

An aerial photograph of a city, likely Zurich, showing a river with a dam, various buildings, and green spaces. A large blue semi-transparent box is overlaid on the left side of the image, containing the title and author information.

A Feedback-Optimization Approach to Resilient Power System Operation

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NREL Workshop on Resilient Autonomous Energy Systems

UNICORN project

A Unified Control Framework for Real-Time Power System operation



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Le réseau
de transport
d'électricité



Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
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Swiss Federal Office of Energy SFOE

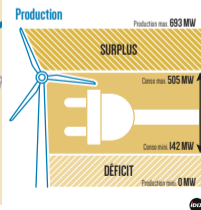
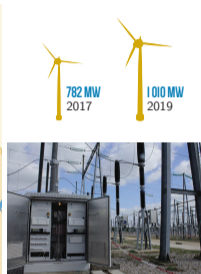
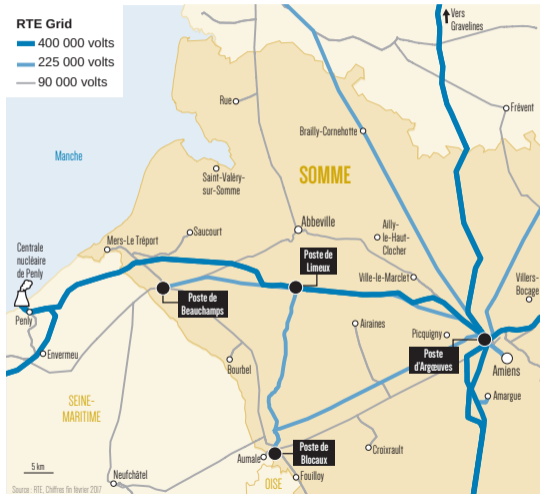
Outline

1. Real-time power system operation
2. Feedback optimization design
3. Numerical experiments: French subtransmission grid
4. Feedback optimization for a resilient power grid
5. Conclusions

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A more responsive grid is needed

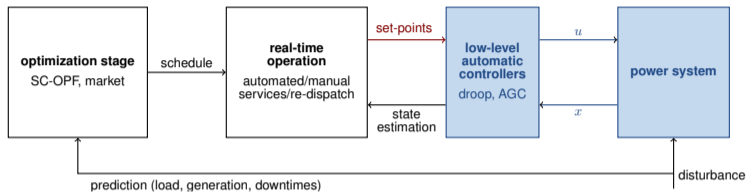


- Larger share of uncontrollable generation
- Distributed generation
- Voltage and line flow constraints

Future real-time operation

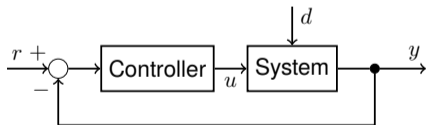
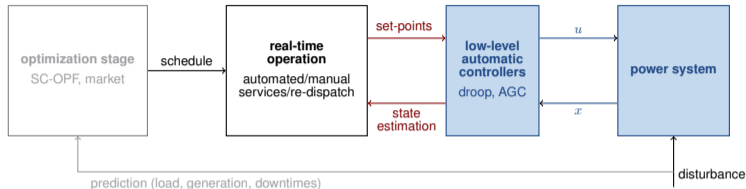
- Online monitoring and measurement
- Real-time operational specifications
- Responsive to fast disturbances

Available actuation (=set-points)



- **Active power curtailment**
 - ramp up/down limits (inverters: 0 s, wind: 20 s in emergency, 60 s otherwise)
- **AVR (Automatic Voltage Regulators) set-points**
 - example: in France, remotely adjusted every 10 s
- **Active power injection from storage**
 - Minimal delay, high flexibility
- **Reactive power injection**
 - any inverter (generators, batteries, loads), hard reactive power limits
- **Tap changers** at the substation transformers

