Temperature Measurements for Absolute Zero and Liquid Nitrogen LN_2

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In this experiment, we measure the temperature of the absolute zero t_0 and liquid nitrogen t_{LN_2}. We then compare our measured values to theoretical predictions and literature values. To achieve this, we calibrate a piezoresistive pressure sensor using two calibration points of different pressure. After carrying out the experiment, we have for the absolute zero $t_0 = -272.9 \pm 0.3°$C and for the temperature of liquid nitrogen $t_{LN_2} = -196.6 \pm 0.2°$C. We conclude that the literature values lie within the uncertainty of our measurements and therefore align with the theoretical model. Hence the experiment should be repeated to take into account sources of errors which went unnoticed and to further optimize the experiment setup.

The Celsius temperature scale was intended to be 0 $°C$ for the freezing point of water and 100 $°C$ for the boiling point of water at normal pressure, equally spaced in between. Since the statistical interpretation of thermodynamics the temperature was considered to be a mass for the average kinetic energy of the particles in a system. A new temperature scale was intended to have its zero point at the lowest possible thermal energy state of these particles, meaning zero would be the lowest possible value on that scale for the temperature. Furthermore it should be spaced like the Celsius scale. This was how the Kelvin scale was defined. We want to relate the Celsius and Kelvin scale to make statements for example about the thermal energy difference between a "heated" system at temperature T in Celsius (thermometers were designed to show the temperature in Celsius) and its lowest possible thermal energy state at zero Kelvin. We will measure the absolute zero temperature t_0 in Celsius so we can equal it to zero Kelvin and get the explicit relation $T_{Kelvin} = t_{Celsius} - t_0$. Using the relation we can easily convert the temperature from one scale to the other.

Purpose of this experiment is the assessment of the temperature for the absolute zero t_0 and liquid nitrogen t_{LN_2}.

The pressure p and volume V of a gas are known to be proportional to each other when the temperature T and number of particles N are fixed. Meaning $pV \propto NT$ (for N, T const.). Experimentally measured, the constant was found to be $k_B = 1.38110^{-23} J/K$, the Boltzmann constant. By writing N as νN_A, with ν the amount of moles and N_A the number of particles per mole, we obtain the well known form of the universal gas law

$$pV = \nu RT \tag{1}$$

where $R = k_B N_A = 8.314$J/mol K

This equation shows that for a constant volume V_0, the pressure p is a linear function of the temperature T. It also implies that since the pressure could be understood as "energy density" (thus non-negative), the temperature T also has to be greater or equal to zero. Here, T is the absolute temperature in Kelvin. The lowest possible value for T according to Eq.(1) is $0K$. If we want to determine the lowest possible value on the Celsius scale t_0 we can rewrite Eq.(1) using the relation between the Kelvin and Celsius scale $T = t - t_0$, where t is the temperature in °C. Dividing Eq.(1) by V and using said relation we arrive at $p = k_B n(t - t_0)$, where n is the particle density of the gas. If we assume that n is constant, yet unknown, we can derive t_0 with two different measurements of the pressure for two different temperatures. This will give us two explicit equations with the two unknown constants n and t_0, which we can solve for t_0.

We measure the pressure of helium gas in a glass bulb p_K and p_E at the temperatures of boiling water t_K and ice water t_E respectively. A first naive approximation for t_0, assuming a constant volume V_0, hence a constant particle density n, is given,

$$t_0 \approx t_0' = -\frac{p_E}{p_K - p_E} t_K \tag{2}$$

Viewed more rigorously, we have to consider different volumes for both measurements. One due to the thermal expansion of the glass bulb when it is being heated. The other due to diffusion of the particles out of the bulb into the glass tube. The glass tube is estimated to be at room temperature t_L and connects the glass bulb to the rest of the experimental set up. The more precise expression for t_0 is

$$t_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{3}$$

where $a = (1+\epsilon)p_E - (1+\epsilon+\gamma t_K)p_K$, $b = \epsilon(p_K - p_E)t_K + (1+\gamma t_K)p_K t_L - p_E(t_L + t_K)$, $c = p_E t_L t_K$

