Online Feedback Optimization with Applications to Power Systems

Florian Dörfler

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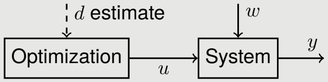
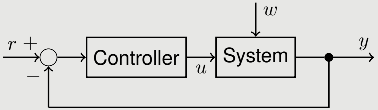
feedforward optimization vs. feedback control

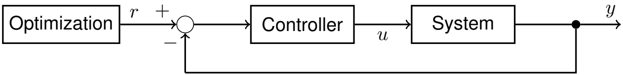


- **complex specifications & decision**
  optimal, constrained, & multivariable
- **strong requirements**
  precise model, full state, disturbance estimate, & computationally intensive



- **simple feedback policies**
  suboptimal, unconstrained, & SISO
- **forgiving nature of feedback**
  measurement driven, robust to uncertainty, fast & agile response

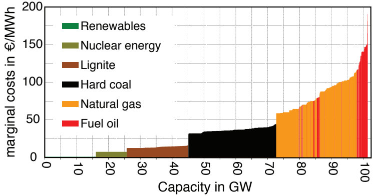
$\rightarrow$ typically **complementary** methods are combined via **time-scale separation**

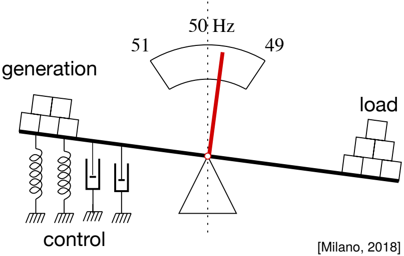


offline & feedforward | **real-time & feedback**

Example: power system balancing

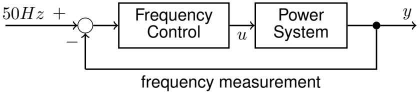
- **offline optimization**: dispatch based on forecasts of loads & renewables





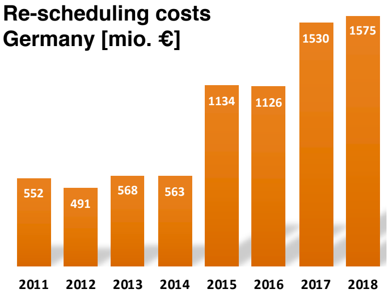
[Milano, 2018]

- **online control** based on frequency



- **re-schedule set-point** to mitigate severe forecasting errors (redispatch, reserve, etc.)

more uncertainty & fluctuations → **infeasible & inefficient** to separate optimization & control

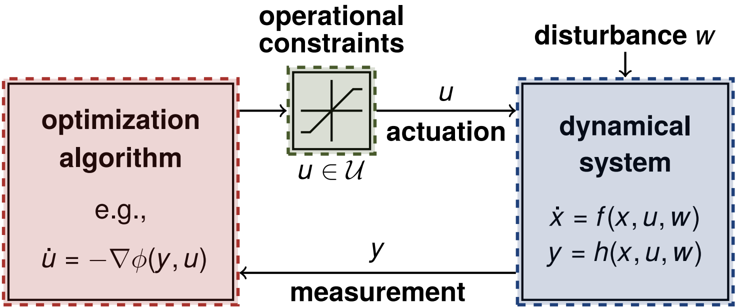


[Bundesnetzagentur, Monitoringbericht 2011-2019]

Synopsis & proposal for control architecture

- **power grid**: separate decision layers hit limits under increasing uncertainty
- similar observations in other **large-scale & uncertain control systems**: process control systems & queuing/routing/infrastructure networks

proposal: **open** (with inputs & outputs) and **online optimization algorithm** (iterative & non-batch) as **feedback** (real-time interconnected) control



Historical roots & conceptually related work

- **process control**: reducing the effect of uncertainty in sucessive optimization
  Optimizing Control [Garcia & Morari, 1981/84], *Self-Optimizing Control* [Skogestad, 2000], *Modifier Adaptation* [Marchetti et. al, 2009], *Real-Time Optimization* [Bonvin, ed., 2017], ...

- **extremum-seeking**: derivative-free but hard for high dimensions & constraints
  [Leblanc, 1922], ...[Wittenmark & Urquhart, 1995], ...[Krstić & Wang, 2000], ..., [Feiling et al., 2018]

- **MPC** with *anytime* guarantees (though for dynamic optimization): real-time MPC [Zeilinger et al. 2009], real-time iteration [Diel et al. 2005], [Feller & Ebenbauer 2017], etc.

- optimal routing, queuing, & congestion control in **communication networks**: e.g., TCP/IP [Kelly et al., 1998/2001], [Low, Paganini, & Doyle 2002], [Srikant 2012], [Low 2017], ...

- **optimization algorithms as dynamic systems**: much early work [Arrow et al., 1958], [Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], ... & recent revival [Holding & Lestas, 2014], [Cherukuri et al., 2017], [Lessard et al., 2016], [Wilson et al., 2016], [Wibisono et al, 2016], ...

