Online Feedback Optimization with Applications to Power Systems

Florian Dörfler

ETH Zürich

European Control Conference 2020
Acknowledgements

Adrian Hauswirth
Saverio Bolognani
Lukas Ortmann

Irina Subotić
Gabriela Hug
Miguel Picallo
Verena Häberle
feedforward optimization vs. feedback control

- **complex specifications & decision**
  optimal, constrained, & multivariable

- **strong requirements**
  precise model, full state, disturbance estimate, & computationally intensive

→ typically *complementary* methods are combined via *time-scale separation*

| offline & feedforward | real-time & feedback |
Example: power system balancing

- **Offline optimization**: dispatch based on forecasts of loads & renewables
  - Graph showing marginal costs in €/MWh against capacity in GW for different energy sources:
    - Renewables
    - Nuclear energy
    - Lignite
    - Hard coal
    - Natural gas
    - Fuel oil

- **Online control** based on frequency
  - $50\,\text{Hz} \pm u$
  - Diagram showing frequency control and power system

- **Re-schedule set-point** to mitigate severe forecasting errors (redispatch, reserve, etc.)

More uncertainty & fluctuations → **infeasible & inefficient** to separate optimization & control

---

[Re-scheduling costs Germany [mio. €]]

<table>
<thead>
<tr>
<th>Year</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
<th>2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costs</td>
<td>552</td>
<td>491</td>
<td>568</td>
<td>563</td>
<td>1134</td>
<td>1126</td>
<td>1530</td>
<td>1575</td>
</tr>
</tbody>
</table>

[Bundesnetzagentur, Monitoringbericht 2011-2019]
Synopsis & proposal for control architecture

- **power grid**: separate decision layers hit limits under increasing uncertainty
- similar observations in other **large-scale & uncertain control systems**: process control systems & queuing/routing/infrastructure networks

**Proposal:** open and online optimization algorithm as feedback control

- with inputs & outputs
- iterative & non-batch
- real-time interconnected

**Optimization Algorithm**

- e.g., \( \dot{u} = -\nabla \phi(y, u) \)

**Dynamical System**

- \( \dot{x} = f(x, u, w) \)
- \( y = h(x, u, w) \)

**Actuation**

- \( u \in U \)

**Measurement**

- \( y \)
Historical roots & conceptually related work

- **process control**: reducing the effect of uncertainty in successive optimization
  *Optimizing Control* [Garcia & Morari, 1981/84], *Self-Optimizing Control* [Skogestad, 2000], *Modifier Adaptation* [Marchetti et. al, 2009], *Real-Time Optimization* [Bonvin, ed., 2017], ...  

- **extremum-seeking**: derivative-free but hard for high dimensions & constraints  
  [Leblanc, 1922], ... [Wittenmark & Urquhart, 1995], ... [Krstić & Wang, 2000], ... , [Feiling et al., 2018]

- **MPC** with *anytime* guarantees (though for dynamic optimization): real-time MPC  
  [Zeilinger et al. 2009], real-time iteration [Diel et al. 2005], [Feller & Ebenbauer 2017], etc.

- optimal routing, queuing, & congestion control in **communication networks**:  
  e.g., TCP/IP [Kelly et al., 1998/2001], [Low, Paganini, & Doyle 2002], [Srikant 2012], [Low 2017], ...

- **optimization algorithms as dynamic systems**: much early work [Arrow et al., 1958], [Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], ... & recent revival [Holding & Lestas, 2014], [Cherukuri et al., 2017], [Lessard et al., 2016], [Wilson et al., 2016], [Wibisono et al, 2016], ...

- recent **system theory** approaches inspired by output regulation [Lawrence et al. 2018]  
  & robust control methods [Nelson et al. 2017], [Colombino et al. 2018]
Theory literature inspired by power systems

- lots of recent theory development stimulated by **power systems** problems
  
  [Simpson-Porco et al., 2013], [Bolognani et al., 2015], [Dall’Anese & Simmoneteto, 2016], [Hauswirth et al., 2016], [Gan & Low, 2016], [Tang & Low, 2017], …

- **early adoption**: KKT control [Jokic et al., 2009]

- literature **kick-started** ~ 2013 by groups from Caltech, UCSB, UMN, Padova, KTH, & Groningen

- **changing focus**: distributed & simple
  → centralized & complex models/methods