An "autonomous" feedback control design problem



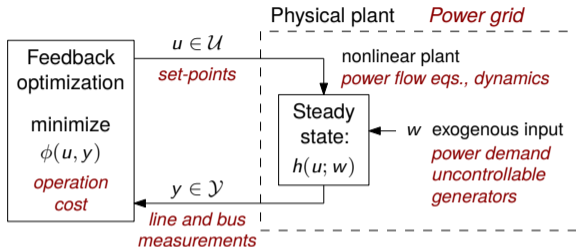
- **Steady state specifications:** solution of a constrained optimization problem
- **Schedule:** known parameter
- **Disturbance:** unknown parameters

→ **Feedback optimization**

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Feedback form of the OPF problem



Steady-state specifications

$$\begin{aligned} &\text{minimize}_{u, y} && \phi(u, y) \\ &\text{subject to} && y \in \mathcal{Y} \\ & && u \in \mathcal{U} \\ & && y = h(u; w) \end{aligned}$$

Optimization perspective

Analysis and design of algorithms with the tools of dynamical systems
but we implement them via the physics

Control perspective

Feedback systems interpreted as solvers of a specific optimization problem **but we require general objective + constraints**

Related: Self-optimizing control, economic MPC, real-time iteration, modifier adaptation, extremum seeking,...

Preprint "Optimization Algorithms as Robust Feedback Controllers" (2021) [↗](#)

Steady-state map $y = h(u; w)$

Chart for the $2n$ -dimensional manifold of **power flow equations**:
invertible map between \mathbb{R}^{2n} and a open subset of \mathcal{M}

Implicit function theorem

If a manifold is defined as

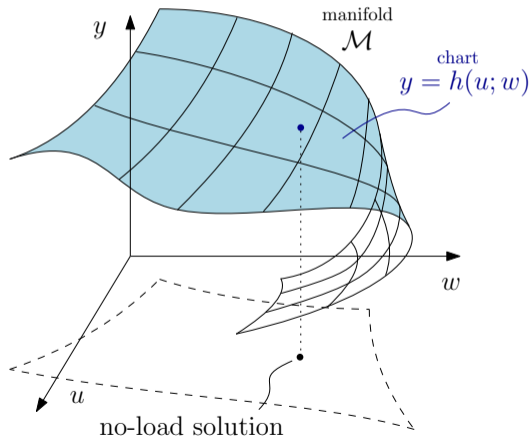
$$\mathcal{M} = \{(u, w, y) \mid F(u, w, y) = 0\}$$

then there exists a continuously differentiable function $y = h(u, w)$ such that

$$F(u, w, h(u, w)) = 0$$

in the open subset where

$$\nabla_y F(u, w, y) \text{ is invertible}$$



Input-output sensitivities $\nabla_{u,w}h(u, w)$

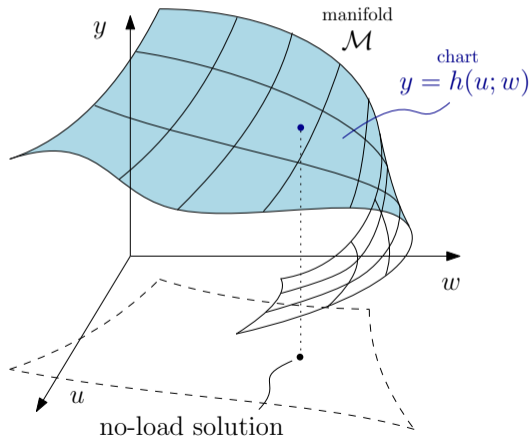
$$\nabla_{u,w}h(u, w) = -(\nabla_y F(u, w, y))^{-1} \nabla_{u,w} F(u, w, y)$$

$\nabla_y F(u, w, y)$ is known as the power flow Jacobian and connected to

- power flow solvability
- voltage collapse

High-voltage PFM

Largest connected component of \mathcal{M} that contains the **no-load solution** and where $\nabla_y F(u, w, y)$ is invertible



