

Power System Stability & Control

Florian Dörfler

Andrej Jokić



dutch institute of systems and control

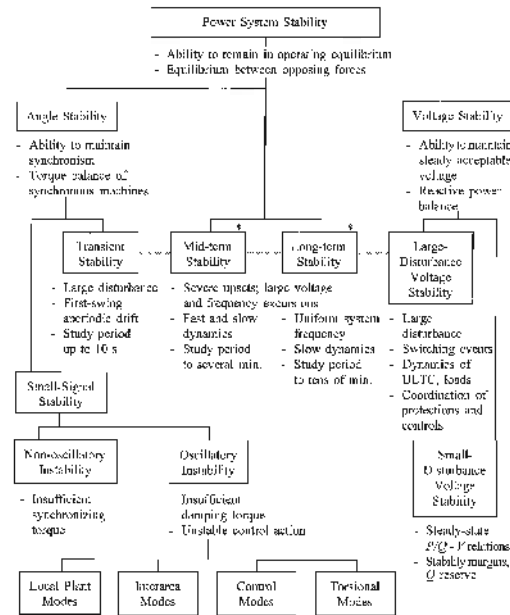


Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich



University of Zagreb

Power system stability & control: have to choose based on



- what Andrej needs
- what I actually know well
- what is interesting from a network perspective rather than from device perspective
- what is relevant for future (smart) power grids with high renewable penetration
- what gives rise to fun distributed control problems
- what you are interested in

Tentative outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

my particular focus is on **networks**

Disclaimers

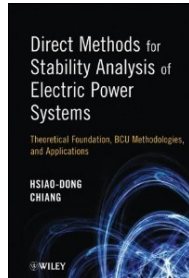
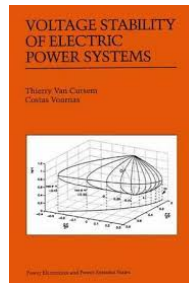
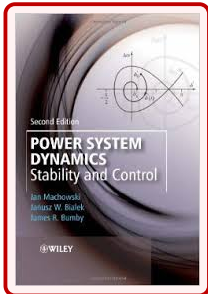
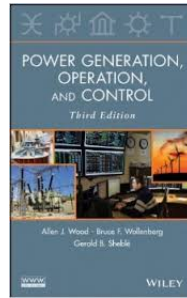
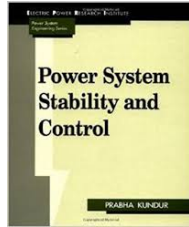
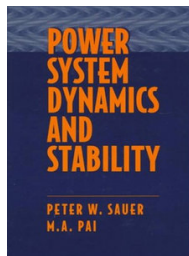
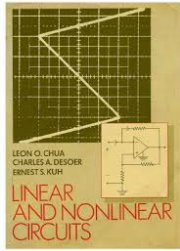
- start off with “boring” modeling before more “sexy” topics
 - start off with basic material & before “cutting edge” work
 - focus on simple models and physical & math intuition
- ⇒ cover fundamentals, convey intuition, & give references for the details

Please ...

- ▶ ask me for further reading about any topic,
- ▶ and interrupt & correct me anytime.

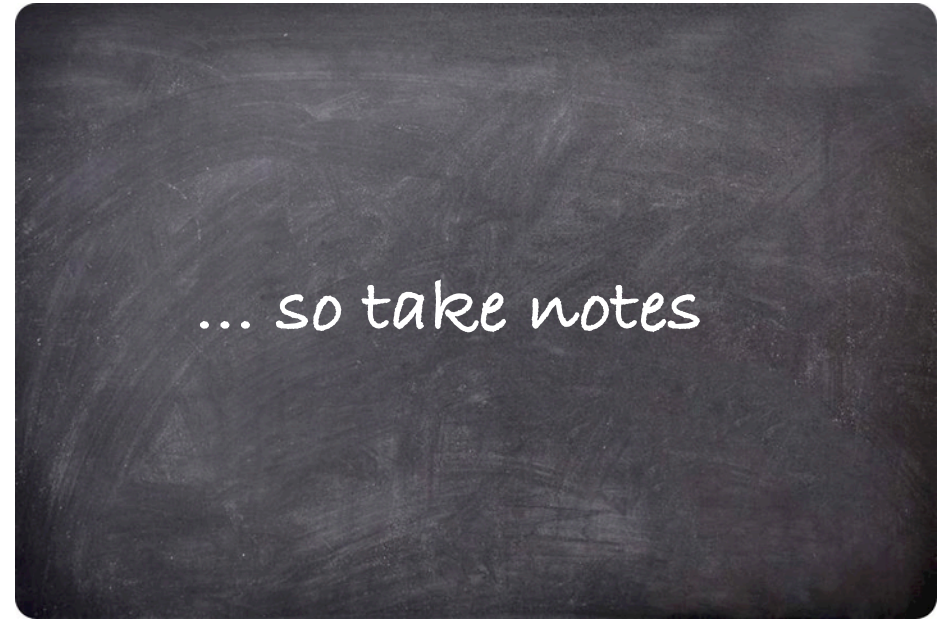
Many references available ... my personal look-up list

... to be complemented by references throughout the lecture



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We will also use the blackboard ...



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... respectively, we will outsource
the blackboard to the exercises

Outline

Brief Introduction

Power Network Modeling

- Circuit Modeling: Network, Loads, & Devices
- Kron Reduction of Circuits
- Power Flow Formulations & Approximations
- Dynamic Network Component Models

Feasibility, Security, & Stability

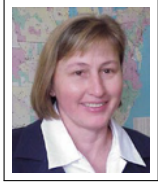
Power System Control Hierarchy

Power System Oscillations

Conclusions

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You will learn to appreciate the following words of wisdom



"Power system research is all about the art of making the right assumptions."

— [Maria Ilic, Lund LCCC Seminar '14]

Circuit Modeling: Network, Loads, & Devices

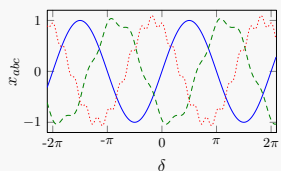
Signal space in three-phase AC circuits

three phase & AC

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} x_a(t+T) \\ x_b(t+T) \\ x_c(t+T) \end{bmatrix}$$

periodic with 0 average

$$\frac{1}{T} \int_0^T x_i(t) dt = 0$$

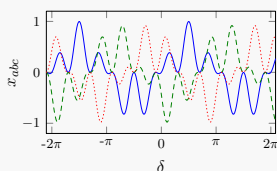


symmetric/balanced

$$= A(t) \begin{bmatrix} \sin(\delta(t)) \\ \sin(\delta(t) - \frac{2\pi}{3}) \\ \sin(\delta(t) + \frac{2\pi}{3}) \end{bmatrix}$$

so that

$$x_a(t) + x_b(t) + x_c(t) = 0$$

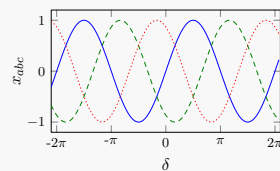


synchronous

$$= A \begin{bmatrix} \sin(\delta_0 + \omega^* t) \\ \sin(\delta_0 + \omega^* t - \frac{2\pi}{3}) \\ \sin(\delta_0 + \omega^* t + \frac{2\pi}{3}) \end{bmatrix}$$

const. freq & amp:

⇒ phasor $Ae^{i(\delta_0 + \omega^* t)}$



Park or $dq0$ -transformation

$$T(\theta) = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

- is unitary $T(\theta)^{-1} = T(\theta)^T$ & maps balanced abc -signal to

$$x_{dq0} = \begin{bmatrix} x_d(t) \\ x_q(t) \\ x_0(t) \end{bmatrix} = T(\theta)x_{abc}(t) = \sqrt{\frac{3}{2}} A(t) \begin{bmatrix} \sin(\delta(t) - \theta) \\ \cos(\delta(t) - \theta) \\ 0 \end{bmatrix}$$

- $T(\omega^* t)$ maps a synchronous signal $x_a(t) = A \sin(\delta_0 + \omega^* t)$ to

$$x_{dq0} = \begin{bmatrix} x_d(t) \\ x_q(t) \\ x_0(t) \end{bmatrix} = T(\omega^* t)x_{abc}(t) = \sqrt{\frac{3}{2}} A \begin{bmatrix} \sin(\delta_0) \\ \cos(\delta_0) \\ 0 \end{bmatrix}$$

- another rotation matrix reduces the signal to q -coordinate $\sqrt{3/2} \cdot A$

Long story short ...

We will work with **single-phase phasor signals** $x(t) = Ae^{i(\delta_0 + \omega^* t)}$ representing the q -phase of a balanced, synchronous, 3-phase AC circuit.

Everything can be extended ... see, e.g., this control-theoretic tutorial:

Modeling of microgrids—from fundamental physics to phasors and voltage sources

Johannes Schiffer^{a,*}, Daniele Zonetti^b, Romeo Ortega^b, Aleksandar Stanković^c, Tevfik Sezi^d, Jörg Raisch^{a,e}

^aTechnische Universität Berlin, Einsteinufer 11, 10587 Berlin, Germany
^bLaboratoire des Signaux et Systèmes, École Supérieure d'Électricité (SUPELEC), Gif-sur-Yvette 91192, France
^cTufts University, Medford, MA 02155, USA
^dSiemens AG, Smart Grid Division, Energy Automation, Humboldtstr. 59, 90459 Nuremberg, Germany
^eMaz-Planck-Institut für Dynamik komplexer technischer Systeme, Sandtorstr. 1, 39106 Magdeburg, Germany

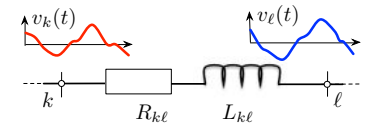
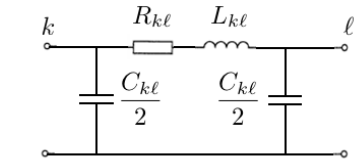
2015
Abstract

Microgrids are an increasingly popular class of electrical systems that facilitate the integration of renewable distributed generation units. Their analysis and controller design requires the development of advanced (typically model-based) techniques naturally posing an interesting challenge to the control community. Although there are widely accepted

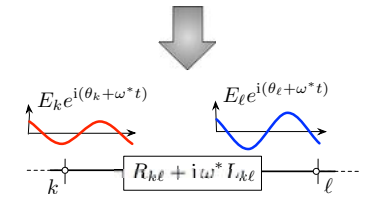
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AC circuits in power networks

- power network modeled by linear **RLC** circuit, e.g., Π -model for
 - transmission lines (mainly inductive)
 - distribution lines (resistive/inductive)
 - cables (capacitive effects)



- we will work in **single-phase**
- quasi-stationary modeling**: harmonic waveforms at nominal frequency ω^*

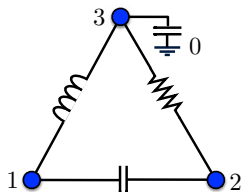


Note: quasi-stationarity assumption can be justified via singular perturbations & modeling can be improved using *dynamic phasors* [A. Stankovic & T. Aydin '00].

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AC circuits – graph-theoretic modeling

- a circuit is a connected & undirected **graph** $G = (\mathcal{V}, \mathcal{E})$
 - $\mathcal{V} = \{1, \dots, n\}$ are the nodes or *buses*
 - buses are partitioned as $\mathcal{V} = \{\text{sources}\} \cup \{\text{loads}\}$
 - the ground is sometimes explicitly modeled as node 0 or $n + 1$
 - $\mathcal{E} \subseteq \{\{i, j\} : i, j \in \mathcal{V}\} = \mathcal{V} \times \mathcal{V}$ are the undirected edges or *branches*
 - edges between distinct nodes $\{i, j\}$ are called *lines*
 - edges $\{i, 0\}$ connecting node i to ground are called *shunts*



$$\mathcal{V} = \{1, 2, 3\}$$

$$\mathcal{E} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{3, 0\}\}$$

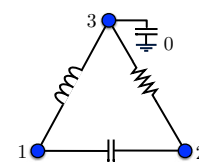
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AC circuits – the network admittance matrix

- $Y = [Y_{ij}] \in \mathbb{C}^{n \times n}$ is the **network admittance matrix** with elements

$$Y_{ij} = \begin{cases} -\frac{1}{Z_{ij}} & \text{for off-diagonal elements } i \neq j \\ \frac{1}{Z_{i,\text{shunt}}} + \sum_{j \neq i} \frac{1}{Z_{ij}} & \text{for diagonal elements } i = j \end{cases}$$

- impedance = resistance + $i \cdot$ reactance: $Z_{ij} = R_{ij} + i \cdot X_{ij}$
- admittance = conductance + $i \cdot$ susceptance: $\frac{1}{Z_{ij}} = G_{ij} + i \cdot B_{ij}$



$$Y = \underbrace{\begin{bmatrix} \frac{1}{Z_{12}} + \frac{1}{Z_{13}} & -\frac{1}{Z_{12}} & -\frac{1}{Z_{13}} \\ -\frac{1}{Z_{12}} & \frac{1}{Z_{12}} + \frac{1}{Z_{23}} & -\frac{1}{Z_{23}} \\ -\frac{1}{Z_{13}} & -\frac{1}{Z_{23}} & \frac{1}{Z_{13}} + \frac{1}{Z_{23}} \end{bmatrix}}_{\text{network Laplacian matrix}} + \underbrace{\begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{1}{Z_{3,\text{shunt}}} \end{bmatrix}}_{\text{diag(shunts)}}$$

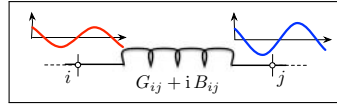
Note quasi-stationary modeling: $Z_{13} = i\omega^* L_{13}$ with nominal frequency ω^*

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AC circuits – basic variables

3 basic variables: voltages & currents

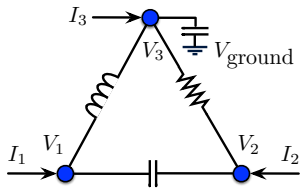
- on nodes: potentials & current injections
- on edges: voltages & current flows



4 quasi-stationary AC phasor coordinates for harmonic waveforms:

- e.g., complex voltage $V = E e^{i\theta}$ denotes $v(t) = E \cos(\theta + \omega^* t)$

where $V \in \mathbb{C}$, $E \in \mathbb{R}_{\geq 0}$, $\theta \in \mathbb{S}^1$, $i = \sqrt{-1}$, and ω^* is nominal frequency



external injections: I_1, I_2, I_3

potentials: V_1, V_2, V_3

reference: $V_{\text{ground}} = 0V$

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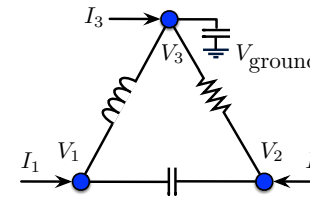
AC circuits – fundamental equations

5 Ohm's law at every branch: $I_{i \rightarrow j} = \frac{1}{Z_{ij}}(V_i - V_j)$

6 Kirchhoff's current law for every bus: $I_i + \sum_j I_{j \rightarrow i} = 0$

7 current balance equations (treating the ground as node with 0V):

$$I_i = -\sum_j I_{j \rightarrow i} = \sum_j \frac{1}{Z_{ij}}(V_i - V_j) = \sum_j Y_{ij} V_j \quad \text{or} \quad \mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$$

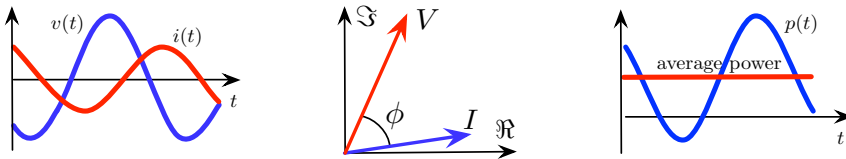


$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Note: all variables are in per unit (p.u.) system, i.e., normalized wrt base voltage 14 / 184

AC circuits – power

(see also exercises)



- voltage phasor: $V = |V|e^{i\theta_V} \Leftrightarrow v(t) = |V| \cos(\omega^* t + \theta_V)$

$$\text{current phasor: } I = |I|e^{i\theta_I} \Leftrightarrow i(t) = |I| \cos(\omega^* t + \theta_I)$$

• instantaneous power:

$$p(t) = v(t) \cdot i(t) = \frac{1}{2} |V| \cdot |I| \cdot \cos(\theta_V - \theta_I) + \frac{1}{2} |V| \cdot |I| \cdot \cos(2\omega^* t + \theta_V + \theta_I)$$

$$\Rightarrow \text{active power (average): } P = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} |V| \cdot |I| \cdot \cos(\phi)$$

$$\Rightarrow \text{reactive power (0-avg): } Q = \frac{1}{T} \int_0^T v(t) \cdot i(t - \frac{T}{4}) dt = \frac{1}{2} |V| \cdot |I| \cdot \sin(\phi)$$

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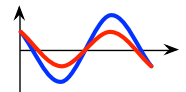
AC circuits – complex power

(see also exercises)

8 active & reactive power in AC circuits:

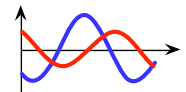
- active (average) power:

$$P = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \cos(\phi)$$



- reactive (0-average) power:

$$Q = \frac{1}{T} \int_0^T v(t) \cdot i(t - T/4) dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \sin(\phi)$$

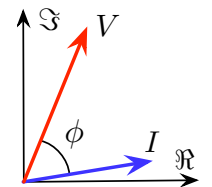


$$\Rightarrow \text{normalize phasors: } V \mapsto 1/\sqrt{2} \cdot |V|e^{i\theta_V}$$

$$\Rightarrow \text{complex power: } S = V \cdot \bar{I} = P + iQ$$

$$= \text{active power} + i \cdot \text{reactive power}$$

$$\Rightarrow \cos(\phi) = P/|S| \text{ is power factor}$$

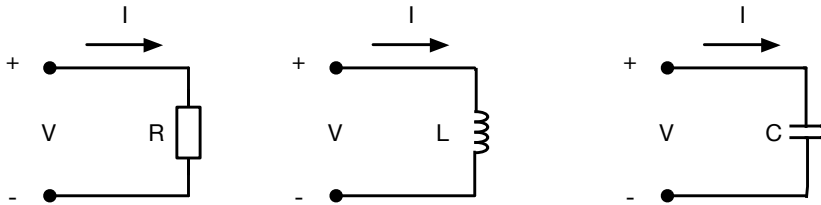


Note: often complex phasors are implicitly normalized $\tilde{V} = 1/\sqrt{2} \cdot E e^{i\theta}$

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AC circuits – power dissipated by RLC loads

details in exercises



Power dissipation $S = V \cdot \bar{I} = P + iQ$ (network sign convention):

$$S = -\frac{1}{2}|I|^2 R$$

$$= -\frac{1}{2} \frac{|V|^2}{R}$$

$$= P < 0$$

$$S = -\frac{1}{2}|I|^2 \cdot i\omega L$$

$$= -i \frac{1}{2} \frac{|V|^2}{\omega L}$$

$$= Q < 0$$

$$S = i \frac{1}{2} \frac{|I|^2}{\omega C}$$

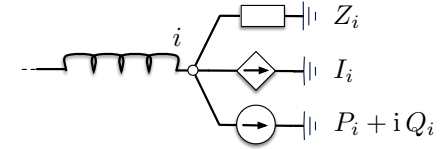
$$= \frac{1}{2} |V|^2 \cdot i\omega C$$

$$= Q > 0$$

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Static models loads

- aggregated **ZIP load model**:
constant impedance **Z** +
constant current **I** +
constant power **P**



- more general **exponential load model**: $\text{power} = \text{const.} \cdot (V/V_{\text{ref}})^{\text{const.}}$
(combinations & variations learned from data)
- various dynamic load models for stability studies ...



“Just use whatever load model fits your mathematics. You will get it wrong anyways.” — [Ian Hiskens, lunch @ Zürich '15]

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Static models for sources

- most common **static load model** is constant active power demand P and constant reactive power demand Q
- conventional **synchronous generators** are controlled to have constant active power output P and voltage magnitude E
- sources interfaced with **power electronics** are typically controlled to have constant active power P and reactive power Q

⇒ common bus device models

- PQ** buses have complex power $S = P + iQ$ specified
- PV** buses have active power P and voltage magnitude E specified
- slack buses** have E and θ specified (not really existent)

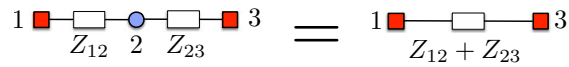
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Kron Reduction of Circuits

Kron reduction

[G. Kron 1939]

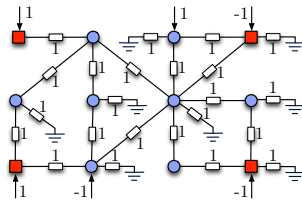
often (almost always) you will encounter Kron-reduced network models



General procedure:

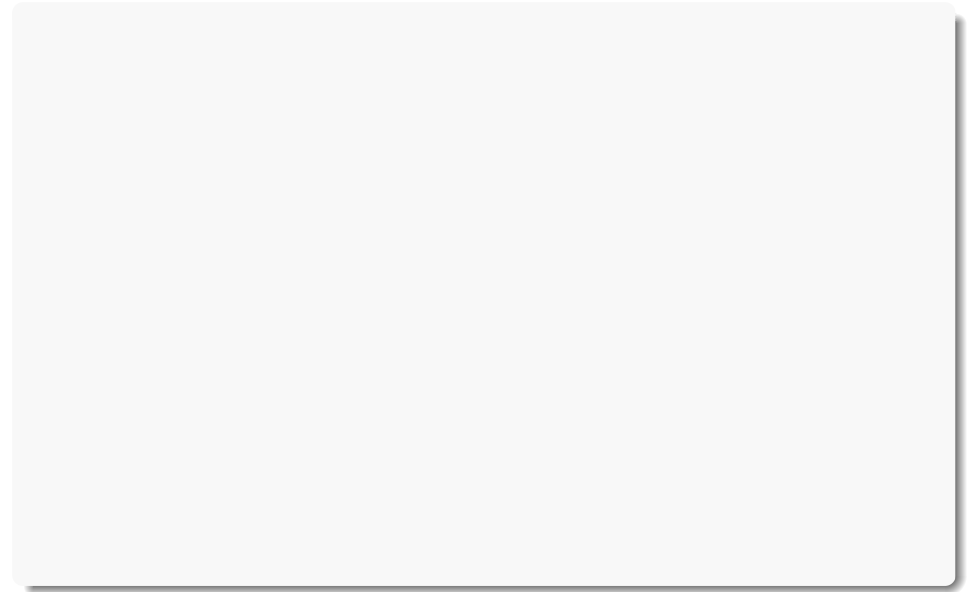
- convert const. power injections locally to shunt impedances $Z = S/V_{ref}^2$
- partition linear current-balance equations via **boundary** & **interior nodes**
(arises naturally, e.g., sources & loads, measurement terminals, etc.)

$$\begin{bmatrix} I_{boundary} \\ I_{interior} \end{bmatrix} = \begin{bmatrix} Y_{boundary} & Y_{bound-int} \\ Y_{bound-int}^T & Y_{interior} \end{bmatrix} \begin{bmatrix} V_{boundary} \\ V_{interior} \end{bmatrix}$$



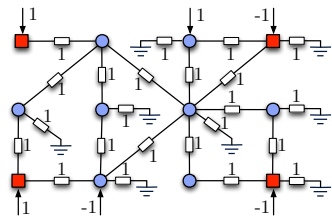
Kron reduction cont'd

on blackboard



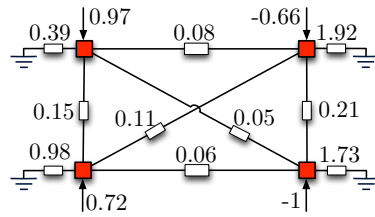
Kron reduction cont'd

- Gaussian elimination of interior voltages



original circuit

$$I = Y \cdot V$$



"equivalent" reduced circuit

$$I_{red} = Y_{red} \cdot V_{boundary}$$

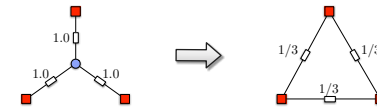
⇒ reduced Y-matrix: $Y_{red} = Y_{boundary} - Y_{bound-int} \cdot Y_{interior}^{-1} \cdot Y_{bound-int}^T$

⇒ reduced injections: $I_{red} = I_{boundary} - Y_{bound-int} \cdot Y_{interior}^{-1} \cdot I_{interior}$

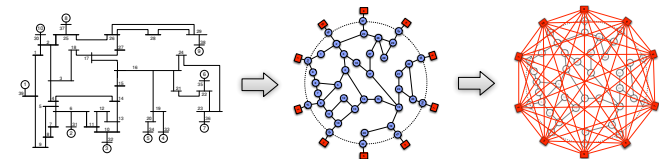
Examples of Kron reduction

algebraic properties are preserved but the network changes significantly

- Star-Δ transformation [A. E. Kennelly 1899, A. Rosen '24]



- Kron reduction of load buses in *IEEE 39 New England power grid*



- ⇒ topology without weights is meaningless!
- ⇒ shunt resistances (loads) are mapped to line conductances
- ⇒ many properties still open [FD & F. Bullo '13, S. Caliskan & P. Tabuada '14]

Kron reduction – so simple yet still full of mysteries

49th IEEE Conference on Decision and Control
December 15-17, 2018
Houston, Texas, USA

The Behavior of Linear Time Invariant RLC Circuits

Erik I. Versteij and Jan C. Willems

Abstract—It is shown that just as we did for a purely resistive network [18], that circuit analysis is very simple if the elements are described not by potentials across and currents through the elements, but rather by the potentials at the nodes and the external currents into the nodes. For example, let v and i be, respectively, the node and external currents, which is more complex than the nodal constitutive laws governing the potential across and current through. However, this description has an advantage in performing the analysis of more complex circuits. These are built by from simple operations like joining two nodes, splitting at nodes, and subdivision.

1. INTRODUCTION: TERMINAL BEHAVIOR
We view electrical circuit as a device that interacts with its environment through a finite number of wires (electrodes)

are compatible with the internal structure of the circuit and component values form a subset $\mathcal{P} \subseteq \mathbb{R}^{(n^2+7n)/2}$, called the terminal behavior of the circuit. \mathcal{P} is of means that the circuit allows the vector functions \mathcal{P} of terminal variables, while \mathcal{P} of \mathcal{P} means that the circuit forbids the vector \mathcal{P} of terminal variables. In [18], it is shown that the terminal potential/current behavior of an interconnection of a finite set of linear nonnegative resistors, inductors and capacitors. The paper is organized as follows: In Section II, the purely resistive network is revisited, and the full characterization we obtained for the behavioral description are stated. The main goal is to extend these to time-invariant

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Characterization and partial synthesis of the behavior of resistive circuits at their terminals

Erkjan van der Schaaf*

Johns Hopkins Institute for Mathematics and Computer Science, University of Groningen, PO Box 407, 9700 MB Groningen, The Netherlands

ARTICLE INFO

ABSTRACT
The external behavior of linear resistive circuits with terminals is characterized as a linear map between maps given by a weighted Laplacian matrix. Conditions are derived for shaping the minimal behavior of the circuit by interconnection with an additional resistive circuit.
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Keywords:
Resistive circuits
Nodal analysis
Partial synthesis by interconnection

1. Introduction

It is originally formulated in [4], and very close to the problem of achievable flow structure addressed in [5]. In [18], indeed, the necessary and sufficient conditions for achieving a certain behavior of linear resistive circuits at their terminals. The paper is heavily inspired by recent work in [18] and [19]. In fact, many of the results obtained in Section 3 on external characterization of linear resistive circuits have an

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Kron Reduction of Graphs With Applications to Electrical Networks

Florian Dörfler and Francesco Bullo

Abstract—Consider a weighted undirected graph and its corresponding Laplacian matrix, possibly augmented with additional diagonal elements corresponding to self-loops. The Kron reduction of the graph is again a graph whose Laplacian matrix is obtained by the Schur complement of the original Laplacian matrix with respect to a specified subset of nodes. The Kron reduction process is analogous to circuit theory and its related disciplines such as electrical impedance homomorphism, smart grid monitoring, transient stability assessment, and analysis of power electronics. Kron reduction is also relevant in other physical domains, in computer-aided applications, and in the reduction of Markov chains. Related concepts have also been studied in purely theoretic problems as the literature on tensor algebra. In this paper we analyze the Kron reduction process from the viewpoint of algebraic graph theory. Specifically, we provide a comprehensive and detailed graph-theoretic analysis of Kron reduction encompassing topological, algebraic, spectral, recursive, and sensitivity analysis. Throughout our theoretic elaborations we especially emphasize the practical applicability of our results to various problems arising in engineering, computation, and linear algebra. Our analysis of Kron reduction leads to several insights both on the mathematical and the

Index Terms—Algebraic graph theory, equivalent circuit, Kron reduction, network-reduced model, Ward equivalent.

graph, its loop Laplacian $\mathcal{L}_{\text{loop}}$, its spectrum, and its effective resistance. Finally, why is the graph reduction process of practical importance and in which application areas? These are some of the questions that motivate this paper.

Electrical networks and the Kron reduction. To illustrate the physical dimension of the problem being introduced above, we consider the associated linear circuit with n nodes, current injections $f \in \mathbb{R}^n$, nodal voltages $V \in \mathbb{R}^n$, branch conductances $A_{ij} > 0$, and shunt conductances $A_{ii} > 0$ connecting node i to the ground. The resulting current-balance equations are $f = \mathcal{L}V$, where the conductance matrix $\mathcal{L} \in \mathbb{R}^{n \times n}$ is the loop Laplacian. In circuit theory and related disciplines it is desirable to obtain a lower-dimensional electrically equivalent network from the viewpoint of certain boundary nodes $\mathcal{O} \subseteq \{1, \dots, n\}$, $|\mathcal{O}| \geq 2$. Let $\mathcal{I} = \{1, \dots, n\} \setminus \mathcal{O}$ denote the set of interior nodes, then, after appropriately labeling the nodes, the current-balance equations can be partitioned as

$$\begin{bmatrix} f_{\mathcal{I}} \\ f_{\mathcal{O}} \end{bmatrix} = \begin{bmatrix} \mathcal{L}_{\mathcal{I}\mathcal{I}} & \mathcal{L}_{\mathcal{I}\mathcal{O}} \\ \mathcal{L}_{\mathcal{O}\mathcal{I}} & \mathcal{L}_{\mathcal{O}\mathcal{O}} \end{bmatrix} \begin{bmatrix} V_{\mathcal{I}} \\ V_{\mathcal{O}} \end{bmatrix} \quad (1)$$

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Brief paper Towards Kron reduction of generalized electrical networks*

Sina Yamaç Caliskan¹, Paulo Tabuada

Department of Electrical Engineering, University of California at Los Angeles, CA 90095-1594, United States

ARTICLE INFO

ABSTRACT
Kron reduction is used to simplify the analysis of multi-machine power system under certain steady state assumptions that underlie the usage of phasors. Using ideas from behavioral system theory, in this paper we show how to perform Kron reduction for a class of electrical networks, called homogeneous electrical networks, without steady state assumptions. The reduced models can then be used to analyze the transient as well as the steady state behavior of these electrical networks.
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Keywords:
Electrical circuits
Graph-theoretic analysis
Identification and model reduction

1. Introduction

Multi-machine power networks are the interconnection of power generators and substations via three-phase transmission lines. This structure can be abstracted as a graph, in which

This reduction, however, is based on the use of phasors and it requires the current and voltage waveforms in each phase to be sinusoidal and with the same frequency. This assumption seems reasonable if we want to study the transient behavior of power systems during which the waveforms are not sinusoidal.

