Feedback Optimization for Real-Time Power System Operation

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Joint work


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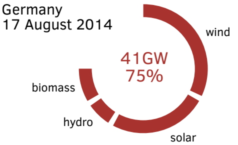
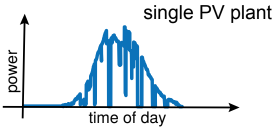
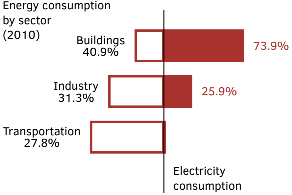
Future power systems: time-varying and uncertain power flows

- **Fluctuating renewable energy sources**
  - dispersed over the grid
  - poor short-range prediction
  - correlated uncertainty
- **Inverter-based generation**
  - control flexibility
  - decreased resilience
  - tighter operating specifications
- **Electric mobility**
  - large additional demand
  - new spatial-temporal patterns

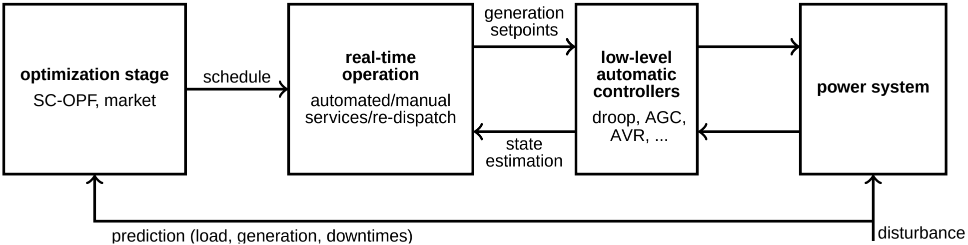






How does the grid accommodate these time-varying and uncertain power flows?

Focus: Real-time operation

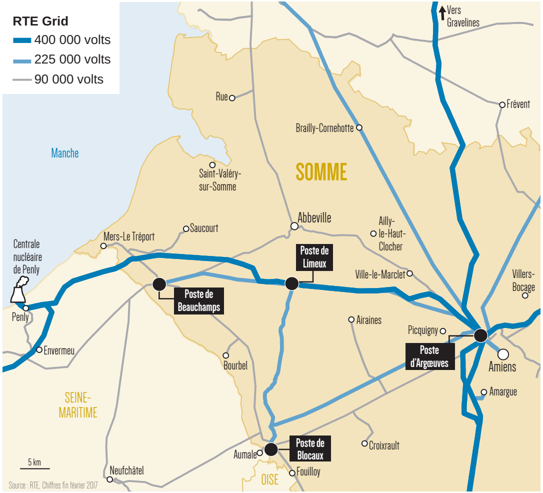


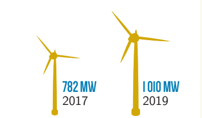
- **optimization stage**
  economic dispatch based on predictions/markets
- **real-time operation**
  unforeseen deviations from schedule (e.g. congestion)
- **low-level automatic control**
  set-point tracking at the individual generators

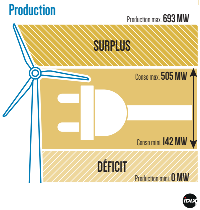
Today's power system operation

- partially automated
- separate mechanisms
- ad-hoc design

Example: Congestion in sub-transmission grid







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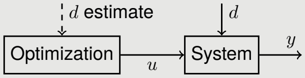
Future real-time operation

- Faster operation
- Online monitoring and measurement
- Real-time operational specifications
- Robust against disturbances

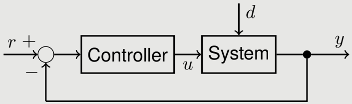
Feedback makes real-time operation effective

Feedforward optimization



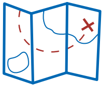
- **complex optimal decision**
- **operational constraints**
- **MIMO (multi-input/output)**
- highly model-based
- computationally intensive

Feedback control



- **robust to model uncertainty**
- **fast response**
- requires exogenous set-points
- suboptimal operation
- unconstrained operation

Proposal: a **feedback optimization** approach to real-time operation
to inherit the best of the two worlds

