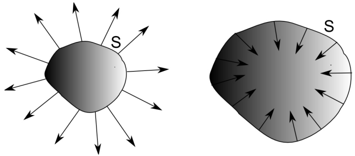
Kirchhoff - Helmholtz integral

- Kirchhoff-Helmholtz integral KHI is valid:
  - in the interior of S
  - in the exterior of S
  - on the surface S with a correction factor of 2

Boundary Elements Method

Boundary Elements Method

typical radiation problem:

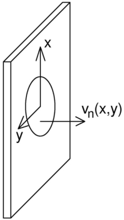


- surface velocity is given as boundary condition
- search for sound pressure field inside or outside of S
- solution with the Boundary Elements Method:
  - discretisation of the radiator surface in n elements
  - with KHI: $\check{p}_{S,i} = \sum_{j=1}^{n} f(\check{p}_{S,j}, \check{v}_{S,j})$
  - solve the system of equations with n unknowns $\rightarrow \check{p}_{S,i}$
  - calculate sound pressure at any point in space with the KHI

Rayleigh Integral

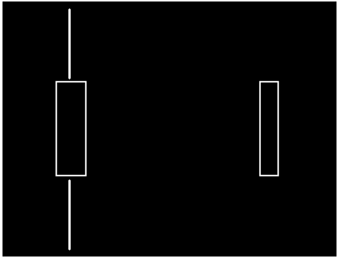
Rayleigh Integral

- radiation of an oscillating piston $\rightarrow$ Kirchhoff-Helmholtz Integral
- special case: oscillating piston mounted in a large and rigid wall
  - wall introduces boundary condition: $v_n = 0$



Rayleigh Integral

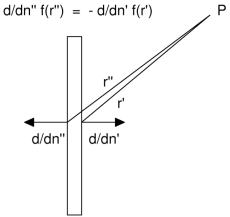
- replace the effect of the wall by a mirror source
- oscillating piston $\rightarrow$ pulsating piston



Rayleigh Integral

evaluation of the Kirchhoff Helmholtz Integral:

$$\check{p}(x,y,z,\omega) = \frac{1}{4\pi}\int_S \left(j\omega\rho_0 \check{v}_S(\omega)\frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega)\frac{\partial}{\partial n}\frac{e^{-j\omega r/c}}{r}\right) \mathrm{d}S$$



contribution of sound pressure $= 0$!

Rayleigh Integral

Kirchhoff Helmholtz Integral simplifies to the Rayleigh Integral:

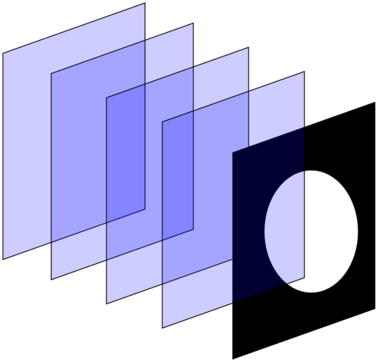
$$\check{p}(x, y, z, \omega) = \frac{j\omega\rho_0}{2\pi} \int_S \check{v}_n(x, y, \omega) \frac{e^{-jkr}}{r} \mathrm{d}S$$

S: *visible* piston surface (front)
v_n: piston velocity

Kirchhoff's approximations

Kirchhoff's approximations: diffraction problems

- screen with aperture:
  - plane wave hits the aperture in a hard screen
  - sound pressure field behind the screen?

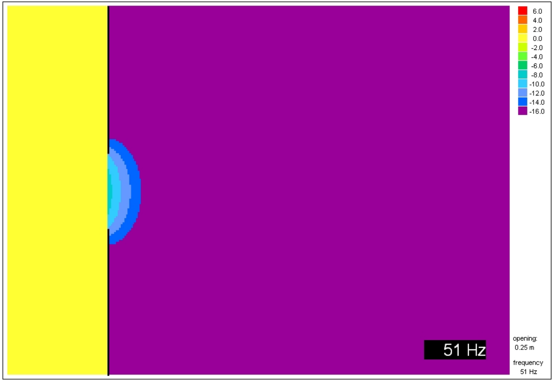


Kirchhoff's approximations: diffraction problems

- solution: application of the Rayleigh Integral
  - needed: sound particle velocity in the aperture
  - Kirchhoff's approximation:
    - assume sound particle velocity as if no screen is present
    - $\rightarrow$ ignore boundaries
    - error decreases with decreasing ratio wavelength / diameter