Power balance eqn's: "power injection = Σ power flows"

1 **complex form:** $S_i = V_i \bar{I}_i = \sum_j V_i \bar{Y}_{ij} \bar{V}_j$ or $S = \text{diag}(V) Y \bar{V}$

⇒ purely quadratic and useful for static calculations & optimization

2 **rectangular form:** insert $V = e + if$ and split real & imaginary parts:

active power: $P_i = \sum_j B_{ij}(e_i f_j - f_i e_j) + G_{ij}(e_i e_j + f_i f_j)$

reactive power: $Q_i = -\sum_j B_{ij}(e_i e_j + f_i f_j) + G_{ij}(e_i f_j - f_i e_j)$

⇒ purely quadratic and useful for homotopy methods & QCQPs

⇒ main complexity is quadratic nonlinearity $V_i \bar{V}_j = [e \quad if] \cdot [e \quad -if]^T$

Power balance eqn's – cont'd

3 **matrix form:** define unit-rank p.s.d. Hermitian matrix $W = V \cdot \bar{V}^T$ with components $W_{ij} = V_i \bar{V}_j$, then power flow is $S_i = \sum_j \bar{Y}_{ij} W_{ij}$

⇒ linear and useful for relaxations in convex optimization problems

TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS

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Convex Relaxation of Optimal Power Flow—Part I: Formulations and Equivalence

Steven H. Low, Fellow, IEEE

Abstract—This tutorial summarizes recent advances in the convex relaxation of the optimal power flow (OPF) problem, focusing on structural properties rather than algorithms. Part I presents two power flow models, formulates OPF and their relaxations in each model, and proves equivalence relationships among them. Part II presents sufficient conditions under which the convex relaxations are exact.

Index Terms—Convex relaxation, optimal power flow, power systems, quadratically constrained quadratic program (QCQP), second-order cone program (SOCP), semidefinite program

SOCQP for radial networks in the branch flow model of [45]. See Remark 6 below for more details. While these convex relaxations have been illustrated numerically in [22] and [23], whether or when they will turn out to be exact is first studied in [24]. Exploiting graph sparsity to simplify the SDP relaxation of OPF is first proposed in [25] and [26] and analyzed in [27] and [28].

Convex relaxation of quadratic programs has been applied to many engineering problems; see, e.g., [29]. There is a rich theory and extensive empirical experiences. Compared with other

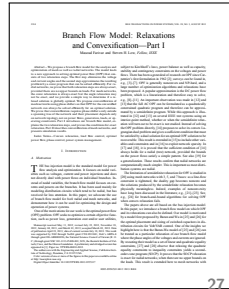
Power balance eqn's – cont'd

4 branch flow eqn's parameterized in flow variables [M. Baran & F. Wu '89]:

- Ohm's law: $V_i - V_j = Z_{ij} I_{i \rightarrow j}$
- branch power flow $i \rightarrow j$: $S_{i \rightarrow j} = V_i \cdot \overline{I_{i \rightarrow j}}$
- power balance at node i :

$$\underbrace{\sum_{k: i \rightarrow k} S_{i \rightarrow k} + Y_{i, \text{shunt}} |V_i|^2}_{\text{outgoing flows}} = S_i + \underbrace{\sum_{j: j \rightarrow i} (S_{j \rightarrow i} - Z_{ij} |I_{i \rightarrow j}|^2)}_{\text{incoming flows}}$$

- **DistFlow formulation** in terms of square magnitude variables $|V_i|^2$ and $|I_{i \rightarrow j}|^2$
(missing angle variables $\angle V_i$ and $\angle I_{i \rightarrow j}$ can sometimes be recovered, e.g., in acyclic case)
- lossless approximation can be solved exactly in acyclic networks (useful for distribution networks)
[M. Baran & F. Wu '89, M. Farivar, L. Chen, & S. Low '13]



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Power balance eqn's – cont'd

5 polar form: insert $V = Ee^{i\theta}$ and split real & imaginary parts:

$$\begin{aligned} \text{active power: } P_i &= \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j) \\ \text{reactive power: } Q_i &= -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j) \end{aligned}$$

⇒ will be our focus these days since ...

- **power system specs** on frequency $\frac{d}{dt}\theta(t)$ and voltage magnitude E
- **dynamics**: generator swing dynamics affect voltage phase angles & voltage magnitudes are controlled to be constant
- **physical intuition**: usual operation near flat voltage profile $V_i \approx 1e^{i\phi}$ which give rise to various insights for analysis & design (later)

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Power flow simplifications & approximations

power flow equations are too complex & unwieldy for analysis & large computations

- ▶ active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- ▶ reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

1 lossless transmission lines $R_{ij}/X_{ij} = -G_{ij}/B_{ij} \approx 0$

$$\begin{aligned} \text{active power: } P_i &= \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) \\ \text{reactive power: } Q_i &= -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) \end{aligned}$$

2 decoupling near operating point $V_i \approx 1e^{i\phi}$: $\begin{bmatrix} \partial P/\partial \theta & \partial P/\partial E \\ \partial Q/\partial \theta & \partial Q/\partial E \end{bmatrix} \approx \begin{bmatrix} * & 0 \\ 0 & * \end{bmatrix}$

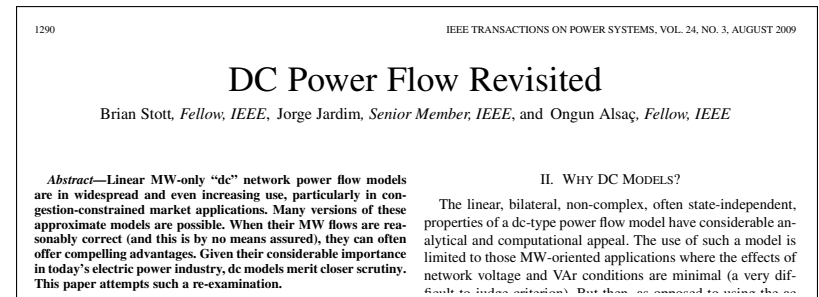
$$\begin{aligned} \text{active power: } P_i &= \sum_j B_{ij} \sin(\theta_i - \theta_j) && \text{(function of angles)} \\ \text{reactive power: } Q_i &= -\sum_j B_{ij} E_i E_j && \text{(function of magnitudes)} \end{aligned}$$

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Power flow simplifications & approximations cont'd

3 linearization for small flows near operating point $V_i \approx 1e^{i\phi}$:

$$\begin{aligned} \text{active power: } P_i &= \sum_j B_{ij} (\theta_i - \theta_j) && \text{known as DC power flow} \\ \text{reactive power: } Q_i &= \sum_j B_{ij} (E_i - E_j) && \text{(if formulated in p.u. system)} \end{aligned}$$



Conclusion on the **most limiting assumption** of DC power flow: $R/X \approx 0$

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Power flow simplifications & approximations cont'd

- ▶ active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- ▶ reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

Multiple variations & combinations of DC power flow

- power flow transformation for constant R/X ratios (see exercise)
- linearization & decoupling at arbitrary operating points [D. Deka et al., '15]
- advanced linearizations especially for reactive power
[S. Bolognani & S. Zampieri '12, B. Gentile et al. '14, J. Simpson-Porco et al. '16]
- linearizations in rectangular coordinates (more accurate for active power)
[R. Baldick '13, S. Bolognani & S. Zampieri '15, S. Dhople et al. '15]



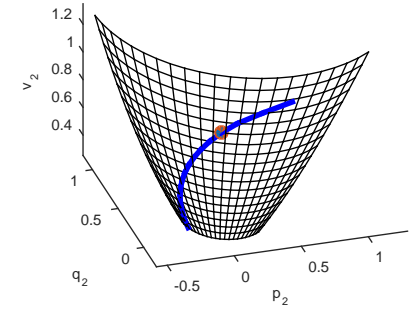
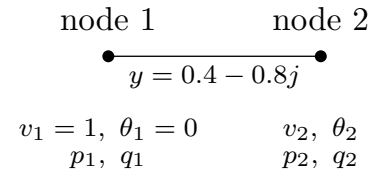
"... plenty of heuristics in industry ... especially for approximation of losses."

— [Bruce Wollenberg, meeting @ Minneapolis '13]

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A unifying geometric perspective

[S. Bolognani & F. Dörfler '15]



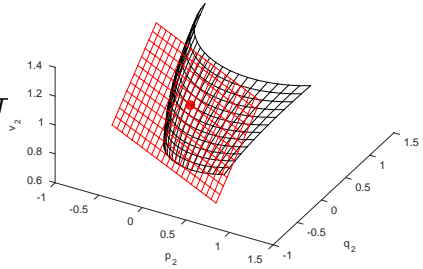
1 variables: all of $x = (E, \theta, p, q)$

2 power flow manifold: $F(x) = 0$

3 normal space spanned by $\left. \frac{\partial F(x)}{\partial x} \right|_{x^*} = A^T$

4 tangent space $A(x - x^*) = 0$
is best linear approximant at x^*

5 accuracy depends on curvature $\frac{\partial^2 F(x)}{\partial x^2}$



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Closer look at implicit formulae $A(x - x^*) = 0$

$$\left[\left(\langle \text{diag } \overline{Y E^*} \rangle + \langle \text{diag } E^* \rangle N \langle Y \rangle \right) \cdot \begin{bmatrix} \text{diag}(\cos \theta^*) & -\text{diag}(E^*) \text{diag}(\sin \theta^*) \\ \text{diag}(\sin \theta^*) & \text{diag}(E^*) \text{diag}(\cos \theta^*) \end{bmatrix} \right]$$

shunt loads

lossy DC flow

rotation \times scaling at operating point

$$\times \underbrace{\begin{bmatrix} v - v^* \\ \theta - \theta^* \end{bmatrix}}_{\text{deviation variables}} = \begin{bmatrix} p - p^* \\ q - q^* \end{bmatrix}$$

deviation variables

where $N = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$ is complex conjugate in real coordinates

and $\langle A \rangle = \begin{bmatrix} \Re(A) & -\Im(A) \\ \Im(A) & \Re(A) \end{bmatrix}$ is complex rotation in real coordinates.

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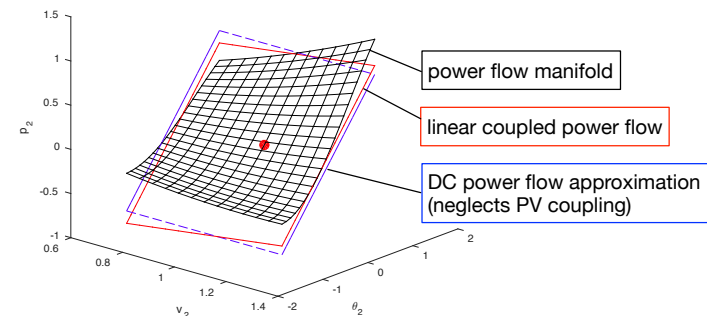
Special cases reveal some old friends I

- flat-voltage/0-injection point: $x^* = (E^*, \theta^*, P^*, Q^*) = (1, 0, 0, 0)$

$$\Rightarrow \text{implicit linearization: } \begin{bmatrix} \Re(Y) & -\Im(Y) \\ -\Im(Y) & \Re(Y) \end{bmatrix} \begin{bmatrix} v \\ \theta \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

is **linear coupled power flow** [D. Deka, S. Backhaus, & M. Chertkov, '15]

$\Rightarrow \Re(Y) = 0$ gives **DC power flow**: $-\Im(Y)\theta = P$ and $-\Im(Y)E = Q$



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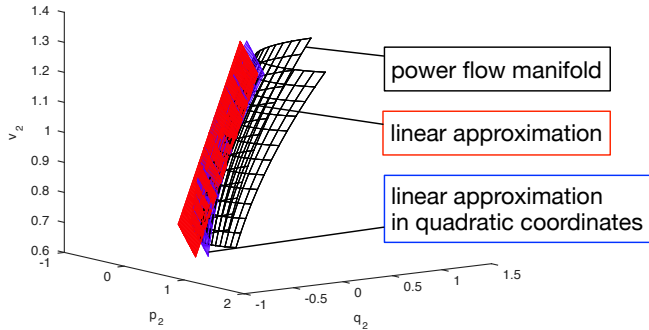
Special cases reveal some old friends II

• **flat-voltage/0-injection point:** $x^* = (E^*, \theta^*, P^*, Q^*) = (1, 0, 0, 0)$

⇒ rectangular coord. ⇒ **rectangular DC flow** [S. Bolognani & S. Zampieri, '15]

• nonlinear change to **quadratic coordinates** from v_h to v_h^2

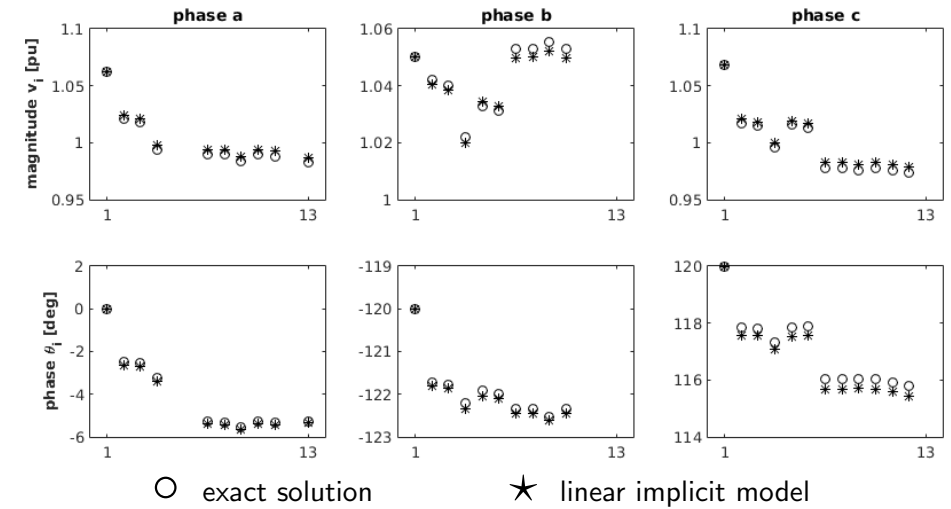
⇒ linearization gives (non-radial) **LinDistFlow** [M.E. Baran & F.F. Wu, '88]



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Accuracy illustrated with unbalanced three-phase IEEE13

can be extended to three-phase, exponential loads, etc.



Matlab/Octave code @ <https://github.com/saveriob/1ACPF>

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Plenty of recent interest in power flow approximations

mainly for the sake of verifying analytic approaches

Fast Power System Analysis via Implicit Linearization of the Power Flow Manifold

Saverio Bolognani and Florian Dörflinger

Abstract—In this paper, we consider the manifold that describes all feasible power flows in a power system as an implicit algebraic relation between nodal voltages (in polar coordinates) and nodal power injections (in rectangular coordinates). We derive the best linear approximation of such a relation around a generic solution of the power flow equations. Our linear approximation is sparse, computationally attractive, and preserves the structure of the power flow. Thanks to the full availability of this approach, the proposed linear implicit model

On the existence and linear approximation of the power flow solution in power distribution networks

Saverio Bolognani and Sandro Zampieri

Abstract—We consider the problem of deriving an explicit approximate solution of the nonlinear power equations that describe a power distribution network. We give sufficient conditions for the existence of a practical solution to the power flow approximation that is linear in demands and generations. For this

Linear Approximations to AC Power Flow in Rectangular Coordinates

Sairaj V. Dhople, Swaroop S. Guggilam, Yu Chen, Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, Minnesota 55455, Email: sdhople.gugg022@UMN.EDU

DC Power Flow Revisited

Brian Stott, Fellow, IEEE, Jorge Jardim, Senior Member, IEEE, and Ungun Alsac, Fellow, IEEE

Abstract—Linear MW-only “dc” network power flow models are in widespread and even increasing use, particularly in congestion-constrained market applications. Many versions of these approximate models are possible. When their MW flows are reasonably correct (and this is by no means assured), they can often offer compelling advantages. Given their considerable importance in today’s electric power industry, do models merit closer scrutiny.

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Once you try to analyze power flow equations

with pen and paper, you will realize . . .



“Maybe we should revisit the way we write power flow equations.” — [Göran Andersson, Santa Fe Grid Science Workshop '15]

Once you work computationally with data, you will see . . .



“The devil introduced the per unit system into power.” — [Peter Sauer, ACC '12]

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Dynamic Network Component Models

Modeling the “essential” network dynamics

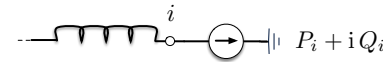
models can be arbitrarily detailed & vary on different time/spatial scales

- 1 active and reactive **power flow**
(e.g., lossless)

$$P_{i,\text{inj}} = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_{i,\text{inj}} = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

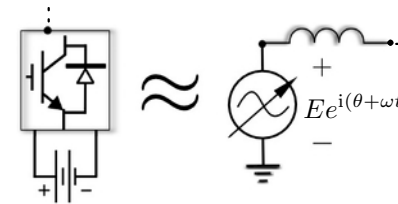
- 2 passive constant power **loads**



$$P_{i,\text{inj}} = P_i = \text{const.}$$

$$Q_{i,\text{inj}} = Q_i = \text{const.}$$

- 3 **inverters**: DC or variable AC sources with power electronics



- (i) have constant/controllable PQ
(max. power-point tracking)

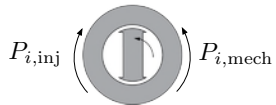
- (ii) or mimic generators
(more later)

Modeling the “essential” synchronous generator dynamics

- 4 electromech. **swing dynamics**
of synchronous machines

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{i,\text{mech}} - P_{i,\text{inj}}$$

$$E_i = \text{const.}$$



(can be derived from first principle model & some (possibly strong) assumptions)

2015 IEEE 54th Annual Conference on Decision and Control (CDC)
December 15-18, 2015, Osaka, Japan

Uses and Abuses of the Swing Equation Model

Sina Y. Caliskan and Paulo Tabuada

Abstract—The swing equation model is widely used in the literature to study a large class problems, including stability analysis of power systems. We show in this paper, by comparison with a first principles model, that the swing equation model may lead to erroneous conclusions when performing stability analysis of power systems, even under small oscillations.

I. INTRODUCTION

The swing equation model is a perfect example of the famous line by George Box and Norman Draper in [2]: “All models are wrong, but some are useful.”. Power engineers

equation for stability analysis under small oscillations we obtain results contradicting a more detailed FP model.

II. SYNCHRONOUS GENERATOR MODELS

In this section, we review two synchronous generator models. The first model is derived from first principles while the second is the traditional swing equation model that is widely used in the literature. After introducing these models, we show how to recover the swing equation model from the

Common variations in dynamic network models

dynamic behavior is very much dependent on load models & generator models

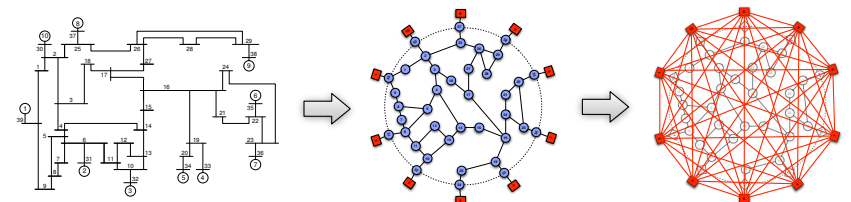
- 1 frequency/voltage-depend. loads
[A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

$$D_i \dot{\theta}_i + P_i = -P_{i,\text{inj}}$$

$$f_i(\dot{V}_i) + Q_i = -Q_{i,\text{inj}}$$

- 2 network-reduced models after Kron reduction of loads
[H. Chiang, F. Wu, & P. Varaiya '94]
(very common but poor assumption: $G_{ij} = 0$)

$$M_i \ddot{\theta}_i + D \dot{\theta}_i = P_{i,\text{mech}} - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) - \underbrace{\sum_j G_{ij} E_i E_j \cos(\theta_i - \theta_j)}_{\text{effect of resistive loads}}$$



Structure-preserving power network model [A. Bergen & D. Hill '81]

without Kron-reduction of load buses

$$\begin{aligned} \dot{\theta}_i &= \omega_i \\ \text{generator swing dynamics: } M_i \dot{\omega}_i &= -D_i \omega_i + P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) \\ Q_i &= -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) \\ \text{frequency-dependent loads: } D_i \dot{\theta}_i &= P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) \\ Q_i &= -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) \end{aligned}$$

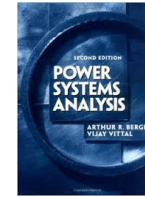
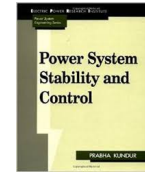
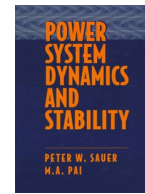
(or inverter-interfaced sources)

- in **academia**: this “baseline model” is typically further simplified: decoupling, linearization, constant voltages, . . .
 - in **industry**: much more detailed models used for grid simulations
- ⇒ **IMHO**: above model captures most interesting network dynamics

Common variations in dynamic network models — cont'd

dynamic behavior is very much dependent on load models & generator models

- higher order generator dynamics [P. Sauer & M. Pai '98] voltages, controls, magnetics etc. (reduction via singular perturbations)
- dynamic & detailed load models [D. Karlsson & D. Hill '94] aggregated dynamic load behavior (e.g., load recovery after voltage step)
- time-domain models [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12] passive Port-Hamiltonian models for machines & RLC circuitry



“Power system research is all about the art of making the right assumptions.”

Lots of current research activity on time-domain models



A port-Hamiltonian approach to power network modeling and analysis
S. Fiaz¹, D. Zanetti¹, R. Ortega¹, J.M.A. Schepers¹, A.J. van der Schaft¹

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1. Introduction
Market liberalization and the ever increasing electricity demand have forced the power system to operate under highly non-linear conditions. This situation has led to the need to revisit the existing modeling, analysis and control techniques that underlie the power system to withstand unexpected contingencies without experiencing voltage or transient instabilities. At the network level power engineers used reduced network models (RNM) where the system is viewed as an n -port described by a set of ordinary differential equations. RNM do not retain the



Towards Kron reduction of generalized electrical networks^{*}
Sina Yamac Caliskan¹, Paulo Tabuada

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1. Introduction
Multi-machine power networks are the interconnection of power generators and actuators via three-phase transmission lines. This structure can be abstracted as a graph, in which

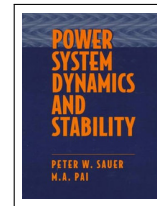
Synchronization of Nonlinear Circuits in Dynamic Electrical Networks With General Topologies
Sainy V. Dhople, Member, IEEE, Brian B. Johnson, Member, IEEE, Florian Dörfler, Member, IEEE, and Abdullah O. Hamada

Abstract—Sufficient conditions are derived for global asymptotic synchronization in a system of identical nonlinear electrical circuits coupled through linear time-invariant (TI) electrical networks. In particular, the conditions we derive apply to settings where the nonlinear circuits are composed of a parallel combination of a gyrator, a current element, and a nonlinear inductor. These conditions are expressed in terms of the Laplacian of the network graph and are independent of the specific nonlinearities of the electrical networks. Uniform networks have identical per-unit-length inductors. Homogeneous electrical networks are characterized by having the same effective inductance. Synchronization in these networks is guaranteed by ensuring the stability of an equivalent circuit-averaged differential system that emphasizes signal differences. The applicability of the synchronization conditions to the linearized and spectral properties of Kron reduction is made explicit in the paper. The validity of the analyzer results is demonstrated with simulations on networks of coupled three-phase circuits.

Index Terms—Kron reduction, nonlinear circuits, synchronization.

1. INTRODUCTION
SYNCHRONIZATION of nonlinear electrical circuits coupled through complex networks is integral to modeling,

On the swing equation . . .



“There is probably more literature on synchronous machines than on any other device in electrical engineering.” — [Peter Sauer & M.A. Pai, Power System Dynamics and Stability '98]



“The swing equation model is a perfect example of the famous line [. . .]: “All models are wrong, but some are useful.””

— [Sina Y. Caliskan and Paulo Tabuada, CDC '15]

Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

- Decoupled Active Power Flow (Synchronization)
- Reactive Power Flow (Voltage Collapse)
- Coupled & Lossy Power Flow
- Transient Rotor Angle Stability

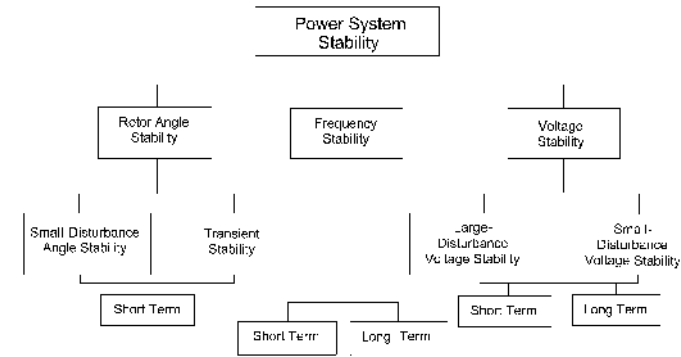
Power System Control Hierarchy

Power System Oscillations

Conclusions

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One system with many dynamics & control problems



"From a practical viewpoint, there are four major analytical problems: ... compute equilibria ... transient stability ... [inter-area] oscillations ... voltage collapse. Of course, theoretically they are all aspects of the one overall stability question." — [David Hill, ISCAS '06]

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prelims on power flow

Preliminary insights on lossless power flow

power flow equations:

$$P_i = \sum_{j=1}^n B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = - \sum_{j=1}^n B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

\Rightarrow solution space: $\mathbb{T}^n \times \mathbb{R}_{\geq 0}^n = (\mathbb{S}^1 \times \dots \times \mathbb{S}^1) \times (\mathbb{R}_{\geq 0} \times \dots \times \mathbb{R}_{\geq 0})$

rotational symmetry:

if θ^* is a solution $\Rightarrow \theta^* + \text{const.} \cdot \mathbf{1}_n$ is another solution

\Rightarrow solution space "modulo rotational symmetry": $\mathbb{T}^n \setminus \mathbb{S}^1 \times \mathbb{R}_{\geq 0}^n$

index shenanigans:

- ▶ active flow $i \rightarrow i = B_{ii} E_i E_j \sin(\theta_i - \theta_i) = 0$ (\Rightarrow can drop index i)
- ▶ reactive flow $i \rightarrow i = -B_{ii} E_i E_j \cos(\theta_i - \theta_i) = -B_{ii} E_i^2$

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Preliminary feasibility conditions for lossless power flow

see exercises for details

power flow equations:

$$P_i = \sum_{j=1}^n B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_i = - \sum_{j=1}^n B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

necessary feasibility condition I:

$$\sum_{i=1}^n P_i = 0 \Leftrightarrow \exists \text{ a solution}$$

- ≜ power balance
- ⇒ typically not true (w/o slack bus) due to unknown load demand
- ⇒ need to consider dynamics

necessary feasibility condition II:

$$\sum_{i=1}^n Q_i \geq 0 \Leftrightarrow \exists \text{ a solution}$$

- ≜ reactive power losses
- ⇒ reactive power must be supplied (for inductive grid w/o shunts)

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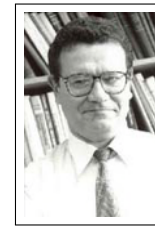
Decoupled Active Power Flow (Synchronization)

Feasibility power flow is crucial for system operation

Given: network parameters & topology and load & generation profile

Q: “∃ an optimal, stable, and robust synchronous operating point ?”

- 1 Security analysis [Araposthatis et al. '81, Wu et al. '80 & '82, Ilić '92, ...]
- 2 Load flow feasibility [Chiang et al. '90, Dobson '92, Lesieutre et al. '99, ...]
- 3 Optimal generation dispatch [Lavaei et al. '12, Bose et al. '12, ...]
- 4 Transient stability [Sastry et al. '80, Bergen et al. '81, Hill et al. '86, ...]
- 5 Inverters in microgrids [Chandorkar et al. '93, Guerrero et al. '09, Zhong '11, ...]
- 6 Complex networks [Hill et al. '06, Strogatz '01, Arenas et al '08, ...]

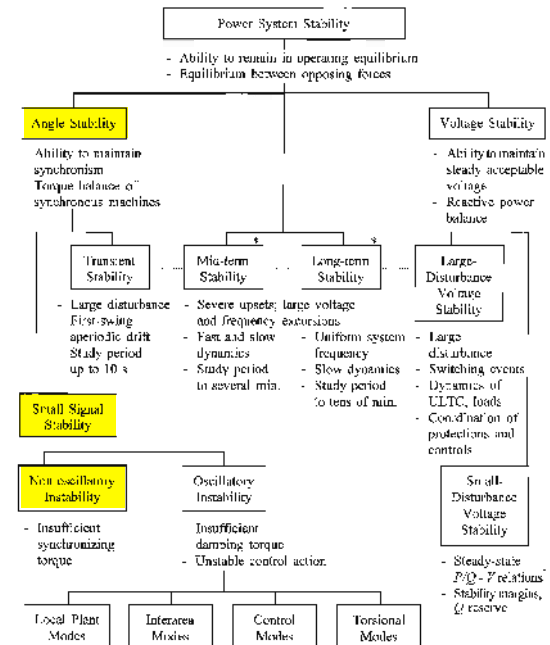


“How do we quantitatively measure feasibility in order to incorporate this attribute in the system design or operation? How do we explicitly describe the region of feasibility in general, and in particular in a large neighborhood around the normal operating injections?”

