A novel technique, to reduce the

Preserves guarantees of

\[ \alpha \leq \text{Compare performance of PAMPC and DAMPC} \]

The predicted state measurement defines a

Monte Carlo simulation study

Parameter, disturbance and constraint sets

Robust state tube: constraint satisfaction

Predicted state tube: worst case performance as MPC cost function

Finite horizon reference tracking, 100 timesteps

Prediction horizon: 8 steps

Contribution

Augment Adaptive MPC with active exploration:

Dual AMPC (DAMPC)

Main features

- Addresses the exploration-exploitation tradeoff: informative data vs. control performance
- Enables reference tracking under uncertain input setpoints
- Preserves guarantees of constraint satisfaction and recursive feasibility

Predicted parameter set

- Using a parameter estimate \( \hat{\theta}_k \), predict the next state measurement as a function of \( u_k \)

\[ x_{k+1} = A(\theta) x_k + B(\theta) u_k + w_k \]

- The predicted state measurement defines a feasible parameter set as

\[ \hat{\theta}_{k+1} = \hat{\theta}_k + \hat{\theta}_k u_k \]

- The identification at the next step can be approximated as

\[ \hat{\theta}_{k+1} = \hat{\theta}_k \cap \hat{\Delta}_{k+1} \]

- Thus, \( \hat{\theta}_{k+1} \) captures the effect of \( u_k \) on identification to be performed in the future

Set-membership identification

- Applicable when systems are subject to bounded noise, even with unknown properties
- Compute feasible parameters at each time step: \( \hat{\Delta}_k = \{ \theta | x_{k+1} = A(\theta) x_k + B(\theta) u_k + w_k \} \)
- Update parameter set such that \( \hat{\theta}_k \) has predefined hyperplane directions and

\[ \hat{\theta}_k = \hat{\theta}_{k-1} \cap \Delta_k \]

- Can be performed by solving linear programs

Safe active exploration

- Construct state tubes using homothetic sets

\[ X_{1|k} = x_{1|k} + \alpha_{1|k} x_0 \]

- Decouple robustness and exploration:

  - Robust state tube: constraint satisfaction
  - Predicted state tube: worst case performance as MPC cost function

Here is a numerical example:

- Monte Carlo simulation study
- Compare performance of PAMPC and DAMPC

We consider a 2nd order system of the form:

\[
\begin{align*}
A_0 &= \begin{bmatrix} 0 & 0.85 \\ 0 & 0.5 \end{bmatrix}, & B_0 &= \begin{bmatrix} 1 \\ 0.4 \end{bmatrix} \\
A_1 &= \begin{bmatrix} 0.2 \\ 0.7 \end{bmatrix}, & B_1 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
A_2 &= \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, & B_2 &= \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}
\end{align*}
\]

Parameter, disturbance and constraint sets

- \( |\theta|_\infty \leq 1 \)
- \( |w|_\infty \leq 0.1 \)
- \( |x|_\infty \leq 3 \)
- \( |\hat{w}|_\infty \leq 2 \)

50 random realizations of true \( \theta \in \Theta_0 \)
4 random disturbance sequences \( w_k \in W \)
Finite horizon reference tracking, 100 timesteps
Prediction horizon: 8 steps

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FEATURES OF ADAPTIVE MPC

- Systems with linear, time-invariant dynamics and parametric uncertainties in the state space matrices and additive disturbances

\[ x_{k+1} = A(\theta) x_k + B(\theta) u_k + w_k \]

- The parameters and disturbances are assumed to be bounded \( w \in W, \theta \in \Theta_0 \)
- Constraint satisfaction ensured using tube MPC
- Uncertain parameters are bounded by sets of non-increasing size using set-membership ID
- Identification is passive (PAMPC)

NUMERICAL EXAMPLE

- Reference tracking

\[ x_{1|k} = x_{1|k} + \alpha_{1|k} x_0 \]

- Decouple robustness and exploration:

  - Robust state tube: constraint satisfaction
  - Predicted state tube: worst case performance as MPC cost function