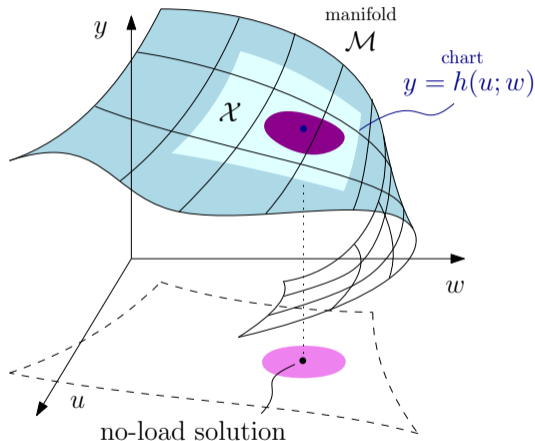
High-voltage PFM

- The high-voltage PFM $\mathcal{M}_{\text{high}}$ can rarely be derived in closed form
 - 2-bus example and little else
- **Inner approximations** are available, but they are usually **conservative**

Running assumption

The operational constraints guarantee that the state of the grid belongs to the high-voltage region

$$(\mathcal{U} \times \mathcal{Y} \times \mathcal{W}) \cap \mathcal{M} \subset \mathcal{M}_{\text{high}}$$



Design of feedback optimizers

Borrow ideas from **iterative optimization algorithms** for **non-convex optimization** and interpret these algorithms as dynamical systems

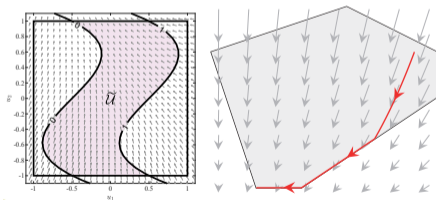
- Gradient Flows
[Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], ...
- Interior-point methods
[Karmarkar, 1984], [Khachian, 1979], [Faybusovich, 1992], ...
- Acceleration & Momentum methods
[Su et al., 2014], [Wibisono et al., 2016], [Krichene et al., 2015], [Wilson et al., 2016], [Lessard et al., 2016], ...
- Saddle-Point Flows
[Arrow et al., 1958], [Kose, 1956], [Feijer & Paganini, 2010], [Cherukuri et al., 2017], [Holding & Lestas, 2014], [Cortés & Niederländer, 2018], [Qu & Li, 2018], ...

Claim: In continuous-time, most algorithms reduce to either (projected) **gradient flows** (w/o momentum) or (projected) **saddle-point** flows.

Example: Projected gradient descent

$$\begin{aligned} & \text{minimize}_{u,y} && \phi(u,y) \\ & \text{subject to} && y \in \mathcal{Y} \quad \text{output constraints} \\ & && u \in \mathcal{U} \quad \text{input saturation} \\ & && y = h(u;w) \quad \text{power flow equations} \end{aligned}$$

$$\tilde{\mathcal{U}} = \mathcal{U} \cap h^{-1}(\mathcal{Y})$$



Projected gradient descent (Hauswirth 2016, Häberle 2020, ...)

Projection on the input and output constraints

$$\dot{u} = \Pi_{\tilde{\mathcal{U}}} \left[-\nabla_u \phi(u,y) - \underbrace{\nabla h(u;w)'}_{\text{model}} \nabla_y \phi(u,y) \right]$$

→ any-time constraint satisfaction

Alternatives:

- Saddle flow (Bognani 2015, Dall'Anese 2018, Bernstein 2019, Colombino 2020, ...)
- Penalty functions (Hauswirth 2017, Tang 2017, Mazzi 2018, ...)