— [J. Jaris & F. Galiana, IEEE PAS '81]

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Our first stab at power system stability



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Synchronization & feasibility of active power flow

sync is crucial for the functionality and operation of the power grid

- **structure-preserving power network model** [A. Bergen & D. Hill '81]:

synchronous machines: $M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

frequency-dependent loads: $D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

- **synchronization** = sync'd frequencies & bounded active power flows

$$\dot{\theta}_i = \omega_{\text{sync}} \quad \forall i \in \mathcal{V} \quad \& \quad |\theta_i - \theta_j| \leq \gamma < \pi/2 \quad \forall \{i, j\} \in \mathcal{E}$$

= active power flow feasibility & security constraints

- **explicit sync frequency:** if sync, then
(by summing over all equations)

$$\omega_{\text{sync}} = \sum_i P_i / \sum_i D_i$$

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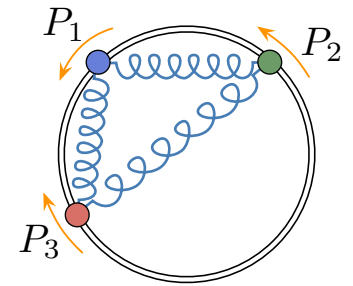
A perspective from coupled oscillators

Mechanical oscillator network

Angles $(\theta_1, \dots, \theta_n)$ evolve on \mathbb{T}^n as

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

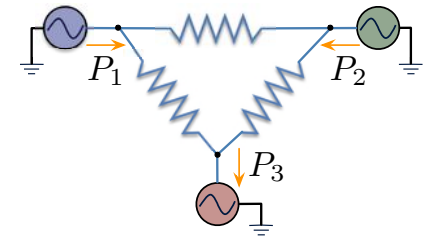
- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $P_i \in \mathbb{R}$
- spring constants $B_{ij} \geq 0$



Structure-preserving power network

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

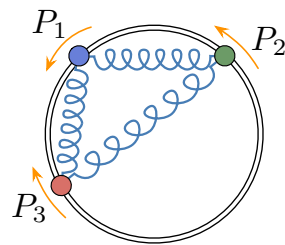
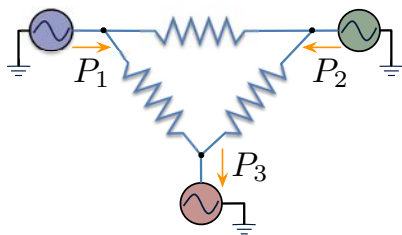
$$D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$



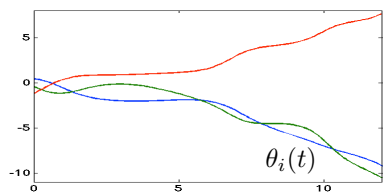
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Phenomenology of sync in power networks

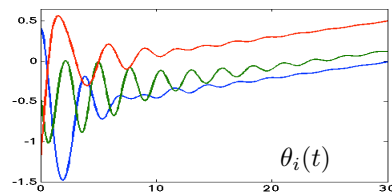
- sync is **crucial for AC power grids**



- sync is a **trade-off**



weak coupling & heterogeneous

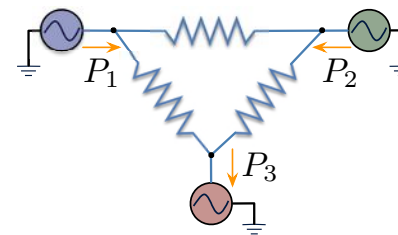


strong coupling & homogeneous

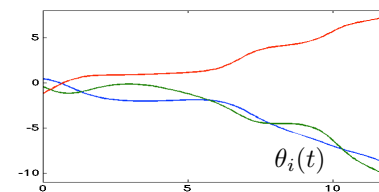
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Phenomenology of sync in power networks

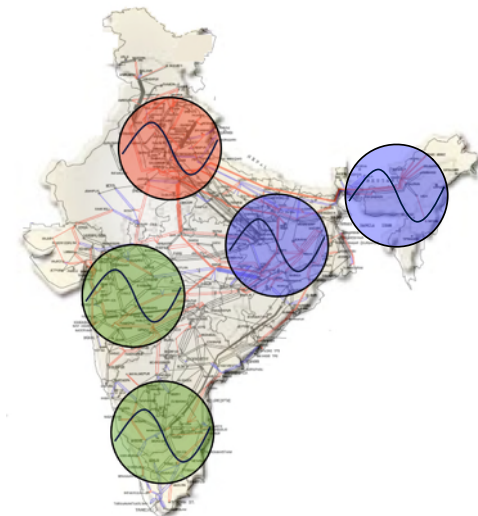
- sync is **crucial for AC power grids**



- sync is a **trade-off**



weak coupling & heterogeneous

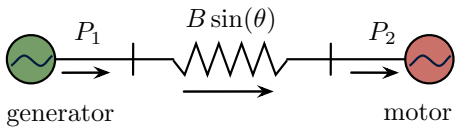


Blackout India July 30/31 2012

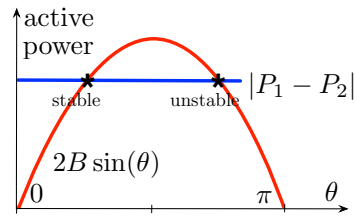
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Back of the envelope calculations for the two-node case

generator connected to identical motor shows bifurcation at difference angle $\theta = \pi/2$



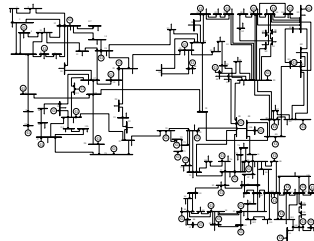
$$M\ddot{\theta} + D\dot{\theta} = P_1 - P_2 - 2B \sin(\theta)$$



\exists stable sync $\Leftrightarrow B > |P_1 - P_2|/2 \Leftrightarrow$ "ntwk coupling > heterogeneity"

Q1: Quantitative generalization to a complex & large-scale network?

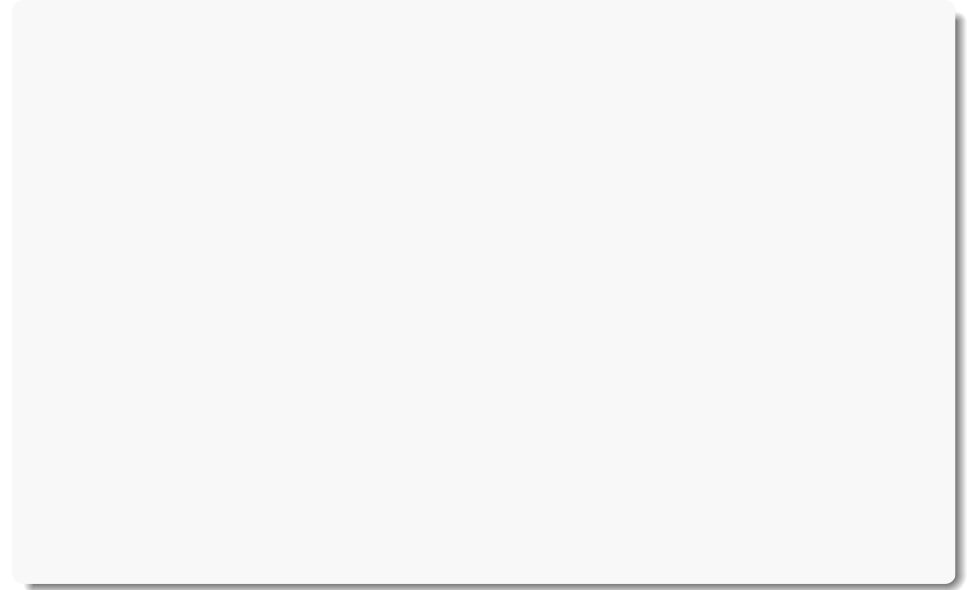
Q2: What are the particular metrics for coupling and heterogeneity?



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Who knows consensus systems?

on blackboard



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Primer on algebraic graph theory

for a connected and undirected graph

Laplacian matrix $L =$ "degree matrix" $-$ "adjacency matrix"

$$L = L^T = \begin{bmatrix} \vdots & \ddots & \vdots & \ddots & \vdots \\ -B_{i1} & \cdots & \sum_{j=1}^n B_{ij} & \cdots & -B_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \end{bmatrix} \geq 0$$

is positive semidefinite with one zero eigenvalue & eigenvector $\mathbf{1}_n$

Notions of connectivity

- spectral: 2nd smallest eigenvalue of L is "algebraic connectivity" $\lambda_2(L)$
- topological: degree $\sum_{j=1}^n B_{ij}$ or degree distribution

Notions of heterogeneity

$$\|P\|_{\mathcal{E},\infty} = \max_{\{i,j\} \in \mathcal{E}} |P_i - P_j|, \quad \|P\|_{\mathcal{E},2} = \left(\sum_{\{i,j\} \in \mathcal{E}} |P_i - P_j|^2 \right)^{1/2}$$

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Synchronization in "complex" networks

for a first-order model — all results generalize locally

$$\dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- 1 **local stability** for equilibria satisfying
(linearization is Laplacian matrix)

$$|\theta_i^* - \theta_j^*| < \pi/2 \quad \forall \{i,j\} \in \mathcal{E}$$

- 2 **necessary sync condition:**
(so that syn'd solution exists)

$$\sum_j B_{ij} \geq |P_i - \omega_{\text{sync}}| \Leftrightarrow \text{sync}$$

- 3 **sufficient sync condition:**

[FD & F. Bullo '12]

$$\lambda_2(L) > \|P\|_{\mathcal{E},2} \Rightarrow \text{sync}$$

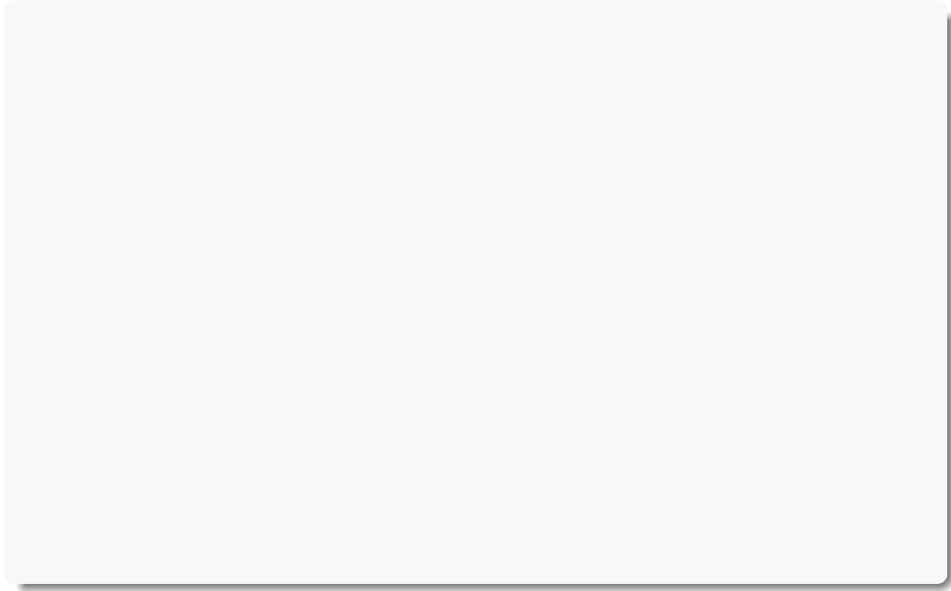
$\Rightarrow \exists$ similar conditions with diff. metrics on coupling & heterogeneity

\Rightarrow **Problem:** sharpest general conditions are conservative

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Can we solve the power flow equations exactly?

on blackboard



A nearly exact sync condition

[FD, M. Chertkov, & F. Bullo '13]

- 1 search equilibrium θ^* with $|\theta_i^* - \theta_j^*| \leq \gamma < \pi/2$ for all $\{i, j\} \in \mathcal{E}$:

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) \quad (*)$$

- 2 consider linear “small-angle” DC approximation of (*):

$$P_i = \sum_j B_{ij}(\delta_i - \delta_j) \quad \Leftrightarrow \quad P = L\delta \quad (**)$$

unique solution (modulo symmetry) of (**) is $\delta^* = L^\dagger P$

- 3 solution ansatz for (*): $\theta_i^* - \theta_j^* = \arcsin(\delta_i^* - \delta_j^*)$ (for a tree)

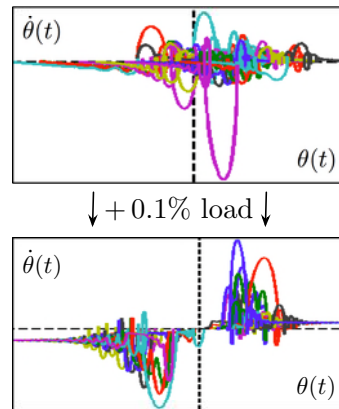
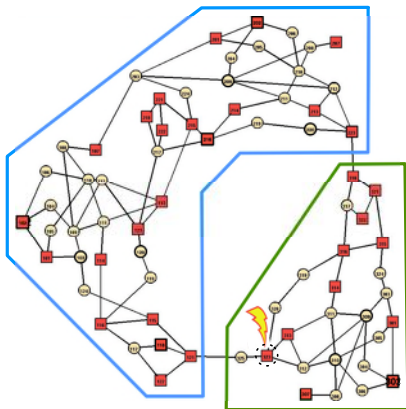
$$P_i = \sum_{j=1}^n a_{ij} \sin(\theta_i - \theta_j) = \sum_{j=1}^n a_{ij} \sin(\arcsin(\delta_i^* - \delta_j^*)) = P_i \quad \checkmark$$

\Rightarrow **Thm:** $\exists \theta^*$ with $|\theta_i^* - \theta_j^*| \leq \gamma \forall \{i, j\} \in \mathcal{E} \Leftrightarrow \|L^\dagger P\|_{\mathcal{E}, \infty} \leq \sin(\gamma)$

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E}, \infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$

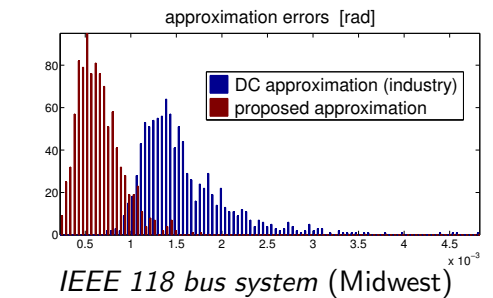
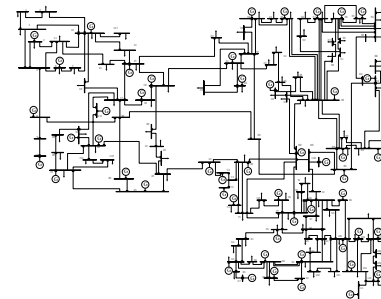


Reliability Test System RTS 96 under two loading conditions

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E}, \infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$



Outperforms conventional DC approximation “on average & in the tail”.

More on power flow approximations

Randomized power network test cases

with 50 % randomized loads and 33 % randomized generation

Randomized test case (1000 instances)	Numerical worst-case angle differences: $\max_{\{i,j\} \in \mathcal{E}} \theta_i^* - \theta_j^* $	Analytic prediction of angle differences: $\arcsin(\ L^\dagger P\ _{\mathcal{E}, \infty})$	Accuracy of condition: $\arcsin(\ L^\dagger P\ _{\mathcal{E}, \infty})$ - $\max_{\{i,j\} \in \mathcal{E}} \theta_i^* - \theta_j^* $
9 bus system	0.12889 rad	0.12893 rad	$4.1218 \cdot 10^{-5}$ rad
IEEE 14 bus system	0.16622 rad	0.16650 rad	$2.7995 \cdot 10^{-4}$ rad
IEEE RTS 24	0.22309 rad	0.22480 rad	$1.7089 \cdot 10^{-3}$ rad
IEEE 30 bus system	0.16430 rad	0.16456 rad	$2.6140 \cdot 10^{-4}$ rad
New England 39	0.16821 rad	0.16828 rad	$6.6355 \cdot 10^{-5}$ rad
IEEE 57 bus system	0.20295 rad	0.22358 rad	$2.0630 \cdot 10^{-2}$ rad
IEEE RTS 96	0.24593 rad	0.24854 rad	$2.6076 \cdot 10^{-3}$ rad
IEEE 118 bus system	0.23524 rad	0.23584 rad	$5.9959 \cdot 10^{-4}$ rad
IEEE 300 bus system	0.43204 rad	0.43257 rad	$5.2618 \cdot 10^{-4}$ rad
Polish 2383 bus system (winter peak 1999/2000)	0.25144 rad	0.25566 rad	$4.2183 \cdot 10^{-3}$ rad

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Discrete control actions to assure sync

① (re)dispatch generation subject to **security constraints**:

find $\theta \in \mathbb{T}^n, u \in \mathbb{R}^{n_l}$ subject to

source power balance:

$$u_i = P_i(\theta)$$

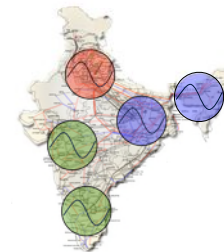
load power balance:

$$P_i = P_i(\theta)$$

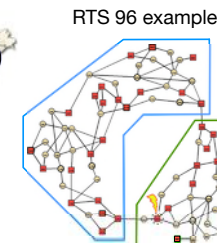
branch flow constraints:

$$|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$$

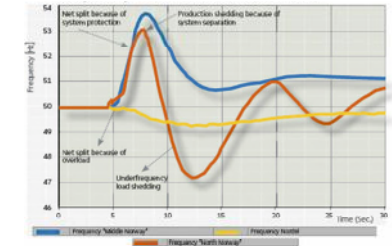
② remedial action schemes: load/production shedding & islanding



India, July 30/31 2012



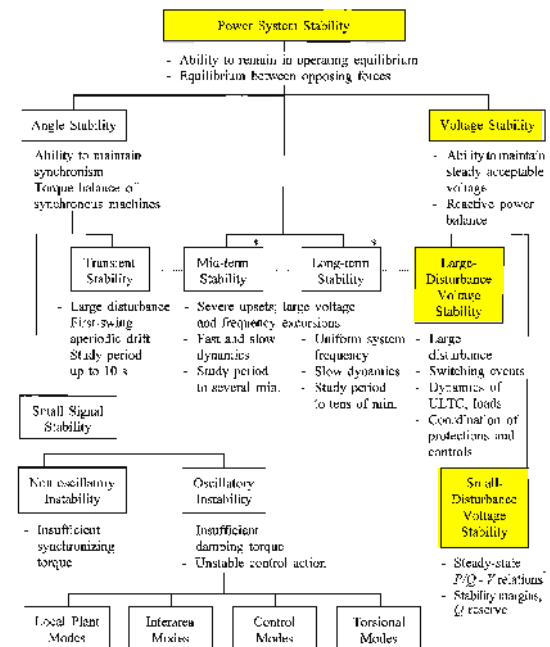
Nordic grid, December 1, 2005 (pacw.org)



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Decoupled Reactive Power Flow (Voltage Collapse)

Apparently a different beast



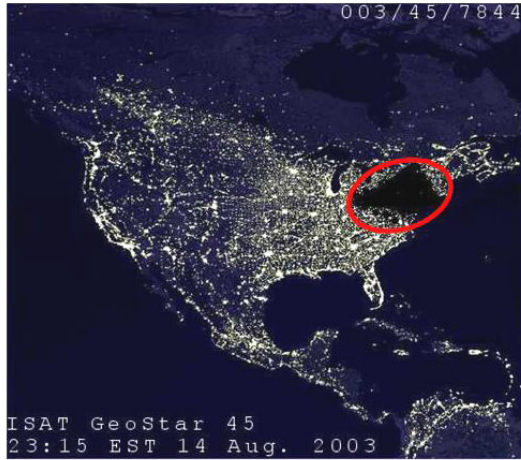
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Voltage collapse in power networks

- **voltage instability:** loading > capacity \Rightarrow voltages drop
"mainly" a reactive power phenomena
- **recent outages:** Québec '96, Scandinavia '03, Northeast '03, Athens '04

"Voltage collapse is still the biggest single threat to the transmission system. It's what keeps me awake at night."

– Phil Harris, CEO PJM.



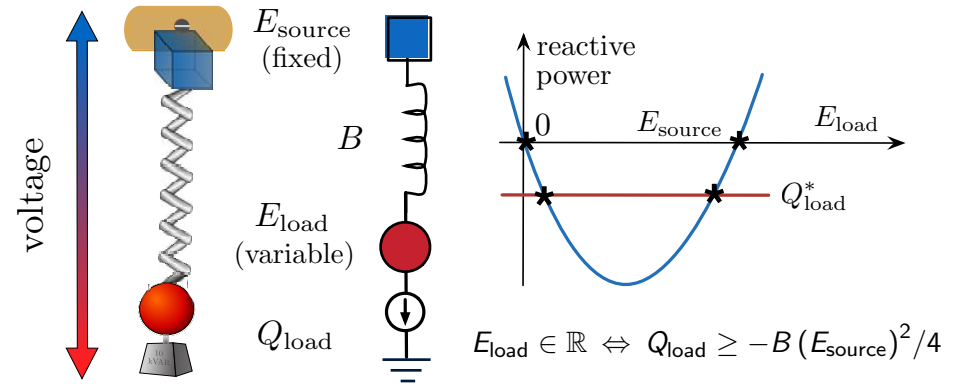
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Back of the envelope calculations for the two-node case

source connected to load shows bifurcation at load voltage $E_{load} = E_{source}/2$

reactive power balance at load:

$$Q_{load} = B E_{load}(E_{load} - E_{source})$$



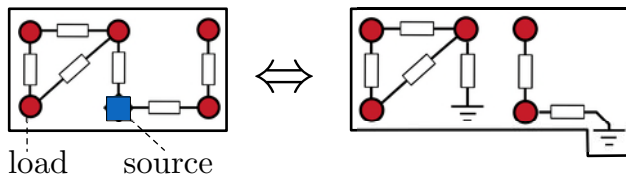
$$\exists \text{ high load voltage solution} \Leftrightarrow (\text{load}) < (\text{network})(\text{source voltage})^2/4$$

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Preliminary insights when going to "complex" networks

- **sources** with constant voltage magnitudes E_i
- **loads** with constant power demand $Q_i(E) = Q_i$

\Rightarrow WLOG assume that network among loads is connected



\Rightarrow reactive power balance: $Q_i = -\sum_j B_{ij} E_i E_j$ or $Q = -\text{diag}(E) B E$

\Rightarrow necessary feasibility condition: $\sum_{i=1}^n Q_i \geq 0 \Leftrightarrow \exists$ a solution

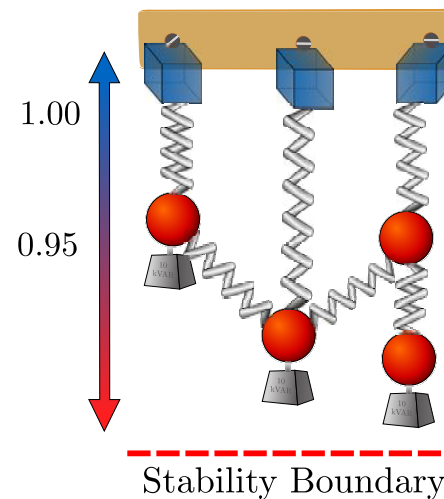
(by summing all equations and using $-E^T B E \geq 0$)

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Intuition extends to complex networks – essential insights

Reactive power balance:

$$Q_i = -\sum_j B_{ij} E_i E_j$$



Suff. & tight cond' for general case [J. Simpson-Porco, FD, & F. Bullo, '16]:

\exists unique high-voltage solution E_{load}
 \Leftrightarrow

$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$

1 nominal (zero load) voltage E_{nom}

$$0 = -\sum_j B_{ij} E_{i,nom} E_{j,nom}$$

2 coord-trafo to solution guess:

$$x_i = E_i / E_{i,nom} - 1$$

3 Picard-Banach iteration $x^+ = f(x)$

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Previous condition “ $\Delta < 1$ ” also predicts voltage deviation

for coupled & lossy power flow

Samples: randomized scenario (50% load and 33% generation variability)

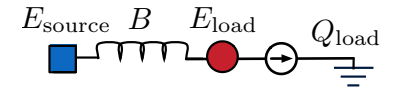
	Numerical	Theoretical	% Error
Randomized test case (1000 instances)	Numerical worst-case voltage deviations: $\delta_{\text{exact}} = \max_i \frac{ E_i - E_i^* }{E_i^*}$	Analytic prediction of voltage deviations: $\delta_- = (1 - \sqrt{1 - \Delta})/2$	Accuracy of prediction: $100 \cdot \frac{\delta_- - \delta_{\text{exact}}}{\delta_{\text{exact}}}$
9 bus system	$5.49 \cdot 10^{-2}$	$5.51 \cdot 10^{-2}$	0.366 %
IEEE 14 bus system	$2.50 \cdot 10^{-2}$	$2.51 \cdot 10^{-2}$	0.200 %
IEEE RTS 24	$3.23 \cdot 10^{-2}$	$3.24 \cdot 10^{-2}$	0.347 %
IEEE 30 bus system	$4.91 \cdot 10^{-2}$	$4.95 \cdot 10^{-2}$	0.806 %
New England 39	$6.26 \cdot 10^{-2}$	$6.30 \cdot 10^{-2}$	0.620 %
IEEE 57 bus system	$1.20 \cdot 10^{-1}$	$1.24 \cdot 10^{-1}$	3.60 %
IEEE RTS 96	$3.43 \cdot 10^{-2}$	$3.44 \cdot 10^{-2}$	0.376 %
IEEE 118 bus system	$2.60 \cdot 10^{-2}$	$2.61 \cdot 10^{-2}$	0.557 %
IEEE 300 bus system	$1.05 \cdot 10^{-1}$	$1.07 \cdot 10^{-1}$	1.76 %
Polish 2383 bus system (winter peak 1999/2000)	$3.99 \cdot 10^{-2}$	$4.02 \cdot 10^{-2}$	0.764 %

A tight & analytic guarantee: typical prediction error of $\sim 1\%$

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More back of the envelope calculations

$$Q_L = B E_L (E_L - E_S)$$



$$\Rightarrow E_L = E_S/2 \left(1 + \sqrt{1 + 4Q_L/(BE_S^2)} \right) = \frac{E_S}{2} \left(1 + \sqrt{1 - Q_L/Q_{\text{crit}}} \right)$$

\Rightarrow Taylor exp. for $Q_L/Q_{\text{crit}} \rightarrow 0$:

$$E_L \approx E_S (1 + Q_L/Q_{\text{crit}})$$

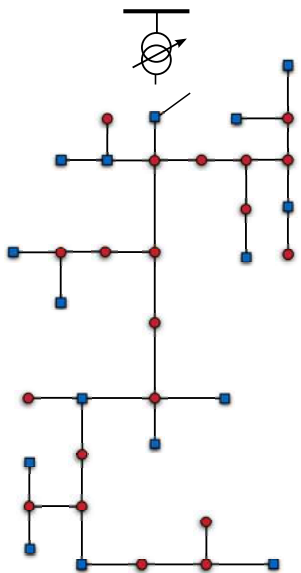
• **general case:** existence & approximation from implicit function thm

- if all loads Q_i are “sufficiently small” [D. Molzahn, B. Lesieutre, & C. DeMarco '12]
- if slack bus has “sufficiently large” E_S [S. Bolognani & S. Zampieri '12 & '14]
- if each source is above a “sufficiently large” E_{source} [B. Gentile et al. '14]
- if previous existence condition is met [J. Simpson-Porco, FD, & F. Bullo, '16]

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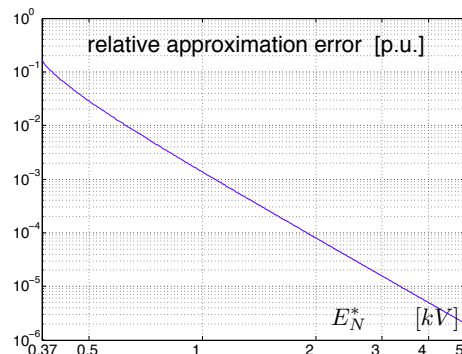
Linear DC approximation extends to complex networks

verification via IEEE 37 bus distribution system (SoCal)



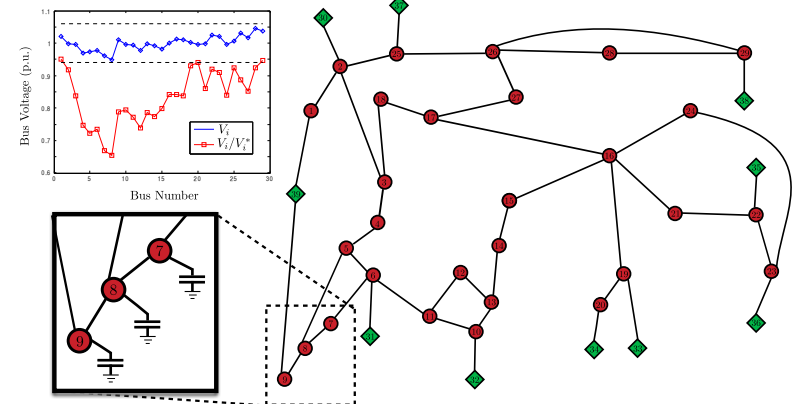
Reactive DC approximation [B. Gentile, J. Simpson-Porco, FD, S. Zampieri, & F. Bullo, '14]:

$$E_L \approx \text{diag}(E_L^*) (\mathbb{1} + Q_{\text{crit}}^{-1} Q_L) + \text{h.o.t.}$$



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Discrete control actions for voltage stability



- 1 shunts support voltage magnitudes, but hide proximity to collapse
 \Rightarrow ratios E_i/E_i^* more useful than per-unit voltages
- 2 $|Q_{\text{crit},89}^{-1}| > |Q_{\text{crit},87}^{-1}|$ means E_8/E_8^* more sensitive to Q_9 than to Q_7
 \Rightarrow place SVC at bus 9 to support E_8 & increase stability margin

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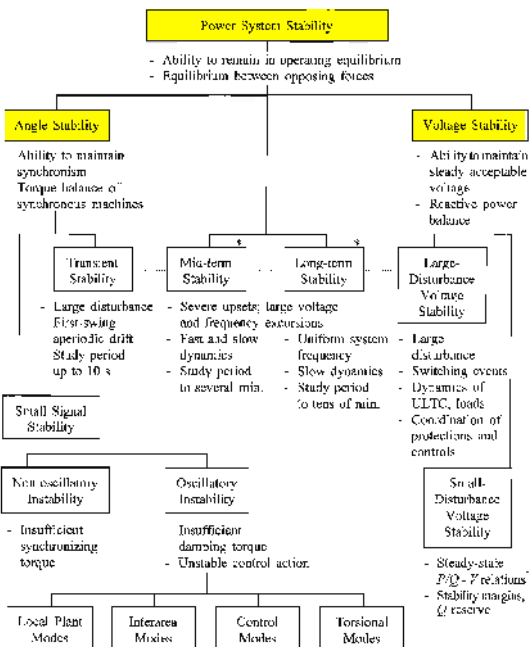
Coupled & Lossy Power Flow

Coupling matters!



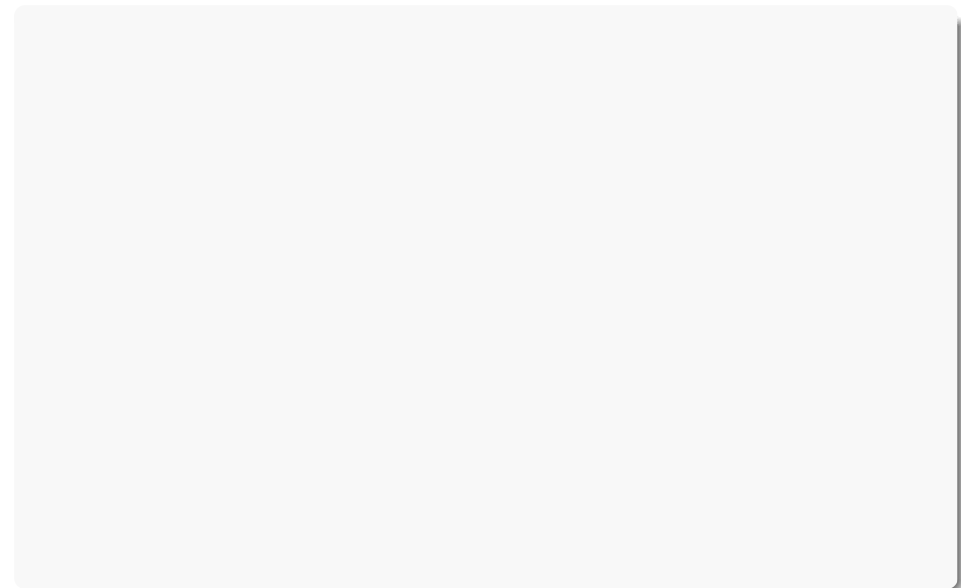
“As systems become more heavily loaded, nonlinearities play an increasingly important role in power system behavior . . . analysis tools should continue to work reliably, even under extreme system conditions . . . the $P - V$ and $Q - \theta$ cross coupling terms become significant.” — [Ian Hiskens, Proc. of IEEE '95]

This is not even really on the map



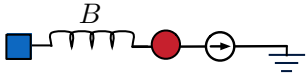
Solving the two-node case

see exercise



Simplest example shows surprisingly complex behavior

- PV source, PQ load, & lossless line



$$P = B E_{\text{source}} E_{\text{load}} \sin(\theta)$$

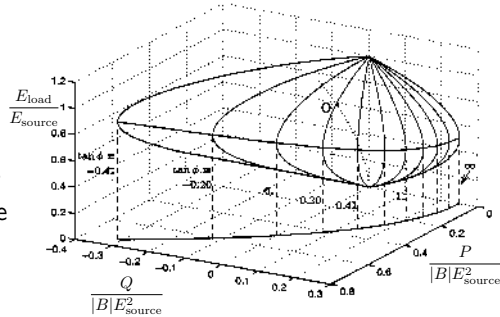
$$Q = B E_{\text{load}}^2 - B E_{\text{source}} E_{\text{load}} \cos(\theta)$$

- after eliminating θ , there exists $E_{\text{load}} \in \mathbb{R}_{\geq 0}$ if and only if

$$P^2 - B E_{\text{source}}^2 Q \leq B^2 E_{\text{source}}^4 / 4$$

- Observations:

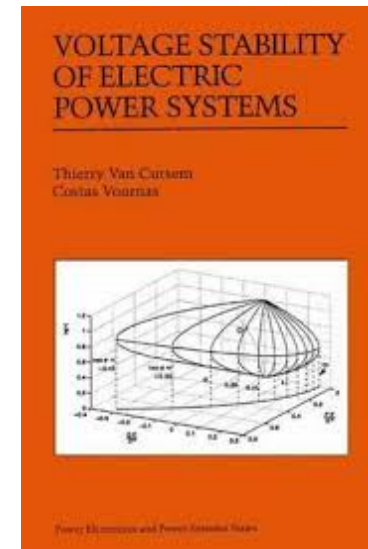
- 1 $P = 0$ case consistent with previous decoupled analysis
- 2 $Q = 0$ case delivers 1/2 transfer capacity from decoupled case
- 3 intermediate cases $Q = P \tan \phi$ give so-called “nose curves”



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Recommended reading to understand a glimpse

at least once in a life-time you should read chapter 2 ...