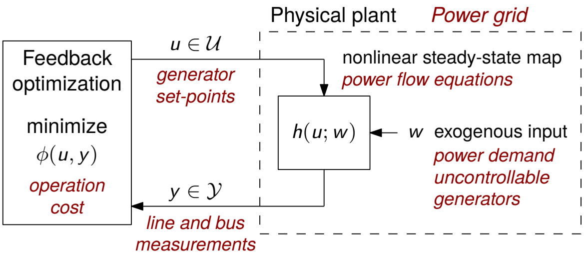


OVERVIEW

1. Feedback optimization design
2. Interconnected dynamics and stability
3. Numerical demo (power systems)

FEEDBACK OPTIMIZATION DESIGN

Steady-state optimization



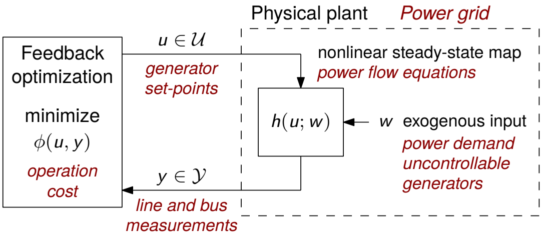
Prototypical optimization problem

$$
\begin{aligned}
\text{minimize}_{u,y} \quad & \phi(u, y) \\
\text{subject to} \quad & y \in \mathcal{Y} \\
& u \in \mathcal{U} \\
& y = h(u; w)
\end{aligned}
$$

From optimization problem to **control design specifications**:

- **Input saturation:** $u \in \mathcal{U}$ at all times (generation capacity, reserves, set-point range)
- **Output feasibility:** $y \in \mathcal{Y}$ (voltage constraints, line flow constraints)
- **Steady state optimality:** convergence to the solution of the OPF (min deviation from schedule)

Design of feedback optimizer $\rightarrow$ continuous-time limit of iterative algorithms



Optimization perspective

Analysis and design of algorithms with the tools of dynamical systems
but implemented via the physics

Control perspective

Feedback systems interpreted as solvers of a specific optimization problem
but with general objective + constraints

Review: Optimization algorithms as dynamical systems

- Gradient Flows on Matrix Manifolds

  [Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], ...
- Interior-point methods

  [Karmarkar, 1984], [Khachian, 1979], [Faybusovich, 1992], ...
- Acceleration & Momentum methods

  [Su et al., 2014], [Wibisono et al, 2016], [Krichene et al., 2015], [Wilson et al., 2016], [Lessard et al., 2016], ...
- Saddle-Point Flows

  [Arrow et al., 1958], [Kose, 1956], [Feijer & Paganini, 2010], [Cherukuri et al., 2017], [Holding & Lestas, 2014], [Cortés & Niederländer, 2018], [Qu & Li, 2018], ...

Claim: In continuous-time, most algorithms reduce to either (projected) **gradient flows** (w/o momentum) or (projected) **saddle-point** flows.

Examples

$$\begin{aligned} \text{minimize}_{u,y} \quad & \phi(u,y) \\ \text{subject to} \quad & y \in \mathcal{Y} \quad \text{output constraints} \\ & u \in \mathcal{U} \quad \text{input saturation} \\ & y = h(u;w) \quad \text{power flow equations} \end{aligned}$$

Main challenge: output constraints

(more precisely: algebraic map / steady state constraint)

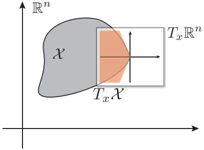
Examples

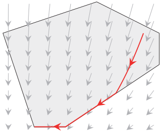
$$\begin{aligned} &\text{minimize}_{u,y} && \phi(u,y) \\ &\text{subject to} && y \in \mathcal{Y} \quad \text{output constraints} \\ & && u \in \mathcal{U} \quad \text{input saturation} \\ & && y = h(u;w) \quad \text{power flow equations} \end{aligned}$$

Main challenge: output constraints

(more precisely: algebraic map / steady state constraint)

$$\begin{aligned} &\text{minimize}_{u} && \phi(u,y) + p(y) \\ &\text{subject to} && u \in \mathcal{U} \\ & && y = h(u;w) \end{aligned}$$





$\mathcal{Y} \rightarrow$ Penalty function (Hauswirth 2017, Tang 2017, Mazzi 2018, ...)

