ETH
Eidgenössische Technische Hochschule Zürich
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Acoustics I: measurements

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measurement tasks

typical measurement tasks

- emission measurements (passive)
  - description of a sound source
  - → sound radiation of a lawn mover
- measurements at a receiver position (passive)
  - description of the strength of a source including the propagation to the receiver
  - → road traffic noise measurement in the living room of a resident
- measurements of a transmission system (often active)
  - description of a transmission system
  - → measurement of the frequency response of a loudspeaker
  - → measurement of the impulse response in a concert hall

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typical measurement tasks

- complete description of a sound field requires:
    - sound pressure at all positions for each moment in time
    - sound particle velocity at all positions for each moment in time
    - however: $grad(p) = -\rho \frac{\partial \vec{v}}{\partial t}$ may be used
- most important quantity: sound pressure
    - ear is sensitive to sound pressure
    - excellent transducers are available

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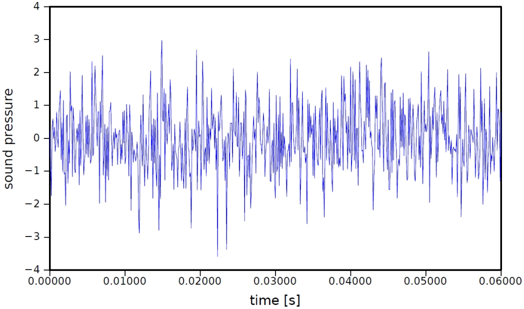
signal attributes

signal attributes

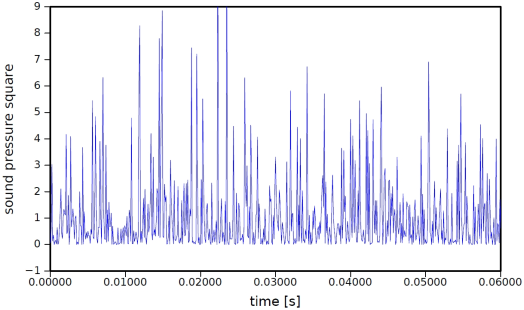
example of a typical sound pressure time history:



- attributes that can be evaluated?

signal attributes

- first step:
  - calculate signal square



signal attributes

attributes / indicators in the squared sound pressure time history:

- peak value
- integral quantities
  - infinite time window with exponential weighting → moving average
  - finite time window → average value
- statistical quantities, e.g. the fraction of the signal duration with sound pressure exceeding a certain threshold

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signal attributes: integral quantities

momentary sound pressure level $L(t)$

$$L(t) = 10\log\left(\frac{1}{RC}\int\limits_{-\infty}^{t}\frac{p^2(\tau)}{p_0^2}e^{\frac{\tau-t}{RC}}\,\mathrm{d}\tau\right) \qquad [\mathrm{dB}]$$

where

RC: time constant, $SLOW = 1$ s, $FAST = 0.125$ s

$p(\tau)$: instantaneous sound pressure

p_0: reference sound pressure $= 2 \times 10^{-5}$ Pa

signal attributes: integral quantities

Equivalent continuous sound pressure level *Leq*

$$Leq = 10 \log \left(\frac{1}{T} \int_0^T \frac{p^2(\tau)}{p_0^2} \mathrm{d}\tau \right) \qquad [\mathrm{dB}]$$

where

T: measurement time interval

$p(\tau)$: instantaneous sound pressure

p_0: reference sound pressure $= 2 \times 10^{-5}$ Pa

signal attributes: integral quantities

Sound exposure level L_E (SEL former designation)

$$L_E = 10\log\left(\frac{1}{1\ \text{sec}}\int_0^T \frac{p^2(\tau)}{p_0^2}\mathrm{d}\tau\right) \qquad [\text{dB}]$$

where

T: measurement time interval

$p(\tau)$: instantaneous sound pressure

p_0: reference sound pressure $= 2\times10^{-5}$ Pa

signal attributes: applications

momentary sound pressure level $L(t)$:

- maximum level with time constant FAST: $Lmax, Fast \rightarrow$ descriptor for shooting noise or the pass-by of road vehicles
- minimum level: $Lmin \rightarrow$ estimation of a stationary signal with occurrence of transient disturbing noise

signal attributes: applications

equivalent continuous sound pressure level Leq:

- characterization of non-stationary sources and signals

signal attributes: applications

sound exposure level L_E, *SEL*:

- measurement of single events such as e.g. train pass-bys

signal attributes: applications

→ software sound level meter (Delphi) demo

filters

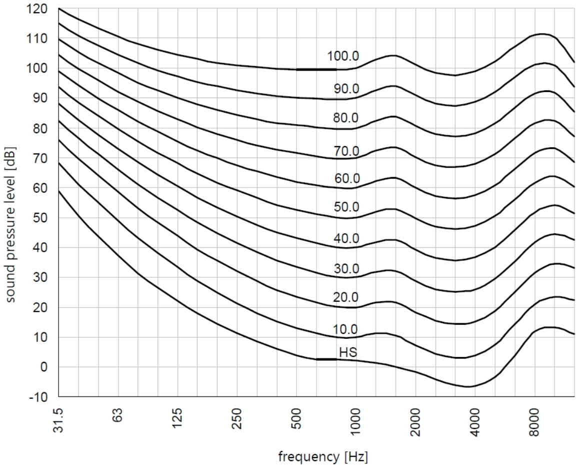
filters

- frequency weighting filters (to mimick frequency response of the ear)
  - A-filter
  - C-filter
- bandpass filters for frequency analysis
  - constant relative bandwidth (perception related)
    - third-octave band filters
    - octave band filters
  - constant absolute bandwidth (technical analysis)
    - narrow band filters (e.g. 1 Hz, 3 Hz, 10 Hz)
    - FFT

frequency weighting filters

frequency weighting filters

equal loudness contours:



level dependent frequency response → A- / C-filter → dB(A) / dB(C)

frequency weighting filters

standard C-filter:

$$T_{\text{C}-\text{Filter}}(s) = \frac{Ks^2}{(s+\omega_1)^2(s+\omega_2)^2}$$

where

$\omega_1 = 1.29 \times 10^2$ [rad/sec]

$\omega_2 = 7.67 \times 10^4$ [rad/sec]

K: adjusted for $|T_{\text{C}-\text{filter}}| = 1$ at 1 kHz

frequency weighting filters

standard A-filter:

$$T_{\mathrm{A-Filter}}(s) = \frac{Ks^4}{(s+\omega_1)^2(s+\omega_2)^2(s+\omega_3)(s+\omega_4)}$$

where

$\omega_1 = 1.29 \times 10^2$ [rad/sec]

$\omega_2 = 7.67 \times 10^4$ [rad/sec]

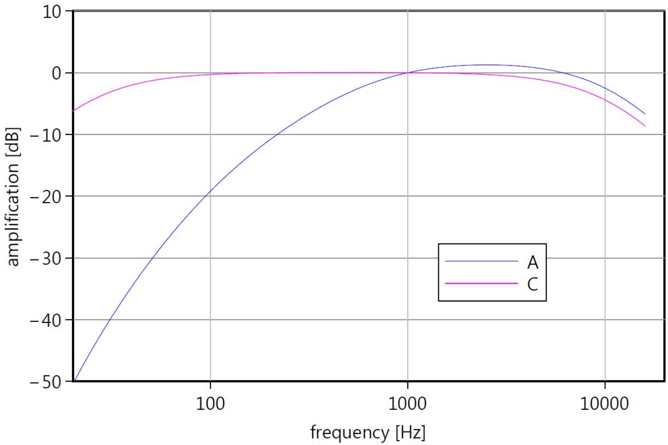
$\omega_3 = 6.77 \times 10^2$ [rad/sec]

$\omega_4 = 4.64 \times 10^3$ [rad/sec]

K: adjusted for $|T_{\mathrm{A-filter}}| = 1$ at 1 kHz

frequency weighting filters

frequency response curves:



bandpass filters:
filters for frequency analysis

frequency analysis by third-octave band filters

- standard filter series with one center frequency f_m at 1000 Hz
- $f_{\mathrm{m},n} = 1000 \cdot (2^{\frac{1}{3}})^n$ for $n = \dots - 2, -1, 0, 1, 2, \dots$
- $B = 0.23 \cdot f_m$, B: bandwidth
- $f_u = f_m \cdot \frac{1}{2^{\frac{1}{6}}}$, f_u: lower limiting frequency
- $f_o = f_m \cdot 2^{\frac{1}{6}}$, f_o: upper limiting frequency

