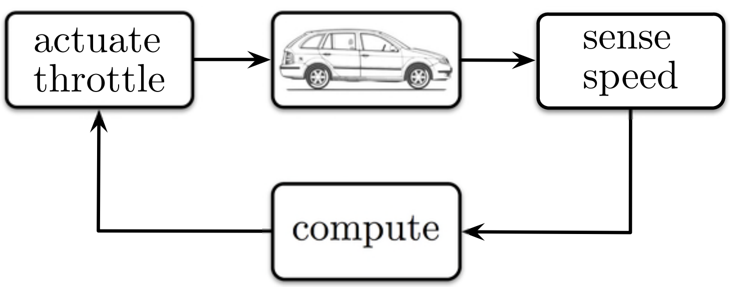
(Complex) Dynamics & (Distributed) Control of (Smart) Power Grids

Winter School on Holistic Modeling & Control of Energy Systems

Florian Dörfler

my motivation for studying power systems

My job description @ETH is "Complex Systems Control"



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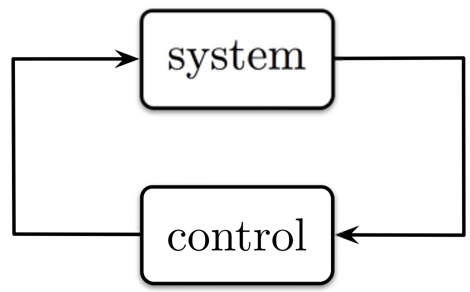
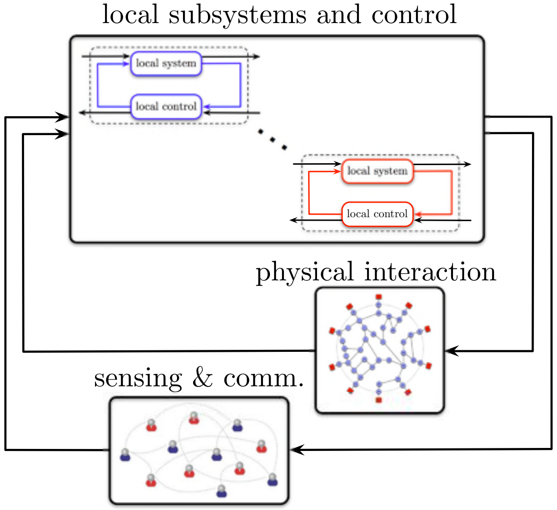


"Simple" control systems are well understood.

"Complexity" can enter in many ways . . .

A "complex" distributed decision making system



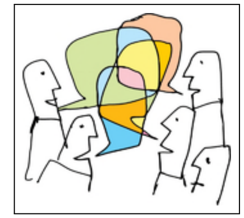
Such distributed systems include **large-scale** physical systems, engineered **multi-agent** systems, & their interconnection in **cyber-physical** systems.

Timely applications of distributed systems control

often the centralized perspective is simply not appropriate


robotic networks


decision making


social networks


sensor networks


self-organization


pervasive computing


traffic networks


smart power grids

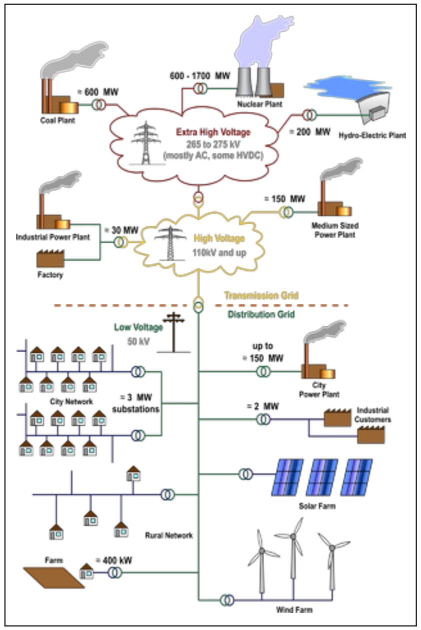
what makes power systems (IMHO) so interesting?

My main application of interest – the power grid


NASA Goddard Space Flight Center

- **Electric energy** is critical for our technological civilization
- Energy supply via **power grid**
- **Complexities**: multiple scales, nonlinear, & non-local

Paradigm shifts in the operation of power networks



Traditional **top to bottom** operation:

- generate/transmit/distribute power
- hierarchical control & operation

Smart & green **power to the people**:

- high renewable penetration
- distributed generation & deregulation
- demand response & load control

Why care about power system dynamics & control?

www.offthegridnews.com

1. increasing renewables & deregulation
2. growing demand & operation at capacity

⇒ increasing volatility & complexity, decreasing robustness margins

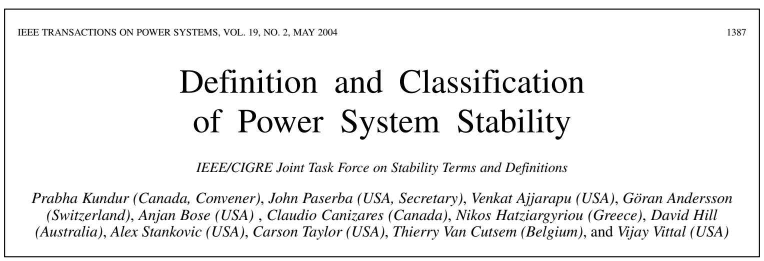
Rapid technological and scientific advances:

1. re-instrumentation: sensors & actuators
2. complex & cyber-physical systems

⇒ cyber-coordination layer for smart grid

⇒ need to understand the **complex** network dynamics & control

One system with many dynamics & control problems

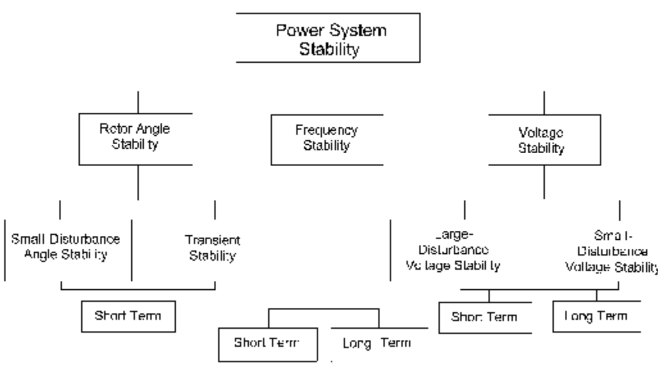


IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 19, NO. 2, MAY 2004 1387

Definition and Classification of Power System Stability

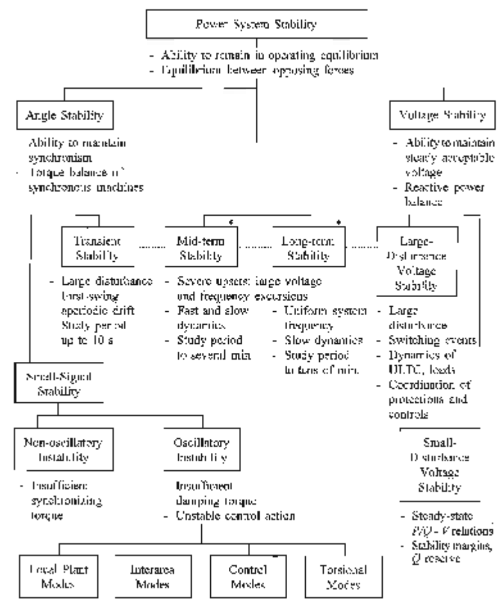
IEEE/CIGRE Joint Task Force on Stability Terms and Definitions

Prabha Kundur (Canada, Convener), John Paserba (USA, Secretary), Venkat Ajjarapu (USA), Göran Andersson (Switzerland), Anjan Bose (USA), Claudio Canizares (Canada), Nikos Hatziargyriou (Greece), David Hill (Australia), Alex Stankovic (USA), Carson Taylor (USA), Thierry Van Cutsem (Belgium), and *Vijay Vittal (USA)*



We have to make a choice based on . . .

many aspects depending on spatial/temporal/state scales, cause & effect, . . .



- what future speakers need and what will be covered by others
- what I actually know well
- what is interesting from a network perspective rather than from device perspective
- what is relevant for future (smart) power grids with high renewable penetration
- what gives rise to fun distributed control problems
- what <u>**you**</u> are interested in

Tentative outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

my particular focus is on networks

Disclaimers

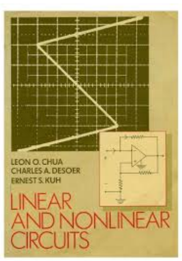
- start-off with "boring" modeling before we get to more "sexy" topics
- we will cover mostly basic material & some recent "cutting edge" work
- we will focus on simple models and developing physical & math intuition
- we will not go deeply into the math though everything is sound

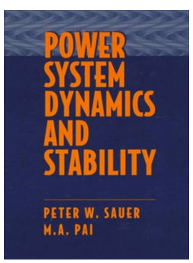
⇒ cover fundamentals, convey intuition, & give references for the details

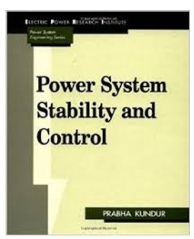
- notation is mostly "standard" (watch out for sign & p.u. conventions)
- ask me for further reading about any topic
- interrupt & correct me anytime

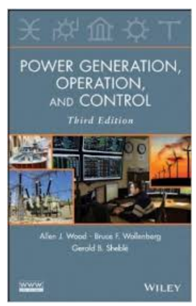
Many references available ... my personal look-up list

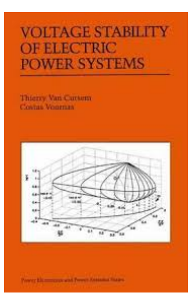
... to be complemented by references throughout the lecture

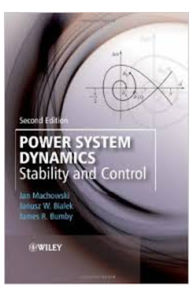


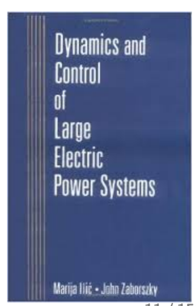












We will also use the blackboard ...

let’s start off with a quiz:

what is your background?

why are you interested in power?

