Fundamentals

introduction
- sound wave phenomena
- special topics in acoustics
- history of acoustics

basic quantities

basic equations
- wave equation
- speed of sound
- Helmholtz equation

types of waves
- plane waves
- spherical waves
- cylindrical waves
- superposition of point sources

reflection
- specular reflection
- diffuse Reflexion

Doppler effect

sonic boom

standing waves

diffraction

dB-scale

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Acoustics I: fundamentals

Kurt Heutschi
2022-12-12

introduction

introduction

- Acoustics: science of sound
  - generation of sound
  - propagation of sound
  - effect of sound on humans and matter
- Sound:
  - mechanical oscillation with wave-like propagation
  - propagation in air
  - propagation in liquids
  - propagation in solid bodies

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introduction

- frequency ranges:
  - infra-sound: $f < 20 Hz$
  - listening range of humans: $20 Hz < f < 20 kHz$
  - ultra-sound: $f > 20 kHz$

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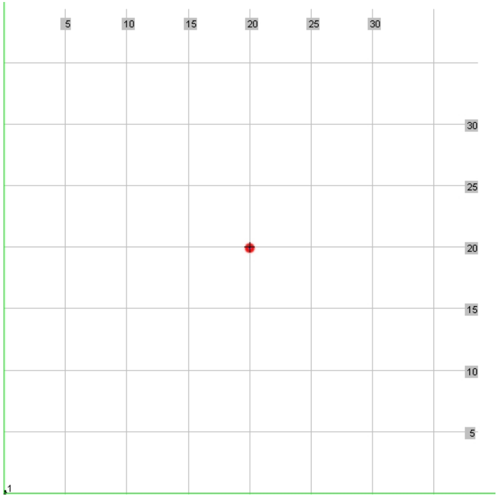
sound wave phenomena

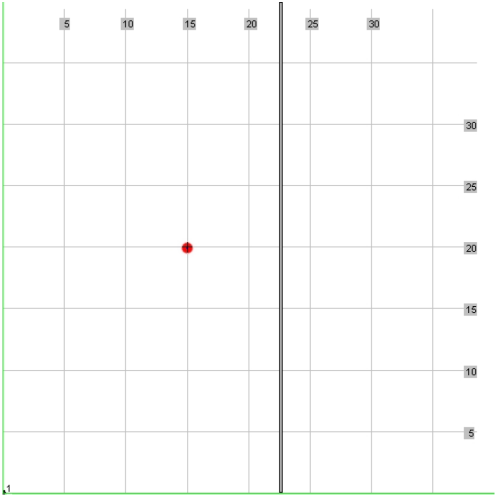
sound wave phenomena

- geometrical spreading
- reflection
- scattering
- diffraction
- interference

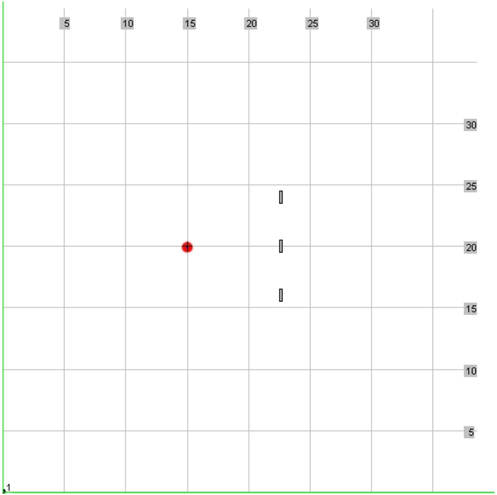
wave phenomena: geometrical spreading



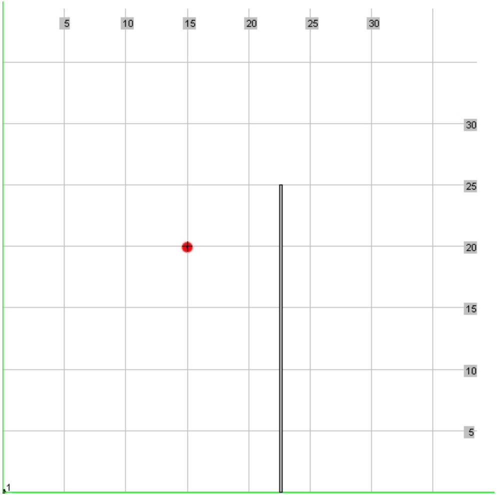
wave phenomena: reflection



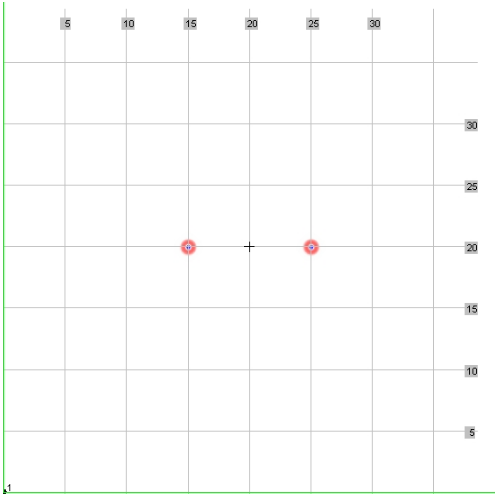
wave phenomena: reflection - scattering



wave phenomena: diffraction



wave phenomena: interference



special topics

special topics

Theoretical acoustics	analytical and numerical sound field calculations.
Nonlinear acoustics	investigation of non-linear effects that come along with very high amplitudes of the field quantities (e.g. explosions or sonic booms).
Underwater acoustics	sound propagation in water, sonar systems, seismic explorations.
Ultrasound	non-destructive test procedures for materials, medical applications.
Vibrations	vibrational behavior of bodies, sound radiation of vibrating structures.
Noise control	description and modeling of noise sources, investigations on noise protection measures.

special topics

Room acoustics assessment, planing and prediction of sound fields in rooms.

Building acoustics noise control in buildings, transmission loss of building structures.

Electroacoustics transducers (microphones, loudspeakers), recording devices, public address systems, signal processing in acoustics.

Acoustics of the ear structure of the ear, characteristics of the ear, perception and subjective evaluation of noise.

history of acoustics

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before Christ

Renaissance woodcut of Pythagoras and the first monochord

- 500 B.C.: Pythagoras: Begin of scientific acoustics:
    - experiments with vibrating strings
    - discovery of the relation between length of strings and pitch of the sound
    - establishment of a relation between numbers and musical intervals

around 0

- Vitruv: De architectura: 10 books for architects:
  - complete manual for the design and the construction of buildings
  - description of possible acoustical problems in theaters:
    - no proper direct sound supply in the audience
    - to much reverberation
    - discrete reflections (echoes)

17th century

- 1630: Marin Mersenne:
  - reliable measurement of speed of sound:
    - boom of canons for optical and acoustical signals
    - result: speed of sound independent of location and sound intensity = 450 m/s
  - quantitative relation between pitch and frequency:
    - experiments with vibrating strings
    - usage of relation: pitch $\sim \frac{1}{\text{stringLength}}$
    - usage of relation: pitch $\sim \sqrt{\text{tension}}$
    - down-scaling for visual inspection
- 1670: Christian Huygens: understanding of sound as a wave phenomenon
  - development of the concept of secondary sources

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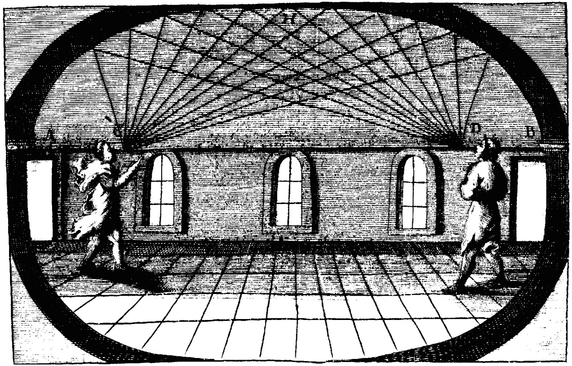
17th century

- 1673: Athanasius Kircher
  - introduction of rays as model of sound propagation in rooms
  - extended studies on the focussing effect of concave structures



18th century

- 1710: Isaac Newton
  - theoretical derivation of the speed of sound
  - value about 16 % too low due to wrong assumption of an isothermal process
- 1711: John Shore
  - invention of the tuning fork
  - → availability of a frequency standard!