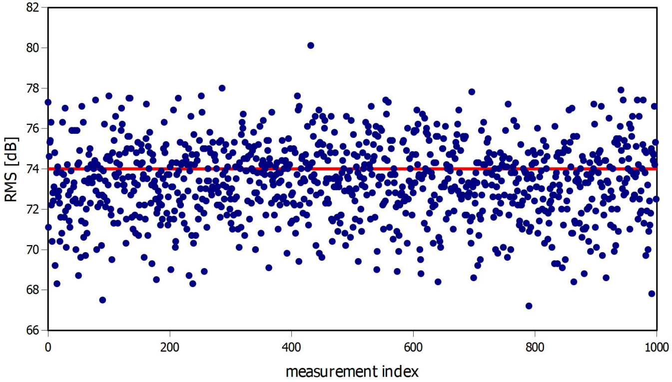
frequency analysis by octave band filters

- standard filter series with one center frequency f_m at 1000 Hz
- $f_{\mathrm{m},n} = 1000 \cdot 2^n$ for $n = \dots - 2, -1, 0, 1, 2, \dots$
- $B = 0.71 \cdot f_m$, B: bandwidth
- $f_u = f_m \cdot \frac{1}{2^{\frac{1}{2}}}$, f_u: lower limiting frequency
- $f_o = f_m \cdot 2^{\frac{1}{2}}$, f_o: upper limiting frequency

uncertainty of measurements of random signals

measurement uncertainty of random signals

example: 0.5 sec-Leq analysis of pink noise evaluated in a band of 10 Hz:



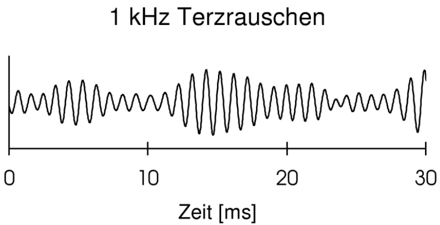
measurement uncertainty of random signals

- frequency band limited analysis of a random signal is inevitably uncertain for finite observation intervals

measurement uncertainty of random signals

- band limitation → subsequent samples are no longer statistically independent
  - neighbor sample does not provide completely new information
- the narrower the analysis bandwidth, the longer the period to wait for a new independent sample

example: 1 kHz third-octave band noise:



measurement uncertainty of random signals

degrees of freedom of a bandlimited random signal:

- random signal
  - bandwidth B
  - analyzing time T
- yields n statistically independent samples

$$n = 2BT$$

- n: degrees of freedom

measurement uncertainty of random signals

given a random signal $u(t)$:

- analysis within the bandwidth B during time T
- $\rightarrow n = 2BT$ statistically independent samples u_i

$$\text{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} u_i^2}$$

- $\sum_{i=1}^{n} u_i^2$ is χ^2 distributed

measurement uncertainty of random signals

error intervals for selected probabilities p:

n	$p = 0.90$	$p = 0.99$
10	$-4.0 \cdots + 2.6$ dB	$-6.6 \cdots + 4.0$ dB
100	$-1.1 \cdots + 0.9$ dB	$-1.7 \cdots + 1.5$ dB
1000	$-0.3 \cdots + 0.3$ dB	$-0.5 \cdots + 0.5$ dB

measurement instruments

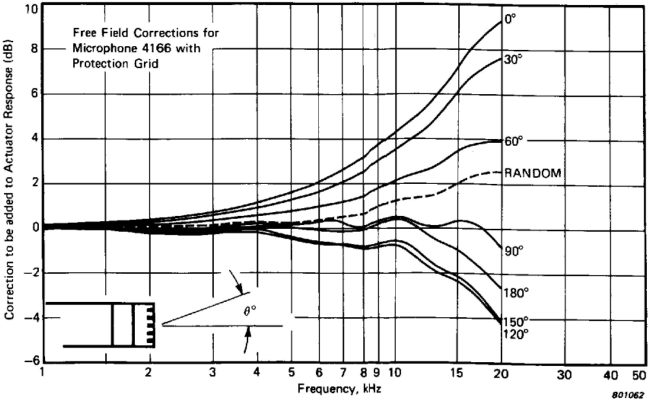
microphones

microphones

- standard sensor: electrostatic sound pressure microphone
- directivity:
  - omnidirectional at low frequencies
  - amplification on axis for high frequencies (sound field distortion)
  - corrections:
    - free field microphones
    - pressure response microphones

microphones: directivity

example: sensitivity as a function of θ for a $1/2$ inch capsule:



microphones

further specifications:

- upper limiting frequency
  - maximal for small membrane area
  - maximal for high membrane stiffness
- self-noise
  - minimal for large membrane area
  - minimal for low membrane stiffness
- maximal sound pressure
  - maximal for small membrane area
  - maximal for high membrane stiffness

microphones

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- maximal sound pressure
  - maximal for small membrane area
  - maximal for high membrane stiffness

microphones

examples:

microphone Ø inch = 2.5cm	dynamic range dB(A)	frequency range Hz
1	10. . .146	2 . . . 18’000
1/2	15. . .146	2 . . . 20’000
1/4	29. . .164	2 . . .100’000

calibrators

calibrators

- microphone calibrators are sound pressure references
- constructions:
    - pistonphone
        - pressure generation by moving pistons
        - operation typically at 250 Hz, 124 dB ($\pm$ 0.15 dB)
        - correction needed for atmospheric pressure (density of air)!
    - acoustical calibrator
        - pressure generation by small loudspeaker
        - operation typically at 1000 Hz, 94 or 114 dB ($\pm$ 0.30 dB)

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calibrators

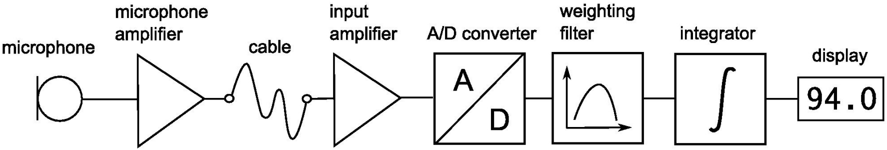
sound level meter

sound level meter

- measurement of sound pressure and evaluation of various signal attributes
  - momentary level, RC = Fast, Slow
  - maximum / minimum level
  - equivalent continuous sound pressure level Leq
  - event level L_E, SEL
  - evaluation in frequency bands or with frequency weighting A/C

sound level meter

block diagram:



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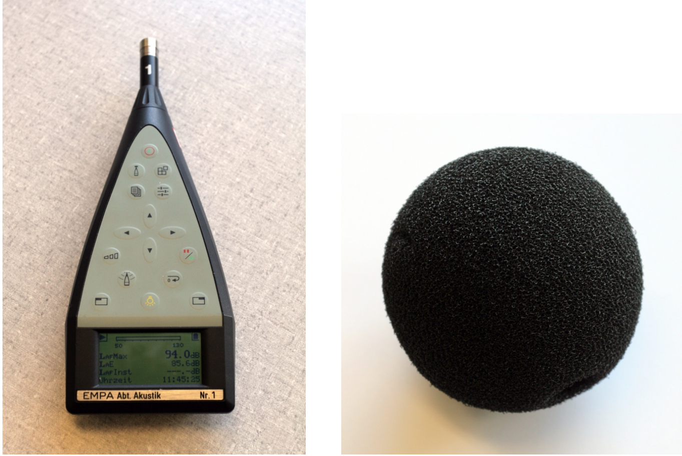


sound level meter and wind screen

sound level meter: precision classes

- IEC specifications for sound level meters:
  - class 1: precise instruments for field applications
  - class 2: survey instruments with larger tolerances
- requirements for measurements relating to Swiss noise legislation:
  - instrument is type approved by *METAS*: Federal Institute of Metrology
  - initial calibration by METAS
  - recalibration every two years

level recorder

level recorder

- registration of the time history of sound pressure level
- instrument specs:
    - typ. dynamic range 80. . .120 dB
    - typ. temporal resolution: 1 ms
    - data available for further evaluations