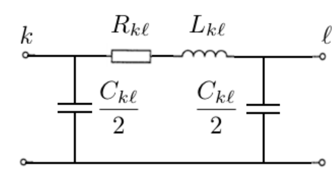
what are your expectations?

Outline

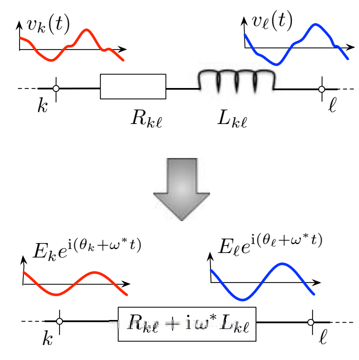
Circuit Modeling: Network, Loads, & Devices

AC circuits – starting from yesterday’s lecture

- power network modeled by linear **RLC** circuit, e.g., Π-model for
  - transmission lines (mainly inductive)
  - distribution lines (resistive/inductive)
  - cables (capacitive effects)



- we will work in **single-phase**, e.g., q-phase of a balanced 3-phase circuit
- **quasi-stationary modeling** at time scales of interest: operation at nominal frequency ω^* with harmonic waveforms
  - phasor signals: $v_k(t) \approx E_k e^{\mathrm{i}(\theta_k + \omega^* t)}$
  - algebraic circuit: $\frac{d}{dt} L_{k\ell} \approx \mathrm{i}\,\omega^* L_{k\ell}$

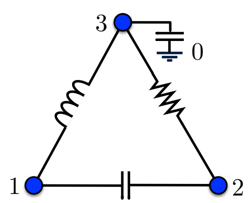


Note: quasi-stationarity assumption can be justified via singular perturbations & modeling can be improved using *dynamic phasors* [A. Stankovic & T. Aydin '00].

AC circuits – graph-theoretic modeling

❶ a circuit is a connected & undirected **graph** $G = (\mathcal{V}, \mathcal{E})$

- $\mathcal{V} = \{1, \ldots, n\}$ are the nodes or *buses*
  - buses are partitioned as $\mathcal{V} = \{\text{sources}\} \cup \{\text{loads}\}$
  - the ground is sometimes explicitly modeled as node 0 or $n+1$
- $\mathcal{E} \subset \{\{i,j\} : i,j \in \mathcal{V}\} = \mathcal{V} \times \mathcal{V}$ are the undirected edges or *branches*
  - edges between distinct nodes $\{i,j\}$ are the *lines*
  - self-edges $\{i,i\}$ (or edges to ground $\{i,0\}$) are the *shunts*



$$\mathcal{V} = \{1,2,3\}$$

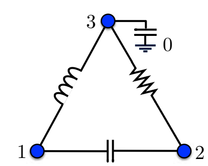
$$\mathcal{E} = \{\{1,2\},\{1,3\},\{2,3\},\{3,3\}\}$$

AC circuits – the network admittance matrix

❷ $Y = [Y_{ij}] \in \mathbb{C}^{n \times n}$ is the **network admittance matrix** with elements

$$Y_{ij} = \begin{cases} -\frac{1}{Z_{ij}} & \text{for off-diagonal elements } i \neq j \\ \frac{1}{Z_{i,\text{shunt}}} + \sum_{j \neq i} \frac{1}{Z_{ij}} & \text{for diagonal elements } i \neq j \end{cases}$$

- impedance = resistance + i · reactance: $Z_{ij} = R_{ij} + \mathrm{i} \cdot X_{ij}$
- admittance = conductance + i · susceptance: $\frac{1}{Z_{ij}} = G_{ij} + \mathrm{i} \cdot B_{ij}$



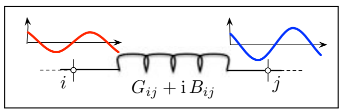
$$Y = \underbrace{\begin{bmatrix} \frac{1}{Z_{12}} + \frac{1}{Z_{13}} & -\frac{1}{Z_{12}} & -\frac{1}{Z_{13}} \\ -\frac{1}{Z_{12}} & \frac{1}{Z_{12}} + \frac{1}{Z_{23}} & -\frac{1}{Z_{23}} \\ -\frac{1}{Z_{13}} & -\frac{1}{Z_{23}} & \frac{1}{Z_{13}} + \frac{1}{Z_{23}} \end{bmatrix}}_{\text{network Laplacian matrix}} + \underbrace{\begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{1}{Z_{3,\text{shunt}}} \end{bmatrix}}_{\text{diag(shunts)}}$$

Note *quasi-stationary* modeling: $Z_{13} = \mathrm{i}\,\omega^* L_{13}$ with nominal frequency ω^*

AC circuits – basic variables

❸ **basic variables**: voltages & currents

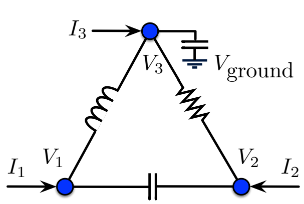
- on nodes: potentials & current injections
- on edges: voltages & current flows



❹ *quasi-stationary* **AC phasor coordinates** for harmonic waveforms:

- e.g., complex voltage $V = E\,e^{\mathrm{i}\,\theta}$ denotes $v(t) = E\cos(\theta + \omega^* t)$

  where $V \in \mathbb{C}$, $E \in \mathbb{R}_{\geq 0}$, $\theta \in \mathbb{S}^1$, $\mathrm{i} = \sqrt{-1}$, and ω^* is nominal frequency



external injections: I_1, I_2, I_3

potentials: V_1, V_2, V_3

reference: $V_{\text{ground}} = 0\text{V}$

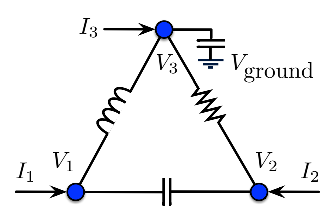
AC circuits – fundamental equations

❺ **Ohm's law** at every branch: $I_{i \to j} = \frac{1}{Z_{ij}}(V_i - V_j)$

❻ **Kirchhoff's current law** for every bus: $I_i + \sum_j I_{j \to i} = 0$

❼ **current balance equations** (treating the ground as node with 0V):

$$I_i = -\sum_j I_{j \to i} = \sum_j \frac{1}{Z_{ij}}(V_i - V_j) = \sum_j Y_{ij} V_j \quad \text{or} \quad I = Y \cdot V$$



$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Note: all variables are in per unit (p.u.) system, i.e., normalized wrt base voltage

AC circuits – average power and power factor

on blackboard

AC circuits – complex power

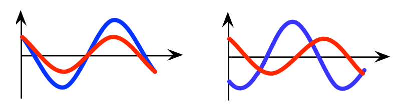
on blackboard

AC circuits – power dissipated by RLC loads

on blackboard

AC circuits – complex power summary

8. **active & reactive power** in AC circuits:



- active (average) power:

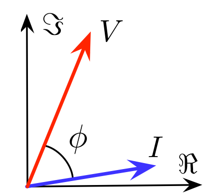
$$P = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt = \frac{1}{2}\cdot|V|\cdot|I|\cdot\cos(\phi)$$

- reactive (0-average) power:

$$Q = \frac{1}{T}\int_0^T v(t)\cdot i(t-T/4)\,dt = \frac{1}{2}\cdot|V|\cdot|I|\cdot\sin(\phi)$$

⇒ normalize voltage & current phasors:

$V \mapsto 1/\sqrt{2}\cdot Ee^{\mathrm{i}\,\theta}$



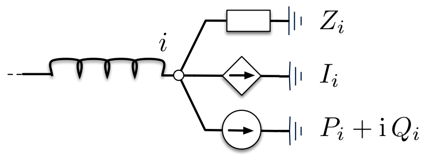
⇒ **complex power:** $S = V\bar{I} = P + \mathrm{i}Q$

$=$ active power $+\,\mathrm{i}\,\cdot$ reactive power

Note: often complex phasors are implicitly normalized $\tilde{V} = 1/\sqrt{2}\cdot Ee^{\mathrm{i}\,\theta}$

Static models for sources & loads

- aggregated **ZIP load model**:
  constant impedance **Z** +
  constant current **I** +
  constant power **P**



- more general **exponential load model**: power $= const. \cdot (V/V_{\text{ref}})^{const.}$
  (combinations & variations learned from data)
- conventional **synchronous generators** are typically controlled
  to have constant active power output P and voltage magnitude E
- sources interfaced with **power electronics** are typically controlled
  to have constant active power P and reactive power Q

⇒ **PQ** buses have complex power $S = P + \mathrm{i}Q$ specified

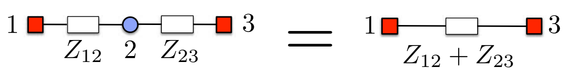
⇒ **PV** buses have active power P and voltage magnitude E specified

⇒ **slack buses** have E and θ specified (not really existent)

Kron Reduction of Circuits

Kron reduction [G. Kron 1939]

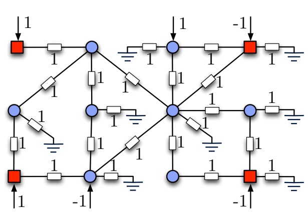
often (almost always) you will encounter Kron-reduced network models



General procedure:

0. convert const. power injections locally to shunt impedances $Z = S/V_{\text{ref}}^2$
1. partition linear current-balance equations via boundary & interior nodes:
   (arises naturally, e.g., sources & loads, measurement terminals, etc.)

$$\begin{bmatrix} I_{\text{boundary}} \\ \hline I_{\text{interior}} \end{bmatrix} = \left[\begin{array}{c|c} Y_{\text{boundary}} & Y_{\text{bound-int}} \\ \hline Y_{\text{bound-int}}^T & Y_{\text{interior}} \end{array}\right] \begin{bmatrix} V_{\text{boundary}} \\ \hline V_{\text{interior}} \end{bmatrix}$$



