On the steady-state behavior of a nonlinear power network model

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Abstract

We consider a dynamic model of a three-phase power system including nonlinear generator dynamics, inverter dynamics transmission line dynamics, and static nonlinear loads. We study conditions under which the power system admits a steady-state behavior characterized by balanced and sinusoidal three-phase AC signals of the same synchronous frequency as well as a power balance for each single device. Necessary and sufficient conditions on the inputs are derived which ensure that the set on which the dynamics of the power system match the desired steady-state behavior is control-invariant. Subsequently, we arrive at a separation between transmission network, generators, and inverters, which allows us to recover the entire steady-state behavior solely from a prescribed operating point of the transmission network. Moreover, we constructively obtain network balance equations typically encountered in power flow analysis and show that the power system admits the desired steady-state if the network balance equations can be solved.

Power Network Model in Stationary Coordinates ($\alpha, \beta$)

Steady-State Specification

- **steady-state behavior specified by synchronous frequency $\omega_0$ and constant amplitude**
  - Generators: $\dot{\theta}_l = \omega_0\lambda_l$, $\dot{p}_l = 0$, $\lambda_l = \omega_0j\lambda_l$, $\dot{q}_l = \omega_0q_l$
  - Inverters: $\dot{q}_{\lambda,k} = 0$, $\lambda_{\lambda,k} = \omega_0j\lambda_{\lambda,k}$, $\dot{q}_{\lambda,k} = \omega_0q_{\lambda,k}$
  - Loads: $\dot{q}_l = \omega_0q_l$
  - Transmission lines: $\lambda_{\lambda,k} = \omega_0j\lambda_{\lambda,k}$

- rotations parametrized by $j = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $\beta = \begin{bmatrix} j & 0 \\ 0 & 1 \end{bmatrix}$

- steady-state specification: $\dot{x} = f_s(x, \omega_0)$

- vector fields coincide (point wise in time) on the set $S$
  $S := \{(x, u, \omega_0) \in \mathbb{R}^{3n} \times \mathbb{R}^{3n} \times \mathbb{R} \mid f(x, u) = f_0(x, \omega_0)\}$

- specifies constant energy in each device

Controlled-Invariance Condition

- **steady-state operation requires $(x, u, \omega_0) \in S$ for all time:**
  - i.e. if and only if $\frac{d}{dt}f(x, u) - f_0(x, \omega_0) = 0$ for all $(x, u, \omega_0) \in S$

- this holds if and only if:
  - inputs $\tau_m$, $v_f$, and $i_{DC}$ are constant
  - $m$ oscillates with synchronous frequency $\omega_0$ and constant amplitude
  - frequency $\omega_0$ is constant

Exploiting the Networked System Structure

- the system admits a non-trivial synchronous steady-state if and only if the network nodal current balance equations have a non-trivial solution

- nodal current balance equations (=power flow equation):
  $P_{\lambda/n} := \{(i_s, i_{ij}, v) \in \mathbb{R}^{2n} \mid (i_s, i_{ij}, 0, 0) + Y_{uv} = 0_n\}$

- network admittance matrix $Y_{\lambda/n} = Y_{\lambda} + Z \bar{C}^T L_T^T$ with $Y_{\lambda} = G_x + \omega_0j L_T C$ (shunt load admittance)

- nonlinear load $l \in L_k = G_{\lambda,k}C_{\lambda,k}L_{\lambda,k} + \frac{l_{\lambda,k}}{L_{\lambda,k}} C_{\lambda,k} C_{\lambda,k} L_{\lambda,k}$

- Transmission line in $T_k$: $\lambda_{\lambda,k} = -R_{\lambda,k} C_{\lambda,k} L_{\lambda,k} + \frac{C_{\lambda,k}}{L_{\lambda,k}} C_{\lambda,k} L_{\lambda,k}$

Summary

- steady-state analysis of a nonlinear three phase power network including synchronous machines, DC/AC inverters, and transmission network

- well-known conditions for steady-state operation can be constructively obtained from first-principles

- constant generator inputs induce oscillatory steady-state described by network balance equations

- non-trivial solution of the power flow equations corresponds to desired steady-state

- separation principle allows to recover the full system state from a solution to the balance equations of the transmission network

References


