ETH zürich

Fully Distributed Peer-to-Peer Optimal Voltage Control with Minimal Model Requirements

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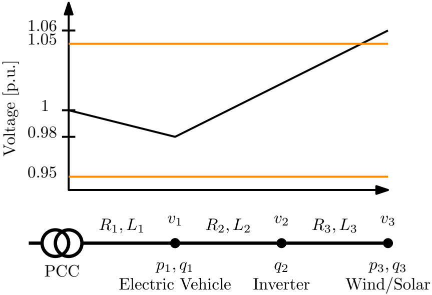
Congestion Leading to Overvoltage



Volt/VAR Problem

for every inverter h find q_h

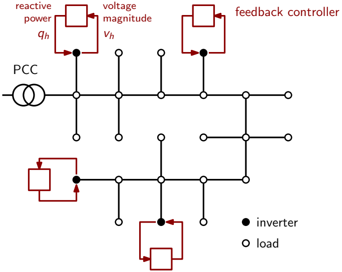
subject to $v_{\min} \leq v_h(q, w) \leq v_{\max}$

$q_{\text{min,h}} \leq q_h \leq q_{\text{max,h}}$

Example:

- Long distribution grid line
- Wind/Solar inject p_3 leads to overvoltage
- Power consumption of EV p_1 is higher than Wind/Solar production p_3

Local Feedback Solutions



Properties

- No communication
- No model information
- Only inverter measurements

But suboptimal

- Jahangiri & Aliprantis (2013)
- Farivar, Zhou, & Chen (2015)
- Yeh, Gayme, & Low (2012)
- Zhu & Liu (2015)
- **IEEE P1547.8**
- Turitsyn et al. (2011)
- Kekatos et al. (2015)
- Li, Gu, & Dahleh (2014)
- Cavraro & Carli (2015)

- **VDE-AR-N 4105 technical req. (2011)**
- **ENTSO-E Draft network code (2012)**

Does communication allow to achieve optimality?

Today: Systematic way of designing communication

The Optimization Problem

$$\begin{aligned} \min_{q} \quad & \frac{1}{2} q^T M q \\ \text{subject to} \quad & v_{\text{min}} \leq v_h(q, w) \leq v_{\text{max}} \\ & q_{\text{min,h}} \leq q_h \leq q_{\text{max,h}} \end{aligned}$$

How to distributedly solve this optimization problem?

Dualizing the Constraints

Multipliers λ and μ

Lagrangian

$$\mathcal{L}(q, \lambda, \mu) = \tfrac{1}{2} q^T M q + \lambda_{\text{max}}^T (v(q, w) - v_{\text{max}}) + \lambda_{\text{min}}^T (v_{\text{min}} - v(q, w)) + \mu_{\text{min}}^T (q_{\text{min}} - q) + \mu_{\text{max}}^T (q - q_{\text{max}})$$

Dual Updates

$$\mathcal{L}(q,\lambda,\mu) = \frac{1}{2}q^T Mq + \lambda_{\max}^T(v(q,w) - v_{\max}) + \lambda_{\min}^T(v_{\min} - v(q,w)) + \mu_{\min}^T(q_{\min} - q) + \mu_{\max}^T(q - q_{\max})$$

λ-Update (Locally integrate the voltage violation)

$$\lambda(t+1) = [\lambda(t) + \alpha \nabla_\lambda \mathcal{L}(q,\lambda,\mu)]_{\geq 0}$$

$$\lambda_{\min}(t+1) = [\lambda_{\min}(t) + \alpha(v_{\min} - v(q,w))]_{\geq 0}$$

$$\lambda_{\max}(t+1) = [\lambda_{\max}(t) + \alpha(v(q,w) - v_{\max})]_{\geq 0}$$

μ-Update (Locally integrate the reactive power violation)

$$\mu(t+1) = [\mu(t) + \gamma \nabla_\mu \mathcal{L}(q,\lambda,\mu)]_{\geq 0}$$

$$\mu_{\min}(t+1) = [\mu_{\min}(t) + \gamma(q_{\min} - q)]_{\geq 0}$$

$$\mu_{\max}(t+1) = [\mu_{\max}(t) + \gamma(q - q_{\max})]_{\geq 0}$$

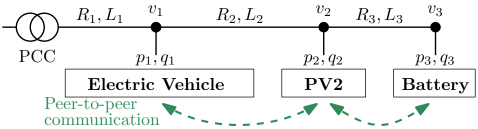
Primal Update and Grid Sparsity

q-Update (unconstrained QP)

$$\mathcal{L}(q,\lambda,\mu) = \frac{1}{2}q^T M q + \lambda_{\max}^T (v(q,w) - v_{\max}) + \lambda_{\min}^T (v_{\min} - v(q,w)) + \mu_{\min}^T (q_{\min} - q) + \mu_{\max}^T (q - q_{\max})$$

