On the steady-state behavior of a nonlinear power network model

Catalin Arghir, Dominic Groß, and Florian Dörfler

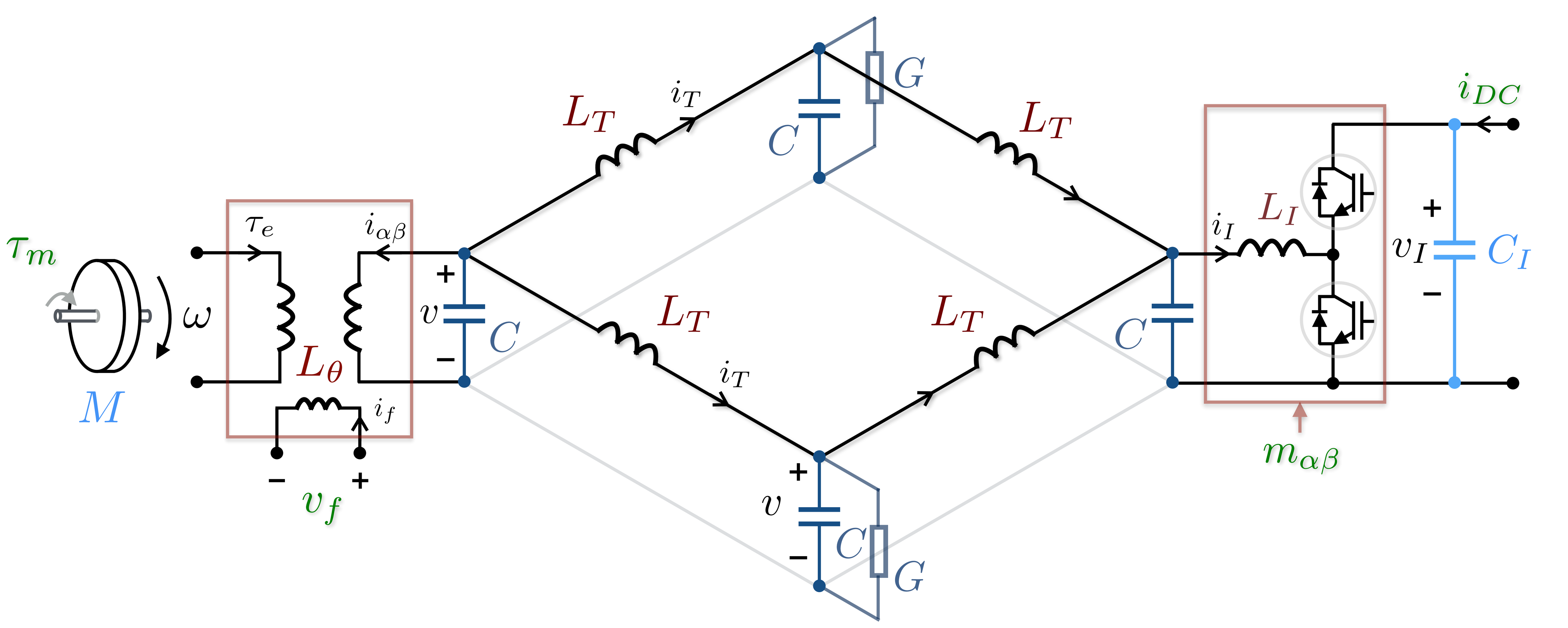
Automatic Control Laboratory, ETH Zürich, Switzerland

Abstract

We consider a dynamic model of a three-phase power system including nonlinear generator dynamics, inverter dynamics transmission line dynamics, and static nonlinear loads. We study conditions under which the power system admits a steady-state behavior characterized by balanced and sinusoidal three-phase AC signals of the same synchronous frequency as well as a power balance for each single device. Necessary and sufficient conditions on the inputs are derived which ensure that the set on which the dynamics of the power system match the desired steady-state behavior is control-invariant. Subsequently, we arrive at a separation between transmission network, generators, and inverters, which allows us to recover the entire steady-state behavior solely from a prescribed operating point of the transmission network. Moreover, we constructively obtain network balance equations typically encountered in power flow analysis and show that the power system admits the desired steady-state if the network balance equations can be solved.

Power Network Model in Stationary Coordinates (α,β)

Hamiltonian dynamics $\dot{x} = (J - R)\nabla\mathcal{H}(x) + gu = f(x,u)$



Hamiltonian (total energy):

$$\mathcal{H}(x) = \frac{1}{2}\left(p^\top M^{-1}p + \lambda^\top \mathcal{L}_\theta^{-1}\lambda + q_I^\top C_I^{-1} q_I + \lambda_I^\top L_I^{-1}\lambda_I + q^\top C^{-1} q + \lambda_T^\top L_T^{-1}\lambda_T\right), \quad \nabla\mathcal{H}(x) = (\omega, i, v_I, i_I, v, i_T)$$

Synchronous machine $k \in \mathbb{G}$:

$$\dot{\theta}_k = {M_k}^{-1} p_k$$

$$\dot{p}_k = -D_k {M_k}^{-1} p_k - \tau_{e,k} + \tau_{m,k}$$

$$\dot{\lambda}_k = -R_k \mathcal{L}_{\theta,k}^{-1}\lambda_k + \begin{bmatrix} C_k^{-1} q_k \\ v_{f,k} \end{bmatrix}$$

$$\dot{q}_k = -G_k C_k^{-1} q_k - \mathcal{I}_s \mathcal{L}_{\theta,k}^{-1}\lambda_k - \mathcal{E}_{q,k} L_T^{-1}\lambda_T$$