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Coupled & lossy power flow in complex networks

▶ active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$

▶ reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

- what makes it so much harder than the previous two node case?
 - losses, mixed lines, cycles, PQ-PQ connections, ...
- much theoretic work, qualitative understanding, & numeric approaches:
 - existence of solutions [Thorp, Schulz, & Ilić '86, Wu & Kumagai '82]
 - solution space [Hiskens & Davy '01, Overbye & Klump '96, Van Cutsem '98, ...]
 - distance-to-failure [Venikov '75, Abe & Isono '76, Dobson '89, Andersson & Hill '93, ...]
 - convex relaxation approaches [Molzahn et al. '12, Dvijotham et al. '15]
- little analytic & quantitative understanding beyond the two-node case

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“Whoever figures that one out [analysis of $n > 2$ node] wins a noble prize!”

— [Peter Sauer, lunch @ UIUC '13]

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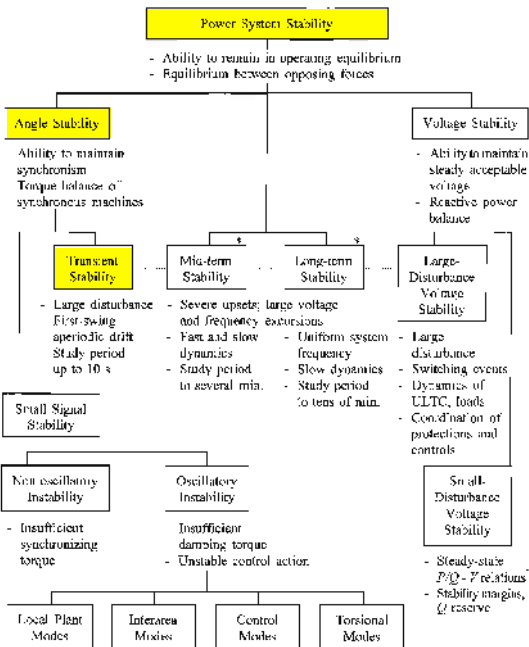
Transient Rotor Angle Stability



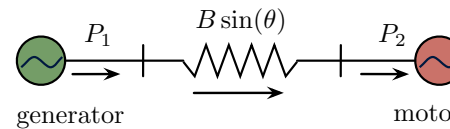
"The crown jewel of power system stability!"

— [Janusz Bialek, skype call '13]

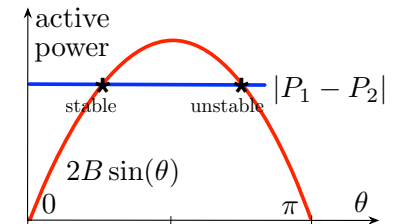
The crown jewel of power system stability



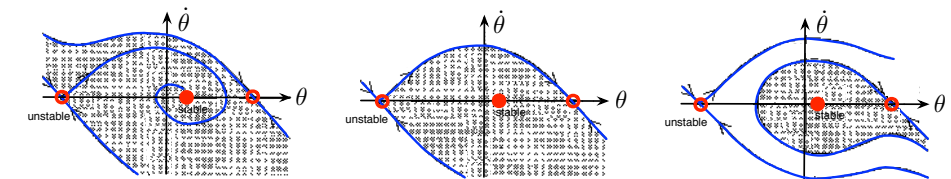
Revisit of the two-node case — the forced pendulum more complex than anticipated



$$M\ddot{\theta} = -D\dot{\theta} + P_1 - P_2 - 2B \sin(\theta)$$



- **Local stability:** ∃ local stable solution ⇔ $B > |P_1 - P_2|/2$
- **Global stability:** depends on gap $B > |P_1 - P_2|/2$ and D/M ratio



$(D/M) > (D/M)_{critical}$

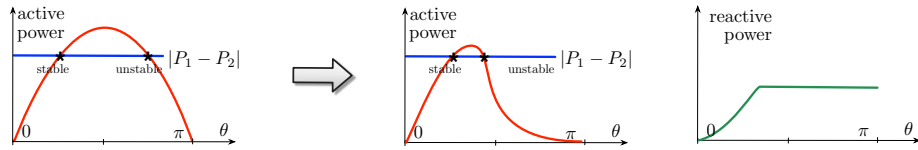
$(D/M) = (D/M)_{critical}$

$(D/M) < (D/M)_{critical}$

Revisit of the two-node case — cont'd

the story is not complete ... some further effects that we swept under the carpet

- **Voltage reduction:** generator needs to provide reactive power for voltage regulation – until saturation, then generator becomes PQ bus



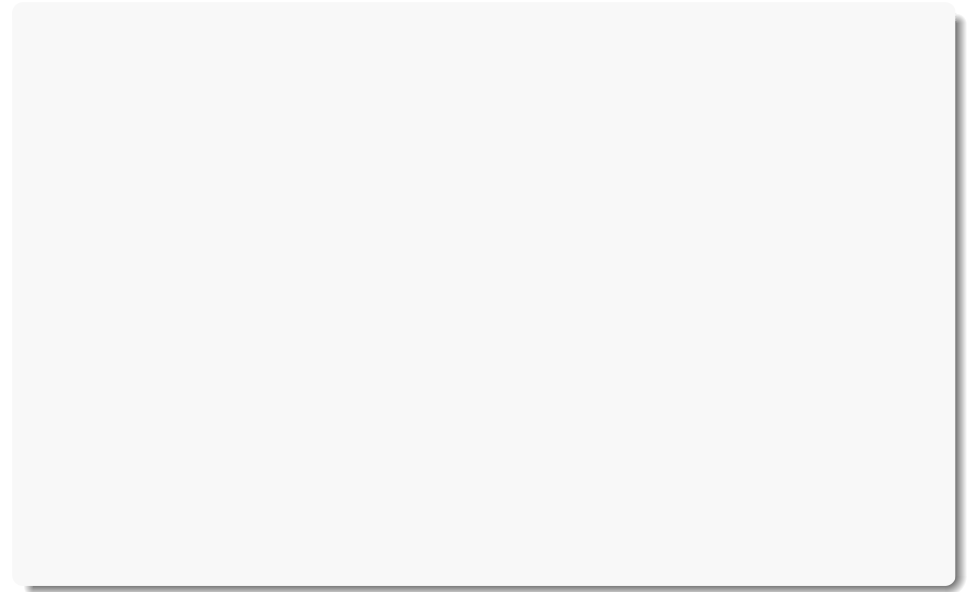
- **Load sensitivity:** different behavior depending on load model: resistive, constant power, frequency-dependent, dynamic, power electronics, ...
- **Singularity-issues** for coupled power flows (load voltage collapse)
- **Losses & higher-order dynamics** change stability properties ...

⇒ quickly run into computational approaches

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Primer on Lyapunov functions

on blackboard



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Transient stability in multi-machine power systems

$$\dot{\theta}_i = \omega_i$$

generators: $M_i \dot{\omega}_i = -D_i \omega_i + P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$

$$Q_i = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

loads: $D_i \dot{\theta}_i = P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$

$$Q_i = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

Challenge (improbable): faster-than-real-time transient stability assessment

Energy function methods for simple lossless models via Lyapunov function

$$V(\omega, \theta, E) = \sum_i \frac{1}{2} M_i \omega_i^2 - \sum_i P_i \theta_i - \sum_i Q_i \log E_i - \sum_{ij} B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

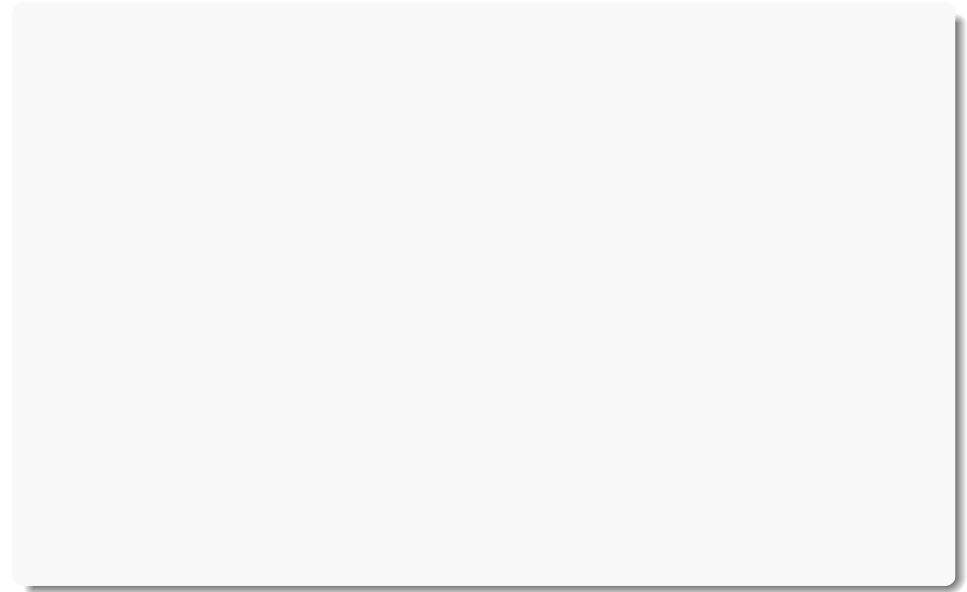
Computational approaches: level sets of energy functions & unstable equilibria, sum-of-squares methods, convex optimization approaches, time-domain simulations, ... (holy grail of power system stability)

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Hamiltonian analysis of the swing equations

more famously known as “energy function analysis”

(see exercise)



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Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

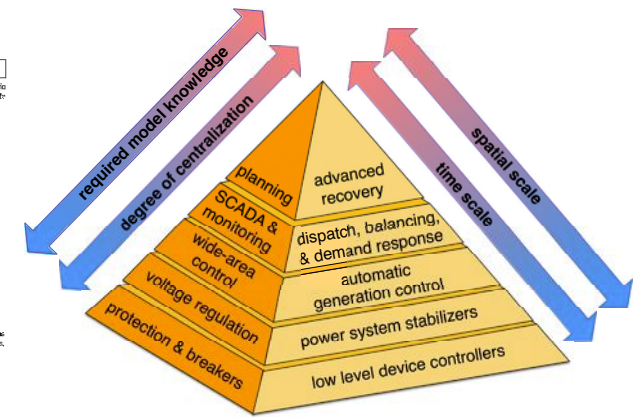
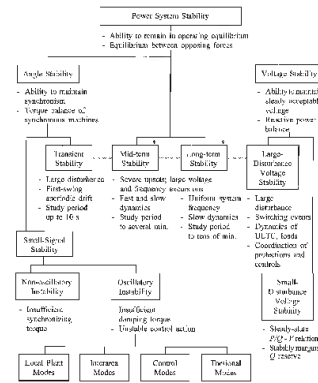
Power System Control Hierarchy

- Primary Control
- Power Sharing
- Secondary control
- Experimental validation (Optional material)

Power System Oscillations

Conclusions

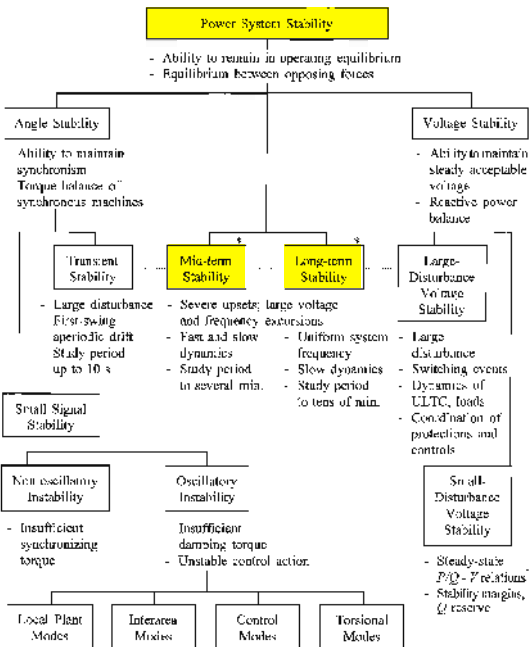
A plethora of control tasks and nested control layers organized in hierarchy and separated by states & spatial/temporal/centralization scales



We will focus on frequency control & primary/secondary/tertiary layers.

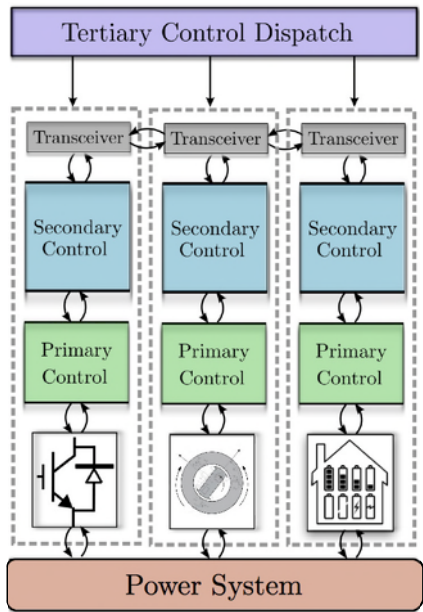
All dynamics & controllers are interacting. Classification & hierarchy are for simplicity.

Where are we on the map?



Objectives

Hierarchical frequency control architecture & objectives



3. **Tertiary control** (offline)
 - Goal: optimize operation
 - Strategy: centralized & forecast
2. **Secondary control** (minutes)
 - Goal: maintain operating point in presence of disturbances
 - Strategy: centralized
1. **Primary control** (real-time)
 - Goal: stabilize frequency & share unknown load
 - Strategy: decentralized

Q: Is this layered & hierarchical architecture still appropriate for tomorrow's power system?

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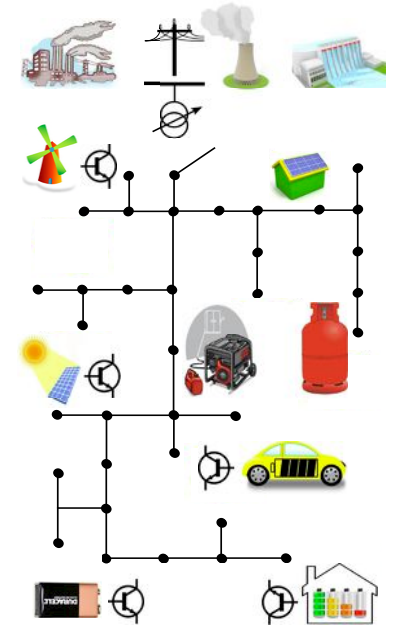
Is this hierarchical control architecture still appropriate?

Some recent developments

- ▶ increasing renewable integration & deregulated energy markets
- ▶ bulk generation replaced by distributed generation
- ▶ synchronous machines replaced by power electronics sources
- ▶ low gas prices & substitutions

Some new problem scenarios

- ▶ alternative spinning reserves: storage, load control, & DER
- ▶ networks of low-inertia & distributed renewable sources
- ▶ small-footprint islanded systems



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Need to adapt the control hierarchy in tomorrow's grid

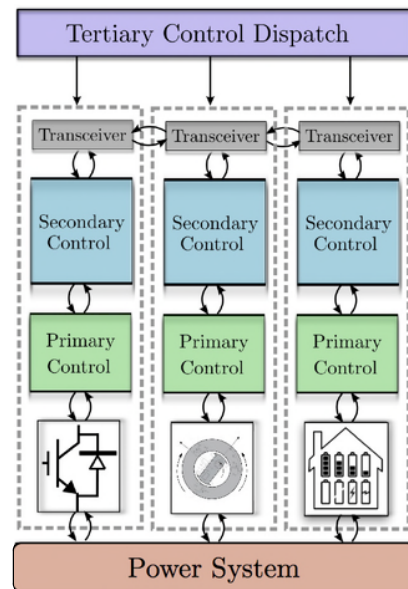
Operational challenges

- ▶ more uncertainty & less inertia
- ▶ more volatile & faster fluctuations
- ▶ plug'n'play control: fast, model-free, & without central authority

Opportunities

- ▶ re-instrumentation: comm & sensors
- ▶ more & faster spinning reserves
- ▶ advances in control of cyber-physical & complex systems

⇒ break vertical & horizontal hierarchy



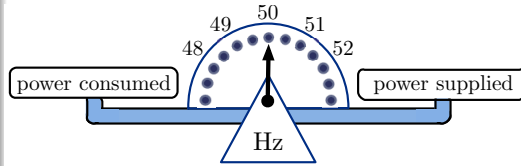
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Primary Control

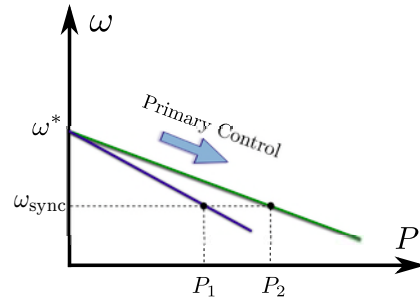
Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$



recall: $\omega_{\text{sync}} = \sum_i P_i^* / D_i$



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Conventional wisdom: physics are naturally stable & sync frequency reveals power imbalance

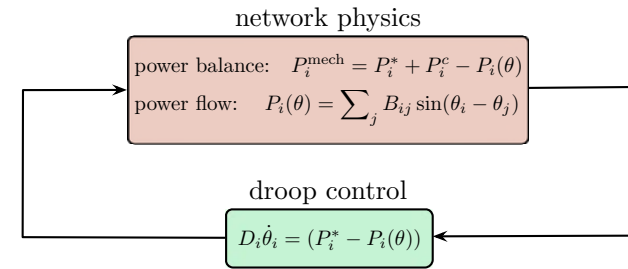
P/θ droop control:

$$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$$

$$\Updownarrow$$

$$D_i \dot{\theta}_i = P_i^* - P_i(\theta)$$

Putting the pieces together...



synchronous machines:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

inverter sources & controllable loads:

$$D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

passive loads &

power-point tracking sources:

$$0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

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Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

active power flow is feasible $\implies \exists$ unique & exp. stable frequency sync

Main **proof ideas** and some **further results**:

- stability via Jacobian & Lyapunov arguments

- synchronization frequency: $\omega_{\text{sync}} = \omega^* + \frac{\sum_{\text{sources}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{sources}} D_i}$
(\propto power balance)

- steady-state power injections: $\mathcal{P}_i = \begin{cases} P_i^* & (\text{load } \#i) \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & (\text{source } \#i) \end{cases}$
(depend on D_i & P_i^*)

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Closed-loop stability?

see exercise

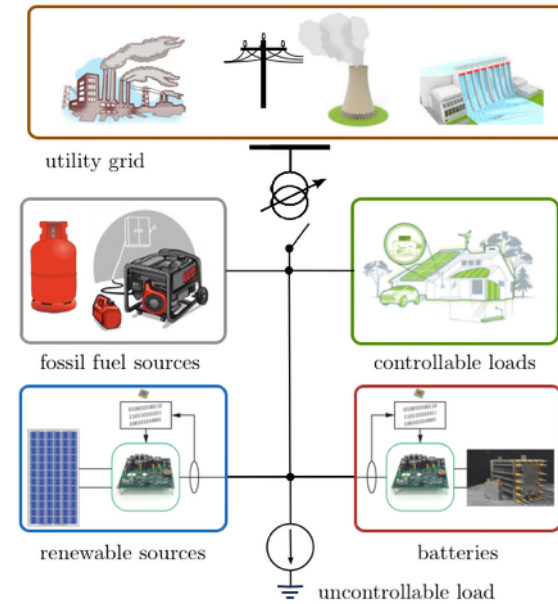
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power sharing & economic optimality under droop control

(sometimes in tertiary layer)

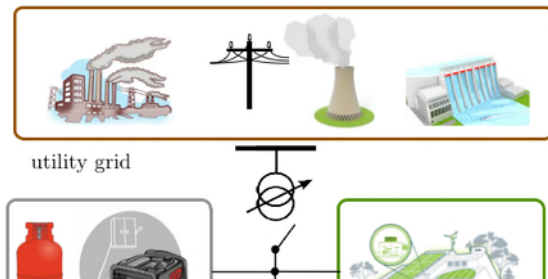
Tertiary control and energy management

an offline resource allocation and scheduling problem



Tertiary control and energy management

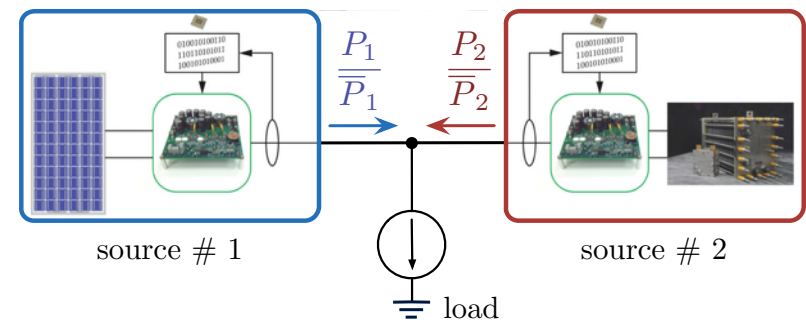
an offline resource allocation and scheduling problem



- minimize {cost of generation, losses, ... }
- subject to
- equality constraints: power balance equations
- inequality constraints: flow/injection/voltage constraints
- logic constraints: commit generators yes/no
- ⋮

Objective I: decentralized proportional load sharing

- 1) Sources have **injection constraints**: $P_i(\theta) \in [0, \bar{P}_i]$
- 2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \bar{P}_j$
- 3) **Fairness**: load should be shared proportionally: $P_i(\theta) / \bar{P}_i = P_j(\theta) / \bar{P}_j$



Objective I: decentralized proportional load sharing

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- 3) **Fairness**: load should be shared proportionally: $P_i(\theta) / \bar{P}_i = P_j(\theta) / \bar{P}_j$

A little calculation reveals in steady state:

$$\frac{P_i(\theta)}{\bar{P}_i} \stackrel{!}{=} \frac{P_j(\theta)}{\bar{P}_j} \Rightarrow \frac{P_i^* - (D_i \omega_{\text{sync}} - \omega^*)}{\bar{P}_i} \stackrel{!}{=} \frac{P_j^* - (D_j \omega_{\text{sync}} - \omega^*)}{\bar{P}_j}$$

... so choose

$$\frac{P_i^*}{\bar{P}_i} = \frac{P_j^*}{\bar{P}_j} \quad \text{and} \quad \frac{D_i}{\bar{P}_i} = \frac{D_j}{\bar{P}_j}$$

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Objective I: decentralized proportional load sharing

- 1) Sources have **injection constraints**: $P_i(\theta) \in [0, \bar{P}_i]$
- 2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \bar{P}_j$
- 3) **Fairness**: load should be shared proportionally: $P_i(\theta) / \bar{P}_i = P_j(\theta) / \bar{P}_j$

Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$D_i / \bar{P}_i = D_j / \bar{P}_j \quad \& \quad P_i^* / \bar{P}_i = P_j^* / \bar{P}_j$$

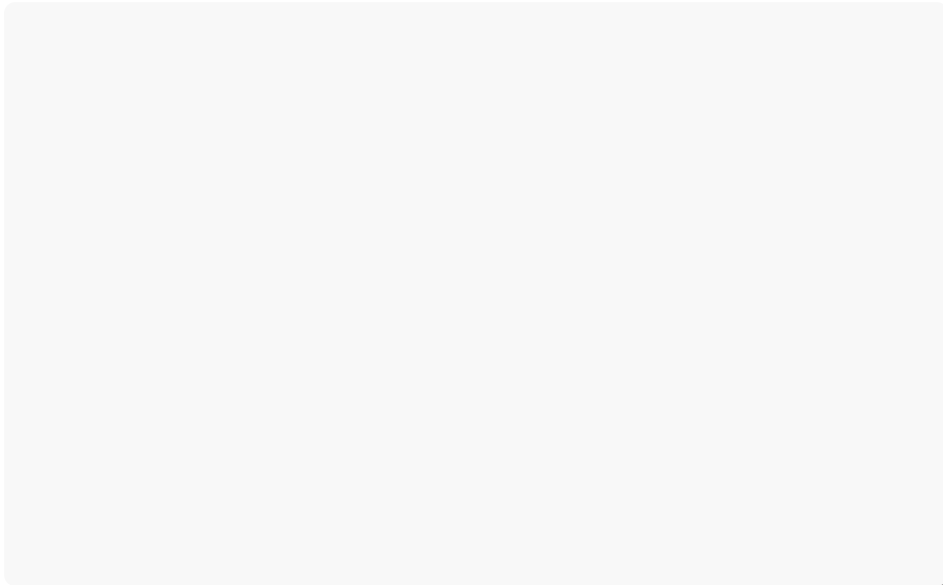
The the following statements hold:

- (i) Proportional load sharing: $P_i(\theta) / \bar{P}_i = P_j(\theta) / \bar{P}_j$
- (ii) Constraints met: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \bar{P}_j \Leftrightarrow P_i(\theta) \in [0, \bar{P}_i]$

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Constraints achieved by fair proportional load sharing

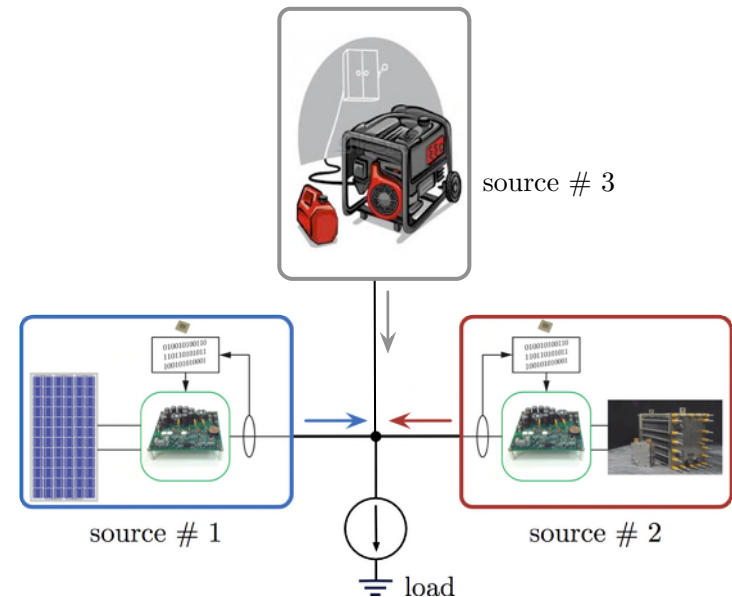
see exercise



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Objective I: fair proportional load sharing

proportional load sharing is not always the right objective



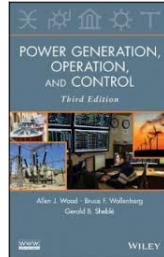
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Objective II: optimal power flow = tertiary control

an offline resource allocation/scheduling problem

minimize {cost of generation, losses, ... }
 subject to
 equality constraints: power balance equations
 inequality constraints: flow/injection/voltage constraints
 logic constraints: commit generators yes/no
 ⋮

Will be discussed more in detail by Andrej.



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Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

minimize $\theta \in \mathbb{T}^n, u \in \mathbb{R}^n$ $J(u) = \sum_{\text{sources}} \alpha_i u_i^2$
 subject to
 source power balance: $P_i^* + u_i = P_i(\theta)$
 load power balance: $P_i^* = P_i(\theta)$
 branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

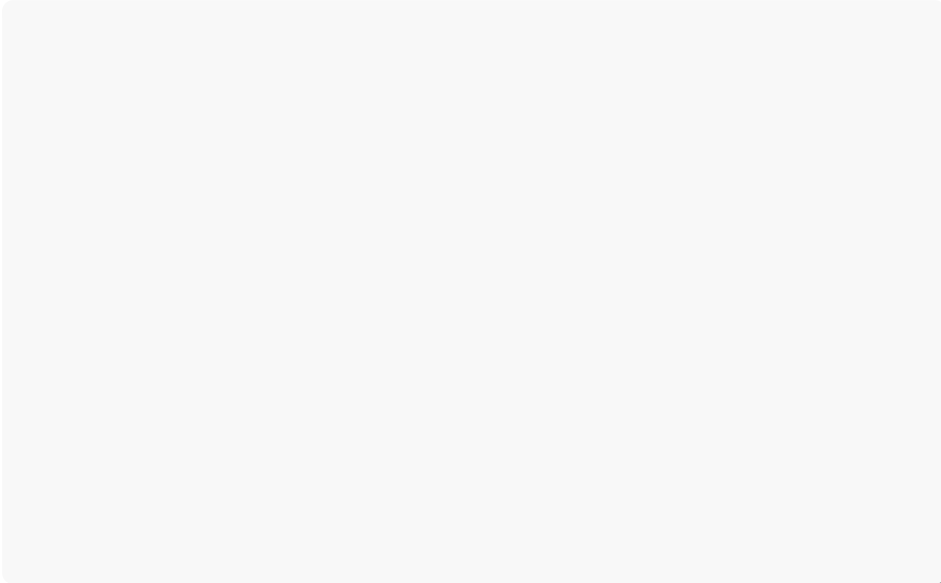
A simpler & equivalent (in the strictly feasible case) problem formulation:

minimize $\theta \in \mathbb{T}^n, u \in \mathbb{R}^n$ $J(u) = \sum_{\text{sources}} \alpha_i u_i^2$
 subject to
 power balance: $\sum_i P_i^* + \sum_i u_i = 0$

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The abc of resource allocation

on blackboard



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Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

minimize $\theta \in \mathbb{T}^n, u \in \mathbb{R}^n$ $J(u) = \sum_{\text{sources}} \alpha_i u_i^2$
 subject to
 source power balance: $P_i^* + u_i = P_i(\theta)$
 load power balance: $P_i^* = P_i(\theta)$
 branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

Unconstrained case: identical marginal costs $\alpha_i u_i^* = \alpha_j u_j^*$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

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Objective II: decentralized dispatch optimization

Insight: droop-controlled system = decentralized optimization algorithm

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

- (i) the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer (θ^*, u^*) .
- (ii) \exists droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ .

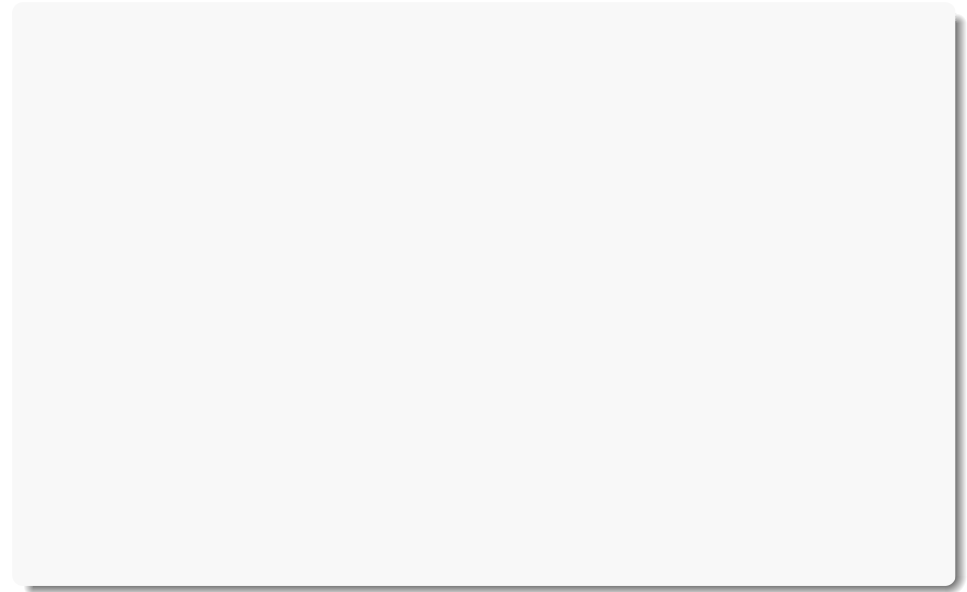
If (i) & (ii) are true, then $\theta_i \sim \theta_i^*$, $u_i^* = -D_i(\omega_{\text{sync}} - \omega^*)$, & $D_i \alpha_i = D_j \alpha_j$.

