Control of Low-Inertia Power Systems: Naive & Foundational Approaches

(extended set of slides)

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Acknowledgements

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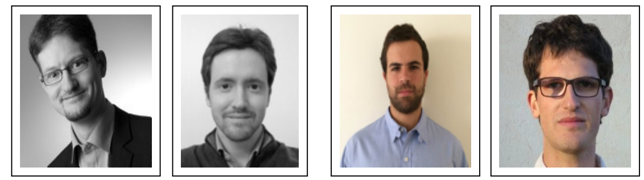

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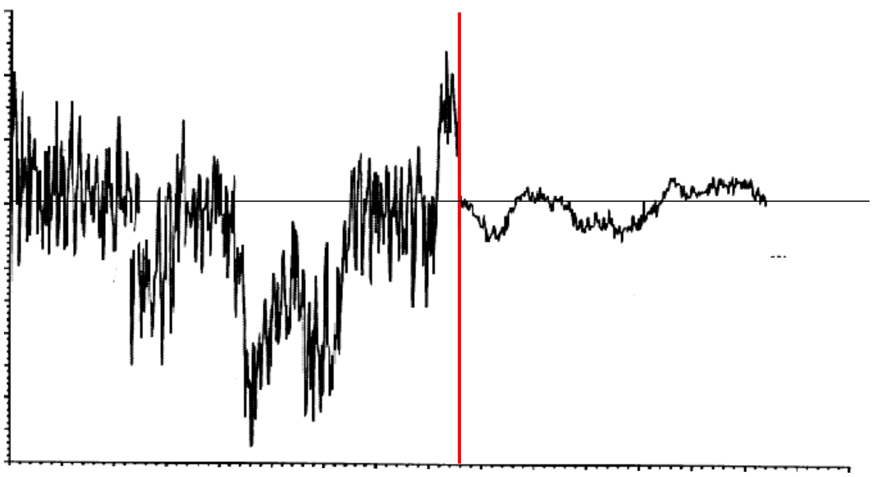
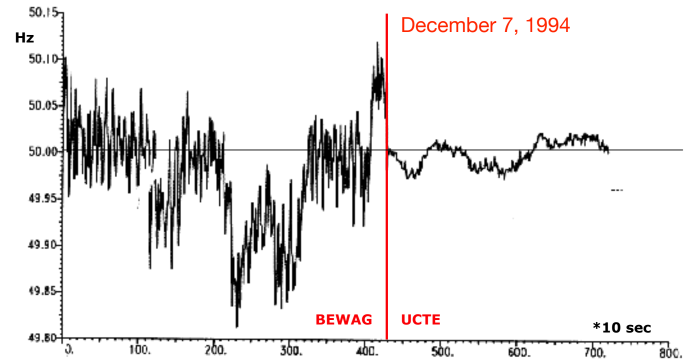
HORIZON 2020

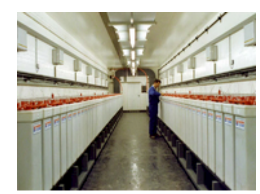
What do we see here?



Frequency of West Berlin when re-connecting to Europe

Source: *Energie-Museum Berlin*





before re-connection: islanded operation based on batteries & single boiler

afterwards connected to European grid based on synchronous generation

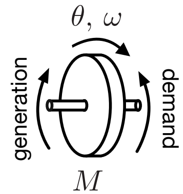
Essentially, the pre/post West Berlin curves date back to. . .

Fact: all of AC power systems built around **synchronous machines** !

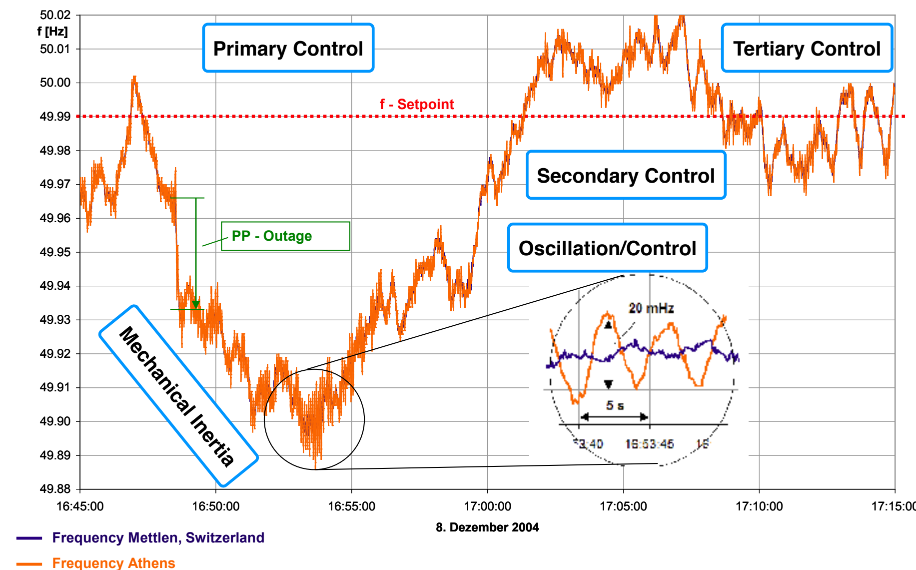
At the heart of it is the generator **swing equation**:

$$M \frac{d}{dt}\omega(t) \; = \; P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

change of kinetic energy $=$ instantaneous power balance



Operation centered around bulk synchronous generation



Source: W. Sattinger, Swissgrid

Renewable/distributed/non-rotational generation on the rise

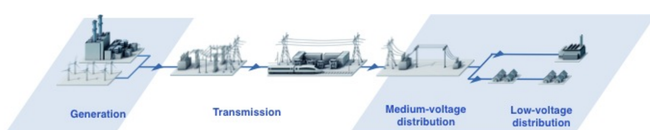
synchronous generator

new workhorse

scaling

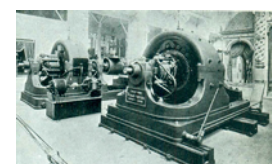
new primary sources

location & distributed implementation



focus today on **non-rotational** generation

The foundation of today's power system



Synchronous machines with rotational **inertia**

$$M\frac{d}{dt}\omega \approx P_{\text{generation}} - P_{\text{demand}}$$

Today's grid operation heavily relies on

1. **robust** stabilization of **frequency** and **voltage** by generator controls
2. **self-synchronization** of machines **through the grid**
3. kinetic energy $\frac{1}{2}M\omega^2$ as **safeguard** against disturbances

We are replacing this solid foundation with . . .

Tomorrow's clean and sustainable power system



Non-synchronous generation connected via **power electronics**

As of today, power electronic converters

1. **lack robust control** for **voltage** and **frequency**
2. **do not** inherently **synchronize** through the grid
3. provide almost **no energy storage**

What could possibly go wrong?

Black System Event in South Australia (Sep 2016)

The Sydney Morning Herald
NATIONAL
State in the dark: South Australia's major power outage

theguardian
South Australia
South Australia blackout: entire state left without power after storms

Key events[1]

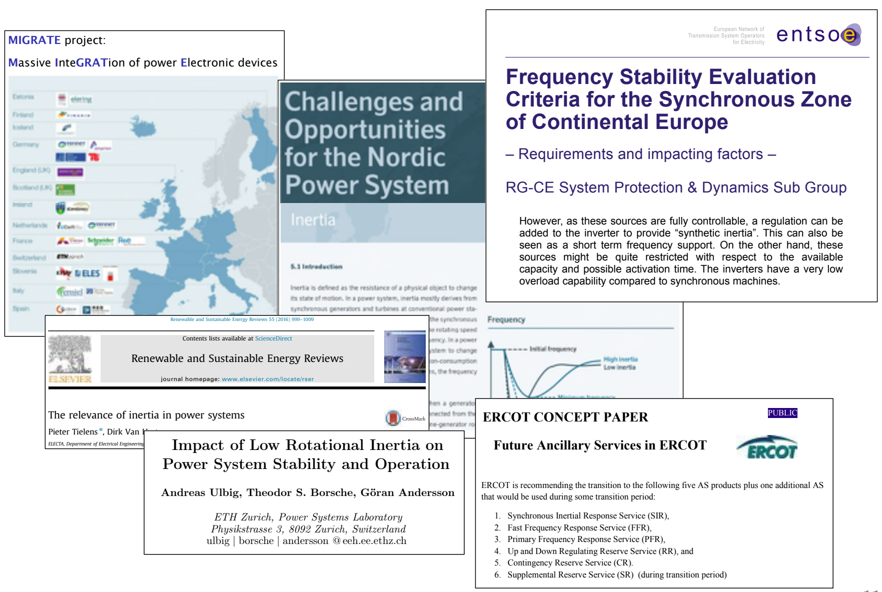
1. intermittent **voltage disturbances** due to line faults
2. loss of **synchronism** between SA and remainder of the grid
3. SA islanded: **frequency collapse** in a **quarter** of a **second**

"**Nine** of the **13** wind farms online **did not ride through** the six **voltage disturbances** experienced during the event."