- recent **system theory** approaches inspired by output regulation [Lawrence et al. 2018] & robust control methods [Nelson et al. 2017], [Colombino et al. 2018]

Theory literature inspired by power systems

- lots of recent theory development stimulated by **power systems** problems

  [Simpson-Porco et al., 2013], [Bolognani et al, 2015], [Dall'Anese & Simmonetto, 2016], [Hauswirth et al., 2016], [Gan & Low, 2016], [Tang & Low, 2017], . . .

  A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems

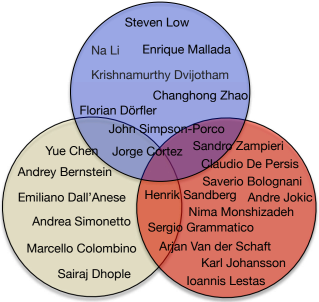
  Daniel K. Molzahn,* *Member, IEEE*, Florian Dörfler,† *Member, IEEE*, Henrik Sandberg,‡ *Member, IEEE*, Steven H. Low,§ *Fellow, IEEE*, Sambuddha Chakrabarti,¶ *Student Member, IEEE*, Ross Baldick,¶ *Fellow, IEEE*, and Javad Lavaei,** *Member, IEEE*

- **early adoption**: KKT control [Jokic et al, 2009]

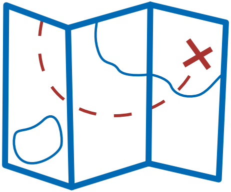
- literature **kick-started** $\sim$ 2013 by groups from Caltech, UCSB, UMN, Padova, KTH, & Groningen

- **changing focus**: distributed & simple $\rightarrow$ centralized & complex models/methods

- **implemented** in microgrids (NREL, DTU, EPFL, . . .) & conceptually also in transactive control pilots (PNNL)



Overview



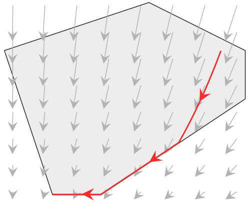
- algorithms & closed-loop stability analysis
- projected gradient flows on manifolds
- robust implementation aspects
- power system case studies throughout

ALGORITHMS & CLOSED-LOOP STABILITY ANALYSIS

Stylized optimization problem & algorithm

simple optimization problem

$$\begin{aligned} \underset{y,u}{\text{minimize}} \quad & \phi(y,u) \\ \text{subject to} \quad & y = h(u) \\ & u \in \mathcal{U} \end{aligned}$$

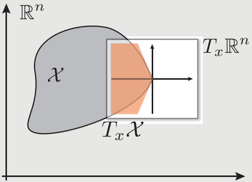


cont.-time **projected gradient flow**

$$\begin{aligned} \dot{u} &= \Pi^g_{\mathcal{U}}\Big(-\nabla\phi\big(h(u),u\big)\Big) \\ &= \Pi^g_{\mathcal{U}}\left(-\left[\tfrac{\partial h}{\partial u} \quad \mathbb{I}\right]\nabla\phi(y,u)\right)\Big|_{y=h(u)} \end{aligned}$$

projected dynamical system

$$\dot{x} \in \Pi^g_{\mathcal{X}}[f](x) \triangleq \underset{v\in T_x\mathcal{X}}{\arg\min} \|v - f(x)\|_{g(x)}$$



- domain $\mathcal{X}$
- vector field f
- metric g
- tangent cone $T\mathcal{X}$

all sufficiently regular†

Fact: a regular† solution $u : [0,\infty] \to \mathcal{X}$ **converges** to critical points if ϕ has Lipschitz gradient & compact sublevel sets.

† regularity conditions made precise later

Algorithm in closed-loop with LTI dynamics

optimization problem

$$\begin{aligned} \underset{y,u}{\text{minimize}} \quad & \phi(y,u) \\ \text{subject to} \quad & y = H_{io}u + R_{io}w \\ & u \in \mathcal{U} \end{aligned}$$

$\rightarrow$ open & scaled projected gradient flow

$$\dot{u} = \Pi_{\mathcal{U}}\Big(-\boldsymbol{\epsilon}\,[H_{io}^T \;\; \mathbb{I}]\,\nabla\phi(\boldsymbol{y},u)\Big)$$

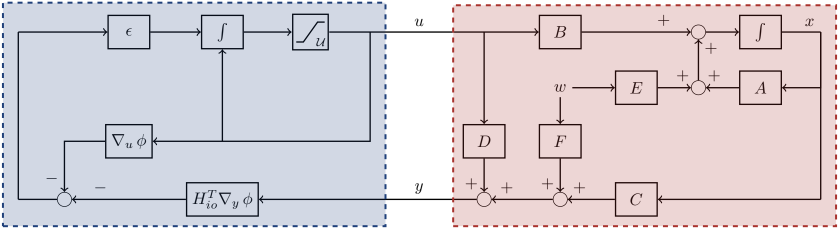
LTI dynamics

$$\begin{aligned} \dot{x} &= Ax + Bu + Ew \\ y &= Cx + Du + Fw \end{aligned}$$

const. disturbance w & steady-state maps

$$x = \underbrace{-A^{-1}B}_{H_{is}}\,u \;\; \underbrace{-A^{-1}E}_{R_{ds}}\,w$$

$$y = \underbrace{\left(D - CA^{-1}B\right)}_{H_{io}}u + \underbrace{\left(F - CA^{-1}E\right)}_{R_{do}}w$$



Stability, feasibility, & asymptotic optimality

Theorem: Assume that

- **regularity** of cost function ϕ: compact sublevel sets & ℓ-Lipschitz gradient
- LTI system asymptotically **stable**: $\exists \tau > 0\,,\, \exists P \succ 0 :\; PA + A^T P \preceq -2\tau P$
- sufficient **time-scale separation** (small gain): $0 < \epsilon < \epsilon^\star \triangleq \frac{2\tau}{\text{cond}(P)} \cdot \frac{1}{\ell \| H_{io} \|}$

Then the closed-loop system is **stable** and **globally converges** to the critical points of the **optimization problem** while remaining **feasible** at all times.

