Estimation of Planck's Quantum of Action h and Measurements on Circuits Containing Diodes

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In this experiment we calculated Planck's quantum of action h, the generated power of an illuminated IR-LED and constructed a light barrier circuit with signal amplification. We measured for the Planck's quantum of action $\langle h \rangle = 5.297 \pm 0.002 \times 10^{-34}$ Js and $P = 2.4 \pm 0.2 \times 10^{-7}$ W for the generated power of an illuminated Infrared-LED. We conclude that the experimentally obtained values for h do not align with the literature value due to abuse of a rather inaccurate formula, several flaws of our equipment and measuring techniques, and also external disturbances. Therefore, the experiment can be improved. For example by using sensors in a blackbox to measure the faint glow of the LED's, a solid definition for the faint glow of the LED and a thorough examination of the electronics used in the experiment.

A semiconductor diode consists of a $p-n$ conjunction, meaning a $p-$layer attached to a $n-$layer. The $p-$layers are specific materials doped with positively charged atoms that are able to bind an electron (they own so called "holes", where electrons can be absorbed into) and the $n-$layers are specific materials doped with negatively charged atoms "carrying" valence electrons (in the so called valence band). For a forward biased diode, connecting the anode (p-region) to the positive side and the cathode (n-region) to the negative side of an applied voltage, the electrons in the $n-$layer "free" themselves from their atoms (move into the so called conduction band) and might even have enough energy to overcome the energy barrier between both layers (due to the initially formed, opposing electrical field caused by the tendency of electrons at the boundary of the $n-$layer to migrate over to the holes at the boundary of the $p-$layer and inducing a slight negative charge distribution on the boundary of the $p-$layer and leaving a positive charge distribution at the boundary of the $n-$layer behind) and can travel into to the $p-$ layer where they are absorbed by the positively charged atoms. When the applied voltage is kept high enough, more free electrons will be "pushed" into the $p-$ layer and the electrons that were already absorbed will be pushed to further away (from the conjunction) positively charged atoms etc. This will form a negative current through the cathode out of the anode.
During this process the electrons are obviously accelerated multiple times and especially when they are absorbed by the positively charged atoms, they are slowed dramatically and thus emit light due to "Bremsstrahlung". For most diodes this emitted light is being absorbed by other atoms in the material, but for special diodes with very thin layers the emitted light can pass through the material. These light transparent diodes are called light emitting diodes (LED's). For certain frequencies the light is visible and thus LED's are the essential part in most light bulbs.

Derived from the photoelectric effect, light is quantized and the light quantas (photons) carry the kinetic energy $h\nu$, where $h = 6.626\,070\,15 \times 10^{-34}$ Js is Planck's quantum of action [1] and ν the frequency of the light. The energy of the photons can be idealized to be equal to the kinetic energy of the electrons eU (electrical energy), gained after being accelerated due to the potential U and lost (emitted) when being bind again by the atoms (absorbed).

$$h\nu = E = eU \implies h = \frac{eU\lambda}{c} \tag{1}$$

where $e \approx 1.602 \times 10^{-19}$ C is the elementary charge [2], λ the wavelength of the light and $c = 299\,792\,458$ m/s the speed of light in vacuum [3]. This equation only holds for the lowest intensity of light, where every electron emits one photon and the entire kinetic energy of the electron is carried on by one photon. The minimum voltage, for which the first photons are being emitted, has to be used. For a higher voltage more photons are being emitted with the same frequency and increasing the intensity of the light. The frequency of the light depends mainly on the material of the diode.

Some LED's can also be used in the reverse way, meaning the electrons absorb light and unbind themselves to travel through the diode and form a negative current, which can be measured. Such a receiver diode can be used as a light sensor.

A transistor consists of $n-p-n$ (or $p-n-p$) layers, where the outer layers are called the collector and emitter respectively. Applying a voltage to the outer layers will not cause any current because a transistor is basically two diodes in reversed bias sandwiched together with the same $p-$layer and one of the inner diodes will block the current. When in addition to the outer voltage, one $n-$layer (emitter) and the $p-$layer (base) of the transistor are used as a forward biased diode, the electrons that overcame the energy barrier will not only move out of the base towards the positive terminal of the inner applied voltage (like for a normal diode) but will mostly move into the $n-$layer(collector), which was not part of the inner diode, and "push" the valance electrons into the conduction band where they are free to move out of the collector towards the positive terminal of the outer applied voltage. One can think of the electrons