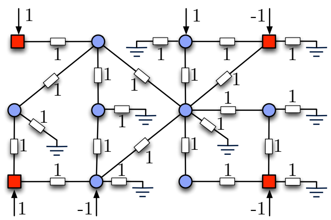
Kron reduction cont'd

on blackboard

Kron reduction cont'd

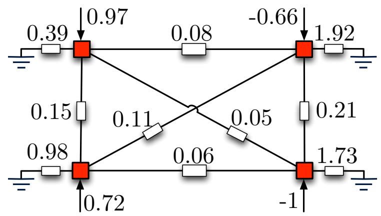
❷ Gaussian elimination of interior voltages:

$$V_{\text{interior}} = Y_{\text{interior}}^{-1}\left(I_{\text{interior}} - Y_{\text{bound-int}}^{T} V_{\text{boundary}}\right)$$



original circuit

$I = Y \cdot V$



"equivalent" reduced circuit

$I_{\text{red}} = Y_{\text{red}} \cdot V_{\text{boundary}}$

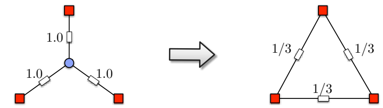
⇒ reduced Y-matrix: $Y_{\text{red}} = Y_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{-1} \cdot Y_{\text{bound-int}}^{T}$

⇒ reduced injections: $I_{\text{red}} = I_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{-1} \cdot I_{\text{interior}}$

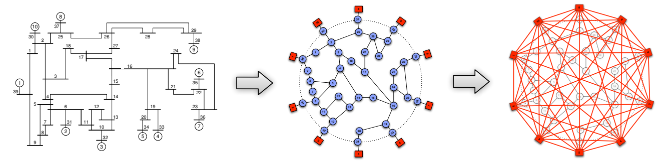
Examples of Kron reduction

algebraic properties are preserved but the network changes significantly

- Star-Δ transformation [A. E. Kennelly 1899, A. Rosen '24]



- Kron reduction of load buses in *IEEE 39 New England power grid*

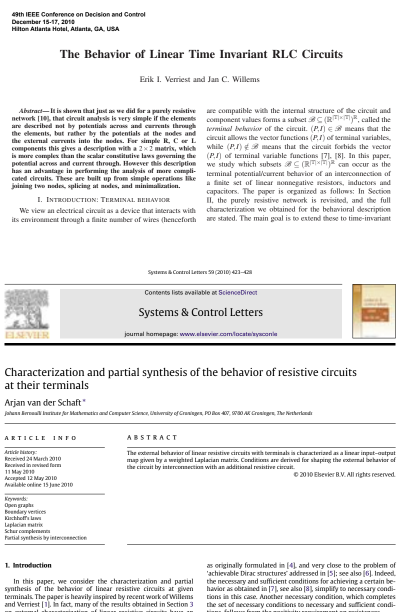


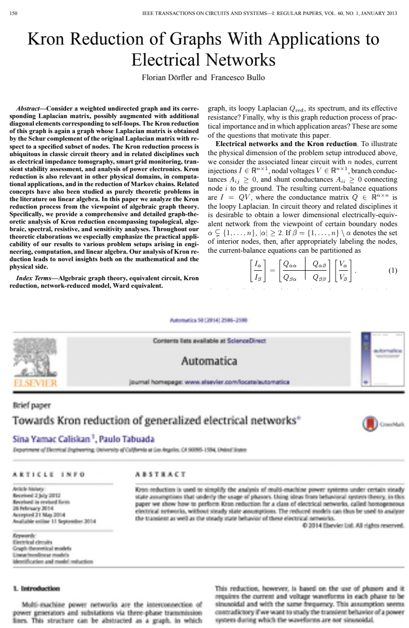
⇒ topology without weights is meaningless!

⇒ shunt resistances (loads) are mapped to line conductances

⇒ many properties still open [FD & F. Bullo '13, S. Caliskan & P. Tabuada '14]

Kron reduction – so simple yet still full of mysteries





Power Flow Formulations & Approximations

Power balance eqn's: "power injection = Σ power flows"

1. **complex form:** $S_i = V_i \overline{I}_i = \sum_j V_i \overline{Y}_{ij} \overline{V}_j$ or $S = \mathrm{diag}(V)\overline{Y}\overline{V}$

   ⇒ purely quadratic and useful for static calculations & optimization

2. **rectangular form:** insert $V = e + if$ and split real & imaginary parts:

   active power: $P_i = \sum_j B_{ij}(e_i f_j - f_i e_j) + G_{ij}(e_i e_j + f_i f_j)$

   reactive power: $Q_i = -\sum_j B_{ij}(e_i e_j + f_i f_j) + G_{ij}(e_i f_j - f_i e_j)$

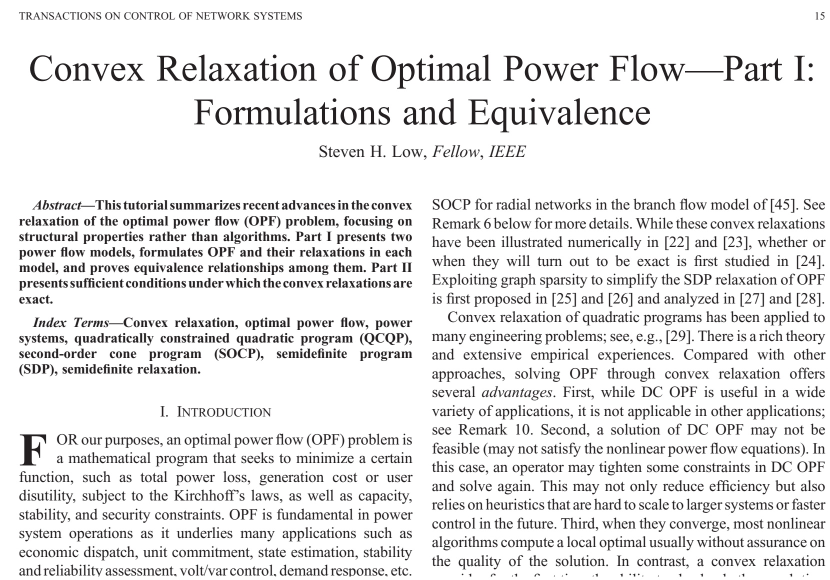
   ⇒ purely quadratic and useful for homotopy methods & QCQPs

3. **matrix form:** define unit-rank p.s.d. Hermitian matrix $W = V \cdot \overline{V}^T$ with components $W_{ij} = V_i \overline{V}_j$, then power flow is $S_i = \sum_j \overline{Y}_{ij} W_{ij}$

   ⇒ linear and useful for relaxations in convex optimization problems

Power balance eqn's – digression

if you're interested in power flow optimization, take a close look at the matrix form



TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS 15

Convex Relaxation of Optimal Power Flow—Part I: Formulations and Equivalence

Steven H. Low, *Fellow, IEEE*

Abstract—**This tutorial summarizes recent advances in the convex relaxation of the optimal power flow (OPF) problem, focusing on structural properties rather than algorithms. Part I presents two power flow models, formulates OPF and their relaxations in each model, and proves equivalence relationships among them. Part II presents sufficient conditions under which the convex relaxations are exact.**

Index Terms—**Convex relaxation, optimal power flow, power systems, quadratically constrained quadratic program (QCQP), second-order cone program (SOCP), semidefinite program (SDP), semidefinite relaxation.**

I. Introduction

For our purposes, an optimal power flow (OPF) problem is a mathematical program that seeks to minimize a certain function, such as total power loss, generation cost or user disutility, subject to the Kirchhoff's laws, as well as capacity, stability, and security constraints. OPF is fundamental in power system operations as it underlies many applications such as economic dispatch, unit commitment, state estimation, stability and reliability assessment, volt/var control, demand response, etc.

SOCP for radial networks in the branch flow model of [45]. See Remark 6 below for more details. While these convex relaxations have been illustrated numerically in [22] and [23], whether or when they will turn out to be exact is first studied in [24]. Exploiting graph sparsity to simplify the SDP relaxation of OPF is first proposed in [25] and [26] and analyzed in [27] and [28].

Convex relaxation of quadratic programs has been applied to many engineering problems; see, e.g., [29]. There is a rich theory and extensive empirical experiences. Compared with other approaches, solving OPF through convex relaxation offers several *advantages*. First, while DC OPF is useful in a wide variety of applications, it is not applicable in other applications; see Remark 10. Second, a solution of DC OPF may not be feasible (may not satisfy the nonlinear power flow equations). In this case, an operator may tighten some constraints in DC OPF and solve again. This may not only reduce efficiency but also relies on heuristics that are hard to scale to larger systems or faster control in the future. Third, when they converge, most nonlinear algorithms compute a local optimal usually without assurance on the quality of the solution. In contrast, a convex relaxation

Power balance eqn's – cont'd

4. **branch flow eqn's** parameterized in flow variables [M. Baran & F. Wu '89]:

   - Ohm's law: $V_i - V_j = Z_{ij} I_{ij}$
   - branch power flow $i \to j$: $S_{ij} = V_i \overline{I_{ij}}$
   - power balance at node i:

$$\underbrace{\sum_{k:\, i \to k} S_{ik}}_{\text{outgoing flows}} + Y_{i,\text{shunt}} |V_i|^2 = S_i + \underbrace{\sum_{j:\, j \to i} (S_{ji} - Z_{ij} |I_{ij}|^2)}_{\text{incoming flows}}$$

- **DistFlow formulation** (or SOCP relaxation) in terms of square magnitude variables $|V_i|^2$ and $|I_{ij}|^2$ (missing angle variables $\angle V_i$ and $\angle I_{ij}$ can sometimes be recovered, e.g., in acyclic case)
- lossless approximation can be solved exactly in acyclic networks (useful for distribution networks) [M. Baran & F. Wu '89, M. Farivar, L. Chen, & S. Low '13]

Branch Flow Model: Relaxations and Convexification—Part I

Power balance eqn's – cont'd

5. **polar form:** insert $V = Ee^{i\theta}$ and split real & imaginary parts:

   active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$

   reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

⇒ will be our focus today since ...