Proof: **LaSalle/Lyapunov** analysis via **singular perturbation** [Saberi & Khalil '84]

$$\Psi_\delta(u,e) \;=\; \delta \cdot \underbrace{e^T P\, e}_{\text{LTI Lyapunov function}} \;+\; (1-\delta) \cdot \underbrace{\phi\big(h(u), u\big)}_{\text{objective function}}$$

with **parameter** $\delta \in (0,1)$ & steady-state **error coordinate** $e = x - H_{is}u - R_{ds}w$

$\rightarrow$ derivative $\dot{\Psi}_\delta(u,e)$ is non-increasing if $\epsilon \leq \epsilon^\star$ and for optimal choice of δ

Example: optimal frequency control

- **dynamic LTI power system model**
  - power balancing **objective**
  - **control** generation set-points
  - unmeasured load **disturbances**
  - **measurements**: frequency + constraint variables (injections & flows)

- ▶ linearized swing dynamics
- ▶ 1st-order turbine-governor
- ▶ primary frequency droop
- ▶ DC power flow approximation

- **optimization problem**

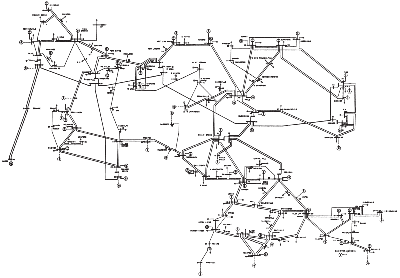
→ **objective**: $\phi(y,u) = \underbrace{\text{cost}(u)}_{\text{economic generation}} + \underbrace{\tfrac{1}{2}\|\max\{0, \underline{y} - y\}\|_{\Xi}^2 + \tfrac{1}{2}\|\max\{0, y - \overline{y}\}\|_{\Xi}^2}_{\text{operational limits (line flows, frequency, ...)}}$

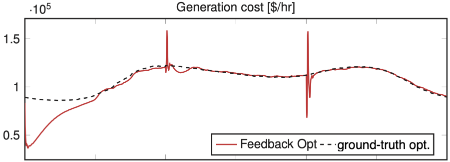
→ **constraints**: actuation $u \in \mathcal{U}$ & steady-state map $y = H_{io}u + R_{do}w$

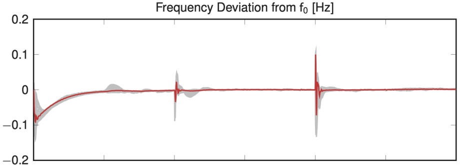
→ **control** $\dot{u} = \Pi_{\mathcal{U}}\left(\ldots \nabla\phi\right) \equiv$ super-charged Automatic Generation Control

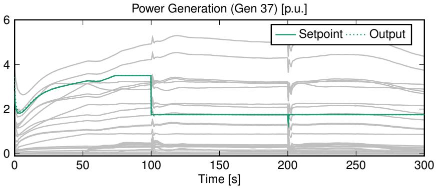
Test case: contingencies in IEEE 118 system

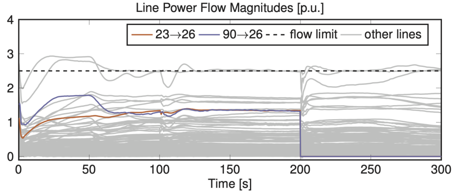
events: generator outage at 100 s & double line tripping at 200 s





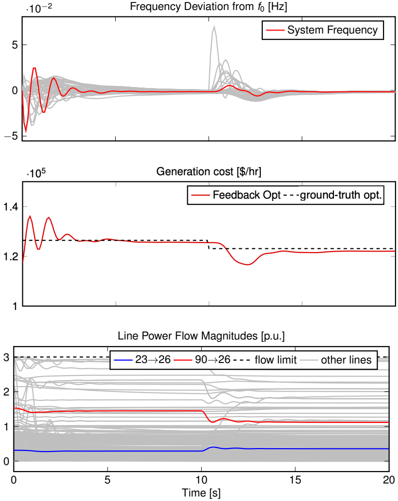




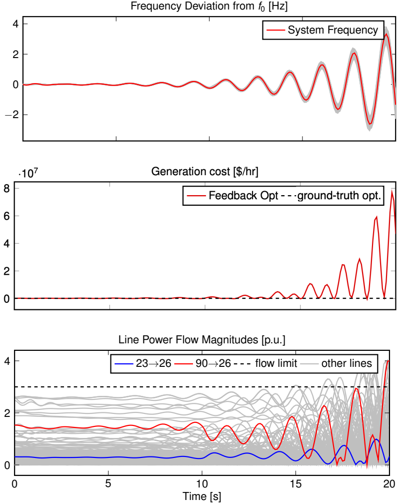


How conservative is $\epsilon < \epsilon^\star$?