Gradient descent flow $\rightarrow$ proportional-like feedback law

$$\dot{u} = \Pi_{\mathcal{U}} \left[-\nabla_u \phi(u,y) - \underbrace{\nabla h(u;w)'}_{\text{model}} \nabla_y \phi(u,y) - \underbrace{\nabla h(u;w)'}_{\text{model}} \nabla p(y) \right]$$

$\rightarrow$ **arbitrarily small output constraint violation**

Examples

$$\begin{aligned} \text{minimize}_{u,y} \quad & \phi(u,y) \\ \text{subject to} \quad & y \in \mathcal{Y} \quad \text{output constraints} \\ & u \in \mathcal{U} \quad \text{input saturation} \\ & y = h(u;w) \quad \text{power flow equations} \end{aligned}$$

Main challenge: output constraints

(more precisely: algebraic map / steady state constraint)

Output constraint representation

$$\mathcal{Y} := \{y \mid g(y) \leq 0\}$$

Lagrangian

$$\mathcal{L}(u,y,\lambda) = \phi(u,y) + \lambda' g(y)$$

Saddle flow **(Bolognani 2015, Dall'Anese 2018, Bernstein 2019, Colombino 2020, ...)**

Primal descent / dual ascent $\rightarrow$ proportional-integral feedback law

$$\begin{cases} \dot{u} = \Pi_{\mathcal{U}}\Big[-\nabla_u \phi(u,y) - \underbrace{\nabla h(u;w)'}_{\text{model}} \nabla_y \phi(u,y) - \underbrace{\nabla h(u;w)'}_{\text{model}} \nabla g(y)' \lambda \Big] \\ \dot{\lambda} = \Pi_{\geq 0}\left[g(y)\right] \end{cases}$$

$\rightarrow$ **asymptotic (exact) constraint satisfaction**

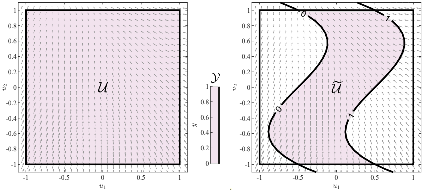
Examples

$$\begin{array}{lll} \text{minimize}_{u,y} & \phi(u,y) & \\ \text{subject to} & y \in \mathcal{Y} & \text{output constraints} \\ & u \in \mathcal{U} & \text{input saturation} \\ & y = h(u;w) & \text{power flow equations} \end{array}$$

Main challenge: output constraints

(more precisely: algebraic map / steady state constraint)

$$\tilde{\mathcal{U}} = \mathcal{U} \cap h^{-1}(\mathcal{Y})$$



Projected gradient descent **(Hauswirth 2016, Häberle 2020, ...)**

Projection on the input and output constraints

$$\dot{u} = \Pi_{\tilde{\mathcal{U}}}\left[-\nabla_u \phi(u,y) - \underbrace{\nabla h(u;w)'}_{\text{model}} \nabla_y \phi(u,y) \right]$$

$\rightarrow$ **any-time constraint satisfaction**

Projected gradient flow via repeated Quadratic Programming

Continuous-time flow

$$\dot{u} = \Pi_{\tilde{\mathcal{U}}}\Big[-\nabla_u \phi(u,y) - \underbrace{\nabla h(u;w)'}_{\text{model}} \nabla_y \phi(u,y) \Big]$$

Assumption

$$\mathcal{U} := \{u \in \mathbb{R}^p \,|\, Au \leq b\}$$
$$\mathcal{Y} := \{y \in \mathbb{R}^n \,|\, Cy \leq d\}$$

Discrete-time approximation

$u^+ = u + \alpha \delta u$ where

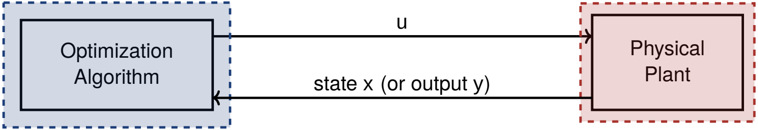
$$\begin{aligned} \delta u := \quad \operatorname{argmin}_v \quad & \|v - (-\nabla_u \phi(u,y) - \nabla h(u;w)' \nabla_y \phi(u,y))\|^2 \\ \text{subject to} \quad & A(u + \alpha v) \leq b \\ & \underbrace{C(y + \alpha \nabla h(u;w)' v) \leq d}_{\text{1st order approx of } h^{-1}(\mathcal{Y}) \text{ centered at the measurement } y}, \end{aligned}$$

Theorem: (Häberle 2020)

LICQ + Lipschitz + differentiability + small α $\rightarrow$ global convergence to the set of local minima

INTERCONNECTED DYNAMICS AND STABILITY ANALYSIS

Gradient-based feedback optimization



Optimization Dynamics

Generalized gradient descent

$$\dot{u} = -Q(u)\left(-\nabla_u \phi(u,y) - \nabla h(u;w)' \nabla_y \phi(u,y)\right)$$

with $Q(u) \succ 0$

Plant Dynamics

Exponentially stable system

$$\dot{x} = f(x,u;w)$$

with steady-state map $x = h(u;w)$

Variations of gradient-based feedback optimization

Theorem **(Hauswirth 2019)**

Assume

- Physical system **exponentially stable** with Lyapunov function $W(u,x)$ s.t.

$$\dot{W}(u,x) \leq -\gamma \|x - h(u;w)\|^2 \qquad \|\nabla_u W(u,x)\| \leq \zeta \|x - h(u;w)\| .$$

- $\phi(u,x)$ has compact level sets and **L-Lipschitz** gradient.