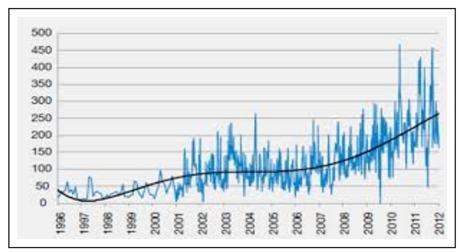
[1] AEMO: Update Report - Black System Event in South Australia on 28 September 2016

Low inertia issues have been broadly recognized

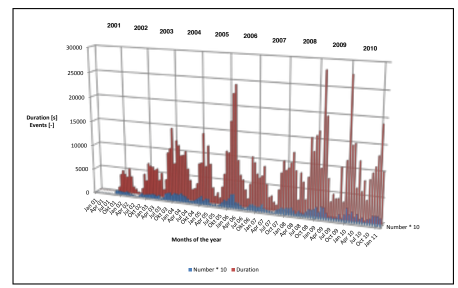
by TSOs, device manufacturers, academia, funding agencies, etc.



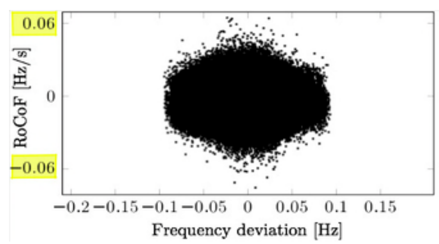
Low-inertia issues close to home



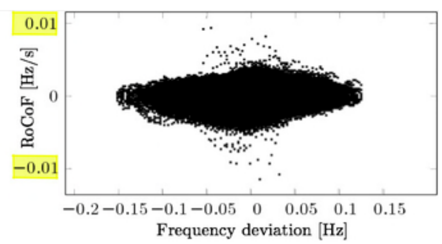
frequency violations in Nordic grid (source: ENTSO-E)



same in Switzerland (source: Swissgrid)



a day in Ireland (source: F. Emiliano)



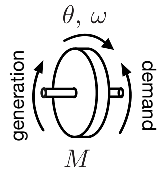
a year in France (source: RTE)

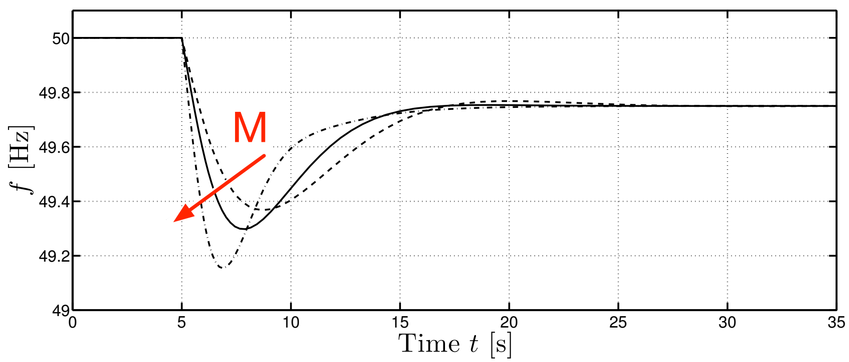
Obvious insight: loss of inertia & frequency stability

We loose our giant electromechanical low-pass filter:

$$\mathbf{M}\,\frac{d}{dt}\,\omega(t) \;=\; P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

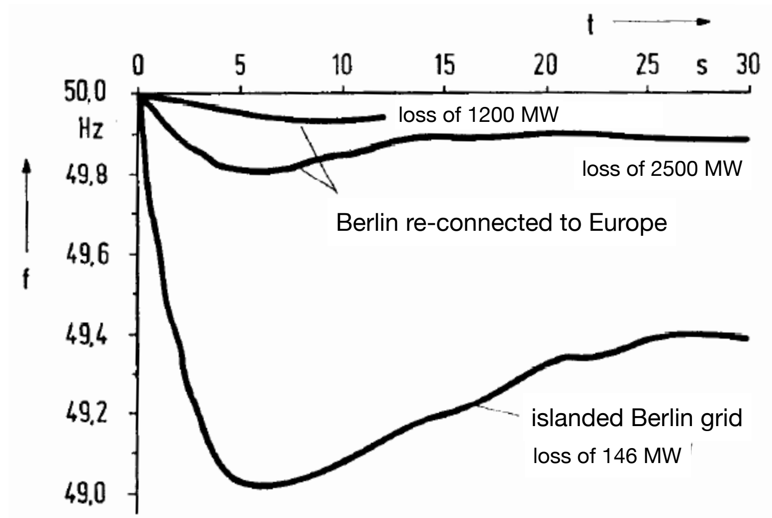
change of kinetic energy = instantaneous power balance





Berlin curves before and after re-connecting to Europe

Source: *Energie-Museum Berlin*



obvious insights lead to obvious (naive) answers

Baseline solution: virtual inertia emulation

$$\mathbf{M}\,\frac{d}{dt}\,\omega(t) \;=\; P_{\text{generation}}(t) - P_{\text{demand}}(t) \;\approx\; \textbf{derivative control} \text{ on } \omega(t)$$

⇒ **focus today:** **where** to do it? **how** to implement it properly?

... we are not just loosing inertia ... **what else** to do ?

Outline

Introduction

System Level: Optimal Placement of Virtual Inertia
network, disturbances, & performance metrics matter

Device Level: Proper Virtual Inertia Emulation Strategy
maybe we should not think about frequency and inertia

A Foundational Control Approach
restart from scratch for low-inertia systems

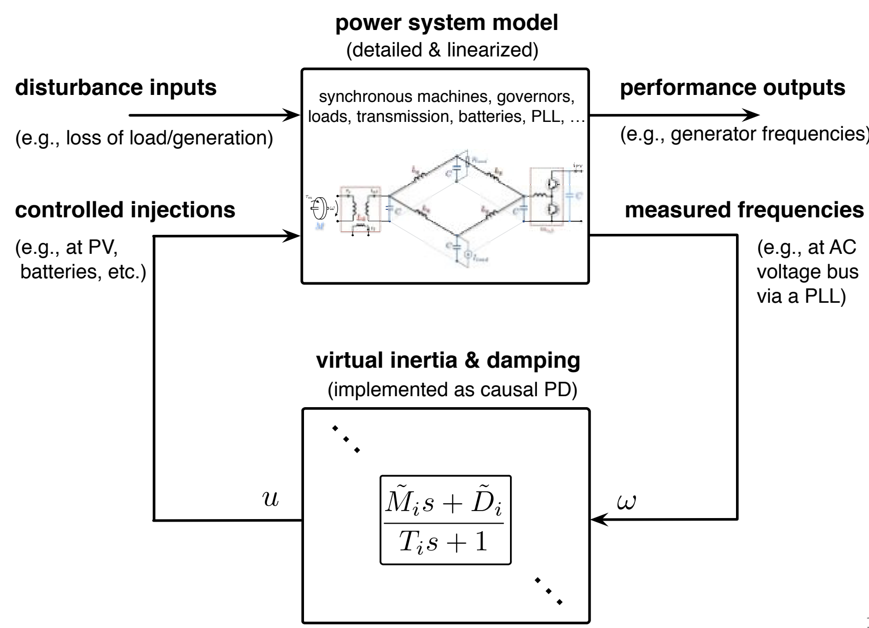
Conclusions

Virtual inertia is becoming a technology and a product

so let's see how we can make use of it

optimal placement of virtual inertia

General power system & inertia emulation model



which metric(s) should our controller optimize ?