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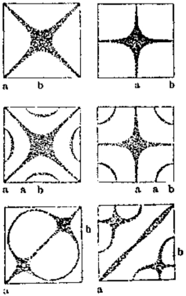
18th century

- 1759: Leonhard Euler
  - publication of the one-dimensional wave equation for sound:

$$\frac{\partial^2 p}{\partial x^2} = \frac{\rho_0}{\kappa P_0}\frac{\partial^2 p}{\partial t^2}$$

18th century

- 1787: E. F. F. Chladni
  - investigations on the vibrational behavior of plates
  - visualizations with sand that accumulates in node lines → Chladni figures



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19th century

- 1810
  - discovery of the adiabatic behavior of sound → varying temperature for fast processes
  - based on this assumption, a correct theoretical derivation of the speed of sound was achieved
- 1818: Augustin Fresnel
  - mathematically correct description of interference and diffraction

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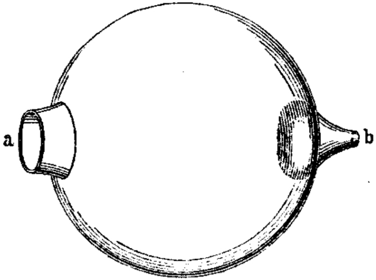
19th century

- 1843: G. S. Ohm
  - Ohm’s law of acoustics:
    - discovery of the ability of the ear to resolve complex tones in the fundamental (pitch) and the harmonics (tone color)
    - insensitivity regarding the phase of the harmonics

19th century

- 1865: H. L. F. von Helmholtz
  - publication of the book: ”Über die Tonempfindung”
    - milestone in knowledge about the human auditory system
  - Helmholtz resonator:



19th century

- 1877: Lord Rayleigh
  - publication of the book: ”Theory of Sound”
    - derivation of theoretical solutions for a variety of classical problems in acoustics
    - calculation of vibrating structures
    - radiation, diffraction and scattering of sound
  - the most relevant theoretical problems are solved!

19th century

- 1877: Thomas Alva Edison
  - invention of the phonograph
    - for the first time possible to store sound for later play-back

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20th century

- 1900: Wallace C. Sabine
  - founder of scientific room acoustics
    - investigations about reverberation of organ tones in the Lecture Room of the Fogg Art Museum in Harvard
    - development of the concept of reverberation time as an indicator to describe the acoustical quality of rooms
    - discovery of the Sabine reverberation time formula:

$$T = \frac{0.16V}{A}$$

    - room acoustical design of Boston Symphony Hall

20th century

- 1920-1940: Harvey Fletcher (Bell Telephone Labs)
    - founder of psychoacoustics
        - investigations on loudness of complex sounds
        - discovery of masking effects

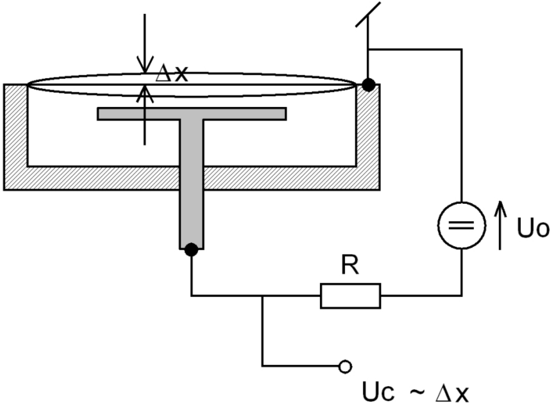
basic quantities

sound pressure

- atmosphere creates a static pressure due to the weight of the air mass
  - atmospheric pressure at sea level: around 1'000 hPa (1000 hectoPascal $=$ 1000 Millibar $=$ 100'000 Newton$/$m^2)
  - $\approx$ 12 Pa atmospheric pressure change per meter height difference

sound pressure

- device for pressure measurement:



sound pressure

sound pressure $p(t)$: quick pressure fluctuations (short term variations of the momentary air pressure):

$$p(t) = P(t) - P_{\text{atm}}$$

where
$P(t)$: momentary air pressure
P_{atm}: atmospheric pressure

sound pressure: typical numerical values

- normal speech in 1 m: $p_{\mathrm{typ,rms}} \approx 0.1$ Pa
- hearing threshold at 1 kHz: $p_{\mathrm{min,rms}} \approx 2 \times 10^{-5}$ Pa
- threshold of pain of the ear: $p_{\mathrm{max,rms}} \approx 100$ Pa

sound particle displacement

- local pressure variations propagate as sound waves
  - in air: longitudinal waves → oscillations of air particles in propagation direction
  - on average the air particles remain at the same location → sound does not transport matter but energy
- sound particle displacement ζ

animation open tube

sound particle displacement: numerical values

- normal speech 1 m / 1 kHz: $\zeta_{\mathrm{typ,rms}} \approx 4 \times 10^{-8}$ m
- hearing threshold at 1 kHz: $\zeta_{\mathrm{min,rms}} \approx 8 \times 10^{-12}$ m
- threshold of pain of the ear at 1 kHz: $\zeta_{\mathrm{max,rms}} \approx 4 \times 10^{-5}$ m

sound particle velocity

sound particle velocity $\vec{v}(t)$:

$$|\vec{v}(t)| = \frac{d\zeta}{dt}$$

- sound particle velocity is a vector and points in direction of propagation

sound particle velocity: typical numerical values

- normal speech in 1 m: $v_{\text{typ,rms}} \approx 2.5 \times 10^{-4}$ m/s
- hearing threshold at 1 kHz: $v_{\text{min,rms}} \approx 5 \times 10^{-8}$ m/s
- threshold of pain of the ear: $v_{\text{max,rms}} \approx 0.25$ m/s

sound intensity

- sound intensity describes the energy transport of a sound wave:
    - energy per second ($=$ power) through an area of 1 m^2 (perpendicular to propagation direction)
- sound intensity is a vector that points in the direction of sound particle velocity

average sound intensity $|\vec{I}|$:

$$|\vec{I}| = \overline{pv} \qquad [W/m^2]$$

note: phase between p and v is relevant!

sound power

average sound power W through an area S:

$$W = \int_S \vec{I} \cdot \mathrm{d}\vec{S} \qquad [W]$$

integrand:

- dot product of the intensity vector $\vec{I}$ and the surface normal of the area element $\mathrm{d}\vec{S}$

if the area S encapsules a source completely, the sound power corresponds to the sound power of the source

→ demo: sound power

sound power

typical sound power values:

	sound power [W]
human voice normal	7×10^{-6}
human voice max.	2×10^{-3}
violin, fortissimo	1×10^{-3}
loudspeaker (10 W el.)	0.1
jackhammer	1
organ, fortissimo	10
orchestra (75 instruments)	70
air plane Boeing 747	6’000
air plane FA-18	200’000

impedance

acoustic impedance Z:

$$Z = \frac{\check{p}}{\check{v}}$$

- $\check{p}$, $\check{v}$: complex amplitudes (pointer representation) contain information about
  - amplitude *and*
  - phase
- Z is usually a complex quantity with non-vanishing imaginary part

volume velocity

volume velocity Q:

$$Q = \int_S \vec{v} \cdot \mathrm{d}\vec{S}$$

integrand:

- dot product of sound particle velocity and the surface normal of the area element $\mathrm{d}\vec{S}$

basic equations

wave equation

wave equation

- → differential equation describing propagation of waves
- compact description of the physics of sound fields

wave equation: derivation

- approach: seeking for formulations that describe the relations between sound pressure and sound particle velocity
  - step 1: formulation of consequences of sound pressure for sound particle velocity
  - step 2: formulation of consequences of sound particle velocity for sound pressure
  - step 3: compilation

wave equation: derivation

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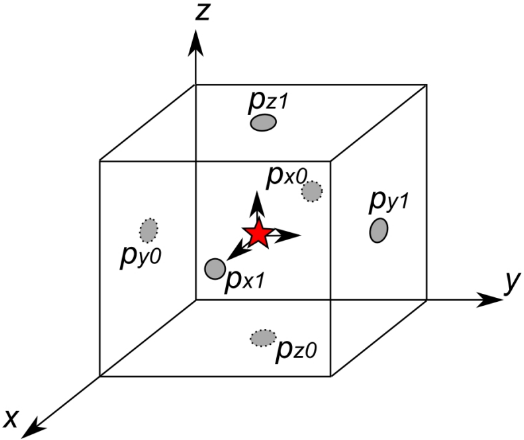
wave equation: $p \rightarrow \vec{v}$

p is given on the sides of the cube $\Delta l \cdot \Delta l \cdot \Delta l$. Consequences for $\vec{v}$?



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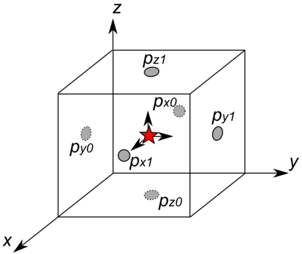
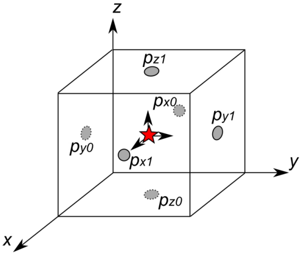


- fundamental physical relation: Newton: $F_{\text{res}} = m \cdot a$
  - $F \leftrightarrow p$
  - $a \leftrightarrow \vec{v}$

wave equation: $p \rightarrow \vec{v}$

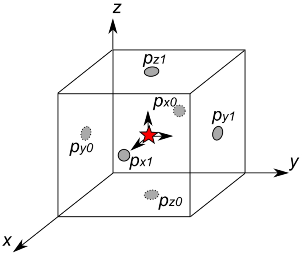


$$F_{\text{res}} = ma$$

in x-direction:

$$\Delta l^2 (p_{x0} - p_{x1}) = m \frac{\Delta v_x}{\Delta t}$$

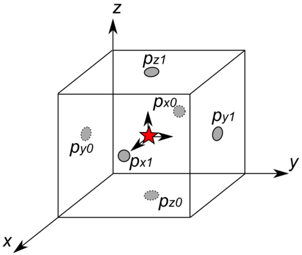
wave equation: $p \to \vec{v}$



$$m = \Delta l^3 \cdot \rho_0$$

$$\Delta l^2 (p_{x0} - p_{x1}) = \Delta l^3 \cdot \rho_0 \frac{\Delta v_x}{\Delta t}$$

wave equation: $p \rightarrow \vec{v}$



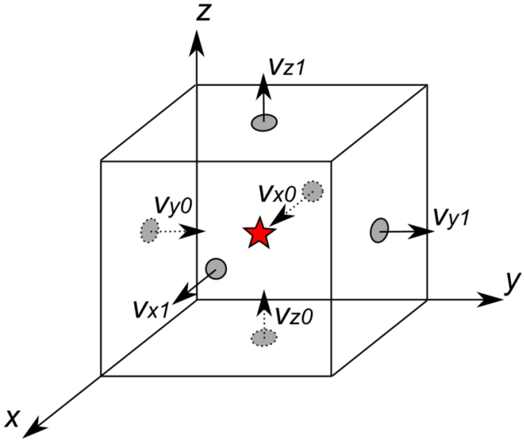
$$\Delta l^2 (p_{x0} - p_{x1}) = \Delta l^3 \cdot \rho_0 \frac{\Delta v_x}{\Delta t} \qquad | : \Delta l^3$$

$$\frac{p_{x0} - p_{x1}}{\Delta l} = \rho_0 \frac{\Delta v_x}{\Delta t}$$

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t}, \quad \frac{\partial p}{\partial y} = -\rho_0 \frac{\partial v_y}{\partial t}, \quad \frac{\partial p}{\partial z} = -\rho_0 \frac{\partial v_z}{\partial t}$$

wave equation: $\vec{v} \rightarrow p$

$\vec{v}$ is given on the sides of the cube $\Delta l \cdot \Delta l \cdot \Delta l$. Consequences for p?



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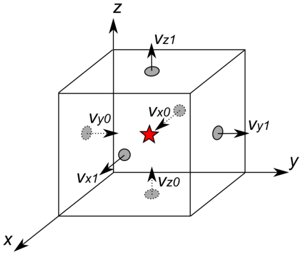

- fundamental physical relation: (adiabatic process): Poisson's law: $P \cdot V^{\kappa} = constant$
  - $\Delta P \leftrightarrow p$
  - $\Delta V \leftrightarrow \vec{v}$

wave equation: $\vec{v} \rightarrow p$

- of interest: consequences of *small* change in volume
- change in volume $\Delta V \leftrightarrow$ change in pressure ΔP?
- *small changes* → linearization of Poisson's law

wave equation: $\vec{v} \rightarrow p$

Poisson's law reformulated:

$$(P_0 + \Delta P)(V_0 + \Delta V)^\kappa = P_0 V_0^\kappa$$

first term:

$$P_0 + \Delta P = P_0 \left(1 + \frac{\Delta P}{P_0}\right)$$

second term (ignoring higher order contributions of the series):

$$(V_0 + \Delta V)^\kappa \approx V_0^\kappa + \Delta V \kappa V_0^{\kappa-1} = V_0^\kappa \left(1 + \kappa \frac{\Delta V}{V_0}\right)$$

wave equation: $\vec{v} \rightarrow p$

approximation inserted in Poisson's law:

$$P_0 \left(1 + \frac{\Delta P}{P_0}\right) V_0^\kappa \left(1 + \kappa \frac{\Delta V}{V_0}\right) \approx P_0 V_0^\kappa$$

$$\left(1 + \frac{\Delta P}{P_0}\right) \left(1 + \kappa \frac{\Delta V}{V_0}\right) \approx 1$$

$$\frac{\Delta P}{P_0} \approx -\kappa \frac{\Delta V}{V_0} - \kappa \frac{\Delta P}{P_0} \frac{\Delta V}{V_0}$$

$\Delta P \cdot \Delta V$ is very small, →

$$\frac{\Delta P}{P_0} \approx -\kappa \frac{\Delta V}{V_0}$$

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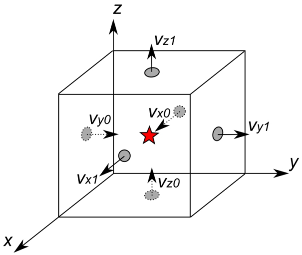
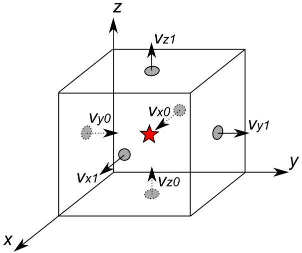