Projected gradient flow via repeated Quadratic Programming

Continuous-time flow

$$\dot{u} = \Pi_{\tilde{\mathcal{U}}} \left[-\nabla_u \phi(u, y) - \underbrace{\nabla h(u; w)'}_{\text{model}} \nabla_y \phi(u, y) \right]$$

Assumption

$$\mathcal{U} := \{u \in \mathbb{R}^p \mid Au \leq b\}$$

$$\mathcal{Y} := \{y \in \mathbb{R}^n \mid Cy \leq d\}$$

Discrete-time approximation

$$u^+ = u + \alpha \delta u \quad \text{where} \quad \delta u := \arg \min_v \quad \|v - (-\nabla_u \phi(u, y) - \nabla h(u; w)' \nabla_y \phi(u, y))\|^2$$

subject to $A(u + \alpha v) \leq b$
 $C(y + \alpha \nabla h(u; w)' v) \leq d,$

1st order approx of $h^{-1}(\mathcal{Y})$ centered at the measurement y

Theorem: V. Häberle et al., “Non-convex Feedback Optimization with Input and Output Constraints,” 2020 [↗](#)

LICQ + Lipschitz + differentiability + small α \rightarrow global convergence to the set of local minima

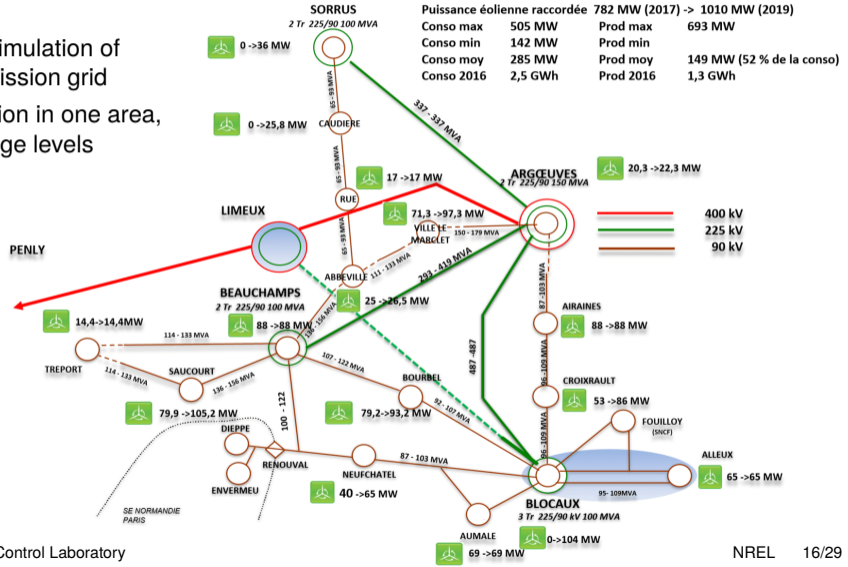
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Benchmark (soon to become public)

Quasi-steady state simulation of entire French transmission grid

Goal: avoid congestion in one area, across different voltage levels



Problem specifications

Inputs

Uncontrollable

- Distributed wind generation (historical worst-case ramp)

Controllable

- Transformers tap-changer position
- Wind generators reactive power injection
- Active power curtailment

Scenario: 225kV-90kV transformer offline

Other scenarios: no tap changers, tighter constraints, higher generation, ...

Output

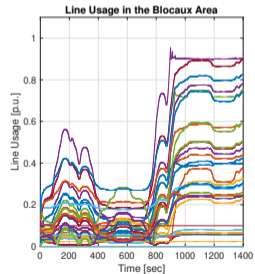
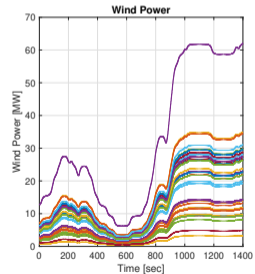
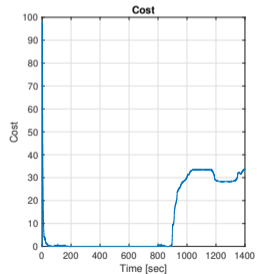
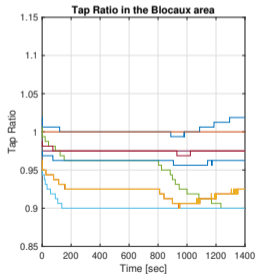
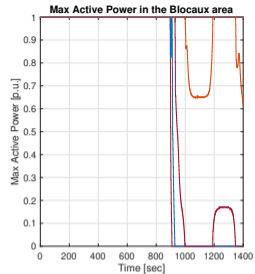
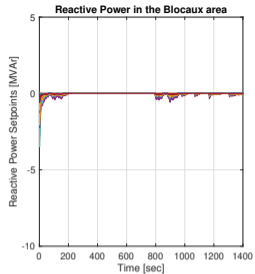
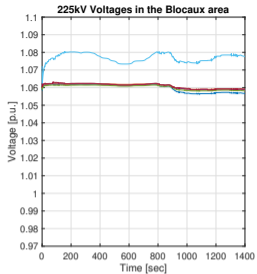
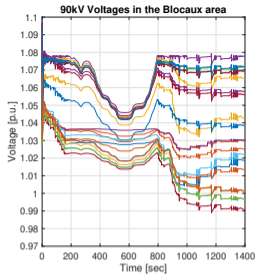
- real-time area state estimation