Then, all trajectories converge to the set of KKT points whenever

$$\sup_{u \in \mathbb{R}^p} \|Q(u)\| < \frac{\gamma}{\zeta L} .$$

- Asymptotically stable equilibrium $\Rightarrow$ strict local minimizer
- Strict local minimizer $\Rightarrow$ stable equilibrium

$\rightarrow$ If ϕ convex and $h(u;w)$ linear, then convergence to set of global minimizers.

Gradient-based Feedback Optimization

Vanilla GD

Choose $Q(u) = \varepsilon \mathbb{I}_n$.
Stability is guaranteed if

$$\varepsilon \leq \frac{\gamma}{\zeta L}$$

$\rightarrow$ **prescription on global control gain**

Projected GD

Control signal u constrained to set $\mathcal{U}$ (in case of actuator saturation).

$$\dot{u} = \Pi_{\mathcal{U}}[-\varepsilon(\nabla_u \phi(u, x) + \nabla h' \nabla_x \phi(u, x))]$$

$\rightarrow$ **stable if $\varepsilon \leq \frac{\gamma}{\zeta L}$ (same bound)**

Newton GD

Choose $Q(u) = (\nabla^2 \phi(h(u; w), u))^{-1}$
(if ϕ μ-strongly convex and twice differentiable)
Stability is guaranteed if

$$\frac{L}{\mu} \leq \frac{\gamma}{\zeta}$$

$\rightarrow$ **invariant under scaling of ϕ**

Not

- Subgradient methods
- Accelerated gradient method

General feedback optimization controllers

General *Slow* Dynamics

$$\dot{u} = \varepsilon g(h(u;w), u, z)$$
$$\dot{z} = \varepsilon k(h(u;w), u, z)$$

- E.g. saddle flows

$\rightarrow$ requires exponential stability (open problem!)

[Qu & Li, 2018]

Theorem

- $(g(x,u,z), k(x,u,z))$ is L-Lipschitz in x
- $(g(h(u;w),u,z), k(h(u;w),u,z))$ is ℓ-Lipschitz
- $\exists$ Lyapunov function $V(u,z)$ for the slow dynamics

$$\dot{V}(u,z) \leq -\mu \|e(u,z)\|^2 \quad \|\nabla V(u,z)\| \leq \kappa \|e(u,z)\|$$

- $\exists$ Lyapunov function $W(x,u)$ for the plant

$$\dot{W}(x,u) \leq -\gamma \|x - h(u;w)\|^2, \quad \|\nabla_u W(x,u)\| \leq \zeta \|x - h(u;w)\|$$

Then, asymptotic stability is guaranteed if

$$\epsilon < \frac{\gamma}{\zeta L(1 + \frac{\kappa \ell}{\mu})}.$$

Singular Perturbation Analysis yields sufficient stability conditions

Weak assumptions on plant

- internal stability
- steady-state sensitivity $\nabla h(u; w)$

Weak assumptions on cost

- Lipschitz gradient
- no convexity required
  (for gradient-based controllers)

- potentially conservative bound, but

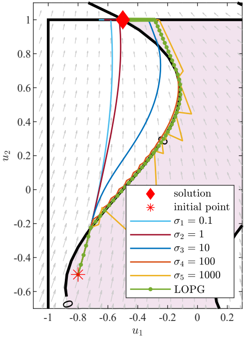
$\rightarrow$ directly **useful for design** of control
(no LMI/IQC stability test)
(Nelson 2017, Colombino 2018)

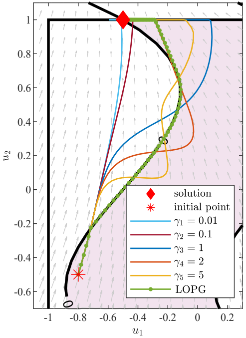
- analysis applicable to many continuous-time optimization algorithms