- **implemented** in microgrids (NREL, DTU, EPFL, …) & conceptually also in transactive control pilots (PNNL)
Overview

- algorithms & closed-loop stability analysis
- projected gradient flows on manifolds
- robust implementation aspects
- power system case studies throughout
ALGORITHMS & CLOSED-LOOP
STABILITY ANALYSIS
Stylized optimization problem & algorithm

**simple optimization problem**

\[
\begin{align*}
\text{minimize} & \quad \phi(y, u) \\
\text{subject to} & \quad y = h(u) \\
& \quad u \in \mathcal{U}
\end{align*}
\]

**cont.-time projected gradient flow**

\[
\dot{u} = \Pi_{\mathcal{U}}^g \left( -\nabla \phi(h(u), u) \right) \\
= \Pi_{\mathcal{U}}^g \left( - \left[ \frac{\partial h}{\partial u} \right] \nabla \phi(y, u) \right) \bigg|_{y=h(u)}
\]

**Fact:** A regular† solution \( u : [0, \infty] \to \mathcal{X} \) converges to critical points if \( \phi \) has Lipschitz gradient & compact sublevel sets.

† Regularity conditions made precise later
Algorithm in closed-loop with LTI dynamics

**optimization problem**

minimize \( \phi(y, u) \)

subject to \( y = H_{io}u + R_{io}w \)

\( u \in \mathcal{U} \)

→ open & scaled projected gradient flow

\[
\dot{u} = \Pi_\mathcal{U} \left( -\epsilon [H_{io}^T \ I] \nabla \phi(y, u) \right)
\]

**LTI dynamics**

\[
\dot{x} = Ax + Bu + Ew
\]

\[
y = Cx + Du + Fw
\]

const. disturbance \( w \) & steady-state maps

\[
x = \underbrace{\left( -A^{-1} B \right) u}_{H_{is}} \quad \underbrace{-A^{-1} E w}_{R_{ds}}
\]

\[
y = \underbrace{(D - C A^{-1} B) u}_{H_{io}} + \underbrace{(F - C A^{-1} E) w}_{R_{do}}
\]
Stability, feasibility, & asymptotic optimality

**Theorem:** Assume that
- **regularity** of cost function $\phi$: compact sublevel sets & $\ell$-Lipschitz gradient
- LTI system asymptotically **stable**: $\exists \tau > 0, \exists P > 0 : PA + A^T P \leq -2\tau P$
- sufficient **time-scale separation** (small gain): $0 < \epsilon < \epsilon^* \triangleq \frac{2\tau}{\text{cond}(P)} \cdot \frac{1}{\ell \|H_{io}\|}$

Then the closed-loop system is **stable** and **globally converges** to the critical points of the **optimization problem** while remaining **feasible** at all times.

**Proof:** LaSalle/Lyapunov analysis via **singular perturbation** [Saberi & Khalil ’84]

\[
\Psi_\delta(u, e) = \delta \cdot e^T P e \quad + \quad (1 - \delta) \cdot \phi(h(u), u)
\]

with **parameter** $\delta \in (0, 1)$ & steady-state **error coordinate** $e = x - H_{is}u - R_{ds}w$

$\rightarrow$ derivative $\dot{\Psi}_\delta(u, e)$ is non-increasing if $\epsilon \leq \epsilon^*$ and for optimal choice of $\delta$
Example: optimal frequency control

- **dynamic LTI power system model**
  - power balancing **objective**
  - control generation set-points
  - unmeasured load **disturbances**
- **measurements**: frequency + constraint variables (injections & flows)

**optimization problem**

→ **objective**: \( \phi(y, u) = \text{cost}(u) + \frac{1}{2} \| \max\{0, y - \bar{y}\} \|_2^2 + \frac{1}{2} \| \max\{0, y - \bar{y}\} \|_2^2 \)
  - economic generation
  - operational limits (line flows, frequency, \ldots)

→ **constraints**: actuation \( u \in U \) & steady-state map \( y = H_{io} u + R_{do} w \)

→ **control** \( \dot{u} = \Pi_U (\ldots \nabla \phi) \equiv \text{super-charged Automatic Generation Control} \)
Test case: contingencies in IEEE 118 system

events: generator outage at 100 s & double line tripping at 200 s
How conservative is $\epsilon < \epsilon^*$?