Constraints

- bus voltage limits
- line current limits
- generator limits
- tap changes

Cost

- \$\$\$ Active power curtailment
- \$ Power losses



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How about emergencies?



→ Overview of the events and causes of the 2003 Italian blackout ↗

03:01	Trip of the 380 kV line Mettlen Lavorgo (CH). Attempts to reclose the line automatically until 3.03:50. Manual re-closure fails at 3:03:50.
03:02-03:08	Attempts to reclose the Mettlen-Lavorgo line. Information exchanges between ETRANS and ATEL and EGL dispatchers.
03:10	ETRANS, by phone, requests a reduction of 300 MW in Italian imports to scheduled values.
03:18-03:22	Exchange of information ETRANS - ATEL -EGL; changes in topology of the Swiss system.
03:21	Italian imports are reduced to 6400 MW
03:25	Trip of the Sils-Soazza 380 kV line (CH)
03:25	Trip of the Airolo Mettlen 220 kV line (CH)

→ 1.2 billion EUR

How about emergencies?



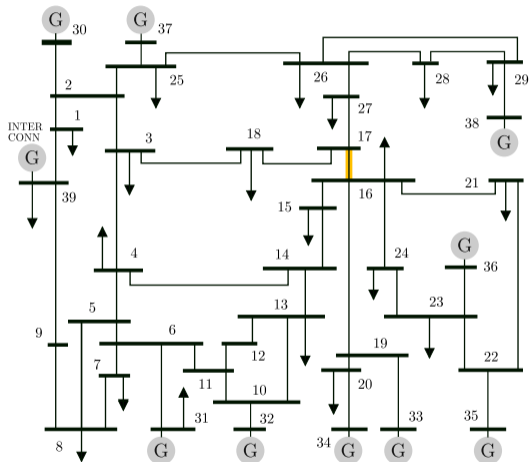
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**Can we frame emergency grid operation
as a feedback optimization problem?**

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Numerical experiment IEEE 39 test grid



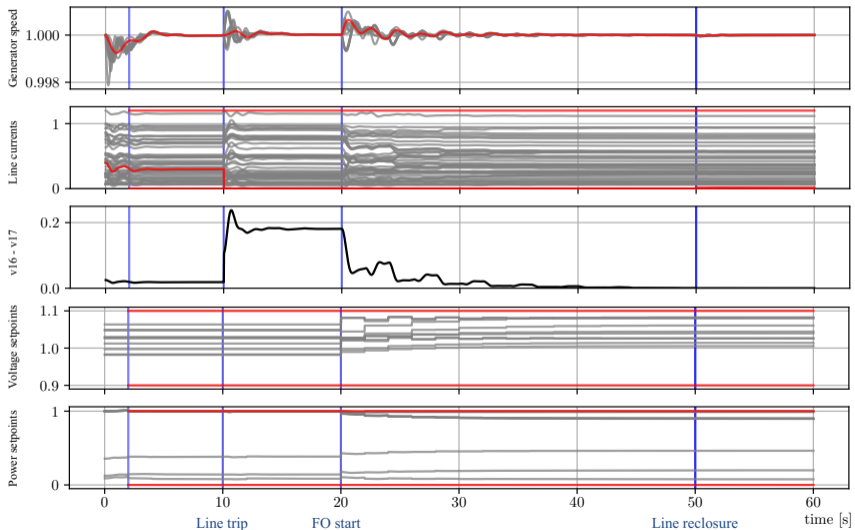
- **Fault:** Line trip of line 16-17
- **Goal:** Reclosure before cascading failure
- Line reclosure requires **small voltage difference** at the breaker

