Power System Stability & Control

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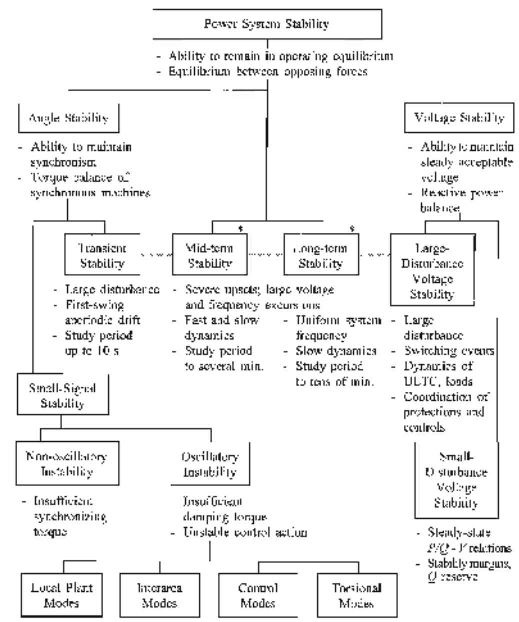
Power system stability & control: have to choose based on



- what Andrej needs
- what I actually know well
- what is interesting from a network perspective rather than from device perspective
- what is relevant for future (smart) power grids with high renewable penetration
- what gives rise to fun distributed control problems
- what **you** are interested in

Tentative outline

my particular focus is on networks

Disclaimers

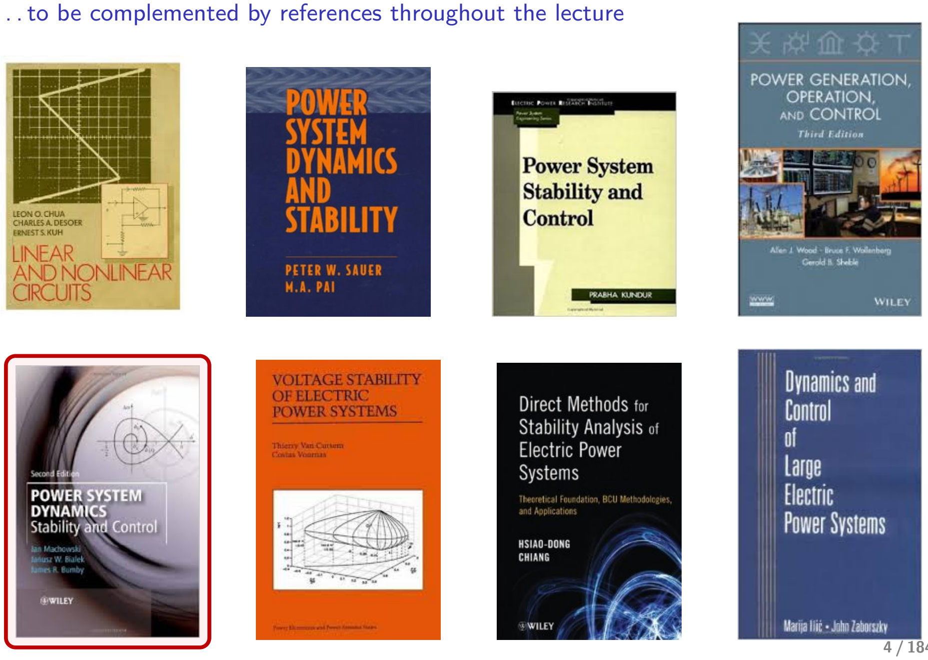
- start off with "boring" modeling before more "sexy" topics
- start off with basic material & before "cutting edge" work
- focus on simple models and physical & math intuition

⇒ cover fundamentals, convey intuition, & give references for the details

Please . . .
- ask me for further reading about any topic,
- and interrupt & correct me anytime.

Many references available . . . my personal look-up list

. . . to be complemented by references throughout the lecture



We will also use the blackboard . . .

. . . respectively, we will outsource the blackboard to the exercises

Outline

You will learn to appreciate the following words of wisdom

"Power system research is all about the art of making the right assumptions."

— [Maria Ilic, Lund LCCC Seminar '14]

Circuit Modeling: Network, Loads, & Devices

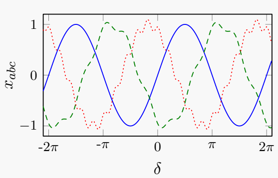
Signal space in three-phase AC circuits

three phase & AC

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} x_a(t+T) \\ x_b(t+T) \\ x_c(t+T) \end{bmatrix}$$

periodic with 0 average

$$\frac{1}{T}\int_0^T x_i(t)dt = 0$$

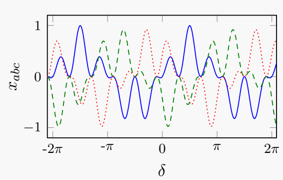


symmetric/balanced

$$= A(t)\begin{bmatrix} \sin(\delta(t)) \\ \sin(\delta(t) - \frac{2\pi}{3}) \\ \sin(\delta(t) + \frac{2\pi}{3}) \end{bmatrix}$$

so that

$$x_a(t)+x_b(t)+x_c(t)=0$$

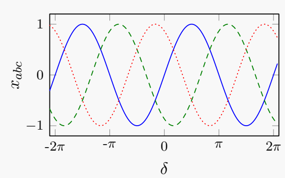


synchronous

$$= A\begin{bmatrix} \sin(\delta_0 + \omega^* t) \\ \sin(\delta_0 + \omega^* t - \frac{2\pi}{3}) \\ \sin(\delta_0 + \omega^* t + \frac{2\pi}{3}) \end{bmatrix}$$

const. freq & amp:

$\Rightarrow$ phasor $Ae^{\mathrm{i}(\delta_0+\omega^* t)}$



Park or *dq*0-transformation

$$T(\theta) = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

- is unitary $T(\theta)^{-1} = T(\theta)^T$ & maps balanced *abc*-signal to

$$x_{dq0} = \begin{bmatrix} x_d(t) \\ x_q(t) \\ x_0(t) \end{bmatrix} = T(\theta)x_{abc}(t) = \sqrt{\frac{3}{2}}A(t)\begin{bmatrix} \sin(\delta(t) - \theta) \\ \cos(\delta(t) - \theta) \\ 0 \end{bmatrix}$$

- $T(\omega^* t)$ maps a synchronous signal $x_a(t) = A\sin(\delta_0 + \omega^* t)$ to

$$x_{dq0} = \begin{bmatrix} x_d(t) \\ x_q(t) \\ x_0(t) \end{bmatrix} = T(\omega^* t)x_{abc}(t) = \sqrt{\frac{3}{2}}A\begin{bmatrix} \sin(\delta_0) \\ \cos(\delta_0) \\ 0 \end{bmatrix}$$

- another rotation matrix reduces the signal to *q*-coordinate $\sqrt{3/2} \cdot A$

Long story short . . .

We will work with **single-phase phasor signals** $x(t) = Ae^{\mathrm{i}(\delta_0+\omega^* t)}$ representing the q-phase of a balanced, synchronous, 3-phase AC circuit.

Everything can be extended . . . see, e.g., this control-theoretic tutorial:

Modeling of microgrids—from fundamental physics to phasors and voltage sources

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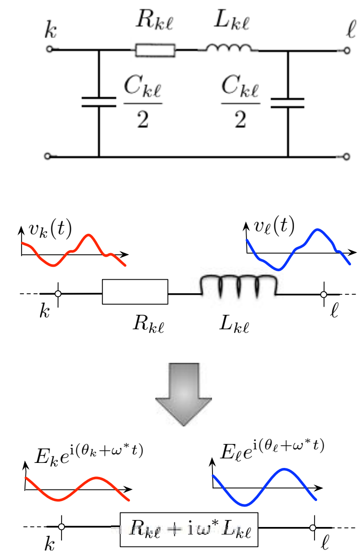
Abstract

Microgrids are an increasingly popular class of electrical systems that facilitate the integration of renewable distributed generation units. Their analysis and controller design requires the development of advanced (typically model-based) techniques naturally posing an interesting challenge to the control community. Although there are widely accepted

y 2015

AC circuits in power networks

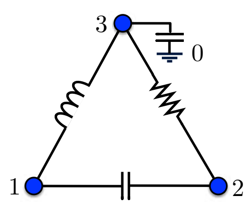
- power network modeled by linear **RLC** circuit, e.g., Π-model for
  - transmission lines (mainly inductive)
  - distribution lines (resistive/inductive)
  - cables (capacitive effects)
- we will work in **single-phase**
- **quasi-stationary modeling**: harmonic waveforms at nominal frequency ω^*
  - phasor signals: $v_k(t) \approx E_k e^{\mathrm{i}(\theta_k+\omega^* t)}$
  - steady-state circuit: $\frac{d}{dt} L_{k\ell} \approx \mathrm{i}\,\omega^* L_{k\ell}$



Note: quasi-stationarity assumption can be justified via singular perturbations & modeling can be improved using *dynamic phasors* [A. Stankovic & T. Aydin '00].

AC circuits – graph-theoretic modeling

1. a circuit is a connected & undirected **graph** $G = (\mathcal{V}, \mathcal{E})$
   - $\mathcal{V} = \{1, \ldots, n\}$ are the nodes or *buses*
     - buses are partitioned as $\mathcal{V} = \{\text{sources}\} \cup \{\text{loads}\}$
     - the ground is sometimes explicitly modeled as node 0 or $n+1$
   - $\mathcal{E} \subseteq \{\{i,j\} : i,j \in \mathcal{V}\} = \mathcal{V} \times \mathcal{V}$ are the undirected edges or *branches*
     - edges between distinct nodes $\{i,j\}$ are called *lines*
     - edges $\{i,0\}$ connecting node i to ground are called *shunts*



$$\mathcal{V} = \{1, 2, 3\}$$

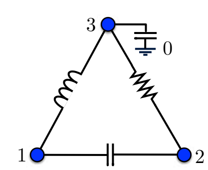
$$\mathcal{E} = \{\{1,2\}, \{1,3\}, \{2,3\}, \{3,3\}\}$$

AC circuits – the network admittance matrix

2. $Y = [Y_{ij}] \in \mathbb{C}^{n\times n}$ is the **network admittance matrix** with elements

$$Y_{ij} = \begin{cases} -\frac{1}{Z_{ij}} & \text{for off-diagonal elements } i \neq j \\ \frac{1}{Z_{i,\text{shunt}}} + \sum_{j\neq i} \frac{1}{Z_{ij}} & \text{for diagonal elements } i = j \end{cases}$$

- impedance = resistance + i · reactance: $Z_{ij} = R_{ij} + \mathrm{i} \cdot X_{ij}$
- admittance = conductance + i · susceptance: $\frac{1}{Z_{ij}} = G_{ij} + \mathrm{i} \cdot B_{ij}$



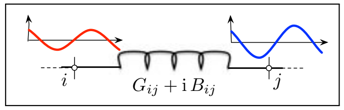
$$Y = \underbrace{\begin{bmatrix} \frac{1}{Z_{12}} + \frac{1}{Z_{13}} & -\frac{1}{Z_{12}} & -\frac{1}{Z_{13}} \\ -\frac{1}{Z_{12}} & \frac{1}{Z_{12}} + \frac{1}{Z_{23}} & -\frac{1}{Z_{23}} \\ -\frac{1}{Z_{13}} & -\frac{1}{Z_{23}} & \frac{1}{Z_{13}} + \frac{1}{Z_{23}} \end{bmatrix}}_{\text{network Laplacian matrix}} + \underbrace{\begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{1}{Z_{3,\text{shunt}}} \end{bmatrix}}_{\text{diag(shunts)}}$$

Note quasi-stationary modeling: $Z_{13} = \mathrm{i}\,\omega^* L_{13}$ with nominal frequency ω^*

AC circuits – basic variables

❸ **basic variables**: voltages & currents

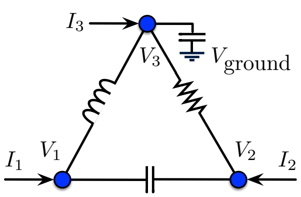
- on nodes: potentials & current injections
- on edges: voltages & current flows



❹ quasi-stationary **AC phasor coordinates** for harmonic waveforms:

- e.g., complex voltage $V = E\,e^{\mathrm{i}\,\theta}$ denotes $v(t) = E\cos(\theta + \omega^* t)$

  where $V \in \mathbb{C}$, $E \in \mathbb{R}_{\geq 0}$, $\theta \in \mathbb{S}^1$, $\mathrm{i} = \sqrt{-1}$, and ω^* is nominal frequency



external injections: I_1, I_2, I_3

potentials: V_1, V_2, V_3

reference: $V_{\text{ground}} = 0\text{V}$

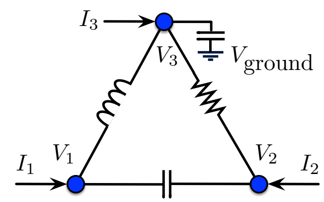
AC circuits – fundamental equations

❺ **Ohm's law** at every branch: $I_{i\to j} = \frac{1}{Z_{ij}}(V_i - V_j)$

❻ **Kirchhoff's current law** for every bus: $I_i + \sum_j I_{j\to i} = 0$

❼ **current balance equations** (treating the ground as node with 0V):

$$I_i = -\sum_j I_{j\to i} = \sum_j \frac{1}{Z_{ij}}(V_i - V_j) = \sum_j Y_{ij} V_j \quad \text{or} \quad I = Y \cdot V$$

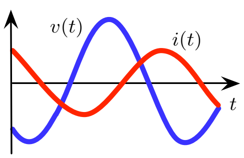


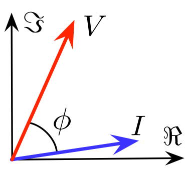
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

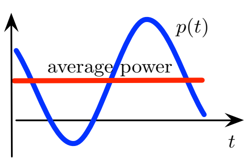
Note: all variables are in per unit (p.u.) system, i.e., normalized wrt base voltage

AC circuits – power

(see also exercises)







- voltage phasor: $V = |V|e^{\mathrm{i}\,\theta_V} \Leftrightarrow v(t) = |V|\cos(\omega^* t + \theta_V)$

  current phasor: $I = |I|e^{\mathrm{i}\,\theta_I} \Leftrightarrow i(t) = |I|\cos(\omega^* t + \theta_I)$

- **instantaneous power**:

$$p(t) = v(t)\cdot i(t) = \frac{1}{2}\cdot|V|\cdot|I|\cdot\cos(\theta_V - \theta_I) + \frac{1}{2}\cdot|V|\cdot|I|\cdot\cos(2\omega^* t + \theta_V + \theta_I)$$

$\Rightarrow$ **active power** (average): $P = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt = \frac{1}{2}\cdot|V|\cdot|I|\cdot\cos(\phi)$

$\Rightarrow$ **reactive power** (0-avg): $Q = \frac{1}{T}\int_0^T v(t)\cdot i(t - \frac{T}{4})\,dt = \frac{1}{2}\cdot|V|\cdot|I|\cdot\sin(\phi)$

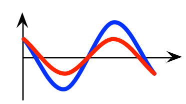
AC circuits – complex power

(see also exercises)

❽ **active & reactive power** in AC circuits:

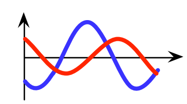
- active (average) power:

$$P = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt = \frac{1}{2}\cdot|V|\cdot|I|\cdot\cos(\phi)$$



- reactive (0-average) power:

$$Q = \frac{1}{T}\int_0^T v(t)\cdot i(t - T/4)\,dt = \frac{1}{2}\cdot|V|\cdot|I|\cdot\sin(\phi)$$

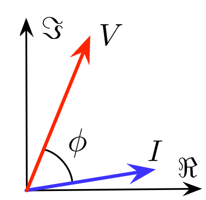


$\Rightarrow$ normalize phasors: $V \mapsto 1/\sqrt{2}\cdot|V|e^{\mathrm{i}\,\theta_V}$

$\Rightarrow$ **complex power:** $S = V\cdot\bar{I} = P + \mathrm{i}Q$

$\quad = $ active power $+\,\mathrm{i}\,\cdot$ reactive power

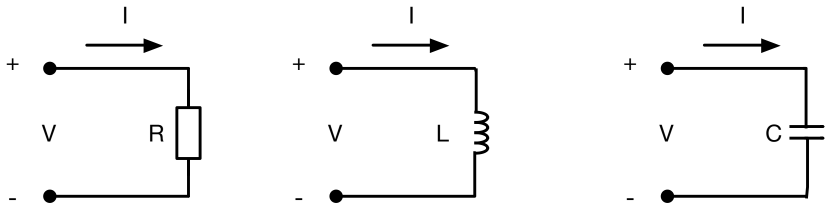
$\Rightarrow$ $\cos(\phi) = P/|S|$ is power factor



Note: often complex phasors are implicitly normalized $\tilde{V} = 1/\sqrt{2}\cdot Ee^{\mathrm{i}\,\theta}$

AC circuits – power dissipated by RLC loads

details in exercises

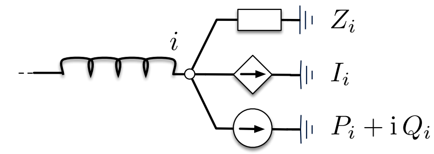


Power dissipation $S = V \cdot \bar{I} = P + \mathrm{i}Q$ (network sign convention):

$S = -\frac{1}{2}\lvert I\rvert^2 R$ $= -\frac{1}{2}\frac{\lvert V\rvert^2}{R}$ $= P < 0$	$S = -\frac{1}{2}\lvert I\rvert^2 \cdot \mathrm{i}\omega L$ $= -\mathrm{i}\frac{1}{2}\frac{\lvert V\rvert^2}{\omega L}$ $= Q < 0$	$S = \mathrm{i}\frac{1}{2}\frac{\lvert I\rvert^2}{\omega C}$ $= \frac{1}{2}\lvert V\rvert^2 \cdot \mathrm{i}\omega C$ $= Q > 0$

Static models loads

- aggregated **ZIP load model**:
  constant impedance **Z** +
  constant current **I** +
  constant power **P**



- more general **exponential load model**: power $= const. \cdot (V/V_{\text{ref}})^{const.}$
  (combinations & variations learned from data)
- various dynamic load models for stability studies ...

"Just use whatever load model fits your mathematics. You will get it wrong anyways." — [Ian Hiskens, lunch @ Zürich '15]

Static models for sources

- most common **static load model** is constant active power demand P and constant reactive power demand Q
- conventional **synchronous generators** are controlled to have constant active power output P and voltage magnitude E
- sources interfaced with **power electronics** are typically controlled to have constant active power P and reactive power Q

$\Rightarrow$ common bus device models

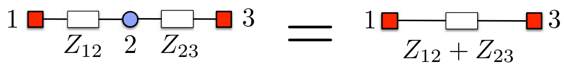
1. **PQ** buses have complex power $S = P + \mathrm{i}Q$ specified
2. **PV** buses have active power P and voltage magnitude E specified
3. **slack buses** have E and θ specified (not really existent)

Kron Reduction of Circuits

Kron reduction

[G. Kron 1939]

often (almost always) you will encounter Kron-reduced network models

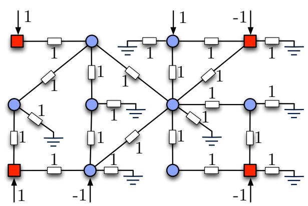


General procedure:

0. convert const. power injections locally to shunt impedances $Z = S/V_{\text{ref}}^2$
1. partition linear current-balance equations via boundary & interior nodes

   (arises naturally, e.g., sources & loads, measurement terminals, etc.)

$$\begin{bmatrix} I_{\text{boundary}} \\ I_{\text{interior}} \end{bmatrix} = \begin{bmatrix} Y_{\text{boundary}} & Y_{\text{bound-int}} \\ Y_{\text{bound-int}}^T & Y_{\text{interior}} \end{bmatrix} \begin{bmatrix} V_{\text{boundary}} \\ V_{\text{interior}} \end{bmatrix}$$

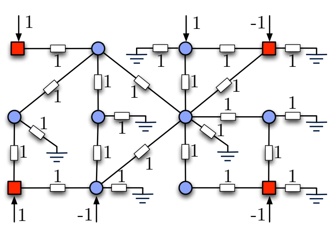


Kron reduction cont'd

on blackboard

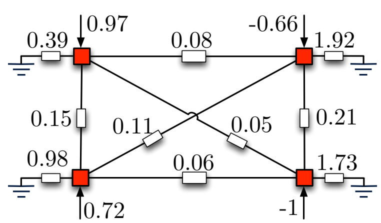
Kron reduction cont'd

2. Gaussian elimination of interior voltages



original circuit

$I = Y \cdot V$



"equivalent" reduced circuit

$I_{\text{red}} = Y_{\text{red}} \cdot V_{\text{boundary}}$

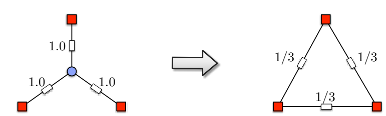
⇒ reduced Y-matrix: $Y_{\text{red}} = Y_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{-1} \cdot Y_{\text{bound-int}}^T$

⇒ reduced injections: $I_{\text{red}} = I_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{-1} \cdot I_{\text{interior}}$

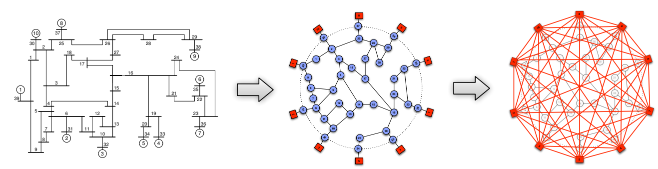
Examples of Kron reduction

algebraic properties are preserved but the network changes significantly

- Star-Δ transformation [A. E. Kennelly 1899, A. Rosen '24]



- Kron reduction of load buses in *IEEE 39 New England power grid*



⇒ topology without weights is meaningless!

⇒ shunt resistances (loads) are mapped to line conductances

⇒ many properties still open [FD & F. Bullo '13, S. Caliskan & P. Tabuada '14]

Kron reduction – so simple yet still full of mysteries

49th IEEE Conference on Decision and Control
December 15-17, 2010
Hilton Atlanta Hotel, Atlanta, GA, USA

The Behavior of Linear Time Invariant RLC Circuits

Erik I. Verriest and Jan C. Willems

IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: REGULAR PAPERS, VOL. 60, NO. 1, JANUARY 2013

Kron Reduction of Graphs With Applications to Electrical Networks

Florian Dörfler and Francesco Bullo

Systems & Control Letters 59 (2010) 423–428

Characterization and partial synthesis of the behavior of resistive circuits at their terminals

Arjan van der Schaft

Automatica 50 (2014) 2586–2590

Brief paper

Towards Kron reduction of generalized electrical networks

Sina Yamac Caliskan, Paulo Tabuada

Power Flow Formulations & Approximations

Power balance eqn's: "power injection = Σ power flows"

1. **complex form:** $S_i = V_i \overline{I}_i = \sum_j V_i \overline{Y}_{ij} \overline{V}_j$ or $S = \mathrm{diag}(V)\overline{Y}\overline{V}$

   ⇒ purely quadratic and useful for static calculations & optimization

2. **rectangular form:** insert $V = e + if$ and split real & imaginary parts:

   active power: $P_i = \sum_j B_{ij}(e_i f_j - f_i e_j) + G_{ij}(e_i e_j + f_i f_j)$

   reactive power: $Q_i = -\sum_j B_{ij}(e_i e_j + f_i f_j) + G_{ij}(e_i f_j - f_i e_j)$

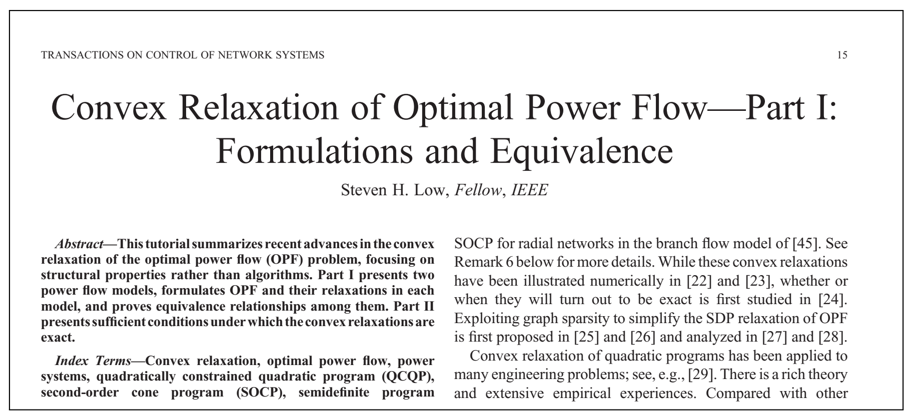
   ⇒ purely quadratic and useful for homotopy methods & QCQPs

⇒ main complexity is quadratic nonlinearity $V_i \overline{V}_j = \begin{bmatrix} e & if \end{bmatrix} \cdot \begin{bmatrix} e & -if \end{bmatrix}^T$

Power balance eqn's – cont'd

3. **matrix form:** define unit-rank p.s.d. Hermitian matrix $W = V \cdot \overline{V}^T$ with components $W_{ij} = V_i \overline{V}_j$, then power flow is $S_i = \sum_j \overline{Y}_{ij} W_{ij}$

   ⇒ linear and useful for relaxations in convex optimization problems



TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS 15

Convex Relaxation of Optimal Power Flow—Part I: Formulations and Equivalence

Steven H. Low, *Fellow, IEEE*

***Abstract*—This tutorial summarizes recent advances in the convex relaxation of the optimal power flow (OPF) problem, focusing on structural properties rather than algorithms. Part I presents two power flow models, formulates OPF and their relaxations in each model, and proves equivalence relationships among them. Part II presents sufficient conditions under which the convex relaxations are exact.**

***Index Terms*—Convex relaxation, optimal power flow, power systems, quadratically constrained quadratic program (QCQP), second-order cone program (SOCP), semidefinite program**

SOCP for radial networks in the branch flow model of [45]. See Remark 6 below for more details. While these convex relaxations have been illustrated numerically in [22] and [23], whether or when they will turn out to be exact is first studied in [24]. Exploiting graph sparsity to simplify the SDP relaxation of OPF is first proposed in [25] and [26] and analyzed in [27] and [28].