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that overcame the energy barrier of the first $p-n$ conjunction, and not moving to the positive terminal of the inner voltage applied to the base and emitter, causing a chain reaction that frees more electrons into the conduction band of the collector making it more fluid for following electrons to wander through the entire transistor from emitter out of the collector and therefor multiplying the initial current from the inner diode. It should be mentioned that the technical current flows in reverse direction into the base and collector out of the emitter (viewed from the outer voltage perspective), hence the names (or as just viewed internally, the collector collects electrons from the emitter).
Transistors can not only be used to amplify an input current, but also as a switch. When the input voltage (base to emitter) is not sufficient to overcome the energy barrier from the emitter to the base of the transistor, there will not be any current flow, also not for the outer circuit, where for example a light bulb could be integrated. Forgetting about the light switch, a switch in general is the foundation of digital electronics, since no current flow (off) or current flow (on) corresponds to 0 and 1 and can be implemented into more complex circuits to operate in binary code.

In this experiment we investigated the behaviour of different LED's as emitter or receiver and usage of transistors as amplifiers in several simple circuits. To make use of our observations we calculated h according to Eq.(1) for LED's with different colors (wavelengths) and their different minimum voltages for the lowest visible intensity. Furthermore we illuminated receiver LED's with different emitter LED's and measured the voltages of the receivers. When there was a voltage created at the receiver we could use the specific emitter and receiver as a simple light barrier and investigated its properties. For latter we had to amplify the current at the receiver LED with a transistor to make effective observations about the received signal strength varying due to us moving and orientating the receiver LED or also blocking the incoming light with our hand.

Methods

Experimental setup. All of the circuits were built on a simple circuit board. We used a power supply to obtain the desired voltage and for our measurements we used a multimeter. The circuits only consisted of LED's, Transistors and Resistors.

In the first task we measured the minimal voltages through LED's of different colors (wavelengths) at the lowest visible intensity (faint glow) of the emitted light. We used a simple Diode-Resistor serial circuit. To begin with we had to calculate the minimal needed series resistor for the maximal allowed current through a diode using

$$\begin{aligned} U_0 = U_R + U_{diode} &= RI_R + U_{diode} \\ \implies I_{max} &\leq (U_0 - U_{diode})/R \\ \implies R &\geq (U_0 - U_{diode})/I_{max} \end{aligned} \tag{2}$$

where $U_0 = 5V$ is the maximal applied voltage to the circuit and I_{max} and the recommended voltage U_{diode} can be read off the data sheets [4] together with the wavelength λ of the diode [4]. We calibrated the series resistor only for the blue (468nm) LED and used the same resistor for all LED's in the first task.

Secondly we used the serial circuit (mentioned above) with a blue LED to illuminate an IR (infrared) receiver LED at a certain distance and measured the open-circuit voltage over the IR LED. Then we measured the voltage over a $R = 10$ $kOhm$ resistor parallel to the IR LED. Knowing $U = RI$, we calculated the generated power of the illuminated IR LED according to

$$P = UI = \frac{U^2}{R} \tag{3}$$

For the serial- (illuminating LED) and parallel (illuminated LED) circuit we then also used two blue LED's both as emitter and receiver, and two IR LED's both as emitter and receiver. In both cases we measured the voltages of the receiving LED. Finally we also measured the voltage of the blue LED when being illuminated by the IR LED (opposite was already measured as mentioned above Eq.(3).

In our last measurement we used again two IR LED's and connected the receiver LED with a transistor (according to Fig. 1 taken from [5]) to amplify the weak signal.

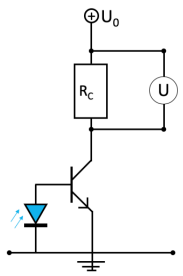


Fig. 1 | Integration of a transistor in the circuit of the receiving LED with $R_c = 10$ $kOhm$

We measured the voltage over the resistor R_c in the receiving circuit when the emitting circuit was supplied with the usual voltage of $5V$.

Eventually we used the last setup as a light barrier (see Appendix A.2) and observed the received signal (voltage over R_c) for different orientations of the receiver IR LED, distances between emitter and receiver LED and when blocking the sent light with our hands.

Data Analysis

If not mentioned otherwise the uncertainties are calculated using Gaussian error propagation (see Appendix A.1). For the first part we measure the voltage of different LED's at faint glow. With these values we are able to perform the necessary calculations to obtain an estimation of Planck's quantum of action h using Eq.(1). The results are displayed in the following Table and Figure.

