# Determining the Specific Charge of the Electron $\frac{e}{m_e}$ Using the Busch Method And Estimation of Earth's Magnetic Field

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March 15, 2024

**In this experiment, we measure the specific charge of the electron using the Busch method and estimate earth's magnetic field and then compare our measured values to literature values. After carrying out the experiment, we have for earth's magnetic field $B_E = 3 \times 10^4 \pm 3 \times 10^4$ nT and for the specific charge of the electron $\frac{e}{m_e} = 2.11 \pm 0.03 \times 10^9 \frac{C}{kg}$ and $\frac{e}{m_e} = 2.15 \pm 0.03 \times 10^9 \frac{C}{kg}$ and $\frac{e}{m_e} = 3.0 \pm 0.1 \times 10^9 \frac{C}{kg}$, respectively since we used three different methods. We conclude that besides the magnetic field of the earth, the literature values do not lie within the uncertainties of our measurements and differ by several orders of magnitude. Therefore the experiment should be repeated with a more precise setup also taking into account sources of errors that were not considered in this experiment.**

Electrons travelling through a magnetic field perpendicular to their direction of motion experience a force, the so-called Lorentz force. Its orientation is again perpendicular to both the direction of motion of the electrons and the magnetic field and can be determined using the right-hand rule. When decomposing the velocity into normal and parallel component, the magnitude of the Lorentz force becomes

$$F = ev_nB \tag{1}$$

where $v_n$ is the normal component to a magnetic field H, e the elementary charge in coulomb and B the magnetic flux density in tesla.[5] From Eq.(1) it follows that the electron in a magnetic field moves along a helical path whose axis is parallel to H. Thus leads to a relationship between the pitch l and orbital period T. Since the electron is accelerated by a voltage of V in the direction of H and by considering the kinetic and potential energy of the electron one establishes the following relationship for the specific charge of the electron

$$\frac{e}{m_e} = \frac{8\pi^2}{\mu_0^2}\frac{V}{l^2H^2} \tag{2}$$

implying that the specific charge can be calculated after determining the pitch l, voltage V and magnetic field H, with given vacuum permeability $\mu_0$. [6]

The electron beam is realized in a Braun tube by applying the anode voltage V. In order to force the electrons on a helical path i.e to generate $v_n$ an alternating voltage is applied to a pair of electrode plates oriented perpendicularly to the beam. The magnetic field H which we are interested in, is generated in a long cylindrical coil which is placed over the Braun tube such that the field H can be assumed homogeneous. By suitable selection of H, the electrons traverse exactly the distance l (pitch) inside the Braun tube before impact on a fluorescent screen by which we are able to tell whether the electrons are focused on a single image point or not. The measurement is then conducted by a second coil i.e the measuring coil.

The magnetic field inside the center of a cylindrical coil is given by

$$H(I) = nI\frac{1}{\sqrt{1 + (\frac{2r}{L})^2}} \tag{3}$$

where I is the current, r the coil radius, L the coil length and n the number of turns per meter.[1]

The total magnetic field $H_0$ inside the Braun tube also contains the vertical component of the earth magnetic field at the location of the experiment $H_n^E$. In order to obtain the correct values for the magnetic field one must therefore account for the correction of the magnetic field by reversing the field polarity i.e reversing the current source from $I^-$ to $I^+$.

$$H_0 = H(\frac{I^+ + I^-}{2}) \qquad H_n^E = H(\frac{I^- - I^+}{2}) \tag{4}$$

where $I^+$ and $I^-$ are the respective mean values of the currents.

Finally we acquire the magnetic field H using a second method. The magnetic field inside the Braun tube is also given by

$$H = \frac{1}{w\mu_0 S}k\alpha \tag{5}$$

where w is the total number of turns of the measuring coil, S the area enclosed by one winding, $\mu_0$ the vacuum permeability and k a galvanometer constant contained in the experiment setup/circuit in $\frac{Vs}{\frac{mm}{m}}$ [1]

## Methods

**Experimental setup.** (see Appendix A.2)
Our experiment is subdivided into three seperate steps.

For our first measurement we use the Braun tube to accelerate electrons onto a fluorescent screen and place a coil around the tube. Then we adjuste the power which goes through the coil until only one dot is visible on the fluorescent screen and measure the respective current. This was performed ten times for each polarity of the supply voltage in order to obtain a mean value with corresponding uncertainty.

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In a second step we measure the magnetic field from the coil. In order to do this place the coil over the measuring coil and measure the induction using a galvanometer by switching the power supply of the first coil on and off.

Finally to be able to use the values obtained from the galvanometer measurements, a calibration was necessary in order to determine k in Eq.(5). A capacitor with known capacity C is charged with voltage V = 10V. By discharging the capacitor the current partially flows through the galvanometer. By measuring the deflection angle of the galvanometer the unknown constant k can be determined and therefore using Eq.(5) the magnetic field can be calculated.

## Data Analysis

If not mentioned otherwise the uncertainties are calculated using Gaussian error propagation. (see Appendix A.1)
For the first part the measured currents are $\langle I^- \rangle = 0.883 \pm 0.003A$ and $\langle I^+ \rangle = 0.875 \pm 0.002A$.
With these values we are able to perform the necessary calculations to obtain the magnetic field H. The results are displayed in the following Tables.

