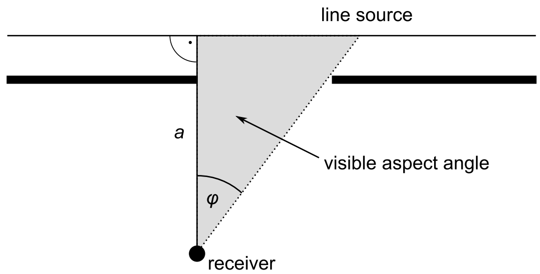
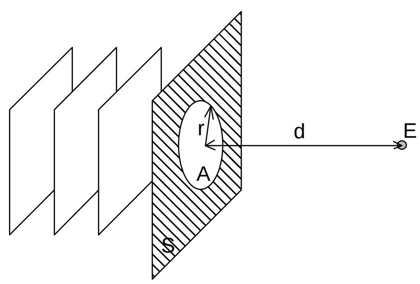
Kurt Heutschi and Reto Pieren, Acoustics I, 2023-05-07

Exercise Acoustics I

1. A incoherently radiating line source with infinite extension is given (see Figure). A section of length l emits the power $K \times l$. A receiver is located in distance a from the line. Due to shielding the segment of the line source visible for the receiver spans an aspect angle φ. Calculate the RMS value of the sound pressure at the receiver as a function of the angle φ.



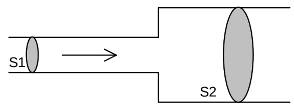
2. An arrangement according to the Figure below is given. A plane wave with frequency f travels from left to right and hits the screen S with a circular opening A of radius r. A receiver E is located in distance d from the center of the opening. By variation of the radius r of the opening, the sound pressure in E can be adjusted. Calculate r for an amplification of 6 dB and an attenuation of 10 dB compared to the sound pressure of the plane wave (free field). The concept of Fresnel zones is helpful to solve this problem. It is assumed that the sound field in the opening corresponds to the plane wave sound field. Boundary effects at the border of the opening are neglected.



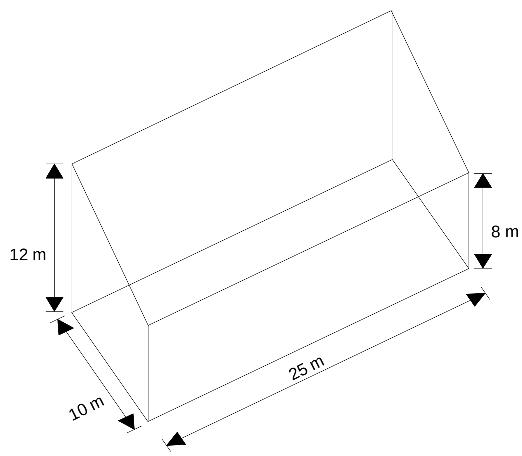
3. At the end of a measurement the sound level meter indicates 50 dB Leq and 70 dB L_E or SEL. How long did the measurement last?

4. An impulsive sound with amplitude 1 Pa (RMS) and duration 10 ms is investigated. Calculate the maximum value of the momentary sound pressure level with the time constants FAST and SLOW and the event level L_E or SEL.

5. A plane wave propagates in a tube of cross sectional area S_1. Calculate the reflection factor R for an abrupt enlargement of the cross sectional area to S_2. It is assumed that the wave lengths are much larger than the diameter of the tube sections and that the tubes extend to infinity. For the solution, an incident, a reflecting and a transmitted wave shall be assumed. At the position of the cross sectional discontinuity, appropriate conditions for sound pressure and sound particle velocity shall be formulated.



6. A room according to the Figure below is given.



The room limiting surfaces have absorption coefficients α as shown in the Table.

	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
floor	0.01	0.01	0.02	0.02	0.03	0.03
walls	0.1	0.2	0.3	0.3	0.3	0.3
ceiling	0.5	0.3	0.2	0.2	0.1	0.1

(a) Calculate the reverberation time in each octave band according to Sabine and Eyring neglecting air absorption.

(b) Calculate the critical distance for the 500 Hz octave band and an omnidirectional radiating point source.

(c) Calculate the change in the critical distance from b) if a reflecting surface is installed directly behind the source.

Solutions to Exercise Acoustics I

1.
The resulting sound pressure is given as integration over the visible section ($a\tan(\varphi)$) of the line source. In distance r, the sound intensity I that stems from the infinitesimal section dx is given as $I = \frac{K\mathrm{d}x}{4\pi r^2}$. For not too low frequencies f and not too small distances r ($fr > 100$) holds $p^2_{\mathsf{rms}} = \rho c I$. For the integration can be written:
$p^2_{\mathsf{rms}}(E) = \int\limits_0^{a\tan(\varphi)} \frac{\rho c K}{4\pi r^2}\mathrm{d}x$ and with $r^2 = a^2 + x^2 \rightarrow p^2_{\mathsf{rms}}(E) = \frac{\rho c K}{4\pi} \int\limits_0^{a\tan(\varphi)} \frac{1}{a^2+x^2}\mathrm{d}x = \frac{\rho c K}{a 4\pi}\varphi$
The proportionality between $p^2_{\mathsf{rms}}(E)$ and the visible aspect angle φ is a very important property of finite line sources.