- **power system specs** on frequency $\frac{d}{dt}\theta(t)$ and voltage magnitude E
- **dynamics:** generator swing dynamics affect voltage phase angles & voltage magnitudes are controlled to be constant
- **physical intuition:** usual operation near flat voltage profile $V_i \approx 1e^{i\phi}$ which will give rise to various insights for analysis & design (later)

Power flow simplifications & approximations

power flow equations are too complex & unwieldy for analysis & large computations

- active power: $P_i \;=\; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i \;=\; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

1. **lossless** transmission lines $R_{ij}/X_{ij} = -G_{ij}/B_{ij} \approx 0$

active power: $P_i \;=\; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$

reactive power: $Q_i \;=\; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$

2. **decoupling** near operating point $V_i \approx 1e^{\mathrm{i}\phi}$: $\begin{bmatrix} \partial P/\partial\theta & \partial P/\partial E \\ \partial Q/\partial\theta & \partial Q/\partial E \end{bmatrix} \approx \begin{bmatrix} \star & 0 \\ 0 & \star \end{bmatrix}$

active power: $P_i \;=\; \sum_j B_{ij} \sin(\theta_i - \theta_j)$ (function of angles)

reactive power: $Q_i \;=\; -\sum_j B_{ij} E_i E_j$ (function of magnitudes)

Power flow simplifications & approximations cont'd

- active power: $P_i \;=\; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i \;=\; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

3. **linearization** for small flows near operating point $V_i \approx 1e^{\mathrm{i}\phi}$:

active power: $P_i \;=\; \sum_j B_{ij} (\theta_i - \theta_j)$ (known as DC power flow)

reactive power: : $Q_i \;=\; \sum_j B_{ij} (E_i - E_j)$ (formulation in p.u. system)

4. Multiple **variations & combinations** are possible
   - linearization & decoupling at arbitrary operating points
   - lines with constant R/X ratios [FD, J. Simpson-Porco, & F. Bullo '14]
   - advanced linearizations [S. Bolognani & S. Zampieri '12, '15, B. Gentile et al. '14]
   - *"plenty of heuristics in the hidden stashes of industry"* (B. Wollenberg '15)

DC power flow assumptions are discussed in every book

1290 IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 24, NO. 3, AUGUST 2009

DC Power Flow Revisited

Brian Stott, *Fellow, IEEE*, Jorge Jardim, *Senior Member, IEEE*, and Ongun Alsaç, *Fellow, IEEE*

***Abstract*—Linear MW-only "dc" network power flow models are in widespread and even increasing use, particularly in congestion-constrained market applications. Many versions of these approximate models are possible. When their MW flows are reasonably correct (and this is by no means assured), they can often offer compelling advantages. Given their considerable importance in today's electric power industry, dc models merit closer scrutiny. This paper attempts such a re-examination.**

***Index Terms*—Congestion revenue rights, contingency analysis, dc power flow, economic dispatch, financial transmission rights, LMP pricing, unit commitment.**

I. Introduction

THIS paper addresses so-called "dc" MW-only power flow modeling, which is of increased interest today because of recent upsurges in its use—mostly in LMP-based market applications where prices are constrained by network conges-

II. Why DC Models?

The linear, bilateral, non-complex, often state-independent, properties of a dc-type power flow model have considerable analytical and computational appeal. The use of such a model is limited to those MW-oriented applications where the effects of network voltage and VAr conditions are minimal (a very difficult-to-judge criterion). But then, as opposed to using the ac power flow model, the perceived advantages of a dc model are as follows.

(a) Its solutions are non-iterative, reliable and unique.
(b) Its methods and software are relatively simple.
(c) Its models can be solved and optimized efficiently, particularly in the demanding area of contingency analysis.
(d) Its network data isminimal and relatively easy to obtain.
(e) Its linearity fits the economic theory on which much of transmission-oriented market design is based.
(f) Its approximated MW flows are reasonably accurate, at least for the heavily loaded branches that might constrain system operation.

Conclusion on the **most limiting assumption** of DC power flow: $R/X \approx 0$

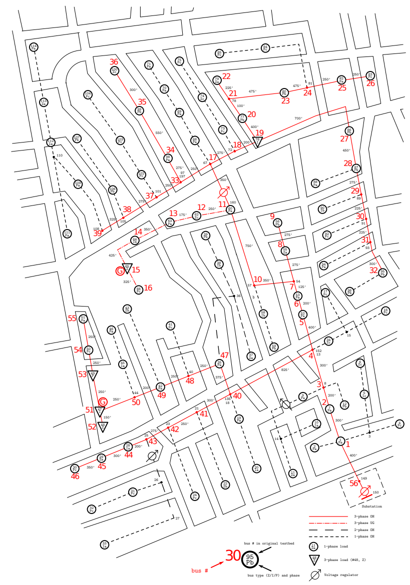
Power flow decoupling for constant (non-zero) R/X ratios

typically a much better assumption (on blackboard)

Advanced approximation method [S. Bolognani & S. Zampieri '15]

- nonlinear power flow equations in complex form
  - power line equations: $YV = I$
  - nodal equation: $S_i = V_i \bar{I}_i$
  - at least one node regulated at a nominal voltage magnitude E_0
- **no assumption** on topology or X & R and no decoupling
- first order **Taylor's expansion** around $E_0 = \infty$ (or zero loading)
  0. existence of flat voltage solution for $E_0 = \infty$
  1. Taylor's terms computed via **implicit function theorem**
  2. nodal currents: $I_i = \frac{\bar{S}_i}{\bar{E}_i} = \frac{\bar{S}_i}{E_0} + \frac{c_i(E_0)}{E_0^2}$
  3. bus voltages: $YV = \frac{\bar{S}}{E_0} + \frac{c(E_0)}{E_0^2}$
  4. $c(E_0)$ bounded in E_0 $\Rightarrow$ **neglect $\frac{c(E_0)}{E_0^2}$ for large E_0**

Advanced approximation method – cont'd



$\Rightarrow$ **LINEAR** power flow formulation:

$$YV = \frac{\bar{S}}{E_0} + \frac{c(E_0)}{E_0^2}$$

$\Rightarrow$ convenient model for **power distribution grids** with lossy lines.

$\Rightarrow$ explicit **approximation bound**:

$$\text{if} \quad E_0^2 > 4\ell_{\max} \|S\|_{\text{tot}}$$

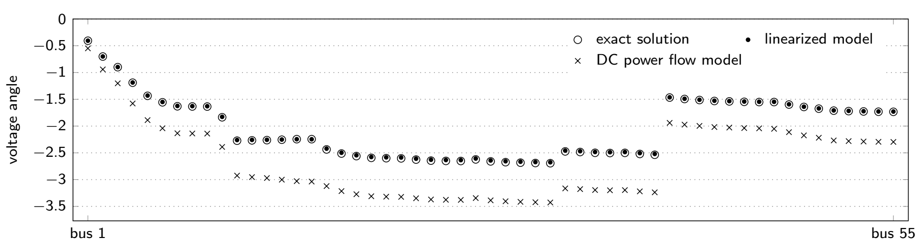
$$\text{then} \quad \left\| \frac{c(E_0)}{E_0^2} \right\| \leq \frac{4\ell_{\max}\|S\|_{\text{tot}}^2}{E_0^2}$$

test feeder and source code available at http://github.com/saveriob/approx-pf

Advanced approximation method – cont'd

- same approximation expressed in **polar coordinates**
  - angles: $\theta = \mathbb{1}_n \theta_0 + \frac{1}{E_0^2} \mathrm{Im}(Y^\dagger \bar{S})$
  - voltage magnitudes: $E = \mathbb{1}_v E_0 + \frac{1}{E_0} \mathrm{Re}(Y^\dagger \bar{S})$

  where $Y^\dagger$ is a pseudoinverse of Y.
- **purely inductive lines** $Y = iB \Rightarrow$ recover **DC power flow model**
- performance evaluation for test feeder:



Dynamic Network Component Models

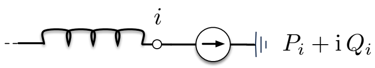
Modeling the "essential" network dynamics

models can be arbitrarily detailed & vary on different time/spatial scales

1. active and reactive **power flow** (e.g., lossless)

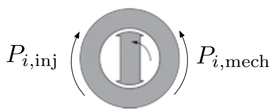
$$P_{i,\text{inj}} = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_{i,\text{inj}} = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

2. passive constant power **loads**



$$P_{i,\text{inj}} = P_i = const.$$
$$Q_{i,\text{inj}} = Q_i = const.$$

3. electromech. **swing dynamics** of synchronous machines



$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{i,\text{mech}} - P_{i,\text{inj}}$$
$$E_i = const.$$

4. **inverters**: DC or variable AC sources with power electronics

(i) have constant/controllable PQ
(ii) or mimic generators with $M = 0$

Common variations in dynamic network models

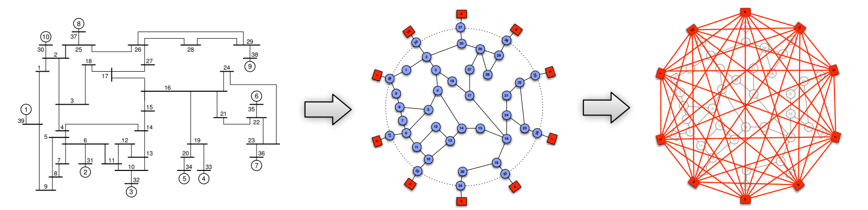
dynamic behavior is very much dependent on load models & generator models

1. frequency/voltage-depend. loads [A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

$$D_i \dot{\theta}_i + P_i = -P_{i,\text{inj}}$$
$$f_i(\dot{V}_i) + Q_i = -Q_{i,\text{inj}}$$