- search for change ΔV in volume caused by $\vec{v}$
- inserted in linearized form of Poisson's law $\rightarrow \Delta P$

wave equation: $\vec{v} \rightarrow p$



volume at time t:

$$V(t) = \Delta l^3$$

volume at time $t + \Delta t$:

$$V(t+\Delta t) = [\Delta l + \Delta t(v_{x1} - v_{x0})] \cdot$$
$$\cdot [\Delta l + \Delta t(v_{y1} - v_{y0})] \cdot [\Delta l + \Delta t(v_{z1} - v_{z0})]$$

wave equation: $\vec{v} \rightarrow p$

$$V(t+\Delta t) \approx \Delta l^3 + \Delta l^2 \Delta t (v_{x1} - v_{x0}) + \\ + \Delta l^2 \Delta t (v_{y1} - v_{y0}) + \Delta l^2 \Delta t (v_{z1} - v_{z0})$$

change in volume during Δt:

$$\Delta V = V(t+\Delta t) - V(t) \approx \Delta l^2 \Delta t (v_{x1} - v_{x0}) + \\ + \Delta l^2 \Delta t (v_{y1} - v_{y0}) + \Delta l^2 \Delta t (v_{z1} - v_{z0})$$

wave equation: $\vec{v} \rightarrow p$

inserted in $\frac{\Delta P}{P_0} \approx -\kappa \frac{\Delta V}{V_0}$:

$$\Delta P \approx \frac{-\kappa P_0}{\Delta l^3} \left[\Delta l^2 \Delta t (v_{x1} - v_{x0}) + \right.$$
$$\left. + \Delta l^2 \Delta t (v_{y1} - v_{y0}) + \Delta l^2 \Delta t (v_{z1} - v_{z0}) \right]$$

$$\frac{\Delta P}{\Delta t} \approx -\kappa P_0 \left(\frac{v_{x1} - v_{x0}}{\Delta l} + \frac{v_{y1} - v_{y0}}{\Delta l} + \frac{v_{z1} - v_{z0}}{\Delta l} \right)$$

wave equation: $\vec{v} \rightarrow p$

$$\frac{\Delta P}{\Delta t} = -\kappa P_0 \left(\frac{v_{x1} - v_{x0}}{\Delta l} + \frac{v_{y1} - v_{y0}}{\Delta l} + \frac{v_{z1} - v_{z0}}{\Delta l} \right)$$

$$\frac{\partial p}{\partial t} = -\kappa P_0 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$\frac{\partial p}{\partial t} = -\kappa P_0 \mathrm{div}(\vec{v})$$

wave equation: compilation

$$A1: \quad \frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t}$$

$$A2: \quad \frac{\partial p}{\partial y} = -\rho_0 \frac{\partial v_y}{\partial t}$$

$$A3: \quad \frac{\partial p}{\partial z} = -\rho_0 \frac{\partial v_z}{\partial t}$$

$$B: \quad \frac{\partial p}{\partial t} = -\kappa P_0 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

- find derivatives of Eq. A relative to x, y, z
- find derivative of Eq. B relative to t

wave equation: compilation

derivatives of Eq. A relative to x, y, z:

$$A1: \quad \rightarrow \frac{\partial^2 p}{\partial x^2} = -\rho_0 \frac{\partial^2 v_x}{\partial t \partial x} \overset{*)}{=} -\rho_0 \frac{\partial^2 v_x}{\partial x \partial t}$$

$$A2: \quad \rightarrow \frac{\partial^2 p}{\partial y^2} = -\rho_0 \frac{\partial^2 v_y}{\partial t \partial y} \overset{*)}{=} -\rho_0 \frac{\partial^2 v_y}{\partial y \partial t}$$

$$A3: \quad \rightarrow \frac{\partial^2 p}{\partial z^2} = -\rho_0 \frac{\partial^2 v_z}{\partial t \partial z} \overset{*)}{=} -\rho_0 \frac{\partial^2 v_z}{\partial z \partial t}$$

*) theorem of Schwarz

wave equation: compilation

derivative of Eq. B relative to t:

$$B: \ \rightarrow \frac{\partial^2 p}{\partial t^2} = -\kappa P_0 \left(\frac{\partial^2 v_x}{\partial x \partial t} + \frac{\partial^2 v_y}{\partial y \partial t} + \frac{\partial^2 v_z}{\partial z \partial t} \right)$$

wave equation: compilation

inserted → wave equation:

$$\frac{\partial^2 p}{\partial t^2} = \frac{\kappa P_0}{\rho_0}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right)$$

or

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{\rho_0}{\kappa P_0}\frac{\partial^2 p}{\partial t^2}$$

wave equation

wave equation:

- is the fundamental equation for the description of sound fields
- has to be fulfilled for each field point in time and space
- specification of specific problem introduces *boundary conditions*
- solution to a specific sound field problem:
  - search sound pressure field $p(x, y, z, t)$, that fulfills:
    - the wave equation
    - all boundary conditions
- note: wave equation made use of the linearized Poisson equation $\Rightarrow$ not valid for large amplitudes!

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speed of sound

speed of sound

- sound field distortion propagates with the speed of sound c
- assumption: one-dimensional propagation: $p = f(x - ct)$ with f: arbitrary function

inserted in equation from above yields:

$$c = \sqrt{\kappa \frac{P_0}{\rho_0}}$$

temperature dependency of c:

$$c \approx 343.2 \sqrt{\frac{T}{293}}$$

speed of sound: wave equation

insertion of c in wave equation:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

or

$$\triangle p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

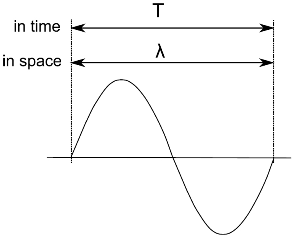
where

$\triangle p$: three-dimensional Laplace operator

Helmholtz equation

sinusoidal waves

- waves with sinusoidal time and space dependency are of special importance in the discussion of theoretical problems
- characterization:
  - amplitude
  - period length T or frequency $f = 1/T$, or angular frequency $\omega = 2\pi f$
  - wave length λ or wave number $k = 2\pi/\lambda$



sinusoidal waves

relation between λ, f, k, ω:

$$\lambda = \frac{c}{f}$$
$$k = \frac{\omega}{c}$$

frequency f	wave length λ
100 Hz	3.4 m
1 kHz	34 cm
10 kHz	3.4 cm

sinusoidal waves: Helmholtz equation

complex writing for sinusoidal oscillations:

$$\underline{p}(location, t) = \check{p}(location) \cdot e^{j\omega t}$$

where:
$\check{p}$: p,check
$\check{p}(location)$: complex, location dependent amplitude function
$e^{j\omega t}$: oscillation term

calculate $\triangle \underline{p}$ and $\frac{\partial^2 \underline{p}}{\partial t^2}$:

$$\triangle \underline{p} = \triangle \check{p} e^{j\omega t}$$

$$\frac{\partial^2 \underline{p}}{\partial t^2} = -\omega^2 \check{p} e^{j\omega t}$$

sinusoidal waves: Helmholtz equation

inserted in the wave equation:

$$\triangle p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

yields the Helmholtz equation:

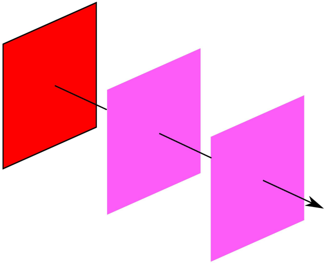
$$\triangle \check{p} + \frac{\omega^2}{c^2}\check{p} = 0$$

complex amplitude function $\check{p}$ is exclusively a function of location → *no explicit time variable*.