**still stable for** $\epsilon = 2\epsilon^*$

**unstable for** $\epsilon = 10\epsilon^*$

**Note:** conservativeness problem dependent & depends, e.g., on penalty scalings
Highlights & comparison of approach

**Weak assumptions on plant**
- internal stability
  - no observability / controllability
  - no passivity or primal-dual structure
- measurements & steady-state I/O map
  - no knowledge of disturbances
  - no full state measurement
  - no dynamic model

**Parsimonious but powerful setup**
- potentially conservative bound, but
  - **minimal assumptions** on optimization problem & plant
- robust & extendable proof
  - nonlinear dynamics
  - time-varying disturbances
  - general algorithms

**Weak assumptions on cost**
- Lipschitz gradient + properness
  - no (strict/strong) convexity required

**take-away:** open online optimization algorithms can be applied in feedback

→ Hauswirth, Bolognani, Hug & Dörfler (2020)
  “Timescale Separation in Autonomous Optimization”
  “Stability of Dynamic Feedback Optimization with Applications to Power Systems”
Nonlinear systems & general algorithms

- **general system dynamics** \( \dot{x} = f(x, u) \) with **steady-state map** \( x = h(u) \)
- **incremental Lyapunov function** \( W(x, u) \) w.r.t error coordinate \( x - h(u) \)

\[
\dot{W}(x, u) \leq -\gamma \|x - h(u)\|^2 \quad \|\nabla_u W(x, u)\| \leq \zeta \|x - h(u)\|
\]

- **variable-metric** \( Q(u) \in \mathbb{S}_+^n \) gradient flow

\[
\dot{u} = -Q(u)^{-1} \nabla \phi(u)
\]
- **examples**: Newton method \( Q(u) = \nabla^2 \phi(u) \)
or mirror descent \( Q(u) = \nabla^2 \psi(\nabla \psi(u)^{-1}) \)
- **stability condition**: \( \frac{\zeta}{\gamma} \cdot \sup_u \|Q(u)^{-1}\| < 1 \)

**non-examples**: bounded-metric or Lipschitz assumption violated

**Similar results for algorithms with memory:**
- **momentum methods** (e.g., heavy-ball)
- (exp. stable) **primal-dual saddle flows**
Highly nonlinear & dynamic test case

- **Nordic system**: case study known for voltage collapse (South Sweden ’83)
- **(static) voltage collapse**: sequence of events $\rightarrow$ saddle-node bifurcation
- **high-fidelity model** of Nordic system
  - RAMSES + Python + MATLAB
  - state: heavily loaded system & large power transfers: north $\rightarrow$ central
  - load buses with Load Tap Changers
  - generators equipped with Automatic Voltage Regulators, Over Excitation Limiters, & speed governor control
Voltage collapse

- **event:** 250 MW load ramp from $t = 500$ s to $t = 800$ s

- **unfortunate control response:** non-coordinated + saturation
  - extra demand is balanced by primary frequency control
  - cascade of activation of over-excitation limiters
  - load tap changers increase power demand at load buses

- **bifurcation:** voltage collapse

- **very hard to mitigate** via conventional controllers

  → apply **feedback optimization to coordinate set-points** of Automatic Voltage Controllers
Voltage collapse averted!

**distance-to-collapse objective**: $\phi = -\log \det(\text{power flow Jacobian})$
PROJECTED GRADIENT FLOWS ON MANIFOLDS
Motivation: steady-state AC power flow

- stationary model
- graphical illustration of AC power flow

**Ohm’s Law**

\[ V = zI \]

**Current Law**

\[ 0 = I_1 + \ldots + I_k \]

**AC power**

\[ S = P + jQ = VI^* \]

**AC power flow equations**

\[ S_k = \sum_{l \in N(k)} \frac{1}{2} V_k (V_k^* - V_l^*) \quad \forall k \in \mathcal{N} \]

- imagine **constraints slicing** this set
  \[ \Rightarrow \text{nonlinear, non-convex, disconnected} \]

- additionally the parameters are ±20% **uncertain** ... this is only the steady state!
Key insights on physical equality constraint

- **AC power flow is complex but takes the form of a smooth manifold**
  - local tangent plane approximations, local invertibility, & generic LICQ
  - regularity (algorithmic flexibility)