2. network-reduced models after Kron reduction of loads [H. Chiang, F. Wu, & P. Varaiya '94] (very common but poor assumption: $G_{ij} = 0$)

$$M_i \ddot{\theta}_i + D \dot{\theta}_i = P_{i,\text{mech}} - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) \underbrace{- \sum_j G_{ij} E_i E_j \cos(\theta_i - \theta_j)}_{\text{effect of resistive loads}}$$



Structure-preserving power network model [A. Bergen & D. Hill '81]

without Kron-reduction of load buses

- **generator swing dynamics:**

$$\dot{\theta}_i = \omega_i$$
$$M_i \dot{\omega}_i = -D_i \omega_i + P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

- **frequency-dependent loads:** (or inverter-interfaced sources)

$$D_i \dot{\theta}_i = P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

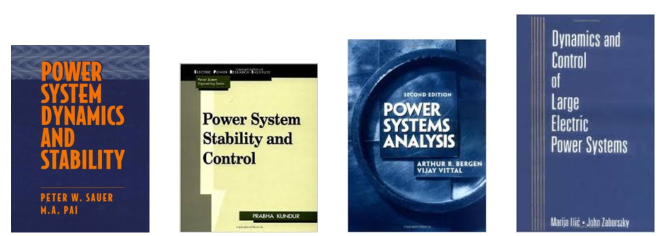
- in **academia:** this "baseline model" is typically further simplified: decoupling, linearization, constant voltages, ...
- in **industry:** much more detailed models used for grid simulations

⇒ **IMHO:** above model captures most interesting network dynamics

Common variations in dynamic network models — cont'd

dynamic behavior is very much dependent on load models & generator models

3. higher order generator dynamics [P. Sauer & M. Pai '98] — voltages, controls, magnetics etc. (reduction via singular perturbations)
4. dynamic & detailed load models [D. Karlsson & D. Hill '94] — aggregated dynamic load behavior (e.g., load recovery after voltage step)
5. time-domain models [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12] — passive Port-Hamiltonian models for machines & RLC circuitry



"Power system research is all about the art of making the right assumptions."

Lots of current research activity on time-domain models

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journal homepage: www.elsevier.com/locate/ejcon

A port-Hamiltonian approach to power network modeling and analysis

S. Fiaz, D. Zonetti, R. Ortega, J.M.A. Scherpen, A.J. van der Schaft

Automatica

Brief paper

Towards Kron reduction of generalized electrical networks

Sina Yamac Caliskan, Paulo Tabuada

Synchronization of Nonlinear Circuits in Dynamic Electrical Networks With General Topologies

Sairaj V. Dhople, *Member, IEEE*, Brian B. Johnson, *Member, IEEE*, Florian Dörfler, *Member, IEEE*, and Abdullah O. Hamadeh

Compositional Transient Stability Analysis of Multimachine Power Networks

Sina Yamac Caliskan and Paulo Tabuada

Outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

- Decoupled Active Power Flow (Synchronization)
- Reactive Power Flow (Voltage Collapse)
- Coupled & Lossy Power Flow
- Transient Rotor Angle Stability

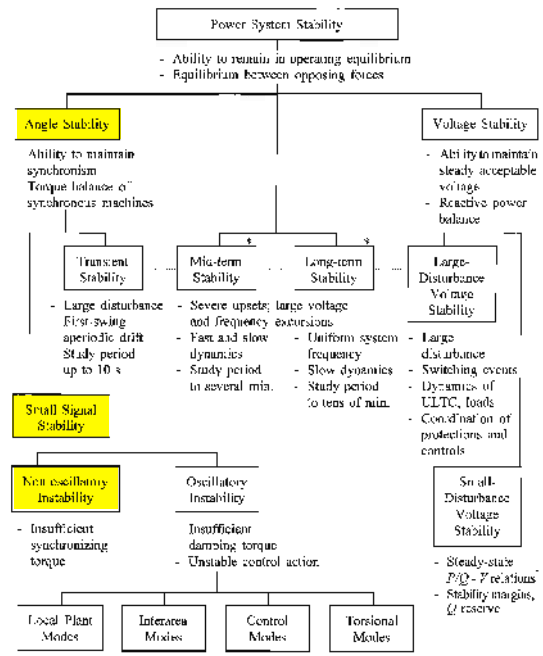
Power System Control Hierarchy

Power System Oscillations

Conclusions

Decoupled Active Power Flow (Synchronization)

Our first stab at power system stability



Preliminary insights on decoupled and lossless power flow

power flow equations:

$$P_i = \sum_{j=1}^{n} B_{ij} \sin(\theta_i - \theta_j)$$

$\Rightarrow$ solution space: $\mathbb{T}^n = \mathbb{S}^1 \times \cdots \times \mathbb{S}^1$

rotational symmetry:

if θ^* is a solution

$\Rightarrow$ $\theta^* + const. \cdot \mathbb{1}_n$ is another solution

$\Rightarrow$ solution space: $\mathbb{T}^n \setminus \mathbb{S}^1$

necessary feasibility condition:

$$\sum_{i=1}^{n} P_i = 0 \;\Leftarrow\; \exists \text{ a solution}$$

(by summing all equations)

$\triangleq$ power balance

$\Rightarrow$ typically not true (w/o slack bus) due to unknown load demand

$\Rightarrow$ need to consider dynamics

Homework: think about the above conditions in coupled and/or lossy case

Synchronization & feasibility of active power flow

basic problem setup

- **structure-preserving power network model** [A. Bergen & D. Hill '81]:
  (simple dynamics & decoupled lossless flows capture essential phenomena)

  synchronous machines: $M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

  frequency-dependent loads: $D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

- **synchronization** = sync'd frequencies & bounded active power flows

  $$\dot{\theta}_i = \omega_{\text{sync}} \;\forall\, i \in \mathcal{V} \quad \& \quad |\theta_i - \theta_j| \leq \gamma < \pi/2 \;\forall\, \{i,j\} \in \mathcal{E}$$

  = active power flow feasibility & security constraints

- **sync is crucial** for the functionality and operation of the power grid

- **explicit sync frequency:** if sync, then $\omega_{\text{sync}} = \sum_i P_i / \sum_i D_i$
  (by summing over all equations)

Synchronization & feasibility of active power flow

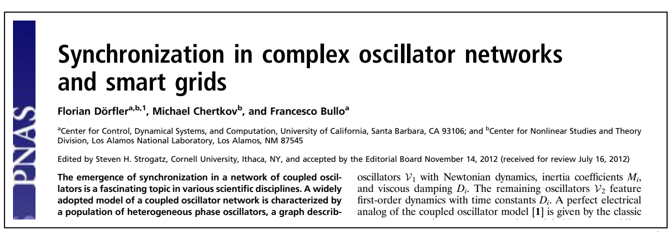
some key questions

Given: network parameters & topology and load & generation profile

Q: "$\exists$ an optimal, stable, and robust sync'd operating point ?"

1. Security analysis [Araposthatis et al. '81, Wu et al. '80 & '82, Ilić '92, ...]
2. Load flow feasibility [Chiang et al. '90, Dobson '92, Lesieutre et al. '99, ...]
3. Optimal generation dispatch [Lavaei et al. '12, Bose et al. '12, ...]
4. Transient stability [Sastry et al. '80, Bergen et al. '81, Hill et al. '86, ...]
5. Inverters in microgrids [Chandorkar et. al. '93, Guerrero et al. '09, Zhong '11, ...]
6. Complex networks [Hill et al. '06, Strogatz '01, Arenas et al '08, ...]

Further reading on sync problem: (my perspective)



PNAS

Synchronization in complex oscillator networks and smart grids

Florian Dörfler[a,b,1], Michael Chertkov[b], and Francesco Bullo[a]

[a]Center for Control, Dynamical Systems, and Computation, University of California, Santa Barbara, CA 93106; and [b]Center for Nonlinear Studies and Theory Division, Los Alamos National Laboratory, Los Alamos, NM 87545

Edited by Steven H. Strogatz, Cornell University, Ithaca, NY, and accepted by the Editorial Board November 14, 2012 (received for review July 16, 2012)

The emergence of synchronization in a network of coupled oscillators is a fascinating topic in various scientific disciplines. A widely adopted model of a coupled oscillator network is characterized by a population of heterogeneous phase oscillators, a graph describ-

oscillators $\mathcal{V}_1$ with Newtonian dynamics, inertia coefficients M_i, and viscous damping D_i. The remaining oscillators $\mathcal{V}_2$ feature first-order dynamics with time constants D_i. A perfect electrical analog of the coupled oscillator model [1] is given by the classic

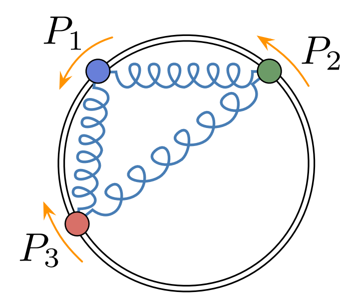
A perspective from coupled oscillators

Mechanical oscillator network

Angles $(\theta_1, \ldots, \theta_n)$ evolve on $\mathbb{T}^n$ as

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

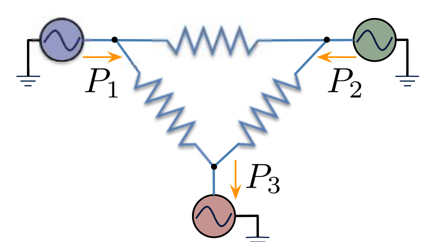
- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $P_i \in \mathbb{R}$
- spring constants $B_{ij} \geq 0$



Structure-preserving power network

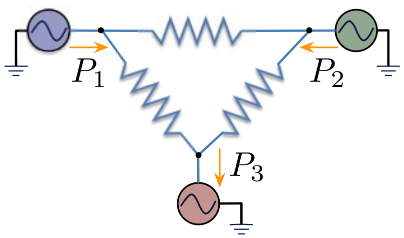
$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

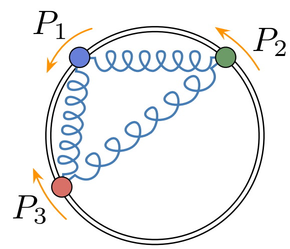
$$D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$