Conventional metrics

disturbance inputs:

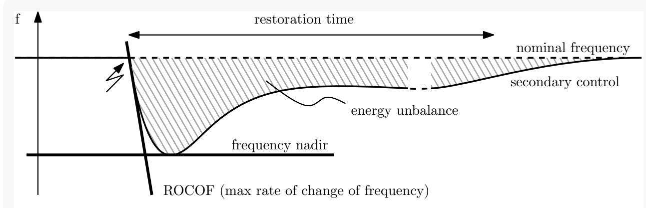
- step (loss of load/generation)
- impulse (line open-/closing)
- noise (renewables & loads)

performance outputs:

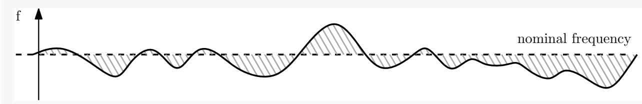
- overshoot (peak signals after fault)
- RoCoF (rate of change of frequency)
- spectrum (damping ratio cones)

re-evaluate scenario?

- hardly tractable for optimization & control design
- metrics & faults justified only in a system dominated by machines
- metrics any useful?

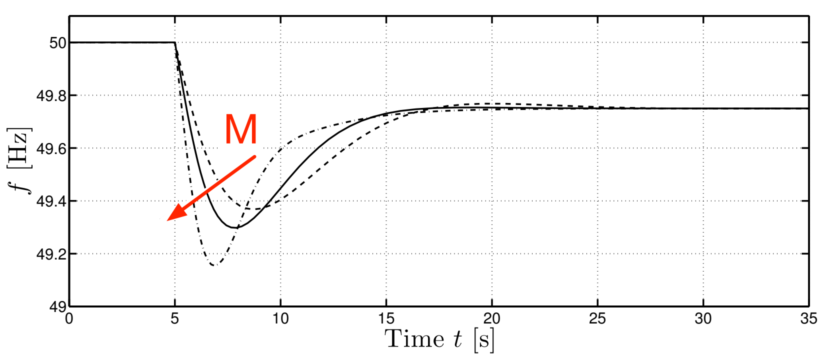


post-fault response in a low-inertia system?

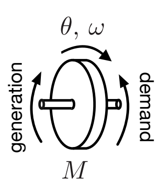


Conventional metrics cont'd: "the more inertia the better"

extrapolation of swing equation: total inertia directly affects RoCoF & frequency nadir

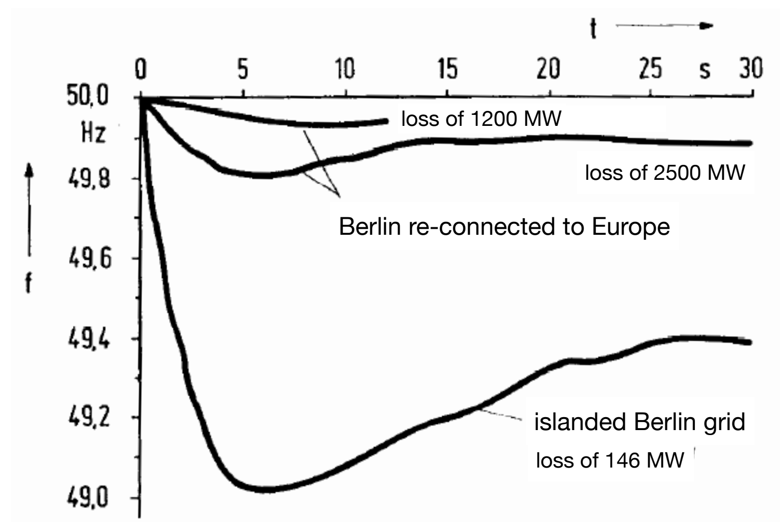


$$\mathbf{M}\,\frac{d}{dt}\omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$



West Berlin curves before & after re-connecting to Europe

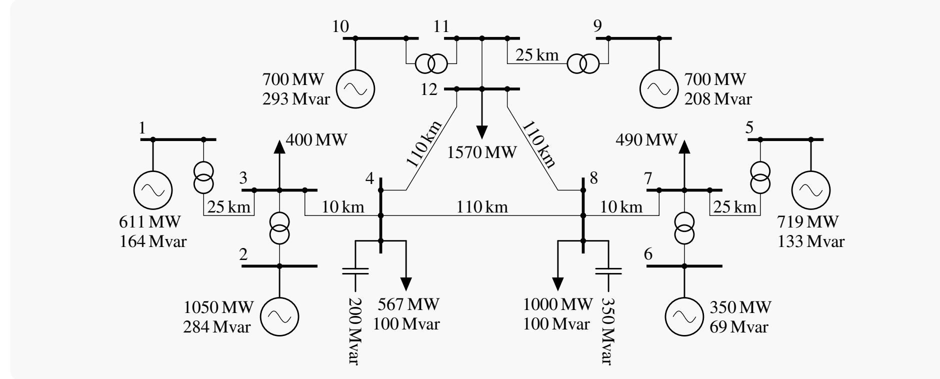
Source: *Energie-Museum Berlin*



are these suitable metrics ?

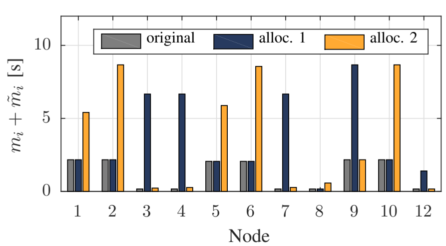
let's look at some simulations

Running example: modified Kundur three-area case study

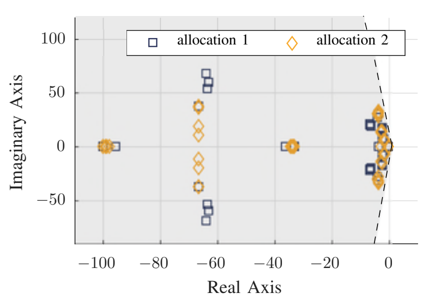


- added third area to standard case
- PLLs at all buses for inertia emulation (overall device response time ∼100ms)
- transformer reactance 0.15 p.u, line impedance (0.0001+0.001i) p.u./km
- original inertia 40s: removed of rotational 28s which can be re-allocated as virtual inertia
- added governors & droop control at all generators

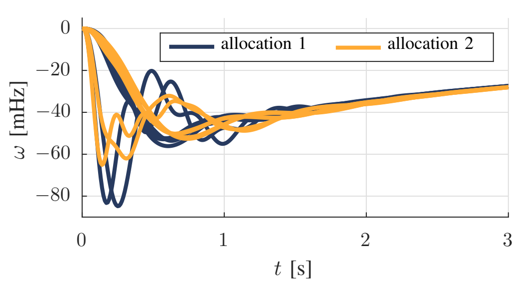
Fact: RoCoF, spectrum, & total inertia are poor metrics



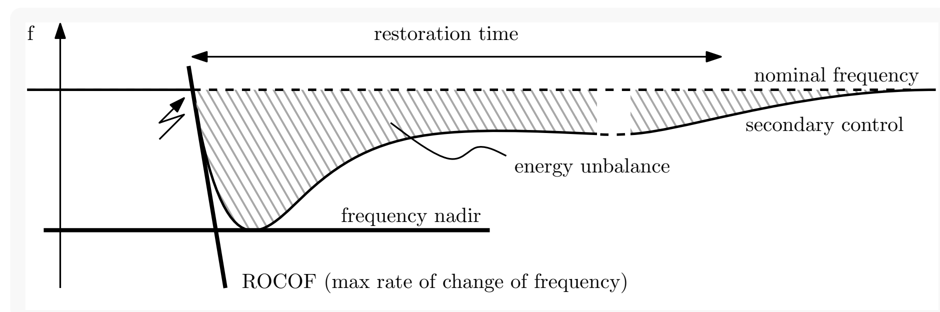
metrics	allocation 1	allocation 2
total inertia	40.85 s	40.85 s
damping ratio	0.1190	0.1206
RoCoF	0.8149 Hz/s	0.8135 Hz/s
ω nadir	-84.8 mHz	-65.1 mHz
peak injection	118.38 MW	7.0446 MW
control effort	15.581	2.699



comparison for 100 MW load step at bus 7



Performance metrics for low-inertia systems

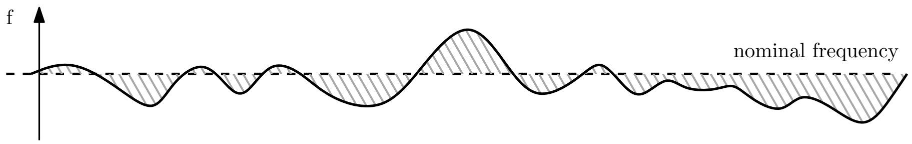


System norm quantifying signal amplifications

disturbances: impulse (fault), step (loss of unit), white noise (renewables) → **system** → **performance outputs:** integral, peak, ROCOF, restoration time, . . .

