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### ETH

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

# Acoustics I: sound field calculations

Reto Pieren 2024

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calculation of a situation specific location- and time dependent sound field (often p)

- conditions for a valid solution:
  - fulfillment of the wave equation or Helmholtz equation
  - fulfillment of the boundary conditions
    - sources
      - boundaries (borders of space)
- analytical solutions for special geometries only
- numerical solutions in the general case:
  - finite elements
  - boundary elements
  - time domain methods such as FDTD

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- ightharpoonup calculation of a situation specific location- and time dependent sound field (often p)
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## Kirchhoff - Helmholtz integral

## Kirchhoff - Helmholtz integral

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### Green's theorem:

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_{S} \left( j\omega \rho_0 \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS$$

S: closed surface

 $\check{v}_S$ : sound particle velocity on and normal to S

 $\check{p}_S$ : sound pressure on S

r: distance of the surface point to the receiver point (x, y, z)

lacktriangle Kirchhoff-Helmholtz integral ightarrow wave field synthesis

## Kirchhoff - Helmholtz integral

Kirchhoff -Helmholtz integral

- Kirchhoff-Helmholtz integral KHI is valid:
  - in the interior of S
  - in the exterior of S
  - on the surface S with a correction factor of 2

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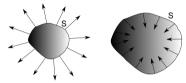
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## Boundary Elements Method

## **Boundary Elements Method**

Boundary Elements Method

### typical radiation problem:



- surface velocity is given as boundary condition
- search for sound pressure field inside or outside of S
- solution with the Boundary Elements Method:
  - discretisation of the radiator surface in n elements
  - with KHI:  $p_{S,i} = \sum_{i=1}^{n} f(p_{S,i}, v_{S,i})$
  - $\triangleright$  solve the system of equations with *n* unknowns  $\rightarrow p_{S,i}$
  - calculate sound pressure at any point in space with the KHI

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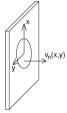
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## Rayleigh Integral

## Rayleigh Integral

### Rayleigh Integral

- $\triangleright$  radiation of an oscillating piston  $\rightarrow$  Kirchhoff-Helmholtz Integral
- special case: oscillating piston mounted in a large and rigid wall
  - $\triangleright$  wall introduces boundary condition:  $v_n = 0$



## Rayleigh Integral

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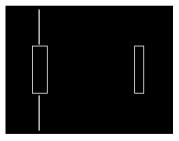
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- replace the effect of the wall by a mirror source
- lacktriangle oscillating piston ightarrow pulsating piston

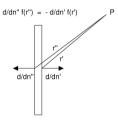


## Rayleigh Integral

Rayleigh Integral

evaluation of the Kirchhoff Helmholtz Integral:

$$\check{p}(x, y, z, \omega) = \frac{1}{4\pi} \int_{S} \left( j\omega \rho_0 \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS$$



contribution of sound pressure = 0!

## Rayleigh Integral

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Kirchhoff Helmholtz Integral simplifies to the Rayleigh Integral:

$$\check{p}(x, y, z, \omega) = \frac{j\omega\rho_0}{2\pi} \int_{S} \check{v}_n(x, y, \omega) \frac{e^{-jkr}}{r} dS$$

S: visible piston surface (front)  $v_n$ : piston velocity

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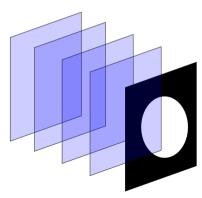
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## Kirchhoff's approximations: diffraction problems

- screen with aperture:
  - plane wave hits the aperture in a hard screen
  - sound pressure field behind the screen?



## Kirchhoff's approximations: diffraction problems

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- solution: application of the Rayleigh Integral
  - needed: sound particle velocity in the aperture
  - Kirchhoff's approximation:
    - assume sound particle velocity as if no screen is present
    - ightharpoonup ightharpoonup ignore boundaries
    - error decreases with decreasing ratio wavelength / diameter

## Kirchhoff's approximations: diffraction problems

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example: sound field of a plane wave behind an aperture of 25 cm diameter

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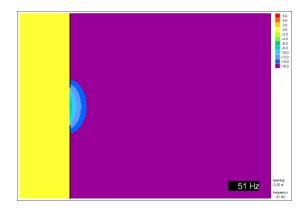
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sound field behind aperture with Kirchhoff's aproximation

## Kirchhoff's approximations: diffraction problems

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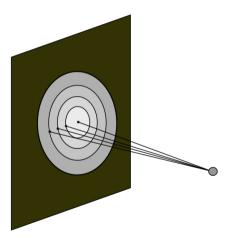
### Rayleigh Integral:

$$\check{p}(x,y,z,\omega) = \frac{j\omega\rho_0}{2\pi} \int_{S} \check{v}_n(x,y,\omega) \frac{e^{-jkr}}{r} dS$$