Inductance matrix:

$$\mathcal{L}_{\theta,k} = \begin{bmatrix} l_{s,k} & 0 & l_{m,k}\cos(\theta_k) \\ 0 & l_{s,k} & l_{m,k}\sin(\theta_k) \\ l_{m,k}\cos(\theta_k) & l_{m,k}\sin(\theta_k) & l_{r,k} \end{bmatrix}$$

DC/AC Inverter $k \in \mathbb{I}$:

$$\dot{q}_{I,k} = -G_{I,k} C_{I,k}^{-1} q_{I,k} + \frac{1}{2}\lambda_{I,k}^\top L_{I,k}^{-1} m_k + i_{DC,k}$$

$$\dot{\lambda}_{I,k} = -R_{I,k} L_{I,k}^{-1}\lambda_{I,k} + C_k^{-1} q_k - \frac{1}{2} C_{I,k}^{-1} q_{I,k} m_k$$

$$\dot{q}_k = -G_k C_k^{-1} q_k - L_{I,k}^{-1}\lambda_{I,k} - \mathcal{E}_{q,k} L_T^{-1}\lambda_T$$

Nonlinear load $k \in \mathbb{L}$:

$$\dot{q}_k = -G_k(\|q_k\|) C_k^{-1} q_k - \mathcal{E}_{q,k} L_T^{-1}\lambda_T$$

Transmission line $k \in \mathbb{T}$:

$$\dot{\lambda}_{T,k} = -R_{T,k} L_{T,k}^{-1}\lambda_{T,k} + \mathcal{E}_{T,k}^\top C^{-1} q$$

Steady-State Specification

- steady-state behavior specified by **synchronous** frequency ω_0 and **constant** amplitude

Generators:

$$\dot{\theta}_k = \omega_0$$
$$\dot{p}_k = 0$$
$$\dot{\lambda}_k = \omega_0 \mathcal{J} \lambda_k$$
$$\dot{q}_k = \omega_0 j q_k$$

Inverters:

$$\dot{q}_{I,k} = 0$$
$$\dot{\lambda}_{I,k} = \omega_0 j \lambda_{I,k}$$
$$\dot{q}_k = \omega_0 j q_k$$

Loads:

$$\dot{q}_k = \omega_0 j q_k$$

Transmission lines:

$$\dot{\lambda}_{T,k} = \omega_0 j \lambda_{T,k}$$

- rotations parametrized by: $j = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $\mathcal{J} = \begin{bmatrix} j & \mathbb{0}_{2\times 1} \\ \mathbb{0}_{1\times 2} & 0 \end{bmatrix}$
- steady-state **specification**: $\dot{x} = f_d(x, \omega_0)$
- **vector fields coincide** (point wise in time) on the set $\mathcal{S}$:

$$\mathcal{S} := \{(x, u, \omega_0) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R} \mid f(x,u) = f_d(x, \omega_0)\}$$

- specifies **constant energy** in each device

Controlled-Invariance Condition

- **steady-state** operation requires $(x, u, \omega_0) \in \mathcal{S}$ **for all time**:
  - ▷ i.e. if and only if $\frac{d}{dt} f(x,u) - f_d(x,\omega_0) = 0$ for all $(x, u, \omega_0) \in \mathcal{S}$
- this holds **if and only if**:
  - ▷ inputs τ_m, v_f, and i_{DC} are **constant**
  - ▷ m oscillates with **synchronous** frequency ω_0 and **constant** amplitude
  - ▷ frequency ω_0 is **constant**

Exploiting the Networked System Structure

- the **system** admits a **non-trivial** synchronous steady-state **if and only if** the **network** nodal current balance equations have a **non-trivial** solution
- nodal current balance equations (=power flow equation):

$$\mathcal{P}_{\omega_0} := \{(i_s, i_I, v) \in \mathbb{R}^{n_p} \mid (i_s, i_I, \mathbb{0}_{2n_l}) + Y_N v = \mathbb{0}_{n_K}\}$$

- **network** admittance matrix $Y_N = Y_v + \mathcal{E} Z_T^{-1} \mathcal{E}^\top$ with
  - ▷ $Y_v = G_v + \omega_0 J_{n_v} C$ (shunt load admittance)
  - ▷ $Z_T = R_T + \omega_0 J_{n_T} L_T$ (branch impedance)
  - ▷ Z_T,Y_v, and Y_N **invertible** due to **dissipation**
- for any $(i_s, i_I, v) \in \mathcal{P}_{\omega_0}$ the **states** and **inputs** of the generators and inverters such that $(x, u) \in \mathcal{S}_{\omega_0}$ holds can be **explicitly** recovered

Summary

- steady-state analysis of a **nonlinear** three phase **power network** including **synchronous machines**, **DC/AC inverters**, and **transmission network**
- well-known **conditions** for steady-state operation can be **constructively** obtained from **first-principles**
- constant generator inputs induce oscillatory steady-state described by network balance equations
- **non-trivial** solution of the power flow equations corresponds to desired steady-state
- separation principle allows to recover the **full system state** from a solution to the balance equations of the **transmission network**

References

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ETH zürich

carghir@control.ee.ethz.ch
gross@control.ee.ethz.ch
dorfler@control.ee.ethz.ch

Automatic Control Laboratory
Institut für Automatik
http://control.ee.ethz.ch