Integral-quadratic coherency performance metric

$$\int_0^\infty x(t)^T Q\, x(t)\, dt$$

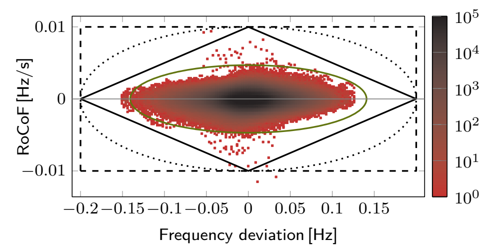


$\mathcal{H}_2$ **system norm** interpretation: $\eta \longrightarrow$ system $\longrightarrow y$

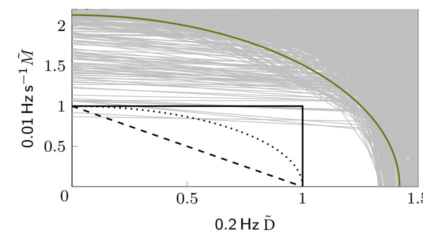
1. **performance output**: $\mathbf{y} = Q^{1/2} x$
2. **impulsive** η (faults) $\longrightarrow$ output energy $\int_0^\infty \mathbf{y}(t)^\mathsf{T}\, \mathbf{y}(t)\, dt$
3. **white noise** η (renewables) $\longrightarrow$ output variance $\lim_{t\to\infty} \mathbb{E}\left(\mathbf{y}(t)^\mathsf{T}\, \mathbf{y}(t)\right)$

Constraints on control inputs

1. **energy constraint**: $\int_0^\infty u^T R\, u\, dt$ directly captured in $\mathcal{H}_2$ framework
2. **power constraint**: $u_i = \tilde{M}_i\, \dot{\omega}_i + \tilde{D}_i\, \omega_i$ must satisfy $\|u_i(t)\|_{\ell_\infty} \leq \overline{u_i}$



European frequency data (source: RTE)



corresponding bounds on gains

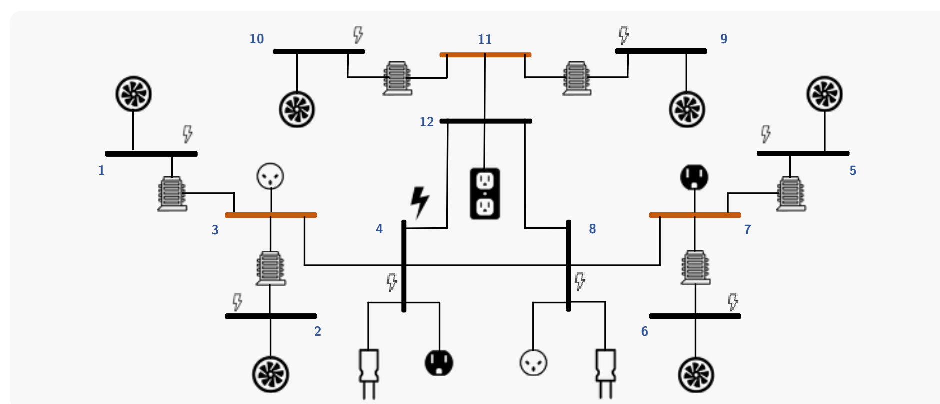
$\Rightarrow$ $\|(\omega_i(t), \dot{\omega}_i(t))\|_p$, $\|(\tilde{D}_i, \tilde{M}_i)\|_q$ bounded $(\frac{1}{p}+\frac{1}{q}=1) \Rightarrow \|u_i(t)\|_{\ell_\infty}$ bounded

3. **budget constraint** for finitely many devices: $\sum_i \overline{u_i} = const.$

(sub)optimize performance and see what we learn

Modified Kundur case study: 3 areas & 12 buses

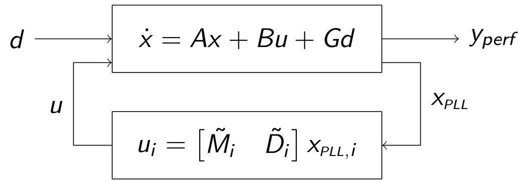
added governors (droop) at generators & PLLs to obtain frequency for inertia emulation



Test case

- **inertia emulation** control via PLL & batteries:

$$u_i = \begin{bmatrix} \tilde{M}_i & \tilde{D}_i \end{bmatrix} x_{PLL,i}$$



- **dynamics:** swing equation, droop via governor & turbine, and PLL

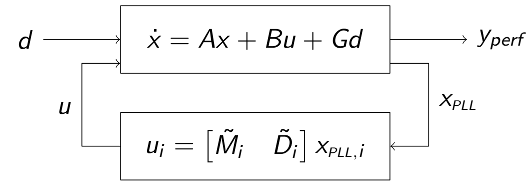
$$\begin{bmatrix} \dot{\delta} \\ \dot{\omega} \\ \dot{x}_{gov} \\ \dot{x}_{PLL} \end{bmatrix} = \underbrace{\begin{bmatrix} A_{sw} & B_{sw}K_{gov} & \mathbb{0} \\ B_{gov} & A_{gov} & \mathbb{0} \\ B_{PLL} & \mathbb{0} & A_{PLL} \end{bmatrix}}_{=A} x + \underbrace{\begin{bmatrix} B_{sw} \\ \mathbb{0} \\ \mathbb{0} \end{bmatrix}}_{=B} u + \underbrace{\begin{bmatrix} B_{sw} \\ \mathbb{0} \\ \mathbb{0} \end{bmatrix}}_{=G} d$$

- **cost** penalizes frequencies, droop control, & inertia emulation effort:

$$\underbrace{\begin{bmatrix} \omega \\ u_{gov} \\ u \end{bmatrix}}_{y_{perf}} = \underbrace{\begin{bmatrix} \mathbb{0} & I & \mathbb{0} & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & K_{gov} & \mathbb{0} \\ \mathbb{0} & \mathbb{0} & \mathbb{0} & \mathbb{0} \end{bmatrix}}_{=Q^{1/2}} x + \underbrace{\begin{bmatrix} \mathbb{0} \\ \mathbb{0} \\ I \end{bmatrix}}_{=R^{1/2}} u$$

Algorithmic approach to desperate & non-convex problem

- **structured** state-feedback with **constraints** on gains
- **computation** $\mathcal{H}_2$ norm, gradient, & projections:



1. observability and controllability Gramians via **Lyapunov equations**

$$(A - BK)^\top P + P(A - BK) + Q + K^\top RK = 0$$
$$(A - BK)L + L(A - BK)^\top + GG^\top = 0$$

2. $\mathcal{H}_2$ **norm** $J = \text{Trace}(G^\top PG)$ and **gradient** $\nabla_K J = 2(RK - B^\top P)L$
3. **projection** on structural & ∞-norm constraint: $\Pi_{\tilde{M},\tilde{D}}[\nabla_K J]$

$\Rightarrow$ $\tilde{M}$ and $\tilde{D}$ can be optimized by **first-order methods**, IPM, SQP, etc.

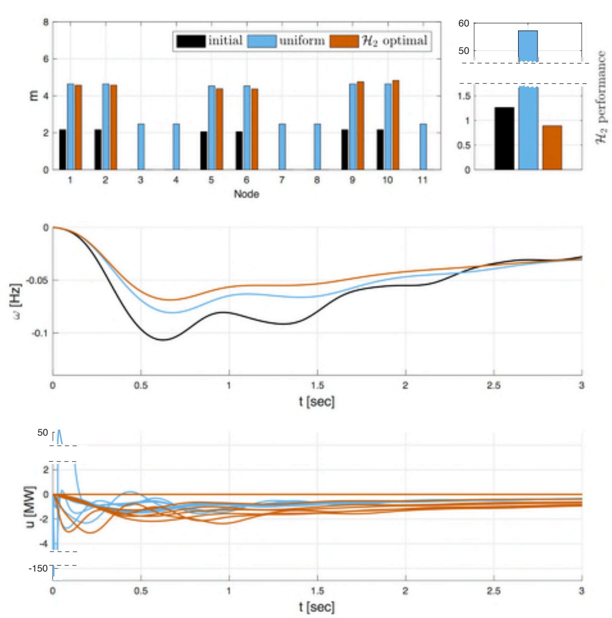
Results & insights for the three-area case study

Optimal allocation:

- location of inertia & damping matters
- outperforms heuristic uniform allocation
- need penalty on droop control effort
- power constraint results in $\tilde{D} \approx 2\tilde{M}$

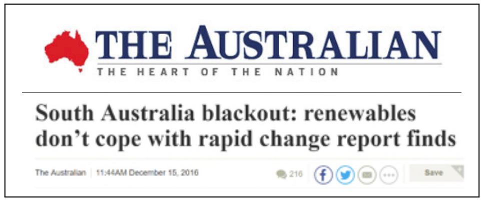
Fault at bus #4:

- strong reduction of frequency deviation
- much less control effort than heuristic



can we make this control design strategy useful ?