analyzers for level statistics

analyzers for level statistics

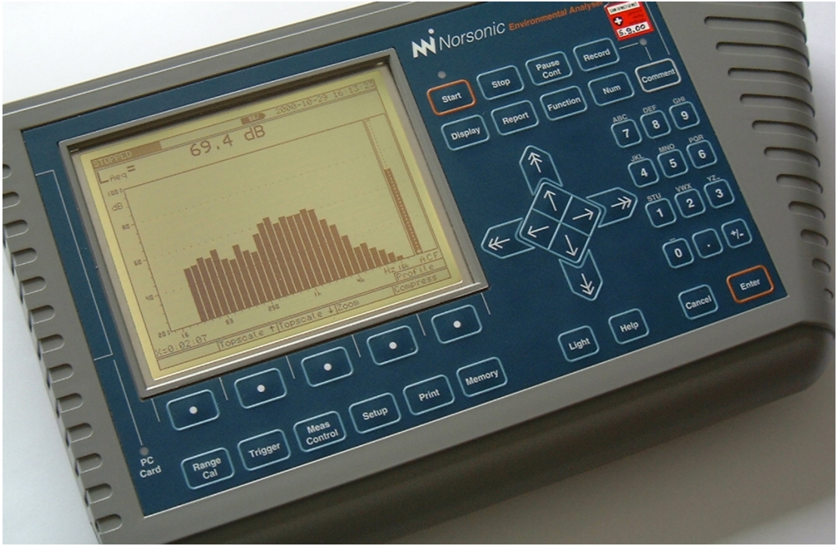
- evaluation of attributes in level statistics, such as:
    - $L1$: level, that is exceeded during 1 % of the measuring time
    - $L50$: level, that is exceeded during 50 % of the measuring time
- statistical levels play a minor role

frequency analyzers

frequency analyzers

- frequency dependent evaluation of the signal attributes
- most widely used: third-octave band analyzers
- today:
  - *handheld*
  - operating in realtime: two-channels from 20 Hz. . .20 kHz

frequency analyzers



sound recorders

sound recorders

- recording of the microphone signal for subsequent analysis
- digital recorders:
  - stand-alone hard-disc recorder
  - audio interface with PC
- formats with data compression (perceptual coders) do not make sense for measuring purposes

reliability of acoustical measurements

reliability of acoustical measurements

discussion of uncertainties in the following measurement task:

- determination of the yearly average sound pressure level on a building facade due to road traffic
  - how to do that?
  - sources of uncertainty?

reliability of acoustical measurements

uncertainties checklist:

- is the source under investigation in a representative condition?
- is the propagation medium in a representative condition?
- is the microphone surrounding representative for the location of interest?
  - local reflections?
  - local screening effects?
- errors due to the calibration and tolerances of the instrument
- errors due to possible unwanted noise
- fundamental uncertainty in the evaluation of random signals

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reliability of acoustical measurements

declaration of results:

- e.g. 63.2 dB(A) ± 0.9 dB(A)
    - number of decimals of a level result should reflect its uncertainty
    - standard uncertainty represents the range for a 66 % probability

special measurements

intensity measurement

intensity measurement

- sound intensity is a vector quantity where $|\vec{I}| = \overline{p(t)\,|\vec{v}(t)|}$:
  - sound pressure → no problem
  - sound particle velocity
    - pressure gradient method
    - hot-wire anemometer

intensity measurement: pressure gradient method

fundamental relation between sound particle velocity and sound pressure:

$$\rho_0 \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x}$$

or:

$$v_x = -\frac{1}{\rho_0} \int \frac{\partial p}{\partial x} \mathrm{d}t$$

- sound particle velocity can be estimated from pressure gradient

intensity measurement: pressure gradient method

approximation of pressure gradient by the pressure difference at two positions in close neighborhood:

$$\frac{\partial p}{\partial x} \approx \frac{p(x) - p(x + \Delta x)}{\Delta x}$$

- optimal choice of Δx?
- practical limitations?

intensity measurement: pressure gradient method: probe

intensity measurement: pressure gradient method: residual intensity

typical values for the **residual intensity** $(L_p - L_I)$ for a plane wave under 90° re. probe axis:

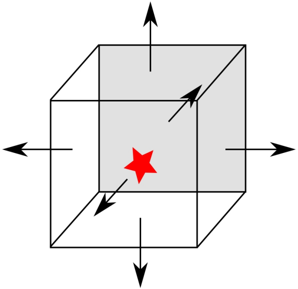
spacer	residual intensity
8.5 mm	$>$ 15 dB for 250 Hz .. 6.3 kHz
12 mm	$>$ 17 dB for 250 Hz .. 5 kHz
50 mm	$>$ 23 dB for 250 Hz .. 1.25 kHz

intensity probe

applications:

- sensor with accurate cosine(ϕ) directivity
- measurement of sound power with suppression of the contribution of unwanted sources