- approximation with Fresnel zones for receivers not too close:
  - ignore small changes of *r*
  - ▶ differentiate phase in classes + (0 degrees) and (180 degrees) only
  - corresponding regions in the aperture: Fresnel zones

## Kirchhoff's approximations: diffraction problems

Fresnel zones in case of circular aperture:



Kirchhoff's approximations

## Kirchhoff's approximations: diffraction problems

Kirchhoff's approximations

$$p \sim \frac{A_1}{r_1} - \frac{A_2}{r_2} + \frac{A_3}{r_3} - \frac{A_4}{r_4} \dots$$

A<sub>i</sub>: area of the *i*-th Fresnel zone

r<sub>i</sub>: average distance to the i-th Fresnel zone

for large apertures:

$$p\sim \frac{A_1}{2r_1}$$

if aperture = 1. Fresnel zone  $\rightarrow$  amplification of +6 dB re. free field

## Fresnel zones for reflection problems

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- reflection at inhomogeneous or finite surfaces:
  - half of the 1. Fresnel zone defines the relevant region on a reflector
  - concept allows for the estimation of situations with:
    - ▶ small reflectors  $F < \frac{A_1}{2} \to p_{\mathsf{refl}} \approx \frac{2F}{A_1} p_{\mathsf{refl}\infty}$
    - inhomogeneous reflectors

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## finite elements

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- common method to solve differential equations by discretization of the field volume
- well suited for:
  - bounded field regions such as vehicle interiors
  - coupled structure/fluid systems, e.g. simulation of airborne sound insulation in the laboratory
  - ightharpoonup simulation of inhomogeneous properties of the medium (c, density)
- not well suited for:
  - radiation in unbounded space

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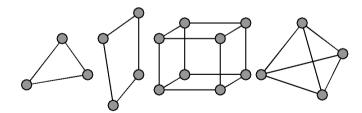
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### finite elements

- discretization of the field volume in finite elements
- establish one equation per element and node
- assembly of the system of equations
- solve the system of equation for each frequency of interest



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## FDTD:

finite differences in the time domain

## Finite Differences in the Time Domain (FDTD)

### **FDTD**

- standard method to find solutions of differential equations numerically

$$ightharpoonup$$
 grad $(p) = -\rho \frac{\partial v}{\partial t}$ 

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standard method to find solutions of differential equations numerically

usage of the fundamental acoustical partial differential equations in the time domain:

• 
$$grad(p) = -\rho \frac{\partial \vec{v}}{\partial t}$$

Newton

$$-\frac{\partial p}{\partial t} = \kappa P_0 \operatorname{div}(\vec{v})$$

Poisson, mass conservation

- strategy:
  - discretisation of simulation domain in space and time
  - replacement of derivatives by differences (space and time)

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- standard method to find solutions of differential equations numerically
- usage of the fundamental acoustical partial differential equations in the time domain:

• 
$$grad(p) = -\rho \frac{\partial \vec{v}}{\partial t}$$

Newton

 $-\frac{\partial p}{\partial t} = \kappa P_0 \operatorname{div}(\vec{v})$ 

Poisson, mass conservation

- strategy:
  - discretisation of simulation domain in space and time
  - replacement of derivatives by differences (space and time)
    - ightharpoonup updating equations in time

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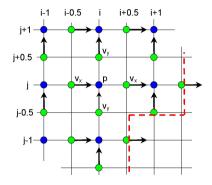
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## finite differences in the time domain (FDTD)

### 2D-formulation:



$$v_x^{\text{new}} = v_x^{\text{old}} - \alpha (p_{\text{right}} - p_{\text{left}})$$
 $p^{\text{new}} = p^{\text{old}} - \beta (v_{\text{xright}} - v_{\text{xleft}}) - \beta (v_{\text{ytop}} - v_{\text{ybottom}})$ 

## finite differences in the time domain (FDTD)

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- typical simulation / calculation:
  - impulse-like pressure distribution as starting condition
  - time-stepwise updating of the field variables at the grid points
- advantages:
  - no system of equation that has to be solved
  - impulse response as a result contains information about all frequencies
- disadvantage:
  - implementation of frequency domain boundary conditions is not straight forward

## finite differences in the time domain (FDTD)

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## finite differences in the time domain (FDTD)

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  - no system of equation that has to be solved
  - impulse response as a result contains information about all frequencies
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# finite differences in the time domain (FDTD)

- computational effort:
  - $\triangleright$  2D-simulation of a region of 200 m  $\times$  40 m
  - $f_{\text{max}} = 2 \text{ kHz} \rightarrow \text{discretization in space: } 0.02 \text{ m}$
  - ightharpoonup mesh size  $10'000 \times 2'000 = 20 \cdot 10^6$  grid points
  - ightharpoonup calculation time  $\rightarrow$  one hour