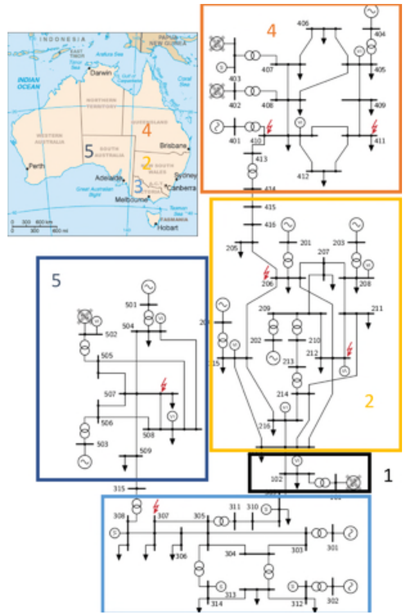
The concerns are not hypothetical: South Australia event



UPDATE REPORT –
BLACK SYSTEM EVENT
IN SOUTH AUSTRALIA ON
28 SEPTEMBER 2016

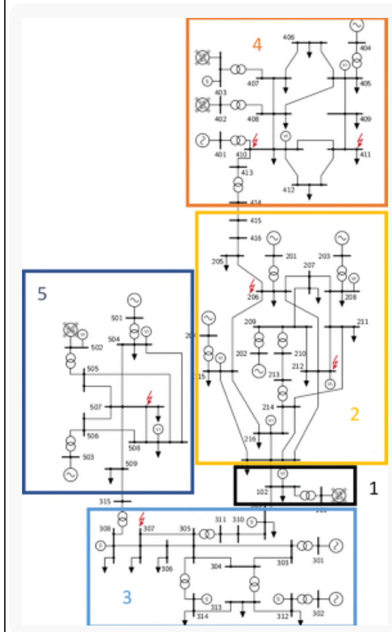
AEMO

AN UPDATE TO THE PRELIMINARY OPERATING INCIDENT REPORT FOR THE NATIONAL ELECTRICITY MARKET.
DATA ANALYSIS AS AT 5.00 PM TUESDAY 11 OCTOBER 2016.



my **conclusions** from official report:
blue area 5 was not resilient due to low inertia and poor wind turbine controls

Control & optimization design scale up to large systems

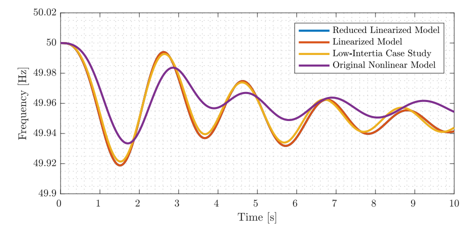


low-inertia **Eastern-Australian grid**:

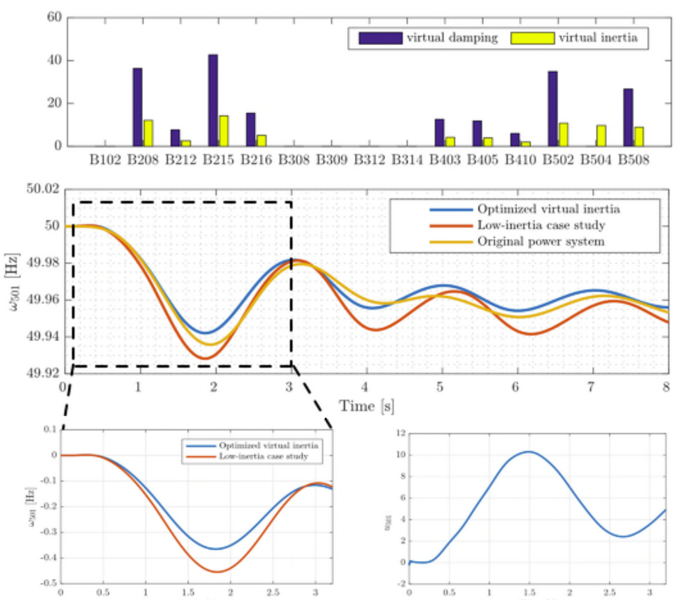
- removed rotational generation at buses 101, 402, 403 and 502
- added controllable power sources with PLLs at 15 buses

tractable model for design:

- linearization of nonlinear model
- balanced reduction to 140 states



$\mathcal{H}_2$-optimal virtual inertia allocation with ℓ_∞ constraints



allocation at core area 2 and critical areas 4 & 5

improves performance of low-inertia & original case

post-fault frequencies & control input well-behaved

placement & metrics matter!

can we get analytic insights ?

Inertia placement in swing equations

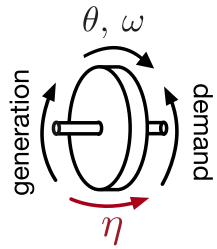
- simplified network swing equation **model**:

$$m_i \ddot{\theta}_i + d_i \dot{\theta}_i = p_{gen,i} - p_{dem,i}$$

generator swing equations

$$p_{dem,i} \approx \sum_j b_{ij} (\theta_i - \theta_j)$$

linearized DC power flow



- likelihood of **disturbance** at $\#i$: $\eta_i \geq 0$ (available from TSO data)
- $\mathcal{H}_2$ performance **metric**: $\int_0^\infty \sum_{i,j} a_{ij}(\theta_i - \theta_j)^2 + \sum_i s_i \dot{\theta}_i^2 \, dt$
- **decision variable** is inertia: $m_i \in [\underline{m_i}\, , \, \overline{m_i}]$

  (additional nonlinearity: enters as m_i^{-1} in constraints & objective)

Closed-form results for cost of primary control

recall: primary control $d_i \dot{\theta}_i$ **effort** was crucial

$$\int_0^\infty \dot{\theta}(t)^\top D \, \dot{\theta}(t) \, dt$$

(computations show that insights *roughly* generalize to other costs)

allocation: the primary control effort $\mathcal{H}_2$ optimization reads equivalently as

$$\underset{m_i}{\text{minimize}} \quad \sum_i \frac{\eta_i}{m_i}$$

$$\text{subject to} \quad \sum_i m_i \leq m_{\text{bdg}}$$

$$\underline{m_i} \leq m_i \leq \overline{m_i}$$

key take-away is **disturbance matching**:

- optimal allocation $m_i^\star \propto \sqrt{\eta_i}$ or $m_i^\star = \min\{m_{\text{bdg}}, \overline{m_i}\}$

⇒ disturbance profile known from historic data, but rare events are crucial

- suggests **robust** $\min_m \max_\eta$ **allocation** to prepare for worst case

⇒ valley-filling solution: $\eta_i^\star / m_i^\star = const.$ (up to constraints)

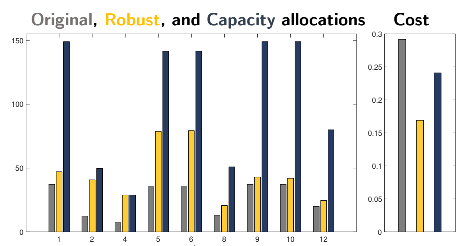
Robust min-max allocation for three-area case study

Scenario: fault (impulse) can occur at any single node

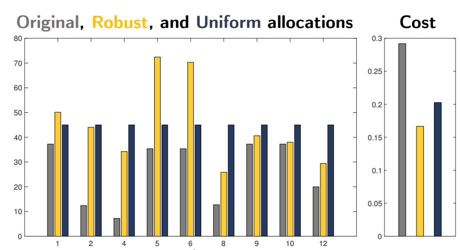
- disturbance set $\eta \in \{e_1 \cup \cdots \cup e_{12}\}$