- includes proportional load sharing $\alpha_i \propto 1/\bar{P}_i$
- similar results hold for strictly convex & differentiable cost

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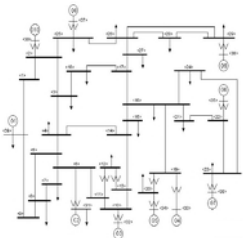
Sketch of the main proof ideas

see exercise

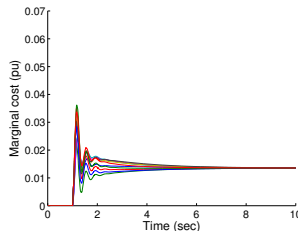


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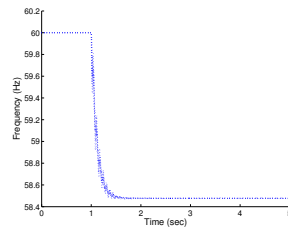
Some quick simulations & extensions



IEEE 39 New England
with load step at 1s



$t \rightarrow \infty$: convergence to
identical marginal costs



$t \rightarrow \infty$: frequency
 \propto power imbalance

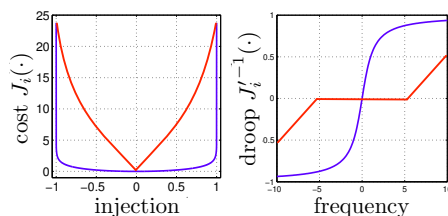
\Rightarrow strictly convex & differentiable cost

$$J(u) = \sum_{\text{sources}} J_i(u_i)$$

\Rightarrow non-linear frequency droop curve

$$J_i'^{-1}(\hat{\theta}_i) = P_i^* - P_i(\theta)$$

\Rightarrow include dead-bands, saturation, etc.

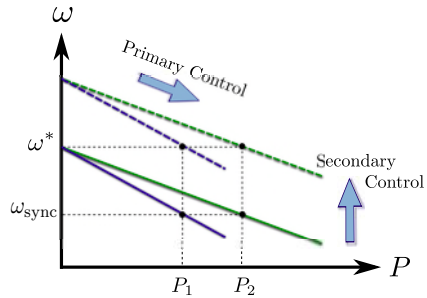


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Secondary Control

Secondary frequency control

- **Problem:** steady-state frequency deviation ($\omega_{\text{sync}} \neq \omega^*$)
- **Solution:** integral control of frequency error
- **Basics** of integral control $\left[\frac{1}{s}\right]$:



1 discrete time: $u_i(t+1) = u_i(t) + k \cdot \dot{\theta}_i(t)$ with gain $k > 0$

2 continuous-time: $u_i(t) = k \cdot \int_0^t \dot{\theta}_i(\tau) d\tau$ or $\dot{u}_i(t) = k \cdot \dot{\theta}_i(t)$

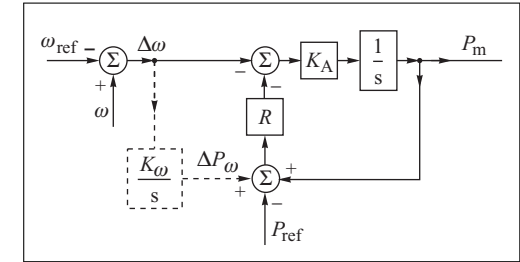
$\Rightarrow \dot{\theta}_i(t)$ is zero in (a possibly stable) steady state

\Rightarrow add additional injection $u_i(t)$ to droop control

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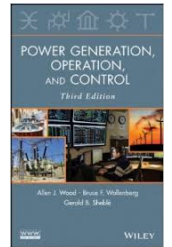
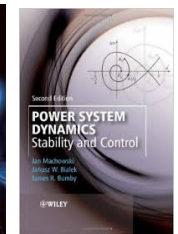
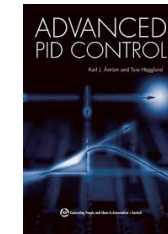
Decentralized secondary integral frequency control

- $\left[\frac{1}{s}\right]$ add local integral controller to every droop controller
- \Rightarrow zero frequency deviation \checkmark
- \Rightarrow nominally globally stabilizing [C. Zhao, E. Mallada, & FD, '14] \checkmark



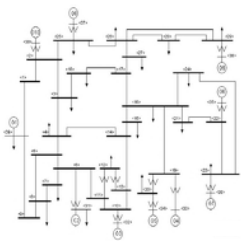
turbine governor integral control loop

- ☹ every integrator induces a 1d equilibrium subspace
- ☹ injections live in subspace of dimension $\#$ integrators
- ☹ load sharing & economic optimality are lost . . .
- ☹ unstable in presence of biased noise [M. Andreasson et al. '14]

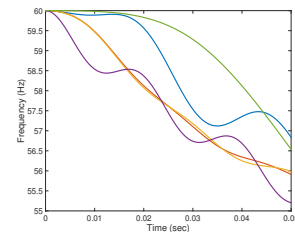


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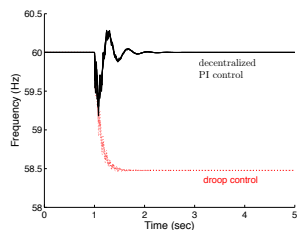
Simulations cont'd



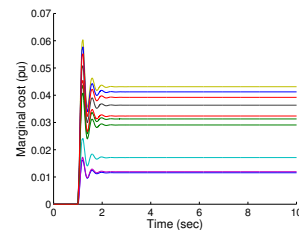
IEEE 39 New England with decentralized PI control



decentralized PI control in presence of biased noise



$t \rightarrow \infty$: decentralized PI control regulates frequency

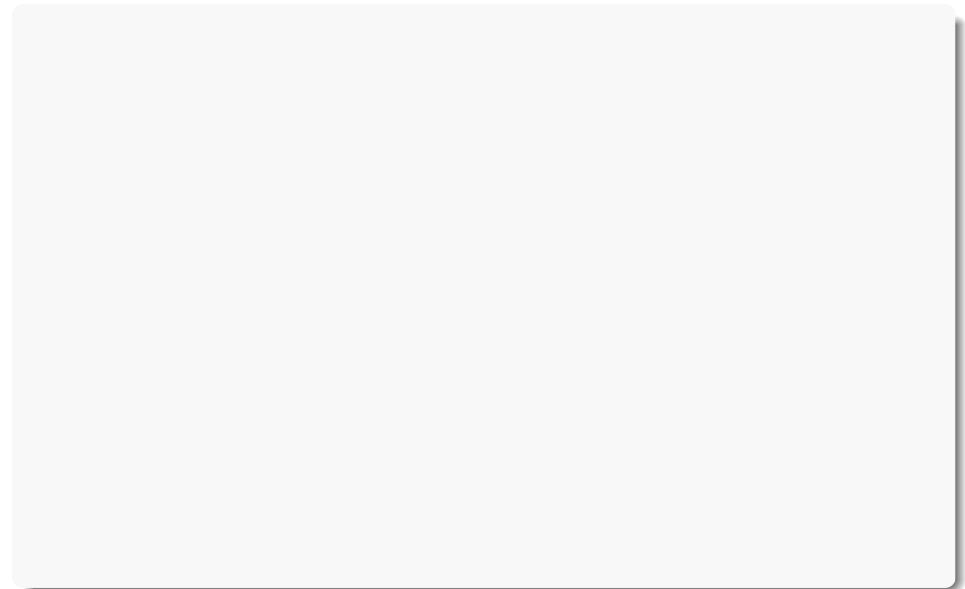


$t \rightarrow \infty$: decentralized PI control is not optimal

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Why does decentralized integral control not work?

see exercise



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Automatic generation control (AGC)

- **ACE** area control error =
 $\{ \text{frequency error} \} +$
 $\{ \text{generation - load - tie-line flow} \}$

$\frac{1}{s}$

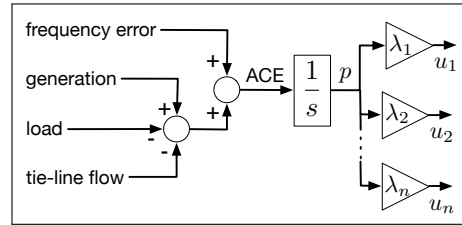
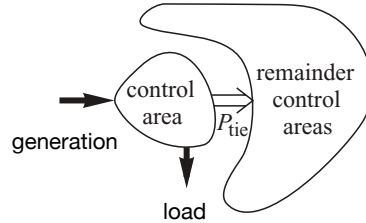
centralized integral control:

$$p(t) = \int_0^t \text{ACE}(\tau) d\tau$$

- **generation allocation:**
 $u_i(t) = \lambda_i p(t)$, where λ_i is
generation participation factor
(in our case $\lambda_i = 1/\alpha_i$)

⇒ assures identical marginal
costs: $\alpha_i u_i = \alpha_j u_j$

- 😊 load sharing & economic
optimality are recovered



AGC implementation

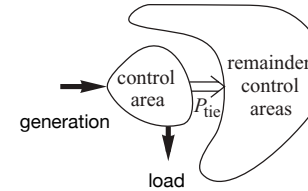
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Drawbacks of conventional secondary frequency control

interconnected systems

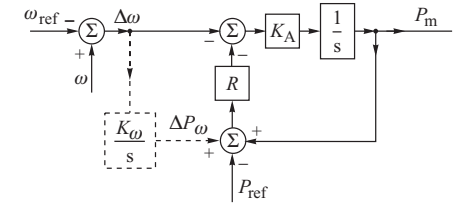
isolated systems

- **centralized** automatic
generation control (AGC)



compatible with econ. dispatch
[N. Li, L. Chen, C. Zhao, & S. Low '13]

- **decentralized** PI control



nominally *globally* stabilizing
[C. Zhao, E. Mallada, & FD, '14]

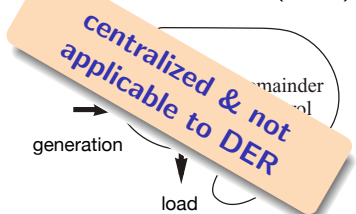
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Drawbacks of conventional secondary frequency control

interconnected systems

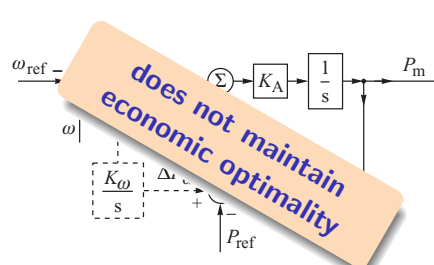
isolated systems

- **centralized** automatic
generation control (AGC)



compatible with econ. dispatch
[N. Li, L. Chen, C. Zhao, & S. Low '13]

- **decentralized** PI control



nominally *globally* stabilizing
[C. Zhao, E. Mallada, & FD, '14]

Distributed energy resources require **distributed (!)** secondary control.

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An incomplete literature review of a busy field

ntwk with unknown disturbances \cup integral control \cup distributed averaging

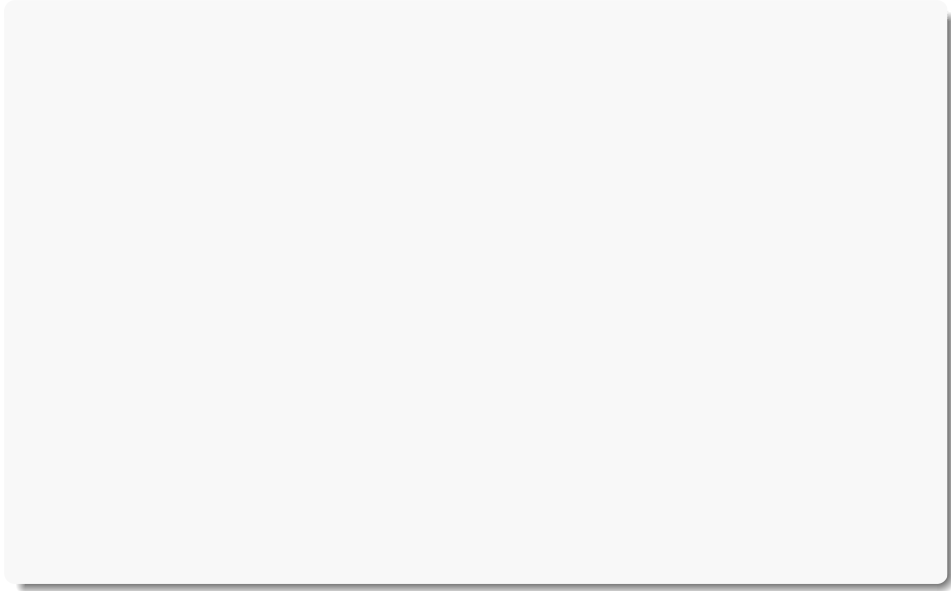
- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcia-Dominguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]

The following idea precedes most references, it's simpler, & it's more robust.

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Let's derive a simple distributed control strategy

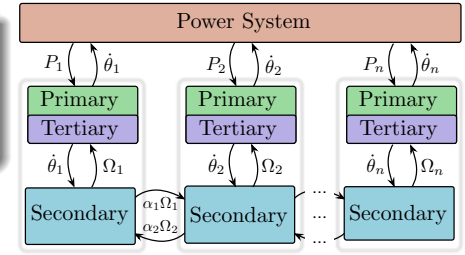
on blackboard



Distributed Averaging PI (DAPI) control

$$D_i \dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i \dot{\Omega}_i = D_i \dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot (\alpha_j \Omega_j - \alpha_i \Omega_i)$$



- no tuning & no time-scale separation: $k_i, D_i > 0$
- recovers optimal dispatch
- distributed & modular: connected comm. network
- has seen many extensions [C. de Persis et al., H. Sandberg et al., J. Schiffer et al., M. Zhu et al., ...]

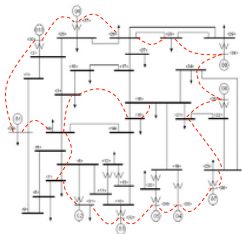
Theorem: stability of DAPI

[J. Simpson-Porco, FD, & F. Bullo '12]

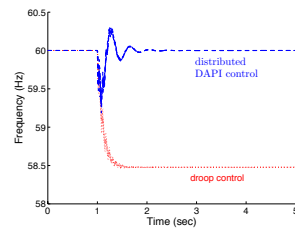
[C. Zhao, E. Mallada, & FD '14]

primary droop controller works
 \iff
 secondary DAPI controller works

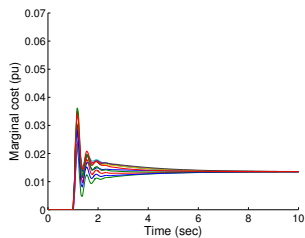
Simulations cont'd



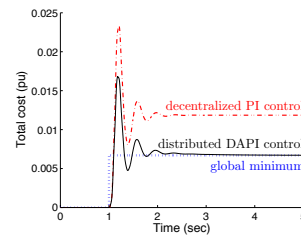
IEEE 39 New England with distributed DAPI control



$t \rightarrow \infty$: DAPI control regulates frequency



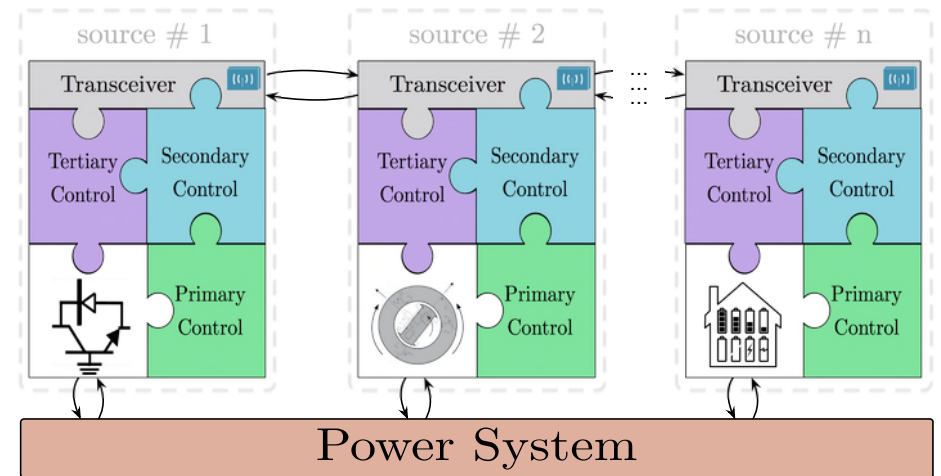
DAPI control synchronizes marginal costs



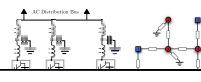
DAPI control minimizes cost with little effort

Plug'n'play architecture

flat hierarchy, distributed, no time-scale separations, & model-free



We can do similar things on the reactive power side

<p>Real-time Decentralized Voltage Control in Distribution Networks Na Li, Guzman Qu, Munther Dahleh</p> <p><i>Abstract</i>—Voltage control plays an important role in the operation of electricity distribution networks, especially when there is a large penetration of renewable energy resources. In this paper, we focus on voltage control through reactive power compensation.</p>	<p>Voltage Stabilization in Microgrids via Quadratic Droop Control John W. Simpson-Porco, Member, IEEE, Florian Dörfler, Member, IEEE, and Francesco Bullo, Fellow, IEEE</p> <p><i>Abstract</i>—We consider the problem of voltage stability and power balancing in islanded multi-source electrical networks with DC/AC inverters ("microgrids"). A droop-based feedback controller is proposed which is quadratic in local voltage magnitudes, allowing for the application of theoretical analysis techniques to the closed-loop system. Operating points of the closed-loop microgrid are in exact</p> 
<p>Voltage stability and reactive power sharing in inverter-based microgrids with consensus-based distributed voltage control Johannes Schiffer, Thomas Seel, Jörg Raisch, Tevfik Sezici</p> <p><i>Abstract</i>—We propose a consensus-based distributed voltage control (DVC), which solves the problem of reactive power sharing in a distributed manner. Essential components in power systems are</p>	<p>Voltage stress minimization by optimal reactive power control Marco Todescato, John W. Simpson-Porco, Florian Dörfler, Ruggiero Carli and Francesco Bullo</p> <p><i>Abstract</i>—A standard operational requirement in power systems is that the voltage magnitudes lie within pre-specified bounds. Conventional engineering wisdom suggests that such a tightly-regulated profile, imposed for system design purposes and good operation of the network, should also guarantee a secure system, operating far from static bifurcation instabilities such as voltage collapse. In general however, these two objectives are distinct and need to be separately enforced. We formulate an optimization problem which maximizes the distance to voltage collapse through injections of reactive power, subject to power flow and operational voltage constraints. By exploiting a linear approximation of the power flow equations, we arrive at a</p>
<p>Equilibrium and Dynamics of Local Voltage Control in Distribution Systems Masoud Farivar, Lijun Chen, Steven Low</p> <p><i>Abstract</i>—We consider a class of local voltage control schemes where the control decision on the reactive power at a bus depends only on the local bus voltage. These local algorithms form a feedback dynamical system and collectively determine the bus voltages of a power network. We show that the dynamical system has a unique equilibrium by interpreting the dynamics as a distributed algorithm for solving a certain convex optimization problem whose unique optimal point is the system equilibrium. Moreover, the objective function serves as a Lyapunov function implying global asymptotic stability of the equilibrium. The optimization based model does not only</p>	<p>Optimal Power Flow Pursuit Emiliano Dall'Anese and Andrea Simonetto</p> <p><i>Abstract</i>—This paper considers distribution networks featuring interfacial distributed energy resources, and develops local feedback controllers that continuously drive the injected power to solutions of AC optimal power flow (OPF). In particular, the controllers update the power setpoints in voltage measurements as well as given (time-varying) targets, and entail elementary operations implementable in least microcontrollers that accompany power-electronic devices of gateways and inverters. The design of the control law is based on suitable linear approximations of the power flow equations as well as Lagrangian regularization. Convergence and OPF-target tracking capabilities of the feeders are analytically established. Overall, the proposal</p>
<p>A distributed control strategy for reactive power compensation in smart microgrids Saverio Bolognani and Sandro Zampieri</p> <p><i>Abstract</i>—We consider the problem of optimal reactive power compensation for the minimization of power distribution losses in a smart microgrid. We first propose an approximate model for the power distribution network, which allows us to cast the problem into the class of convex quadratic, linearly constrained, optimization problems. We then consider the specific problem of commanding the microgrids connected to the microgrid, in order to achieve the optimal injection of reactive power. For this task, we design a randomized gossip-like optimization algorithm. We show how a distributed approach is possible, where micrograders need to have only a partial knowledge of the problem parameters and of the state, and can perform</p>	<p>with correct reformulations/approximations of the OPF, and utilize iterative primal-dual-type methods to decompose the solution of the OPF task across devices [8], [13], [14].</p> <p>OPF approaches have been successfully applied to optimize the operation of transmission systems. However, the time required to collect all the problem inputs (e.g., loads across the network and available RES power) and solve the OPF task may not be consistent with underlying distribution-system dynamics. For example, Figure 1 provides a snapshot of the leading of five secondary transformers located in a distribution feeder in Anatolia, CA [15]; in this case, it is apparent</p>

Much recent work on reactive power control

- heuristic linear Q/E droop: $(E_i - E_i^*) \propto (Q_i^* - Q_i(E))$ sometimes with integrator & nonlinearities [J. Simpson-Porco et. al. '16]
- reactive power sharing DAPI [J. Simpson-Porco et. al. '15, J. Schiffer et al. '16]

$$\kappa_i \dot{e}_i = \sum_{j \in \text{sources}} a_{ij} \cdot (Q_i / \bar{Q}_i - Q_j / \bar{Q}_j) - \varepsilon e_i$$
- voltage regulation [M. Farivar et al. '13]: $\kappa_i \dot{e}_i = E_i - E_i^*$
- loss minimization: minimize $\sum_{\{i,j\} \in \mathcal{E}} B_{ij} (E_i - E_j)^2$ [N. Li et al. '14]
- robustness margins: maximize det (Jacobian) [M. Todescato et al. '16]
- maximize reactive reserves s.t. flat voltage profile $E_i \approx 1$ [RTE France]

Main distinction to active power: while each of these objectives is individually feasible, they are also all **mutually exclusive** ...

A great unifying perspective on secondary control

pretty much incorporating everything that we've discussed this far

A unifying energy-based approach to optimal frequency and market regulation in power grids
Tjerk Stegink and Claudio De Persis and Arjan van der Schaft

Abstract—In this paper we provide a unifying energy-based approach to the modeling, analysis and control of power systems and markets, which is based on the port-Hamiltonian framework. Using a primal-dual gradient method applied to the social welfare problem, a distributed dynamic pricing algorithm in port-Hamiltonian form is obtained. By interconnection with the physical model a closed-loop port-Hamiltonian system is obtained, whose properties are exploited to prove asymptotic

A modular design of incremental Lyapunov functions for microgrid control with power sharing
C. De Persis and N. Monshizadeh

Abstract—In this paper we contribute a theoretical framework that sheds a new light on the problem of microgrid analysis and control. The starting point is an energy function comprising the kinetic energy associated with the elements that emulate the rotating machinery and terms taking into account the reactive

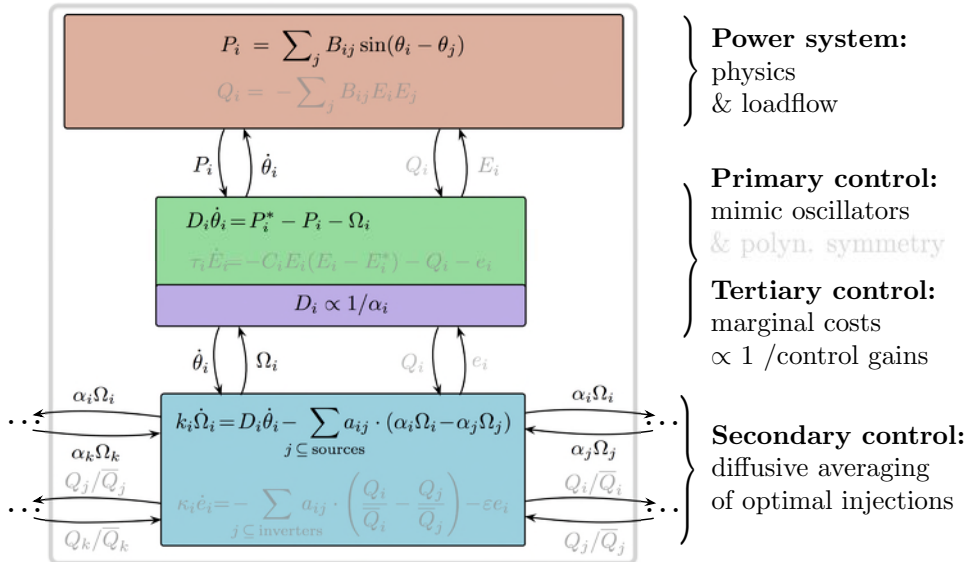
additional requirement of achieving zero frequency deviation with respect to the nominal value (e.g. 50 Hz), under the assumption that the voltages amplitudes are regulated to be constant. The second problem we consider is to minimize the total (quadratic) generation cost in the presence of a constant unknown and uncontrollable power consumption, while achieving zero frequency deviation. In the sequel, this

these quantities are sinusoidal terms depending on the voltage phasor relative phases. As a result, mathematical models of microgrids reduce to high-order oscillators interconnected via sinusoidal coupling. Moreover the coupling weights depend on

plug-and-play experiments

Plug'n'play architecture

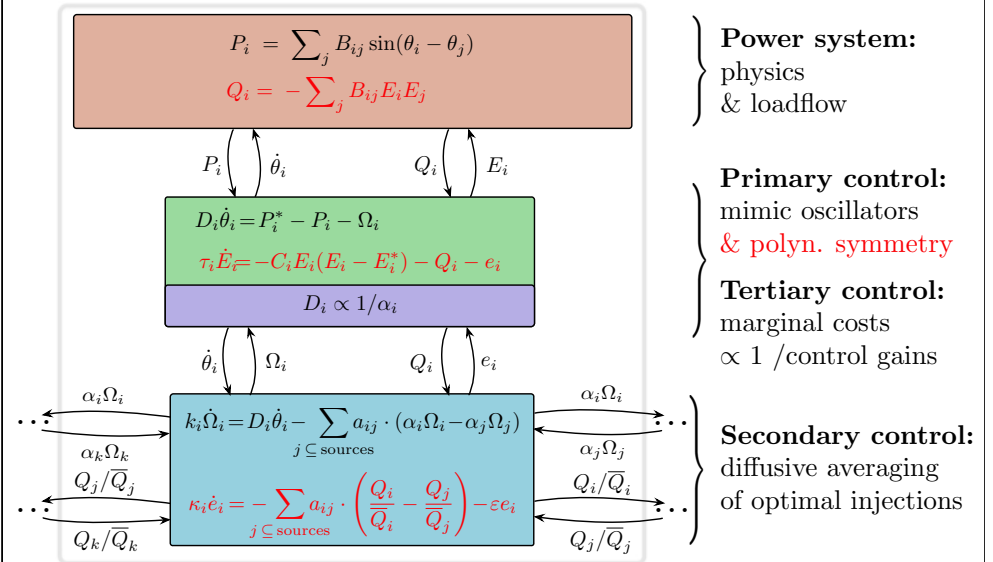
recap of detailed signal flow (active power only)



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Plug'n'play architecture

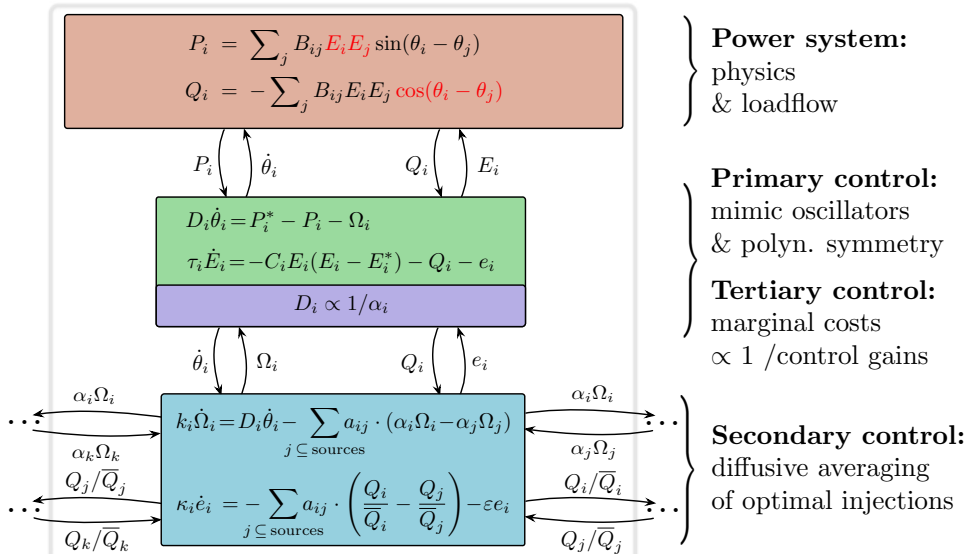
similar results for **decoupled reactive power flow** [J. Simpson-Porco, FD, & F. Bullo '13 - '15]



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Plug'n'play architecture

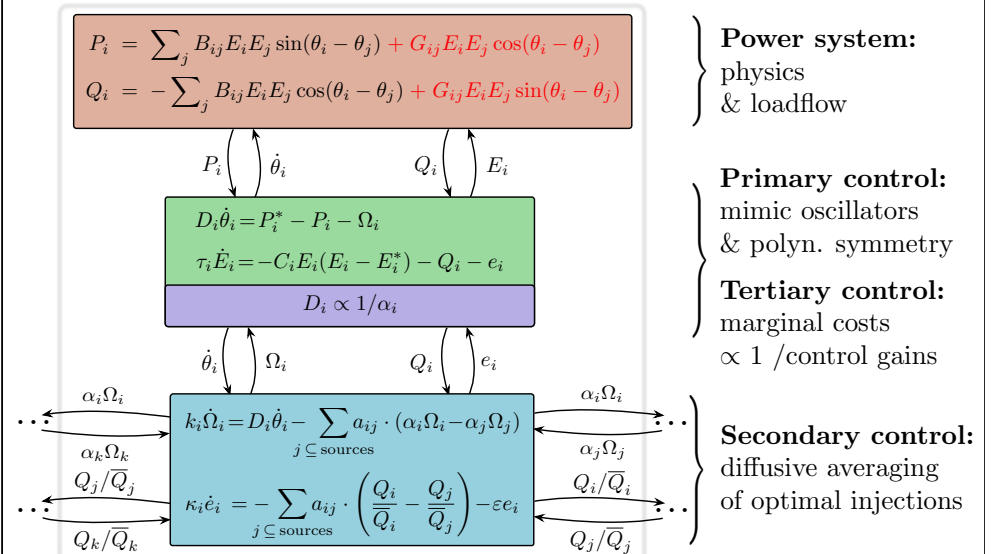
can all be proved also in the **coupled case** [N. Monshizadeh & C. de Persis, '15]



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Plug'n'play architecture

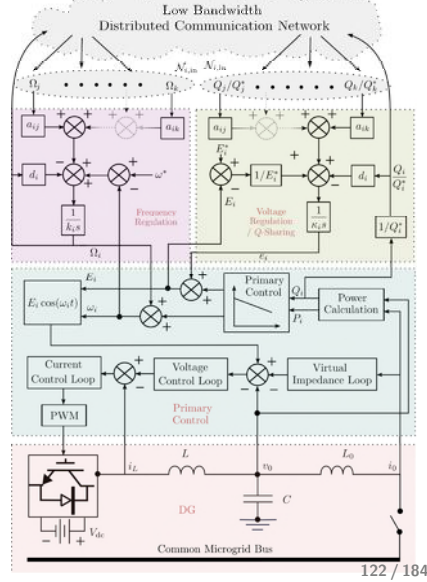
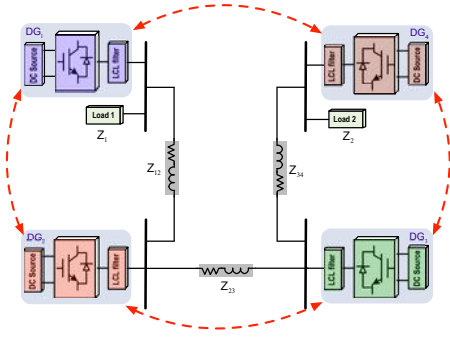
experiments also work well in the **lossy case**



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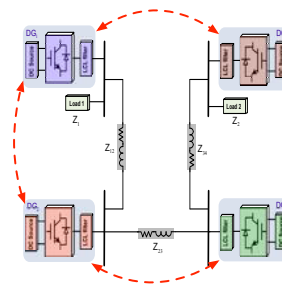
Experimental validation of control & opt. algorithms

in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University

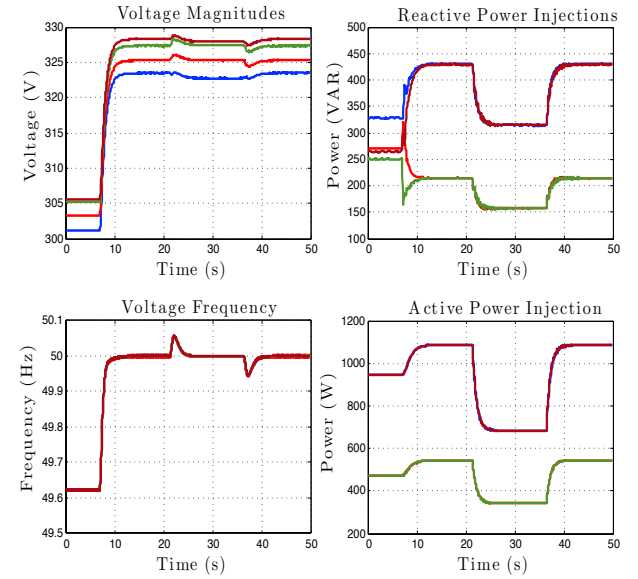


Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing



$t \in [0s, 7s]$: primary & tertiary control
 $t = 7s$: secondary control activated
 $t = 22s$: load # 2 unplugged
 $t = 36s$: load # 2 plugged back

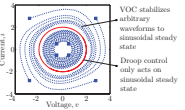


There are also many exciting alternatives to droop control

Uncovering Droop Control Laws Embedded Within the Nonlinear Dynamics of Van der Pol Oscillators

Mohit Sinha, Florian Dörfler, Member, IEEE,
 Brian B. Johnson, Member, IEEE, and Sairaj V. Dhople, Member, IEEE

Abstract—This paper examines the dynamics of power-electronic inverters in islanded microgrids that are controlled to emulate the dynamics of Van der Pol oscillators. The general strategy of controlling inverters to emulate the behavior of nonlinear oscillators presents a compelling time-domain alternative to ubiquitous droop control methods which presume the existence of a quasi-steady sinusoidal steady state and operate on phase quantities. We present two main results in this work. First, by leveraging the method of periodic averaging, we demonstrate that droop laws are an asymptotically embedded control law. Second, we establish the global convergence of amplitude and phase dynamics in a nonlinear network of distributed inverters controlled as Van der Pol oscillators. Furthermore, under a set of nonrestrictive decoupling approximations, we derive sufficient conditions for local asymptotic stability of desirable equilibria of the linearized amplitude and phase dynamics.