still **stable for** $\epsilon = 2\,\epsilon^\star$



unstable for $\epsilon = 10\,\epsilon^\star$



Note: conservativeness problem dependent & depends, e.g., on penalty scalings

Highlights & comparison of approach

Weak assumptions on plant

- internal stability
- → no observability / controllability
- → no passivity or primal-dual structure
- measurements & steady-state I / O map
- → no knowledge of disturbances
- → no full state measurement
- → no dynamic model

Weak assumptions on cost

- Lipschitz gradient + properness
- → no (strict/strong) convexity required

Parsimonious but powerful setup

- potentially conservative bound, but
- → **minimal assumptions** on optimization problem & plant
- **robust & extendable proof**
- → nonlinear dynamics
- → time-varying disturbances
- → general algorithms

take-away: open online optimization algorithms can be applied in feedback

→ Hauswirth, Bolognani, Hug & Dörfler (2020)
"Timescale Separation in Autonomous Optimization"

→ Menta, Hauswirth, Bolognani, Hug & Dörfler (2018)
"Stability of Dynamic Feedback Optimization with Applications to Power Systems"

Nonlinear systems & general algorithms

- **general system dynamics** $\dot{x} = f(x,u)$ with **steady-state map** $x = h(u)$
- **incremental Lyapunov function** $W(x,u)$ w.r.t **error coordinate** $x - h(u)$

$$\dot{W}(x,u) \le -\gamma \, \|x - h(u)\|^2 \qquad \|\nabla_u W(x,u)\| \le \zeta \, \|x - h(u)\|$$

- **variable-metric** $Q(u) \in \mathbb{S}^n_+$ gradient flow

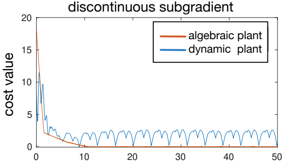
$$\dot{u} = -\,Q(u)^{-1}\,\nabla\phi(u)$$

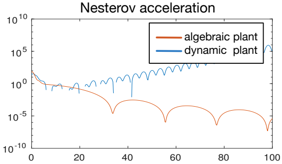
- **examples**: Newton method $Q(u) = \nabla^2\phi(u)$ or mirror descent $Q(u) = \nabla^2\psi(\nabla\psi(u)^{-1})$
- **stability condition**: $\frac{\zeta\ell}{\gamma} \cdot \sup_u \|Q(u)^{-1}\| \, < \, 1$

Similar results for algorithms with memory:

- **momentum methods** (e.g., heavy-ball)
- (exp. stable) **primal-dual saddle flows**

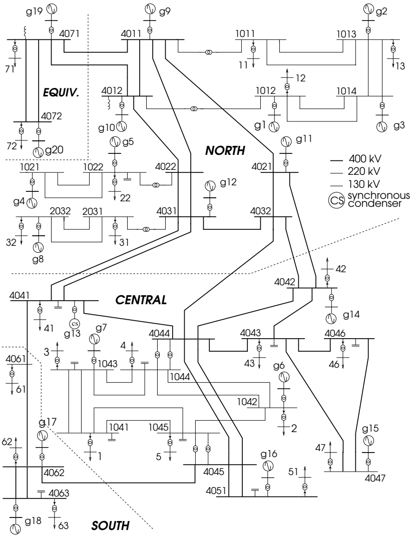
non-examples: bounded-metric or Lipschitz assumption violated



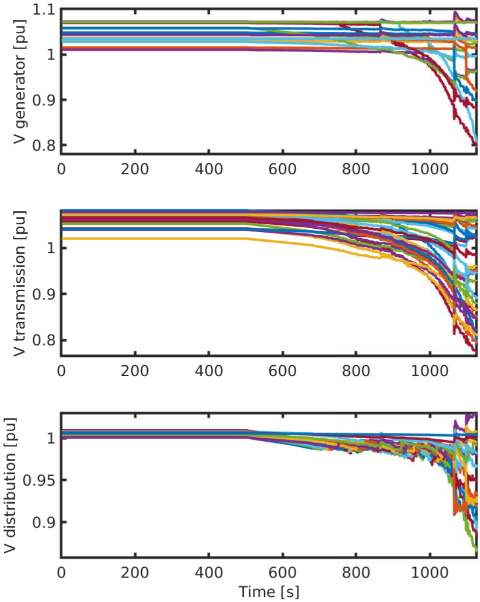


Highly nonlinear & dynamic test case

- **Nordic system**: case study known for voltage collapse (South Sweden '83)
- (static) **voltage collapse**: sequence of events → saddle-node bifurcation
- **high-fidelity model** of Nordic system
  - RAMSES + Python + MATLAB
  - state: heavily loaded system & large power transfers: north → central
  - load buses with Load Tap Changers
  - generators equipped with Automatic Voltage Regulators, Over Excitation Limiters, & speed governor control