⇒ **min/max** over convex hull

- inertia **capacity constraints**
- robust inertia allocation **outperforms heuristic** max-capacity allocation
- results become **intuitive**: valley-filling property
- same for uniform allocation



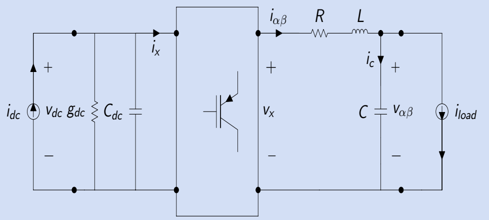
allocation subject to capacity constraints



allocation subject to the budget constraint

Outline

Averaged power converter model



DC cap & AC filter equations:

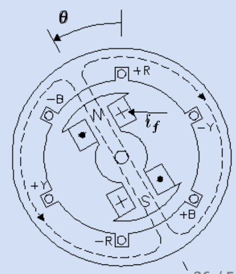
$$C_{dc}\dot{v}_{dc} = -G_{dc}v_{dc} + i_{dc} - \frac{1}{2}m^\top i_{\alpha\beta}$$
$$L\dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} + \frac{1}{2}mv_{dc} - v_{\alpha\beta}$$
$$C\dot{v}_{\alpha\beta} = -i_{load} + i_{\alpha\beta}$$

modulation: $v_x = \frac{1}{2}mv_{dc}$, $i_x = \frac{1}{2}m^\top i_{\alpha\beta}$

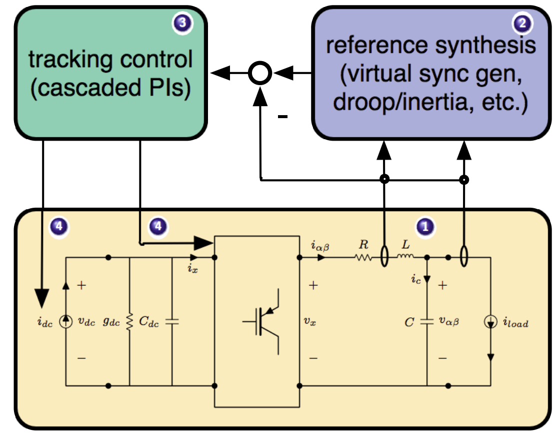
control/dist. inputs: (i_{dc}, i_{load})

synchronous generator:
mechanical
+ stator flux
+ AC cap

$$\dot{\theta} = \omega$$
$$M\dot{\omega} = -D\omega + \tau_m + i_{\alpha\beta}^\top L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$
$$L_s\dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} - v_{\alpha\beta} - \omega L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$
$$C\dot{v}_{\alpha\beta} = -i_{load} + i_{\alpha\beta}$$



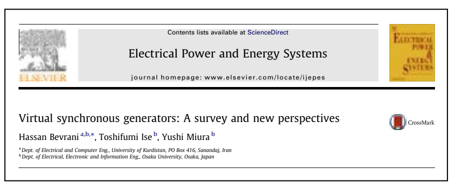
Standard power electronics control would continue by



1. acquiring & processing of **AC measurements**
2. synthesis of **references** (voltage/current/power)
3. **track** error signals at converter terminals
4. **actuation** via modulation (inner loop) and/or via DC source (outer loop)

I guess you can see the **problems building up** . . .

Challenges in power converter implementations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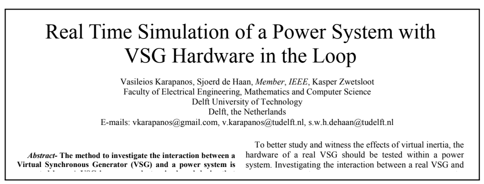
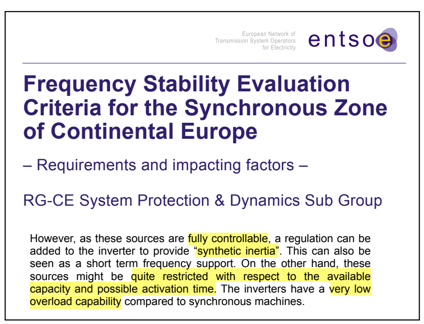
Virtual synchronous generators: A survey and new perspectives
Hassan Bevrani, Toshifumi Ise, Yushi Miura


Real Time Simulation of a Power System with VSG Hardware in the Loop

Vasileios Karapanos, Sjoerd de Haan, Member, IEEE, Kasper Zwetsloot
Faculty of Electrical Engineering, Mathematics and Computer Science
Delft University of Technology
Delft, the Netherlands

1. **delays** in measurement acquisition, signal processing, & actuation
2. **accuracy** in AC measurements (averaging over multiple cycles)
3. **constraints** on currents, voltages, power, etc.
4. **certificates** on stability, robustness, & performance


entsoe

Frequency Stability Evaluation Criteria for the Synchronous Zone of Continental Europe

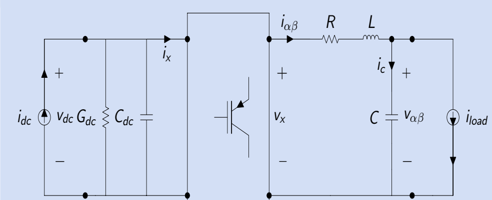
– Requirements and impacting factors –

RG-CE System Protection & Dynamics Sub Group

However, as these sources are fully controllable, a regulation can be added to the inverter to provide "synthetic inertia". This can also be seen as a short term frequency support. On the other hand, these sources might be quite restricted with respect to the available capacity and possible activation time. The inverters have a very low overload capability compared to synchronous machines.

let's do **something smarter** . . .

See the similarities & the differences ?



DC cap & AC filter equations:

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$$L\dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} + \frac{1}{2}mv_{dc} - v_{\alpha\beta}$$
$$C\dot{v}_{\alpha\beta} = -i_{load} + i_{\alpha\beta}$$

modulation: $v_x = \frac{1}{2}mv_{dc}$, $i_x = \frac{1}{2}m^\top i_{\alpha\beta}$

passive: $(i_{dc}, i_{load}) \rightarrow (v_{dc}, v_{\alpha\beta})$

synchronous generator:
mechanical
+ stator flux
+ AC cap

$$\dot{\theta} = \omega$$
$$M\dot{\omega} = -D\omega + \tau_m + i_{\alpha\beta}^\top L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$
$$L_s\dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} - v_{\alpha\beta} - \omega L_m i_f \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$
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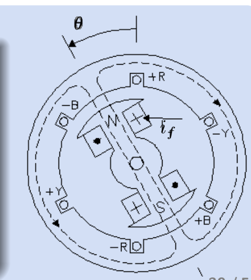
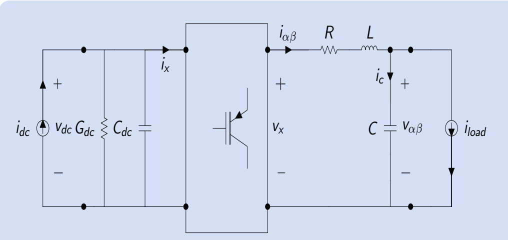


Model matching ($\neq$ emulation) as inner control loop



DC cap & AC filter equations:

$$C_{dc}\dot{v}_{dc} = -G_{dc}v_{dc} + i_{dc} - \frac{1}{2}m^\top i_{\alpha\beta}$$

$$L\dot{i}_{\alpha\beta} = -Ri_{\alpha\beta} + \frac{1}{2}mv_{dc} - v_{\alpha\beta}$$

$$C\dot{v}_{\alpha\beta} = -i_{load} + i_{\alpha\beta}$$

matching control: $\dot{\theta} = K_m \cdot v_{dc}\ ,\ m = \hat{m} \cdot \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$ with $K_m, \hat{m} > 0$

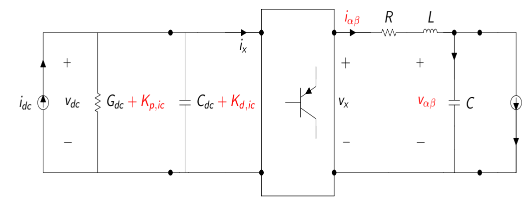
$\Rightarrow$ **equivalent** inertia $M = \frac{C_{dc}}{K_m^2}$, droop/dissipation $D = \frac{G_{dc}}{K_m^2}$, torque $\tau_m = \frac{i_{dc}}{K_m}$, field current $i_f = \frac{\hat{m}}{K_m L_m}$, & imbalance signal $\omega = K_m \cdot v_{dc}$