I. INTRODUCTION
 A nonlinear inverter-based microgrid is a collection of heterogeneous DC energy resources, e.g., photovoltaic (PV) arrays, fuel cells, and energy-storage devices, interfaced to an AC electric distribution network, and operated independently from the bulk power system. Energy conversion is typically

Voltage and frequency control of islanded microgrids: a plug-and-play approach

Stefano Rivera¹, Fabio Sarzo¹ and Giancarlo Ferrari-Trecate¹

¹Dipartimento di Ingegneria Industriale e dell'Informazione, Università degli Studi di Pavia, raffaele.rossetti@unipv.it, Corresponding author

Abstract—In this paper we propose a new decentralized control scheme for islanded microgrids (IMGs) composed by the interconnection of Distributed Generation Units (DGUs). Local controllers regulate voltage and frequency at the Point of Common Coupling (PCC) of each DGU and they are able to guarantee stability of the overall IMG. The control design procedure is decentralized, since, besides the global scalar quantities, the stability of a local controller and the information on the corresponding DGU and lines connected to it. Most important, our design procedure enables Plug-and-Play (PnP) physical connection to it to have to reuse their local controllers. We study the performance of the proposed controllers simulating different scenarios in Matlab/Simulink and using indexes proposed in IEEE standards.

I. INTRODUCTION
 In recent years, research on islanded microgrids (IMG) has received major attention. IMGs are self-sufficient microgrids composed by several Distributed Generation Units (DGUs) and designed to operate safely and reliably in absence of a connection with the main grid. Besides fostering the use of renewable generation, IMGs help distribution generation

Synchronization of Nonlinear Oscillators in an LTI Electrical Power Network

Brian B. Johnson, Member, IEEE, Sairaj V. Dhople, Member, IEEE, Abdullhah O. Hamadeh, and Philip T. Koenig, Fellow, IEEE

Abstract—Sufficient conditions are derived for the global asymptotic synchronization of a class of identical nonlinear oscillators coupled through a linear time-invariant network. In particular, we focus on systems whose oscillators are connected to a common node through identical branch impedances. For such networks, it is shown that the synchronization condition is independent of the number of oscillators and the value of the load impedance connected to the common node. Theoretical findings are then leveraged to control a system of parallel single-phase voltage source inverters using an impedance-based, low-bandwidth control approach. The ensuing paradigm aims at providing a systematic, time-invariant, and independent of system load, and self-facilitating a modular design approach between the synchronization condition and independent of the number of oscillators. We present both simulation and experimental results to validate the analytical results and demonstrate the proposed application.

Index Terms—Inverter control, microgrid, nonlinear oscillators, synchronization.

Synchronization of Oscillators Coupled through a Network with Dynamics: A Constructive Approach with Applications to the Parallel Operation of Voltage Power Supplies

Leonardo A. B. Torres, Member, IEEE, João P. Hespanha, Fellow, IEEE, and Jeff Mecklin

Abstract—We consider the problem of synchronizing a group of oscillators coupled by a network that is modeled by a multi-input multi-output (MIMO) system. We recently introduced a new system and also to control feedback with a view on decentralization of the synthesis procedure. More specifically, we developed a Plug-and-Play (PnP) design algorithm where the synthesis of a local controller for a DGU requires parameters of transmission lines connected to it, the knowledge of two global scalar parameters, but not specific information about any other DGU. This implies that when a DGU is plugged in or out, only DGUs physically connected to it have to re-tune their local controllers.

PnP control design for general linear constrained systems is typically proposed in [1], [2]. PnP design for IMGs is however different since it is based on the concept of neutral interactions [10] rather than on robustness against subsystem coupling. Furthermore, for achieving neutral interactions among DGUs, we exploit Quasi-Stationary Line (QSL) approximations of time-domain [11].

(optional material)

what can we do better?

algorithms, detailed models,
cyber-physical aspects, ...

many groups out there push
all these directions heavily

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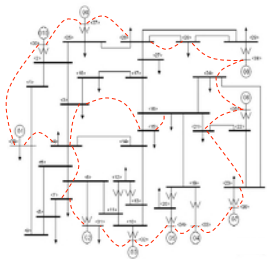
Variation I:

Europe: no centralized dispatch
but trade in **energy markets**

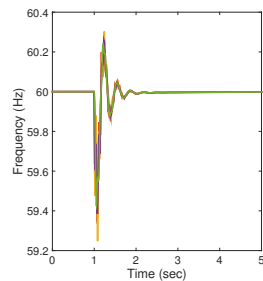


game-theoretic formulation
of optimal secondary control

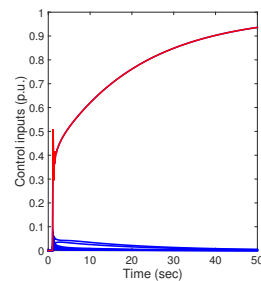
Some strong motivations for game-theoretic perspective



IEEE 39 New England with
distributed DAPI control



DAPI control with cheating of generator # 10



A simple (illegal) cheating strategy for generator #10:

- 1 report wrong injection $u_{10}(t) = 0$ to all neighbors in comm network
 - 2 do not average neighbor values $a_{10,j} = 0$ for all j
- ⇒ generator #10 alone picks up net load & regulates the frequency
⇒ need an incentive scheme so that everybody plays “best response”

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Market formulation of secondary control

[FD & S. Grammatico '16]

Competitive spot market:

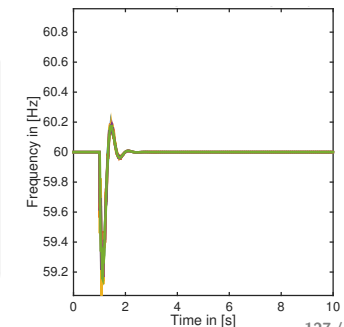
- 1 given a prize λ , player i bids
$$u_i^* = \underset{u_i}{\operatorname{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$$
- 2 market clearing prize λ^* from
$$0 = \sum_i P_i^* + u_i^* = \sum_i P_i^* + J_i'^{-1}(\lambda^*)$$

Broadcast controller:

- 1 convex measurement:
$$k \cdot \dot{\lambda}(t) = \sum_i C_i \dot{\theta}_i(t)$$
- 2 local allocation:
$$u_i(t) = J_i'^{-1}(\lambda(t))$$

Auction (dual decomposition):

- 1
$$u_i^+ = \underset{u_i}{\operatorname{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$$
 - 2
$$\lambda^+ = \lambda - \epsilon (\sum_i P_i^* + u_i^+) = \lambda - \epsilon \cdot \omega_{\text{sync}}$$
- ⇒ converges to optimal economic dispatch



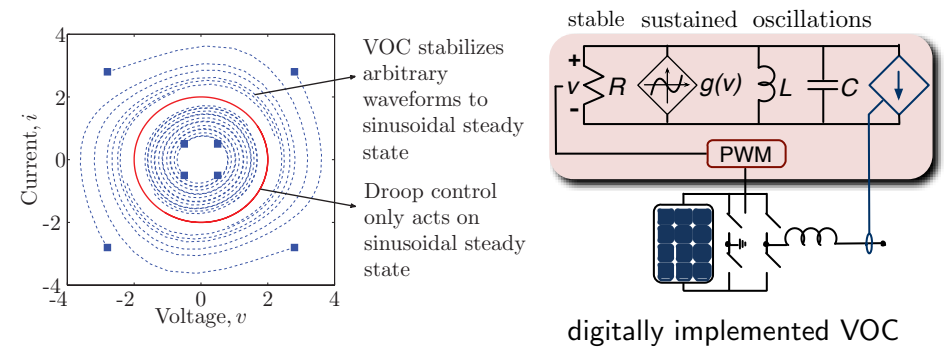
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Variation II:

VOC: virtual oscillator control instead of primary droop control

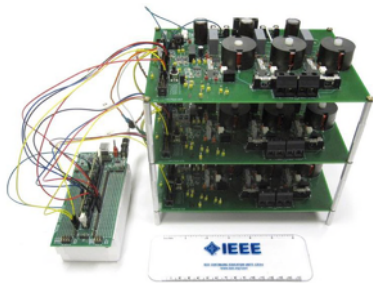
Removing the assumptions of droop control

- **idealistic assumptions:** quasi-stationary operation & phasor coordinates
- ⇒ **future grids:** more power electronics, more renewables, & less inertia
- ⇒ **Virtual Oscillator Control:** control inverters as limit cycle oscillators [Torres, Moehlis, & Hespanha '12, Johnson, Dhople, Hamadeh, & Krein '13]

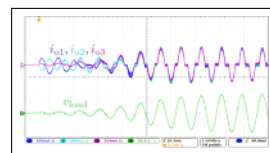


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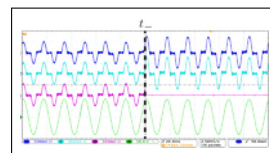
Plug'n'play Virtual Oscillator Control (VOC)



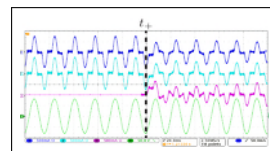
Oscilloscope plots:



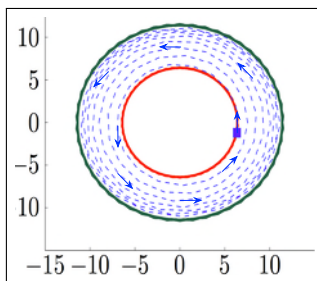
emergence of synchrony



removal of inverter



addition of inverter



change of setpoint

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Crash course on planar limit cycle oscillators

$$L \frac{d}{dt} i = v$$

$$C \frac{d}{dt} v = -Rv - g(v) - i - i_{\text{grid}}$$

⇒ normalized coordinates

$$\ddot{v} + v + \epsilon k_1 g'(v) \cdot \dot{v} = \epsilon k_2 u$$

Liénard's limit cycle condition

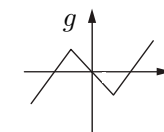
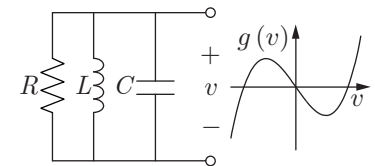
for virtual oscillator with $u = 0$:

if $\epsilon = \sqrt{L/C} \rightarrow 0$

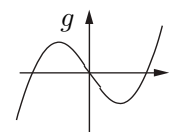
⇒ $\mathcal{O}(\epsilon)$ close to harmonic oscillator

if damping $g'(v)$ is negative near origin & positive elsewhere

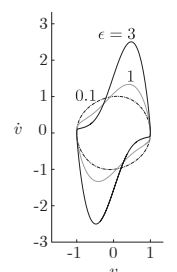
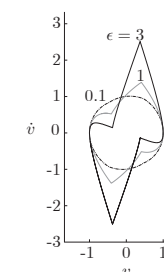
⇒ unique & stable limit cycle



deadzone

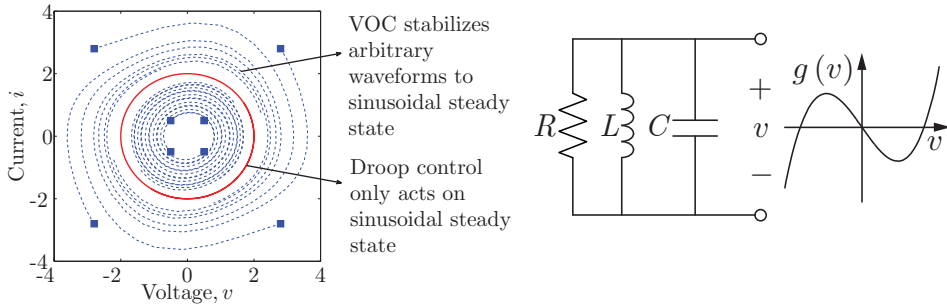


Van der Pol



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Backward compatibility to droop [M. Sinha, FD, B. Johnson, & S. Dhople, '14]



⇒ transf. to polar coordinates, averaging, & generalized power definitions

Thm: in vicinity of the limit cycle:

$$\dot{\theta} = \text{constant} \cdot (\text{reactive power})$$

VOC ⊃ droop:

$$r - r^* = \text{constant} \cdot (P^* - \text{active power})$$

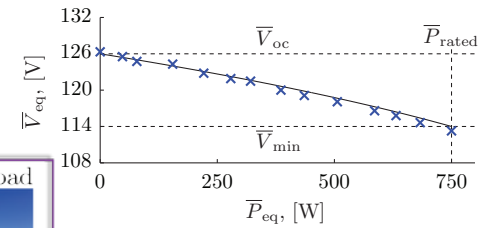
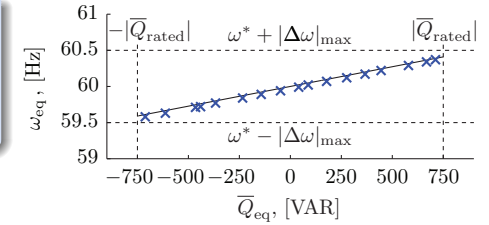
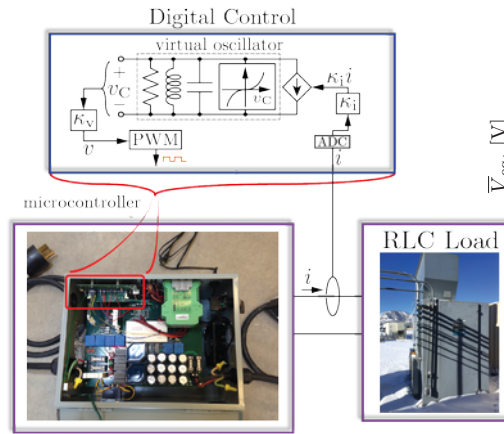
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Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

1 **VOC ⊃ droop:**

$$\dot{\theta} = \text{constant} \cdot (\text{reactive power})$$

$$r - r^* = \text{constant} \cdot (P^* - \text{active power})$$



analytic vs. measured droop curves of VOC

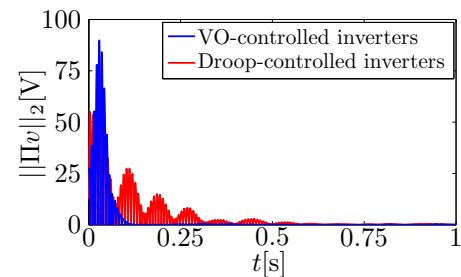
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Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

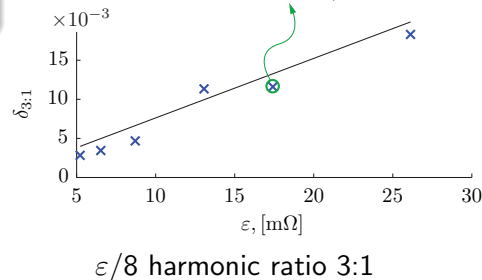
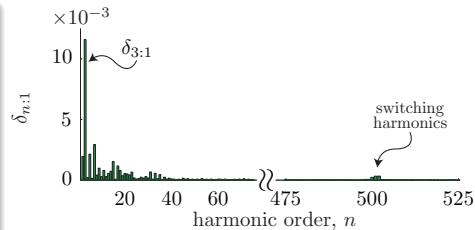
1 **VOC ⊃ droop**

2 **VOC $\xrightarrow{\varepsilon \rightarrow 0}$ harmonic oscillator**
with $\varepsilon/8$ harmonic ratio 3:1

3 **VOC: faster & better transients**
than droop-controlled inverters



synchronization error: VOC vs. droop

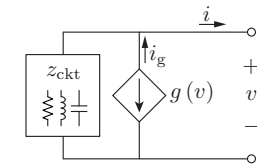


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Analysis of VOC system [S. Dhople, B. Johnson, FD, & A. Hamadeh '13]

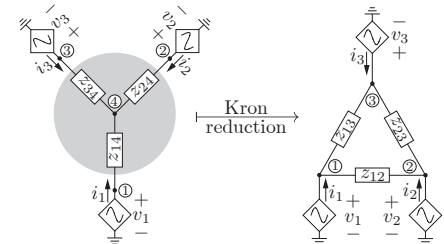
Nonlinear oscillators:

- passive circuit impedance $z_{\text{ckt}}(s)$
- active current source $g(v)$



Co-evolving network:

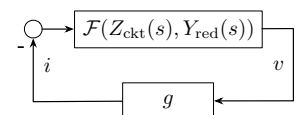
- RLC network & loads are LTI
- Kron reduction: eliminate loads



Stability analysis:

- homogeneity assumption: identical reduced oscillators
- Lure system formulation
- incremental IQC analysis

⇒ sync for strong coupling



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Variation III:

can we turn tertiary optimization directly into continuous control?

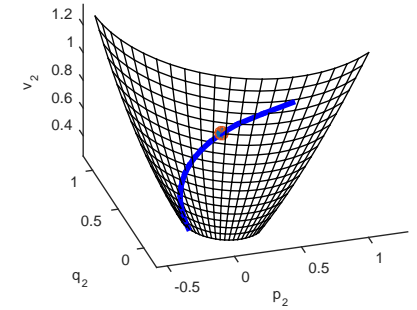


preview on **online optimization**

The power flow manifold & linear tangent approximation

$$\begin{array}{cc} \text{node 1} & \text{node 2} \\ \bullet & \text{---} \bullet \\ & y = 0.4 - 0.8j \end{array}$$

$$\begin{array}{cc} v_1 = 1, \theta_1 = 0 & v_2, \theta_2 \\ p_1, q_1 & p_2, q_2 \end{array}$$

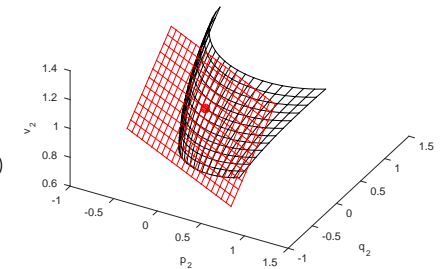


1 **power flow manifold:** $F(x) = 0$

2 **normal space** spanned by $\frac{\partial F(x)}{\partial x} \Big|_{x^*}$

3 **tangent space:** $\frac{\partial F(x)}{\partial x} \Big|_{x^*}^T (x - x^*) = 0$

⇒ sparse & implicit model is **structure-preserving** → distributed control

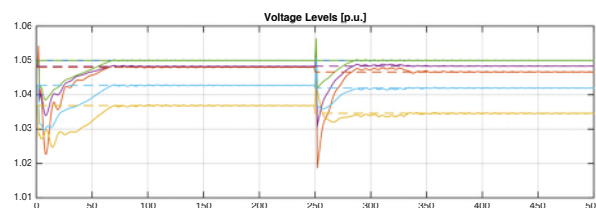
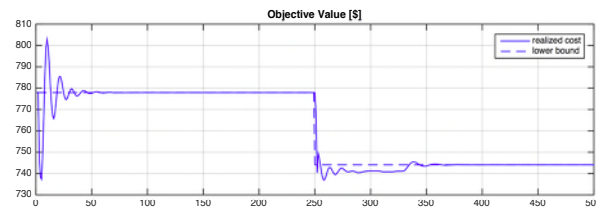
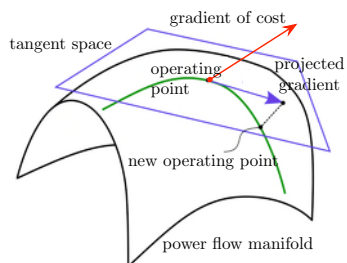


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Online optimization on power flow manifold

with Adrian Hauswirth, Saverio Bolognani, & Gabriela Hug

- **manifold optimization** → gradient flow on power flow manifold
- **online optimization** → controller realizes gradient flow in closed loop



applied to optimal voltage control in IEEE 30 grid_{136 / 184}

Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Causes for Oscillations

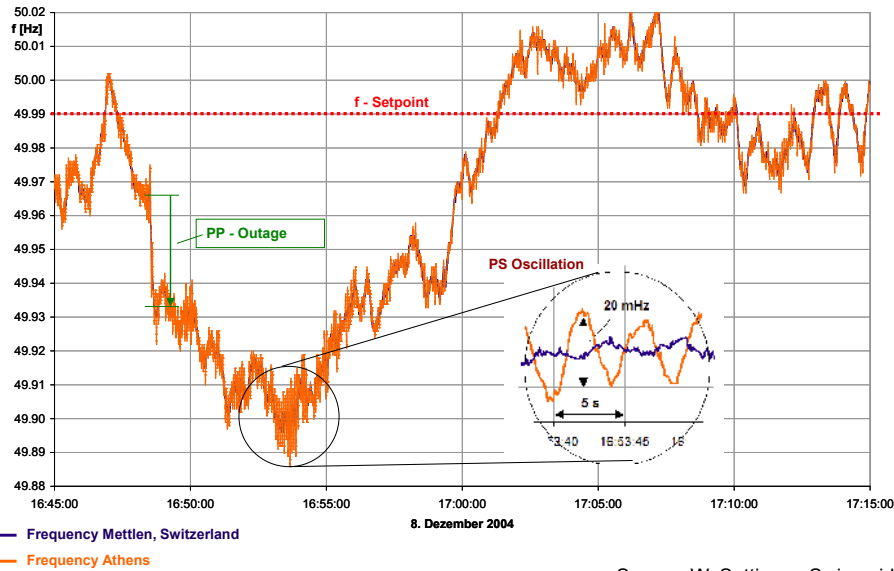
Slow Coherency Modeling

Inter-Area Oscillations & Wide-Area Control

Conclusions

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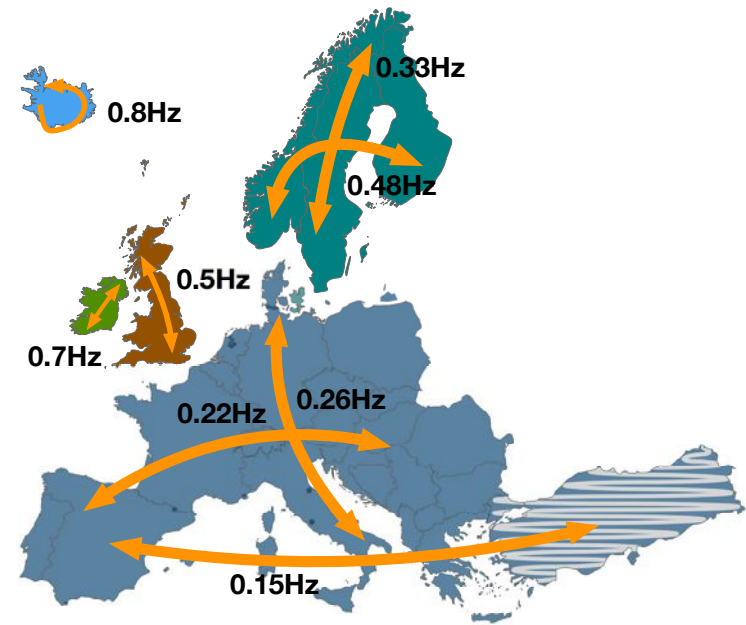
Frequency time-series reveals inter-area oscillations



Source: W. Sattinger, Swissgrid

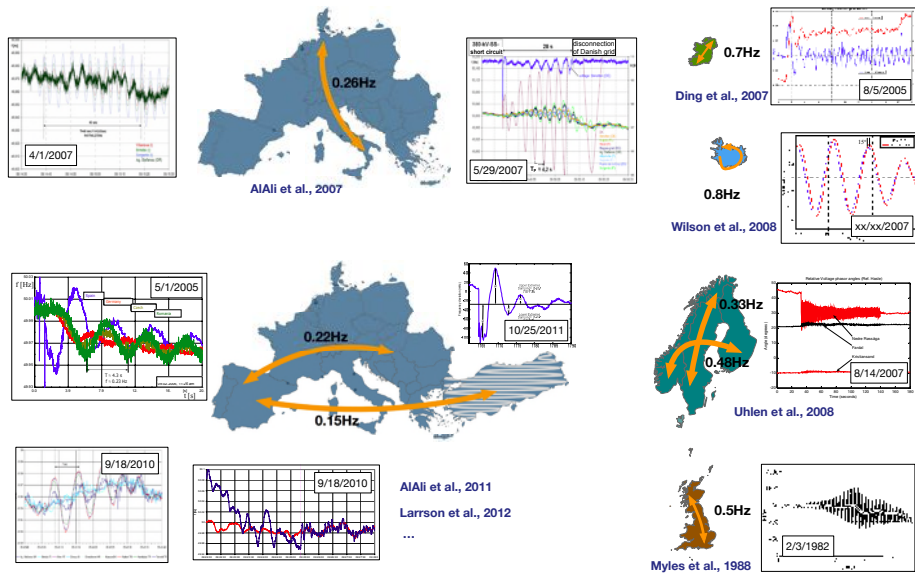
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A few typical inter-area oscillations in Europe



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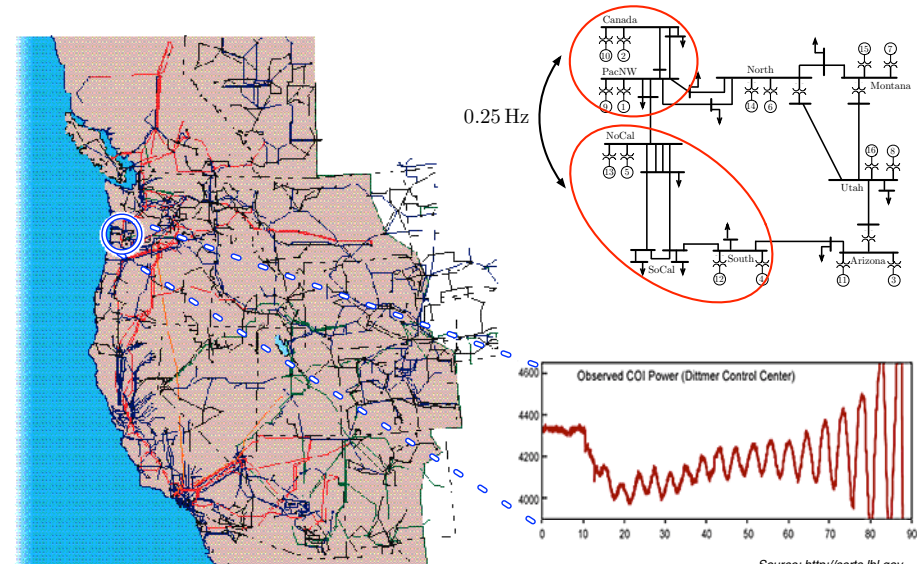
A closer look at some European incidents