  → Hauswirth, Bolognani, Hug, & Dörfler (2015)
  "Fast power system analysis via implicit linearization of the power flow manifold"

  → Bolognani & Dörfler (2018)
  "Generic Existence of Unique Lagrange Multipliers in AC Optimal Power Flow"

- **AC power flow is attractive steady state** for ambient physical dynamics
  - physics enforce feasibility even for non-exact (e.g., discrete) updates
  - robustness (algorithm & model)

  → Gross, Arghir, & Dörfler (2018)
  "On the steady-state behavior of a nonlinear power system model"
Feedback optimization on the manifold

challenging **specifications** on closed-loop trajectories:
1. stay on manifold at all times
2. satisfy constraints at all times
3. converge to optimal solution

prototypical optimal power flow

\[
\begin{align*}
\text{minimize} & \quad \phi(x) \\
\text{subject to} & \quad x \in \mathcal{X} = \mathcal{M} \cap \mathcal{K} \\
\phi : \mathbb{R}^n & \rightarrow \mathbb{R} \quad \text{objective function} \\
\mathcal{M} \subset \mathbb{R}^n & \quad \text{AC power flow manifold} \\
\mathcal{K} \subset \mathbb{R}^n & \quad \text{operational constraints}
\end{align*}
\]

feedback optimization algorithm
\[
\dot{x} = \Pi^g_{\mathcal{X}} (-\nabla \phi(x))
\]

physical steady-state power system (AC power flow)
\[
S_k + w_k = \sum_{\ell} \frac{1}{z_{k\ell}} (V^*_k - V^*_\ell)
\]

projection of trajectory on feasible cone
Simple low-dimensional case studies . . .

... can have **simple** feasible sets

... or can have **really complex** sets

![Diagram showing simple feasible sets and complex sets](image)

**Application** demands sophisticated **level of generality**!
### Theorem:
Consider a Carathéodory solution \( x : [0, \infty) \to \mathcal{X} \) of the initial value problem
\[
\dot{x} = \Pi_{\mathcal{X}}^g (-\nabla \phi(x)) , \quad x(0) = x_0 \in \mathcal{X}.
\]
If \( \phi \) has compact sublevel sets on \( \mathcal{X} \), then \( x(t) \) converges to the set of critical points of \( \phi \) on \( \mathcal{X} \).

### Hidden assumption:
existence, uniqueness, & completeness of Carathéodory solution \( x(t) \in \mathcal{X} \) in absence of convexity, Euclidean space, …?

\[
\mathcal{X} = \{ x : \|x\|_2^2 = 1 , \|x\|_1 \leq \sqrt{2} \}
\]

### Regularity conditions

<table>
<thead>
<tr>
<th>Regularity conditions</th>
<th>Constraint set</th>
<th>Vector field</th>
<th>Metric</th>
<th>Manifold</th>
</tr>
</thead>
<tbody>
<tr>
<td>existence of Krasovski</td>
<td>loc. compact</td>
<td>loc. bounded</td>
<td>bounded</td>
<td>( C^1 )</td>
</tr>
<tr>
<td>existence of Carathéodory</td>
<td>Clarke regular</td>
<td>( C^0 )</td>
<td>( C^0 )</td>
<td>( C^1 )</td>
</tr>
<tr>
<td>uniqueness of solutions</td>
<td>prox regular</td>
<td>( C^{0,1} )</td>
<td>( C^{0,1} )</td>
<td>( C^{1,1} )</td>
</tr>
</tbody>
</table>

→ Hauswirth, Bolognani, & Dörfler (2018)
“Projected Dynamical Systems on Irregular Non-Euclidean Domains for Nonlinear Optimization”

→ Hauswirth, Bolognani, Hug, & Dörfler (2016)
“Projected gradient descent on Riemannian manifolds with applications to online power system optimization”
ROBUST IMPLEMENTATION ASPECTS
Robust implementation of projections

- **projection & integrator** $\rightarrow$ **windup**
  - **robust anti-windup** approximation
  - saturation often “for free” by physics

- **disturbance** $\rightarrow$ **time-varying domain**
  - **temporal tangent cone** & vector field
  - ensure suff. regularity & tracking certificates