Knowing t_0, we can derive the temperature of the helium gas for a measured pressure. We can approximate t_{LN_2} by measuring the pressure p_N according to

$$t_{LN_2} \approx t'_{LN_2} = (1 - \frac{p_N}{p_E}) t_0 \tag{4}$$

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where we assumed the volume to be unchanged (V_0). Making accurate assumptions for the volume of the helium gas like above, we get the more precise expression

$$t_{LN2} = \frac{At_0 + p_N}{A - \gamma p_N} \tag{5}$$

where $A = \frac{p_E}{t_E - t_0} + \frac{\epsilon(p_E - p_N)}{t_L - t_0}$

In order to measure the pressures of the helium gas we used a piezoresistive sensor which outputs a voltage U linear to the pressure. This means, that the pressure is also a linear function of the voltage, which we can write as

$$p = p_0 + CU \tag{6}$$

where p_0 and C are yet unknown constants. Determining p_0 and C by measuring U for two different pressures and solving the two equations for p_0 and C, will leave us with a function (Eq.(6)) to calculate any pressure for an outputted voltage U.

Methods

Experimental setup. (see Appendix A.2)
Our experiment is subdivided in into three different sections.

First, we calibrate the pressure sensor by measuring temperature and pressure in the lab at the time of the experiment (U_L, p_L) as a first calibration point. This was done with the help of a barometer and thermometer installed in the lab. Since the scale of the barometer is in mmHg we convert the pressure to mbar for further calculation using the rounded conversion factor $c = 1.33322$. Next we use the experiment setup consisting of glass bulb connected with the pressure sensor through glass tube as well as a rotary vane pump which is used to evacuate the glass bulb, additionally a balloon filled with helium creates an over pressure. By evacuating the glass bulb one gets a new pressure point at very low pressure, due to the over pressure created by the He-balloon we may assume $p_t = 0.1 \pm 0.1$mbar, furthermore the pressure sensor yields a corresponding voltage U_t, such that (U_t, p_t) provides the second calibration point used to fit a linear function through the two data points which contains all the information for the calibration of the sensor and therefore the calculation of the pressure using Eq.(6).

In a second step we measure the absolute zero temperature. To do that we evacuate the glass bulb several times as mentioned above. Then the glass bulb is heated up by vapor from boiling water t_K, which was given by the pressure in the lab at the time of the experiment. When the pressure in the bulb is constant we obtain $(t_K$(from before)$, U_K, p_K$ (using Eq.(6))). After that we cool the glass bulb with ice water to 0° C ($= t_E$) and consequently obtain $(t_E, U_E, p_E$ (using Eq.(6))).

This leaves us with the necessary data to calculate the absolute zero temperature using Eqs. (2) and (3) and allows for a comparison of our experimentally obtained values the literature value of $t_0 = -273.150 \pm 0.005$°C.

Finally we measure the temperature of liquid nitrogen using the same setup by submerging the bulb completely in the liquid.

Results and Data Analysis

The results of the first measurement for the calibration of the pressure sensor are presented in table 1. If not mentioned otherwise, the uncertainties are calculated using Gaussian error propagation (see Appendix A.1)

Table 1 | Experimental results for the measurement of temperature, pressure and voltage in the lab (t_L, U_L, p_L) at the time of the experiment. As well as the voltage and pressure at low pressure (U_t, p_L) for the second data point and their uncertainties. The uncertainty of the temperature and pressure in the lab are acquired in similar fashion by halving the last decimal from the readout of the thermometer and barometer respectively. Whereas the uncertainties of the sensor voltages and pressure at low pressure p_t are taken from the provided lab manual. [3] [4]

measuring point	lab pressure	low pressure
t (°C)	18.5 ± 0.5	-
U (mV)	135.04 ± 0.08	-15.34 ± 0.08
p (mbar)	947.5 ± 0.2	0.1 ± 0.1

With these values a linear fit can be performed and therefore the pressure sensor calibrated for Eq.(6).