Kirchhoff's approximations: diffraction problems

example: sound field of a plane wave behind an aperture of 25 cm diameter



sound field behind aperture with Kirchhoff's aproximation

Kirchhoff's approximations: diffraction problems

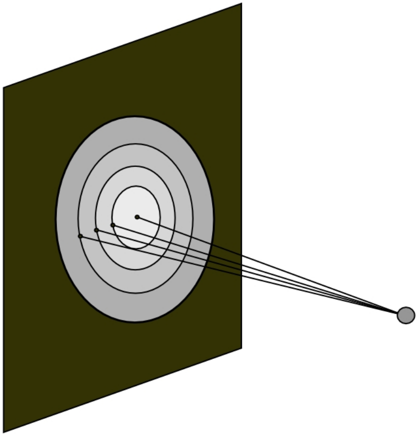
Rayleigh Integral:

$$\check{p}(x,y,z,\omega) = \frac{j\omega\rho_0}{2\pi}\int_S \check{v}_n(x,y,\omega)\frac{e^{-jkr}}{r}\mathrm{d}S$$

- approximation with Fresnel zones for receivers not too close:
  - ignore small changes of r
  - differentiate phase in classes + *(0 degrees)* and - *(180 degrees)* only
  - corresponding regions in the aperture: Fresnel zones

Kirchhoff's approximations: diffraction problems

Fresnel zones in case of circular aperture:



Kirchhoff's approximations: diffraction problems

$$p \sim \frac{A_1}{r_1} - \frac{A_2}{r_2} + \frac{A_3}{r_3} - \frac{A_4}{r_4} \ldots$$

A_i: area of the i-th Fresnel zone
r_i: average distance to the i-th Fresnel zone

for large apertures:

$$p \sim \frac{A_1}{2r_1}$$

if aperture $=$ 1. Fresnel zone $\rightarrow$ amplification of $+6$ dB re. free field

Fresnel zones for reflection problems

- reflection at inhomogeneous or finite surfaces:
  - half of the 1. Fresnel zone defines the relevant region on a reflector
  - concept allows for the estimation of situations with:
    - small reflectors $F < \frac{A_1}{2} \rightarrow p_{\mathrm{refl}} \approx \frac{2F}{A_1} p_{\mathrm{refl}\infty}$
    - inhomogeneous reflectors

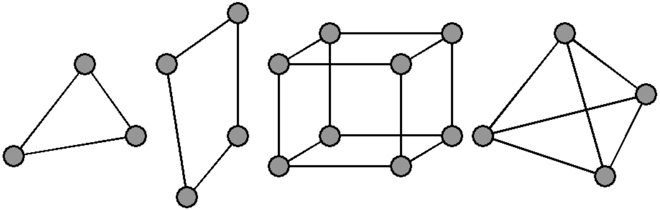
finite elements

finite elements

- common method to solve differential equations by discretization of the field volume
- well suited for:
  - bounded field regions such as vehicle interiors
  - coupled structure/fluid systems, e.g. simulation of airborne sound insulation in the laboratory
  - simulation of inhomogeneous properties of the medium (c, density)
- not well suited for:
  - radiation in unbounded space

finite elements

- discretization of the field volume in finite elements
- establish one equation per element and node
- assembly of the system of equations
- solve the system of equation for each frequency of interest



FDTD:
finite differences in the time domain

Finite Differences in the Time Domain (FDTD)

- standard method to find solutions of differential equations numerically
- usage of the fundamental acoustical partial differential equations in the time domain:
  - $grad(p) = -\rho \frac{\partial \vec{v}}{\partial t}$ Newton
  - $-\frac{\partial p}{\partial t} = \kappa P_0 \mathrm{div}(\vec{v})$ Poisson, mass conservation
- strategy:
  - discretisation of simulation domain in space and time
  - replacement of derivatives by differences (space and time)
    - $\rightarrow$ updating equations in time