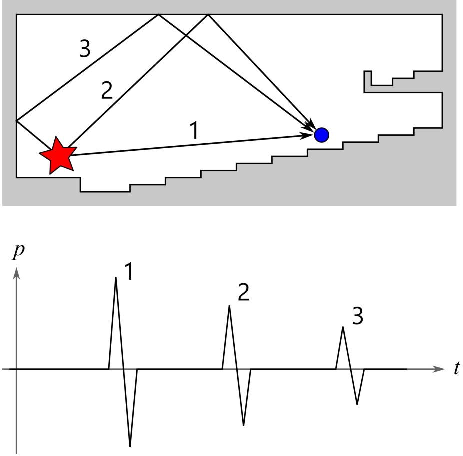
$$W = \int_S \vec{I}\mathrm{d}\vec{S} \qquad [W]$$



impulse response measurements

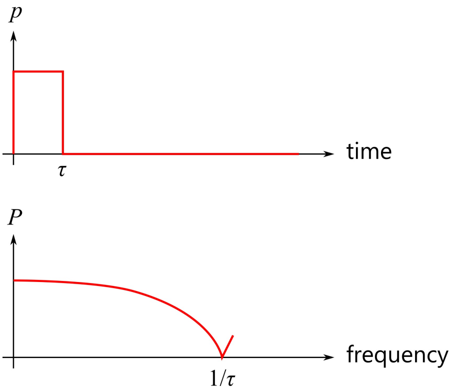
motivation

impulse response between a source and a receiver resolves different propagation paths:



difficulty

excitation impulse has to be short:



challenge:
how to excite the system with sufficient energy?

excitation

possible excitations:

- pistol shots (spectrum?, directivity?)
- bursting balloons (spectrum?, directivity?)
- electrical sparks (spectrum?, directivity?)

preferred choice:

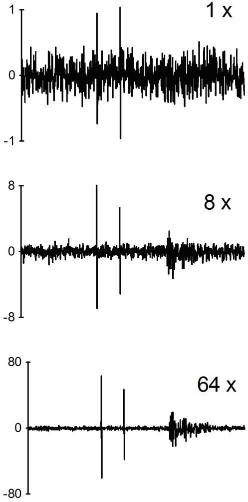
- loudspeakers (often poor S/N → trick?)

averaging process

- possible strategy to overcome poor S/N: *averaging*
- repetition of single impulse measurements (only for time-invariant systems!)

increase of wanted signal:	+6 dB per doubling
increase of noise:	+3 dB per doubling
S/N improvement:	+3 dB per doubling

averaging process: summation of several measurements



averaging process

- multiple averaging over single impulse measurements is very time consuming
    - wait until system response has dropped to sufficiently low values (in rooms: seconds)
    - averaging numbers $O(10'000)$ are needed
- correlation methods implement averaging processes very efficiently

correlation methods

correlation functions

- system input signal: $x(t)$
- system output signal: $y(t)$

auto-correlation function:

$$R_{xx}(\tau) = \frac{1}{2T} \int_{-T}^{+T} x(t-\tau)x(t)\mathrm{d}t \qquad (T \to \infty)$$

cross-correlation function:

$$R_{xy}(\tau) = \frac{1}{2T} \int_{-T}^{+T} x(t-\tau)y(t)\mathrm{d}t \qquad (T \to \infty)$$

Wiener-Hopf-equation

for a system with impulse response $h(t)$:

$$R_{xy}(\tau) = h(t) * R_{xx}(\tau)$$

if $R_{xx}(\tau)$ corresponds to a Dirac pulse, the relation simplifies to

$$R_{xy}(\tau) = h(t)$$

signals with dirac-like ACF?

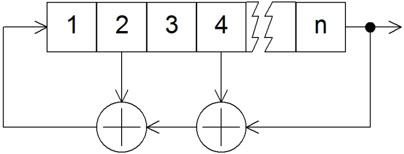
signals with dirac-like ACF

signals with dirac-like ACF:

- Dirac pulse
- white noise
- periodically continued maximum length sequences (pseudo random sequences)

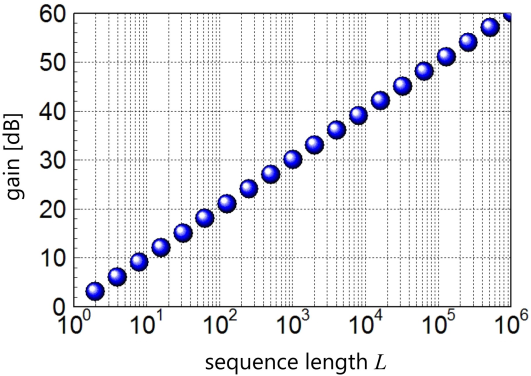
maximum length sequences

- two-valued signal (+1,-1) for lengths $L = 2^n - 1$ with (n: integer > 1)
- white spectrum
- optimal crest-factor = 1 (= 0 dB)
- fast algorithm (Hadamard transformation) for the correlation calculation
- generation with help of shift registers with appropriate feed-back



benefit

- correlation with sequence of length L corresponds to averaging L times a single impulse measurement
- gain or S/N improvement $= \log_2(L) \cdot 3$ dB