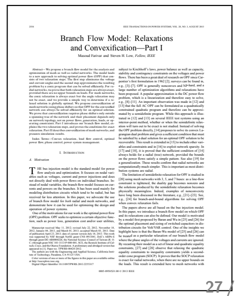
Convex relaxation of quadratic programs has been applied to many engineering problems; see, e.g., [29]. There is a rich theory and extensive empirical experiences. Compared with other

Power balance eqn's – cont'd

4. **branch flow eqn's** parameterized in flow variables [M. Baran & F. Wu '89]:
   - Ohm's law: $V_i - V_j = Z_{ij} I_{i \to j}$
   - branch power flow $i \to j$: $S_{i \to j} = V_i \cdot \overline{I_{i \to j}}$
   - power balance at node i:

$$\underbrace{\sum_{k:\, i \to k} S_{i \to k}}_{\text{outgoing flows}} + Y_{i,\text{shunt}} |V_i|^2 = S_i + \underbrace{\sum_{j:\, j \to i} \left(S_{j \to i} - Z_{ij} |I_{i \to j}|^2 \right)}_{\text{incoming flows}}$$

- **DistFlow formulation** in terms of square magnitude variables $|V_i|^2$ and $|I_{i \to j}|^2$ (missing angle variables $\angle V_i$ and $\angle I_{i \to j}$ can sometimes be recovered, e.g., in acyclic case)
- lossless approximation can be solved exactly in acyclic networks (useful for distribution networks) [M. Baran & F. Wu '89, M. Farivar, L. Chen, & S. Low '13]



Power balance eqn's – cont'd

5. **polar form:** insert $V = E e^{i\theta}$ and split real & imaginary parts:

active power:	$P_i =$	$\sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
reactive power:	$Q_i =$	$-\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

$\Rightarrow$ will be our focus these days since ...

- **power system specs** on frequency $\frac{d}{dt}\theta(t)$ and voltage magnitude E
- **dynamics:** generator swing dynamics affect voltage phase angles & voltage magnitudes are controlled to be constant
- **physical intuition:** usual operation near flat voltage profile $V_i \approx 1 e^{i\phi}$ which give rise to various insights for analysis & design (later)

Power flow simplifications & approximations

power flow equations are too complex & unwieldy for analysis & large computations

- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

1. **lossless** transmission lines $R_{ij}/X_{ij} = -G_{ij}/B_{ij} \approx 0$

active power:	$P_i =$	$\sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$
reactive power:	$Q_i =$	$-\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$

2. **decoupling** near operating point $V_i \approx 1 e^{i\phi}$: $\begin{bmatrix} \partial P/\partial\theta & \partial P/\partial E \\ \partial Q/\partial\theta & \partial Q/\partial E \end{bmatrix} \approx \begin{bmatrix} \star & 0 \\ 0 & \star \end{bmatrix}$

active power:	$P_i =$	$\sum_j B_{ij} \sin(\theta_i - \theta_j)$	(function of angles)
reactive power:	$Q_i =$	$-\sum_j B_{ij} E_i E_j$	(function of magnitudes)

Power flow simplifications & approximations cont'd

3. **linearization** for small flows near operating point $V_i \approx 1 e^{i\phi}$:

active power:	$P_i =$	$\sum_j B_{ij}(\theta_i - \theta_j)$	known as **DC power flow**
reactive power: :	$Q_i =$	$\sum_j B_{ij}(E_i - E_j)$	(if formulated in p.u. system)

1290 IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 24, NO. 3, AUGUST 2009

DC Power Flow Revisited

Brian Stott, *Fellow, IEEE*, Jorge Jardim, *Senior Member, IEEE*, and Ongun Alsaç, *Fellow, IEEE*

***Abstract*—Linear MW-only "dc" network power flow models are in widespread and even increasing use, particularly in congestion-constrained market applications. Many versions of these approximate models are possible. When their MW flows are reasonably correct (and this is by no means assured), they can often offer compelling advantages. Given their considerable importance in today's electric power industry, dc models merit closer scrutiny. This paper attempts such a re-examination.**

II. WHY DC MODELS?

The linear, bilateral, non-complex, often state-independent, properties of a dc-type power flow model have considerable analytical and computational appeal. The use of such a model is limited to those MW-oriented applications where the effects of network voltage and VAr conditions are minimal (a very dif-

Conclusion on the **most limiting assumption** of DC power flow: $R/X \approx 0$

Power flow simplifications & approximations cont'd

- active power: $P_i \;=\; \sum_j B_{ij}E_iE_j \sin(\theta_i-\theta_j) + G_{ij}E_iE_j\cos(\theta_i-\theta_j)$
- reactive power: $Q_i \;=\; -\sum_j B_{ij}E_iE_j \cos(\theta_i-\theta_j) + G_{ij}E_iE_j\sin(\theta_i-\theta_j)$

4. Multiple **variations & combinations** of DC power flow
   - power flow transformation for constant R/X ratios (see exercise)
   - linearization & decoupling at arbitrary operating points [D. Deka et al., '15]
   - advanced linearizations especially for reactive power [S. Bolognani & S. Zampieri '12, B. Gentile et al. '14, J. Simpson-Porco et al. '16]
   - linearizations in rectangular coordinates (more accurate for active power) [R. Baldick '13, S. Bolognani & S. Zampieri '15, S. Dhople et al. '15]

"...plenty of heuristics in industry ... especially for approximation of losses."

— [Bruce Wollenberg, meeting @ Minneapolis '13]

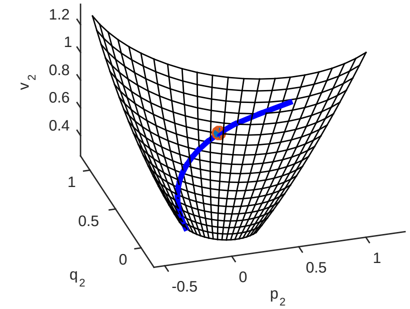
A unifying geometric perspective [S. Bolognani & F. Dörfler '15]

node 1 — node 2

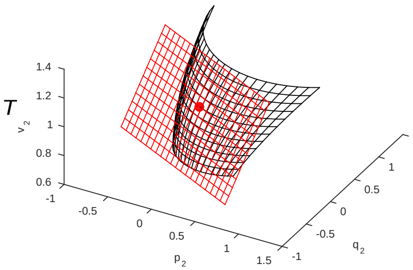
$y = 0.4 - 0.8j$

$v_1 = 1,\ \theta_1 = 0$ $\quad$ $v_2,\ \theta_2$

$p_1,\ q_1$ $\quad$ $p_2,\ q_2$



1. **variables:** all of $x = (E, \theta, p, q)$
2. **power flow manifold:** $F(x) = \mathbb{0}$
3. **normal space** spanned by $\left.\frac{\partial F(x)}{\partial x}\right|_{x^*} = A^T$
4. **tangent space** $A(x-x^*) = \mathbb{0}$ is best linear approximant at x^*
5. **accuracy** depends on curvature $\frac{\partial^2 F(x)}{\partial x^2}$



Closer look at implicit formulae $A(x-x^*) = \mathbb{0}$

$$\left[\left(\langle \mathrm{diag}\,\overline{YE^*}\rangle + \langle \mathrm{diag}\,E^*\rangle N\langle Y\rangle\right)\cdot\begin{bmatrix}\mathrm{diag}(\cos\theta^*) & -\mathrm{diag}(E^*)\,\mathrm{diag}(\sin\theta^*)\\ \mathrm{diag}(\sin\theta^*) & \mathrm{diag}(E^*)\,\mathrm{diag}(\cos\theta^*)\end{bmatrix}\right]$$

shunt loads — lossy DC flow — rotation $\times$ scaling at operating point

$$\times\begin{bmatrix}v-v^*\\ \theta-\theta^*\end{bmatrix} = \begin{bmatrix}p-p^*\\ q-q^*\end{bmatrix}$$

deviation variables

where $N = \begin{bmatrix} I & 0\\ 0 & -I\end{bmatrix}$ is complex conjugate in real coordinates

and $\langle A\rangle = \begin{bmatrix}\Re(A) & -\Im(A)\\ \Im(A) & \Re(A)\end{bmatrix}$ is complex rotation in real coordinates.

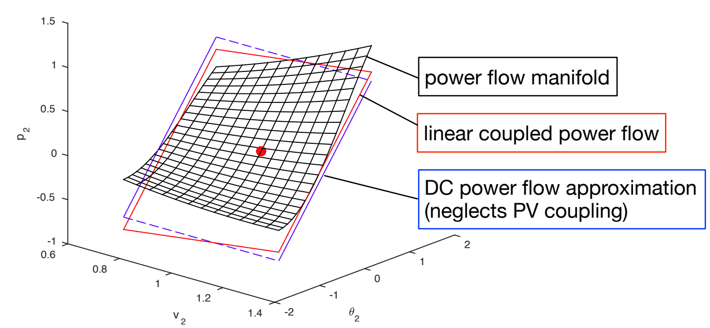
Special cases reveal some old friends I

- **flat-voltage/0-injection point:** $x^* = (E^*, \theta^*, P^*, Q^*) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

$\Rightarrow$ implicit linearization: $\begin{bmatrix}\Re(Y) & -\Im(Y)\\ -\Im(Y) & \Re(Y)\end{bmatrix}\begin{bmatrix}v\\ \theta\end{bmatrix} = \begin{bmatrix}p\\ q\end{bmatrix}$

is **linear coupled power flow** [D. Deka, S. Backhaus, & M. Chertkov, '15]

$\Rightarrow$ $\Re(Y) = \mathbb{0}$ gives **DC power flow:** $-\Im(Y)\theta = P$ and $-\Im(Y)E = Q$



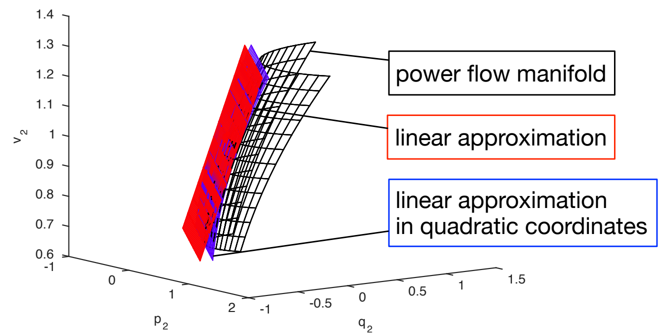
Special cases reveal some old friends II

- **flat-voltage/0-injection point:** $x^* = (E^*, \theta^*, P^*, Q^*) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

$\Rightarrow$ rectangular coord. $\Rightarrow$ **rectangular DC flow** [S. Bolognani & S. Zampieri, '15]

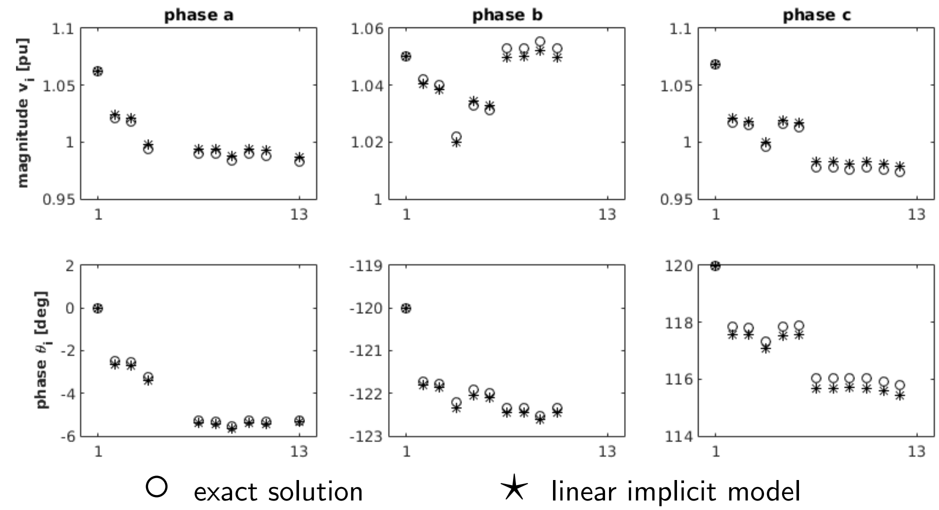
- nonlinear change to **quadratic coordinates** from v_h to v_h^2

$\Rightarrow$ linearization gives (non-radial) **LinDistFlow** [M.E. Baran & F.F. Wu, '88]



Accuracy illustrated with unbalanced three-phase IEEE13

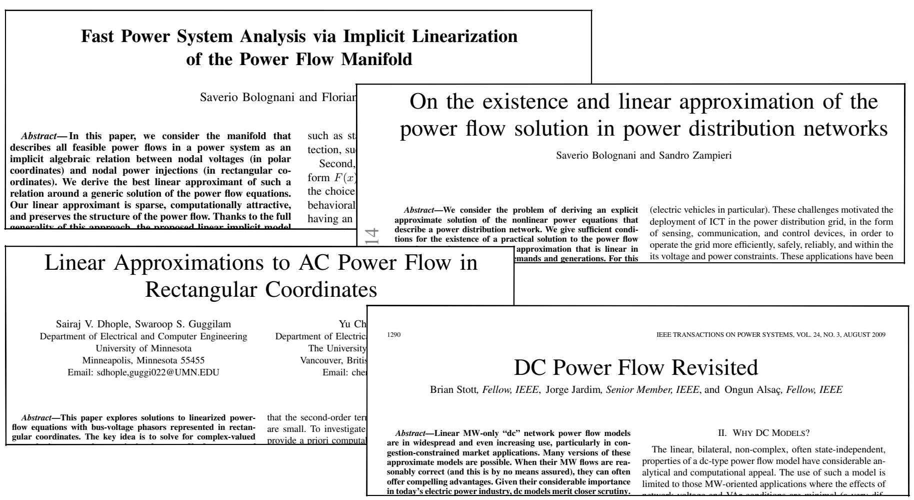
can be extended to three-phase, exponential loads, etc.



Matlab/Octave code @ `https://github.com/saveriob/1ACPF`

Plenty of recent interest in power flow approximations

mainly for the sake of verifying analytic approaches



Fast Power System Analysis via Implicit Linearization of the Power Flow Manifold

Saverio Bolognani and Florian

On the existence and linear approximation of the power flow solution in power distribution networks

Saverio Bolognani and Sandro Zampieri

Linear Approximations to AC Power Flow in Rectangular Coordinates

Sairaj V. Dhople, Swaroop S. Guggilam, Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, Minnesota 55455

DC Power Flow Revisited

Brian Stott, *Fellow, IEEE*, Jorge Jardim, *Senior Member, IEEE*, and Ongun Alsaç, *Fellow, IEEE*

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 24, NO. 3, AUGUST 2009

Once you try to analyze power flow equations with pen and paper, you will realize . . .

"Maybe we should revisit the way we write power flow equations." — [Göran Andersson, Santa Fe Grid Science Workshop '15]

Once you work computationally with data, you will see . . .

"The devil introduced the per unit system into power." — [Peter Sauer, ACC '12]

Dynamic Network Component Models

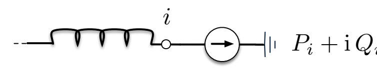
Modeling the "essential" network dynamics

models can be arbitrarily detailed & vary on different time/spatial scales

1. active and reactive **power flow** (e.g., lossless)

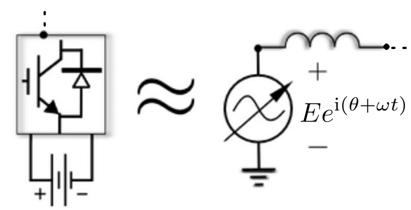
$$P_{i,\text{inj}} = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_{i,\text{inj}} = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

2. passive constant power **loads**



$$P_{i,\text{inj}} = P_i = const.$$
$$Q_{i,\text{inj}} = Q_i = const.$$

3. **inverters**: DC or variable AC sources with power electronics

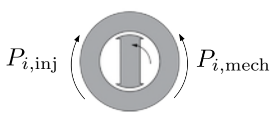


(i) have constant/controllable PQ (max. power-point tracking)

(ii) or mimic generators (more later)

Modeling the "essential" synchronous generator dynamics

4. electromech. **swing dynamics** of synchronous machines



$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{i,\text{mech}} - P_{i,\text{inj}}$$
$$E_i = const.$$

(can be derived from first principle model & some (possibly strong) assumptions)

2015 IEEE 54th Annual Conference on Decision and Control (CDC)
December 15-18, 2015. Osaka, Japan

Uses and Abuses of the Swing Equation Model

Sina Y. Caliskan and Paulo Tabuada

***Abstract*— The swing equation model is widely used in the literature to study a large class problems, including stability analysis of power systems. We show in this paper, by comparison with a first principles model, that the swing equation model may lead to erroneous conclusions when performing stability analysis of power systems, even under small oscillations.**

I. Introduction

The swing equation model is a perfect example of the famous line by George Box and Norman Draper in [2]: "All models are wrong, but some are useful.". Power engineers equation for stability analysis under small oscillations we obtain results contradicting a more detailed FP model.

II. Synchronous Generator Models

In this section, we review two synchronous generator models. The first model is derived from first principles while the second is the traditional swing equation model that is widely used in the literature. After introducing these models, we show how to recover the swing equation model from the

Common variations in dynamic network models

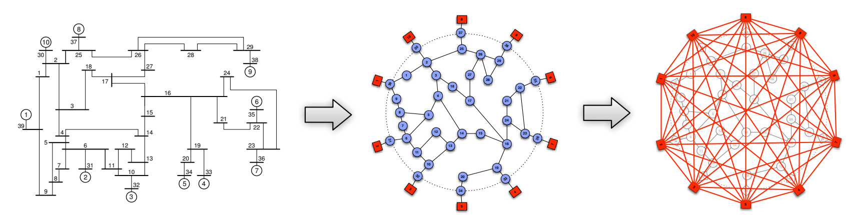
dynamic behavior is very much dependent on load models & generator models

1. frequency/voltage-depend. loads [A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

$$D_i \dot{\theta}_i + P_i = -P_{i,\text{inj}}$$
$$f_i(\dot{V}_i) + Q_i = -Q_{i,\text{inj}}$$

2. network-reduced models after Kron reduction of loads [H. Chiang, F. Wu, & P. Varaiya '94] (very common but poor assumption: $G_{ij} = 0$)

$$M_i \ddot{\theta}_i + D \dot{\theta}_i = P_{i,\text{mech}} - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) \underbrace{- \sum_j G_{ij} E_i E_j \cos(\theta_i - \theta_j)}_{\text{effect of resistive loads}}$$



Structure-preserving power network model [A. Bergen & D. Hill '81]

without Kron-reduction of load buses

- **generator swing dynamics:**

$$\dot{\theta}_i = \omega_i$$
$$M_i\dot{\omega}_i = -D_i\omega_i + P_i - \sum\nolimits_j B_{ij}E_iE_j\sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij}E_iE_j\cos(\theta_i - \theta_j)$$

- **frequency-dependent loads:** (or inverter-interfaced sources)

$$D_i\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}E_iE_j\sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij}E_iE_j\cos(\theta_i - \theta_j)$$

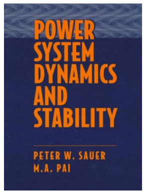
- in **academia:** this "baseline model" is typically further simplified: decoupling, linearization, constant voltages, ...
- in **industry:** much more detailed models used for grid simulations

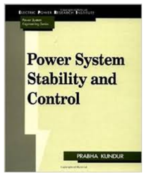
⇒ **IMHO:** above model captures most interesting network dynamics

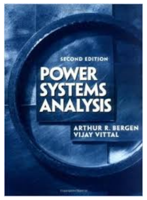
Common variations in dynamic network models — cont'd

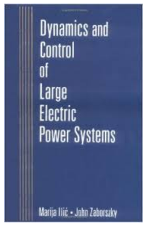
dynamic behavior is very much dependent on load models & generator models

3. higher order generator dynamics [P. Sauer & M. Pai '98] — voltages, controls, magnetics etc. (reduction via singular perturbations)
4. dynamic & detailed load models [D. Karlsson & D. Hill '94] — aggregated dynamic load behavior (e.g., load recovery after voltage step)
5. time-domain models [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12] — passive Port-Hamiltonian models for machines & RLC circuitry



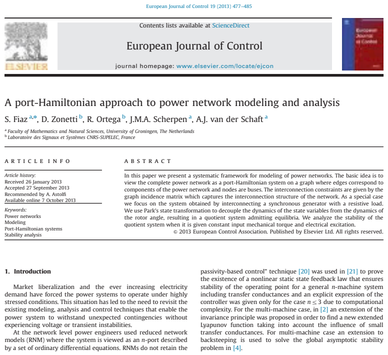






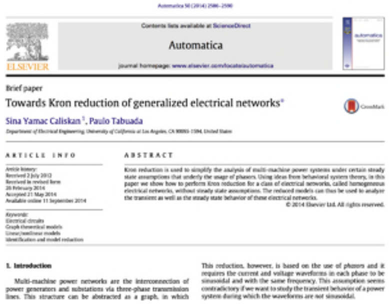
"Power system research is all about the art of making the right assumptions."

Lots of current research activity on time-domain models


European Journal of Control

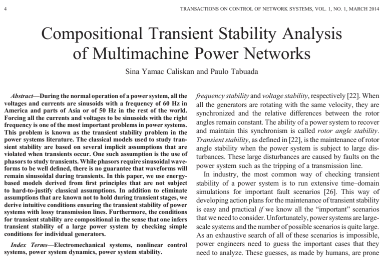
A port-Hamiltonian approach to power network modeling and analysis

S. Fiaz, D. Zonetti, R. Ortega, J.M.A. Scherpen, A.J. van der Schaft

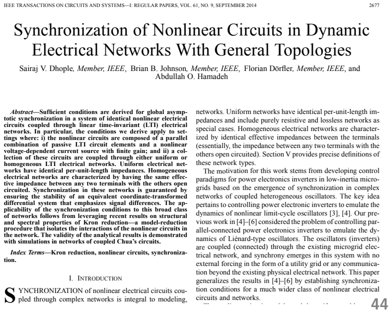

Automatica

Towards Kron reduction of generalized electrical networks

Sina Yamac Caliskan, Paulo Tabuada

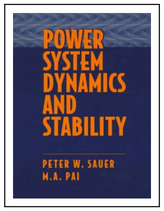

Compositional Transient Stability Analysis of Multimachine Power Networks

Sina Yamac Caliskan and Paulo Tabuada


Synchronization of Nonlinear Circuits in Dynamic Electrical Networks With General Topologies

Sairaj V. Dhople, *Member, IEEE*, Brian B. Johnson, *Member, IEEE*, Florian Dörfler, *Member, IEEE*, and Abdullah O. Hamadeh

On the swing equation ...



"There is probably more literature on synchronous machines than on any other device in electrical engineering." — [Peter Sauer & M.A. Pai, Power System Dynamics and Stability '98]

"The swing equation model is a perfect example of the famous line [...]: "All models are wrong, but some are useful.""

— [Sina Y. Caliskan and Paulo Tabuada, CDC '15]

Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

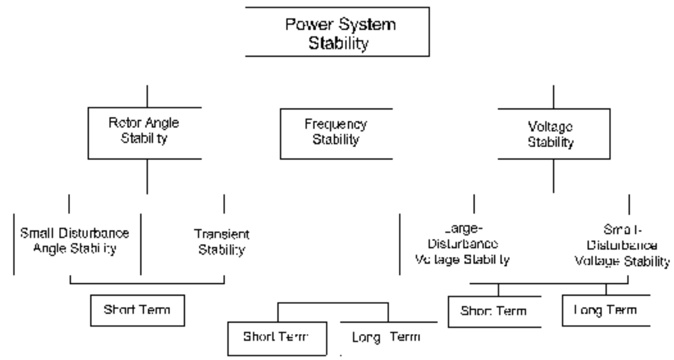
- Decoupled Active Power Flow (Synchronization)
- Reactive Power Flow (Voltage Collapse)
- Coupled & Lossy Power Flow
- Transient Rotor Angle Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

One system with many dynamics & control problems



"From a practical viewpoint, there are four major analytical problems: ... compute equilibria ... transient stability ... [inter-area] oscillations ... voltage collapse. Of course, theoretically they are all aspects of the one overall stability question." — [David Hill, ISCAS '06]

prelims on power flow

Preliminary insights on lossless power flow

power flow equations:

$$P_i = \sum_{j=1}^{n} B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum_{j=1}^{n} B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

$\Rightarrow$ solution space: $\mathbb{T}^n \times \mathbb{R}^n_{\geq 0} = (\mathbb{S}^1 \times \cdots \times \mathbb{S}^1) \times (\mathbb{R}_{\geq 0} \times \cdots \times \mathbb{R}_{\geq 0})$

rotational symmetry:

if θ^* is a solution $\Rightarrow$ $\theta^* + const. \cdot \mathbb{1}_n$ is another solution

$\Rightarrow$ solution space "modulo rotational symmetry": $\mathbb{T}^n \setminus \mathbb{S}^1 \times \mathbb{R}^n_{\geq 0}$

index shenanigans:

- active flow $i \to i = B_{ii} E_i E_j \sin(\theta_i - \theta_i) = 0$ ($\Rightarrow$ can drop index i)
- reactive flow $i \to i = -B_{ii} E_i E_j \cos(\theta_i - \theta_i) = -B_{ii} E_i^2$

Preliminary feasibility conditions for lossless power flow

see exercises for details

power flow equations:

$$P_i = \sum_{j=1}^{n} B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_i = -\sum_{j=1}^{n} B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

necessary feasibility condition I:

$\sum_{i=1}^{n} P_i = 0 \Leftarrow \exists$ a solution

$\triangleq$ power balance

$\Rightarrow$ typically not true (w/o slack bus) due to unknown load demand

$\Rightarrow$ need to consider dynamics

necessary feasibility condition II:

$\sum_{i=1}^{n} Q_i \geq 0 \Leftarrow \exists$ a solution

$\triangleq$ reactive power losses

$\Rightarrow$ reactive power must be supplied

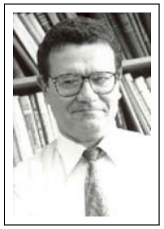
(for inductive grid w/o shunts)

Feasibility power flow is crucial for system operation

Given: network parameters & topology and load & generation profile

Q: "$\exists$ an optimal, stable, and robust synchronous operating point ?"

1. Security analysis [Araposthatis et al. '81, Wu et al. '80 & '82, Ilić '92, ...]
2. Load flow feasibility [Chiang et al. '90, Dobson '92, Lesieutre et al. '99, ...]
3. Optimal generation dispatch [Lavaei et al. '12, Bose et al. '12, ...]
4. Transient stability [Sastry et al. '80, Bergen et al. '81, Hill et al. '86, ...]
5. Inverters in microgrids [Chandorkar et. al. '93, Guerrero et al. '09, Zhong '11,...]
6. Complex networks [Hill et al. '06, Strogatz '01, Arenas et al '08, ...]

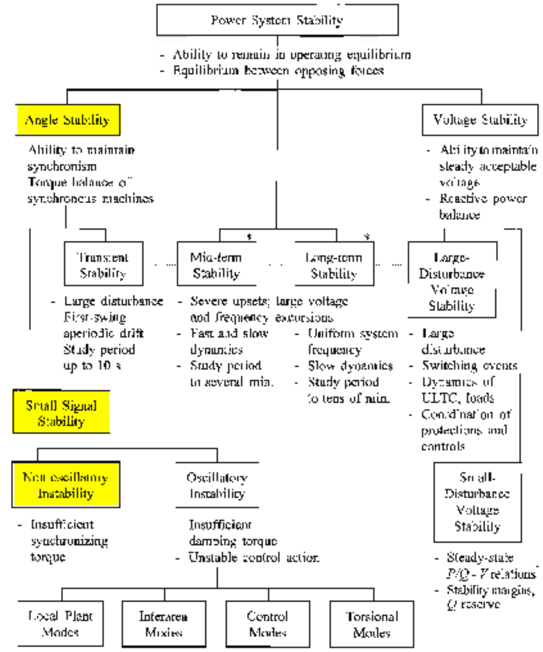


"How do we quantitatively measure feasibility in order to incorporate this attribute in the system design or operation? How do we explicitly describe the region of feasibility in general, and in particular in a large neighborhood around the normal operating injections?"