- **handling uncertainty** when enforcing **non-input constraints**: $x \in \mathcal{X}$ or $y \in \mathcal{Y}$
  - cannot measure state $x$ directly
  - Kalman filtering: estimation & separation
  - cannot enforce constraints on $y = h(u)$ by projection (not actuated & $h(\cdot)$ unknown)
  - soft penalty or dualization + grad flows (inaccurate, violations, & strong assumptions)
  - project on $1^{st}$ order prediction of $y = h(u)$
    - $y^+ \approx h(u) + \epsilon \frac{\partial h}{\partial u} w$
    - measured | steady-state | feasible descent
      - I/O sensitivity | direction
  - $\Rightarrow$ global convergence to critical points

$\rightarrow$ Häberle, Hauswirth, Ortmann, Bolognani, & Dörfler (2020)
“Enforcing Output Constraints in Feedback-based Optimization”
$\rightarrow$ Hauswirth, Subotić, Bolognani, Hug, & Dörfler (2018)
“Time-varying Projected Dynamical Systems with Applications...”
Tracking performance under disturbances

- Generator
- Synchronous Condensor
- Solar
- Wind

5.3.2 30 Bus Power Flow Test Case
To investigate the capabilities of our new scheme under time-varying generation limits in \[\text{MVAr}\], and voltage limits in \[\text{p.u.}\]. The system base power is fixed to 100MVA.

Tracking performance under disturbances
net demand: load, wind, & solar (discontinuous)

Active power injection [MW]

Bus voltages [p.u.]

Branch current magnitudes [p.u.]

Aggregated demand [MW]
Optimality despite disturbances & uncertainty

- transient trajectory **feasibility**
- practically **exact tracking** of ground-truth optimizer
  (omniscient & no computation delay)
- **robustness** to model mismatch
  (asymptotic optimality under wrong model)

### Table 5.3: Model Uncertainty and Optimization Results

<table>
<thead>
<tr>
<th>Model Uncertainty</th>
<th>Offline Optimization</th>
<th>Feedback Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feasible?</td>
<td>(\phi - \phi^*)</td>
</tr>
<tr>
<td>loads (\pm 40%)</td>
<td>no</td>
<td>94.6</td>
</tr>
<tr>
<td>line params (\pm 20%)</td>
<td>yes</td>
<td>0.19</td>
</tr>
<tr>
<td>2 line failures</td>
<td>no</td>
<td>-0.12</td>
</tr>
</tbody>
</table>

### Conclusion:

*simple algorithm performs extremely well & robust*
EXPERIMENTS
21 min **experiment** with events

- \( t = 3 \text{ min: control turned ON} \)
- \( t \in [11, 14] \text{ min: } P_{\text{batt}} = 0 \text{ kW} \)

**base-line controllers**

decentralized nonlinear proportional droop control (IEEE 1547.2018)

**comparison** of three controllers

- decentralized control
- feedforward optimization
- feedback optimization

\[
\begin{align*}
q_i(t + 1) &= f_i(v_i(t)) \\
1 &\leq v_i, q_i, q_{\text{max}}, q_{\text{min}} \\
&\leq v_{\text{max}}, v_{\text{min}}, q_{\text{max}}, q_{\text{min}}
\end{align*}
\]
Decentralized feedback control

decentralized nonlinear proportional droop control

**constraint violations** due to local control saturation & lack of coordination
Successive feedforward optimization
centralized, omniscient, & successively updated at high sampling rate

performs well but persistent **constraint violation** due to model uncertainty
Feedback optimization

primal-dual flow with 10 s sampling time requiring only model I/O sensitivity $\nabla h$ (or an estimate)

excellent performance & model-free(!) since $\nabla h(u)$ approximated by $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
CONCLUSIONS
Conclusions

Summary

- open & online feedback optimization algorithms as controllers
- **approach**: projected dynamical systems & time-scale separation
- **unified framework**: broad class of systems, algorithms, & programs
- illustrated throughout with non-trivial **power systems** case studies

Ongoing work & open directions

- **analysis**: robustness, performance, stochasticity, sampled-data
- **algorithms**: 0th-order, sensitivity estimation, distributed, minmax
- **power systems**: more experiments, virtual power plant extensions
- **further app’s**: seeking optimality in uncertain & constrained systems

*It works much better than it should! We still need to fully grasp why?*
Thanks!

Florian Dörfler
http://control.ee.ethz.ch/~floriand
[link] to related publications