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Blackout of August 10, 1996

instability of the 0.25 Hz mode in the Western interconnected system



Source: <http://certs.lbl.gov>

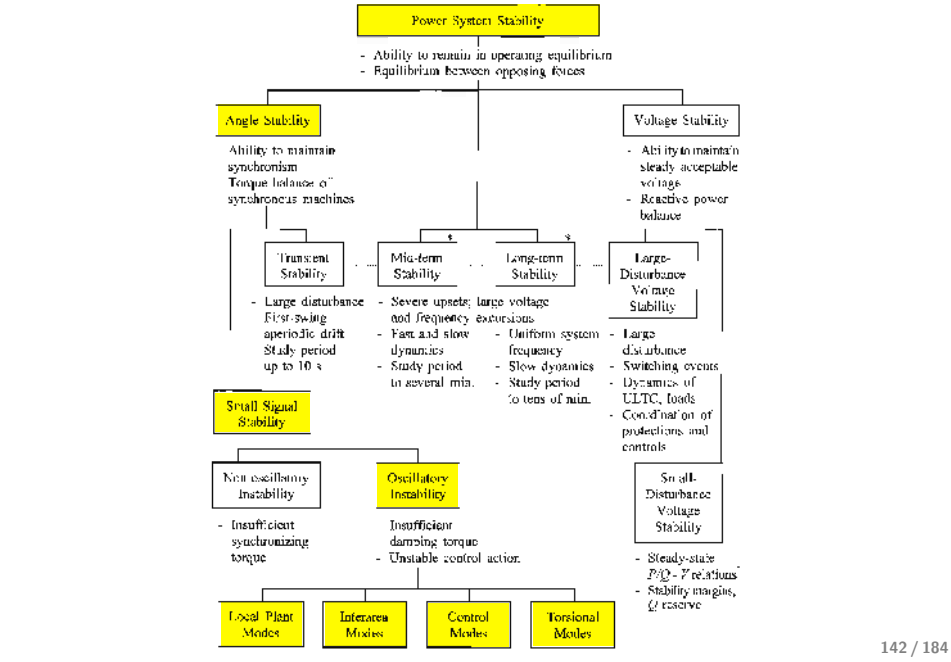
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Recent developments putting oscillations in the spotlight

- | | |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Europe:</p> <ul style="list-style-type: none"> ▶ transmission network upgrades & expansion, ▶ renewable generation in remote locations, & ▶ deregulated markets ... | <p>United States:</p> <ul style="list-style-type: none"> ▶ sparse grid with load & generation hubs, ▶ aging transmission infrastructure, & ▶ long power transfers ... |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

<p>Impact of Increasing Wind Power Generation on the North-South Inter-Area Oscillation Mode in the European ENTSO-E System</p> <p>Sahadeo Alam¹, Torsten Haase², Ibrahim Neme³, Harald Weber⁴</p> <p>¹Institute of Electrical Power Engineering, University of Braunschweig ²Energy Energy Hamburg ³Department of Electrical Engineering, Ain Shams University, Egypt ⁴Germany (Tel.: 0049-381-4007125; e-mail: Sahadeo.alam@tu-bs.de)</p> <p>Abstract: After the enlargement of the European ENTSO-E power system towards Italy at the end of 2010, the East-West Inter-Area Oscillation mode in the enlarged European ENTSO-E power system has been identified in the frequency range of 0.1 Hz ($\omega = 7.8$ rad/s) accompanied by insufficient damping. By the end of 2012, more than 100 GW of wind generation capacity had been installed across Europe, representing about 20% of the peak demand of ENTSO-E power system. In this paper, the impact of large-scale wind power penetration on the European ENTSO-E system on the North-South Inter-Area Oscillation mode using a detailed dynamic model of the European ENTSO-E system is investigated by gradually replacing the power generated by the synchronous generators in the system either full-scale Constant or Doubly-Fed Induction Generator (DFIG) wind turbines. Because the whole system is extended nonlinearly, the analysis method in this paper is nonlinear. However, the damping behavior of linearized oscillations of the whole system are analyzed in detail using the analysis method in this paper. The model was created using MATLAB/SIMULINK software.</p>	<p>Oscillation behaviour of the enlarged European power system under deregulated energy market conditions</p> <p>M. Kurth¹, E. Weidner²</p> <p>¹Department of Electric Drive Systems, Royal Institute of Technology (KTH), Stockholm, Sweden ²Research 11 Energy 2006, received 17 March 2011 Available online 26 May 2011</p> <p>Abstract:</p> <p>This paper deals with optimal coordinated control of several high-voltage direct current (HVDC) links based on an extended model of large power systems. The model of the power system is extended by the same topology system identification techniques. An optimal controller is designed based on the extended model with the aim to improve the damping in the system. The main contribution of the paper is the development of a new method which uses global Phase-locked loop (PLL) signals for coordinated damping control of multiple HVDC links. The paper explains the controllability and observability of the extended PLL signals and the special aspects of optimal controller obtained from PMU. The PLL signals are used to replace the current state of the model. The state of the system, which is dependent on the state and the PLL signals, are independent of the system equilibrium and therefore makes it possible to use state-feedback control, i.e., coordinated control. The method is applied to the Cigre Bends 22 line system including two HVDC links. The simulation results show that the damping can be significantly increased. Copyright © 2011 John Wiley & Sons, Ltd.</p>
<p>Impact of long distance power links on the dynamic security of Large Interconnected Power Systems</p> <p>J. Lehner, T. Weiskopf, C. Schaffhardt</p> <p>Institute of Process Engineering and Power Plant Technology (ITP), Universität Stuttgart Stuttgart, Germany; (e-mail: lehner@itp.uni-stuttgart.de)</p> <p>Abstract: Due to deregulated energy market conditions and the planned extension of the UCTE power system towards Eastern Europe, as well as towards the Middle East and North Africa to close the so called "Mediterranean Ring", the oscillation damping behavior of the UCTE power system is gaining more and more importance. Within the present paper, the oscillation damping behavior of the enlarged UCTE power system after the commissioning of the Turkish power system is analyzed, using time and frequency domain methods. Firstly the Turkish power system is analyzed in a separate network in related operation as one unit in a study. Subsequently the enlarged UCTE system, including the Turkish power system is analyzed. Differences in the oscillation behaviors are shown, precision system coordination are identified and measures to solve occurring problems are given and discussed.</p>	<p>Optimal coordinated control of multiple HVDC links for power oscillation damping based on model identification</p> <p>Robert Eriksson¹ and Lennart Söder²</p> <p>¹Department of Electric Drive Systems, Royal Institute of Technology (KTH), Stockholm, Sweden ²Department of Automatic Control, Royal Institute of Technology (KTH), Stockholm, Sweden</p> <p>SUMMARY</p> <p>This paper deals with optimal coordinated control of several high-voltage direct current (HVDC) links based on an extended model of large power systems. The model of the power system is extended by the same topology system identification techniques. An optimal controller is designed based on the extended model with the aim to improve the damping in the system. The main contribution of the paper is the development of a new method which uses global Phase-locked loop (PLL) signals for coordinated damping control of multiple HVDC links. The paper explains the controllability and observability of the extended PLL signals and the special aspects of optimal controller obtained from PMU. The PLL signals are used to replace the current state of the model. The state of the system, which is dependent on the state and the PLL signals, are independent of the system equilibrium and therefore makes it possible to use state-feedback control, i.e., coordinated control. The method is applied to the Cigre Bends 22 line system including two HVDC links. The simulation results show that the damping can be significantly increased. Copyright © 2011 John Wiley & Sons, Ltd.</p>
<p>Oscillation Behaviour of the Enlarged UCTE Power System Including the Turkish Power System</p> <p>J. Lehner, T. Weiskopf, C. Schaffhardt</p> <p>Institute of Process Engineering and Power Plant Technology (ITP), Universität Stuttgart Stuttgart, Germany; (e-mail: lehner@itp.uni-stuttgart.de)</p> <p>Abstract: Due to deregulated energy market conditions and the planned extension of the UCTE power system towards Eastern Europe, as well as towards the Middle East and North Africa to close the so called "Mediterranean Ring", the oscillation damping behavior of the UCTE power system is gaining more and more importance. Within the present paper, the oscillation damping behavior of the enlarged UCTE power system after the commissioning of the Turkish power system is analyzed, using time and frequency domain methods. Firstly the Turkish power system is analyzed in a separate network in related operation as one unit in a study. Subsequently the enlarged UCTE system, including the Turkish power system is analyzed. Differences in the oscillation behaviors are shown, precision system coordination are identified and measures to solve occurring problems are given and discussed.</p>	<p>Impact of Low Rotational Inertia on Power System Stability and Operation</p> <p>Andreas Urbach, Theodor S. Borsche, Göran Andersson</p> <p>ETH Zurich, Power Systems Laboratory Physikstrasse 5, 8092 Zurich, Switzerland (e-mail: theodor.urbach@ethz.ch)</p> <p>Abstract: Large-scale deployment of Renewable Energy Sources (RES) has led to significant penetration shares of variable RES in power systems worldwide. RES units, notably inverter-connected wind turbines and photovoltaics (PV), that are not able to provide rotational inertia, are effectively displacing conventional generators and their rotating machinery. The traditional assumption that grid inertia is sufficiently high with only small variations over time is thus not valid for power systems with high RES shares. This has implications for frequency dynamics and power system stability and operation. Frequency dynamics are faster in power systems with low rotational inertia, making frequency control and power system operation more challenging. This paper investigates the impact of low rotational inertia on power system stability and operation, contributes new analysis insights and offers mitigation options for low inertia impacts.</p>

Where are we on the map?



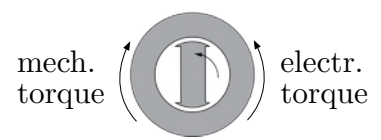
Causes for Oscillations

Why do power systems oscillate?

power network dynamics ≈ coupled, forced, & heterogeneous pendula

generator **torque balance**:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = \text{mech.} - \text{electr. torque}$$

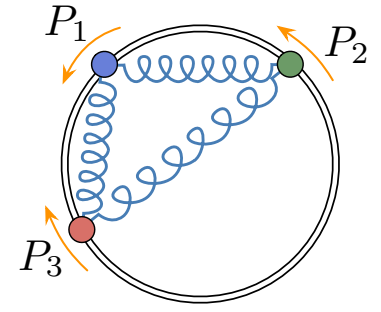


≈ electro-mechanical oscillator

coupled **swing equations**:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

≈ coupled, forced, & heterogeneous pendula



linearized at equilibrium ($\theta^*, \dot{\theta}^*, P^*$):

$$M \ddot{\theta} + D \dot{\theta} + L \theta = P$$

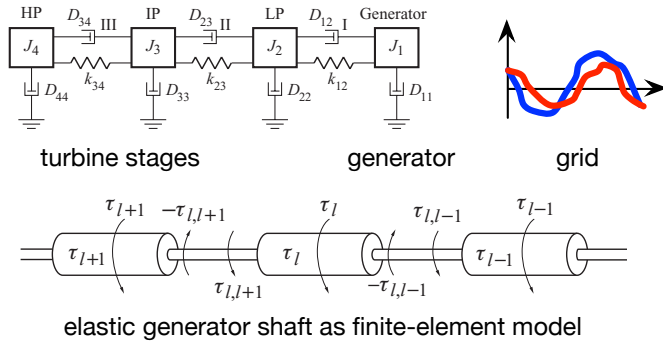
where M, D are inertia and damping matrices & L is network Laplacian

Torsional oscillations in power networks

essentially a (subsynchronous) resonance phenomenon

⇒ arise from interplay of

- electrical oscillations
- flexible mechanical shaft models
- generator-turbine coupling



⇒ subsynchronous resonance phenomena often arise in wind turbines 144 / 184

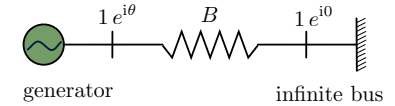
Local oscillations and their control

Automatic Voltage Regulator (AVR):

- objective: generator voltage = *const.*

⇒ diminishing damping & sync torque $\frac{\partial P}{\partial \theta}$

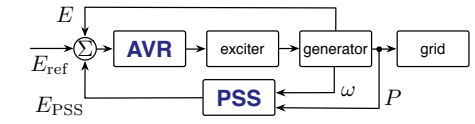
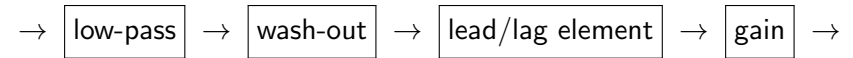
⇒ can result in oscillatory instability



Power System Stabilizer (PSS):

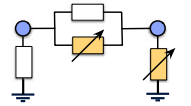
- objective: net damping positive

- typical control design:



Flexible AC Transmission Systems (FACTS) or HVDC:

- control by “modulating” transmission line parameters
- either connected in series with a line or as shunt device



Control-induced oscillations and their control

- **short story:** multiple local controllers interact in an adverse way

- **system-theoretic reason:** power system has unstable zeros

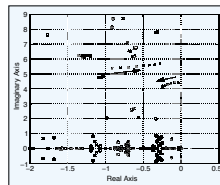
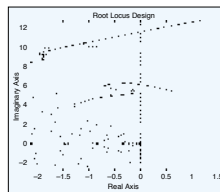
⇒ trade-off: high-gain (local stability) vs. low-gain control (avoid zeros)

⇒ numerous tuning rules & heuristics for decentralized PSS design

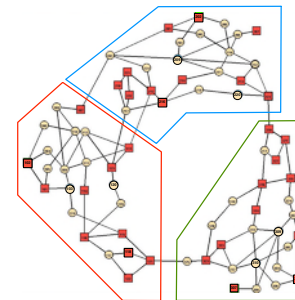
By Joe H. Chow, Juan J. Sanchez-Gasca, Haoxing Ren, and Shaopeng Wang



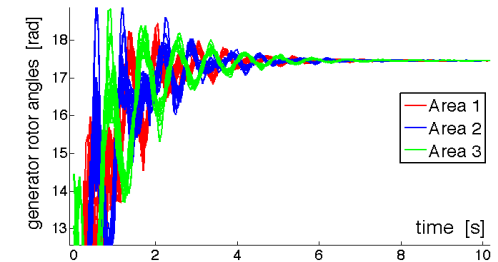
Power System Damping Controller Design
Using Multiple Input Signals



Inter-area oscillations in power networks arise due to



RTS 96 power network



swing dynamics

- 1 **topology:** modular & clustered
- 2 **heterogeneity** in responses (inertia M_i & damping D_i)
- 3 **power transfers** between areas (weaken coupling)
- 4 **interaction** of multiple local controllers

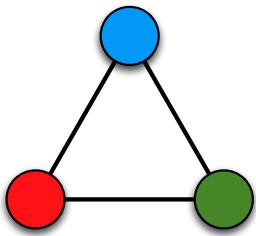
Taxonomy of electro-mechanical oscillations

- Synchronous generator = electromech. oscillator \Rightarrow **local oscillations**:
 - = single generator oscillates relative to the rest of the grid
 - ☹ torsional oscillations induced by mechanical/electrical/flexible coupling
 - ☹ AVR control induces unstable local oscillations
 - ☺ typically damped by local feedback via PSSs
- Power system = complex oscillator network \Rightarrow **inter-area oscillations**:
 - = groups of generators oscillate relative to each other
 - ☹ poorly tuned local PSSs result in unstable inter-area oscillations
 - ☹ inter-area oscillations are only poorly controllable by local feedback
- Consequences of **recent developments**:
 - ☹ increasing power transfers outpace capacity of transmission system
 - \Rightarrow ever more lightly damped electromechanical inter-area oscillations
 - ☹ technological opportunities for **wide-area control (WAC)**

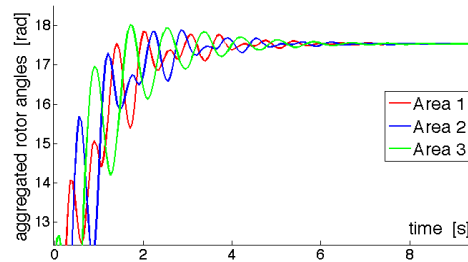
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Slow Coherency Modeling

Slow coherency and area aggregation



aggregated RTS 96 model



swing dynamics of aggregated model

Aggregate model of lower dimension & with less complexity for

- 1 analysis and insights into inter-area dynamics [Chow and Kokotovic '85]
- 2 measurement-based id of equivalent models [Chakraborty et al.'10]
- 3 remedial action schemes [Xu et. al. '11] & wide-area control (later today)

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How to find the areas?

a crash course in spectral partitioning

- given: an undirected, connected, & weighted **graph**
- **partition**: $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, and $\mathcal{V}_1, \mathcal{V}_2 \neq \emptyset$

- **cut** is the size of a partition: $J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij}$
- \Rightarrow if $x_i = 1$ for $i \in \mathcal{V}_1$ and $x_j = -1$ for $j \in \mathcal{V}_2$, then

$$J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij} = \frac{1}{2} \sum_{i,j=1}^n a_{ij} (x_i - x_j)^2 = \frac{1}{2} x^T L x$$

- combinatorial **min-cut** problem: minimize $x \in \{-1, 1\}^n \setminus \{-\mathbf{1}_n, \mathbf{1}_n\}$ $\frac{1}{2} x^T L x$
 - **relaxed problem**: minimize $y \in \mathbb{R}^n, y \perp \mathbf{1}_n, \|y\|_2 = 1$ $\frac{1}{2} y^T L y$
- \Rightarrow minimum is *algebraic connectivity* λ_2 and minimizer is *Fiedler vector* v_2
- **heuristic**: $x_i = \text{sign}(y_i) \Rightarrow$ "spectral partition"

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A quick example

```
% choose a graph size
n = 1000;

% randomly assign the nodes to two groups
x = randperm(n);
group_size = 450;
group1 = x(1:group_size);
group2 = x(group_size+1:end);

% assign probabilities of connecting nodes
p_group1 = 0.5;
p_group2 = 0.4;
p_between_groups = 0.1;

% construct adjacency matrix
A(group1, group1) = rand(group_size,group_size) < p_group1;
A(group2, group2) = rand(n-group_size,n-group_size) < p_group2;
A(group1, group2) = rand(group_size, n-group_size) < p_between_groups;
A = triu(A,1); A = A + A';

% can you see the groups?
subplot(1,3,1); spy(A);

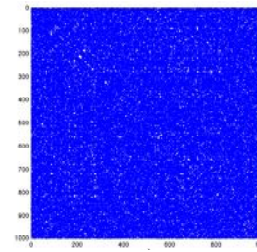
% construct Laplacian and its spectrum
L = diag(sum(A))-A;
[V D] = eigs(L, 2, 'SA');

% plot the components of the algebraic connectivity sorted by magnitude
subplot(1,3,2); plot(sort(V(:,2)), '-');

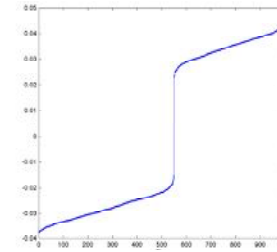
% partition the matrix accordingly and spot the communities
[ignore p] = sort(V(:,2));
subplot(1,3,3); spy(A(p,p));
```

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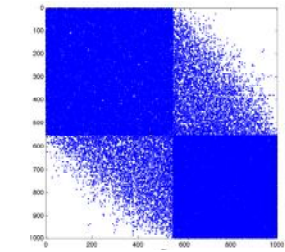
A quick example – cont'd



adjacency matrix



Fiedler vector v_2



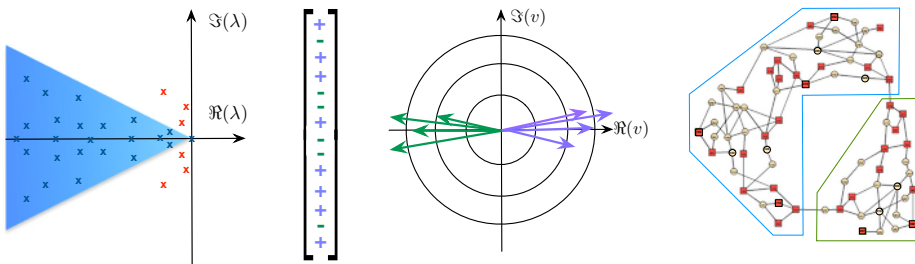
re-arranged adj. matrix

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Classical power system partitioning \approx spectral partitioning

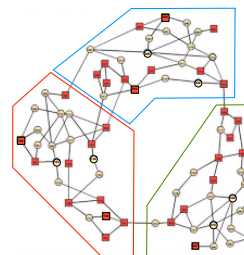
- 1 construct a linear model $\dot{x} = Ax$
- 2 recall solution via eigenvalues λ_i and left/right eigenvectors w_i and v_i :

$$x(t) = \sum_i v_i e^{\lambda_i t} \cdot w_i^T x_0 = \sum_i \{\text{mode \#}i\} \cdot \{\text{contribution from } x_0\}$$
- 3 look at poorly damped complex conjugate mode pairs
- 4 look at angle & frequency components of eigenvectors
- 5 group the generators according to their polarity in eigenvectors

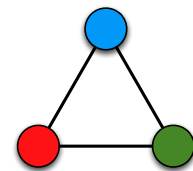


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Setup in slow coherency



original model



aggregated model

- network r given **areas**
(from spectral partition [Chow et al. '85 & '13])

- small **sparsity parameter**:

$$\delta = \frac{\max_{\alpha} (\sum \text{external connections in area } \alpha)}{\min_{\alpha} (\sum \text{internal connections in area } \alpha)}$$

- **inter-area dynamics** by center of inertia:

$$y_{\alpha} = \frac{\sum_{i \in \alpha} M_i \theta_i}{\sum_{i \in \alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

- **intra-area dynamics** by area differences:

$$z_{i-1}^{\alpha} = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

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Linear transformation & time-scale separation

Swing equation \implies singular perturbation standard form

$$M\ddot{\theta} + D\dot{\theta} + L\theta = 0 \implies \begin{cases} \frac{d}{dt_s} \begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta} z \\ \sqrt{\delta} \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & A & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \end{cases}$$

Slow motion given by center of inertia:

$$y_\alpha = \frac{\sum_{i \in \alpha} M_i \theta_i}{\sum_{i \in \alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

Fast motion given by intra-area differences:

$$z_{i-1}^\alpha = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

Slow time scale: $t_s = \delta \cdot t$ "max internal area degree"

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Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s} \begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta} z \\ \sqrt{\delta} \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & A & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a \ddot{\varphi} + D_a \dot{\varphi} + L_{\text{red}} \varphi = 0$$

Properties of aggregated model

[D. Romeres, FD, & F. Bullo, '13]

- $M_a = \begin{bmatrix} \ddots & & \\ & \sum_{i \in \alpha} M_i & \\ & & \ddots \end{bmatrix}$ and $D_a = \begin{bmatrix} \ddots & & \\ & \sum_{i \in \alpha} D_i & \\ & & \ddots \end{bmatrix}$
- $L_{\text{red}} =$ "inter-area Laplacian" + "intra-area contributions"
= positive semidefinite Laplacian with possibly negative weights

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Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s} \begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta} z \\ \sqrt{\delta} \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & A & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a \ddot{\varphi} + D_a \dot{\varphi} + L_{\text{red}} \varphi = 0$$

Singular perturbation approximation

[D. Romeres, FD, & F. Bullo, '13]

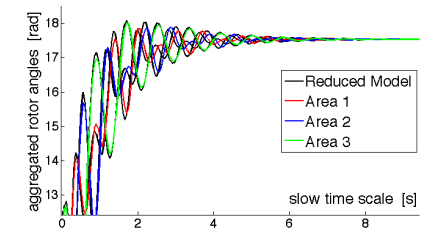
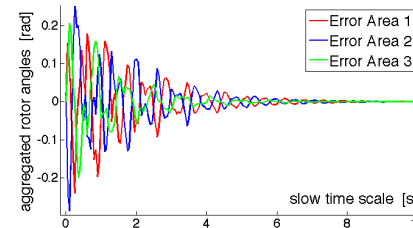
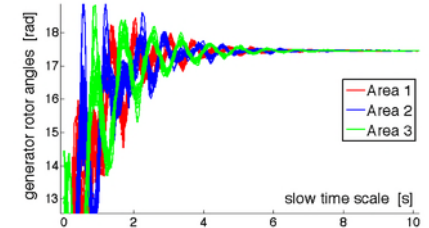
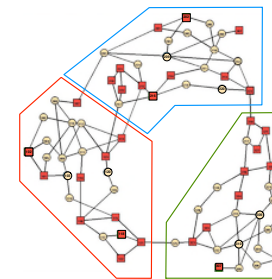
There exist δ^* sufficiently small such that for $\delta \leq \delta^*$ and for all $t > 0$:

$$\begin{bmatrix} y(t_s) \\ \dot{y}(t_s) \end{bmatrix} = \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}), \quad \begin{bmatrix} z(t_s) \\ \dot{z}(t_s) \end{bmatrix} = \tilde{A} \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}).$$

center of inertia \approx solution of aggregated swing equation

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RTS 96 swing dynamics revisited

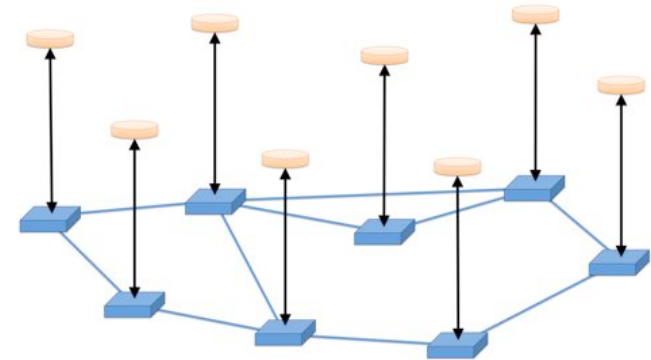


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Inter-Area Oscillations & Wide-Area Control

Remedies against electro-mechanical oscillations

conventional control

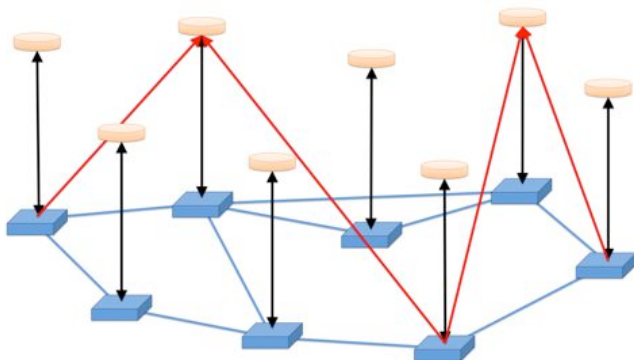


- blue layer: interconnected generators
- fully decentralized control implemented locally
 - ☺ effective against local oscillations
 - ☹ ineffective against inter-area oscillations

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Remedies against electro-mechanical oscillations

wide-area control (WAC)

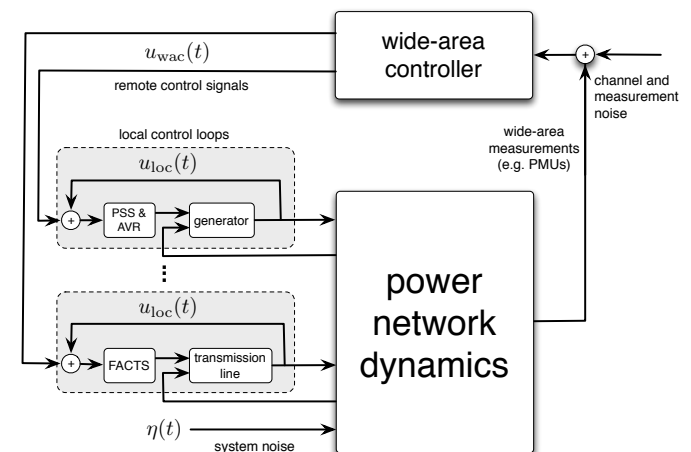


- blue layer: interconnected generators
- fully decentralized control implemented locally
- distributed wide-area control using remote signals

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Setup in wide-area control

- 1 remote control signals & remote measurements (e.g., PMUs)
- 2 excitation (PSS & AVR) and power electronics (FACTS) actuators
- 3 communication backbone network



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Debated: do we need **distributed** wide-area control
or can we get away with **fully decentralized** control?

Transactions on Power Systems, Vol. 7, No. 1, February 1992 97

**DAMPING STRUCTURE AND SENSITIVITY
IN THE NORDEL POWER SYSTEM**

Bo E. Eliasson
Operational Department,
Sydkraft AB, Sweden

David J. Hill
Department of Electrical Engineering & Computer
Science,
University of Newcastle, Australia

Abstract - To enhance the inherent damping of power systems due to generators and loads, a variety of stabilizer configurations can be used for the generators, SVCs and HVDC links. A study is made of how the overall damping matrix is built up from these contributions. This is used to develop a technique for systematic siting of damping equipment in power systems with several poorly damped modes in a given frequency window. This technique is applied to the NORDEL system. Special emphasis is given to handling very large systems, voltage dependent loads and alternative measurement schemes.

The hierarchy of models enables preliminary studies on smaller models to establish general ideas of siting and signal schemes for PSSs and SVCs in order to improve the damping of slow system wide modes with a smaller number of free parameters when coordinated tuning is performed. Then the process can be repeated with more insight on the large models.

A novel feature of the presentation of results for large systems is to graphically superimpose mass scaled eigenvectors and sensitivity information on network diagrams. (No large tables are used.) The results have revealed several interesting features of the

"The above reasoning implies that if observability is small, so is also controllability. The benefits of remote signals for power system damping should thus be marginal." [follow-up comments by G. Andersson & T. Smed, '92]

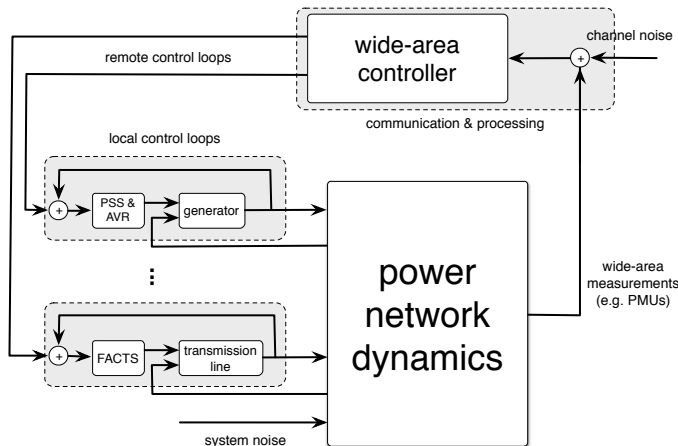
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conventional analysis & wide-area control (based on spectral methods)

I will be a little provocative ...

Canonical setup in wide-area control

local actuators, remote measurements, & communication backbone

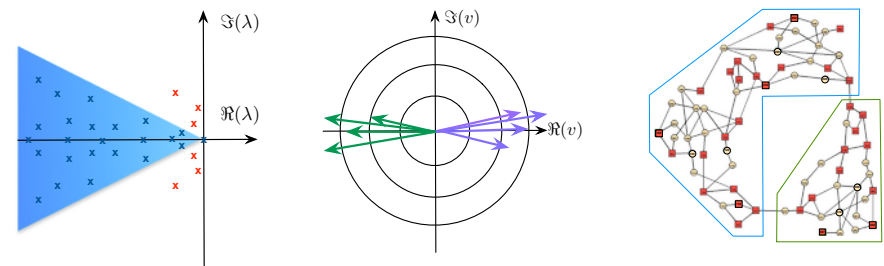


- ⇒ **problem I:** signal selection (sensors & actuators)
- ⇒ **problem II:** WAC design (subject to control signals)

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Recall: spectral analysis reveals critical modes & areas

- 1 recall solution of $\dot{x} = Ax$: $x(t) = \sum_i \underbrace{v_i e^{\lambda_i t}}_{\text{mode } \#i} \cdot \underbrace{w_i^T x_0}_{\text{contribution from } x_0}$
- 2 analyze eigenvectors & participation factors of weakly damped modes
- 3 spectral partitioning reveals coherent groups in eigenvectors polarities



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Which sensors and actuators ?