— [J. Jaris & F. Galiana, IEEE PAS '81]

Decoupled Active Power Flow (Synchronization)

Our first stab at power system stability



Synchronization & feasibility of active power flow

sync is crucial for the functionality and operation of the power grid

- **structure-preserving power network model** [A. Bergen & D. Hill '81]:

synchronous machines: $M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

frequency-dependent loads: $D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

- **synchronization** = sync'd frequencies & bounded active power flows

$$\dot{\theta}_i = \omega_{\text{sync}}\ \forall\, i \in \mathcal{V} \quad \& \quad |\theta_i - \theta_j| \leq \gamma < \pi/2\ \forall\,\{i,j\} \in \mathcal{E}$$

= active power flow feasibility & security constraints

- **explicit sync frequency:** if sync, then $\omega_{\text{sync}} = \sum_i P_i / \sum_i D_i$

(by summing over all equations)

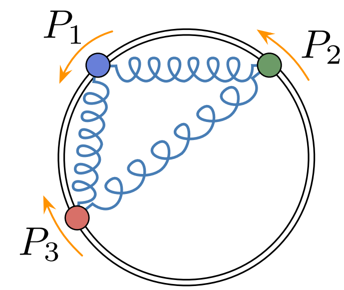
A perspective from coupled oscillators

Mechanical oscillator network

Angles $(\theta_1, \dots, \theta_n)$ evolve on $\mathbb{T}^n$ as

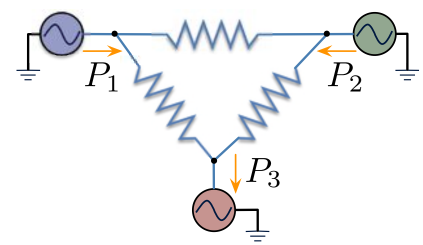
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $P_i \in \mathbb{R}$
- spring constants $B_{ij} \geq 0$



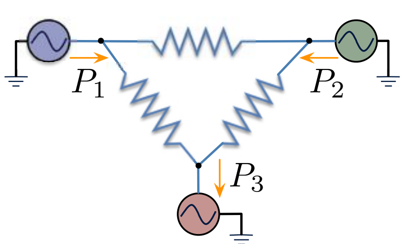
Structure-preserving power network

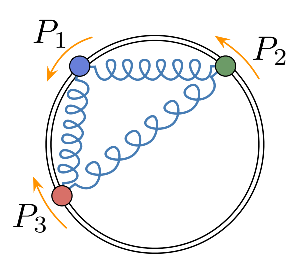
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$
$$D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$



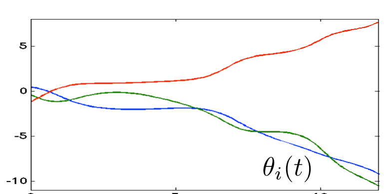
Phenomenology of sync in power networks

- sync is **crucial for AC power grids**

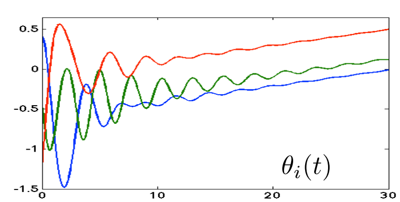




- sync is a **trade-off**



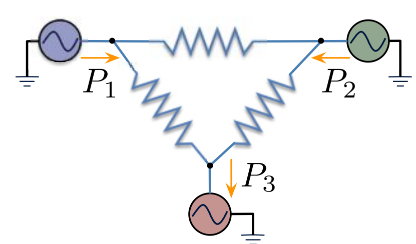
weak coupling & heterogeneous

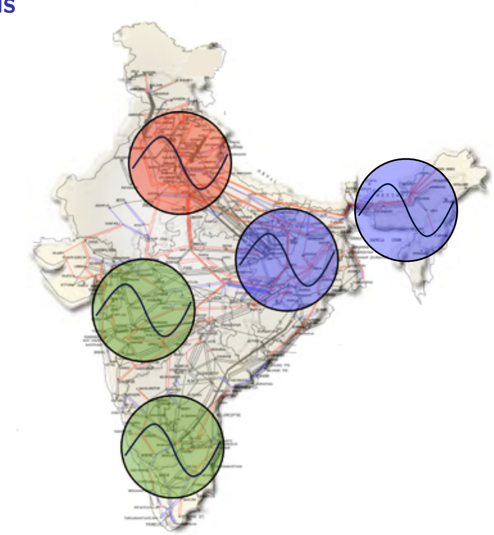


strong coupling & homogeneous

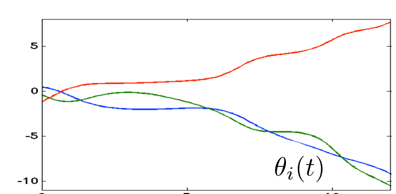
Phenomenology of sync in power networks

- sync is **crucial for AC power grids**





- sync is a **trade-off**

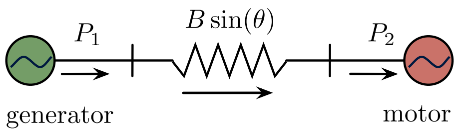


weak coupling & heterogeneous

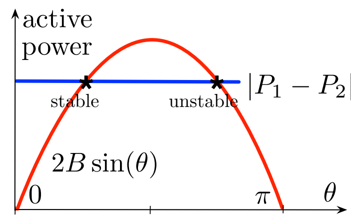
Blackout India July 30/31 2012

Back of the envelope calculations for the two-node case

generator connected to identical motor shows bifurcation at difference angle $\theta = \pi/2$



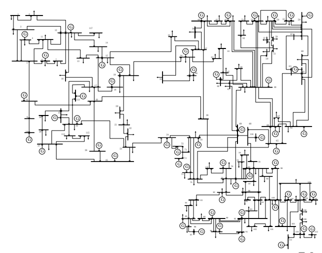
$$M\ddot{\theta} + D\dot{\theta} = P_1 - P_2 - 2B\sin(\theta)$$



$\exists$ stable sync $\Leftrightarrow$ $B > |P_1 - P_2|/2$ $\Leftrightarrow$ "ntwk coupling $>$ heterogeneity"

Q1: Quantitative generalization to a complex & large-scale network?

Q2: What are the particular metrics for coupling and heterogeneity?



Who knows consensus systems?

on blackboard

Primer on algebraic graph theory

for a connected and undirected graph

Laplacian matrix $L =$ "degree matrix" $-$ "adjacency matrix"

$$L = L^T = \begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -B_{i1} & \cdots & \sum_{j=1}^{n} B_{ij} & \cdots & -B_{in} \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix} \geq 0$$

is positive semidefinite with one zero eigenvalue & eigenvector $\mathbb{1}_n$

Notions of connectivity

- spectral: 2nd smallest eigenvalue of L is "algebraic connectivity" $\lambda_2(L)$
- topological: degree $\sum_{j=1}^{n} B_{ij}$ or degree distribution

Notions of heterogeneity

$\|P\|_{\mathcal{E},\infty} = \max_{\{i,j\}\in\mathcal{E}} |P_i - P_j|$, $\quad \|P\|_{\mathcal{E},2} = \left(\sum_{\{i,j\}\in\mathcal{E}} |P_i - P_j|^2\right)^{1/2}$

Synchronization in "complex" networks

for a first-order model — all results generalize locally

$$\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$

1. **local stability** for equilibria satisfying $|\theta_i^* - \theta_j^*| < \pi/2 \; \forall \, \{i,j\} \in \mathcal{E}$
   (linearization is Laplacian matrix)
2. **necessary sync condition**: $\sum_j B_{ij} \geq |P_i - \omega_{\text{sync}}| \Leftarrow$ sync
   (so that syn'd solution exists)
3. **sufficient sync condition**: $\lambda_2(L) > \|P\|_{\mathcal{E},2} \quad \Rightarrow \quad$ sync
   [FD & F. Bullo '12]

$\Rightarrow$ $\exists$ similar conditions with diff. metrics on coupling & heterogeneity

$\Rightarrow$ **Problem:** sharpest general conditions are conservative

Can we solve the power flow equations exactly?

on blackboard

A nearly exact sync condition [FD, M. Chertkov, & F. Bullo '13]

1. search equilibrium θ^* with $|\theta_i^* - \theta_j^*| \leq \gamma < \pi/2$ for all $\{i,j\} \in \mathcal{E}$:

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) \qquad (\star)$$

2. consider linear "small-angle" DC approximation of $(\star)$:

$$P_i = \sum_j B_{ij}(\delta_i - \delta_j) \quad \Leftrightarrow \quad P = L\delta \qquad (\star\star)$$

unique solution (modulo symmetry) of $(\star\star)$ is $\delta^* = L^\dagger P$

3. solution ansatz for $(\star)$: $\boxed{\theta_i^* - \theta_j^* = \arcsin(\delta_i^* - \delta_j^*)}$ **(for a tree)**

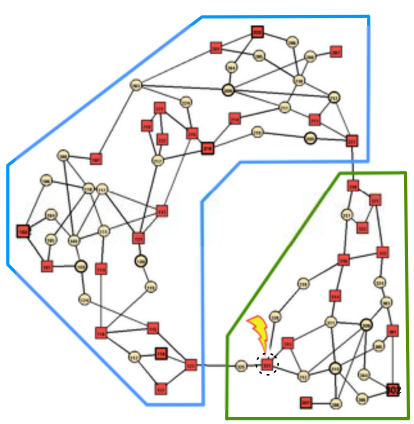
$$P_i = \sum_{j=1}^n a_{ij} \sin(\theta_i - \theta_j) = \sum_{j=1}^n a_{ij} \sin\big(\arcsin(\delta_i^* - \delta_j^*)\big) = P_i \quad \checkmark$$

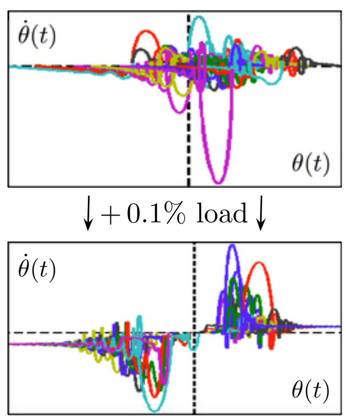
$\Rightarrow$ **Thm:** $\exists\, \theta^*$ with $|\theta_i^* - \theta_j^*| \leq \gamma \;\forall \{i,j\} \in \mathcal{E} \quad \Leftrightarrow \quad \|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$



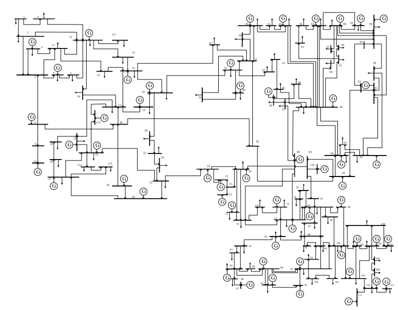


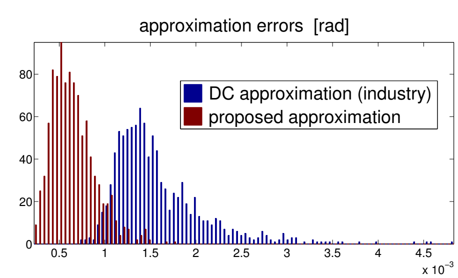
Reliability Test System RTS 96 under two loading conditions

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$





IEEE 118 bus system (Midwest)

Outperforms conventional DC approximation "on average & in the tail".

More on power flow approximations

Randomized power network test cases

with 50 % randomized loads and 33 % randomized generation

Randomized test case (1000 instances)	Numerical worst-case angle differences: $\max_{\{i,j\}\in\mathcal{E}} \lvert\theta_i^* - \theta_j^*\rvert$	Analytic prediction of angle differences: $\arcsin(\lVert L^\dagger P\rVert_{\mathcal{E},\infty})$	Accuracy of condition: $\arcsin(\lVert L^\dagger P\rVert_{\mathcal{E},\infty}) - \max_{\{i,j\}\in\mathcal{E}} \lvert\theta_i^* - \theta_j^*\rvert$
9 bus system	0.12889 rad	0.12893 rad	$4.1218 \cdot 10^{-5}$ rad
IEEE 14 bus system	0.16622 rad	0.16650 rad	$2.7995 \cdot 10^{-4}$ rad
IEEE RTS 24	0.22309 rad	0.22480 rad	$1.7089 \cdot 10^{-3}$ rad
IEEE 30 bus system	0.16430 rad	0.16456 rad	$2.6140 \cdot 10^{-4}$ rad
New England 39	0.16821 rad	0.16828 rad	$6.6355 \cdot 10^{-5}$ rad
IEEE 57 bus system	0.20295 rad	0.22358 rad	$2.0630 \cdot 10^{-2}$ rad
IEEE RTS 96	0.24593 rad	0.24854 rad	$2.6076 \cdot 10^{-3}$ rad
IEEE 118 bus system	0.23524 rad	0.23584 rad	$5.9959 \cdot 10^{-4}$ rad
IEEE 300 bus system	0.43204 rad	0.43257 rad	$5.2618 \cdot 10^{-4}$ rad
Polish 2383 bus system (winter peak 1999/2000)	0.25144 rad	0.25566 rad	$4.2183 \cdot 10^{-3}$ rad

Discrete control actions to assure sync

1. (re)dispatch generation subject to **security constraints**:

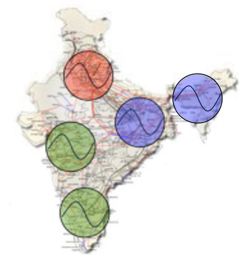
$$\text{find}_{\,\theta\in\mathbb{T}^n,\, u\in\mathbb{R}^{n_I}} \quad \text{subject to}$$

source power balance: $u_i = P_i(\theta)$

load power balance: $P_i = P_i(\theta)$

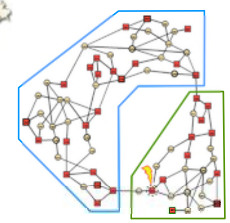
branch flow constraints: $\lvert\theta_i - \theta_j\rvert \le \gamma_{ij} < \pi/2$

2. **remedial action schemes**: load/production shedding & islanding

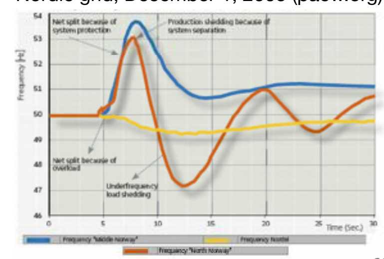


India, July 30/31 2012

RTS 96 example

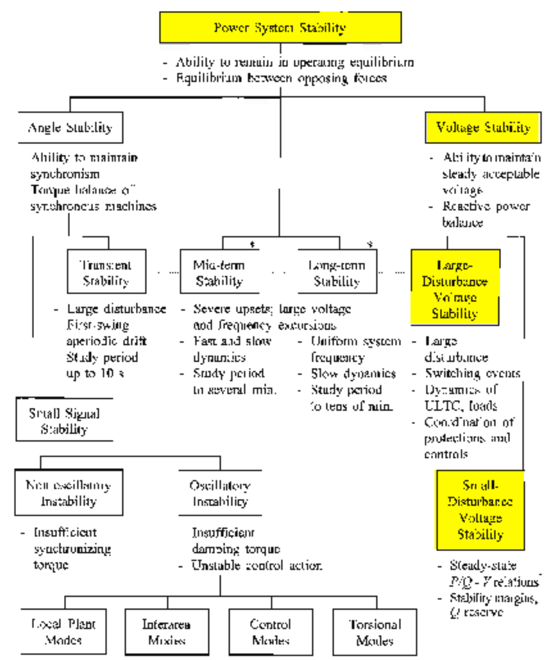


Nordic grid, December 1, 2005 (pacw.org)



Decoupled Reactive Power Flow (Voltage Collapse)

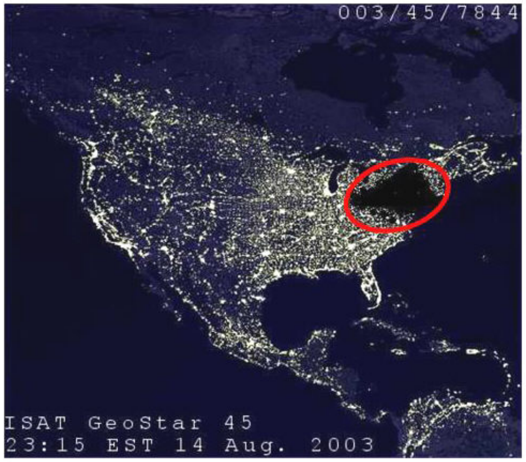
Apparently a different beast



Voltage collapse in power networks

- **voltage instability**: loading > capacity ⇒ voltages drop
  "mainly" a reactive power phenomena
- **recent outages:** Québec '96, Scandinavia '03, Northeast '03, Athens '04

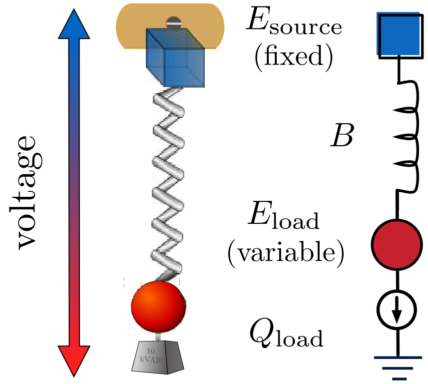
> *"Voltage collapse is still the biggest single threat to the transmission system. It's what keeps me awake at night."*
>
> – Phil Harris, CEO PJM.

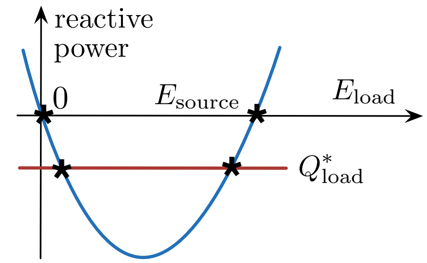


Back of the envelope calculations for the two-node case

source connected to load shows bifurcation at load voltage $E_{\text{load}} = E_{\text{source}}/2$

reactive power balance at load: $Q_{\text{load}} = B\, E_{\text{load}}(E_{\text{load}} - E_{\text{source}})$





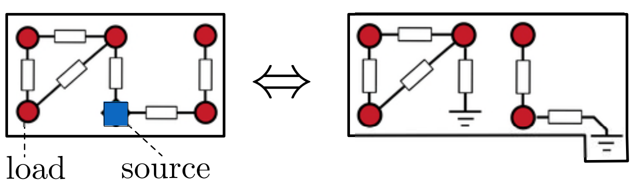
$E_{\text{load}} \in \mathbb{R} \Leftrightarrow Q_{\text{load}} \geq -B\,(E_{\text{source}})^2/4$

∃ high load voltage solution ⇔ (load) < (network)(source voltage)$^2/4$

Preliminary insights when going to "complex" networks

- **sources** with constant voltage magnitudes E_i
- **loads** with constant power demand $Q_i(E) = Q_i$

⇒ WLOG assume that network among loads is connected



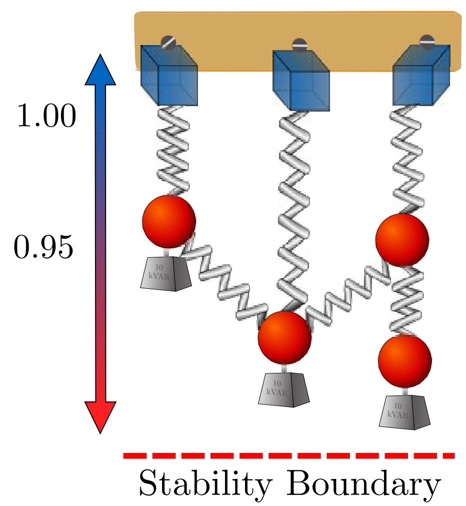
⇒ reactive power balance: $Q_i = -\sum_j B_{ij} E_i E_j$ or $Q = -\,\text{diag}(E) B E$

⇒ necessary feasibility condition: $\sum_{i=1}^{n} Q_i \geq 0 \;\Leftarrow\; \exists$ a solution

(by summing all equations and using $-E^T BE \geq 0$)

Intuition extends to complex networks – essential insights

Reactive power balance:
$Q_i = -\sum_j B_{ij} E_i E_j$



Suff. & tight cond' for general case [J. Simpson-Porco, FD, & F. Bullo, '16]:

∃ unique high-voltage solution E_{load}

⇔

$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$

1. nominal (zero load) voltage E_{nom}
   $$0 = -\sum_j B_{ij}\, E_{i,\text{nom}}\, E_{j,\text{nom}}$$
2. coord-trafo to solution guess:
   $$x_i = E_i / E_{i,\text{nom}} - 1$$
3. Picard-Banach iteration $x^+ = f(x)$

Previous condition “$\Delta < 1$” also predicts voltage deviation

for coupled & lossy power flow

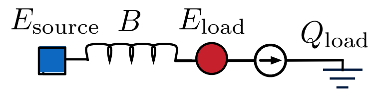
Samples: randomized scenario (50% load and 33% generation variability)

Randomized test case (1000 instances)	**Numerical** Numerical worst-case voltage deviations: $\delta_{\text{exact}} = \max_i \frac{\lvert E_i - E_i^* \rvert}{E_i^*}$	**Theoretical** Analytic prediction of voltage deviations: $\delta_- = (1-\sqrt{1-\Delta})/2$	**% Error** Accuracy of prediction: $100 \cdot \frac{\delta_- - \delta_{\text{exact}}}{\delta_{\text{exact}}}$
9 bus system	$5.49 \cdot 10^{-2}$	$5.51 \cdot 10^{-2}$	0.366 %
IEEE 14 bus system	$2.50 \cdot 10^{-2}$	$2.51 \cdot 10^{-2}$	0.200 %
IEEE RTS 24	$3.23 \cdot 10^{-2}$	$3.24 \cdot 10^{-2}$	0.347 %
IEEE 30 bus system	$4.91 \cdot 10^{-2}$	$4.95 \cdot 10^{-2}$	0.806 %
New England 39	$6.26 \cdot 10^{-2}$	$6.30 \cdot 10^{-2}$	0.620 %
IEEE 57 bus system	$1.20 \cdot 10^{-1}$	$1.24 \cdot 10^{-2}$	3.60 %
IEEE RTS 96	$3.43 \cdot 10^{-2}$	$3.44 \cdot 10^{-2}$	0.376 %
IEEE 118 bus system	$2.60 \cdot 10^{-2}$	$2.61 \cdot 10^{-2}$	0.557 %
IEEE 300 bus system	$1.05 \cdot 10^{-1}$	$1.07 \cdot 10^{-2}$	1.76 %
Polish 2383 bus system (winter peak 1999/2000)	**$3.99 \cdot 10^{-2}$**	**$4.02 \cdot 10^{-2}$**	**0.764 %**

A tight & analytic guarantee: typical prediction error of $\sim 1\%$

More back of the envelope calculations

$$Q_L = B\, E_L (E_L - E_S)$$



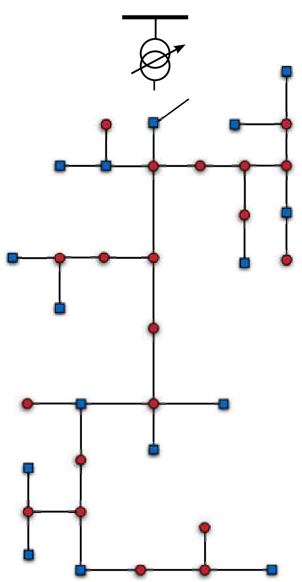
$$\Rightarrow E_L = E_S/2 \left(1 + \sqrt{1 + 4Q_L/(BE_S^2)}\right) = \frac{E_S}{2}\left(1 + \sqrt{1 - Q_L/Q_{\text{crit}}}\right)$$

$\Rightarrow$ Taylor exp. for $Q_L/Q_{\text{crit}} \to 0$: $E_L \approx E_S\,(1 + Q_L/Q_{\text{crit}})$

- **general case**: existence & approximation from implicit function thm
  - if all loads Q_i are “sufficiently small” [D. Molzahn, B. Lesieutre, & C. DeMarco ’12]
  - if slack bus has “sufficiently large” E_S [S. Bolognani & S. Zampieri ’12 & ’14]
  - if each source is above a “sufficiently large” E_{source} [B. Gentile et al. ’14]
  - if previous existence condition is met [J. Simpson-Porco, FD, & F. Bullo, ’16]

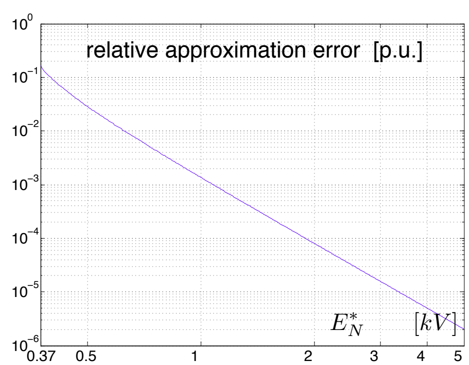
Linear DC approximation extends to complex networks

verification via *IEEE 37 bus distribution system* (SoCal)

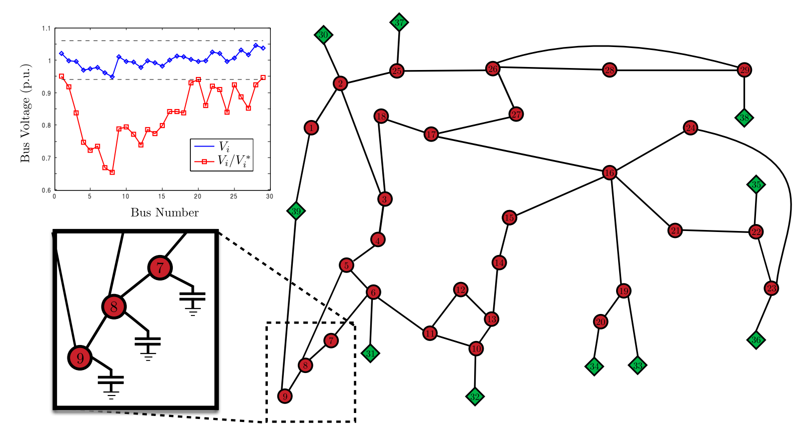


Reactive DC approximation [B. Gentile, J. Simpson-Porco, FD, S. Zampieri, & F. Bullo, ’14]:

$$E_L \approx \text{diag}(E_L^*)\,(\mathbb{1} + Q_{\text{crit}}^{-1} Q_L) + \text{h.o.t.}$$



Discrete control actions for voltage stability



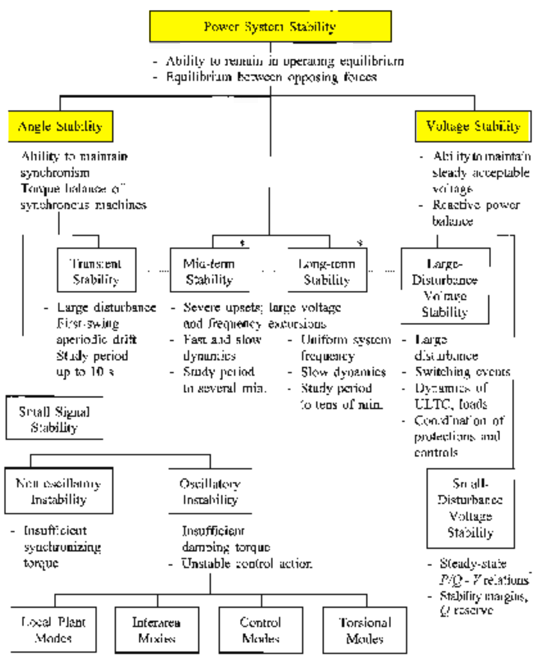
1. shunts support voltage magnitudes, but hide proximity to collapse
   $\Rightarrow$ ratios E_i/E_i^* more useful than per-unit voltages
2. $\lvert Q_{\text{crit},89}^{-1}\rvert > \lvert Q_{\text{crit},87}^{-1}\rvert$ means E_8/E_8^* more sensitive to Q_9 then to Q_7
   $\Longrightarrow$ place SVC at bus 9 to support E_8 & increase stability margin

Coupled & Lossy Power Flow

Coupling matters!