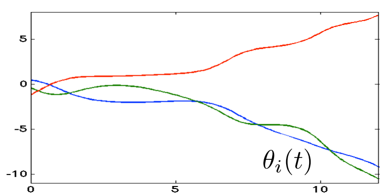
Phenomenology of sync in power networks

- sync is **crucial for AC power grids**

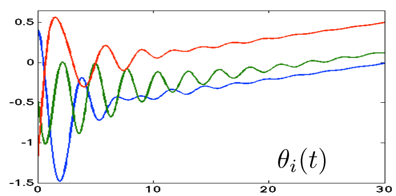




- sync is a **trade-off**



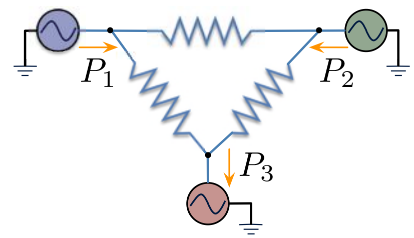
weak coupling & heterogeneous

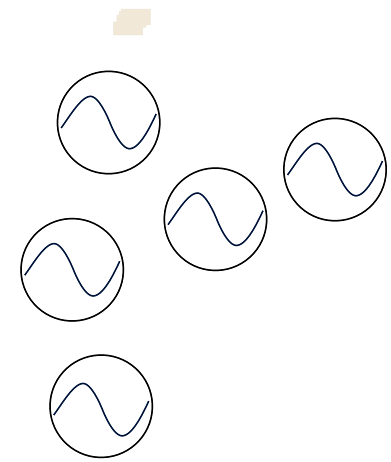


strong coupling & homogeneous

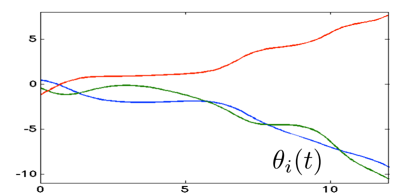
Phenomenology of sync in power networks

- sync is **crucial for AC power grids**





- sync is a **trade-off**



weak coupling & heterogeneous

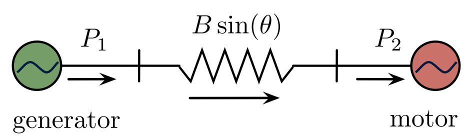
Blackout India July 30/31 2012

Derivation of a two-bus toy model

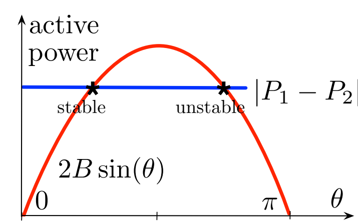
on blackboard

Back of the envelope calculations for the two-node case

generator connected to identical motor shows bifurcation at difference angle $\theta = \pi/2$



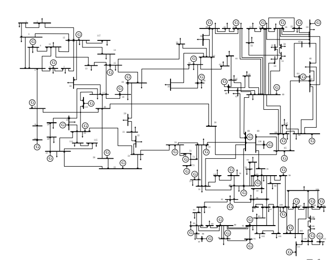
$$M\ddot{\theta} + D\dot{\theta} = P_1 - P_2 - 2B\sin(\theta)$$



$\exists$ stable sync $\Leftrightarrow$ $B > |P_1 - P_2|/2$ $\Leftrightarrow$ "ntwk coupling > heterogeneity"

Q1: Quantitative generalization to a complex & large-scale network?

Q2: What are the particular metrics for coupling and heterogeneity?



Some properties of Laplacian matrices

on blackboard

Who knows consensus systems?

on blackboard

Primer on algebraic graph theory

for a connected and undirected graph

Laplacian matrix $L =$ "degree matrix" − "adjacency matrix"

$$L = L^T = \begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -B_{i1} & \cdots & \sum_{j=1}^n B_{ij} & \cdots & -B_{in} \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix} \geq 0$$

is positive semidefinite with one zero eigenvalue & eigenvector $\mathbb{1}_n$

Notions of connectivity

- spectral: 2nd smallest eigenvalue of L is "algebraic connectivity" $\lambda_2(L)$
- topological: degree $\sum_{j=1}^n B_{ij}$ or degree distribution

Notions of heterogeneity

$\|P\|_{\mathcal{E},\infty} = \max_{\{i,j\}\in\mathcal{E}} |P_i - P_j|$, $\|P\|_{\mathcal{E},2} = \left(\sum_{\{i,j\}\in\mathcal{E}} |P_i - P_j|^2\right)^{1/2}$

Synchronization in "complex" networks

for a first-order model — all results generalize locally

$$\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij} \sin(\theta_i - \theta_j)$$

1. **local stability** for equilibria satisfying $|\theta_i^* - \theta_j^*| < \pi/2 \ \forall \{i,j\} \in \mathcal{E}$
   (linearization is Laplacian matrix)
2. **necessary sync condition**: $\sum_j B_{ij} \geq |P_i - \omega_{\text{sync}}| \Leftarrow$ sync
   (so that syn'd solution exists)
3. **sufficient sync condition**: $\lambda_2(L) > \|P\|_{\mathcal{E},2} \quad \Rightarrow \quad$ sync
   [FD & F. Bullo '12]

⇒ ∃ similar conditions with diff. metrics on coupling & heterogeneity

⇒ **Problem:** sharpest general conditions are conservative

Can we solve the power flow equations exactly?

on blackboard

A nearly exact sync condition

[FD, M. Chertkov, & F. Bullo '13]

1. search equilibrium θ^* with $|\theta_i^* - \theta_j^*| \leq \gamma < \pi/2$ for all $\{i,j\} \in \mathcal{E}$:

$$P_i = \sum\nolimits_j B_{ij} \sin(\theta_i - \theta_j) \qquad (\star)$$

2. consider linear "small-angle" DC approximation of $(\star)$:

$$P_i = \sum\nolimits_j B_{ij}(\delta_i - \delta_j) \quad \Leftrightarrow \quad P = L\delta \qquad (\star\star)$$

unique solution (modulo symmetry) of $(\star\star)$ is $\delta^* = L^\dagger P$

3. solution ansatz for $(\star)$: $\boxed{\theta_i^* - \theta_j^* = \arcsin(\delta_i^* - \delta_j^*)}$ **(for a tree)**

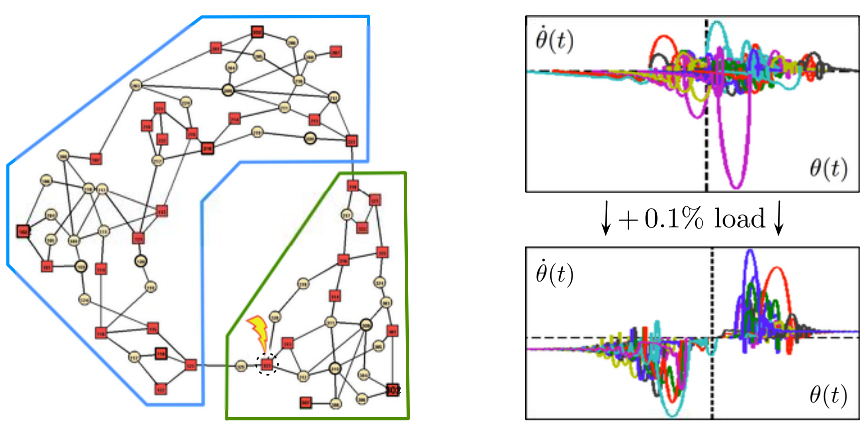
$$P_i = \sum\nolimits_{j=1}^n a_{ij} \sin(\theta_i - \theta_j) = \sum\nolimits_{j=1}^n a_{ij} \sin\big(\arcsin(\delta_i^* - \delta_j^*)\big) = P_i \quad \checkmark$$

$\Rightarrow$ **Thm:** $\exists\, \theta^*$ with $|\theta_i^* - \theta_j^*| \leq \gamma \;\forall \{i,j\} \in \mathcal{E} \;\Leftrightarrow\; \|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$

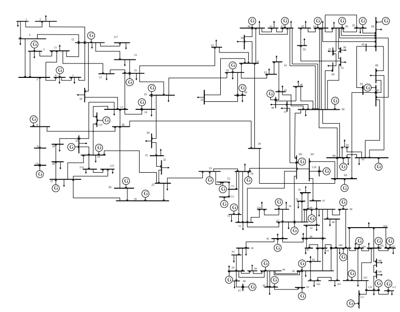


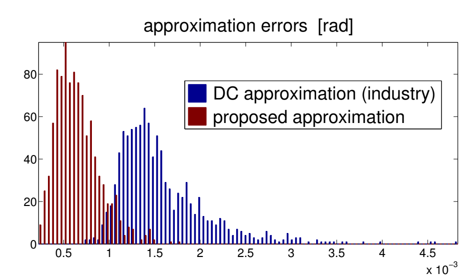
Reliability Test System RTS 96 under two loading conditions

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$





IEEE 118 bus system (Midwest)

Outperforms conventional DC approximation "on average & in the tail".