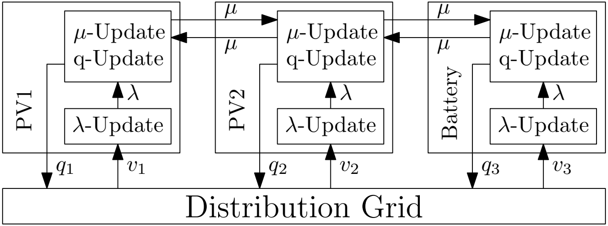
$$\begin{aligned} q(t+1) &= \arg\min \ \mathcal{L}(q, \lambda(t+1), \mu(t+1)) \\ &= I(\lambda_{\min} - \lambda_{\max}) + G(\mu_{\min} - \mu_{\max}) \end{aligned}$$

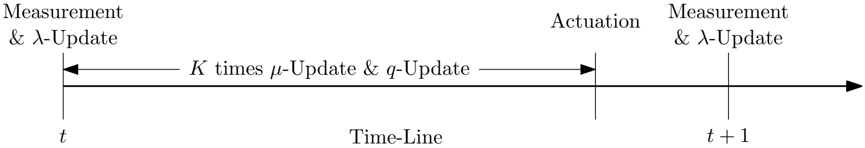
$$G = \begin{bmatrix} 48.3 & -40.7 & 0 \\ -40.7 & 61.8 & -18.7 \\ 0 & -18.7 & 19.1 \end{bmatrix}$$



G is sparse $\Rightarrow$ sparse communication

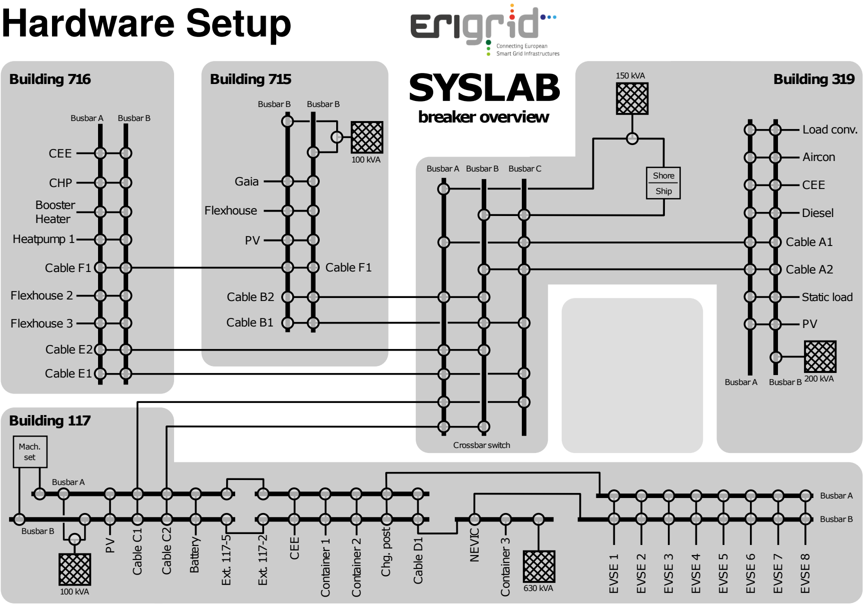
Block Diagram & Time-Line



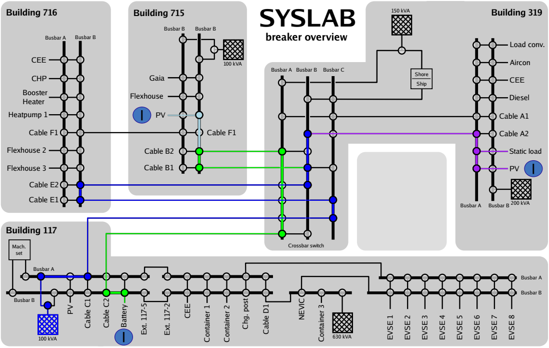


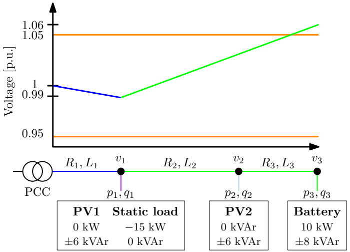
- Inverters take local voltage magnitude measurements
- Only dual multipliers μ are send to neighbours
- Guaranteed convergence from distributed optimization theory