"As systems become more heavily loaded, nonlinearities play an increasingly important role in power system behavior . . . analysis tools should continue to work reliably, even under extreme system conditions . . . the $P - V$ and $Q - \theta$ cross coupling terms become significant." — [Ian Hiskens, Proc. of IEEE '95]

This is not even really on the map

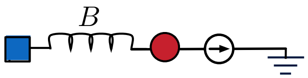


Solving the two-node case

see exercise

Simplest example shows surprisingly complex behavior

- PV source, PQ load, & lossless line

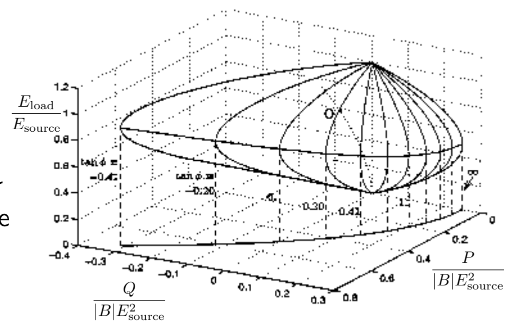


$$P = B\, E_{\text{source}}\, E_{\text{load}} \sin(\theta)$$
$$Q = B\, E_{\text{load}}^2 - B\, E_{\text{source}}\, E_{\text{load}} \cos(\theta)$$

- after eliminating θ, there exists $E_{\text{load}} \in \mathbb{R}_{\geq 0}$ if and only if

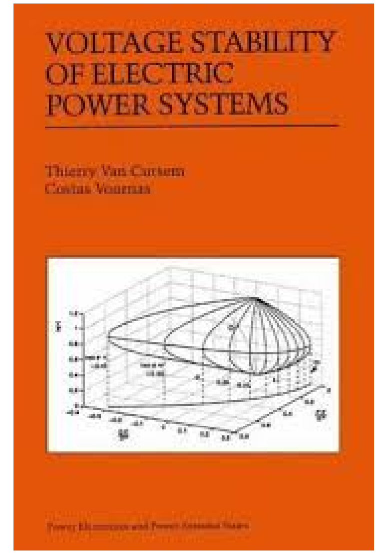
$$P^2 - B\, E_{\text{source}}^2\, Q \leq B^2 E_{\text{source}}^4 \,/\, 4$$

- Observations:
  1. $P = 0$ case consistent with previous decoupled analysis
  2. $Q = 0$ case delivers 1/2 transfer capacity from decoupled case
  3. intermediate cases $Q = P \tan\phi$ give so-called "nose curves"



Recommended reading to understand a glimpse

at least once in a life-time you should read chapter 2 . . .



Coupled & lossy power flow in complex networks

- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

- what makes it so much harder than the previous two node case?
  losses, mixed lines, cycles, PQ-PQ connections, . . .
- much theoretic work, qualitative understanding, & numeric approaches:
  - existence of solutions [Thorp, Schulz, & Ilić '86, Wu & Kumagai '82]
  - solution space [Hiskens & Davy '01, Overbye & Klump '96, Van Cutsem '98, . . .]
  - distance-to-failure [Venikov '75, Abe & Isono '76, Dobson '89, Andersson & Hill '93, . . .]
  - convex relaxation approaches [Molzahn et al. '12, Dvijotham et al. '15]
- little analytic & quantitative understanding beyond the two-node case

"Whoever figures that one out [analysis of $n > 2$ node] wins a noble prize!"

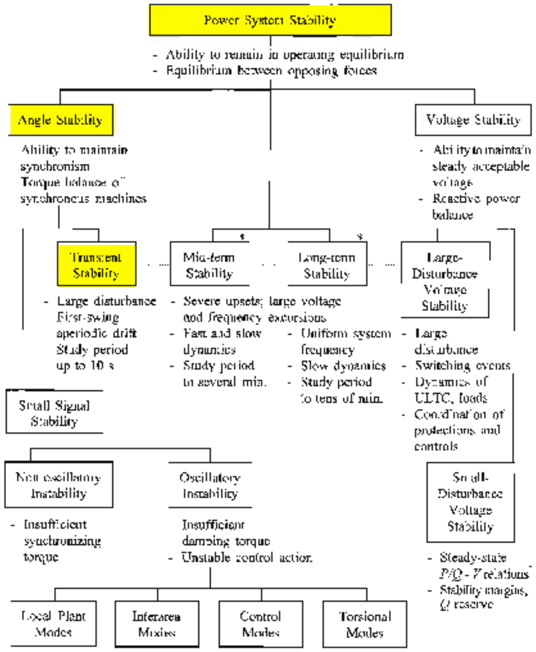
— [Peter Sauer, lunch @ UIUC '13]

Transient Rotor Angle Stability

"The crown jewel of power system stability!"

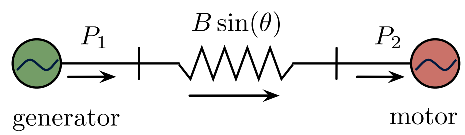
— [Janusz Bialek, skype call '13]

The crown jewel of power system stability

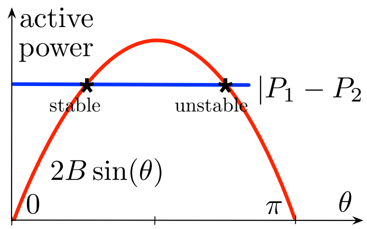


Revisit of the two-node case — the forced pendulum

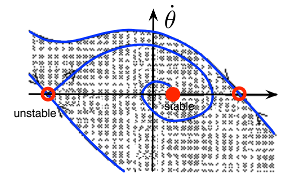
more complex than anticipated



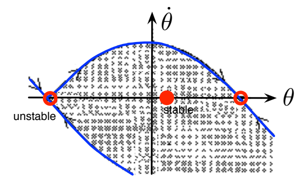
$$M\ddot{\theta} = -D\dot{\theta} + P_1 - P_2 - 2B\sin(\theta)$$



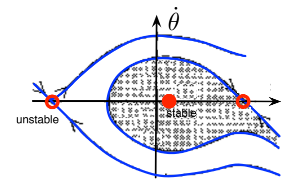
- **Local stability**: $\exists$ local stable solution $\Leftrightarrow$ $B > |P_1 - P_2|/2$
- **Global stability**: depends on gap $B > |P_1 - P_2|/2$ and D/M ratio



$(D/M) > (D/M)_{\text{critical}}$



$(D/M) = (D/M)_{\text{critical}}$

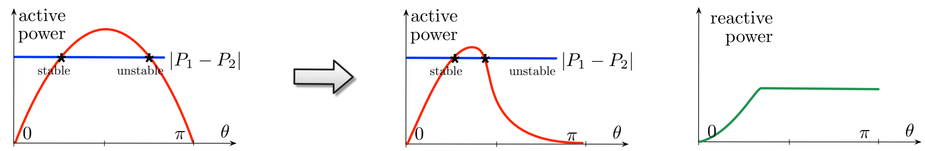


$(D/M) < (D/M)_{\text{critical}}$

Revisit of the two-node case — cont'd

the story is not complete . . . some further effects that we swept under the carpet

- **Voltage reduction**: generator needs to provide reactive power for voltage regulation – until saturation, then generator becomes PQ bus



- **Load sensitivity**: different behavior depending on load model: resistive, constant power, frequency-dependent, dynamic, power electronics, . . .
- **Singularity-issues** for coupled power flows (load voltage collapse)
- **Losses & higher-order dynamics** change stability properties . . .

$\Rightarrow$ quickly run into computational approaches

Primer on Lyapunov functions

on blackboard

Transient stability in multi-machine power systems

generators:

$$\dot{\theta}_i = \omega_i$$
$$M_i \dot{\omega}_i = -D_i \omega_i + P_i - \sum\nolimits_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

loads:

$$D_i \dot{\theta}_i = P_i - \sum\nolimits_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

Challenge (improbable): faster-than-real-time transient stability assessment

Energy function methods for simple lossless models via Lyapunov function

$$V(\omega, \theta, E) = \sum\nolimits_i \frac{1}{2} M_i \omega_i^2 - \sum\nolimits_i P_i \theta_i - \sum\nolimits_i Q_i \log E_i - \sum\nolimits_{ij} B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

Computational approaches: level sets of energy functions & unstable equilibria, sum-of-squares methods, convex optimization approaches, time-domain simulations, . . . (holy grail of power system stability)

Hamiltonian analysis of the swing equations

more famously known as "energy function analysis" (see exercise)

Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

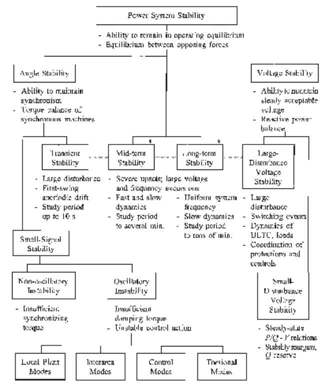
- Primary Control
- Power Sharing
- Secondary control
- Experimental validation
- (Optional material)

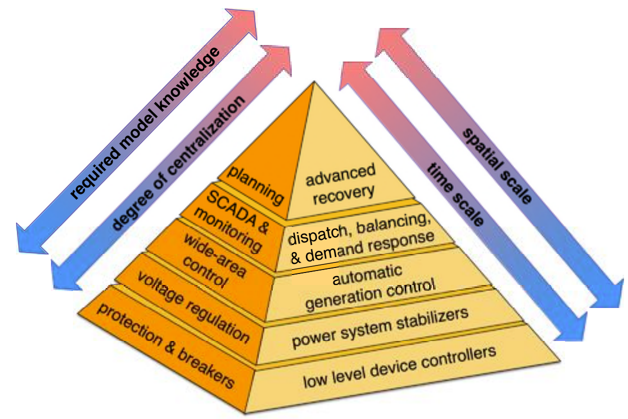
Power System Oscillations

Conclusions

A plethora of control tasks and nested control layers

organized in hierarchy and separated by states & spatial/temporal/centralization scales

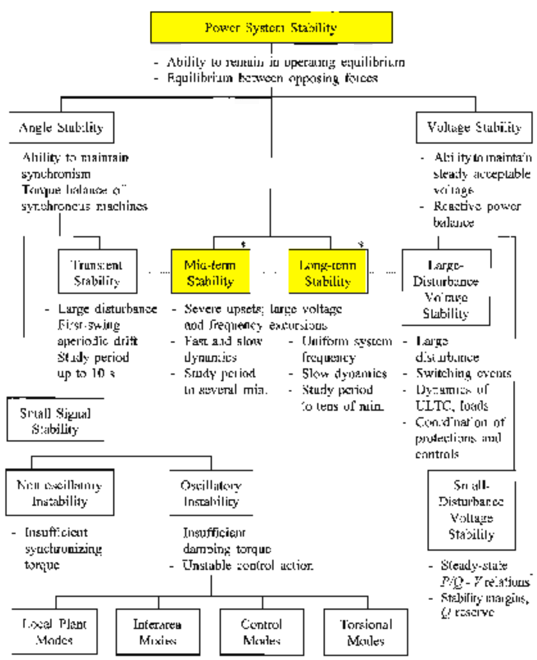




We will focus on frequency control & primary/secondary/tertiary layers.

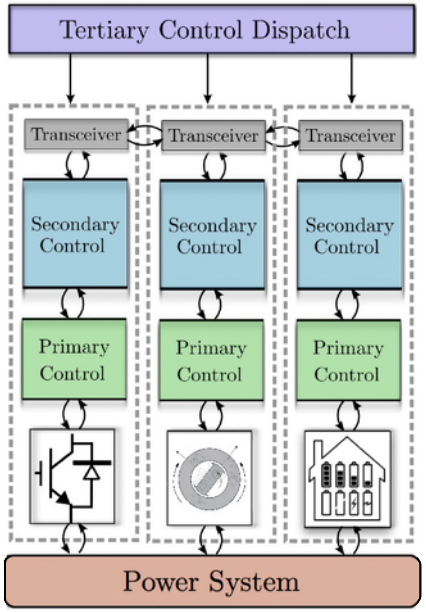
All dynamics & controllers are interacting. Classification & hierarchy are for simplicity.

Where are we on the map?



Objectives

Hierarchical frequency control architecture & objectives



3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (minutes)
   - Goal: maintain operating point in presence of disturbances
   - Strategy: centralized
1. **Primary control** (real-time)
   - Goal: stabilize frequency & share unknown load
   - Strategy: decentralized

Q: Is this layered & hierarchical architecture still appropriate for tomorrow's power system?

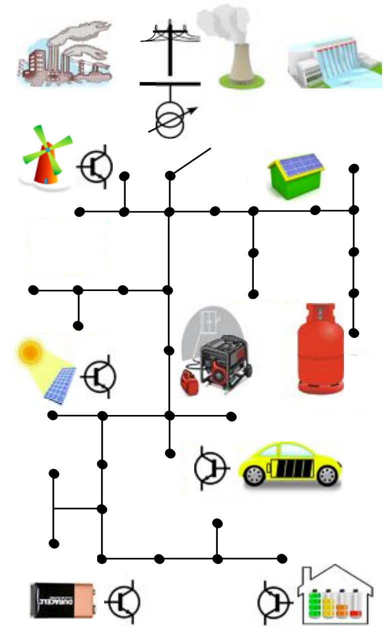
Is this hierarchical control architecture still appropriate?

Some recent developments

- increasing renewable integration & deregulated energy markets
- bulk generation replaced by distributed generation
- synchronous machines replaced by power electronics sources
- low gas prices & substitutions

Some new problem scenarios

- alternative spinning reserves: storage, load control, & DER
- networks of low-inertia & distributed renewable sources
- small-footprint islanded systems



Need to adapt the control hierarchy in tomorrow's grid

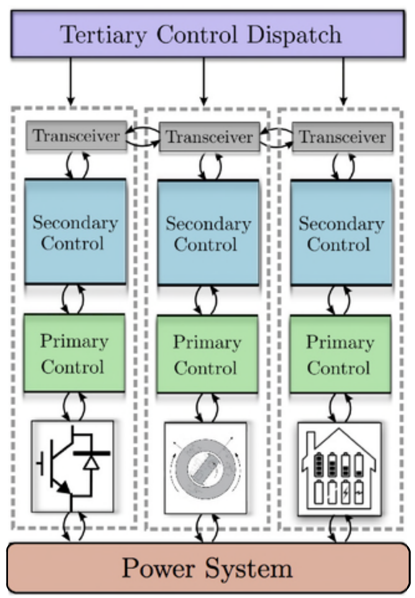
☹perational challenges

- more uncertainty & less inertia
- more volatile & faster fluctuations
- plug'n'play control: fast, model-free, & without central authority

☺pportunities

- re-instrumentation: comm & sensors
- more & faster spinning reserves
- advances in control of cyber-physical & complex systems

⇒ **break** vertical & horizontal **hierarchy**

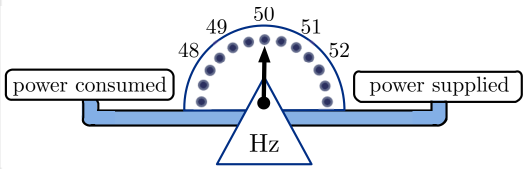


Primary Control

Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i \ddot{\theta} + D_i \dot{\theta}_i$$
$$= P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

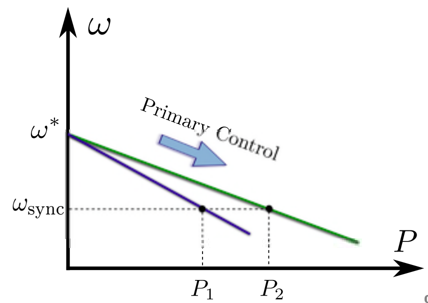


Conventional wisdom: physics are naturally stable & sync frequency reveals power imbalance

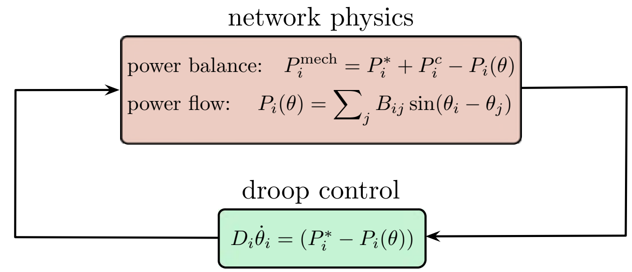
recall: $\omega_{\text{sync}} = \sum_i P_i^* / D_i$

$P/\dot{\theta}$ droop control:

$$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$$
$$\Updownarrow$$
$$D_i \dot{\theta}_i = P_i^* - P_i(\theta)$$



Putting the pieces together...



synchronous machines: $M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

inverter sources & controllable loads: $D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

passive loads & power-point tracking sources: $0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

active power flow is feasible $\Longrightarrow$ $\exists$ unique & exp. stable frequency sync

Main **proof ideas** and some **further results**:

- stability via Jacobian & Lyapunov arguments
- synchronization frequency: ($\propto$ power balance)
  $$\omega_{\text{sync}} = \omega^* + \frac{\sum_{\text{sources}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{sources}} D_i}$$
- steady-state power injections: (depend on D_i & P_i^*)
  $$\mathcal{P}_i = \begin{cases} P_i^* & \text{(load } \#i) \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & \text{(source } \#i) \end{cases}$$

Closed-loop stability?

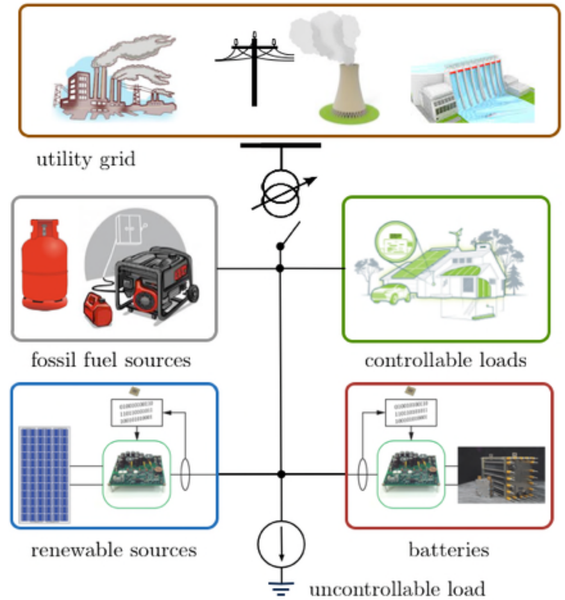
see exercise

power sharing & economic optimality under droop control

(sometimes in tertiary layer)

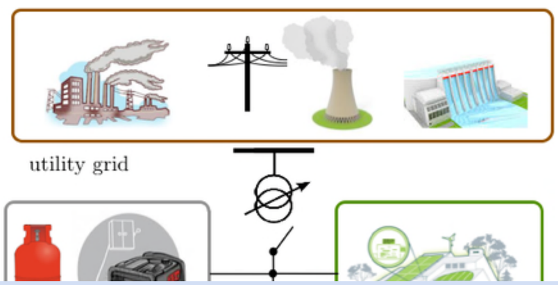
Tertiary control and energy management

an offline resource allocation and scheduling problem



Tertiary control and energy management

an offline resource allocation and scheduling problem



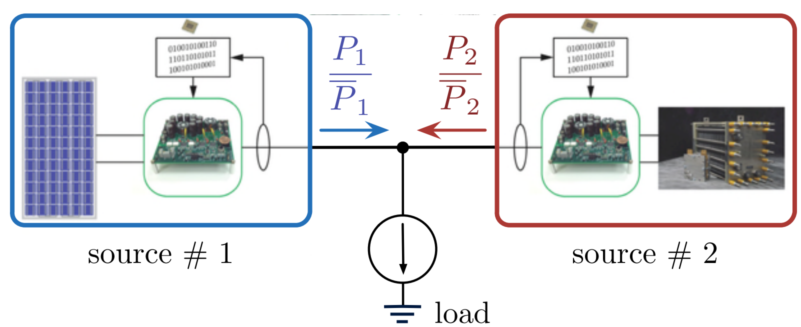
minimize	{cost of generation, losses, ...}
subject to	
equality constraints:	power balance equations
inequality constraints:	flow/injection/voltage constraints
logic constraints:	commit generators yes/no
	⋮

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left|\sum_{\text{loads}} P_j^*\right| \leq \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) / \overline{P}_i = P_j(\theta) / \overline{P}_j$



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3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

A little calculation reveals in steady state:

$$\frac{P_i(\theta)}{\overline{P}_i} \overset{!}{=} \frac{P_j(\theta)}{\overline{P}_j} \quad \Rightarrow \quad \frac{P_i^* - (D_i \omega_{\text{sync}} - \omega^*)}{\overline{P}_i} \overset{!}{=} \frac{P_j^* - (D_j \omega_{\text{sync}} - \omega^*)}{\overline{P}_i}$$

. . . so choose

$$\frac{P_i^*}{\overline{P}_i} = \frac{P_j^*}{\overline{P}_j} \quad \text{and} \quad \frac{D_i}{\overline{P}_i} = \frac{D_j}{\overline{P}_j}$$

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$\boxed{D_i / \overline{P}_i = D_j / \overline{P}_j \ \ \& \ \ P_i^* / \overline{P}_i = P_j^* / \overline{P}_j}$$

The the following statements hold:

(i) Proportional load sharing: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

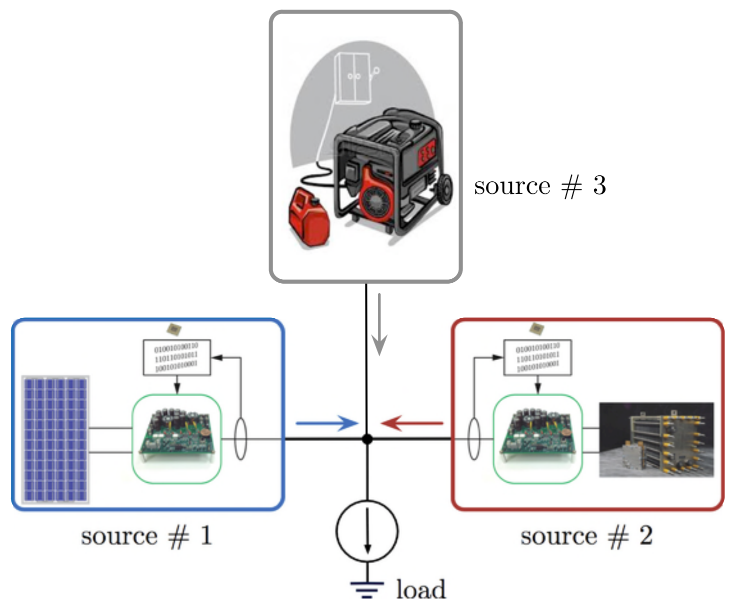
(ii) Constraints met: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \overline{P}_j \ \Leftrightarrow \ P_i(\theta) \in [0, \overline{P}_i]$

Constraints achieved by fair proportional load sharing

see exercise

Objective I: fair proportional load sharing

proportional load sharing is not always the right objective

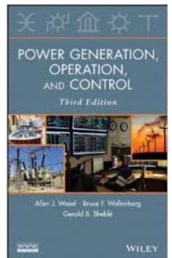


Objective II: optimal power flow = tertiary control

an offline resource allocation/scheduling problem

minimize	{cost of generation, losses, ...}
subject to	
equality constraints:	power balance equations
inequality constraints:	flow/injection/voltage constraints
logic constraints:	commit generators yes/no
	⋮

Will be discussed more in detail by Andrej.



Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

minimize $_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}}$	$J(u) = \sum_{\text{sources}} \alpha_i u_i^2$
subject to	
source power balance:	$P_i^* + u_i = P_i(\theta)$
load power balance:	$P_i^* = P_i(\theta)$
branch flow constraints:	$\lvert\theta_i - \theta_j\rvert \le \gamma_{ij} < \pi/2$

A simpler & equivalent (in the strictly feasible case) problem formulation:

minimize $_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}}$	$J(u) = \sum_{\text{sources}} \alpha_i u_i^2$
subject to	
power balance:	$\sum_i P_i^* + \sum_i u_i = 0$

The *abc* of resource allocation

on blackboard

Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

minimize $_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}}$	$J(u) = \sum_{\text{sources}} \alpha_i u_i^2$
subject to	
source power balance:	$P_i^* + u_i = P_i(\theta)$
load power balance:	$P_i^* = P_i(\theta)$
branch flow constraints:	$\lvert\theta_i - \theta_j\rvert \le \gamma_{ij} < \pi/2$

Unconstrained case: identical marginal costs $\alpha_i u_i^* = \alpha_j u_j^*$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

Objective II: decentralized dispatch optimization

Insight: droop-controlled system $=$ decentralized optimization algorithm

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

(i) the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer (θ^*, u^*).

(ii) $\exists$ droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ.

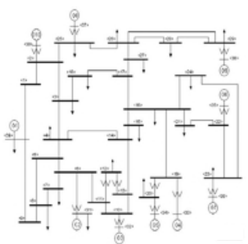
If (i) & (ii) are true, then $\theta_i \sim \theta_i^*$, $u_i^* = -D_i(\omega_{\text{sync}} - \omega^*)$, & $\boxed{D_i\alpha_i = D_j\alpha_j}$.

