REMARK: All exercises are referred to in the lecture slides, at places indicated by the sign  **, what helps to give them a proper context.**

Exercise 1

A function $f : \mathbb{R} \to \mathbb{R}$ is convex on an interval $[a, b]$, $a < b$, if for any $x_1, x_2 \in [a, b]$ and α with $0 \leq \alpha \leq 1$ we have $f(x_1 + \alpha(x_2 - x_1)) \leq (1 - \alpha)f(x_1) + \alpha f(x_2)$. Starting from the above definition, show the following:

a) $f : \mathbb{R} \to \mathbb{R}$ is convex on $[a, b]$, if and only if

$$\frac{f(x) - f(a)}{x - a} \leq \frac{f(b) - f(a)}{b - a} \leq \frac{f(b) - f(x)}{b - x} \tag{1}$$

for all $x \in (a, b)$.

b) Use results from (a) to show: *i*) A differentiable function of one variable is convex on an interval if and only if its derivative is monotonically non-decreasing on that interval. *ii*) A differentiable function of one variable is concave on an interval if and only if its derivative is monotonically non-increasing on that interval.

d) A non-decreasing offer curve in case of perfect competition (price-takers) implies convex (possibly non-differentiable) cost function, while non-increasing bid curve implies concave (possibly non-differentiable) benefit function.

Exercise 2

Let the bids be piecewise constant functions (constant on intervals with non-empty interior) which are non-decreasing for supply bids and non-increasing for demand bids. Formulate the market clearing problem as an optimization problem (primal). Remark: from previous exercise we know it has to be convex optimization problem.

Exercise 3

Consider a BRP (e.g., a microgrid registered as BRP) with the following portfolio

- m generators, where i-th generator is characterised by: $C_i(p_i)$ as the production cost function; $\underline{p}_i$ and $\overline{p}_i$ as lower and upper bounds on power production, respectively;
- n controllable loads, $\{B_i(d_i), \underline{d}_i, \overline{d}_i\}_{i=1,\ldots,n}$; $B_i(d_i)$ is benefit function; $\underline{d}_i$ and $\overline{d}_i$ are lower and upper limit for consumption;
- aggregated price inelastic power injection g.

Let p_{EX} denote the total (aggregated) net power injection from a BRP into the grid, and let λ denote the corresponding electricity price. Consider the following two approaches for calculating market bid curve $\beta_{BRP}(p_{EX})$ for the BRP.

Approach I Treat λ (market price) as parameter which varies in some interval, and calculate p_{EX} by solving the following optimization problem

$$\min_{\{p_i\},\{d_j\},p_{EX}} \sum_{i=1}^{m} C_i(p_i) - \sum_{j=1}^{n} B_j(d_j) - \lambda p_{EX}$$

$$\text{subject to} \sum_{i=1}^{m} p_i - \sum_{j=1}^{n} d_j + q = p_{EX}$$

$$\underline{p}_i \leq p_i \leq \overline{p}_i,\ i = 1, \ldots, m$$

$$\underline{d}_j \leq d_j \leq \overline{d}_j\ j = 1, \ldots, n$$

Create the bid curve by setting $\beta(p_{EX}) = \lambda$, for each solution pair (λ, p_{EX}).

Approach II Treat p_{EX} as parameter which which varies in some interval and calculate the Lagrange multiplier λ related to the constraint (♣) in the

Lagrange dual problem to the following (primal) optimization problem

$$\begin{aligned} \min_{\{p_i\},\{d_j\},p_{EX}} \quad & \sum_{i=1}^{m} C_i(p_i) - \sum_{j=1}^{n} B_j(d_j) \\ \text{subject to} \quad & \sum_{i=1}^{m} p_i - \sum_{j=1}^{n} d_j + q = p_{EX} \qquad (\clubsuit) \\ & \underline{p}_i \leq p_i \leq \overline{p}_i, \; i = 1, \ldots, m \\ & \underline{d}_j \leq d_j \leq \overline{d}_j \; j = 1, \ldots, n \end{aligned}$$

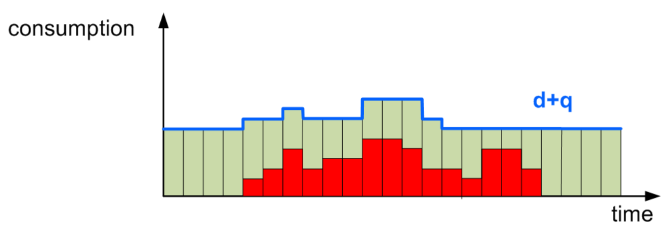
Create the bid curve by setting $\beta(p_{EX}) = \lambda$, for each solution pair (λ, p_{EX}).

Show equivalence between Approach I and Approach II.