**Table 1** | Calculated magnetic flux density of earth's magnetic field at experiment location (47.40779°N, 8.51098°E, 546.3m MSL ) $B_E$ as well as the literature value $B_{E,L}$ with respective uncertainties. [2]

| | Magnetic Flux Density in nT |
|---|---|
| $B_E$ | $3 \times 10^4 \pm 3 \times 10^4$ |
| $B_{E,L}$ | $4.32244 \times 10^4 \pm 0.1570 \times 10^4$ [3] |

**Table 2** | Calculated magnetic field strength inside Braun tube after systematic error correction caused by earth's magnetic field. $H(\langle I^+ \rangle)$ and $H(\langle I^- \rangle)$ are calculated using Eq.(3) whereas $H'$ is calculated using Eq.(5).

| | Magnetic Field Strength in $\frac{A}{m}$ |
|---|---|
| $H(\langle I^- \rangle)$ | $4500 \pm 100$ |
| $H(\langle I^+ \rangle)$ | $5260 \pm 10$ |
| $H'$ | $5310 \pm 20$ |

**Table 3** | By Eq.(2) calculated specific charge of the electron using the beforehand acquiered data (Table 2), For $l = 9.3 \pm 0.05$cm and $V = 10 \pm 0.5$V. Furthermore the literature value is also displayed.

| | Charge to Mass Ratio in $\frac{C}{kg}$ |
|---|---|
| $\frac{e}{m_e}(H(\langle I^- \rangle))$ | $2.11 \pm 0.03 \times 10^9$ |
| $\frac{e}{m_e}(H(\langle I^+ \rangle))$ | $2.15 \pm 0.03 \times 10^9$ |
| $\frac{e}{m_e}(H')$ | $3.0 \pm 0.1 \times 10^9$ |
| $\frac{e}{m_e}$ | $1.75882001024 \times 10^{11} \pm 5.3 \times 10^3$ [4] |

For visualization purposes the results are displayed in the following graph.

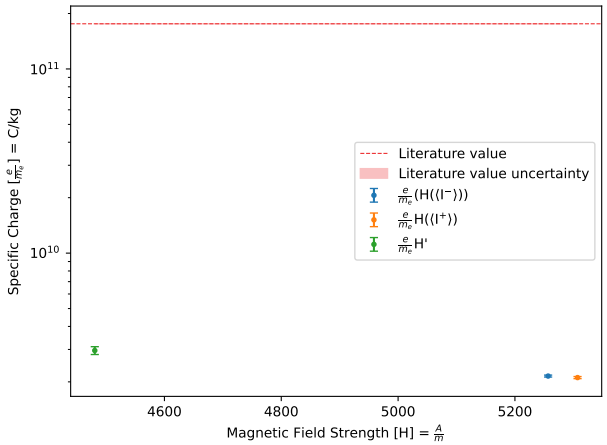



**Fig. 1** | In order to visualize the accuracy of the experiment, the negative specific charge is displayed, thus a log scaling on the y-axis is possible. The figure shows the specific charge in dependancy of the magnetic field strength inside the Braun tube as well as the literature value independent of the magnetic field strength with its uncertainty.

## Discussion of Results

The results of the experiment are generally not accurate and come with great errors. The literature value for the magnetic flux density of earth's magnetic field at the location of the experiment $B_E^n$ is within the uncertainty of our measurement but $B_E$ comes with an error of same magnitude as the mean value itself, therefore one can not be content with this result and its accuracy.

Regarding the specific charge of the electron the literature value does not lie within the uncertainties of our measurements and differs by several orders of magnitude. As one might observe in Figure 1 is the fact that the last method is closer to the literature value than the previous method but still very far off.

## Conclusion

In this experiment we measured the specific charge of the electron using the Busch method and estimated earth's magnetic field. These values allowed us to test the accuracy of the equations provided as well as the accuracy of the experiment setup by comparing to literature values. Since we were not able to perform any error analysis on the Braun tube itself and since great voltages are involved in the experiment that very likely might have changed some parameters due to heating up of the wiring and measuring instruments, our precision was limited. Therefore the real uncertainty of our values is likely higher but regardless the value for the specific charge of the electron differs by several orders of magnitude from the literature value. Therefore we conclude that the experiment setup precision, thus the experiment setup itself needs to be improved for a re-evaluation of the theoretical model.

# References


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# A Appendix

## A.1 Calculation of Uncertainties Using Gaussian Error Propagation

For uncorrelated variables $x_1, x_2, ..., x_n$ with uncertainties $\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}$ and a function $f(x_1, x_2, ..., x_n)$, the uncertainty of $f$ is given by

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2} \tag{6}$$

## A.2 Schemes and Diagrams

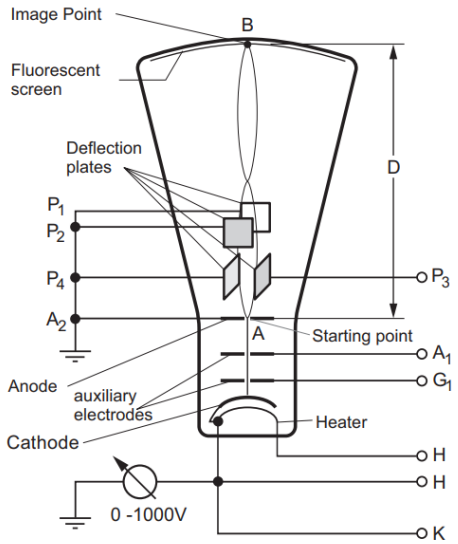



**Fig. A.2.1** | Braun tube scheme with wiring instructions for the experiment setup.

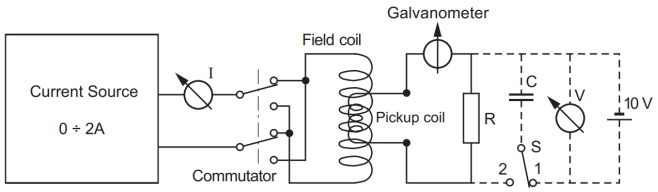



**Fig. A.2.2** | Circuit used in first and second step of the experiment, where the second step includes the dotted part of the circuit in the figure.