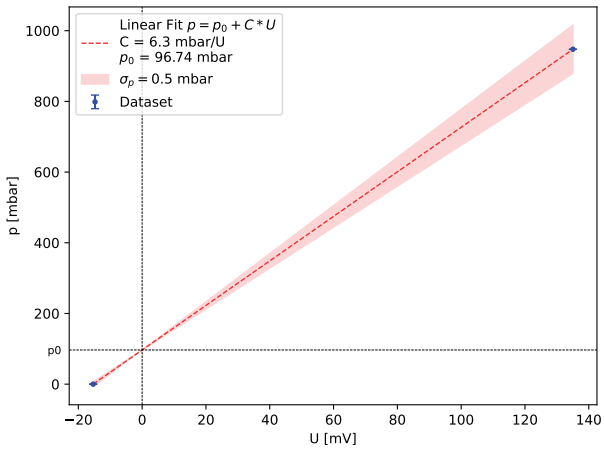


Fig. 1 | Linear fit through the two provided calibration points with the uncertainty showing the dependence of pressure in mbar on the voltage in mV.

With the calibrated pressure sensor we now measure two more points in order to calculate the absolute zero using Eq.(2) for a first estimation and Eq.(3) for the more exact value.

Table 2 | Measured and calculated values for temperature, voltage and pressure for boiling point of water (t_K, U_K, p_K) and ice water (t_E, U_E, p_E), where $t_E = 0$°C was assumed [3].

measuring point	boiling pt. of water	ice water
t (°C)	98.133 ± 0.004	0 ± 0.5
U (mV)	138.86 ± 0.08	98.22 ± 0.08
p (mbar)	971.5 ± 0.5	715.5 ± 0.5

With this, all the necessary information is acquired for the calculation of the absolute zero temperature. For a first rough estimate we calculated $t_0' = -274.3 \pm 0.5$ °C, which turned out to be rather accurate. Finally we obtained $t_0 = -272.9 \pm 0.3$ °C for the absolute zero temperature through our measurements.
For the temperature of liquid nitrogen we got $t_{LN_2} = -196.6 \pm 0.2$ °C and $t_{LN_2,th} = -196.9 \pm 0.2$ °C which was calculated exactly as t_{LN_2} with the only difference that the literature value for the absolute zero was used instead of our measured absolute zero temperature.

Discussion
We are generally satisfied with the precision of the calibration of the experimental setup since the literature value for the absolute zero lies within the uncertainty of our measurements.

The literature values are $t_{0,L} = -273.150 \pm 0.005$ °C for the absolute zero and $t_{LN_2,L} = -195.79 \pm 0.005$°C for the temperature of liquid nitrogen. [1] [2]

The literature value for the absolute zero lies within the uncertainty interval of our measured value $t_0 = -272.9 \pm 0.3$°C. [2] Further uncertainties that were not included in the calculations were the uncertainty of the thermal expansion coefficient γ, the uncertainty of the proportionality factor for the volume of the glass tube ϵ and that we sometimes did not wait long enough for the output of the pressure sensor to stabilize. The values are too close to make a first good guess to why the measured value is slightly **higher** (less negative) and **not** slightly lower (more negative) than the literature value, regarding Eq.(3).

Consider Eq.(2) instead, we can see that for a slightly higher pressure p_E we would get a slightly lower temperature t_0'. A higher temperature for $t_E = 0$°C would lead to such an increase in p_E (since $p \approx const(t - t_0)$). We put tap water into our ice filled bucket to increase thermal contact with the glass bulb, which may have lead to a temperature t_E higher than 0°C.

The literature value for the temperature of the boiling point liquid nitrogen at pressure of 1013.2 mbar is -195.79 ± 0.005°C.[1] Our experiment was conducted at the pressure $p_L = 947.5 \pm 0.2$ mbar. A lower ambient pressure should result in a lower boiling point, because the gas is less compressed, meaning the particles can "free themselves" faster (with less kinetic energy). We could not find any more precise information.