- Linear control system:** $\dot{x} = Ax + Bu$, $y = Cx$
 - B with column b_j = control location # j
 - C with row c_j^T = sensor location # j
 - A : eigenvalues λ_i and orthonormal right & left eigenvectors v_i & w_i^*
- Diagonalization:** $x = Vz = [v_1 \dots v_n] z$, $z = Wx = [w_1 \dots w_n]^* x$

$$\Rightarrow \dot{z} = \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{=WAV} z + \underbrace{\begin{bmatrix} \vdots & & \\ \dots & w_i^* b_j & \dots \\ \vdots & & \end{bmatrix}}_{=WB} u, \quad y = \underbrace{\begin{bmatrix} \vdots & & \\ \dots & c_i^* v_j & \dots \\ \vdots & & \end{bmatrix}}_{=CV} uz$$
- Controllability of mode i by input $j \triangleq \cos(\angle(w_i, b_j)) = \frac{w_i^* b_j}{\|w_i\| \|b_j\|}$
- Observability of mode i by sensor $j \triangleq \cos(\angle(c_i, v_j)) = \frac{c_i^* v_j}{\|c_i\| \|v_j\|}$

Modal signal selection metrics

Assessment of Two Methods to Select Wide-Area Signals for Power System Damping Control

Annissa Heniche, Member, IEEE, and Innocent Kamwa, Fellow, IEEE

Abstract—In this paper, two different approaches are applied to the Hydro-Québec network in order to select the most effective signals to damp inter-area oscillations. The damping is obtained by static var compensator (SVC) and synchronous condenser (SC) modulation. The robustness analysis, the simulations, and statistical results show, unambiguously, that in the case of wide-area signals, the geometric approach is more reliable and useful than the residues approach. In fact, this study shows that the best robustness and performances are always obtained with the stabilizer configuration using the signals recommended by the geometric approach. In addition, the results confirm that wide-area control is more effective than local control for damping inter-area oscillations.

The results concern only the Hydro-Québec network, it is important to notice that a statistical analysis was realized. This analysis allowed the test of the two approaches using 1140 different configurations of the network.

The aims of this paper are on one hand to show that the two measures do not provide the same conclusion in terms of control loop selection and on the other hand to demonstrate the efficiency and reliability of one measure in comparison to the other. To do that, the two measures were applied in order to select the most effective control loops for damping the 0.6-Hz inter-area

- geometric criteria** [H.M.A. Hamdan & A.M.A. Hamdan '87]:
 - e.g., modal controllability: effect of control input # j on mode # i
 - frequency criteria** [M. Tarokh '92]: modal residues of transfer function
- \Rightarrow suboptimal procedures with many requirements: (i) identification of critical modes, (ii) sensor/actuator catalog, (iii) **combinatorial** evaluation

Decentralized WAC control design ...

- ... subject to structural constraints is **tough**
- ... usually handled with **suboptimal heuristics** in MIMO case

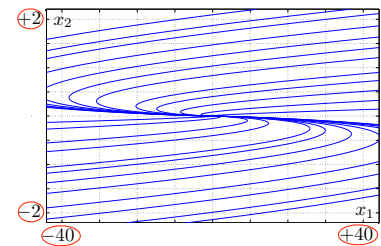
<p>Robust and coordinated tuning of power system stabiliser gains using sequential linear programming R.A. Jabir¹, B.C. Pur², N. Mortins³, J.C.R. Ferraz⁴ ¹Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2BX, UK ²Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2BX, UK ³Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2BX, UK ⁴Department of Electrical and Electronic Engineering, Imperial College London, London SW7 2BX, UK</p>	<p>Decentralized Power System Stabilizer Design Using Linear Parameter Varying Approach Wenhang Qiu, Student Member, IEEE, Vijay Vittal, Fellow, IEEE, and Mustafa Khambadkar, Senior Member, IEEE</p>	<p>Simultaneous Coordinated Tuning of PSS and FACTS Damping Controllers in Large Power Systems Lu Jun Guo and Ibrahim Ertik, Member, IEEE</p>
<p>Robust and Low Order Power Oscillation Damper Design Through Polynomial Control Dimitris D. Sotiriadis, Student Member, IEEE, and Nikolaos C. Paf, Senior Member, IEEE</p>	<p>Robust Pole Placement: Stabilizer Design Using Linear Matrix Inequalities Changping Zhu, Member, IEEE, and Binbin Han, Member, IEEE</p>	<p>Robust Power System Stabilizer Design Using H_∞ Loop Shaping Approach Changping Zhu, Member, IEEE, Mustafa Khambadkar, Senior Member, IEEE, Vijay Vittal, Fellow, IEEE, and Wenhang Qiu, Student Member, IEEE</p>

Challenges in wide-area control

- signal selection** is combinatorial
- decentralized control** is suboptimal
- identification** of critical modes is somewhat *ad hoc*

What information is contained in the spectrum of a *non-normal* matrix ?

Example: $\dot{x} = \begin{bmatrix} -1 & 10^2 \\ 0 & -1 \end{bmatrix} x$



Today [X. Wu, FD, & M. Jovanovic '15]:

- \Rightarrow performance metric: variance amplification of stochastic system
- \Rightarrow simultaneously optimize performance & control architecture
- \Rightarrow fully decentralized & nearly optimal controller

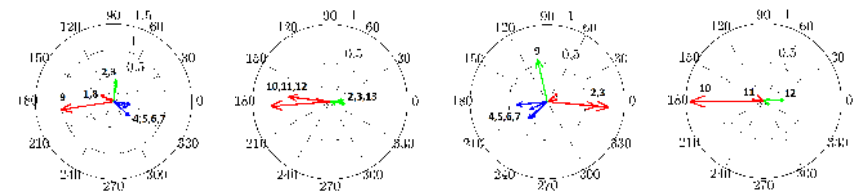
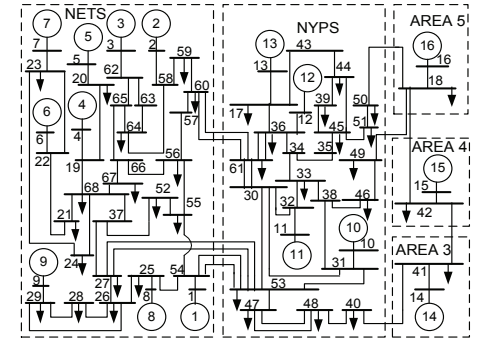
running case study: New England – New York

Case study: New England – New York test system

- **model features** (242 states):

- sub-transient generator models [Singh et. al. '14]
- open loop is unstable
- exciters & tuned PSSs

- frequency & damping ratios of dominant **inter-area modes**



1.1Hz @ 3.8%

1.3Hz @ 4.2%

1.1Hz @ 4.7%

1.3Hz @ 4.9%

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variance amplification as performance metric

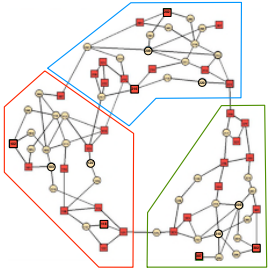
$$\int_0^{\infty} x(t)^T Q x(t) dt$$

Primer on \mathcal{H}_2 - norms

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Slow coherency performance objectives

- recall **sources** for inter-area oscillations:



- linearized **swing equation**: $M\ddot{\theta} + D\dot{\theta} + L\theta = P$
- mechanical energy**: $\frac{1}{2} \dot{\theta}^T M \dot{\theta} + \frac{1}{2} \theta^T L \theta$
- heterogeneities** in topology, power transfers, & machine responses (inertia & damp)

⇒ performance **objective** = energy of homogeneous network:

$$x^T Q x = \dot{\theta}^T M \dot{\theta} + \theta^T (I_n - (1/n) \cdot \mathbf{1}_{n \times n}) \theta$$

- other choices possible: center of inertia, inter-area differences, etc.

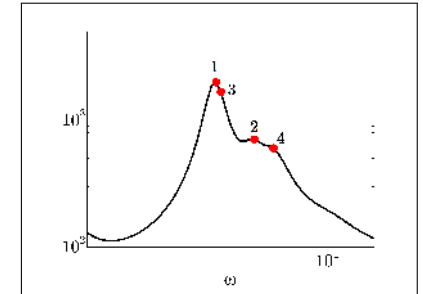
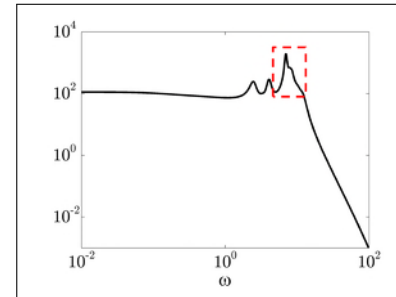
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Input-output analysis in \mathcal{H}_2 -metric

- linear system with **white noise input**: $\dot{x} = Ax + B_1 \eta$
- energy of homogeneous network as **performance output**: $z = Q^{1/2} x$
- steady-state variance** of the output is given by the \mathcal{H}_2 -norm

$$\|G\|_{\mathcal{H}_2}^2 := \lim_{t \rightarrow \infty} \mathbb{E} \left(x(t)^T Q x(t) \right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \|G(j\omega)\|_{\text{HS}}^2 d\omega$$

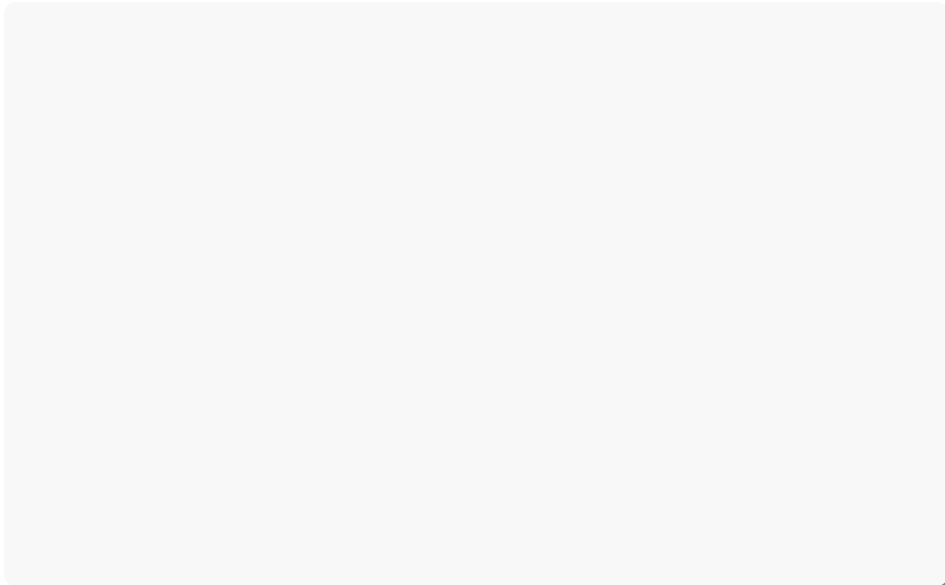
- power spectral density** $\|G(j\omega)\|_{\text{HS}}^2$ reveals NE-NY inter-area modes



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\mathcal{H}_2 - norms for consensus-like systems

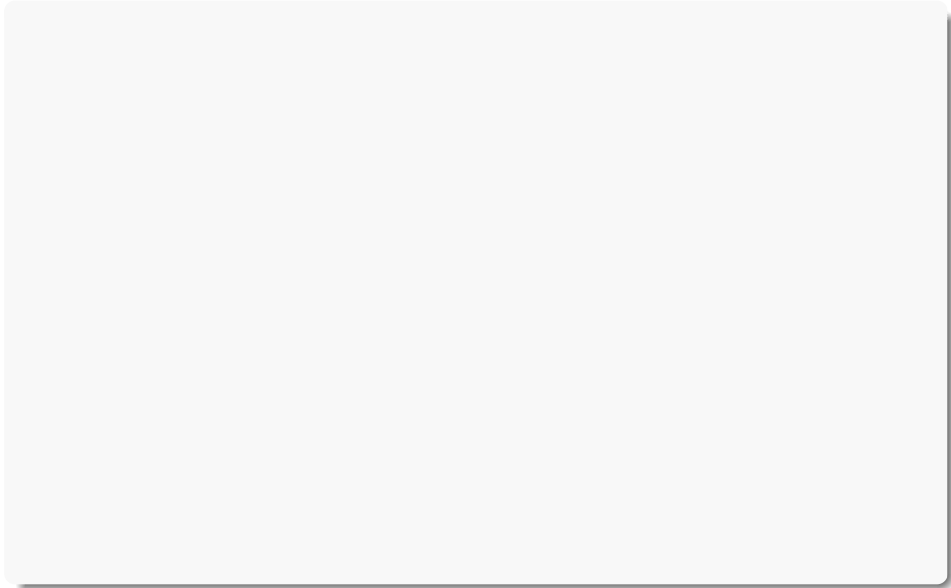
see exercise



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**sparsity-promoting
optimal control**

Primer on Linear Quadratic Control (LQR)



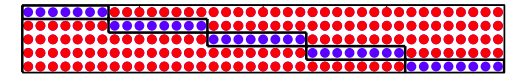
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Optimal linear quadratic regulator (LQR)

- **model:** linearized ODE dynamics $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t)$
- **control:** memoryless linear state feedback $u = -Kx(t)$
- **optimal centralized control** with quadratic \mathcal{H}_2 -performance index:

$$\begin{aligned} &\text{minimize } J(K) \triangleq \lim_{t \rightarrow \infty} \mathbb{E} \left\{ x(t)^T Q x(t) + u(t)^T R u(t) \right\} \\ &\text{subject to} \\ &\text{linear dynamics: } \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ &\text{linear control: } u(t) = -Kx(t), \\ &\text{stability: } (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

(no structural constraints on K)



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Sparsity-promoting optimal LQR

[Lin, Fardad, & Jovanović, '13]

simultaneously optimize performance & architecture

$$\begin{aligned} &\text{minimize } \lim_{t \rightarrow \infty} \mathbb{E} \left\{ x(t)^T Q x(t) + u(t)^T R u(t) \right\} + \gamma \cdot \text{card}(K) \\ &\text{subject to} \\ &\text{linear dynamics: } \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ &\text{linear control: } u(t) = -Kx(t), \\ &\text{stability: } (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

⇒ for $\gamma = 0$: standard optimal control (typically not sparse)

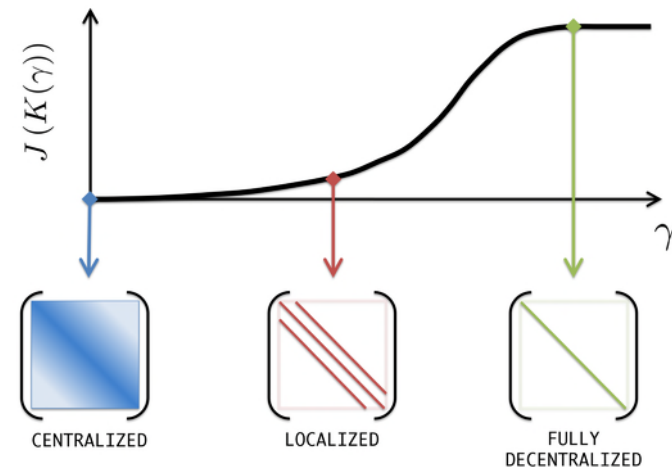
⇒ for $\gamma > 0$: sparsity is promoted (problem is combinatorial)

⇒ $\text{card}(K)$ convexified by weighted ℓ_1 -norm $\sum_{i,j} w_{ij} |K_{ij}|$

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Parameterized family of feedback gains

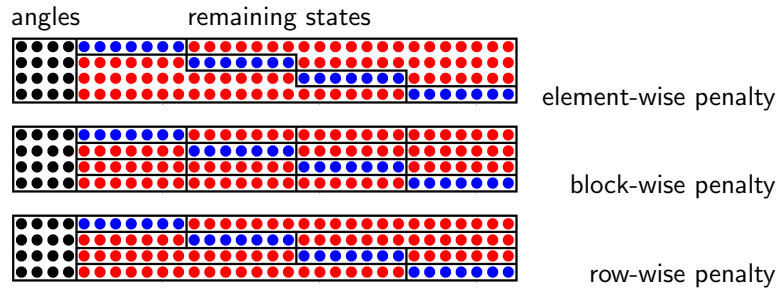
$$K(\gamma) = \arg \min_K \left(J(K) + \gamma \cdot \sum_{i,j} w_{ij} |K_{ij}| \right)$$



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Algorithmic approach in an nutshell (detailed in back-up slides)

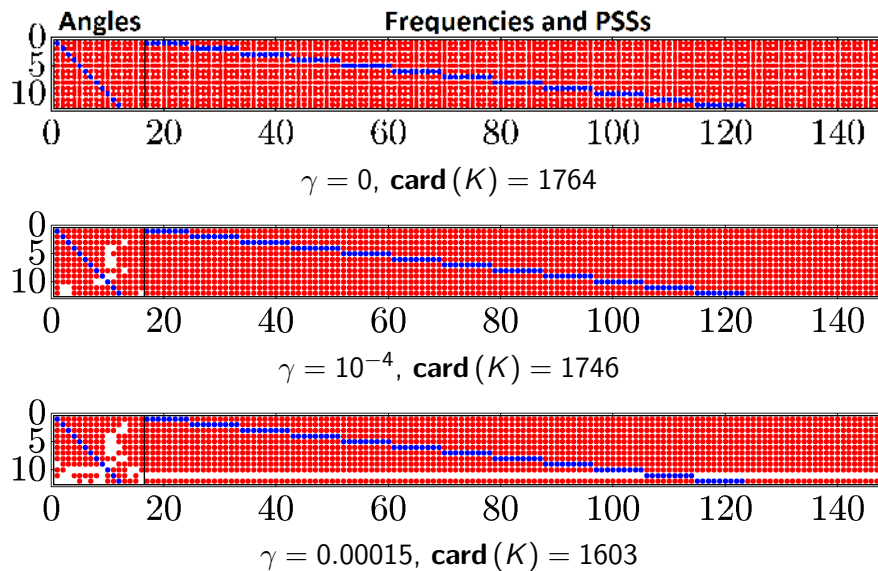
- 1 Algebraic formulation via Gramian and Lyapunov equation
 - 2 Non-convexity in K : use homotopy path in γ & ADMM
-
- 3 Rotational symmetry: remove absolute angle by COI transformation
 - 4 Block/row-sparsity-promoting optimal control



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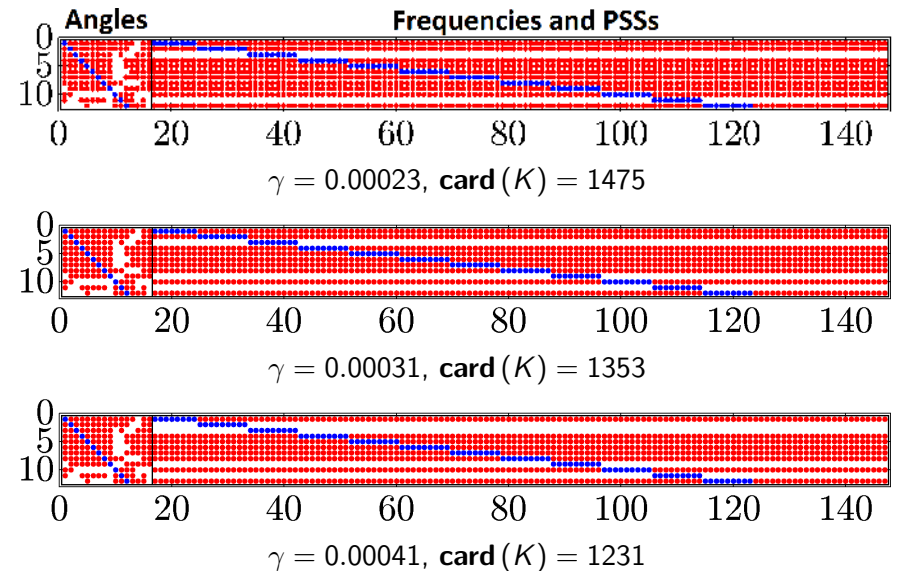
sparsity-promoting control of inter-area oscillations

Sparsity-promoting control architecture



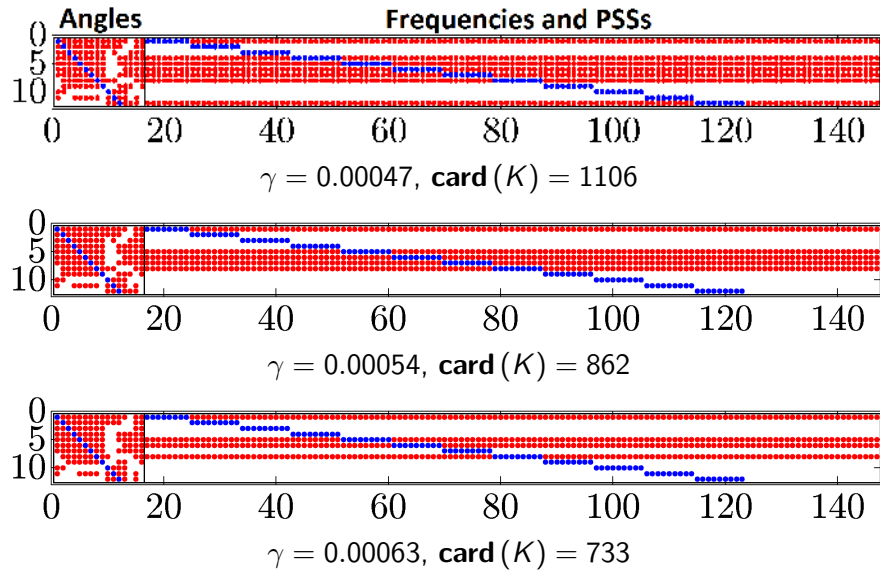
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Sparsity-promoting control architecture



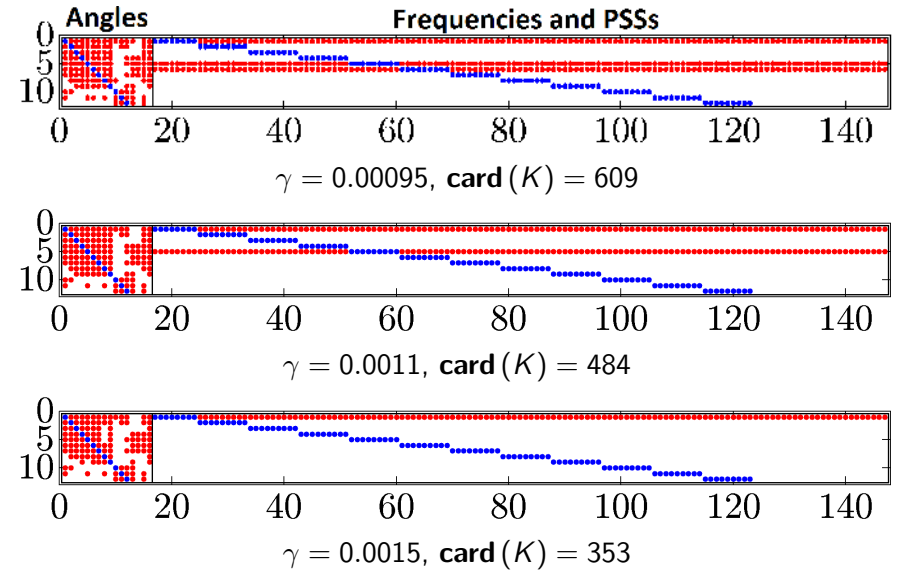
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Sparsity-promoting control architecture



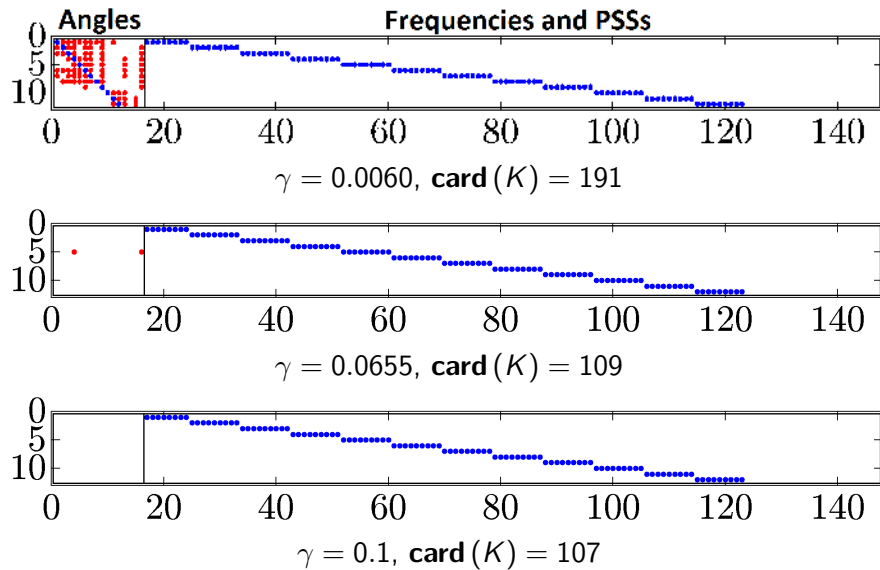
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Sparsity-promoting control architecture



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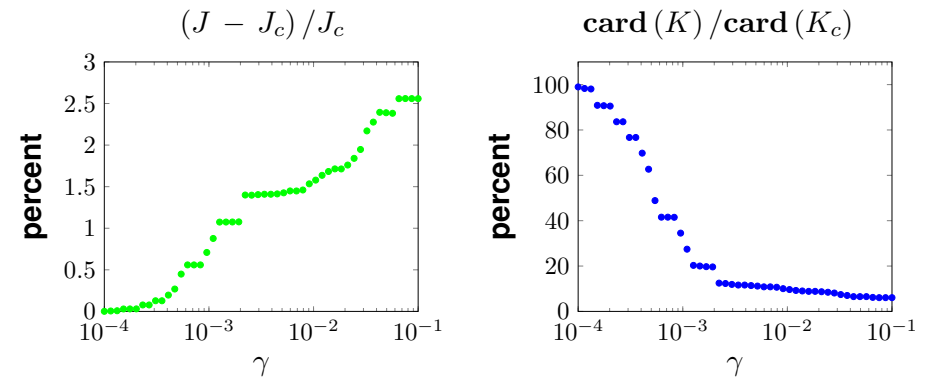
Sparsity-promoting control architecture



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Performance vs. sparsity

Q = energy of homogeneous network, $R = I_n$, $\gamma \in [10^{-4}, 0.1]$

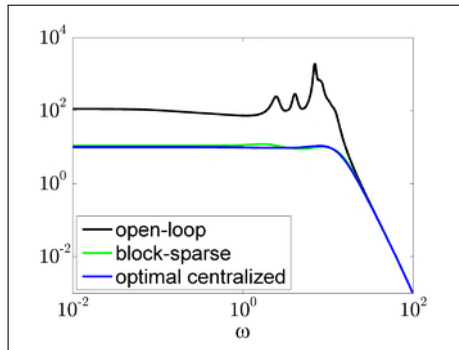


$\gamma = 0.1 \Rightarrow \begin{cases} 2.6\% & \text{relative performance loss} \\ 6.1\% & \text{non-zero elements in } K \end{cases}$

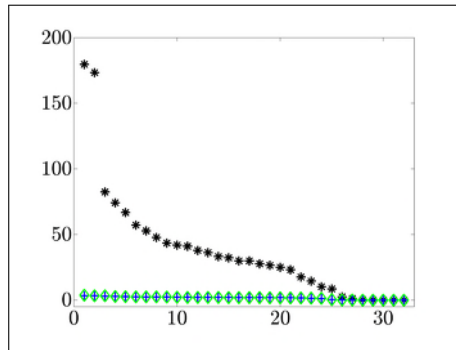
\Rightarrow fully decentralized control is nearly optimal !

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Performance comparison of different approaches



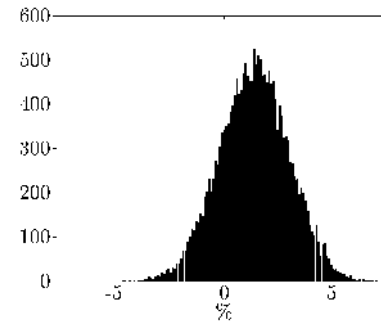
power spectral density



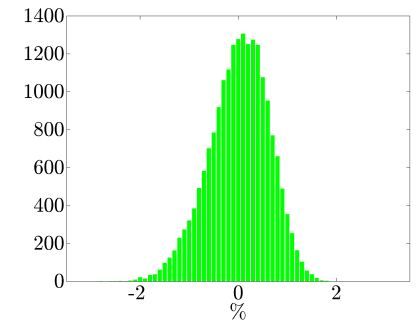
spectrum of covariance matrix

Robustness: optimal control reduces sensitivity

nominal controller applied to 20,000 operating points with $\pm 20\%$ randomized loading



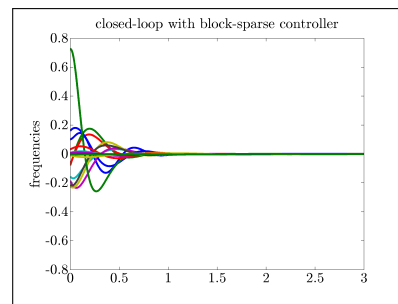
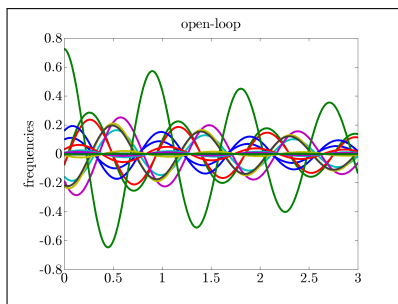
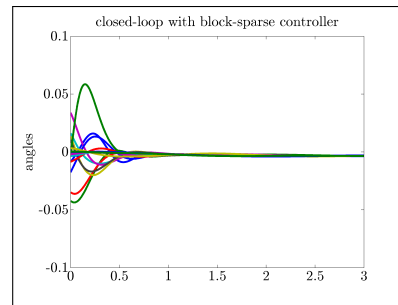
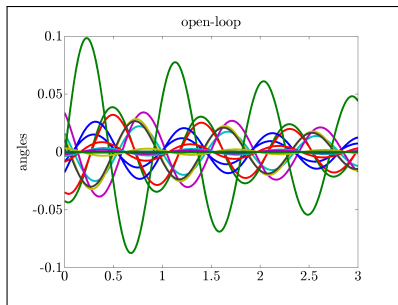
open-loop system



block-sparse controller

⇒ optimal (decentralized) control reduces sensitivity

Eye candy: time-domain simulations



Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

Looking for data, toolboxes, & test cases

- **Matpower (static)** for (optimal) power flow & static models
<http://www.pserc.cornell.edu//matpower/>
- **Matpower (dynamic)** with generator models
<http://www.kios.ucy.ac.cy>
- **Power System Toolbox** for dynamics & North American models
http://www.eps.ee.kth.se/personal/vanfretti/pst/Power_System_Toolbox_Webpage/PST.html
- **IEEE Task Force PES PSDPC SCS**: New York, Brazil, Australian grids etc.; <http://www.sel.eesc.usp.br/ieee/>
- **ObjectStab** for Modelica for dynamics & models
<https://github.com/modelica-3rdparty/ObjectStab>
- **More freeware**: MatDyn, PSAT, THYME, Dome, ...
http://ewh.ieee.org/cmte/pspace/CAMS_taskforce/
- **Other**: many test cases in papers, reports, task forces, ...

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Conclusions

Brief Introduction

Power Network Modeling

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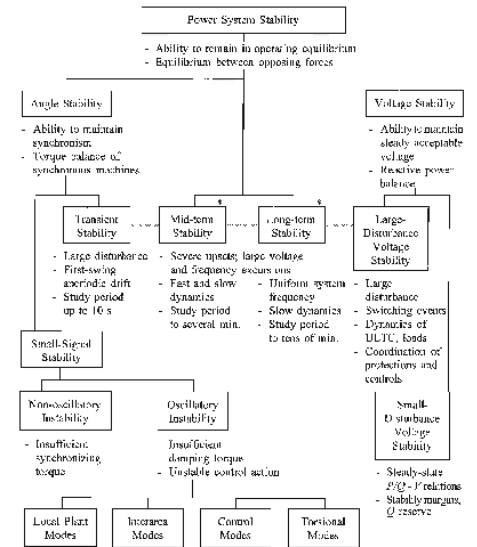
Power System Control Hierarchy

Power System Oscillations

Conclusions

Obviously, there is a lot more ...

I hope I could give you a little insight into a few interesting problems.



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final words of wisdom