- includes proportional load sharing $\alpha_i \propto 1/\overline{P}_i$
- similar results hold for strictly convex & differentiable cost

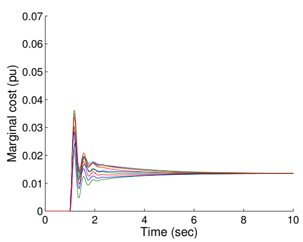
Sketch of the main proof ideas

see exercise

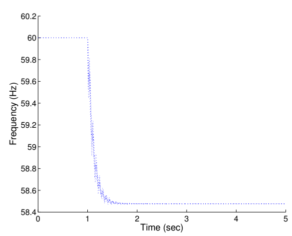
Some quick simulations & extensions



IEEE 39 New England with load step at 1s



$t \to \infty$: convergence to identical marginal costs

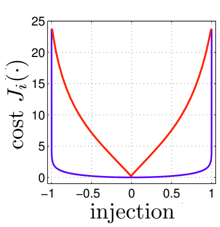


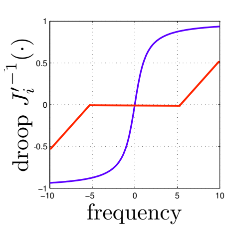
$t \to \infty$: frequency $\propto$ power imbalance

$\Rightarrow$ strictly convex & differentiable cost $J(u) = \sum_{\text{sources}} J_i(u_i)$

$\Rightarrow$ non-linear frequency droop curve $J_i'^{-1}(\dot\theta_i) = P_i^* - P_i(\theta)$

$\Rightarrow$ include dead-bands, saturation, etc.

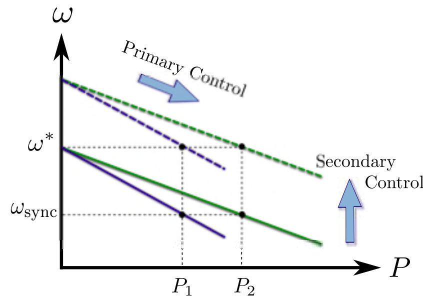




Secondary Control

Secondary frequency control

- **Problem:** steady-state frequency deviation ($\omega_{\text{sync}} \neq \omega^*$)
- **Solution:** integral control of frequency error



- **Basics** of integral control $\boxed{\frac{1}{s}}$:

  1. discrete time: $u_i(t+1) = u_i(t) + k \cdot \dot{\theta}_i(t)$ with gain $k > 0$
  2. continuous-time: $u_i(t) = k \cdot \int_0^t \dot{\theta}_i(\tau)\, d\tau$ or $\dot{u}_i(t) = k \cdot \dot{\theta}_i(t)$

$\Rightarrow$ $\dot{\theta}_i(t)$ is zero in (a possibly stable) steady state

$\Rightarrow$ add additional injection $u_i(t)$ to droop control

Decentralized secondary integral frequency control

$\boxed{\frac{1}{s}}$ add local integral controller to every droop controller

$\Rightarrow$ zero frequency deviation ✓

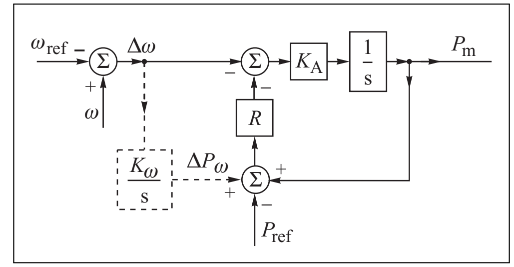
$\Rightarrow$ nominally globally stabilizing [C. Zhao, E. Mallada, & FD, '14] ✓

☹ every integrator induces a 1d equilibrium subspace

☹ injections live in subspace of dimension # integrators

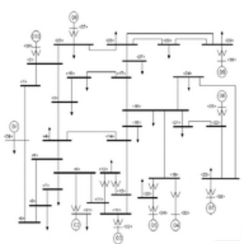
☹ load sharing & economic optimality are lost ...

☹ unstable in presence of biased noise [M. Andreasson et al. '14]

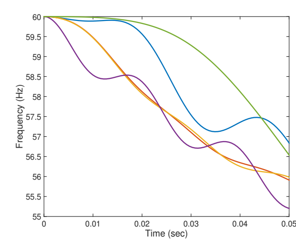


turbine governor integral control loop

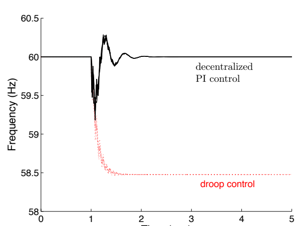
Simulations cont'd



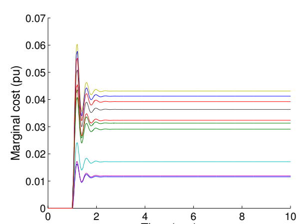
IEEE 39 New England with decentralized PI control



decentralized PI control in presence of biased noise



$t \to \infty$: decentralized PI control regulates frequency



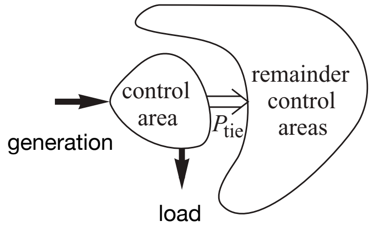
$t \to \infty$: decentralized PI control is not optimal

Why does decentralized integral control not work?

see exercise

Automatic generation control (AGC)

- **ACE** area control error = { frequency error } + { generation - load - tie-line flow }

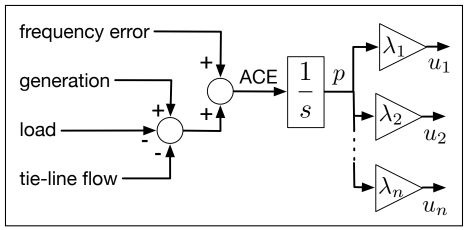


$\frac{1}{s}$ **centralized integral control:**

$$p(t) = \int_0^t \mathrm{ACE}(\tau)\, d\tau$$

- **generation allocation:** $u_i(t) = \lambda_i p(t)$, where λ_i is generation participation factor (in our case $\lambda_i = 1/\alpha_i$)

⇒ assures identical marginal costs: $\alpha_i u_i = \alpha_j u_j$

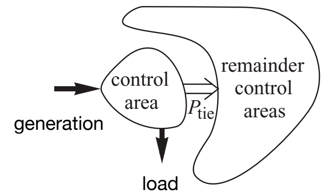


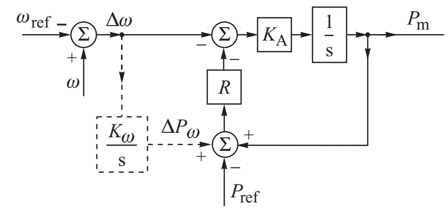
AGC implementation

☺ load sharing & economic optimality are recovered

Drawbacks of conventional secondary frequency control

interconnected systems	isolated systems
• **centralized** automatic generation control (AGC)	• **decentralized** PI control



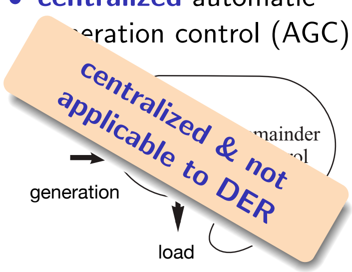


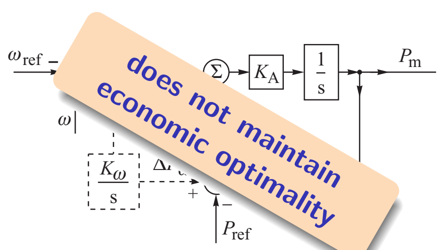
compatible with econ. dispatch [N. Li, L. Chen, C. Zhao, & S. Low '13]

nominally *globally* stabilizing [C. Zhao, E. Mallada, & FD, '14]

Drawbacks of conventional secondary frequency control

interconnected systems	isolated systems
• **centralized** automatic generation control (AGC)	• **decentralized** PI control





compatible with econ. dispatch [N. Li, L. Chen, C. Zhao, & S. Low '13]

nominally *globally* stabilizing [C. Zhao, E. Mallada, & FD, '14]

Distributed energy resources require **distributed** (**!**) secondary control.

An incomplete literature review of a busy field

ntwk with unknown disturbances ∪ integral control ∪ distributed averaging

- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcıa-Domınguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]

The following idea precedes most references, it's simpler, & it's more robust.

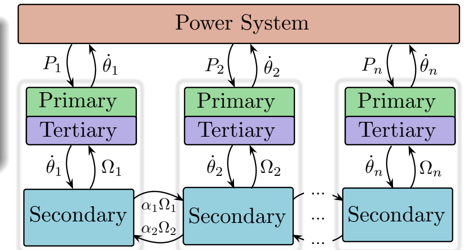
Let's derive a simple distributed control strategy

on blackboard

Distributed Averaging PI (DAPI) control

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot (\alpha_i\Omega_i - \alpha_j\Omega_j)$$



- no tuning & no time-scale separation: $k_i, D_i > 0$
- recovers optimal dispatch
- distributed & modular: connected comm. network
- has seen many extensions [C. de Persis et al., H. Sandberg et al., J. Schiffer et al., M. Zhu et al., ...]

Theorem: stability of DAPI

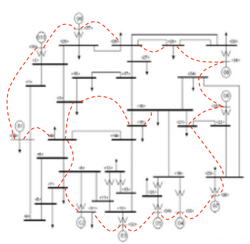
[J. Simpson-Porco, FD, & F. Bullo '12]
[C. Zhao, E. Mallada, & FD '14]

primary droop controller works

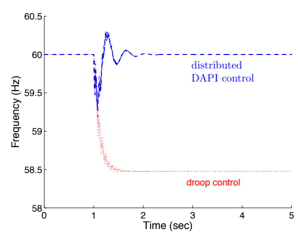
$\iff$

secondary DAPI controller works

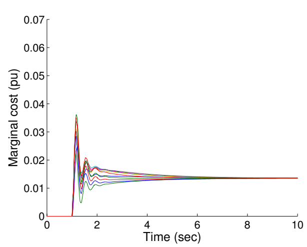
Simulations cont'd



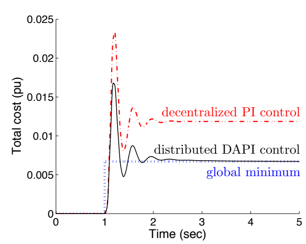
IEEE 39 New England with distributed DAPI control



$t \to \infty$: DAPI control regulates frequency



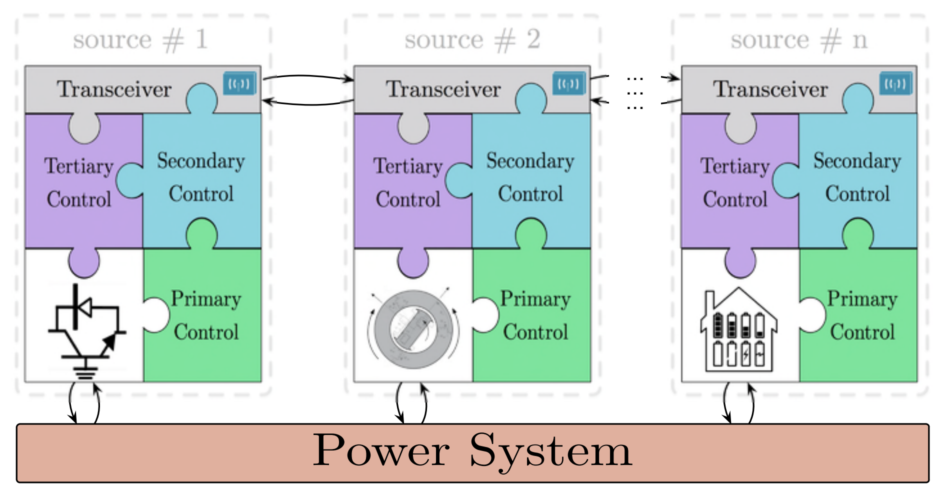
DAPI control synchronizes marginal costs



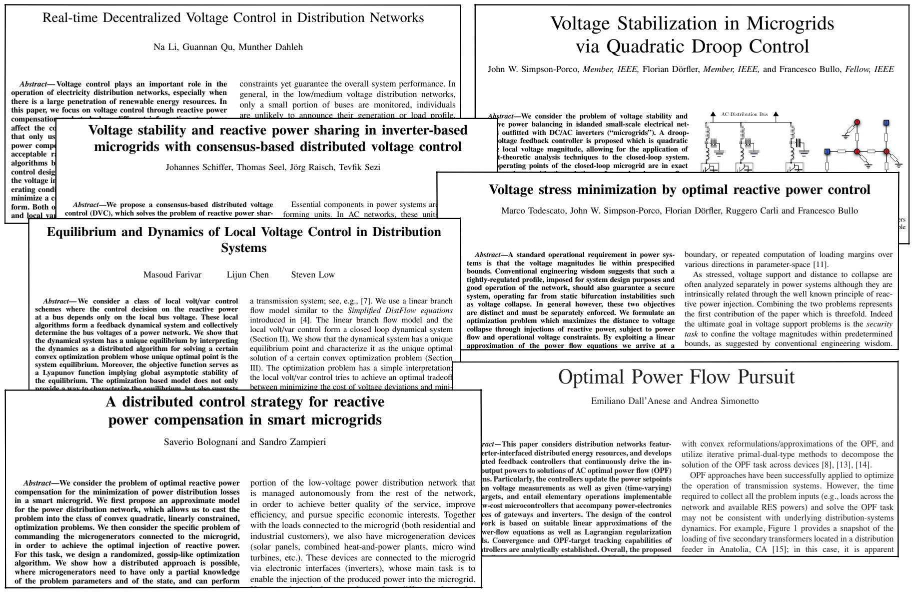
DAPI control minimizes cost with little effort

Plug'n'play architecture

flat hierarchy, distributed, no time-scale separations, & model-free



We can do similar things on the reactive power side


Real-time Decentralized Voltage Control in Distribution Networks
Na Li, Guannan Qu, Munther Dahleh

Voltage Stabilization in Microgrids via Quadratic Droop Control
John W. Simpson-Porco, Member, IEEE, Florian Dörfler, Member, IEEE, and Francesco Bullo, Fellow, IEEE

Voltage stability and reactive power sharing in inverter-based microgrids with consensus-based distributed voltage control
Johannes Schiffer, Thomas Seel, Jörg Raisch, Tevfik Sezi

Voltage stress minimization by optimal reactive power control
Marco Todescato, John W. Simpson-Porco, Florian Dörfler, Ruggero Carli and Francesco Bullo

Equilibrium and Dynamics of Local Voltage Control in Distribution Systems
Masoud Farivar, Lijun Chen, Steven Low

Optimal Power Flow Pursuit
Emiliano Dall'Anese and Andrea Simonetto

A distributed control strategy for reactive power compensation in smart microgrids
Saverio Bolognani and Sandro Zampieri

Much recent work on reactive power control

- heuristic linear Q/E droop: $(E_i - E_i^*) \propto (Q_i^* - Q_i(E))$
  sometimes with integrator & nonlinearities [J. Simpson-Porco et. al. '16]
- reactive power sharing DAPI [J. Simpson-Porco et. al. '15, J. Schiffer et al. '16]

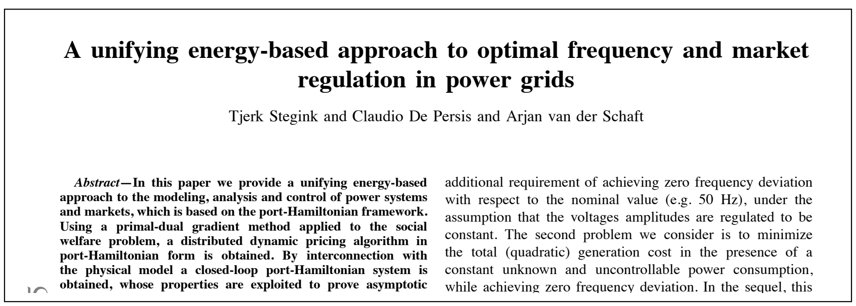
$$\kappa_i \dot{e}_i = \sum_{j \subseteq \text{sources}} a_{ij} \cdot \left(Q_i/\overline{Q_i} - Q_j/\overline{Q_j}\right) - \varepsilon e_i$$

- voltage regulation [M. Farivar et al. '13]: $\kappa_i \dot{e}_i = E_i - E_i^*$
- loss minimization: minimize $\sum_{\{i,j\}\in\mathcal{E}} B_{ij}(E_i - E_j)^2$ [N. Li et al. '14]
- robustness margins: maximize det (Jacobian) [M. Todescato et al. '16]
- maximize reative reserves s.t. flat voltage profile $E_i \approx 1$ [RTE France]

Main distinction to active power: while each of these objectives is individually feasible, they are also all **mutually exclusive** . . .

A great unifying perspective on secondary control

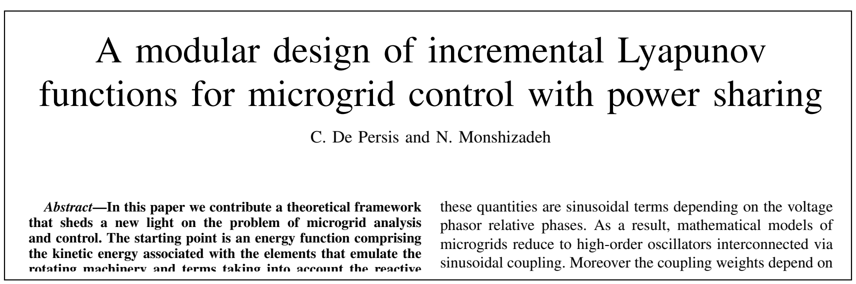
pretty much incorporating everything that we've discussed this far


A unifying energy-based approach to optimal frequency and market regulation in power grids

Tjerk Stegink and Claudio De Persis and Arjan van der Schaft

***Abstract*—In this paper we provide a unifying energy-based approach to the modeling, analysis and control of power systems and markets, which is based on the port-Hamiltonian framework. Using a primal-dual gradient method applied to the social welfare problem, a distributed dynamic pricing algorithm in port-Hamiltonian form is obtained. By interconnection with the physical model a closed-loop port-Hamiltonian system is obtained, whose properties are exploited to prove asymptotic**

additional requirement of achieving zero frequency deviation with respect to the nominal value (e.g. 50 Hz), under the assumption that the voltages amplitudes are regulated to be constant. The second problem we consider is to minimize the total (quadratic) generation cost in the presence of a constant unknown and uncontrollable power consumption, while achieving zero frequency deviation. In the sequel, this


A modular design of incremental Lyapunov functions for microgrid control with power sharing

C. De Persis and N. Monshizadeh

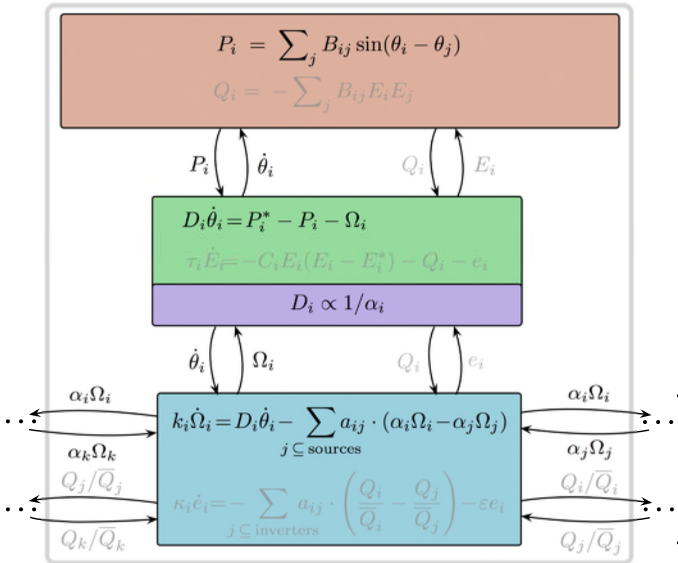
***Abstract*—In this paper we contribute a theoretical framework that sheds a new light on the problem of microgrid analysis and control. The starting point is an energy function comprising the kinetic energy associated with the elements that emulate the rotating machinery and terms taking into account the reactive**

these quantities are sinusoidal terms depending on the voltage phasor relative phases. As a result, mathematical models of microgrids reduce to high-order oscillators interconnected via sinusoidal coupling. Moreover the coupling weights depend on

plug-and-play experiments

Plug'n'play architecture

recap of detailed signal flow (active power only)



Power system: physics & loadflow

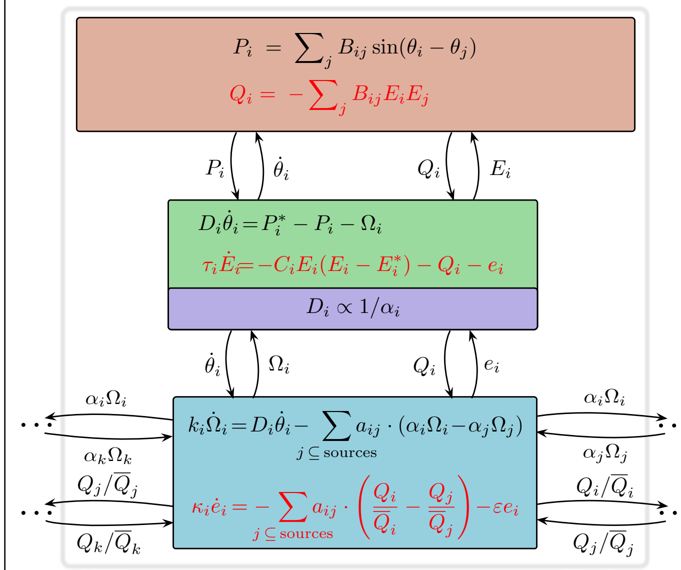
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs ∝ 1 /control gains

Secondary control: diffusive averaging of optimal injections

Plug'n'play architecture

similar results for decoupled reactive power flow [J. Simpson-Porco, FD, & F. Bullo '13 - '15]



Power system: physics & loadflow

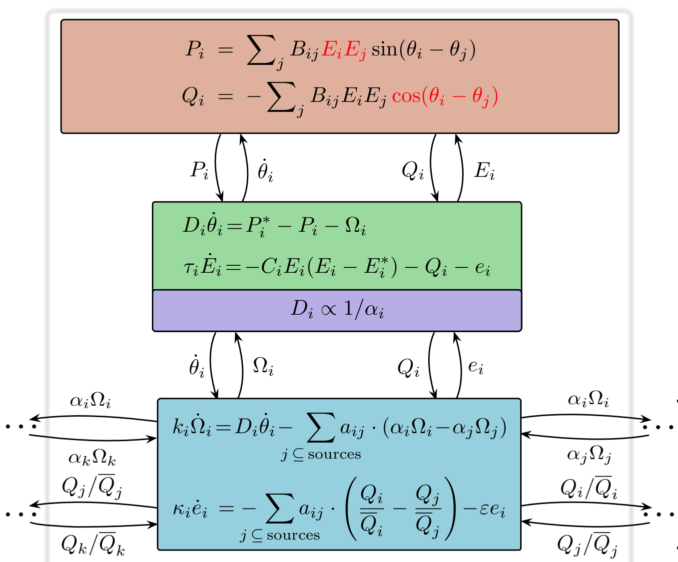
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs ∝ 1 /control gains

Secondary control: diffusive averaging of optimal injections

Plug'n'play architecture

can all be proved also in the coupled case [N. Monshizadeh & C. de Persis, '15]



Power system: physics & loadflow

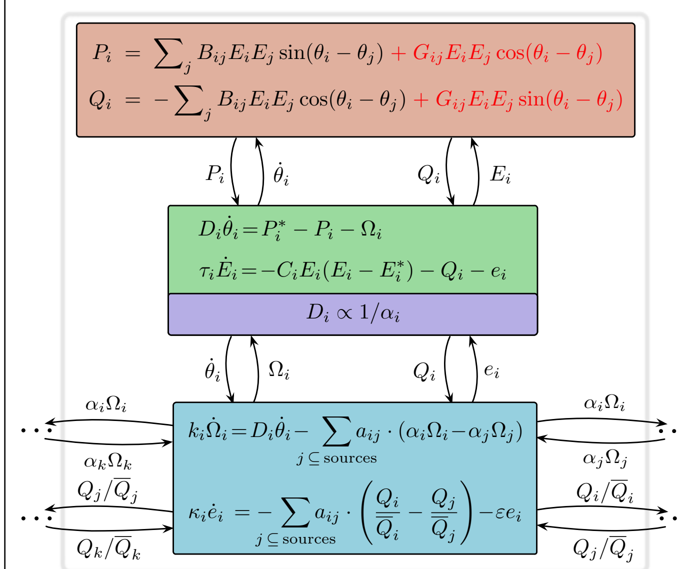
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs ∝ 1 /control gains

Secondary control: diffusive averaging of optimal injections

Plug'n'play architecture

experiments also work well in the lossy case



Power system: physics & loadflow

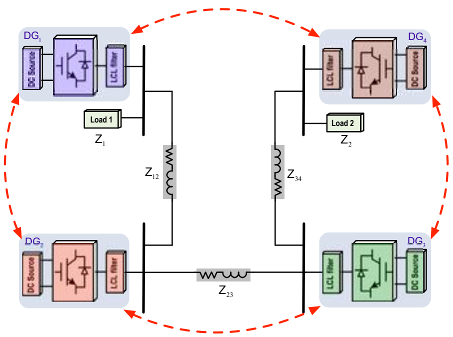
Primary control: mimic oscillators & polyn. symmetry

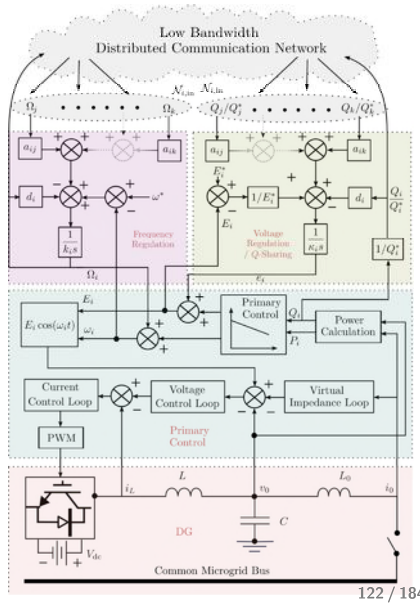
Tertiary control: marginal costs ∝ 1 /control gains

Secondary control: diffusive averaging of optimal injections

Experimental validation of control & opt. algorithms

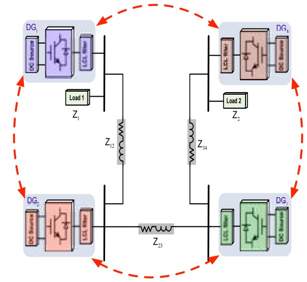
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University

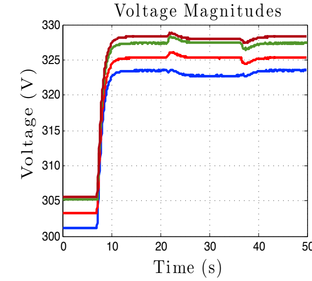


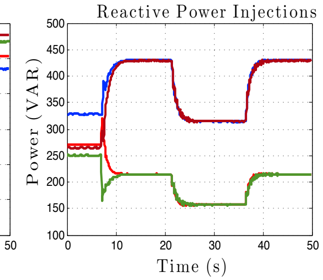


Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing





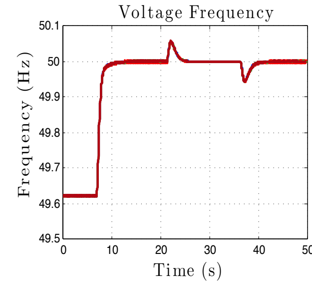


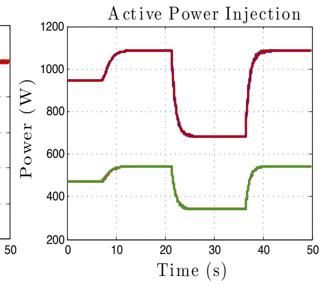
$t \in [0s, 7s]$: primary & tertiary control