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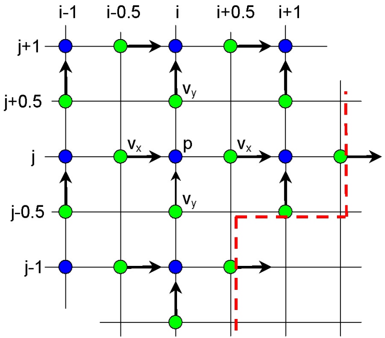
finite differences in the time domain (FDTD)

2D-formulation:



$$
\begin{aligned}
v_x^{\text{new}} &= v_x^{\text{old}} - \alpha \left(p_{\text{right}} - p_{\text{left}} \right) \\
p^{\text{new}} &= p^{\text{old}} - \beta \left(v_{x\text{right}} - v_{x\text{left}} \right) - \beta \left(v_{y\text{top}} - v_{y\text{bottom}} \right)
\end{aligned}
$$

finite differences in the time domain (FDTD)

- typical simulation / calculation:
  - impulse-like pressure distribution as starting condition
  - time-stepwise updating of the field variables at the grid points
- advantages:
  - no system of equation that has to be solved
  - impulse response as a result contains information about all frequencies
- disadvantage:
  - implementation of frequency domain boundary conditions is not straight forward

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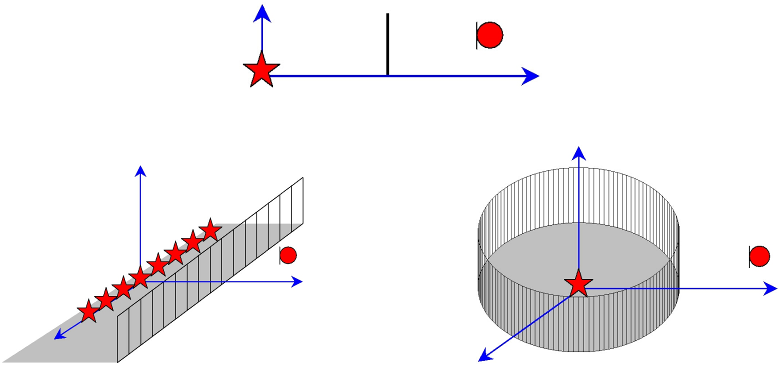
finite differences in the time domain (FDTD)

- computational effort:
  - 2D-simulation of a region of 200 m $\times$ 40 m
  - $f_{\mathrm{max}} = 2$ kHz $\rightarrow$ discretization in space: 0.02 m
  - mesh size 10'000 $\times$ 2'000 $= 20 \cdot 10^6$ grid points
  - calculation time $\rightarrow$ one hour

2-/3-D simulations

- mapping of 3-dimensional geometries onto 2 independent coordinates:
  - translation invariant situation
  - rotation invariant situation

2-/3-D simulations



2-/3-D simulations

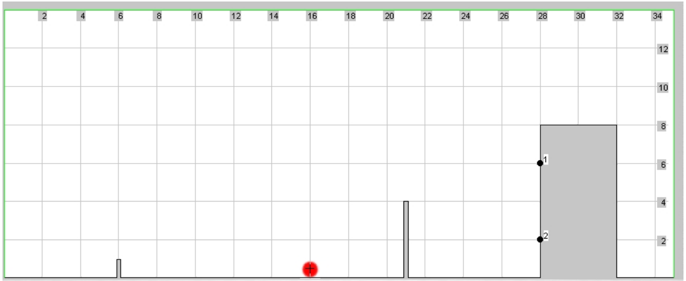
- translation invariant situation
  - cartesian coordinate system
  - situation geometry does not change in y-direction
  - all derivatives of the sound field equations with respect to y-direction are set to 0
  - simulated source = coherent line source with extension in y-direction
  - coherent - incoherent line source??