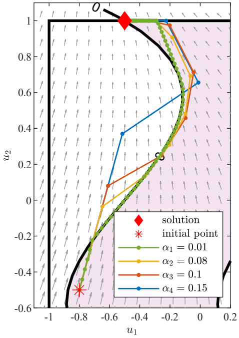
Further works: see discussion in Lawrence 2019

Comparison and tradeoffs

	Penalty	Saddle-flow	Projected gradient
Feasibility	arbitrarily small violation	asymptotic feasibility	any-time feasibility
Controller	“proportional”	“proportional-integral”	quadratic programming
Stability	steep penalty limits speed	requires exp. stability	simple gain limit



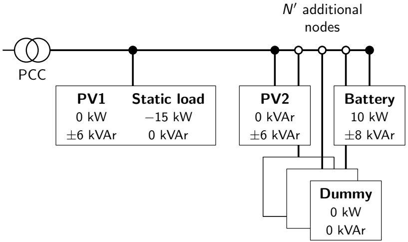




Scalability (preliminary results)

Experimental demo (Ortmann 2020)

Volt/VAR regulation a distribution feeder



- **input:** reactive power
- **input constraints:** $\mathcal{U} = [q_{\min} q_{\max}]$
- **output:** voltages
- **output constraints:** $\mathcal{Y} = [v_{\min} v_{\max}]$

Comparison:

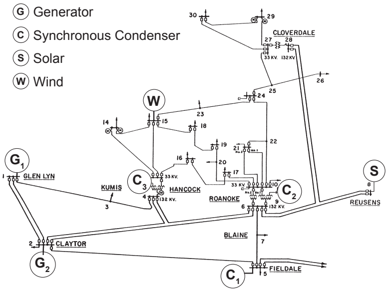
- dualization of $\mathcal{U} \rightarrow$ **saddle flow**
- projection on $\mathcal{U} \rightarrow$ **projected gradient**

	Projected gradient		Saddle-flow
nodes	QP iterations	actuation steps	actuation steps
3	35	46	258 ($\alpha = 40$)
7	566	46	2975 ($\alpha = 3.1$)
10	1417	47	5972 ($\alpha = 1.6$)
30	14515	51	>30000
100	19848	61	>30000

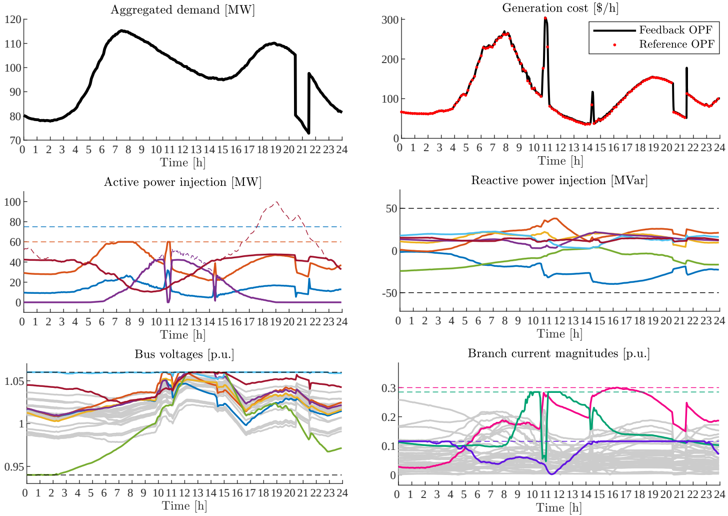
Actuation steps are expensive, use them well!

NUMERICAL DEMO (POWER SYSTEMS)

Real-time operation of IEEE 30-bus system (projected gradient)



		Unit type	Set-points
(blue)	G_1	Generator (AVR, freq. control)	v
(orange)	G_2	Generator (AVR)	p, v
(yellow)	C_1	Synchronous condenser	v
(green)	C_2	Synchronous condenser	v
(light blue)	C_3	Synchronous condenser	v
(purple)	S	Solar farm	p, q
(dark red)	W	Wind farm	p, q



CONCLUSIONS

Highlights

- **Real-time operation of power systems** can be automated via feedback optimization
- Feedback optimization design taps into **iterative nonlinear optimization algorithms**
- **Closed-loop stability** via time-scale separation
  - applies to many optimization algorithms
  - yields direct design specifications
- Fundamental challenge: satisfaction of **output constraints**
  - Penalty functions may compromise stability
  - Dualization seems harder to tune and deteriorates the trajectory (oscillations, windup)
  - Projected gradient via QP scales well, is easy to tune, and yields (almost) anytime feasibility
    $\rightarrow$ no need for another integrator in the control loop

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