FO formalization

$$\begin{aligned} & \min_{P_G, V_G} \|v_{16} - v_{17}\|^2 \\ & \text{subject to} \quad 0 \leq P_{G_i} \leq P_{G_i}^{\max} \\ & \quad V_{G_i}^{\min} \leq V_{G_i} \leq V_{G_i}^{\max} \\ & \quad v^{\min} \leq v_i \leq v^{\max} \\ & \quad 0 \leq f_{ij} \leq f_{ij}^{\max} \end{aligned}$$

Full dynamic simulation

DynPSSimPy [↗](#) (by Gianni Hotz)



Feedback optimization for emergencies

The problem is... it shouldn't work!

Grid dynamics not at steady-state between set-point updates

- Design/certify stable interconnections (LTI systems)
Lawrence, Simpson-Porco, Mallada, "*Linear-convex optimal steady-state control*", 2020 [↗](#)
Colombino, Dall'Anese, Bernstein, "*Online opt. as a feedback controller: stability and tracking*," 2018 [↗](#)
Bianchin et al. "*Time-varying optimization of LTI systems via projected primal-dual gradient flows*," 2021 [↗](#)
- **Quantify sufficient time-scale separation (nonlinear grid dynamics)**

Wrong (pre-fault) model during contingency

- Rely on the inherent robustness of feedback optimization (performance tradeoff, computation)
Colombino, Simpson-Porco, Bernstein, "*Towards robustness guarantees for feedback-based opt.*," 2019 [↗](#)
L. Ortmann et al., "*Experimental validation of feedback optimization in power distribution grids*," 2020 [↗](#)
- **Online sensitivity estimation**

Time-scale separation analysis

A. Hauswirth, S. Bolognani, G. Hug, F. Dörfler, IEEE TAC, 2021 

Optimization Dynamics

The cost function $\phi(u, x)$ has

- compact level sets
- **L -Lipschitz** gradient.

Unconstrained gradient descent

$$\dot{u} = -\alpha \left(-\nabla_u \phi(u, x) - \nabla h(u; w)' \nabla_y \phi(u, x) \right)$$

Plant Dynamics

Exponentially stable system

$$\dot{x} = f(x, u; w) \quad (\text{steady state } x = h(u; w))$$

with Lyapunov function $W(u, x)$ such that

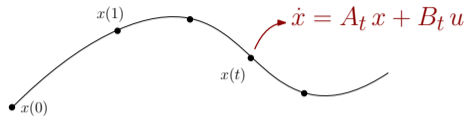
$$\begin{aligned} \dot{W}(u, x) &\leq -\gamma \|x - h(u; w)\|^2 \\ \|\nabla_u W(u, x)\| &\leq \zeta \|x - h(u; w)\|. \end{aligned}$$

Then, all trajectories converge to the set of KKT points whenever

$$\alpha < \frac{\gamma}{\zeta L}.$$

(Similar results for projected gradient, saddle flow, Newton flow. **Not** for subgradient, accelerated gradient.)