Voltage collapse

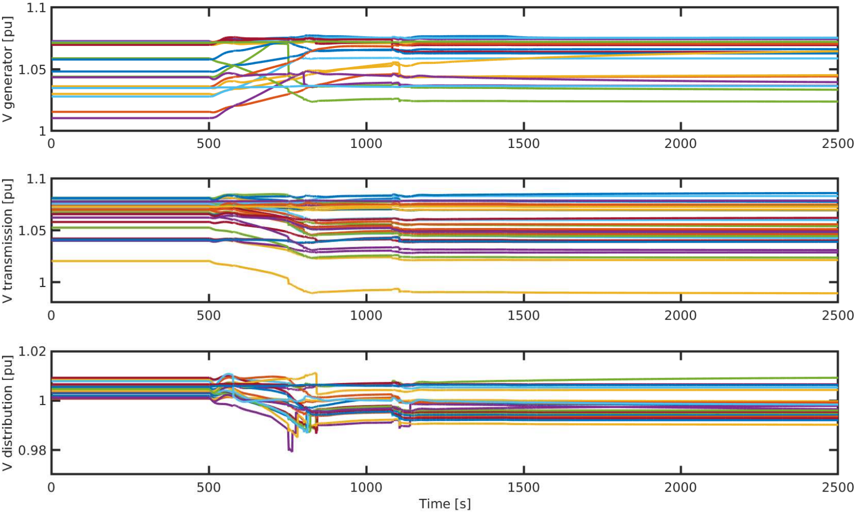


- **event:** 250 MW load ramp from $t = 500$ s to $t = 800$ s
- **unfortunate control response**: non-coordinated + saturation
  - extra demand is balanced by primary frequency control
  - cascade of activation of over-excitation limiters
  - load tap changers increase power demand at load buses
- **bifurcation**: voltage collapse
- **very hard to mitigate** via conventional controllers

$\rightarrow$ apply **feedback optimization to coordinate set-points** of Automatic Voltage Controllers

Voltage collapse averted !

distance-to-collapse objective : $\phi = -\log\det(\text{power flow Jacobian})$

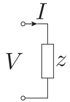


PROJECTED GRADIENT FLOWS ON MANIFOLDS

Motivation: steady-state AC power flow

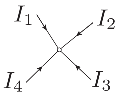
- **stationary model**

Ohm's Law



$V = zI$

Current Law



$0 = I_1 + \ldots + I_k$

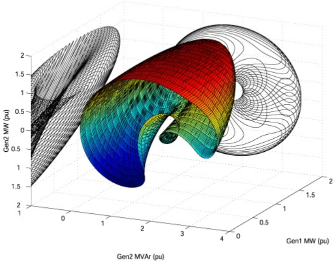
AC power $S = P + jQ = VI^*$

AC power flow equations

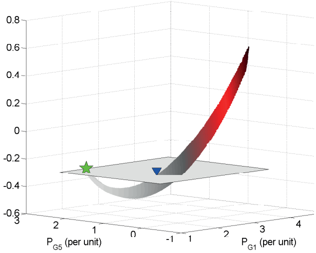
$$S_k = \sum_{l \in N(k)} \frac{1}{z_{kl}^*} V_k (V_k^* - V_l^*) \quad \forall k \in \mathcal{N}$$

- imagine **constraints slicing** this set
  $\Rightarrow$ nonlinear, non-convex, disconnected
- additionally the parameters are $\pm 20\%$ **uncertain** . . . this is only the steady state!

- **graphical illustration** of AC power flow

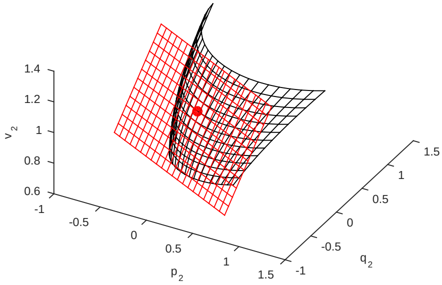


[Hiskens, 2001]



[Molzahn, 2016]

Key insights on physical equality constraint



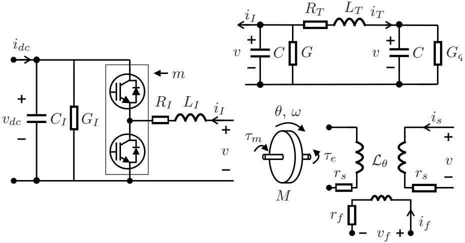
- AC power flow is complex but takes the form of a **smooth manifold**

→ local tangent plane approximations, local invertibility, & generic LICQ

→ **regularity** (algorithmic flexibility)

→ Hauswirth, Bolognani, Hug, & Dörfler (2015) **"Fast power system analysis via implicit linearization of the power flow manifold"**

→ Bolognani & Dörfler (2018) **"Generic Existence of Unique Lagrange Multipliers in AC Optimal Power Flow"**