More on power flow approximations

Randomized power network test cases

with 50 % randomized loads and 33 % randomized generation

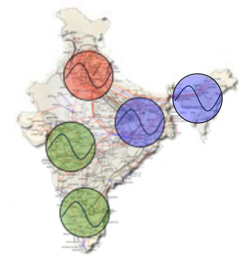
Randomized test case (1000 instances)	Numerical worst-case angle differences: $\max_{\{i,j\}\in\mathcal{E}} \lvert\theta_i^* - \theta_j^*\rvert$	Analytic prediction of angle differences: $\arcsin(\lVert L^\dagger P\rVert_{\mathcal{E},\infty})$	Accuracy of condition: $\arcsin(\lVert L^\dagger P\rVert_{\mathcal{E},\infty}) - \max_{\{i,j\}\in\mathcal{E}} \lvert\theta_i^* - \theta_j^*\rvert$
9 bus system	0.12889 rad	0.12893 rad	$4.1218 \cdot 10^{-5}$ rad
IEEE 14 bus system	0.16622 rad	0.16650 rad	$2.7995 \cdot 10^{-4}$ rad
IEEE RTS 24	0.22309 rad	0.22480 rad	$1.7089 \cdot 10^{-3}$ rad
IEEE 30 bus system	0.16430 rad	0.16456 rad	$2.6140 \cdot 10^{-4}$ rad
New England 39	0.16821 rad	0.16828 rad	$6.6355 \cdot 10^{-5}$ rad
IEEE 57 bus system	0.20295 rad	0.22358 rad	$2.0630 \cdot 10^{-2}$ rad
IEEE RTS 96	0.24593 rad	0.24854 rad	$2.6076 \cdot 10^{-3}$ rad
IEEE 118 bus system	0.23524 rad	0.23584 rad	$5.9959 \cdot 10^{-4}$ rad
IEEE 300 bus system	0.43204 rad	0.43257 rad	$5.2618 \cdot 10^{-4}$ rad
Polish 2383 bus system (winter peak 1999/2000)	0.25144 rad	0.25566 rad	$4.2183 \cdot 10^{-3}$ rad

Discrete control actions to assure sync

1. (re)dispatch generation subject to **security constraints**:

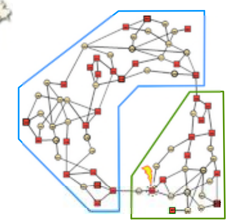
$$\begin{aligned} &\text{find}_{\,\theta\in\mathbb{T}^n,\,u\in\mathbb{R}^{n_I}} \quad \text{subject to} \\ &\text{source power balance:} && u_i = P_i(\theta) \\ &\text{load power balance:} && P_i = P_i(\theta) \\ &\text{branch flow constraints:} && \lvert\theta_i - \theta_j\rvert \le \gamma_{ij} < \pi/2 \end{aligned}$$

2. **remedial action schemes**: load/production shedding & islanding

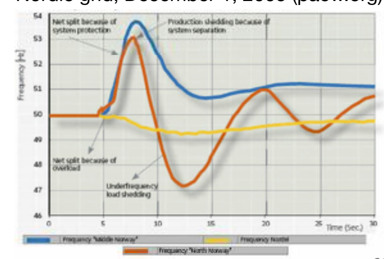


India, July 30/31 2012

RTS 96 example

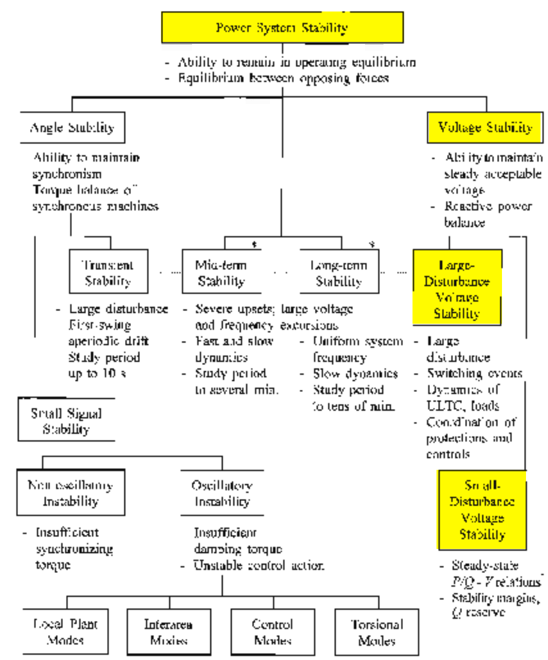


Nordic grid, December 1, 2005 (pacw.org)



Decoupled Reactive Power Flow (Voltage Collapse)

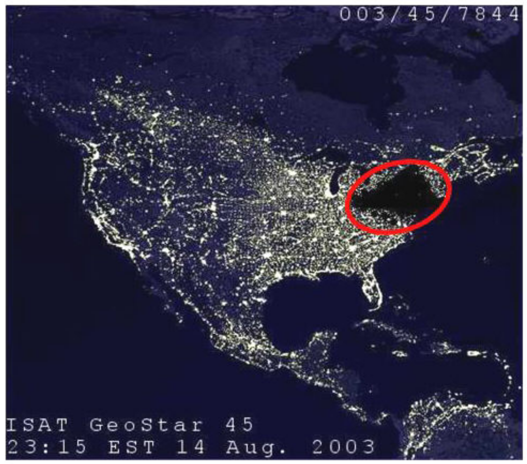
Apparently a different beast



Voltage collapse in power networks

- **voltage instability**: loading > capacity ⇒ voltages drop
  "mainly" a reactive power phenomena
- **recent outages:** Québec '96, Scandinavia '03, Northeast '03, Athens '04

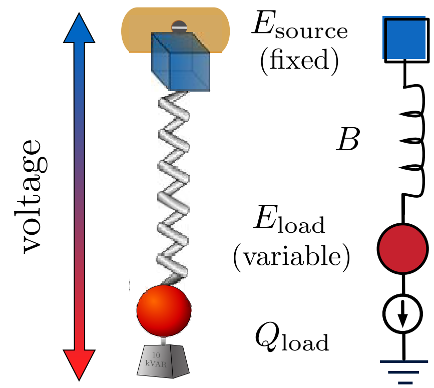
> *"Voltage collapse is still the biggest single threat to the transmission system. It's what keeps me awake at night."*
> – Phil Harris, CEO PJM.

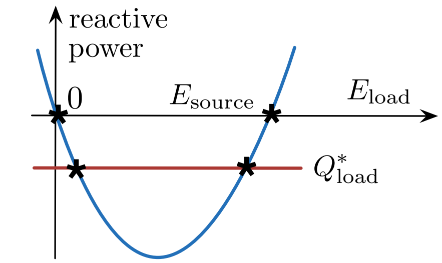


Back of the envelope calculations for the two-node case

source connected to load shows bifurcation at load voltage $E_{\text{load}} = E_{\text{source}}/2$

reactive power balance at load: $Q_{\text{load}} = B\,E_{\text{load}}(E_{\text{load}} - E_{\text{source}})$





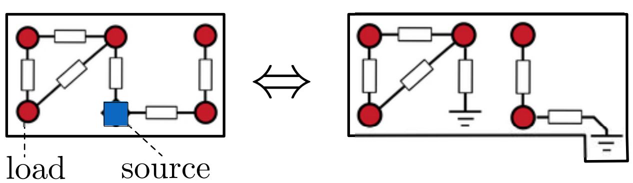
$E_{\text{load}} \in \mathbb{R} \Leftrightarrow Q_{\text{load}} \geq -B\,(E_{\text{source}})^2/4$

∃ high load voltage solution ⇔ (load) < (network)(source voltage)^2/4

Preliminary insights when going to "complex" networks

- **sources** with constant voltage magnitudes E_i
- **loads** with constant power demand $Q_i(E) = Q_i$

⇒ WLOG assume that network among loads is connected



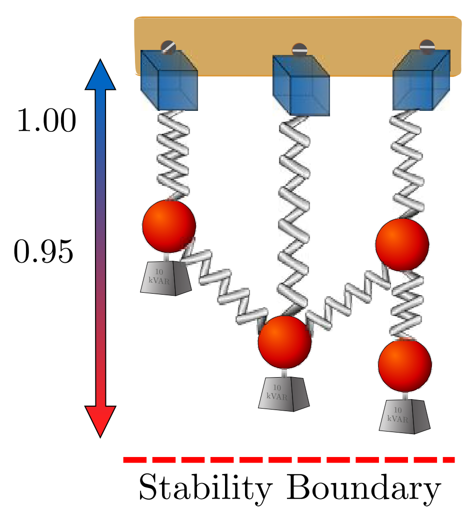
⇒ reactive power balance: $Q_i = -\sum_j B_{ij} E_i E_j$ or $Q = -\operatorname{diag}(E) B E$

⇒ necessary feasibility condition: $\sum_{i=1}^{n} Q_i \geq 0 \Leftarrow \exists$ a solution

(by summing all equations and using $-E^T B E \geq 0$)

Intuition extends to complex networks – essential insights

Reactive power balance:
$Q_i = -\sum_j B_{ij} E_i E_j$



Suff. & tight cond' for general case [J. Simpson-Porco, FD, & F. Bullo, '14]:

∃ unique high-voltage solution E_{load}

⇔

$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$

Intuition extends to complex networks – details

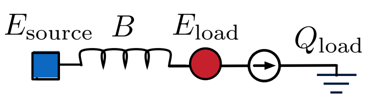
on blackboard

Intuition extends to complex networks – details cont'd

on blackboard

More back of the envelope calculations

$$Q_L = B\,E_L(E_L - E_S)$$



Exact soln: $E_L = \frac{E_S}{2}\left(1+\sqrt{1+4Q_L/(BE_S^2)}\right) = \frac{E_S}{2}\left(1+\sqrt{1-\frac{Q_L}{Q_{\text{crit}}}}\right)$

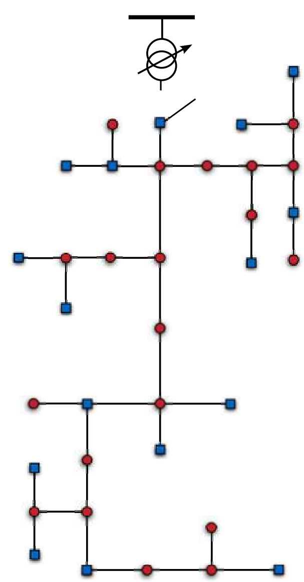
$\Rightarrow$ Taylor exp. for $\frac{Q_L}{Q_{\text{crit}}} \to 0$: $$E_L \approx E_S\left(1-\frac{1}{4}\frac{Q_L}{Q_{\text{crit}}}\right)$$

- **General case**: existence & approximation from implicit function thm
  - if all loads Q_i are "sufficiently small" [D. Molzahn, B. Lesieutre, & C. DeMarco '12]
  - if slack bus has "sufficiently large" E_S [S. Bolognani & S. Zampieri '12 & '14]
  - if each source is above a "sufficiently large" E_{source} [B. Gentile et al. '14]
  - if previous existence condition is met [J. Simpson-Porco, FD, & F. Bullo, '14]