types of waves

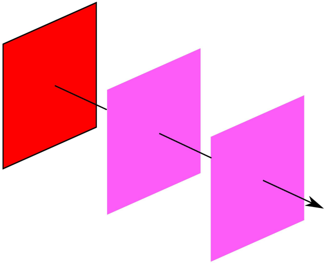
plane waves

plane waves



- excitation by a plane surface
- propagation in one direction only
- wave fronts = plane surfaces
- sound field variables p and $\vec{v}$ depend on one coordinate only
- no divergence in space
- example:
  - waves at low frequencies propagating in a tube

plane waves



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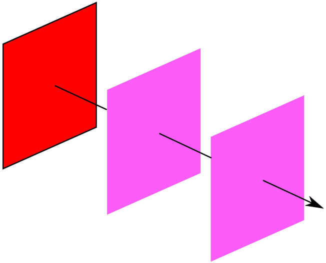

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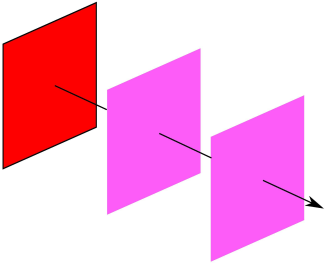

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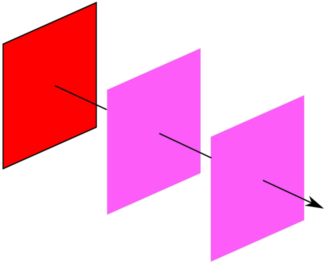

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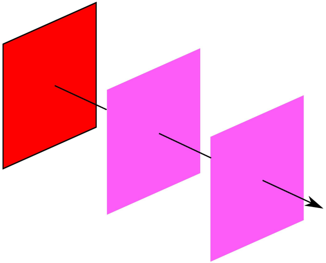

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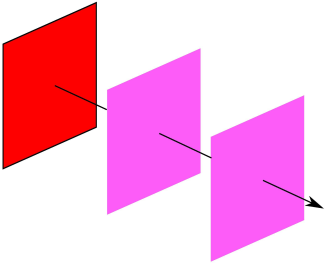

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plane waves

plane waves fulfill the one-dimensional wave equation:

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

all solutions $p(x,t)$ have the form

$$p(x,t) = \mathrm{f}(ct \pm x)$$

where:
$\mathrm{f}(ct - x)$: wave traveling in positive x-direction (→ right)
$\mathrm{f}(ct + x)$: wave traveling in negative x-direction (→ left)

plane waves

sinusoidal plane wave (sound pressure) in positive x-direction in complex representation:

$$\underline{p}(x,t) = \hat{p} e^{j(-kx+\phi)} e^{j\omega t}$$

where
$\hat{p}$: pressure amplitude
ϕ: initial phase

assumption for sound particle velocity:

$$\underline{v}_x(x,t) = \check{v}_x e^{j\omega t}$$

where
$\check{v}_x$: complex, location dependent amplitude function

plane waves

inserted in

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t} \qquad \text{Newton}$$

yields:

$$\underline{v}_x(x, t) = \frac{1}{\rho c}\underline{p}(x, t)$$

sound pressure and sound particle velocity are in phase, the ratio of their amplitudes (impedance) is

$$Z_0 = \frac{\check{p}}{\check{v}} = \rho c$$

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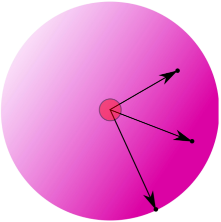

- excited by a point source
- propagate radially in all directions
- wave fronts are spherical surfaces
- due to symmetry reasons → p and $\vec{v}$ depend on radius only
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- example:
  - wave radiated by a pulsating sphere

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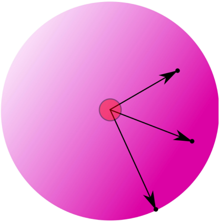

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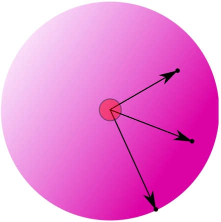

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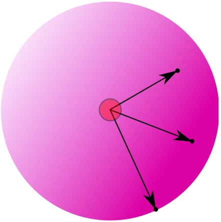

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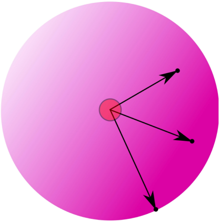

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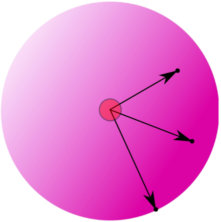

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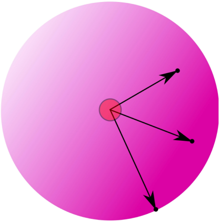

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spherical waves

guess for sound pressure p as function of radius r:

$$\underline{p}(r,t) = \frac{1}{r} \cdot \underline{p}_{\text{plane.wave}} = \frac{1}{r}\hat{p}e^{j(-kr+\phi)}e^{j\omega t}$$

verification with help of Helmholtz equation in spherical coordinates:

$$\frac{\partial^2 \check{p}}{\partial r^2} + \frac{2}{r}\frac{\partial \check{p}}{\partial r} + k^2\check{p} = 0$$

insertion → o.k.

spherical waves

with

$$\frac{\partial p}{\partial x} = -\rho_0 \frac{\partial v_x}{\partial t} \qquad \text{Newton}$$

the sound particle velocity in radial direction is found as:

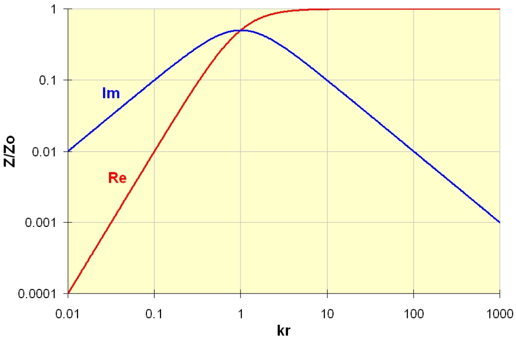
$$\underline{v}_r(r,t) = \underline{p}(r,t)\left(\frac{1}{\rho c} + \frac{1}{j\omega\rho r}\right)$$

for the impedance Z_s follows

$$Z_s = \frac{\check{p}}{\check{v}} = \rho c \frac{jkr}{1+jkr}$$

spherical waves

impedance:



- proximity effect for sound particle velocity sensors (e.g. cardioid microphones)
- $r \to \infty \Rightarrow Z_{\text{spherical.wave}} = Z_{\text{plane.wave}}$

sound pressure and sound power of a point source

sound power W of an omnidirectional point source:

$$W = \int_S \vec{I} \mathrm{d} \vec{S}$$

if S is the surface of a sphere with radius r, $|\vec{I}(r)|$ is constant:

$$W = I(r) 4\pi r^2$$

sound pressure and sound power of a point source

far field approximation:

$$Z = \frac{p(r)}{v(r)} = \rho_0 c$$

and therefore:

$$v(r) = \frac{p(r)}{\rho_0 c}$$

and:

$$I(r) = p_{\mathrm{rms}}(r) v_{\mathrm{rms}}(r) = \frac{p_{\mathrm{rms}}^2(r)}{\rho_0 c}$$

sound pressure and sound power of a point source

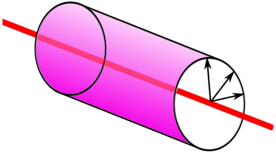
and finally:

$$W = \frac{p_{\mathrm{rms}}^2(r)}{\rho_0 c} 4\pi r^2$$

near-field?