- AC power flow is **attractive steady state** for ambient physical dynamics

→ physics enforce feasibility even for non-exact (e.g., discrete) updates

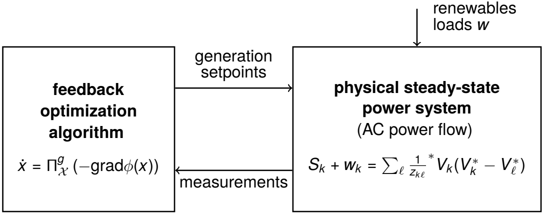
→ **robustness** (algorithm & model)

→ Gross, Arghir, & Dörfler (2018) **"On the steady-state behavior of a nonlinear power system model"**

Feedback optimization on the manifold

challenging **specifications on closed-loop trajectories**:

1. stay on manifold at all times
2. satisfy constraints at all times
3. converge to optimal solution

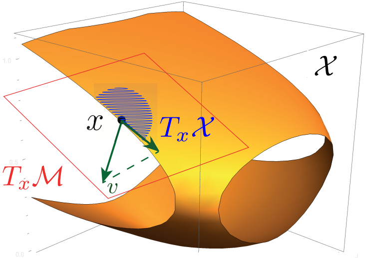


prototypical optimal power flow

minimize $\quad \phi(x)$

subject to $\quad x \in \mathcal{X} = \mathcal{M} \cap \mathcal{K}$

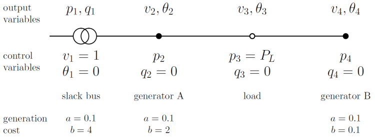
$\phi : \mathbb{R}^n \to \mathbb{R}$	objective function
$\mathcal{M} \subset \mathbb{R}^n$	AC power flow manifold
$\mathcal{K} \subset \mathbb{R}^n$	operational constraints

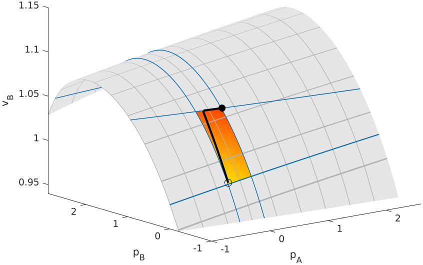


projection of trajectory on **feasible cone**

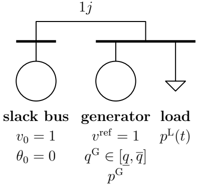
Simple low-dimensional case studies . . .

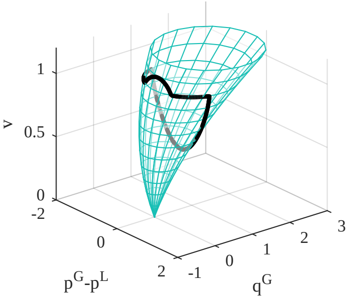
. . . can have **simple** feasible sets





. . . or can have **really complex** sets





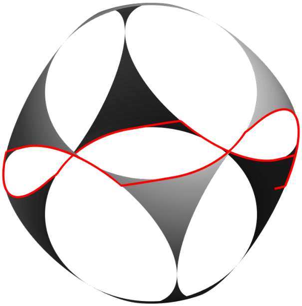
application demands sophisticated **level of generality** !

Projected dynamical systems on irregular domains

Theorem: Consider a Carathéodory solution $x : [0, \infty) \to \mathcal{X}$ of the initial value problem

$$\dot{x} = \Pi^g_{\mathcal{X}} \left(-\text{grad}\phi(x)\right) , \quad x(0) = x_0 \in \mathcal{X} .$$

If ϕ has compact sublevel sets on $\mathcal{X}$, then $x(t)$ converges to the set of critical points of ϕ on $\mathcal{X}$.



$$\mathcal{X} = \{x : \|x\|_2^2 = 1 , \|x\|_1 \leq \sqrt{2}\}$$

Hidden assumption: existence, uniqueness, & completeness of Carathéodory solution $x(t) \in \mathcal{X}$ in absence of convexity, Euclidean space, ...?

regularity conditions	constraint set	vector field	metric	manifold
existence of Krasovski	loc. compact	loc. bounded	bounded	C^1
existence of Carathéodory	Clarke regular	C^0	C^0	C^1
uniqueness of solutions	prox regular	$C^{0,1}$	$C^{0,1}$	$C^{1,1}$

$\longrightarrow$ Hauswirth, Bolognani, & Dörfler (2018)
"Projected Dynamical Systems on Irregular Non-Euclidean Domains for Nonlinear Optimization"

$\longrightarrow$ Hauswirth, Bolognani, Hug, & Dörfler (2016)
"Projected gradient descent on Riemanniann manifolds with applications to online power system optimization"

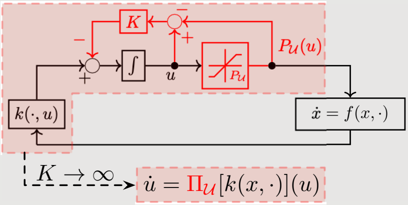
ROBUST IMPLEMENTATION ASPECTS

Robust implementation of projections

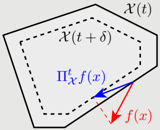
- projection & integrator → **windup**

→ **robust anti-windup** approximation

→ saturation often "for free" by physics



- disturbance → **time-varying domain**



- ▸ **temporal tangent cone** & vector field
- ▸ ensure suff. regularity & tracking certificates