MLS Demo

MLS demo

application hints

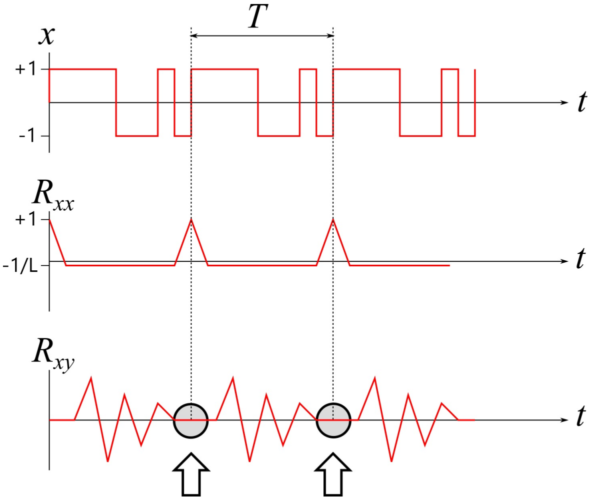
applications hints

to be considered in MLS measurements:

- periodicity of the maximum length sequence
- validity of time-invariance property of the system under test
- linearity of the system (inclusive excitation, e.g. loudspeaker)

periodicity of the ML sequence

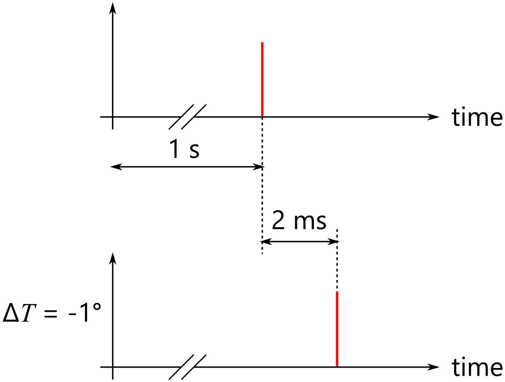
possible aliasing by the periodic repetition of the maximum length sequence:



time-invariance of the system

fundamental requirement for averaging: time-invariance of the system!

caution: e.g. temperature dependency of the speed of sound: $\Delta T = -1°$ $\rightarrow \Delta c = -0.6$ m/s

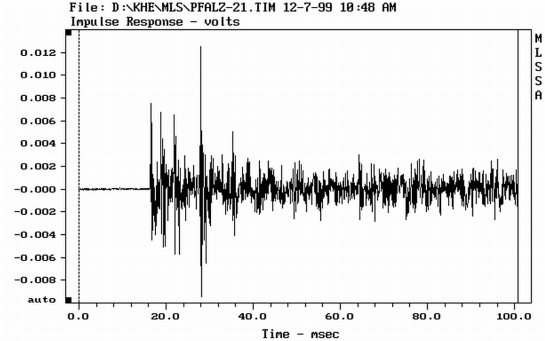


linearity of the system

- air and boundaries are linear in good approximation
- serious problem: exciting loudspeaker
- correlation process is extremely sensitive to nonlinear distortion
- results in increased background noise in the impulse response
- optimal S/N is obtained for moderate loudspeaker volume

evaluation of an impulse response measurement

- any disturbing noise shows up homogeneously distributed over the whole impulse response
- discussion of S/N is possible in a signal-free section: e.g. before the direct sound component



reverberation time measurements

measurement tasks
signal attributes
filters
frequency weighting filters
bandpass filters
measuring random signals
measurement instruments
reliability of measurements
special measurements
intensity measurement
impulse response measurements
reverberation time measurements
time - bandwidth uncertainty
back

reverberation time measurements

- reverberation time RT: decrease of energy density to 10^{-6} of the initial value after muting the exciting source
- RT is the most important global indicator for the acoustical characterisation of a room
- standard measurement:
  - pink noise excitation by a loudspeaker
  - recording of the sound pressure decay after muting the source
  - third-octave or octave band filtering
  - typical range for the evaluation: time T' for decay from -5 dB…-35 dB $\rightarrow RT = 2T'$