Time-scale separation analysis



- solve **Lyapunov eq.** $A_t^\top P_t + P_t A_t \preceq I$
- **Lyapunov fcn.** $W(u, x) = \|x - h(u)\|_{P_t}^2$
- $\dot{W}(u, x) \leq -\|x - h(u)\|^2$ ($\rightarrow \gamma = 1$)
- **Linear steady state** $h(u) = H u$
- $\zeta = \|P_t H\|$

$$\rightarrow \alpha \leq \frac{1}{L \|P_t H\|} \quad \forall t$$

Plant Dynamics

Exponentially stable system

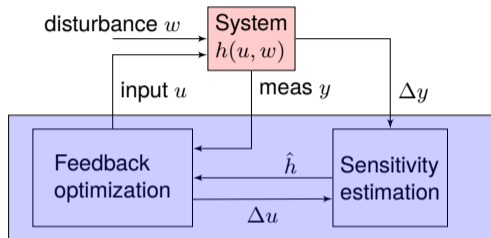
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- + bound on α relatively uniform in x
- very conservative certificate
- ? better data-driven estimates γ and ζ

Online sensitivity estimation



Best online estimate

$$\hat{h}_{t+1} = \arg \min_{\hat{h}} \left\| \hat{h} - \hat{h}_t \right\|_{\frac{\Sigma_t^{-1}}{\|\Delta u_t\|_2^2}}^2 + \left\| \Delta y_t - U_{\Delta, t} \hat{h} \right\|_{\frac{\Sigma_{m, t}^{-1}}{\|\Delta u_t\|_2^2}}^2$$

→ Kalman-like update

$$\hat{h}_{t+1} = \hat{h}_t + K_t (\Delta y_t - U_{\Delta, t} \hat{h}_t)$$

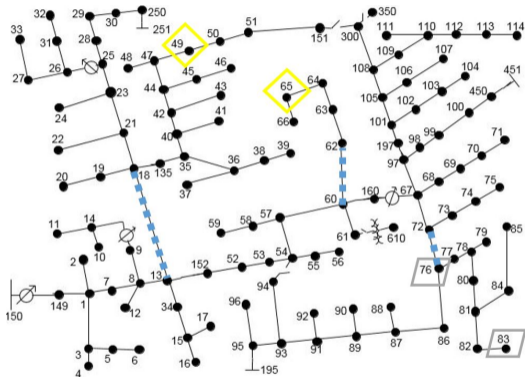
$$\Sigma_{t+1} = (1 - K_t U_{\Delta, t}) \Sigma_t + \Sigma_{p, t} \|\Delta u_t\|_2^2,$$

Proposition

Strong-monotonicity of the optimization flow + **persistently exciting** input $u_t \Rightarrow$

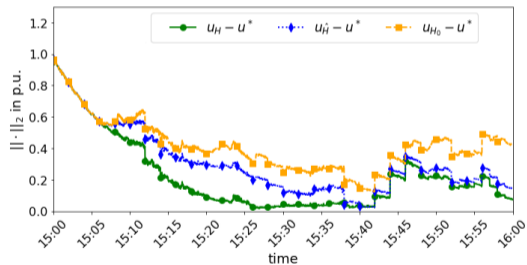
$$\lim_{t \rightarrow \infty} \mathbb{E}[\|h_t - \hat{h}_t\|_2^2] \rightarrow 0 \quad \lim_{t \rightarrow \infty} \mathbb{E}[\|h_t - \hat{h}_t\|_2^2] \rightarrow C_h < \infty \quad \lim_{t \rightarrow \infty} \mathbb{E}[\|u_t - u^*(d_t)\|_2^2] \rightarrow C_u \leq \dots$$

Online sensitivity estimation



IEEE 123 test case with

- modified line impedance
- nonlinear regime



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Highlights

- **Real-time operation of power systems** can be automated via **feedback optimization**
 - UNICORN numerical testbed will be published by the end of the year – get in touch!
- Feedback optimization design taps into **iterative nonlinear optimization algorithms**

- Numerical experiments show that feedback optimization can produce complex multi-input/multi-objective **restorative actions** in response to **fast contingencies**
- **Open problem 1:** tighter **stability certificates** that
 - can be used as design guidelines
 - are uniform in the system working point
- **Open problem 2:** **online sensitivity estimation** to enable
 - model-free design
 - robustness to unforeseen system changes



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