2.
If the opening of the screen corresponds exactly to the first Fresnel zone, the sound pressure in E doubles compared to the free field case (+6 dB). The radius r of the first Fresnel zone is defined by the condition $r^2 + d^2 = (d + \frac{\lambda}{2})^2 \rightarrow r_{+6} = \sqrt{d\lambda + \frac{\lambda^2}{4}}$ where λ is the wave length, $\lambda = \frac{c}{f}$, c: speed of sound, f: frequency. For smaller radii of the opening, the sound pressure at the receiver decreases. An attenuation of 10 dB relative to free field corresponds to the reduction of the opening area to 1/6 of the area of the first Fresnel zone. For the radius follows: $r_{-10} \approx \frac{1}{\sqrt{6}}\sqrt{d\lambda + \frac{\lambda^2}{4}}$

3.
$10\log(\frac{1}{T}) = -20$ dB $\rightarrow T = 100$ s

4.
the continuous sound pressure level after 10 ms is found as:

$$L(10\mathsf{ms}) = 10\log\left(\frac{1}{RC}\int_0^{10\mathsf{ms}} \frac{1\mathsf{Pa}^2}{p_0^2} e^{\frac{\tau - 10\mathsf{ms}}{RC}} d\tau\right)$$
$$= 10\log\left(\frac{1}{RC}\frac{1}{p_0^2}\int_{-0.01}^{0} e^{\frac{\tau}{RC}} d\tau\right) = 10\log\left(\frac{1 - e^{\frac{-0.01}{RC}}}{p_0^2}\right)$$

Alternatively the algorithm for the evaluation of the moving square average may be used:

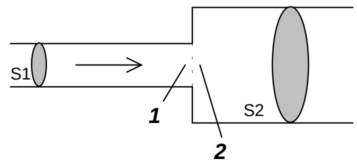
$$x^2_{\mathsf{rms}}(t + \Delta t) \approx x^2_{\mathsf{rms}}(t) + \frac{x^2(t+\Delta t) - x^2_{\mathsf{rms}}(t)}{\frac{RC}{\Delta t}}$$

As the length of the impulse is much smaller than the two time constants 125 ms and 1000 ms, the sampling interval can be chosen as $\Delta t = 10\mathsf{ms}$. With this:

$$x^2_{\mathsf{rms}}(10\mathsf{ms}) \approx \frac{1\mathsf{Pa}^2}{\frac{RC}{10\mathsf{ms}}}$$

With *FAST* the maximum level reaches 83 dB, with *SLOW* the maximum is 74 dB.

5.
Three waves have to be considered: incident (e), reflected (r) and transmitted (t).



The following conditions have to be fulfilled:

1. continuity of pressure: the total sound pressure at position 1 has to be equal to the total sound pressure at position 2 (any pressure discontinuity would correspond to a large force and induce an equalization movement.) $\rightarrow p_t = p_e + p_r$
2. conservation of mass: incident volume flow = transmitted volume flow (volume flow = sound particle velocity $\times$ cross sectional area) $\rightarrow v_e S_1 - v_r S_1 = v_t S_2$

For plane waves with impedance $Z_0 = \rho_0 c$, the conservation of mass can be written as

$$\frac{p_e}{Z_0} S_1 - \frac{p_r}{Z_0} S_1 = \frac{p_t}{Z_0} S_2 \tag{1}$$

Insertion of the pressure continuity condition in (1) yields:

$$p_e(S_1 - S_2) = p_r(S_1 + S_2) \tag{2}$$

and the reflection factor R is found as:

$$R = \frac{p_r}{p_e} = \frac{S_1 - S_2}{S_1 + S_2} \tag{3}$$

6.(a)
The following values are found as total absorption:

	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
floor (250 m^2)	2.5	2.5	5.0	5.0	7.5	7.5
walls (700 m^2)	70	140	210	210	210	210
ceiling (270 m^2)	135	81	54	54	27	27
total	207.5	223.5	269	269	244.5	244.5

With a room volume of 2'500 m^3, the reverberation time according to Sabine calculates as

	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
Sabine-reverberation time [sec]	1.93	1.79	1.49	1.49	1.64	1.64

With a total room surface area of 1220 m^2 the average (area-weighted) absorption coefficients are found as:

	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
average absorption coefficient	0.17	0.18	0.22	0.22	0.20	0.20

With this the reverberation time according to Eyring is found as:

	125 Hz	250 Hz	500 Hz	1 kHz	2 kHz	4 kHz
Eyring-reverberation time [sec]	1.76	1.62	1.32	1.32	1.47	1.47

(b)
critical distance r_H:

$$r_H = \sqrt{\frac{269}{16\pi}} = 2.3\mathsf{m} \tag{4}$$

(c)
It can be assumed that the diffuse field is unaltered, but the direct sound is amplified by 3 dB. From this follows $r'_H = r_H \times 1.4 = 3.2$ m.