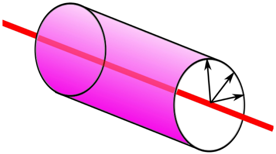
cylindrical waves

cylindrical waves



- line source
- propagate radially perpendicular to the line source
- wave fronts are cylinder surfaces
- due to symmetry → p and $\vec{v}$ depend on radius only
- divergence in space
- example:
  - hum noise of a high voltage power line

cylindrical waves



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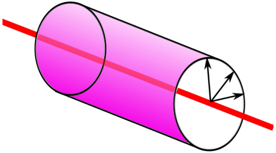

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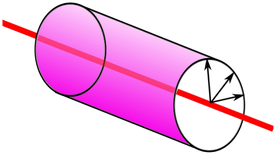

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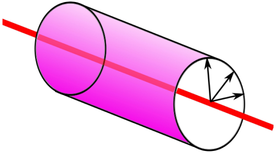

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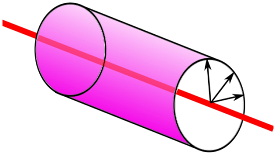

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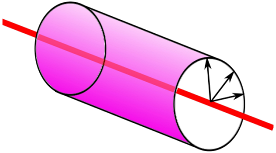

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verification with help of the Helmholtz equation in cylindrical coordinates

similar impedance curve as for spherical waves, however near-field / far-field transition for somewhat smaller kr values.

overview:
wave types

overview: wave types

	plane wave	spherical wave	cylindrical wave
p	const	$\sim \frac{1}{r}$	$\sim \frac{1}{\sqrt{r}}$
Z	ρc	near/far	near/far

superposition of point sources

superposition of point sources

- determination of total sound pressure stemming from several point sources
- application of the superposition principle (linear acoustics assumed)
- coherent sources:
  - phase sensitive summation of sound pressure and sound particle velocity
  - $\underline{p}_{\text{tot}} = \sum_{i=1}^{N} \underline{p}_i$
  - $\rightarrow$ constructive and destructive interference possible
- incoherent sources:
  - energetic summation → sum of the mean square values of sound pressure or sound particle velocity
  - $p^2_{\text{rms,tot}} = \sum_{i=1}^{N} p^2_{rms,i}$

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superposition of point sources

- examples of coherent summation?
- examples of incoherent summation?

superposition of point sources

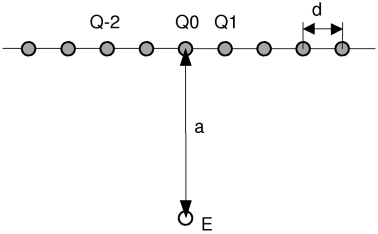
- examples of coherent sources:
  - a source and its mirror source
  - several transformers that emit 100 Hz due to magnetostriction
  - a pair of stereo loudspeakers emitting the same signal
- examples of incoherent sources
  - several machines in a factory building
  - cars on a road

incoherent point sources

incoherent point sources along a straight line

infinite line of incoherent point sources

situation:



contribution of source *n*:

$$p_{\mathrm{rms},n}^2 = \frac{K}{a^2 + (nd)^2}$$

where
K: constant (source strength)

infinite line of incoherent point sources

superposition of all contributions:

$$p^2_{\mathrm{rms,tot}} = \sum_{n=-\infty}^{+\infty} p^2_{\mathrm{rms},n} = K \frac{1}{d^2} \sum_{n=-\infty}^{+\infty} \frac{1}{\frac{a^2}{d^2} + n^2}$$

with series representation of coth:

$$\coth x = \frac{1}{x} + \frac{2x}{\pi^2} \sum_{n=1}^{+\infty} \frac{1}{\frac{x^2}{\pi^2} + n^2}$$

follows:

$$p^2_{\mathrm{rms,tot}} = \frac{K}{d^2} \frac{\pi d}{a} \coth\left(\frac{\pi a}{d}\right) = \frac{K\pi}{ad} \coth\left(\frac{\pi a}{d}\right)$$

infinite line of incoherent point sources

discussion → two cases:

- $\frac{\pi a}{d}$ small (→ small distances)
  - $\coth\left(\frac{\pi a}{d}\right) \approx \frac{d}{\pi a}$
  - $p^2_{\text{rms,tot}} \approx \frac{K}{a^2}$
  - $p_{\text{rms,tot}} \approx \frac{\sqrt{K}}{a}$
  - spherical wave behavior
- $\frac{\pi a}{d}$ large (→ large distances)
  - $\coth\left(\frac{\pi a}{d}\right) \approx 1$
  - $p^2_{\text{rms,tot}} \approx \frac{K\pi}{ad}$
  - $p_{\text{rms,tot}} \approx \sqrt{\frac{K\pi}{d}}\frac{1}{\sqrt{a}}$
  - cylindrical wave behavior

transition (both approximations identical):

$$a = \frac{d}{\pi}$$

incoherent point sources distributed over an area

area of incoherent point sources

situation:

- incoherent point sources spread over a rectangular area
  - length: L
  - width: B

sound pressure as a function of distance a:

$a < B/\pi$	behavior of a plane wave
$B/\pi < a < L/\pi$	behavior of a cylindrical wave
$L/\pi < a$	behavior of a spherical wave

coherent point sources along a straight line

infinite line of coherent point sources

- phase sensitive summation
- simplified calculation with help of Fresnel zones (sections with path length differences $< \lambda/2$)
- result: remaining contribution stems from half of the first zone → only a small section is relevant
- for a line of point sources of finite length: line source behavior is valid up to large distances

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reflection of sound waves at hard boundaries

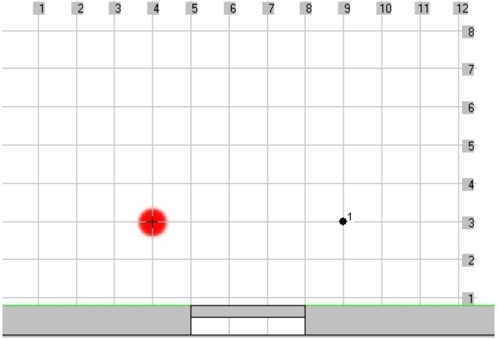
reflection of sound waves

any impedance discontinuity of the medium results in a partial reflection of an incident sound wave

- specular reflection
  - occurs at plane, large and impedance-homogeneous surfaces
- diffuse reflection
  - occurs at structured or impedance-inhomogeneous surfaces
- scattering
  - occurs at small objects

specular reflection

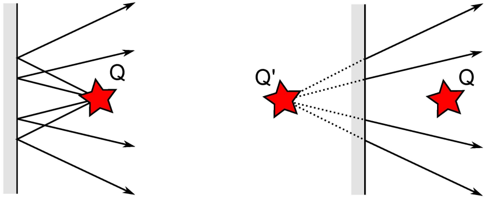
specular reflection



FDTD simulation: plane surface

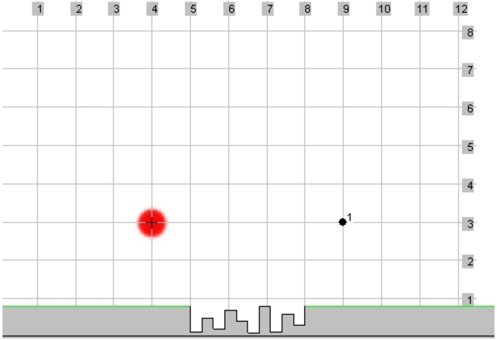
specular reflection

- plane, rigid reflector → specular reflection
- reflector → boundary condition: $v_n = 0$
- solution: introduction of a mirror source:
  - reflected contribution seems to stem from the *mirror source*



diffuse reflection

diffuse reflection



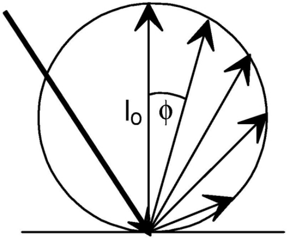
FDTD simulation: structured surface

diffuse reflection

diffuse reflection:

- directivity often idealized according to Lambert's law:

$$I_{\text{refl.}}(\phi) = I_0 \cos\phi$$



Doppler effect

Doppler effect

Doppler effect:

- shift of signal frequency due to movement of source or receiver
- examples:
  - vehicles passing-by
  - simultaneous radiation of low and high frequencies by a loudspeaker membrane
  - Leslie cabinets of Hammond organs

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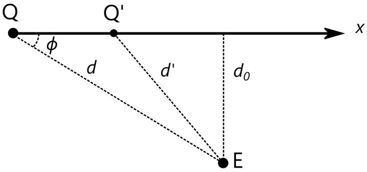
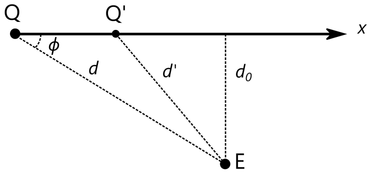
Doppler effect

calculation of frequency shift:

- Q: source
    - moves with speed v_Q in x-direction
    - emits a tone of frequency f_0
- E: receiver
    - at rest, in distance d under angle ϕ
    - received frequency f?