Hardware Setup



Hardware Setup

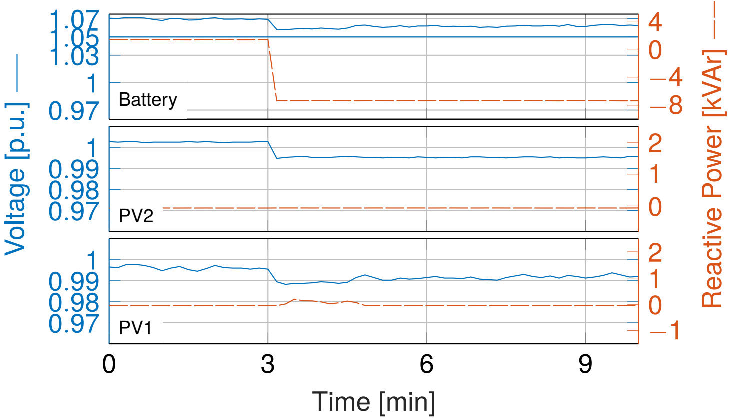




Active Power and Reactive Power Capabilities

Battery:	P=10 kW	Q=±8 kVAr
Static Load:	P=15 kW	Q=0 kVAr
PV Inverter:	P=0 kW	Q=±6 kVAr

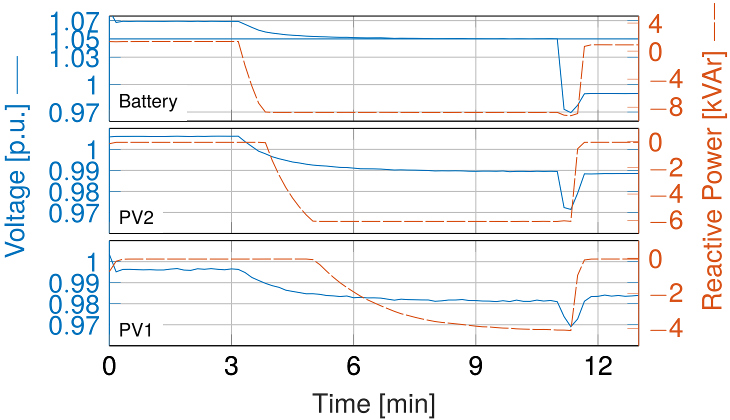
Experimental Results - Droop Control



Reactive power of PVs not used ⇒ persistant overvoltage

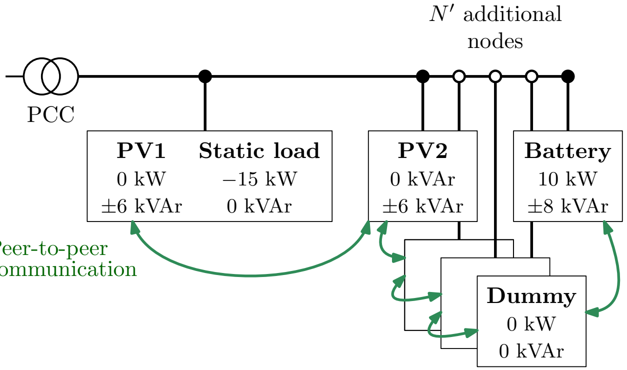
Local control at PV1 makes overvoltage worse

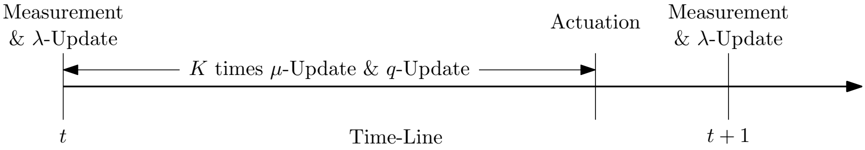
Exp. Results - Distributed Control



Convergence to the allowed voltage band

Scalability





Necessary Communication Steps

	$K \gg 1$		$K = 1$
nodes	communication steps	actuation steps	actuation steps
3	35	46	258 ($\alpha = 40$)
7	566	46	2975 ($\alpha = 3.1$)
10	1417	47	5972 ($\alpha = 1.6$)
30	14515	51	>30000
100	19848	61	>30000

Scaling of Communication Steps

- More nodes more communication steps
- No exponential rise though
  $\Rightarrow$ The algorithm is scalable

Necessary Actuation Steps

Actuation steps required for convergence, for different values of K (communication steps) in a network of 30 nodes.

K	100	300	1000	3000	10000	30000
actuation steps	724	211	67	39	50	51

Scaling of Actuation Steps

- Faster convergence with larger K
- K should be as large as possible
- But it doesn't need to be very large

Conclusions

Take-Away Messages

- **All local control strategies** can make over/undervoltages worse
- **Peer-to-peer communication** enables convergence to optimal solution
- **Communication** should happen more often then actuation
- **Scales** nicely with number of nodes

People Involved

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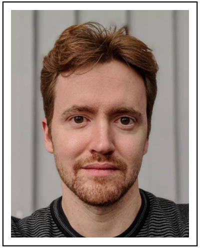

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Smart Grid Infrastructures