# 2-/3-D simulations

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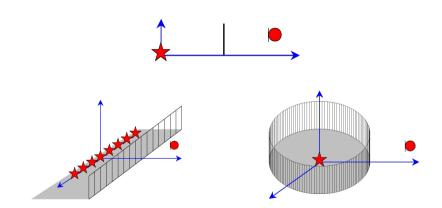
example: railway line

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- ▶ mapping of 3-dimensional geometries onto 2 independent coordinates:
  - translation invariant situation
  - rotation invariant situation

# 2-/3-D simulations

#### **FDTD**



# 2-/3-D simulations

**FDTD** 

- translation invariant situation
  - cartesian coordinate system
  - situation geometry does not change in y-direction
  - lack all derivatives of the sound field equations with respect to y-direction are set to 0
  - $\triangleright$  simulated source = coherent line source with extension in v-direction
  - coherent incoherent line source??

# 2-/3-D simulations

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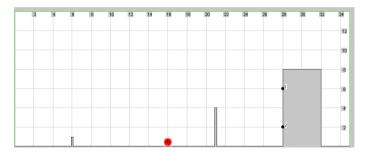
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- rotation invariant situation
  - cylindrical coordinate system
  - lacktriangle situation geometry does not change with angle  $\phi$
  - $\blacktriangleright$  all derivatives of the sound field equations with respect to  $\phi\text{-direction}$  are set to 0
  - simulated source = point source in the origin
  - lacktriangle caution: reflections lead to focusing effects at the source position ightarrow only strictly propagating waves allowed

# finite differences in the time domain (FDTD)

example: road traffic situation

## example: road traffic situation



road traffic noise situation

## reflection at noise barrier

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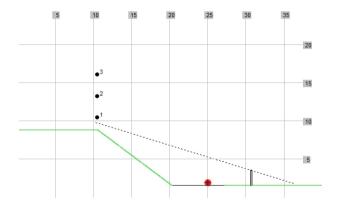
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#### noise barrier

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reflection at noise barrier

finite differences in the time domain (FDTD)

example: Hardbrücke, effect of absorbing layer at the bottom of bridge



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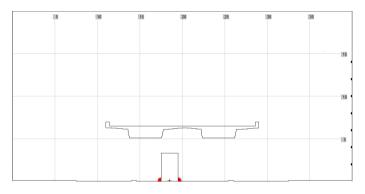
Hardbrücke example: railway lin

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# finite differences in the time domain (FDTD)

## reflecting bridge:

Hardbrücke

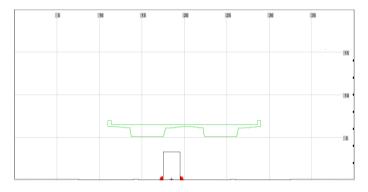


Hardbrücke - reflecting

# finite differences in the time domain (FDTD)

## absorbing bridge:

#### Hardbrücke



Hardbrücke - absorbing

# finite differences in the time domain (FDTD)

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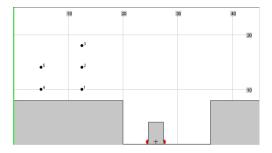
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example: railway line cutting



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acoustical holography

# acoustical holography

## acoustical holography

acoustical holography

Kirchhoff-Helmholtz integral is valid for arbitrary surfaces

$$\check{p}(x,y,z,\omega) = \frac{1}{4\pi} \int_{S} \left( j\omega \rho_0 \check{v}_S(\omega) \frac{e^{-j\omega r/c}}{r} + \check{p}_S(\omega) \frac{\partial}{\partial n} \frac{e^{-j\omega r/c}}{r} \right) dS$$

▶ for specifically designed surfaces further simplifications are possible

## acoustical holography

for a plane S that closes in infinity

acoustical holography



sound pressure in the right half space is given as:

$$\check{p}(x, y, z, \omega) = j \int_{S} \check{p}_{S}(\omega) \cos \phi \left(1 - \frac{j}{kr}\right) \frac{e^{-jkr}}{\lambda r} dS$$

## acoustical holography

acoustical holography

- $\triangleright$  equation from above describes  $p \neq 0$  in 3D-space by a  $p \neq 0$  representation on a 2D-plane
- ightharpoonup ightharpoonup principle of holography
- holography in practical applications:
  - simultaneous determination of sound pressure distribution (amplitude and phase) at discrete grid points on a suitable plane
  - usage of microphone arrays
  - sequential sampling by using a fixed reference (phase)
  - ightharpoonup ightharpoonup complete information about the 3D field

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