$\Rightarrow$ **pros:** uses physical storage, uses DC measurements, & remains passive

Further properties of machine matching control

1. base for **outer loops**

$\Rightarrow$ i_{dc} = **PD**(v_{dc}) gives virtual inertia & damping



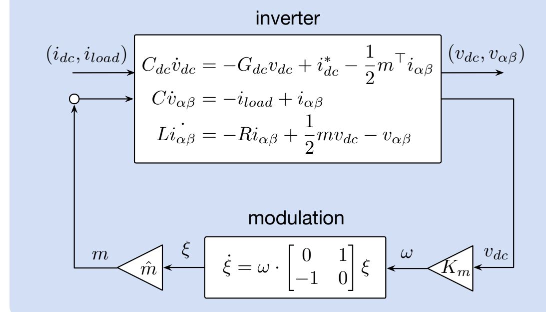
2. droop slopes & nose curves, & further outer loops $\hat{m}(\|v_{\alpha\beta}\|)$

3. reformulation of

$$m = \hat{m} \cdot \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

as adaptive **oscillator**:

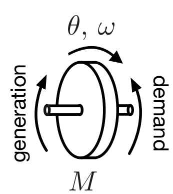
$$\dot{m} = K_m\, v_{dc} \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} m$$



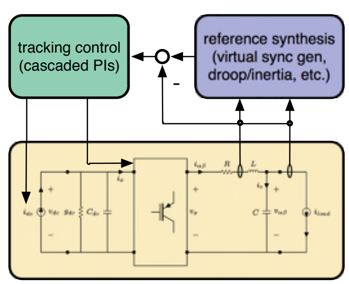
Summary: bottlenecks to inertia emulation

power system model on **grid level:**

$$M\frac{d}{dt}\omega = P_{\text{generation}} - P_{\text{demand}}$$



inertia emulation on **device level:**

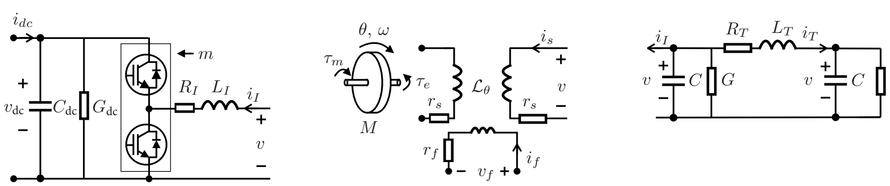


- **I/O mismatch:** none of the converter inputs or outputs are present in the swing-equation, e.g., frequency is not a state in the converter
- **inertia emulation** à la PD problematic both in theory & practice

$\Rightarrow$ maybe **matching control** $\dot{m} = K_m\, v_{dc} \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} m$ was quite clever ?

Outline

Low-inertia power system model from first principles



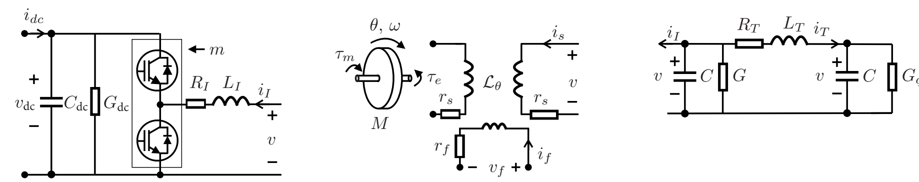
- balanced three-phase system
  - (α, β) coordinates
- synchronous machines
  - first principle, 5th order
- DC/AC inverters
  - averaged-switched
- nonlinear loads $G(\|v\|)$
- voltage bus charge dynamics
- dynamic transmission lines: Π-model

Port-Hamiltonian model

$$\dot{x} = \Big(J(x,u) - R(x)\Big)\,\nabla H(x) + g(x)u$$

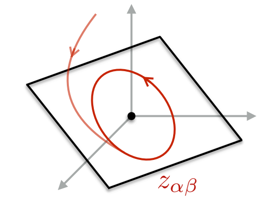
nonlinear & large, but insightful

Desired steady-state locus & control specifications



steady-state specifications for nonlinear system:

- synchronous frequency
- constant amplitude
- three-phase balanced



AC quantities v, i_s, i_I, i_T:

$$\dot{z}_{\alpha\beta} = \omega_0 \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} z_{\alpha\beta}$$

rotor angles: $\dot{\theta} = \omega_0$

DC quantities v_{dc}, v_f, ω: $\dot{z} = 0$

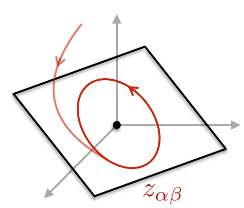
desired dynamics: $\dot{x} = f_{\text{des}}(x, \omega_0)$

controls i_{dc}, m, τ_m, i_f to be found

Proving the obvious (?)

- **steady-state locus**: physics & desired closed-loop vector field coincide (point-wise in time) on set

$$\mathcal{S} := \big\{(x, u, \omega_0) : \ f_{\text{phys}}(x,u) = f_{\text{des}}(x, \omega_0)\big\}$$



- **control-invariance**: steady-state operation $(x, u, \omega_0) \in \mathcal{S}$ for all time **if and only if**

  1. **synchronous frequency** ω_0 is constant
  2. **network** satisfies power flow equations with impedances $R + \omega_0 JL$
  3. at each **generator**: constant torque τ_m & excitation i_f
  4. at each **inverter**: constant DC current i_{dc} & inverter duty cycle with constant amplitude & synchronous frequency: $\dot{m} = \omega_0 \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} m$

  ⇒ **internal models** & feedforward **input-to-steady-state map**

Reduction to a tractable model for synthesis

- **internal oscillator model** for inverter duty cycle with inputs ω_m, $\hat{m}$

$$\dot{\theta}_I = \omega_m, \qquad m = \hat{m} \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

- **model reduction** steps

  1. **rotating coordinate frame** with synchronous frequency ω_0

     ⇒ time scales of AC quantities scaled by $1/\omega_0$

  2. **DC/AC time-scale separation** via singular perturbation ($\epsilon \to 0$)

     slow DC variables: $x_r = (\theta, \omega, i_f, \theta_I, v_{dc})$, $\quad \dot{x}_r = f_z(x_r, z_{\alpha,\beta}, u)$

     fast AC variables: $z_{\alpha,\beta} = (i_s, i_I, v, i_T)$, $\quad \epsilon\, \dot{z}_{\alpha,\beta} = f_{\alpha,\beta}(x_r, z_{\alpha,\beta}, u)$

  3. reformulation via **relative angles** δ with respect to synchronous motion

Insights from reduced model: $v_{dc} \propto$ power imbalance

- **nonlinear reduced order model** in rotating frame:

$$
\begin{aligned}
\dot{\theta} &= \omega \\
M\dot{\omega} &= -D\omega + \tau_m - \tau_e(x_r, u) \\
L_f \dot{i}_f &= -R_f i_f + v_f - v_{EMF}(x_r, u) \\
\dot{\theta}_I &= \omega_m \\
C_{dc}\dot{v}_{dc} &= -G_{dc} v_{dc} + i_{dc} - i_{sw}(x_r, u)
\end{aligned}
$$

- **interconnection** via τ_e, i_{sw}, v_{EMF}
- **analogies**: suggest matching control: $\omega_m \sim v_{dc}$

generator	inverter	interpretation
$\frac{1}{2}M\omega^2$	$\frac{1}{2}C_{dc}v_{dc}^2$	energy stored in device
τ_m	i_{dc}	energy supply
τ_e	i_{sw}	energy flow to grid
ω	v_{dc}	power imbalance

Completing the control design

Thus far:

1. desired **steady-state locus** requires internal oscillator model
2. **converter/generator analogies** suggest model matching control

Remaining steps:

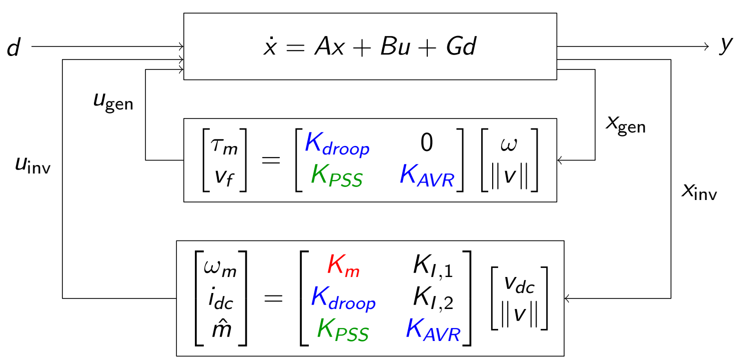
3. **robustness & stability under interconnection** requires local feedback passification with respect to an incremental energy function

$$H_{\text{des}}(x) = \frac{1}{2}\omega^T M\omega + \frac{1}{2}(i_f - i_f^*)^T L_f (i_f - i_f^*) + \frac{1}{2}(v_{dc} - v_{dc}^*)^T C_{dc}(v_{dc} - v_{dc}^*)^T + \ldots$$

$\Rightarrow$ associated passifying control is a scaled AC droop & DC droop

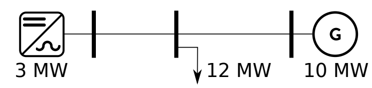
4. **performance** requires design of structured & optimal MIMO control

Decentralized MIMO control architecture



- **states** $x = (\delta, \omega, i_f, v_{dc}, \|v\|)$ & **output** $y = (\omega, v_{dc}, \|v\|)$
- included measurement devices for **AC voltage magnitude** $\|v\|$
- $\mathcal{H}_2$**-optimal tuning** of decentralized MIMO converter controller

Illustrative conceptual example



test case:

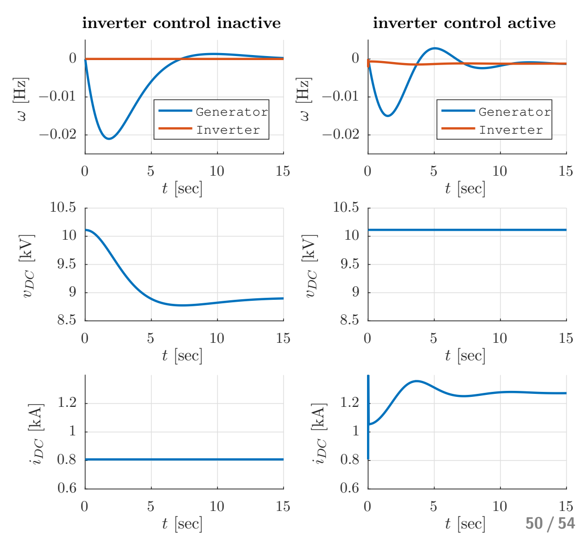
- generator & inverter
- impedance load
- 10% load increase at $t=0$

no inverter control:

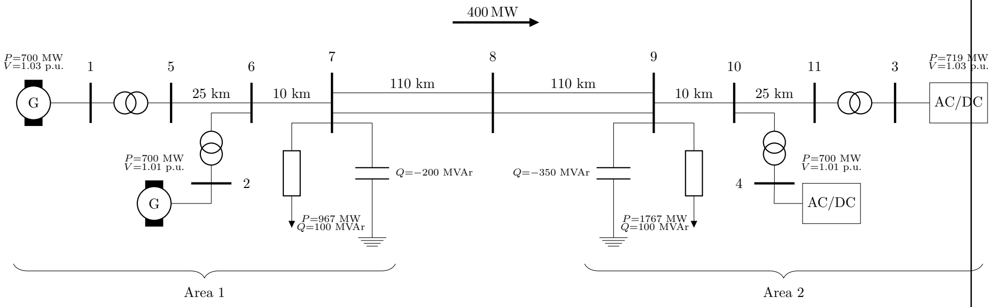
- ω_m and i_{dc} constant
- power imbalance: ω_G, v_{dc}
- governor stabilizes ω_G

controlled inverter:

- reduced peak in ω_G
- v_{dc} stabilized via i_{dc}
- ω_m and ω_G synchronize

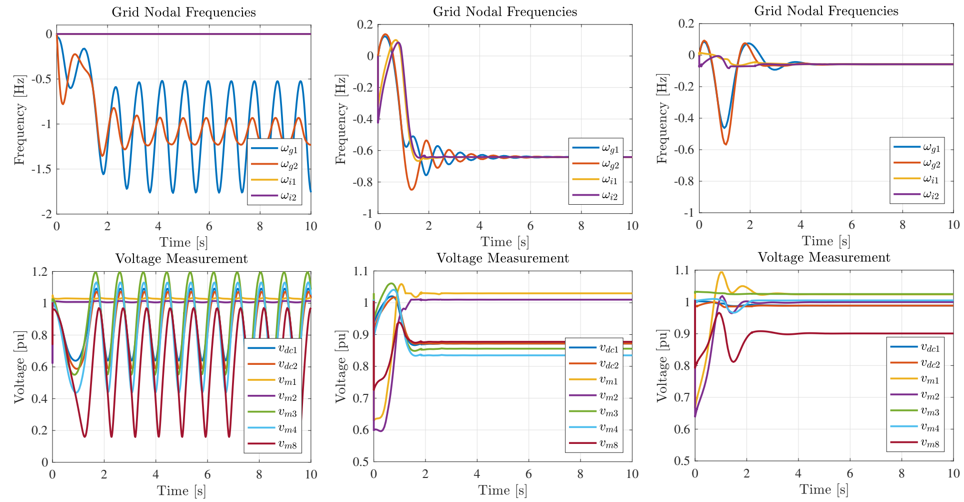


Modified Kundur two-area case study



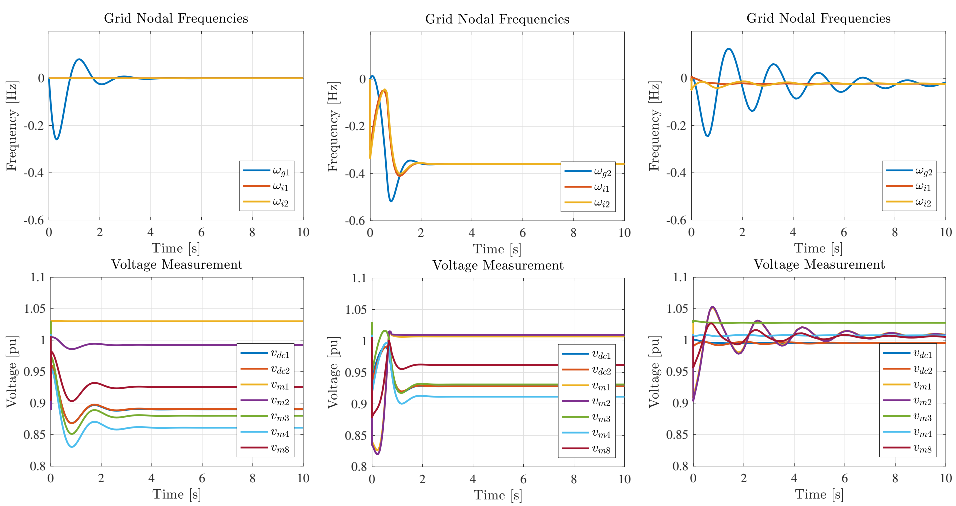
- standard line parameters and power flows
- synchronous machines with **droop control** and **voltage regulator**
- two **synchronous machines replaced by DC/AC inverters**
- all **dirt effects** modeled: saturation, nonlinearities, etc.
- simulation scenarios: **load step** (×2) & **outage** of synchronous machine

Scenario: load step & different converter controllers



feedforward control (power point tracking)	**matching control** & un-tuned MIMO gains	**$\mathcal{H}_2$-optimal control** (all gains tuned)

Scenario: outage of a synchronous machine



feedforward control (power point tracking)	**matching control** & un-tuned MIMO gains	**$\mathcal{H}_2$-optimal control** (all gains tuned)

conclusions

Conclusions on virtual inertia emulation

Where to do it?

1. $\mathcal{H}_2$-optimal (non-convex) allocation
2. numerical approach via gradient computation
3. closed-form results for cost of primary control

How to do it?

1. down-sides of naive inertia emulation
2. machine matching reveals power imbalance in DC voltage

What else to do?

1. first-principle low-inertia system model
2. nonlinear steady-state control specifications
3. reduction to tractable model for synthesis
4. first promising controller synthesis:
   internal model + matching + passifying + $\mathcal{H}_2$ performance loops