2-/3-D simulations

- rotation invariant situation
  - cylindrical coordinate system
  - situation geometry does not change with angle ϕ
  - all derivatives of the sound field equations with respect to ϕ-direction are set to 0
  - simulated source = point source in the origin
  - caution: reflections lead to focusing effects at the source position $\rightarrow$ only strictly propagating waves allowed

finite differences in the time domain (FDTD)

example: road traffic situation



road traffic noise situation

reflection at noise barrier

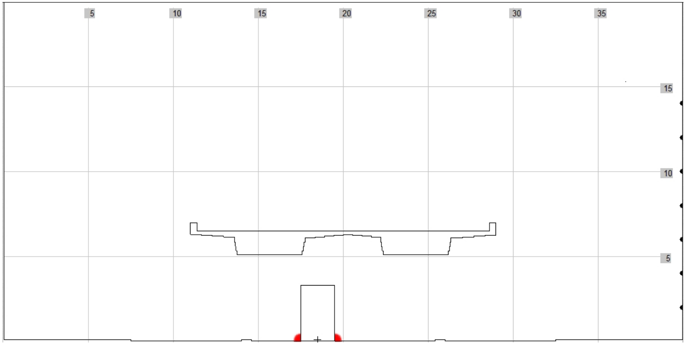
reflection at noise barrier

finite differences in the time domain (FDTD)

example: Hardbrücke, effect of absorbing layer at the bottom of bridge

finite differences in the time domain (FDTD)

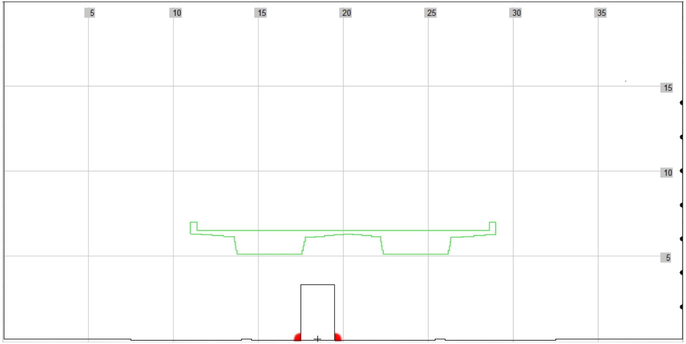
reflecting bridge:



Hardbrücke - reflecting

finite differences in the time domain (FDTD)

absorbing bridge:



Hardbrücke - absorbing

finite differences in the time domain (FDTD)

example: railway line cutting

finite differences in the time domain (FDTD)

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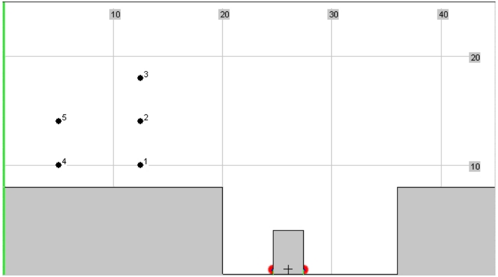

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acoustical holography

acoustical holography

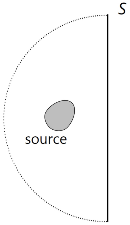
- Kirchhoff-Helmholtz integral is valid for arbitrary surfaces

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_S \left(j\omega\rho_0 \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) \mathrm{d}S$$

- for specifically designed surfaces further simplifications are possible

acoustical holography

for a plane S that closes in infinity



sound pressure in the right half space is given as:

$$\check{p}(x,y,z,\omega) = j \int_S \check{p}_S(\omega) \cos\phi \left(1 - \frac{j}{kr}\right) \frac{e^{-jkr}}{\lambda r} \mathrm{d}S$$

acoustical holography

- equation from above describes $\check{p}$ in 3D-space by a $\check{p}$ representation on a 2D-plane
- $\rightarrow$ principle of holography
- holography in practical applications:
  - simultaneous determination of sound pressure distribution (amplitude and phase) at discrete grid points on a suitable plane
  - usage of microphone arrays
  - sequential sampling by using a fixed reference (phase)
  - $\rightarrow$ complete information about the 3D field

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