Table 1 | Table containing the experimental results of our first measurement. We calculate Planck's quantum of action h using Eq.(1) for LED's of different wavelength λ in nm and their voltage at faint glow U_{faint} in V with an uncertainty of 0.0005V. Additionally the voltage $U_{10\mu A}$ of

the LED's at $10\mu A$ are displayed in V, also with an uncertainty of 0.0005V. Lastly the mean value for h $\langle h \rangle$ is also displayed. The uncertainty for the wavelength is taken from the respective data sheets [4].

$\lambda \pm 1$ (nm)	$U_{10\mu A}$ (V)	U_{faint} (V)	h (10^{-34}Js)
468	2.338	2.220	5.552 ± 0.001
515	2.120	1.846	5.081 ± 0.001
565	1.703	1.743	5.263 ± 0.002
592	1.686	1.722	5.448 ± 0.002
660	1.445	1.458	5.143 ± 0.002
$\langle h \rangle$			5.297 ± 0.002

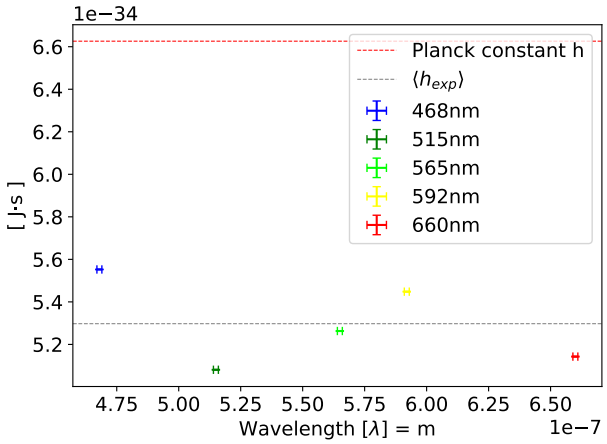


Fig. 2 | In order to visualize the accuracy of our measurements the figure shows the unit of action in Js in dependency of the wavelength of each LED used calculated using Eq.(1). Furthermore the mean value of our experimentally obtained results as well as the literature value for Planck's quantum of action h is displayed as a constant value.

In a second step we calculated the generated power of an illuminated IR-LED. Using Eq.(3) we acquired $P = 2.4 \pm 0.2 \times 10^{-7}$W

Then we illuminated the receiver LED with different emitter LED's and measured the voltages of the receivers. The results are displayed in the following Table.

Table 2 | Table containing the measured voltage at the receiver diode. (Blue: 468nm)

Emitter Diode	Receiver Diode	$U \pm 0.0005$ (V)
IR	IR	0.882
Blue	Blue	1.856
IR	Blue	0.066
Blue	IR	0.708

This roughly verifies the quantization of light. The intensity of the IR diode does not matter for generating any current at the blue diode. The frequency of the IR diode is too low and thus the individual photons do not carry enough energy to free an electron with higher binding energy in the "blue diode" material than in the "IR diode" material. The voltages in general do not depend on the intensity (at least for our experiment conducted with normal intensities) of the light. Increasing the intensity of the blue diode illuminating the IR diode will cause more electrons to be freed due to a higher rate of incoming photons but the individual electron will have the usual kinetic energy and will not lead to higher measured "stoppage voltage". The photoelectric current nevertheless increases rather proportionally with the intensity because more electrons are freed to flow.

The results of our observations with the light barrier formed by two IR diodes can be summed up simply. The generated current is proportional to the intensity. Putting a hand between the diodes will block the signal. The light sensor of the diodes are at their tips and the received signal is the strongest with the tips pointing towards each other. Disorientating the tips will lead to a decrease of the received signal somehow depending on the angle. The intensity of the emitted light decreases with $\frac{1}{r^2}$, where r is the distance to the light source, because at constant power P of the emitter diode the power of the light distributes for isotropic sources equally over a larger area of the sphere $A = 4\pi r^2$ and the intensity is given through $I = \frac{P}{A}$.

Discussion of Results

The literature value for Planck's quantum of action is $h = 6.62607015 \times 10^{-34}$ Js. This does not lie within any of our experimentally determined values for h using Eq.(1) for different wavelengths. However the values are of same magnitude as the literature value and the mean $\langle h_{exp} \rangle$ deviates by approximately 20% from the literature value h.