$t = 7s$: secondary control activated

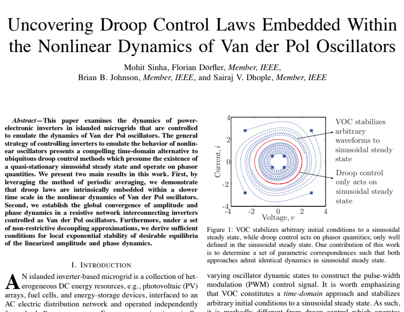
$t = 22s$: load # 2 unplugged

$t = 36s$: load # 2 plugged back

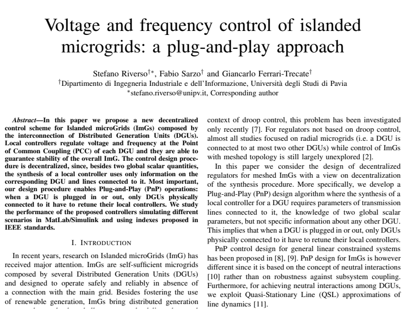




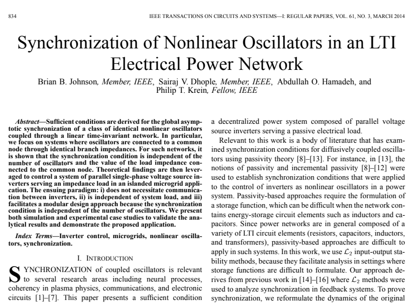
There are also many exciting alternatives to droop control


Uncovering Droop Control Laws Embedded Within the Nonlinear Dynamics of Van der Pol Oscillators

Mohit Sinha, Florian Dörfler, Member, IEEE, Brian B. Johnson, Member, IEEE, and Sairaj V. Dhople, Member, IEEE

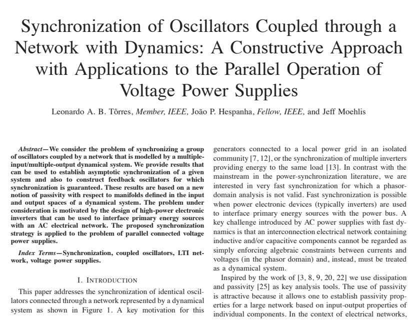

Voltage and frequency control of islanded microgrids: a plug-and-play approach

Stefano Riverso, Fabio Sarzo and Giancarlo Ferrari-Trecate


IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: REGULAR PAPERS, VOL. 61, NO. 3, MARCH 2014

Synchronization of Nonlinear Oscillators in an LTI Electrical Power Network

Brian B. Johnson, Member, IEEE, Sairaj V. Dhople, Member, IEEE, Abdullah O. Hamadeh, and Philip T. Krein, Fellow, IEEE


Synchronization of Oscillators Coupled through a Network with Dynamics: A Constructive Approach with Applications to the Parallel Operation of Voltage Power Supplies

Leonardo A. B. Tôrres, Member, IEEE, João P. Hespanha, Fellow, IEEE, and Jeff Moehlis

(optional material)

what can we do better?

algorithms, detailed models, cyber-physical aspects, ...

many groups out there push all these directions heavily

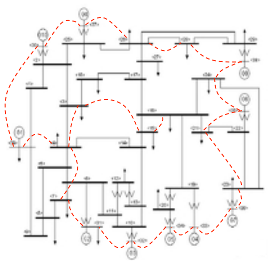
Variation I:

Europe: no centralized dispatch but trade in **energy markets**

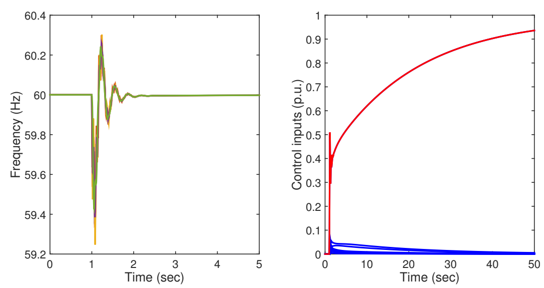
$\Downarrow$

game-theoretic formulation of optimal secondary control

Some strong motivations for game-theoretic perspective



IEEE 39 New England with distributed DAPI control



DAPI control with cheating of generator # 10

A simple (illegal) cheating strategy for generator #10:

1. report wrong injection $u_{10}(t) = 0$ to all neighbors in comm network
2. do not average neighbor values $a_{10,j} = 0$ for all j

$\Rightarrow$ generator #10 alone picks up net load & regulates the frequency

$\Rightarrow$ need an incentive scheme so that everybody plays "best response"

Market formulation of secondary control [FD & S. Grammatico '16]

Competitive spot market:

1. given a prize λ, player i bids
   $u_i^\star = \underset{u_i}{\text{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$
2. market clearing prize $\lambda^\star$ from
   $0 = \sum_i P_i^* + u_i^\star = \sum_i P_i^* + J_i'^{-1}(\lambda^\star)$

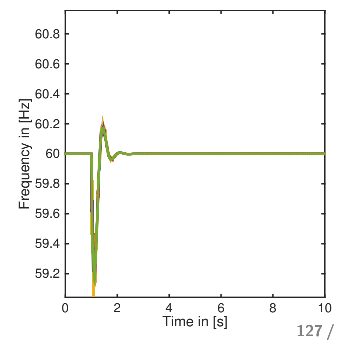
Broadcast controller:

1. convex measurement:
   $k \cdot \dot{\lambda}(t) = \sum_i C_i \dot{\theta}_i(t)$
2. local allocation:
   $u_i(t) = J_i'^{-1}(\lambda(t))$

Auction (dual decomposition):

1. $u_i^+ = \underset{u_i}{\text{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$
2. $\lambda^+ = \lambda - \epsilon \left(\sum_i P_i^* + u_i^+\right) = \lambda - \epsilon \cdot \omega_{\text{sync}}$

$\Rightarrow$ converges to optimal economic dispatch



Variation II:

VOC: virtual oscillator control

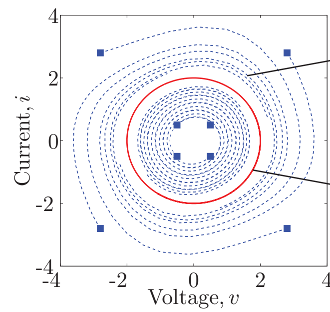
instead of primary droop control

Removing the assumptions of droop control

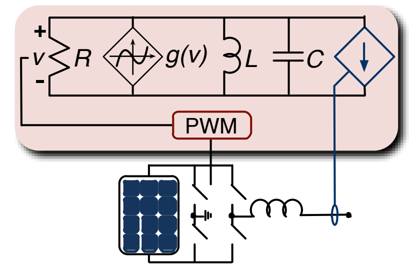
- **idealistic assumptions:** quasi-stationary operation & phasor coordinates

⇒ **future grids:** more power electronics, more renewables, & less inertia

⇒ **Virtual Oscillator Control:** control inverters as limit cycle oscillators
[Torres, Moehlis, & Hespanha '12, Johnson, Dhople, Hamadeh, & Krein '13]

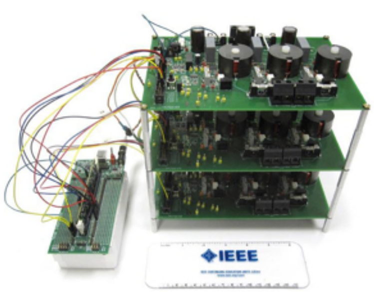


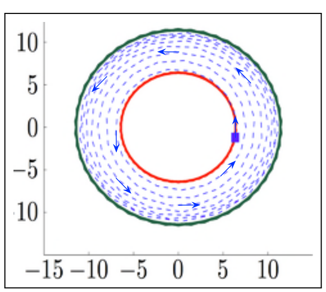
stable sustained oscillations



digitally implemented VOC

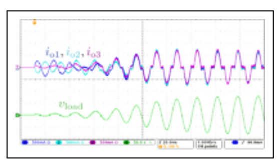
Plug'n'play **V**irtual **O**scillator **C**ontrol (VOC)



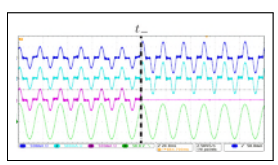


change of setpoint

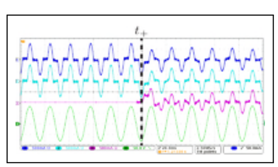
Oscilloscope plots:



emergence of synchrony



removal of inverter



addition of inverter

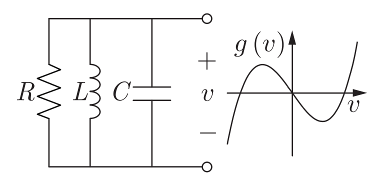
Crash course on planar limit cycle oscillators

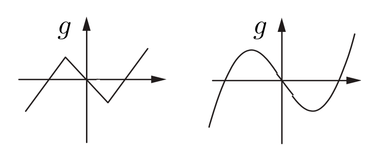
$$L\frac{d}{dt}i = v$$

$$C\frac{d}{dt}v = -Rv - g(v) - i - i_{\text{grid}}$$

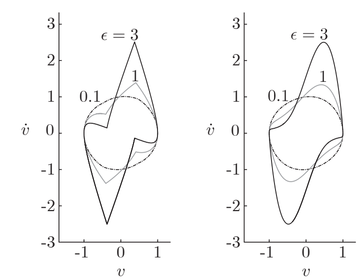
⇒ normalized coordinates

$$\ddot{v} + v + \varepsilon k_1 g'(v) \cdot \dot{v} = \varepsilon k_2 u$$





deadzone Van der Pol



Liénard's limit cycle condition
for virtual oscillator with $u = 0$:

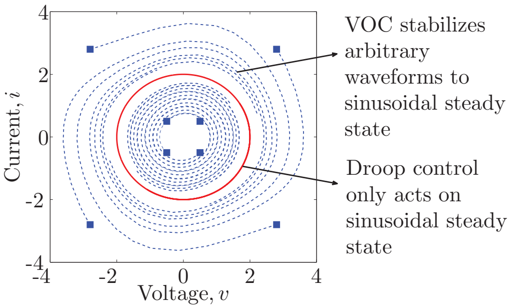
if $\varepsilon = \sqrt{L/C} \to 0$

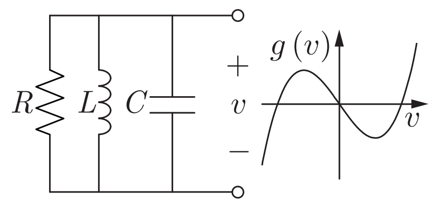
⇒ $\mathcal{O}(\varepsilon)$ close to harmonic oscillator

if damping $g'(v)$ is negative near origin & positive elsewhere

⇒ unique & stable limit cycle

Backward compatibility to droop [M. Sinha, FD, B. Johnson, & S. Dhople, '14]





⇒ transf. to polar coordinates, averaging, & generalized power definitions

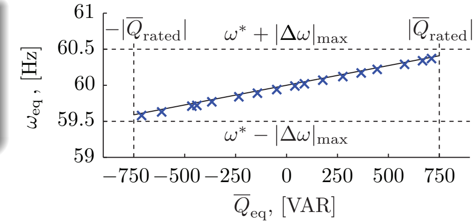
Thm: in vicinity of the limit cycle: **VOC ⊃ droop:**

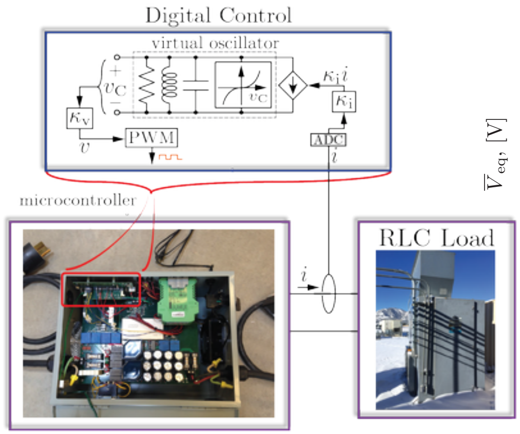
$$\dot{\theta} = constant \cdot (\text{reactive power})$$
$$r - r^* = constant \cdot (P^* - \text{active power})$$

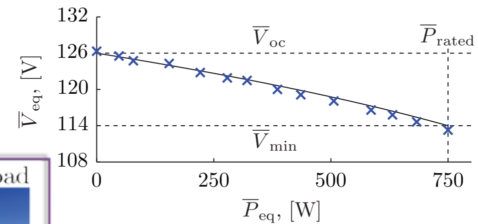
Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

1 VOC ⊃ droop:

$$\dot{\theta} = constant \cdot (\text{reactive power})$$
$$r - r^* = constant \cdot (P^* - \text{active power})$$







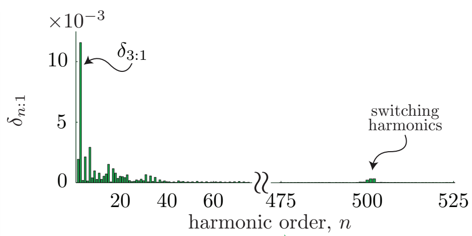
analytic vs. measured droop curves of VOC

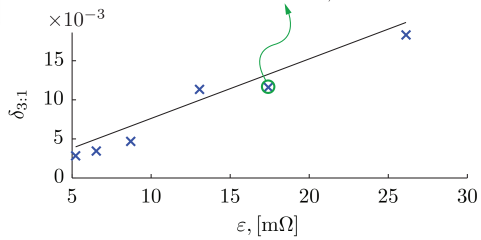
Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

1 VOC ⊃ droop

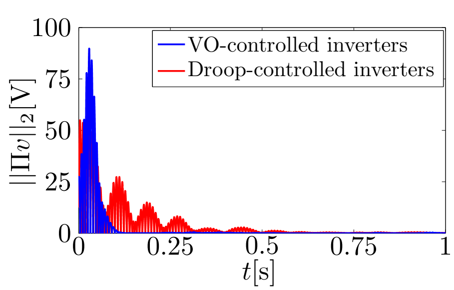
2 VOC $\xrightarrow{\varepsilon \to 0}$ harmonic oscillator
with $\varepsilon/8$ harmonic ratio 3:1

3 VOC: faster & better transients
than droop-controlled inverters





$\varepsilon/8$ harmonic ratio 3:1

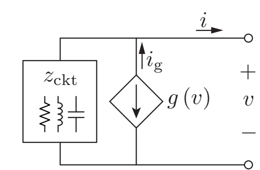


synchronization error: VOC vs. droop

Analysis of VOC system [S. Dhople, B. Johnson, FD, & A. Hamadeh '13]

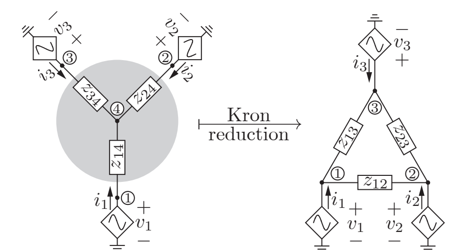
Nonlinear oscillators:

- passive circuit impedance $z_{ckt}(s)$
- active current source $g(v)$



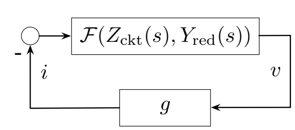
Co-evolving network:

- RLC network & loads are LTI
- Kron reduction: eliminate loads



Stability analysis:

- homogeneity assumption: identical reduced oscillators
- Lure system formulation
- incremental IQC analysis



⇝ sync for strong coupling

Variation III:

can we turn tertiary optimization directly into continuous control?

$\Downarrow$

preview on **online optimization**

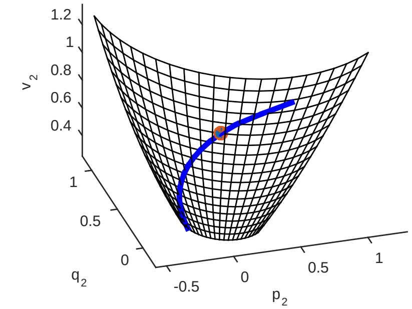
The power flow manifold & linear tangent approximation

node 1 — node 2

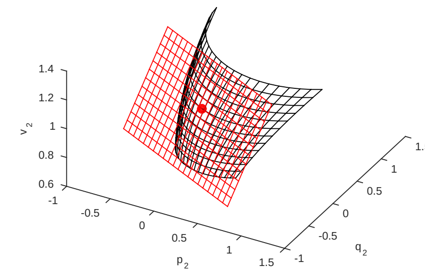
$y = 0.4 - 0.8j$

$v_1 = 1,\ \theta_1 = 0$ $\quad$ $v_2,\ \theta_2$

$p_1,\ q_1$ $\quad$ $p_2,\ q_2$



1. **power flow manifold:** $F(x) = \mathbb{0}$
2. **normal space** spanned by $\frac{\partial F(x)}{\partial x}\Big|_{x^*}$
3. **tangent space**: $\frac{\partial F(x)}{\partial x}\Big|_{x^*}^{T} (x - x^*) = \mathbb{0}$

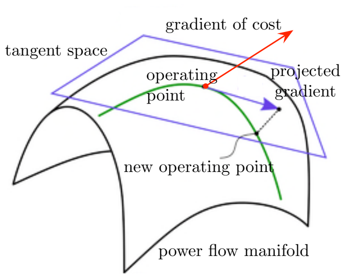


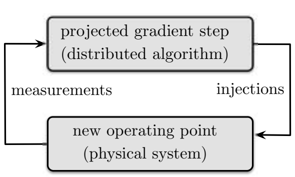
$\Rightarrow$ sparse & implicit model is **structure-preserving** $\rightarrow$ distributed control

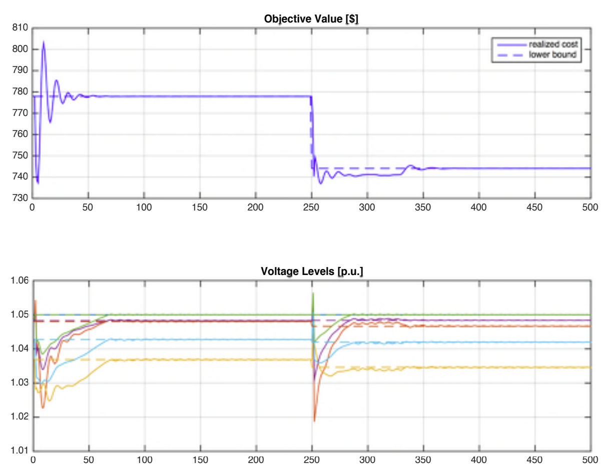
Online optimization on power flow manifold

with Adrian Hauswirth, Saverio Bolognani, & Gabriela Hug

- **manifold optimization** $\rightarrow$ gradient flow on power flow manifold
- **online optimization** $\rightarrow$ controller realizes gradient flow in closed loop



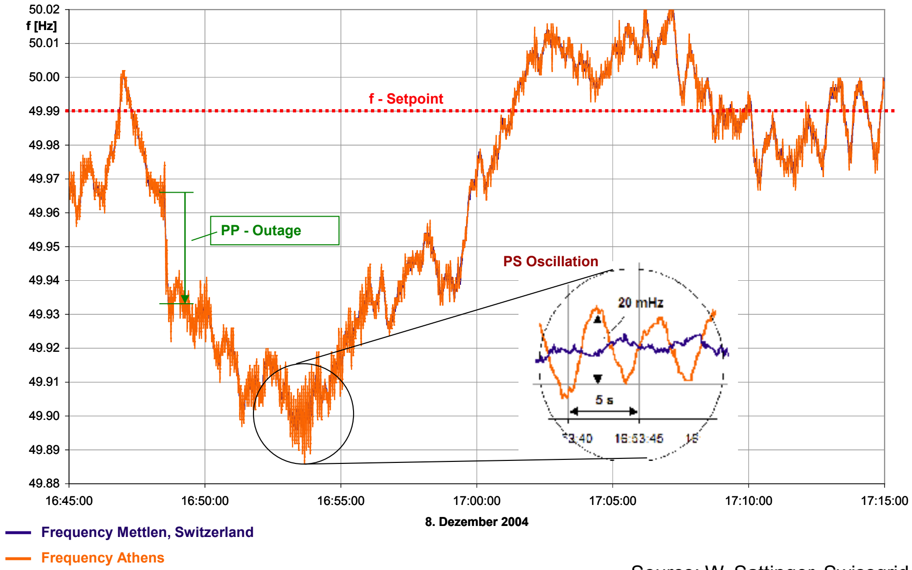




applied to optimal voltage control in IEEE 30 grid

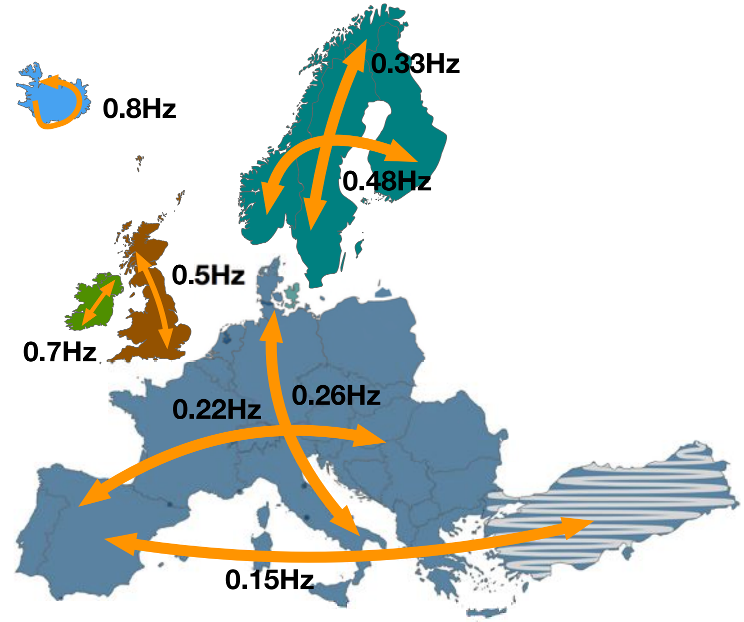
Outline

Frequency time-series reveals inter-area oscillations

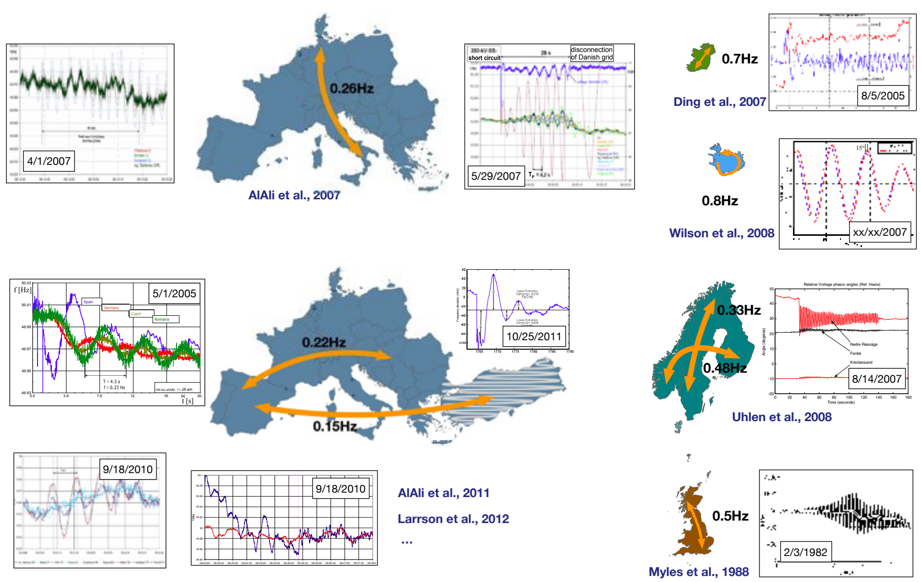


Source: W. Sattinger, Swissgrid

A few typical inter-area oscillations in Europe

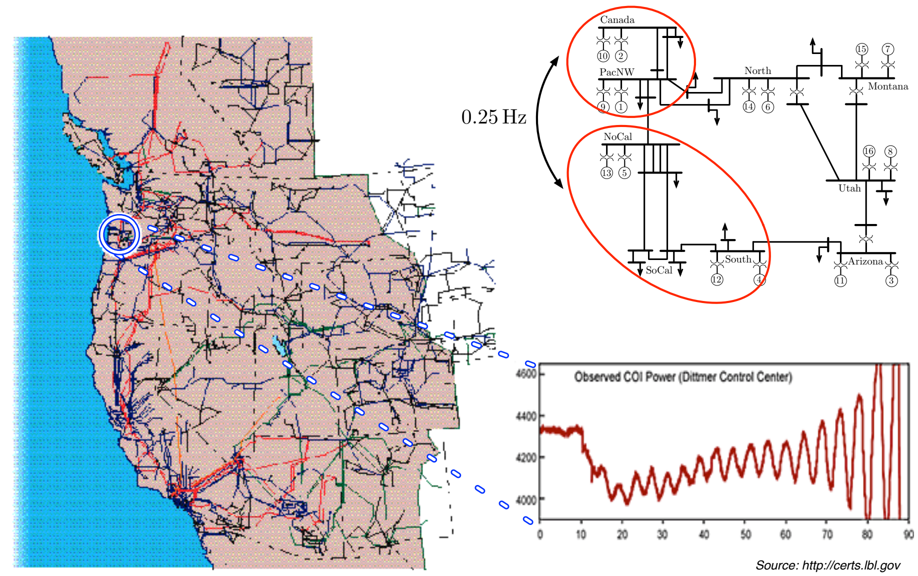


A closer look at some European incidents



Blackout of August 10, 1996

instability of the 0.25 Hz mode in the Western interconnected system



Source: http://certs.lbl.gov

Recent developments putting oscillations in the spotlight

Europe:
- transmission network upgrades & expansion,
- renewable generation in remote locations, &
- deregulated markets ...

United states:
- sparse grid with load & generation hubs,
- aging transmission infrastructure, &
- long power transfers ...