Exercise 4

The goal of this exercise is to illustrate a case when the load factor cannot be one. Recall that the load factor defined over some finite time horizon is given by

$$\text{load factor} = \frac{\text{average demand}}{\text{peak demand}}.$$



We make the following definitions

- $p(k)$=controllable power production at time k
- $q(k)$=uncontrollable load or negated uncontrollable power
- $d(k)$=controllable load

- $C(p)$=cost function for producing at power level p
- $B(d)$=benefit function of consuming at power level d

Consider time horizon $k \in \{1, 2, \ldots, N\}$ and suppose that the controllable load is energy constrained in a sense that the following constraint has to hold $\sum_{k=1}^{N} d(k) = E_N$, for some given positive E_N. Suppose that the power profile of uncontrollable load q over the horizon is known, that is, we know $\mathbf{q} = (q(1), \ldots, q(N))$. Formulate optimization problem in which the goal is to maximize the social welfare over the considered time horizon, taking into account the energy constraint of a controllable load. Note that the solution to this optimization problem coincides to the result of market-based scheduling under perfect competition. The tasks are as follow:

a) Suppose that $C(\cdot)$ is strictly convex and $B(\cdot)$ strictly concave. Consider the optimal power production/consumption profile over the time horizon. Show that if q is not constant over the time horizon, the load factor is necessarily smaller than 1.

b) With $B(\cdot) \equiv 0$ and $C(\cdot)$ strictly convex, optimal load shifting of energy constrained loads leads to power factor 1 even with q not being constant.

Exercise 5

Related to the slides on “Nodal pricing”. Consider nodal pricing with *DC* power flow. Prove that the *congestion revenue* (merchandise surplus) is always nonnegative.

Exercise 6

Consider simple power system presented in Figures 1 and 2 with the following characteristics

- The bids (incremental costs) for generators at nodes A, B and C: $\beta_A(p_A) = 25 + 0.02p_A$, $\beta_B(p_B) = 30 + 0.02p_B$, $\beta_C(p_C) = 35 + 0.02p_C$
- Load is price inelastic with values indicated on the figures.
- All three lines are identical.

For the two scenarios from the figures ((1) No line flow limits; (2) Power flow in line $A - B$ constrained to $\leq$ 100MW), calculate the set of nodal prices, the corresponding power production levels and power flows in lines. Use DC load flow model. Note: the final results are also presented in the figure (λ_A, λ_B and λ_C denote the prices).

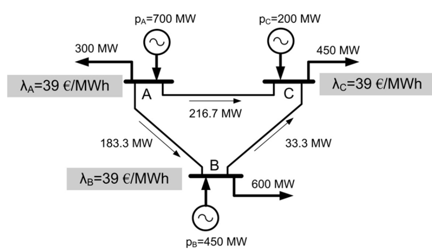


Figure 1: No line flow constraints.

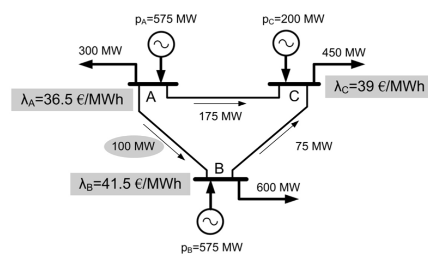


Figure 2: Power flow in line $A - B$ constrained to $\leq$ 100MW.

Exercise 7

This is a MATLAB exercise. For network with topology presented on Figure 3 and with the numerical data given in the tables below, calculate: nodal prices, zonal prices, PTDFs for transactions of choice. The coefficients a_i and b_i in the right table below define the cost functions of generator at node i: $C_i(p_i) = a_i p_i^2 + b_i p_i$.

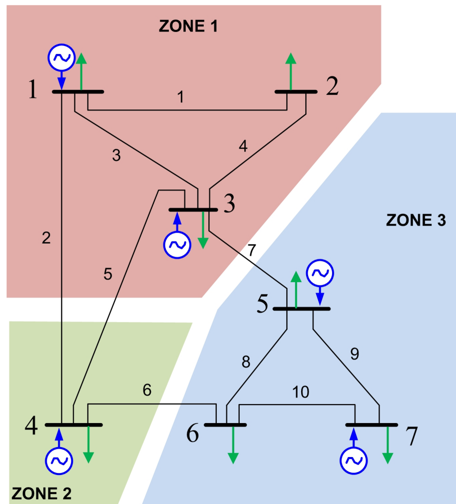


Figure 3: Net topology with line and node labels.

line i-j	x_{ij}	flow limit
1-2	0.0576	100
1-4	0.092	100
1-3	0.17	100
2-3	0.0586	100
3-4	0.1008	100
4-6	0.072	100
3-5	0.0625	100
3-5	0.161	100
3-5	0.085	100
3-5	0.0856	100

node i	a_i	b_i	load
1	0.13	1.73	88
2	-	-	87
3	0.13	1.86	64
4	0.09	2.13	110
5	0.10	2.39	147
6	-	-	203
7	0.12	2.53	172

Exercise 8

Show that $ACE_i = 0$ for each control area i, implies that $\Delta f = 0$ (frequency deviation is zero) and that total power exchanges among control areas as at scheduled values.

Hint: Write down the equations for a simple example (e.g. in the Figure 4),

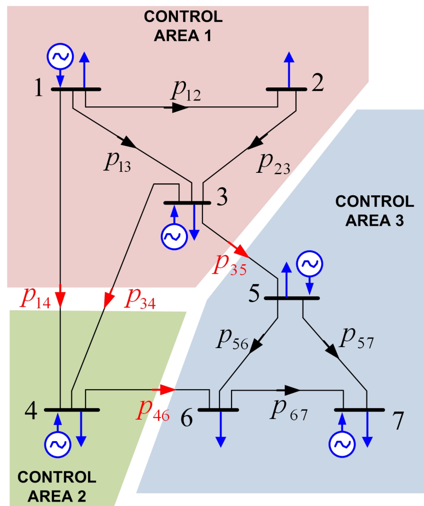


Figure 4: Network example.

and generalize.