Doppler effect



- Q emits a first maximum at time $t = 0$
  - arrival at the receiver at time $t = d/c$
- Q' emits a second maximum at $t = 1/f_0$
  - arrival at the receiver at $t = 1/f_0 + d'/c$

Doppler effect

time interval T between the two maxima at the receiver:

$$T = \left(\frac{1}{f_0} + \frac{d'}{c}\right) - \frac{d}{c}$$

frequency f at the receiver

$$f = \frac{1}{T} = \frac{1}{\frac{1}{f_0} - \frac{d-d'}{c}}$$

with d':

$$d' = \sqrt{d^2 - 2dv_Q T_0 \cos\phi + v_Q^2 T_0^2}$$

Doppler effect

for $\phi = 0$ the formula simplifies to:

$$d' = d - v_Q T_0$$

and the frequency at the receiver becomes:

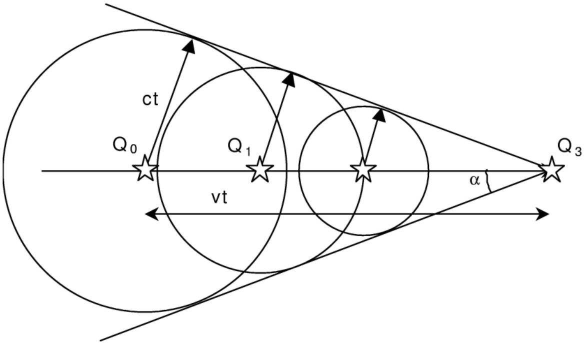
$$f = f_0 \frac{c}{c - v_Q}$$

sonic boom

sonic boom

- sonic boom generated by sources with speed $v > c$
- examples:
  - air planes
  - projectiles
- high signal amplitudes due to wave front steepening

sonic boom: Mach's cone



$$\sin\alpha = \frac{c}{v} \qquad\qquad (v > c)$$

standing waves

standing waves

perfect standing waves occur in case of:

- superposition of plane waves traveling in opposite directions with
    - identical frequency
    - identical amplitude

standing waves

wave 1 →: $\underline{p}_1(x,t) = \hat{p} e^{j(\omega t - kx)}$

wave 2 ←: $\underline{p}_2(x,t) = \hat{p} e^{j(\omega t + kx)}$

$$\begin{aligned}
\underline{p}_{\text{tot}}(x,t) &= \underline{p}_1(x,t) + \underline{p}_2(x,t) \\
&= \hat{p} e^{j\omega t}\left(e^{-jkx} + e^{jkx}\right) \\
&= \hat{p} e^{j\omega t}\left(\cos(-kx) + j\sin(-kx) + \right. \\
&\quad \left. + \cos(kx) + j\sin(kx)\right) \\
&= \hat{p} e^{j\omega t} 2\cos(kx)
\end{aligned}$$

- no propagating wave
- harmonic oscillation with local $\cos(kx)$-modulation
  - maxima
  - minima

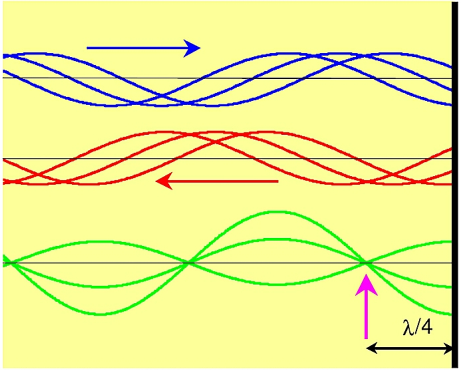
standing waves

example: plane wave is reflected at a hard surface (sound pressure is shown):

movement of sound particles:

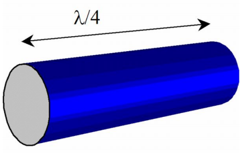
standing waves

standing wave in front of a rigid reflector
sound pressure:



standing waves: $\lambda/4$ resonator

tube open on one side, closed at the other end:



- tube forces a pressure minimum at the open end
- creates a local sound field discontinuity
- equalization by strong pressure increase inside the tube

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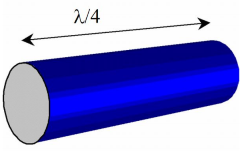

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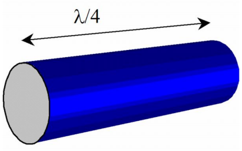

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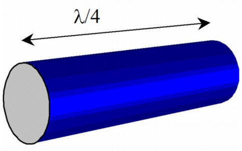

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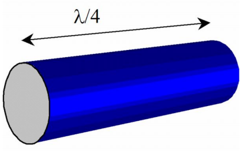

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example: maximum sensitivity of the human ear between 3…4 kHz due to a $\frac{\lambda}{4}$ resonance of the ear canal (length 2.5 cm).

diffraction phenomena

diffraction phenomena

- sound waves are bent around corners (diffracted)
- sound reaches a receiver even in case of interrupted sight line
- diffraction process corresponds to low-pass filtering

diffraction phenomena: Maekawa's formula

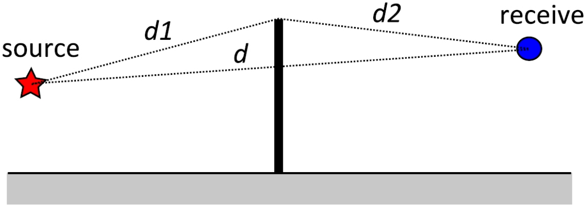
attenuation A_H due to an obstacle:

$$A_H = 10\log\left(3 + 20\frac{z}{\lambda/2}\right) \qquad [\mathrm{dB}]$$

where

λ: wavelength

z: path length difference source - edge of the barrier - receiver and source - receiver $z = d_1 + d_2 - d$



dB-scale
level quantities

dB-scale: level quantities

dB-scale:

$$\text{level} = 10 \cdot \log\left(\frac{\text{powerX}}{\text{powerY}}\right) \quad [\text{dB}]$$

acoustical quantities proportional to power:

- sound pressure square p^2
- sound particle velocity square v^2
- sound intensity I
- sound power W

dB-scale: level quantities

applications of the dB-scale:

- comparison of quantities
  - e.g. quantity X is 3 dB larger than quantity Y
- expression of a quantity in relation to a reference
  - sound pressure level $L_p = 10 \cdot \log\left(\frac{p}{2 \cdot 10^{-5}\text{Pa}}\right)^2$
  - sound intensity level $L_I = 10 \cdot \log\left(\frac{I}{10^{-12}\text{W/m}^2}\right)$
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dB-scale: level quantities

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dB-scale: level quantities

consequences of the dB-scale:

- multiplication of quantities corresponds to an addition in the dB-scale
  - example: amplification of the power by a factor of 2 corresponds to a level increase by $+3$ dB
- audible range is mapped onto the sound pressure level interval L_p: $0 \ldots 120$ dB
- constant loudness variation corresponds to a constant dB step

dB-scale: level quantities

subtlety of the dB-scale:

0dB, +1dB, 0dB, +3dB, 0dB, +6dB, 0dB, +10dB, 0dB

dB-scale: level quantities

subtlety of the dB-scale:

level difference	perception
< 2 dB	not audible
$2 \ldots 4$ dB	just audible
$5 \ldots 10$ dB	clearly audible
> 10 dB	very convincing

dB-scale: level quantities

typical sound pressure level values:

sound source	sound pressure level
speech in 2 m	60 dB
road traffic in 10 m *	70 dB
air plane in 100 m	120 dB

* 1000 vehicles/h, 80 km/h

dB-scale: level quantities

calculations with decibel quantities:

- caution: logarithmic quantity
- multiplication of underlying physical quantities → addition of dB quantities
- summation of underlying physical quantities → addition of the linear quantities
  - $L_{W,tot} = 10 \log \left(10^{0.1 L_{W1}} + 10^{0.1 L_{W2}}\right)$

dB-scale: level quantities

calculations with decibel quantities: important values of the $\log_{10}$ function:

a	$\log(a)$	$10\log(a)$	$10\log(a^2)$
0.01	-2	-20	-40
0.1	-1	-10	-20
0.5	$\approx$ -0.3	$\approx$ -3	$\approx$ -6
1	0	0	0
2	$\approx$ 0.3	$\approx$ 3	$\approx$ 6
3	$\approx$ 0.5	$\approx$ 5	$\approx$ 10
10	1	10	20
100	2	20	40
1000	3	30	60
10000	4	40	80

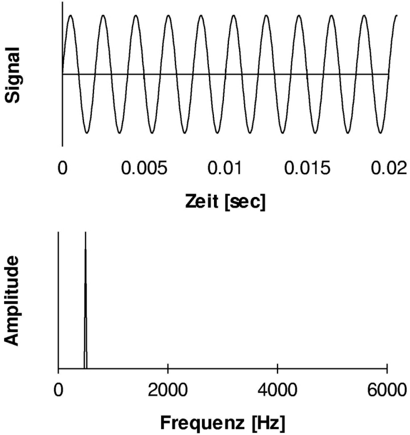
distance dependency of prototype waves in dB scale

sound pressure or intensity variation for a doubling of distance:

	plane wave	spherical wave	cylindrical wave
ΔL	0 dB	-6 dB	-3 dB

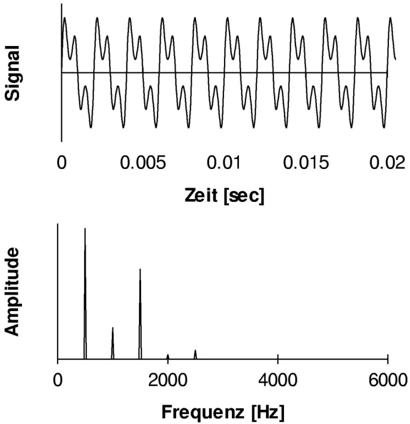
signal prototypes

pure tone: time course and spectrum



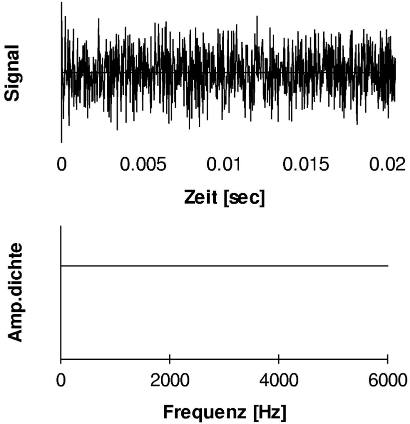
pure tone 440 Hz

complex tonal sound: time course and spectrum



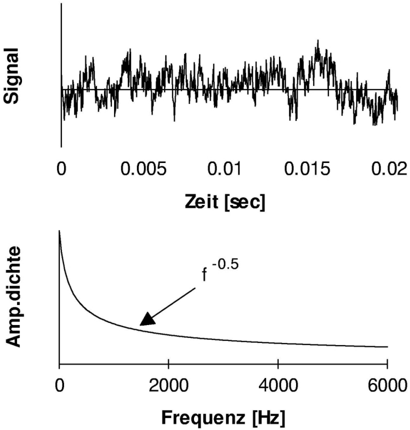
complex tonal sound 440 Hz + 3. + 5. harmonic

white noise: time course and spectrum



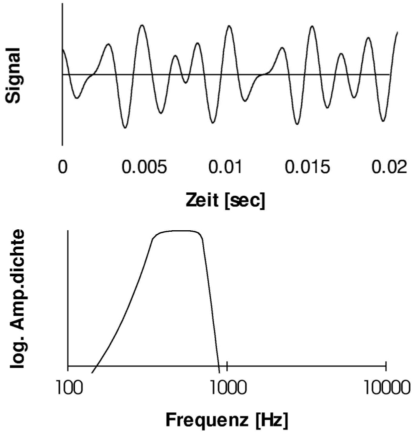
white noise

pink noise: time course and spectrum

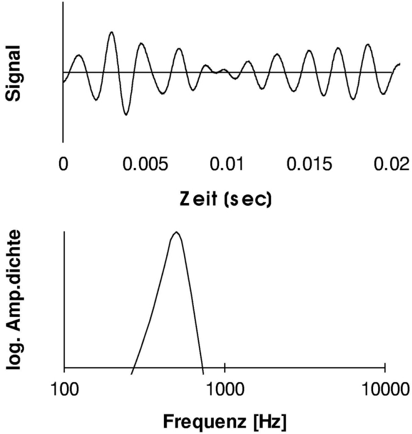


pink noise

500 Hz octave band filtered noise: time course and spectrum

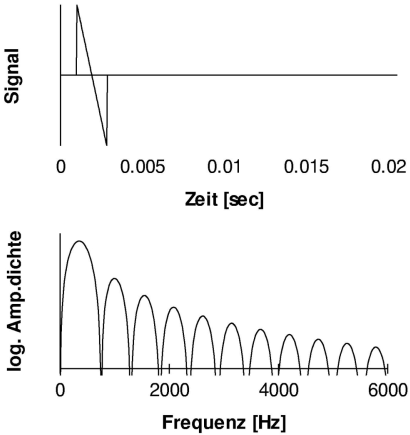


500 Hz third octave band filtered noise: time course and spectrum

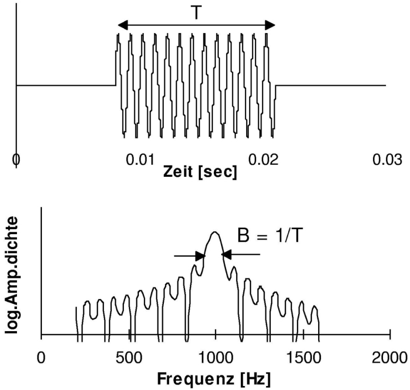


sweeping third octave band noise

bang: time course and spectrum

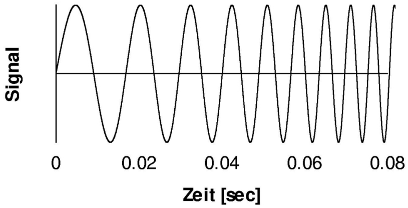


tone burst: time course and spectrum



bursts 440 Hz: 1, 2, 4, 8, 16, 32, 64, 128, 256 cycles

sweep: time course



sinus sweep

eth-acoustics-1