→ Hauswirth, Dörfler, & Teel (2020)
"Anti-Windup Approximations of Oblique Projected Dynamical Systems for Feedback-based Optimization"

- handling **uncertainty** when enforcing **non-input constraints**: $x \in \mathcal{X}$ or $y \in \mathcal{Y}$

- ▸ **cannot measure** state x directly

→ Kalman filtering: estimation & separation

- ▸ **cannot enforce constraints** on $y = h(u)$ by projection (not actuated & $h(\cdot)$ unknown)

→ soft penalty or dualization + grad flows (inaccurate, violations, & strong assumptions)

→ project on **1st order prediction** of $y = h(u)$

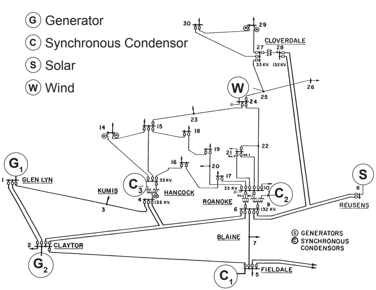
$$y^+ \approx \underbrace{h(u)}_{\text{measured}} + \epsilon \underbrace{\frac{\partial h}{\partial u}}_{\substack{\text{steady-state} \\ \text{I/O sensitivity}}} \underbrace{w}_{\substack{\text{feasible descent} \\ \text{direction}}}$$

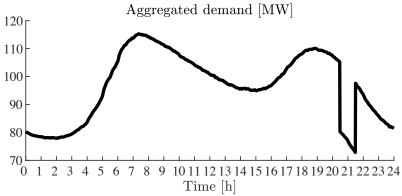
⇒ global convergence to critical points

→ Häberle, Hauswirth, Ortmann, Bolognani, & Dörfler (2020)
"Enforcing Output Constraints in Feedback-based Optimization"

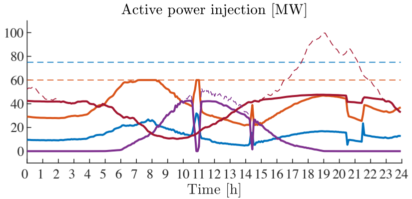
→ Hauswirth, Subotić, Bolognani, Hug, & Dörfler (2018)
"Time-varying Projected Dynamical Systems with Applications ..."

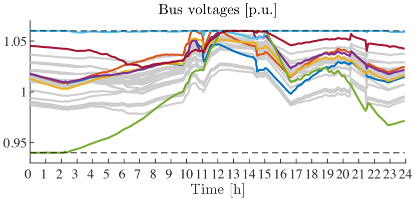
Tracking performance under disturbances

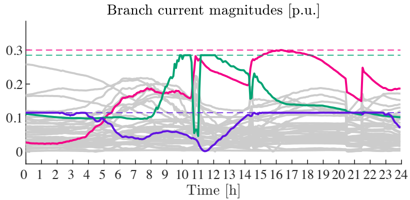




net demand: load, wind, & solar (discontinuous)

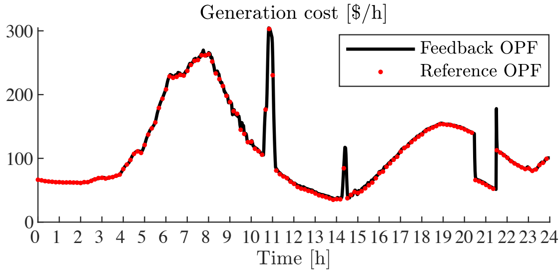






Optimality despite disturbances & uncertainty

- transient trajectory **feasibility**
- practically **exact tracking** of ground-truth optimizer
  (omniscient & no computation delay)
- **robustness** to model mismatch
  (asymptotic optimality under wrong model)

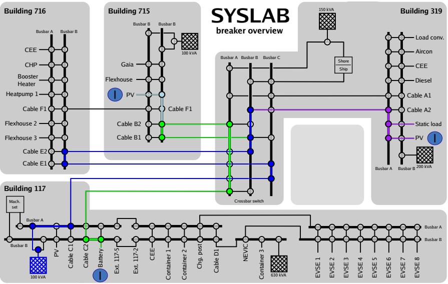


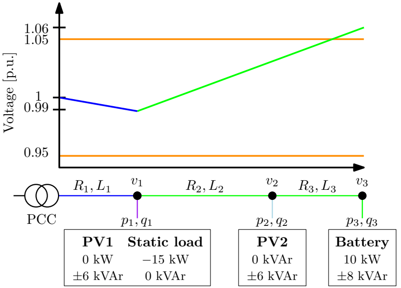
	offline optimization			feedback optimization		
model uncertainty	feasible ?	$\phi - \phi^*$	$\|v - v^*\|$	feasible ?	$\phi - \phi^*$	$\|v - v^*\|$
loads $\pm 40\%$	no	94.6	0.03	yes	0.0	0.0
line params $\pm 20\%$	yes	0.19	0.01	yes	0.01	0.003
2 line failures	no	-0.12	0.06	yes	0.19	0.007

conclusion: simple algorithm performs extremely well & robust

EXPERIMENTS

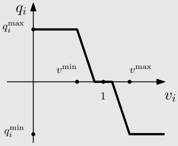
Experimental case study @ DTU





- 21 min **experiment** with events
  - $t = 3$ min: control turned ON
  - $t \in [11, 14]$ min: $P_{\text{batt}} = 0$ kW
- **base-line** controllers

  decentralized nonlinear proportional droop control (IEEE 1547.2018)