Impact of Increasing Wind Power Generation on the North-South Inter-Area Oscillation Mode in the European ENTSO-E System
Salaheddin Alali, Torsten Haase, Ibrahim Nassar, Harald Weber

Oscillation behaviour of the enlarged European power system under deregulated energy market conditions
M. Kurth, E. Welfonder

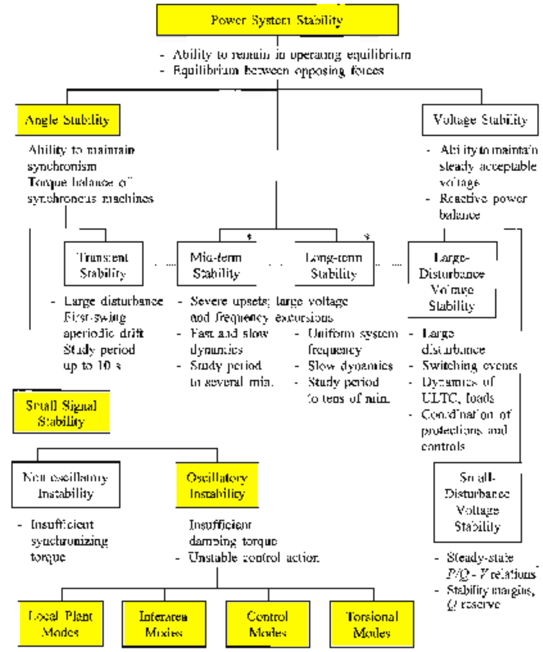
Optimal coordinated control of multiple HVDC links for power oscillation damping based on model identification
Robert Eriksson and Lennart Söder

Impact of long distance power transits on the dynamic security of Large Interconnected Power Systems

Oscillation Behaviour of the Enlarged UCTE Power System Including the Turkish Power System
J. Lehner, T. Weissbach, G. Scheffknecht

Impact of Low Rotational Inertia on Power System Stability and Operation
Andreas Ulbig, Theodor S. Borsche, Göran Andersson

Where are we on the map?



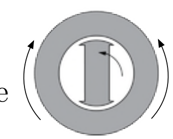
Causes for Oscillations

Why do power systems oscillate?

power network dynamics ≈ coupled, forced, & heterogeneous pendula

generator **torque balance**:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = \text{mech.} - \text{electr. torque}$$

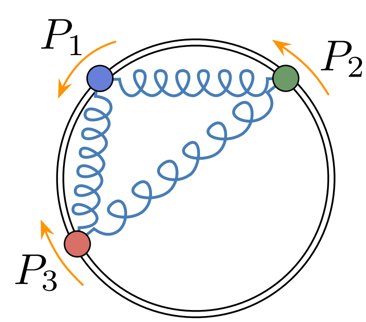


≈ electro-mechanical oscillator

coupled **swing equations**:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$

≈ coupled, forced, & heterogeneous pendula



linearized at equilibrium $(\theta^*, \dot{\theta}^*, P^*)$:

$$M\ddot{\theta} + D\dot{\theta} + L\theta = P$$

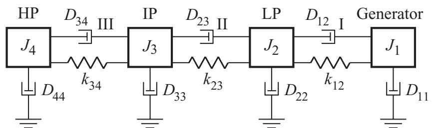
where M, D are inertia and damping matrices & L is network Laplacian

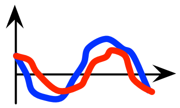
Torsional oscillations in power networks

essentially a (subsynchronous) resonance phenomenon

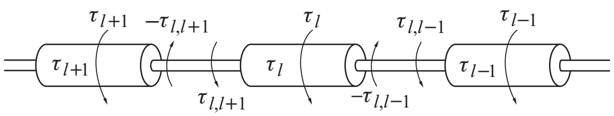
$\Rightarrow$ arise from interplay of

- electrical oscillations
- flexible mechanical shaft models
- generator-turbine coupling





turbine stages generator grid

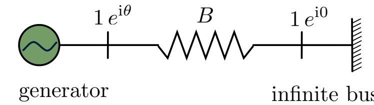


elastic generator shaft as finite-element model

$\Rightarrow$ subsynchronous resonance phenomena often arise in wind turbines

Local oscillations and their control

Automatic Voltage Regulator (AVR):

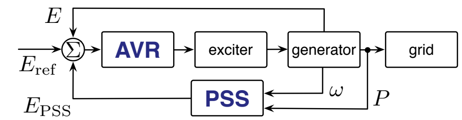


- objective: generator voltage = *const*.

$\Rightarrow$ diminishing damping & sync torque $\frac{\partial P}{\partial \theta}$

$\Rightarrow$ can result in oscillatory instability

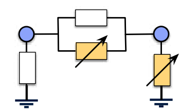
Power System Stabilizer (PSS):



- objective: net damping positive
- typical control design:

$\rightarrow$ low-pass $\rightarrow$ wash-out $\rightarrow$ lead/lag element $\rightarrow$ gain $\rightarrow$

Flexible AC Transmission Systems (FACTS) or HVDC:



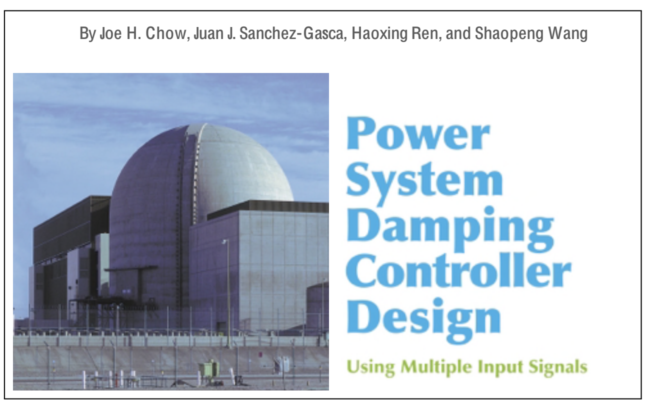
- control by "modulating" transmission line parameters
- either connected in series with a line or as shunt device

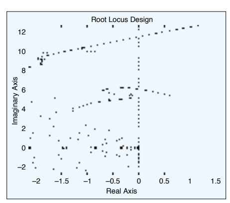
Control-induced oscillations and their control

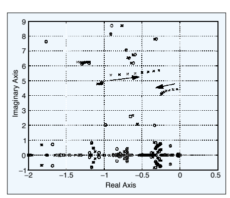
- **short story:** multiple local controllers interact in an adverse way
- **system-theoretic reason:** power system has unstable zeros

$\Rightarrow$ trade-off: high-gain (local stability) vs. low-gain control (avoid zeros)

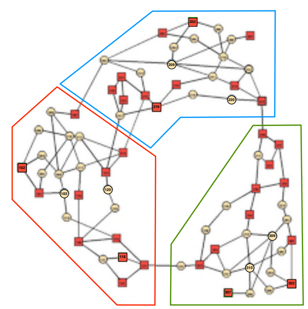
$\Rightarrow$ numerous tuning rules & heuristics for decentralized PSS design



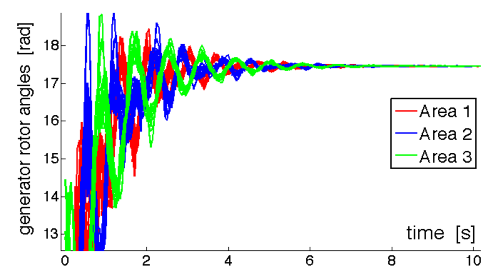




Inter-area oscillations in power networks arise due to



RTS 96 power network



swing dynamics

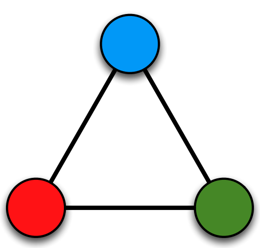
1. **topology**: modular & clustered
2. **heterogeneity** in responses (inertia M_i & damping D_i)
3. **power transfers** between areas (weaken coupling)
4. **interaction** of multiple local controllers

Taxonomy of electro-mechanical oscillations

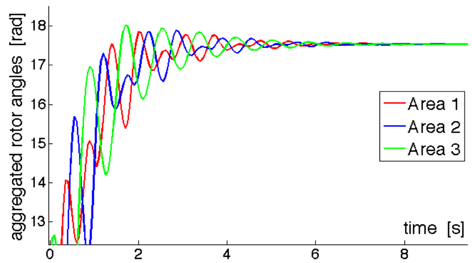
- Synchronous generator = electromech. oscillator ⇒ **local oscillations**:
  - = single generator oscillates relative to the rest of the grid
  - ☹ torsional oscillations induced by mechanical/electrical/flexible coupling
  - ☹ AVR control induces unstable local oscillations
  - ☺ typically damped by local feedback via PSSs
- Power system = complex oscillator network ⇒ **inter-area oscillations**:
  - = groups of generators oscillate relative to each other
  - ☹ poorly tuned local PSSs result in unstable inter-area oscillations
  - ☹ inter-area oscillations are only poorly controllable by local feedback
- Consequences of **recent developments**:
  - ☹ increasing power transfers outpace capacity of transmission system
  - ⇒ ever more lightly damped electromechanical inter-area oscillations
  - ☺ technological opportunities for **wide-area control (WAC)**

Slow Coherency Modeling

Slow coherency and area aggregation



aggregated RTS 96 model



swing dynamics of aggregated model

Aggregate model of lower dimension & with less complexity for

1. analysis and insights into inter-area dynamics [Chow and Kokotovic '85]
2. measurement-based id of equivalent models [Chakrabortty et.al.'10]
3. remedial action schemes [Xu et. al. '11] & wide-area control (later today)

How to find the areas?

a crash course in spectral partitioning

- given: an undirected, connected, & weighted **graph**
- **partition:** $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, and $\mathcal{V}_1, \mathcal{V}_2 \neq \emptyset$
- **cut** is the size of a partition: $J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij}$

⇒ if $x_i = 1$ for $i \in \mathcal{V}_1$ and $x_j = -1$ for $j \in \mathcal{V}_2$, then

$$J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij} = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij}(x_i - x_j)^2 = \frac{1}{2} x^T L x$$

- combinatorial **min-cut** problem: $\text{minimize}_{x \in \{-1,1\}^n \setminus \{-\mathbb{1}_n, \mathbb{1}_n\}} \frac{1}{2} x^T L x$
- **relaxed problem:** $\text{minimize}_{y \in \mathbb{R}^n,\, y \perp \mathbb{1}_n,\, \|y\|_2 = 1} \frac{1}{2} y^T L y$

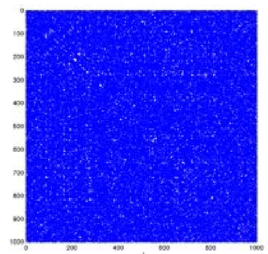
⇒ minimum is *algebraic connectivity* λ_2 and minimizer is *Fiedler vector* v_2

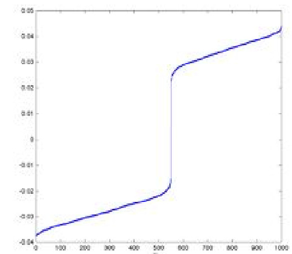
- **heuristic:** $x_i = \text{sign}(y_i)$ ⇒ "spectral partition"

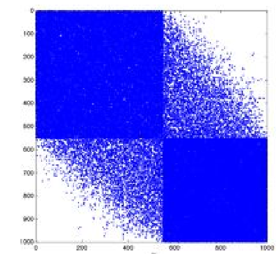
A quick example

```
% choose a graph size
n = 1000;

% randomly assign the nodes to two grous
x = randperm(n);
group_size = 450;
group1 = x(1:group_size);
group2 = x(group_size+1:end);

% assign probabilities of connecting nodes
p_group1 = 0.5;
p_group2 = 0.4;
p_between_groups = 0.1;

% construct adjacency matrix
A(group1, group1) = rand(group_size,group_size) < p_group1;
A(group2, group2) = rand(n-group_size,n-group_size) < p_group2;
A(group1, group2) = rand(group_size, n-group_size) < p_between_groups;
A = triu(A,1); A = A + A';

% can you see the groups?
subplot(1,3,1); spy(A);

% construct Laplacian and its spectrum
L = diag(sum(A))-A;
[V D] = eigs(L, 2, 'SA');

% plot the components of the algebraic connectivity sorted by magnitude
subplot(1,3,2); plot(sort(V(:,2)), '.-');

% partition the matrix accordingly and spot the communities
[ignore p] = sort(V(:,2));
subplot(1,3,3); spy(A(p,p));
```


A quick example – cont'd


adjacency matrix


Fiedler vector v_2

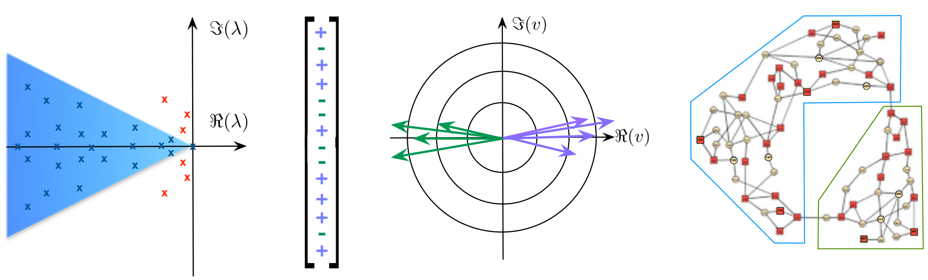

re-arranged adj. matrix

Classical power system partitioning ≈ spectral partitioning

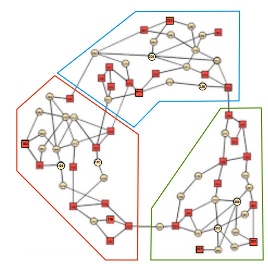
1. construct a linear model $\dot{x} = Ax$
2. recall solution via eigenvalues λ_i and left/right eigenvectors w_i and v_i:

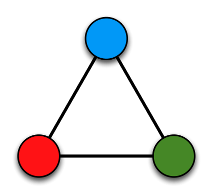
$$x(t) = \sum_i v_i e^{\lambda_i t} \cdot w_i^T x_0 = \sum_i \{\text{mode } \#i\} \cdot \{\text{contribution from } x_0\}$$

3. look at poorly damped complex conjugate mode pairs
4. look at angle & frequency components of eigenvectors
5. group the generators according to their polarity in eigenvectors



Setup in slow coherency


original model


aggregated model

- network r given **areas**
  (from spectral partition [Chow et al. '85 & '13])
- small **sparsity parameter**:

$$\delta = \frac{\max_\alpha(\Sigma \text{ external connections in area } \alpha)}{\min_\alpha(\Sigma \text{ internal connections in area } \alpha)}$$

- **inter-area dynamics** by center of inertia:

$$y_\alpha = \frac{\sum_{i\in\alpha} M_i \theta_i}{\sum_{i\in\alpha} M_i}, \quad \alpha \in \{1,\dots,r\}$$

- **intra-area dynamics** by area differences:

$$z_{i-1}^\alpha = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1,\dots,r\}$$

Linear transformation & time-scale separation

Swing equation $\Longrightarrow$ singular perturbation standard form

$$M\ddot{\theta} + D\dot{\theta} + L\theta = 0 \quad \Longrightarrow \quad \left\{ \frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \right.$$

Slow motion given by center of inertia:

$$y_\alpha = \frac{\sum_{i\in\alpha} M_i \theta_i}{\sum_{i\in\alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

Fast motion given by intra-area differences:

$$z_{i-1}^{\alpha} = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

Slow time scale: $t_s = \delta \cdot t \cdot$ "max internal area degree"

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\, \ddot{\varphi} + D_a\, \dot{\varphi} + L_{\mathrm{red}}\, \varphi = 0$$

Properties of aggregated model [D. Romeres, FD, & F. Bullo, '13]

1. $M_a = \begin{bmatrix} \ddots & & \\ & \sum_{i\in\alpha} M_i & \\ & & \ddots \end{bmatrix}$ and $D_a = \begin{bmatrix} \ddots & & \\ & \sum_{i\in\alpha} D_i & \\ & & \ddots \end{bmatrix}$
2. L_{red} = "inter-area Laplacian" + "intra-area contributions"
   = positive semidefinite Laplacian with possibly negative weights

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\, \ddot{\varphi} + D_a\, \dot{\varphi} + L_{\mathrm{red}}\, \varphi = 0$$

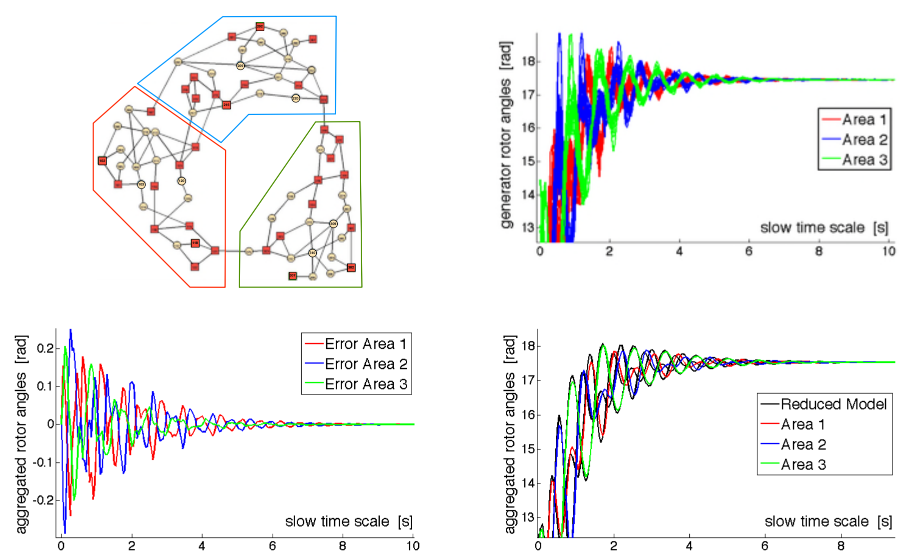
Singular perturbation approximation [D. Romeres, FD, & F. Bullo, '13]

There exist δ^* sufficiently small such that for $\delta \leq \delta^*$ and for all $t > 0$:

$$\begin{bmatrix} y(t_s) \\ \dot{y}(t_s) \end{bmatrix} = \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}), \quad \begin{bmatrix} z(t_s) \\ \dot{z}(t_s) \end{bmatrix} = \tilde{A} \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}).$$

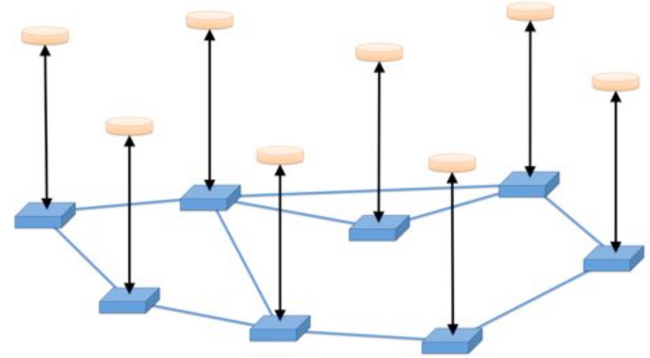
center of inertia $\approx$ solution of aggregated swing equation

RTS 96 swing dynamics revisited



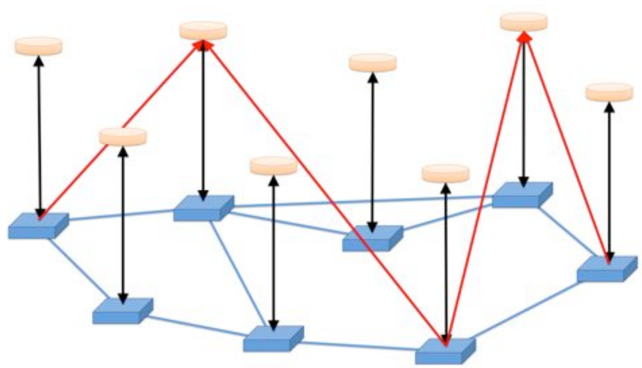
Inter-Area Oscillations & Wide-Area Control

Remedies against electro-mechanical oscillations
conventional control



- blue layer: interconnected generators
- fully decentralized control implemented locally
  - ☺ effective against local oscillations
  - ☹ ineffective against inter-area oscillations

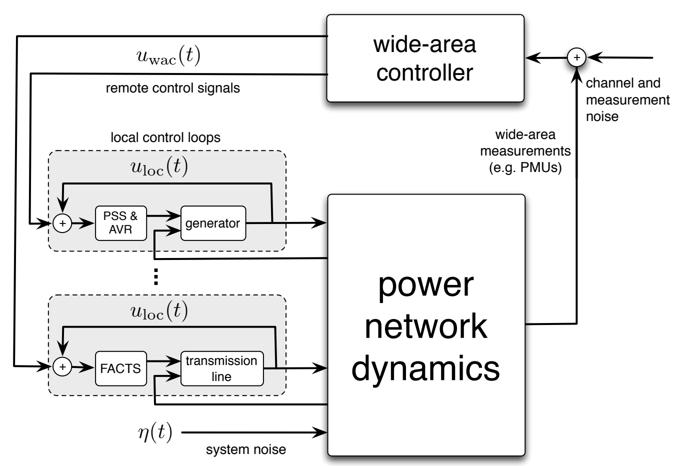
Remedies against electro-mechanical oscillations
wide-area control (WAC)



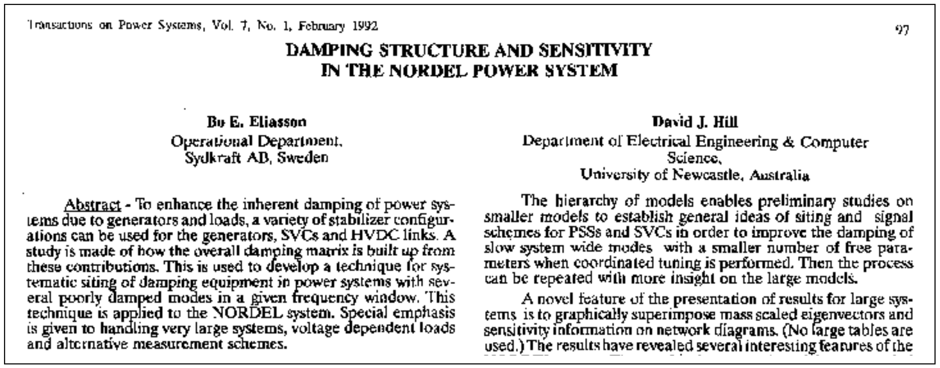
- blue layer: interconnected generators
- fully decentralized control implemented locally
- distributed wide-area control using remote signals

Setup in wide-area control

1. remote control signals & remote measurements (e.g., PMUs)
2. excitation (PSS & AVR) and power electronics (FACTS) actuators
3. communication backbone network



Debated: do we need **distributed** wide-area control or can we get away with **fully decentralized** control?


Transactions on Power Systems, Vol. 7, No. 1, February 1992 97

DAMPING STRUCTURE AND SENSITIVITY IN THE NORDEL POWER SYSTEM

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Sydkraft AB, Sweden

David J. Hill
Department of Electrical Engineering & Computer Science,
University of Newcastle, Australia

Abstract - To enhance the inherent damping of power systems due to generators and loads, a variety of stabilizer configurations can be used for the generators, SVCs and HVDC links. A study is made of how the overall damping matrix is built up from these contributions. This is used to develop a technique for systematic siting of damping equipment in power systems with several poorly damped modes in a given frequency window. This technique is applied to the NORDEL system. Special emphasis is given to handling very large systems, voltage dependent loads and alternative measurement schemes.

The hierarchy of models enables preliminary studies on smaller models to establish general ideas of siting and signal schemes for PSSs and SVCs in order to improve the damping of slow system wide modes with a smaller number of free parameters when coordinated tuning is performed. Then the process can be repeated with more insight on the large models.

A novel feature of the presentation of results for large systems is to graphically superimpose mass scaled eigenvectors and sensitivity information on network diagrams. (No large tables are used.) The results have revealed several interesting features of the

"The above reasoning implies that if observability is small, so is also controllability. The benefits of remote signals for power system damping should thus be marginal." [follow-up comments by G. Andersson & T. Smed, '92]

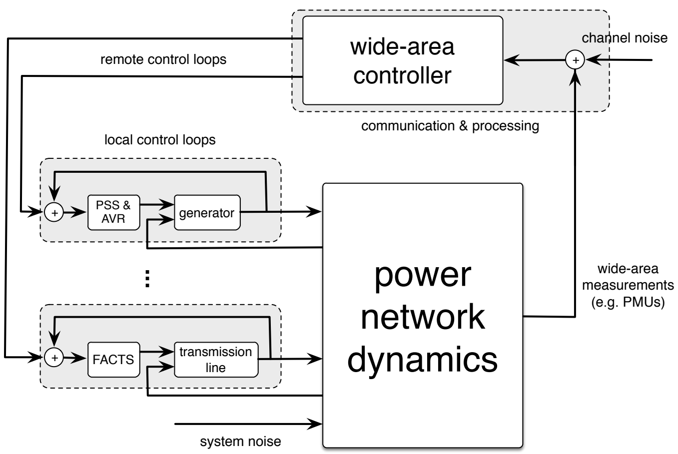
conventional analysis & wide-area control

(based on spectral methods)

I will be a little provocative . . .

Canonical setup in wide-area control

local actuators, remote measurements, & communication backbone

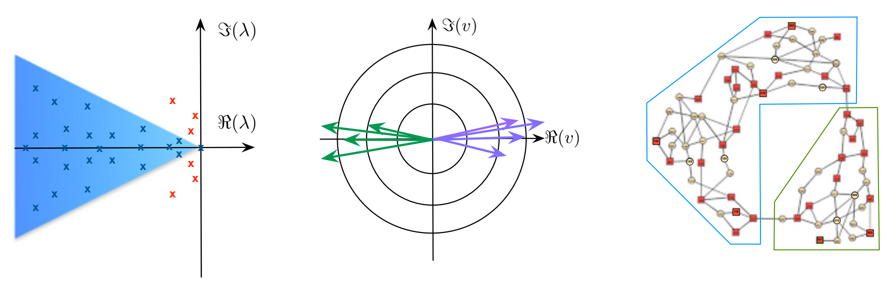


⇒ **problem I:** signal selection (sensors & actuators)

⇒ **problem II:** WAC design (subject to control signals)

Recall: spectral analysis reveals critical modes & areas

1. recall solution of $\dot{x} = Ax$: $x(t) = \sum_i \underbrace{v_i e^{\lambda_i t}}_{\text{mode } \#i} \cdot \underbrace{w_i^T x_0}_{\text{contribution from } x_0}$
2. analyze eigenvectors & participation factors of weakly damped modes
3. spectral partitioning reveals coherent groups in eigenvectors polarities



Which sensors and actuators ?

1. **Linear control system:** $\dot{x} = Ax + Bu \ , \ y = Cx$
   - B with column b_j = control location $\#j$
   - C with row c_j^T = sensor location $\#j$
   - A: eigenvalues λ_i and orthonormal right & left eigenvectors v_i & w_i^*
2. **Diagonalization:** $x = Vz = \begin{bmatrix} v_1 & \dots & v_n \end{bmatrix} z \ , \ z = Wx = \begin{bmatrix} w_1 & \dots & w_n \end{bmatrix}^* x$

$$\Rightarrow \dot{z} = \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{=WAV} z + \underbrace{\begin{bmatrix} & \vdots & \\ \dots & w_i^* b_j & \dots \\ & \vdots & \end{bmatrix}}_{=WB} u \ , \ y = \underbrace{\begin{bmatrix} & \vdots & \\ \dots & c_i^* v_j & \dots \\ & \vdots & \end{bmatrix}}_{=CV} uz$$

3. Controllability of mode i by input $j \triangleq \cos\left(\angle(w_i \, , \, b_j)\right) = \frac{w_i^* b_j}{\|w_i\| \|b_j\|}$
4. Observability of mode i by sensor $j \triangleq \cos\left(\angle(c_i \, , \, v_j)\right) = \frac{c_i^* v_j}{\|c_i\| \|v_j\|}$

Modal signal selection metrics

Assessment of Two Methods to Select Wide-Area Signals for Power System Damping Control

Annissa Heniche, *Member, IEEE*, and Innocent Kamwa, *Fellow, IEEE*

***Abstract*—In this paper, two different approaches are applied to the Hydro-Québec network in order to select the most effective signals to damp inter-area oscillations. The damping is obtained by static var compensator (SVC) and synchronous condenser (SC) modulation. The robustness analysis, the simulations, and statistical results show, unambiguously, that in the case of wide-area signals, the geometric approach is more reliable and useful than the residues approach. In fact, this study shows that the best robustness and performances are always obtained with the stabilizer configuration using the signals recommended by the geometric approach. In addition, the results confirm that wide-area control is more effective than local control for damping inter-area oscillations.**

the results concern only the Hydro-Québec network, it is important to notice that a statistical analysis was realized. This analysis allowed the test of the two approaches using 1140 different configurations of the network.