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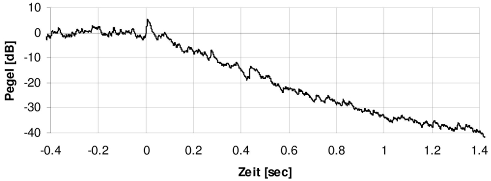
reverberation time measurements

- problem with the standard measurement:
    - randomness of the decay process → uncertainty in the evaluation



Schroeder inverse integration

- randomness in the decay curve asks for averaging over several measurements $s^2(t)$

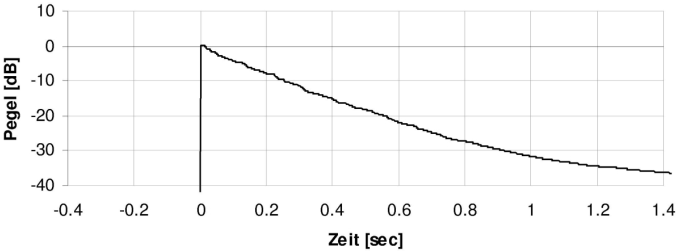
derived by Schroeder:

$$\langle s^2(t)\rangle \sim \int_t^\infty r^2(\tau)\mathrm{d}\tau$$

where
$\langle s^2(t)\rangle$: average of all possible decays of the squared time response
$r^2(t)$: squared impulse response of the room for the selected source and microphone position

Schroeder inverse integration

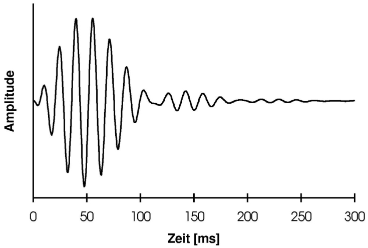
example Schroeder inverse integration:



reverberation time measurements

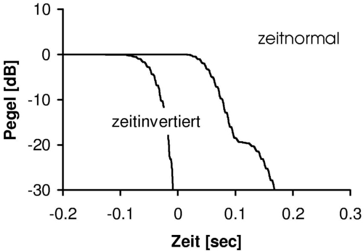
- evaluation of short reverberation times with small filter bandwidths:
  - narrow bandpass filters (lowest third octaves) → significant temporal broadening of the impulse
  - filter decay may be slower than room decay
  - necessary condition: $B \cdot T60 > 16$

e.g. impulse response of a 63 Hz third octave band filter:



reverberation time measurements

- impulse response of a bandpass filter is asymmetrical
- with respect to decay it is beneficial to invert time axis (frequency behavior is unaltered!):
  - digital filter with inverted time axis or
  - backwards play-back of recorded impulse responses
- $\rightarrow$ new relaxed condition: $B \cdot T60 > 4$



time - bandwidth uncertainty principle

time - bandwidth uncertainty

- bandpass filtering with bandwidth Δf of an impulse response results in a temporal broadening Δt (-3 dB points) where

$$\Delta t \cdot \Delta f \geq 0.5$$

time - bandwidth uncertainty

- challenge: find 'optimal' filter function for $\Delta t \cdot \Delta f = 0.5$:

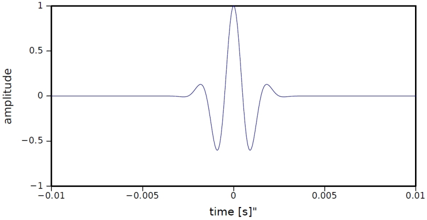
$$G(\omega) = 0.5\frac{\sqrt{\pi}}{\alpha}\left(e^{-(\omega+\omega_0)^2/4\alpha^2} + e^{-(\omega-\omega_0)^2/4\alpha^2}\right)$$

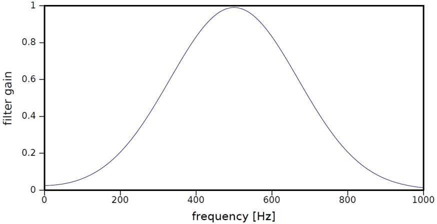
where
ω_0: filter center angular frequency in rad/s
$\alpha = \frac{\Delta\omega}{\sqrt{2\pi}}$
$\Delta\omega$: filter bandwidth in rad/s

time - bandwidth uncertainty

- impulse response of 'optimal' filter: Gabor pulse

$$g(t) = e^{-\alpha^2 t^2} \cos(\omega_0 t)$$





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