Conclusion
We made an experiment to measure the absolute zero temperature t_0 on the Celsius scale in a first step and the temperature of liquid nitrogen t_{LN_2} in a second step. Our experimental set up, consisting of a glass bulb with helium, a piezoresistive sensor, different surroundings to change the temperature of the glass bulb and more, included a lot of small uncertainties. Nevertheless our measured value for $t_0 = -272.9 \pm 0.3$°C is very close to the literature value of $t_{0,L} = -273.150 \pm 0.005$°C. Our measured value for liquid nitrogen, which was boiling, $t_{LN2} = -196.6 \pm 0.2$°C is also very close to the literature value of the boiling point of liquid nitrogen $t_{LN2,L} = -195.79 \pm 0.005$°C, knowing that the boiling point varies with pressure. We could not find any more precise information regarding the literature value for the boiling point of liquid nitrogen at lab pressure.

We conclude this experiment to be an accurate way to measure the absolute zero and temperature of liquid nitrogen. Regarding the cooling of the bulb to $t_E = 0$°C with ice and tap water, we assume that the glass bulb was at a slightly higher temperature since tap water is not at $0°C$. Mathematically, a higher temperature for t_E would shift the estimation for t_0 towards the literature value and might result in an even more precise measurement. An optimization for the future could be to find a better way than ice with water in a bucket to cool down the glass bulb to 0°C. Furthermore one could evacuate longer and wait longer for the output of the pressure sensor to stabilize or maybe someone has access to even better devices than a rotary vacuum pump and a piezoresistive pressure sensor. Another possibility to reduce the scale of the errors could be to scale the experiment by using a larger glass bulb.

References

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A Appendix

A.1 Calculation of Uncertainties Using Gaussian Error Propagation

For uncorrelated variables $x_1, x_2, ..., x_n$ with uncertainties $\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}$ and a function $f(x_1, x_2, ..., x_n)$, the uncertainty of f is given by

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2} \tag{7}$$

A.2 Experimental Setup Scheme

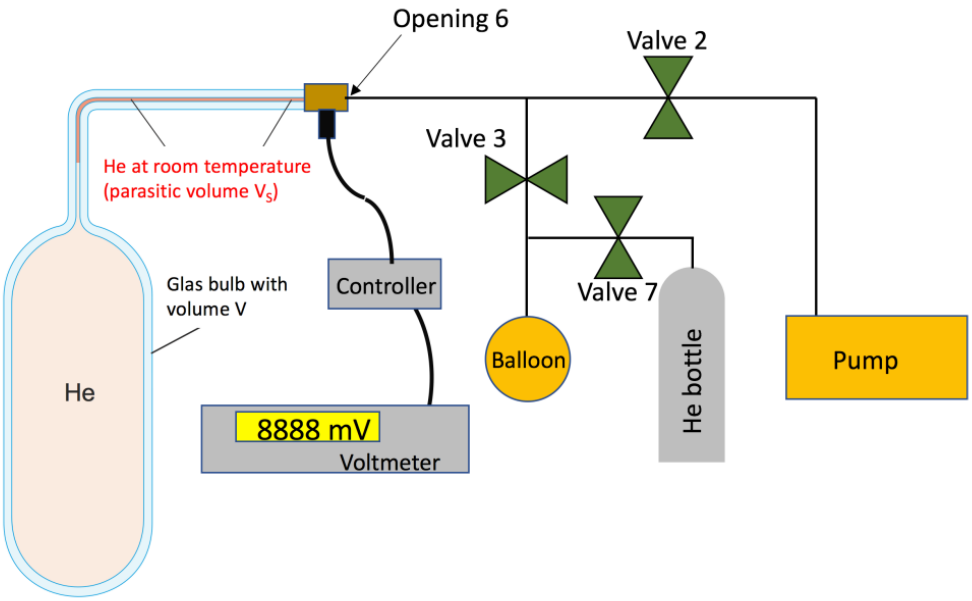


Fig. A1 | Experimental setup. [3]