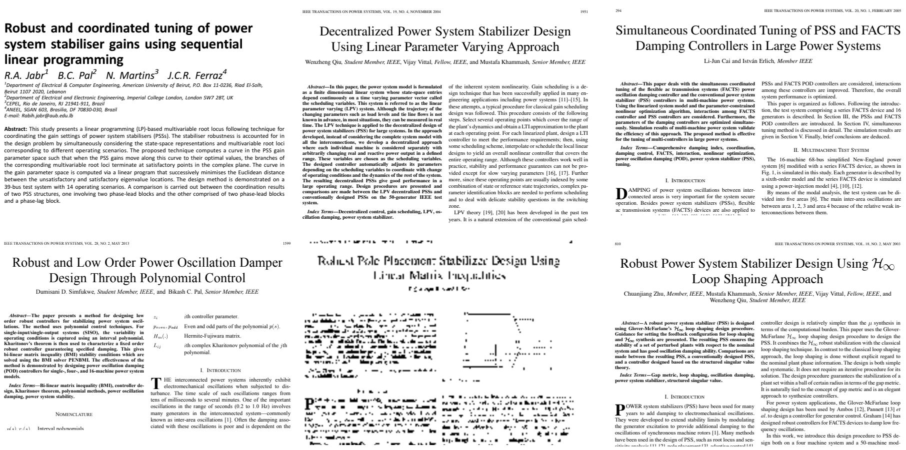
The aims of this paper are on one hand to show that the two measures do not provide the same conclusion in terms of control loop selection and on the other hand to demonstrate the efficiency and reliability of one measure in comparison to the other. To do that, the two measures were applied in order to select the most effective control loops for damping the 0.6-Hz inter-area

1. **geometric criteria** [H.M.A. Hamdan & A.M.A. Hamdan '87]:
   - e.g., modal controllability: effect of control input $\#j$ on mode $\#i$
2. **frequency criteria** [M. Tarokh '92]: modal residues of transfer function

$\Rightarrow$ suboptimal procedures with many requirements: (i) identification of critical modes, (ii) sensor/actuator catalog, (iii) **combinatorial** evaluation

Decentralized WAC control design ...

- ...subject to structural constraints is **tough**
- ...usually handled with **suboptimal heuristics** in MIMO case



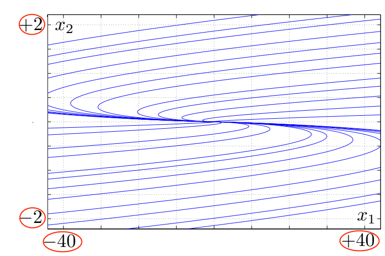
$\Rightarrow$ signal selection is combinatorial & control design is suboptimal

Challenges in wide-area control

1. **signal selection** is combinatorial
2. **decentralized control** is suboptimal
3. **identification** of critical modes is somewhat *ad hoc*

What information is contained in the spectrum of a *non-normal* matrix?

Example: $\dot{x} = \begin{bmatrix} -1 & 10^2 \\ 0 & -1 \end{bmatrix} x$



Today [X. Wu, FD, & M. Jovanovic '15]:

$\Rightarrow$ performance metric: variance amplification of stochastic system

$\Rightarrow$ simultaneously optimize performance & control architecture

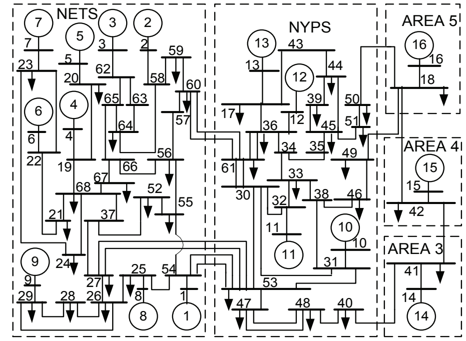
$\Rightarrow$ fully decentralized & nearly optimal controller

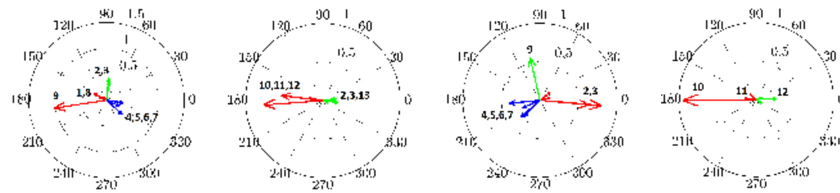
running case study:

New England – New York

Case study: New England – New York test system

- **model features** (242 states):
  - sub-transient generator models [Singh et. al. '14]
  - open loop is unstable
  - exciters & tuned PSSs
- frequency & damping ratios of dominant **inter-area modes**





1.1Hz @ 3.8% 1.3Hz @ 4.2% 1.1Hz @ 4.7% 1.3Hz @ 4.9%

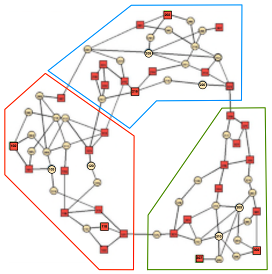
variance amplification as performance metric

$$\int_0^\infty x(t)^T Q\, x(t)\, dt$$

Primer on $\mathcal{H}_2$ - **norms**

Slow coherency performance objectives

- recall **sources** for inter-area oscillations:



- linearized **swing equation**: $M\ddot{\theta} + D\dot{\theta} + L\theta = P$
- **mechanical energy**: $\frac{1}{2}\,\dot{\theta} M \dot{\theta} + \frac{1}{2}\,\theta^T L \theta$
- **heterogeneities** in topology, power transfers, & machine responses (inertia & damp)

$\Rightarrow$ performance **objective** = energy of homogeneous network:

$$x^T Q\, x \;=\; \dot{\theta}^T M\, \dot{\theta} \;+\; \theta^T \big(I_n - (1/n)\cdot \mathbb{1}_{n\times n}\big)\,\theta$$

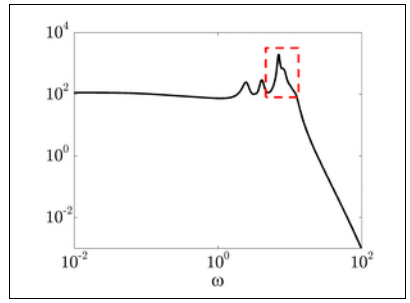
- other choices possible: center of inertia, inter-area differences, etc.

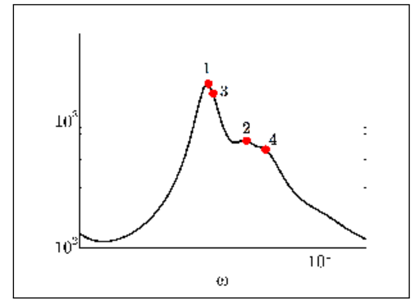
Input-output analysis in $\mathcal{H}_2$ - metric

- linear system with **white noise input**: $\dot{x} = Ax + B_1\eta$
- energy of homogeneous network as **performance output**: $z = Q^{1/2}x$
- **steady-state variance** of the output is given by the $\mathcal{H}_2$-norm

$$\|G\|^2_{\mathcal{H}_2} \;:=\; \lim_{t\to\infty} \mathbb{E}\left(x(t)^T\, Q\, x(t)\right) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} \|G(\mathrm{j}\,\omega)\|^2_{\mathrm{HS}}\,\mathrm{d}\omega$$

- **power spectral density** $\|G(\mathrm{j}\,\omega)\|^2_{\mathrm{HS}}$ reveals NE-NY inter-area modes





$\mathcal{H}_2$ - norms for consensus-like systems

see exercise

sparsity-promoting optimal control

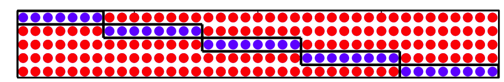
Primer on **L**inear **Q**uadratic **C**ontrol (**LQR**)

Optimal linear quadratic regulator (LQR)

- **model:** linearized ODE dynamics $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t)$
- **control:** memoryless linear state feedback $u = -Kx(t)$
- **optimal centralized control** with quadratic $\mathcal{H}_2$ - performance index:

$$\begin{aligned} \text{minimize} \quad & J(K) \triangleq \lim_{t\to\infty} \mathbb{E}\left\{x(t)^T Qx(t) + u(t)^T Ru(t)\right\} \\ \text{subject to} \quad & \\ \text{linear dynamics:} \quad & \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ \text{linear control:} \quad & u(t) = -Kx(t), \\ \text{stability:} \quad & (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

(no structural constraints on K)



Sparsity-promoting optimal LQR

[Lin, Fardad, & Jovanović, '13]

simultaneously optimize performance & architecture

$$\begin{aligned} \text{minimize} \quad & \lim_{t\to\infty} \mathbb{E}\left\{x(t)^T Qx(t) + u(t)^T Ru(t)\right\} + \gamma \cdot \text{card}(K) \\ \text{subject to} \quad & \\ \text{linear dynamics:} \quad & \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ \text{linear control:} \quad & u(t) = -Kx(t), \\ \text{stability:} \quad & (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

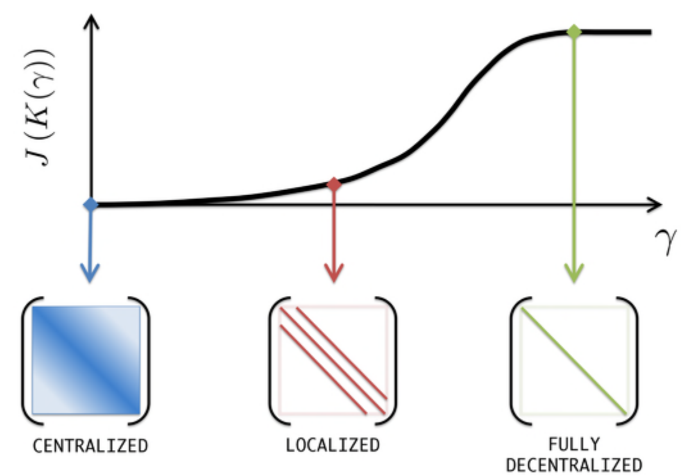
⇒ for $\gamma = 0$: standard optimal control (typically not sparse)

⇒ for $\gamma > 0$: sparsity is promoted (problem is combinatorial)

⇒ card(K) convexified by weighted ℓ_1-norm $\sum_{i,j} w_{ij}|K_{ij}|$

Parameterized family of feedback gains

$$K(\gamma) = \arg\min_K \left(J(K) + \gamma \cdot \textstyle\sum_{i,j} w_{ij}|K_{ij}|\right)$$

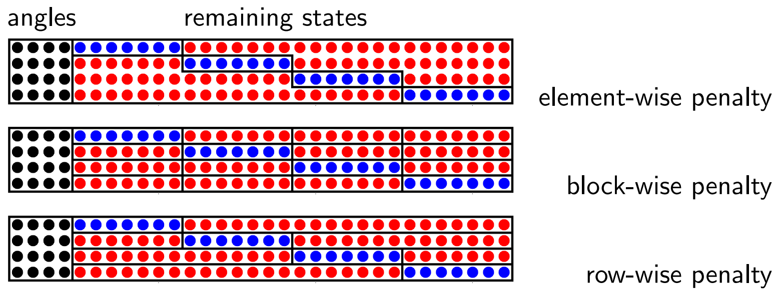


Algorithmic approach in an nutshell (detailed in back-up slides)

1. **Algebraic formulation** via Gramian and Lyapunov equation
2. **Non-convexity** in K: use homotopy path in γ & ADMM

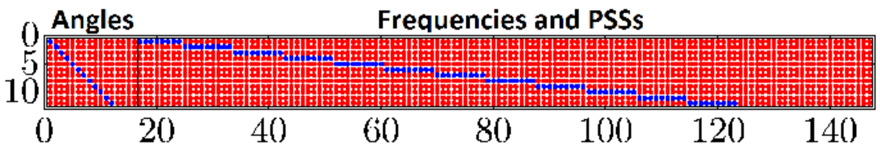
---

3. **Rotational symmetry:** remove absolute angle by COI transformation
4. **Block/row-sparsity**-promoting optimal control

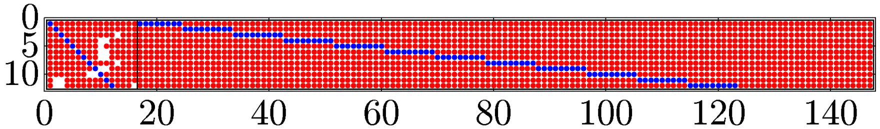


sparsity-promoting control of inter-area oscillations

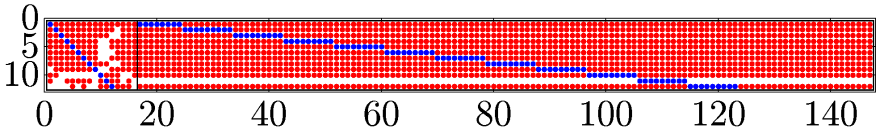
Sparsity-promoting control architecture



$\gamma = 0$, **card** $(K) = 1764$

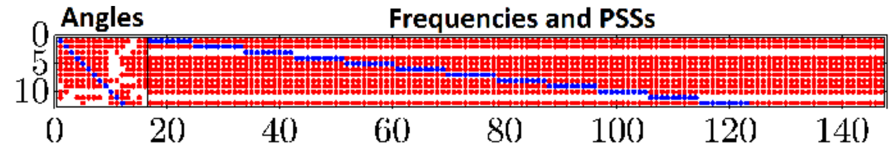


$\gamma = 10^{-4}$, **card** $(K) = 1746$

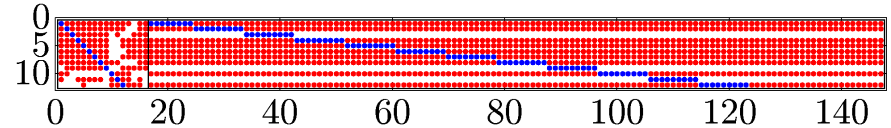


$\gamma = 0.00015$, **card** $(K) = 1603$

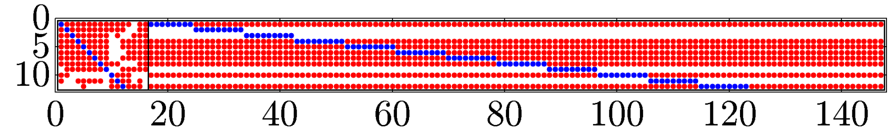
Sparsity-promoting control architecture



$\gamma = 0.00023$, **card** $(K) = 1475$

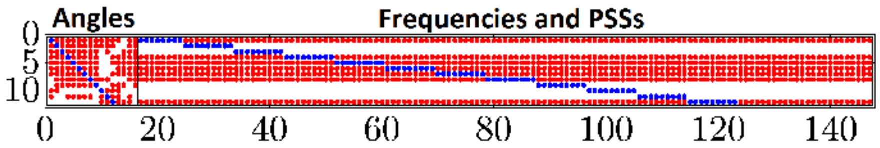


$\gamma = 0.00031$, **card** $(K) = 1353$

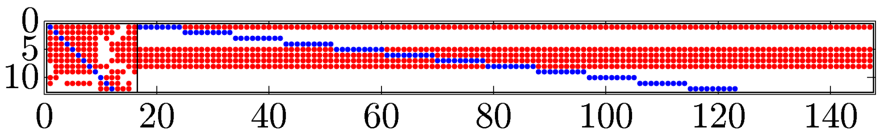


$\gamma = 0.00041$, **card** $(K) = 1231$

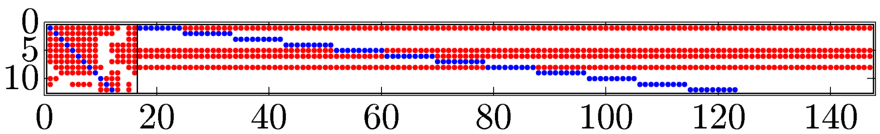
Sparsity-promoting control architecture



$\gamma = 0.00047$, **card** $(K) = 1106$

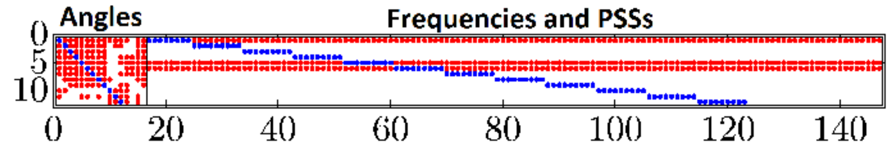


$\gamma = 0.00054$, **card** $(K) = 862$

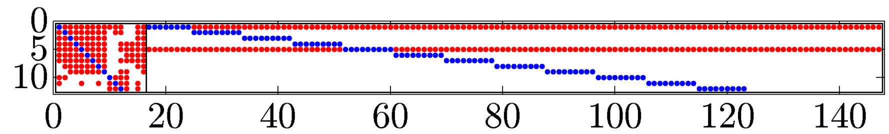


$\gamma = 0.00063$, **card** $(K) = 733$

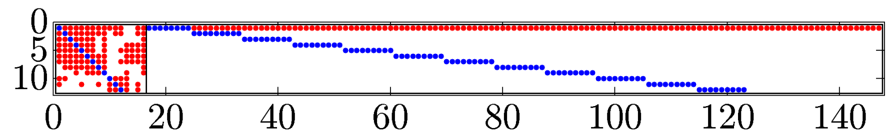
Sparsity-promoting control architecture



$\gamma = 0.00095$, **card** $(K) = 609$

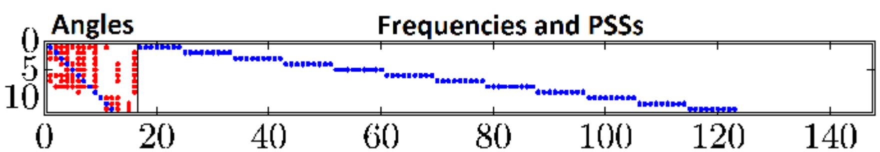


$\gamma = 0.0011$, **card** $(K) = 484$

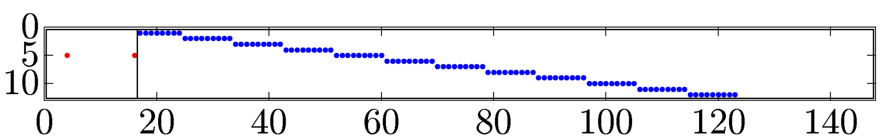


$\gamma = 0.0015$, **card** $(K) = 353$

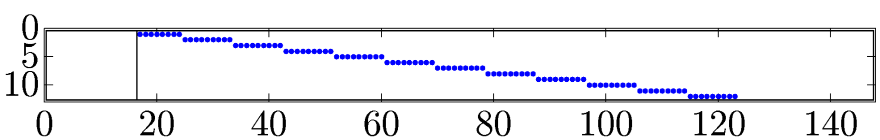
Sparsity-promoting control architecture



$\gamma = 0.0060$, **card** $(K) = 191$



$\gamma = 0.0655$, **card** $(K) = 109$

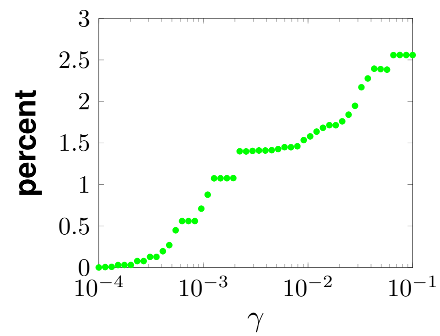


$\gamma = 0.1$, **card** $(K) = 107$

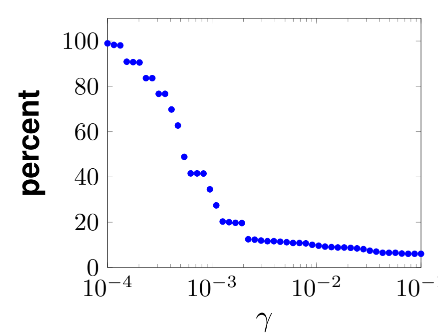
Performance vs. sparsity

$Q =$ energy of homogeneous network , $R = I_n$, $\gamma \in [10^{-4}, 0.1]$

$(J - J_c)/J_c$



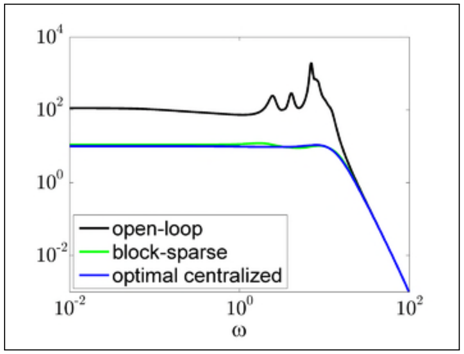
card (K) / **card** (K_c)



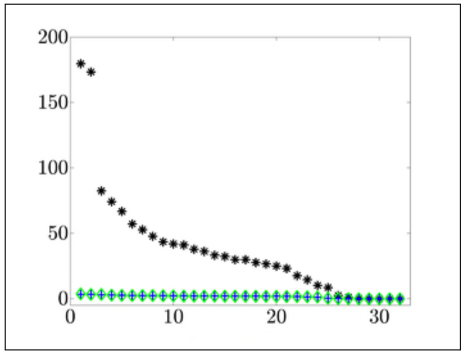
$\gamma = 0.1 \Rightarrow \begin{cases} 2.6\,\% & \text{relative performance loss} \\ 6.1\,\% & \text{non-zero elements in } K \end{cases}$

$\Rightarrow$ fully decentralized control is nearly optimal **!**

Performance comparison of different approaches



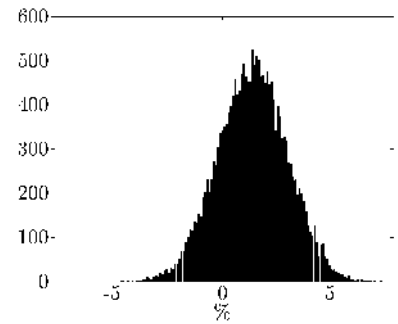
power spectral density



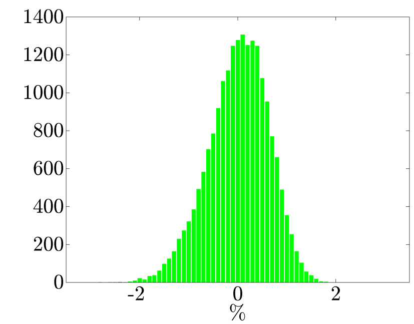
spectrum of covariance matrix

Robustness: optimal control reduces sensitivity

nominal controller applied to 20, 000 operating points with ±20% randomized loading



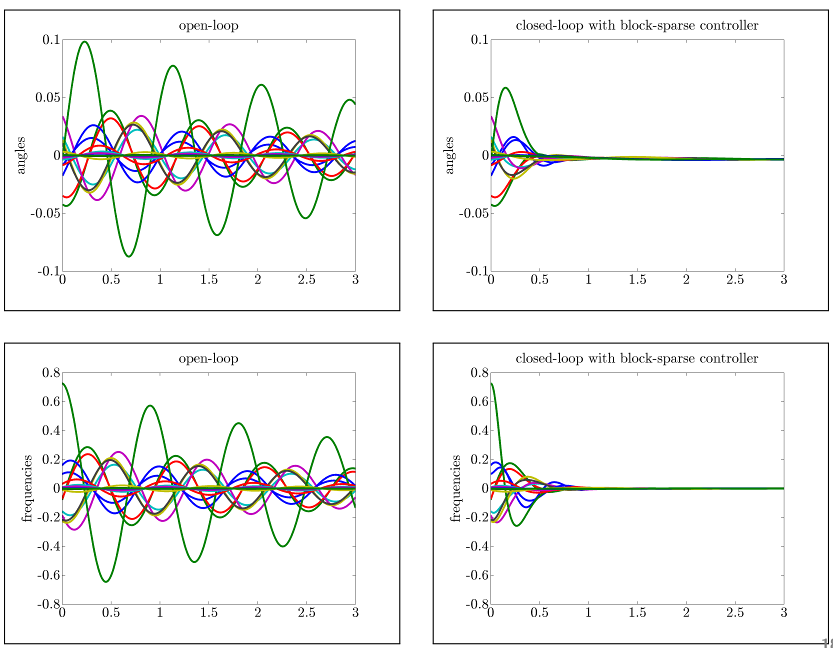
open-loop system



block-sparse controller

⇒ optimal (decentralized) control reduces sensitivity

Eye candy: time-domain simulations



Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

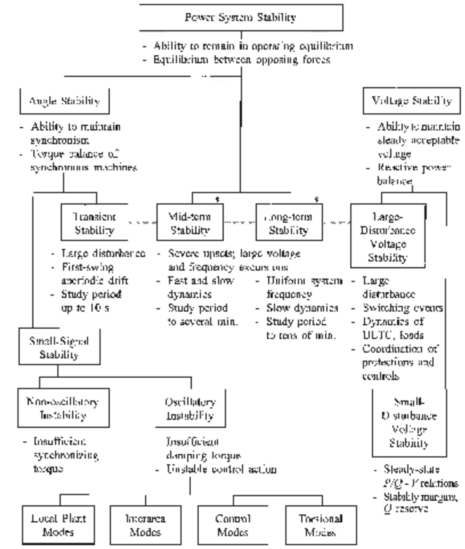
Conclusions

Looking for data, toolboxes, & test cases

- **Matpower (static)** for (optimal) power flow & static models
  `http://www.pserc.cornell.edu//matpower/`
- **Matpower (dynamic)** with generator models
  `http://www.kios.ucy.ac.cy`
- **Power System Toolbox** for dynamics & North American models
  `http://www.eps.ee.kth.se/personal/vanfretti/pst/Power_System_Toolbox_Webpage/PST.html`
- **IEEE Task Force PES PSDPC SCS**: New York, Brazil, Australian grids etc.; `http://www.sel.eesc.usp.br/ieee/`
- **ObjectStab** for Modelica for dynamics & models
  `https://github.com/modelica-3rdparty/ObjectStab`
- **More freeware:** MatDyn, PSAT, THYME, Dome, . . .
  `http://ewh.ieee.org/cmte/psace/CAMS_taskforce/`
- **Other:** many test cases in papers, reports, task forces, . . .

Conclusions

Obviously, there is a lot more . . .



I hope I could give you a little insight into a few interesting problems.

final words of wisdom