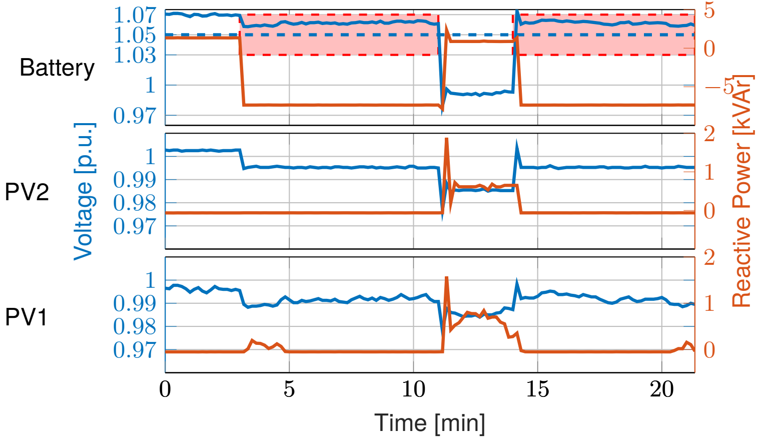


- **comparison** of three controllers
  - decentralized control
  - feedforward optimization
  - feedback optimization

→ Ortmann, Hauswirth, Caduff, Dörfler, & Bolognani (2020) **"Experimental Validation of Feedback Optimization in Power Distribution Grids"**

Decentralized feedback control

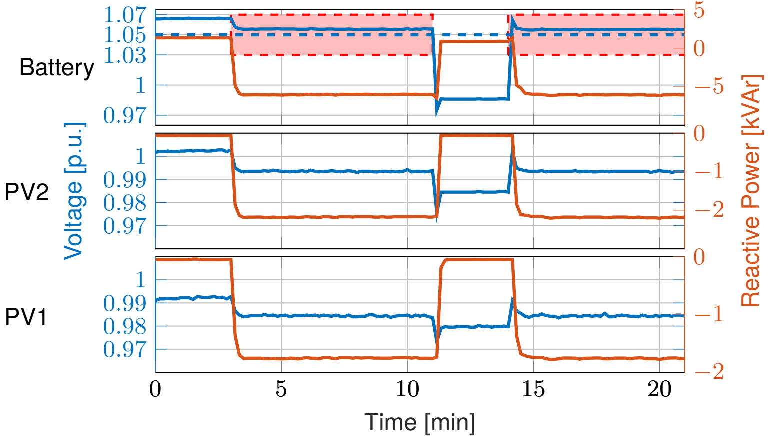
decentralized nonlinear proportional droop control



constraint violations due to local control saturation & lack of coordination

Successive feedforward optimization

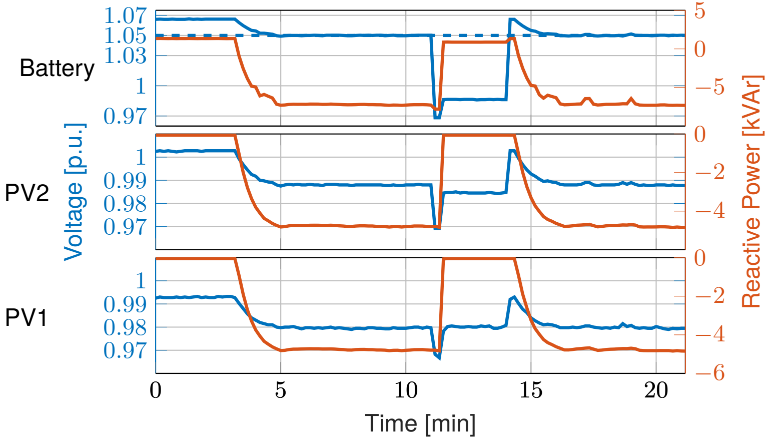
centralized, omniscient, & successively updated at high sampling rate



performs well but persistent **constraint violation** due to model uncertainty

Feedback optimization

primal-dual flow with 10 s sampling time requiring only model I/O sensitivity ∇h (or an estimate)



excellent performance & **model-free**(!) since $\nabla h(u)$ approximated by $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

CONCLUSIONS

Conclusions

Summary

- open & online **feedback optimization** algorithms as controllers
- **approach:** projected dynamical systems & time-scale separation
- **unified framework**: broad class of systems, algorithms, & programs
- illustrated throughout with non-trivial **power systems** case studies

Ongoing work & open directions

- **analysis**: robustness, performance, stochasticity, sampled-data
- **algorithms**: 0th-order, sensitivity estimation, distributed, minmax
- **power systems**: more experiments, virtual power plant extensions
- **further app's**: seeking optimality in uncertain & constrained systems

It works much better than it should ! We still need to fully grasp why ?

Thanks !

Florian Dörfler

http://control.ee.ethz.ch/~floriand

[link] to related publications