The difference observed is due to the fact that many systematic errors are present in the experimental setup used. The voltage at faint glow was obtained by looking at the diode and reading off the multimeter. The problem this poses is firstly the definition of faint glow which is whether clear nor quantifiable but on the other hand also by using the eye to measure the beforehand doubtful defined faint glow. With that comes a large uncertainty how one looks at the diode from above since the angle may also alter one's perception. Furthermore this all was conducted in a room with a lot of noise in terms of scattered light from the environment.

The error sources named above lead to a rather random deviation but since we observe a one-sided effect, that is a negative deviation from h of about 20%, they cannot justify this systematic error. One explanation could be the neglection of smaller energy losses (apart from being absorbed into the "holes") of the electrons due to interactions with atoms from the complex inner structure of the material which also slightly slows them and makes them radiate photons with a lower frequency (infrared light), which would lead to a an even higher average frequency of the emitted photons (when added to the frequency of the visible light) and thus to an even smaller value for h ($h\nu = E$). So this can not be it. Considering that the electrons absorbed in the $p-$layer have a lower binding energy than in the $n-$layer leads to a loss of energy that is even higher than just the electrical energy eU. Since the wavelength is given, the only way to account for this greater energy loss $E > eU$ would be a larger approximation for h and therefor shifting our empir-

ical value up to the literature value.

To further reduce the deviation of our values from h, the measurements should be conducted using sensors oriented at a right angle to the LED inside some sort of black-box where no scattered light is present. We also conclude Eq. (1) to not be a precise modulation for the energy emitted by electrons travelling from the $n-$ into the $p-$layer.

Conclusion

In this experiment we estimated Planck's quantum of action h using Eq.(1). The literature value does not lie within the uncertainty of our experimental results due to imprecise modulation through Eq. (1) of reality, several flaws of our measurement methods and external disturbances.

Furthermore we determine the generated power of an illuminated IR-Diode and finally construct a light barrier circuit with signal amplification.

There are many ways one might improve this experiment. For example a more precise definition of faint glow and its measurement with some sensors in a blackbox, also considering microscopic effects which occur during the illumination of a LED. Therefore one might have to conduct an examination of the electronics at hand to account for error sources which went unnoticed. A more accurate theoretical formula could also be used accounting for electrodynamics in materials thus maybe also including unconsidered behaviours of the electrons (different binding energies for different atoms).

References

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A Appendix

A.1 Calculation of Uncertainties Using Gaussian Error Propagation

For uncorrelated variables $x_1, x_2, ..., x_n$ with uncertainties $\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}$ and a function $f(x_1, x_2, ..., x_n)$, the uncertainty of f is given by

$$\sigma_f = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2} \tag{4}$$

A.2 Schemes and Diagrams

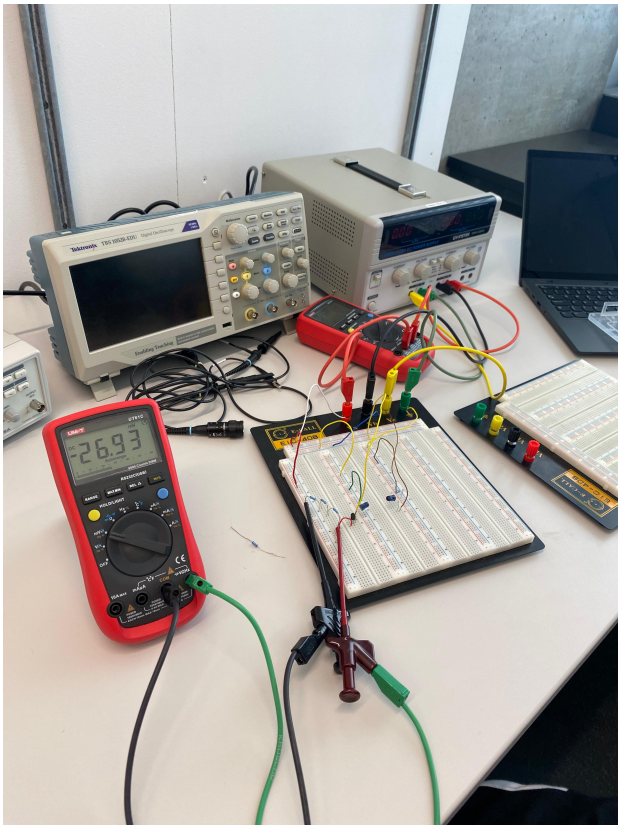


Fig. A.2.1 | During the lab constructed light barrier circuit on a breadboard. The multimeter at left measures the by transistor amplified signal. The other multimeter measures the current in the circuit of the emitter LED to make sure that the current stays below I_{max}