$\Rightarrow$ 1st order approximation: $$E_L \approx \operatorname{diag}(E_L^*)\left(\mathbb{1}-\frac{1}{4}Q_{\text{crit}}^{-1}Q_L\right)$$

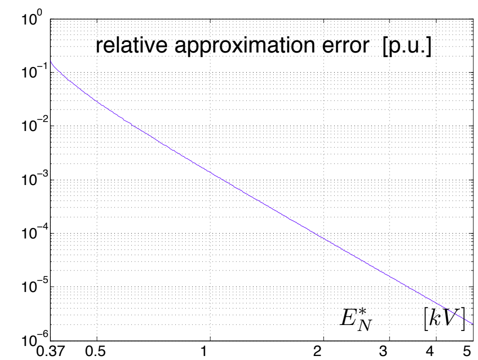
Linear DC approximation extends to complex networks

verification via *IEEE 37 bus distribution system* (SoCal)

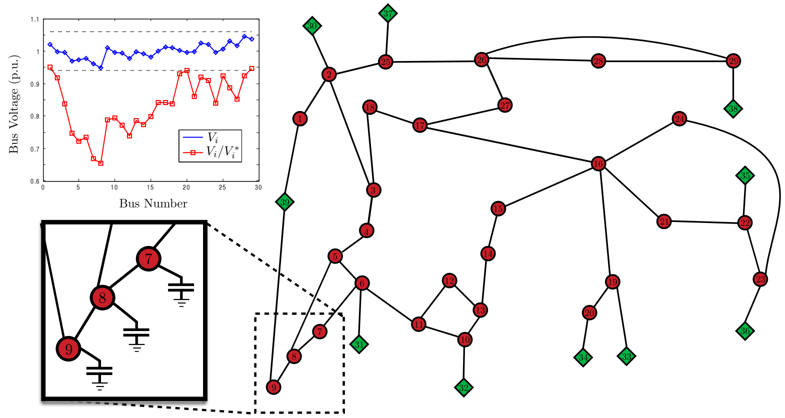


Reactive DC approximation [B. Gentile, J. Simpson-Porco, FD, S. Zampieri, & F. Bullo, '14]:

$$E_L \approx \operatorname{diag}(E_L^*)\left(\mathbb{1}+\tfrac{1}{4}Q_{\text{crit}}^{-1}Q_L\right) + \text{h.o.t.}$$



Discrete control actions for voltage stability

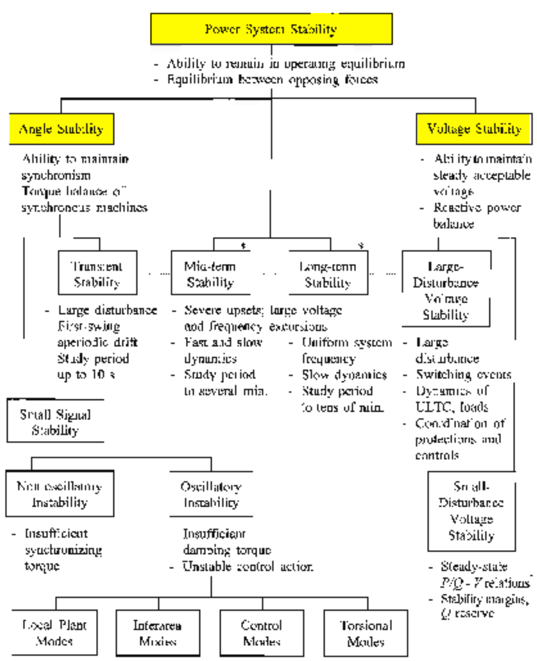


1. shunts support voltage magnitudes, but hide proximity to collapse
   $\Rightarrow$ ratios E_i/E_i^* more useful than per-unit voltages
2. $|Q^{-1}_{\text{crit},89}| > |Q^{-1}_{\text{crit},87}|$ means E_8/E_8^* more sensitive to Q_9 then to Q_7

$\Longrightarrow$ place SVC at bus 9 to support E_8 & increase stability margin

Coupled & Lossy Power Flow

This is not even really on the map

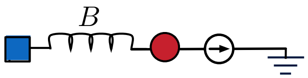


Solving the two-node case

on blackboard

Simplest example shows surprisingly complex behavior

- PV source, PQ load, & lossless line

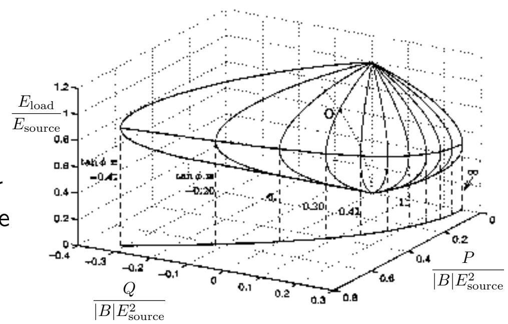


$$P = B\, E_{\text{source}}\, E_{\text{load}} \sin(\theta)$$
$$Q = B\, E_{\text{load}}^2 - B\, E_{\text{source}}\, E_{\text{load}} \cos(\theta)$$

- after eliminating θ, there exists $E_{\text{load}} \in \mathbb{R}_{\geq 0}$ if and only if

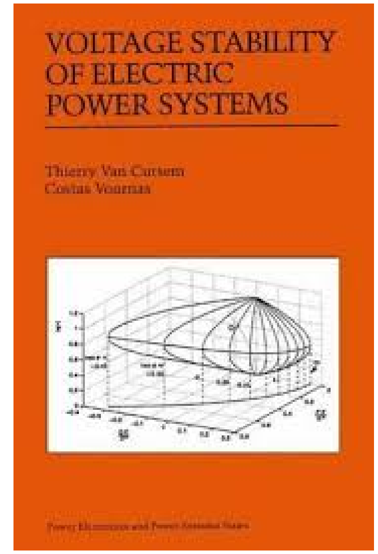
$$P^2 - B\, E_{\text{source}}^2\, Q \leq B^2 E_{\text{source}}^4 / 4$$

- Observations:
  1. $P = 0$ case consistent with previous decoupled analysis
  2. $Q = 0$ case delivers 1/2 transfer capacity from decoupled case
  3. intermediate cases $Q = P \tan\phi$ give so-called "nose curves"



Recommended reading to understand a glimpse

at least once in a life-time you should read chapter 2 ...



Coupled & lossy power flow in complex networks

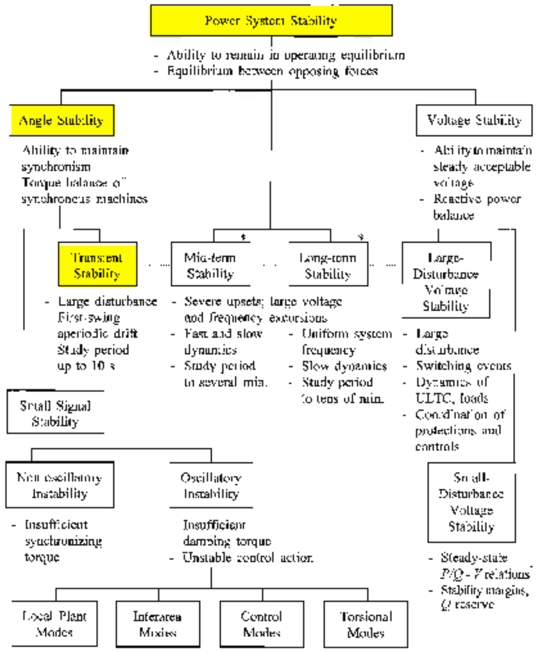
- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

- what makes it so much harder than the previous two node case?
  losses, mixed lines, cycles, PQ-PQ connections, ...
- much theoretic work, qualitative understanding, & numeric approaches:
  - existence of solutions [Thorp, Schulz, & Ilić '86, Wu & Kumagai '82]
  - solution space [Hiskens & Davy '01, Overbye & Klump '96, Van Cutsem '98, ...]
  - distance-to-failure [Venikov '75, Abe & Isono '76, Dobson '89, Andersson & Hill '93, ...]
  - convex relaxation approaches [Molzahn, Lesieutre, & DeMarco '12]
- little analytic & quantitative understanding beyond the two-node case

"Whoever figures that one out wins a noble prize!" Pete Sauer

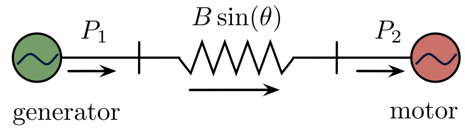
Transient Rotor Angle Stability

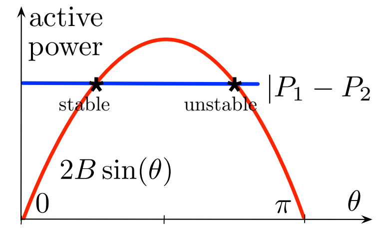
The crown jewel of power system stability



Revisit of the two-node case — the forced pendulum

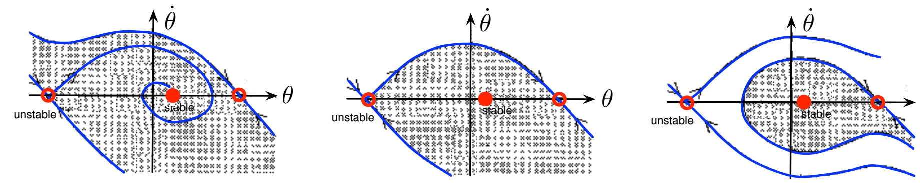
more complex than anticipated





$$\dot{\theta} = \omega$$
$$M\dot{\omega} = -D\omega + P_1 - P_2 - 2B\sin(\theta)$$

- **Local stability**: $\exists$ local stable solution $\Leftrightarrow$ $B > |P_1 - P_2|/2$
- **Global stability**: depends on gap $B > |P_1 - P_2|/2$ and D/M ratio

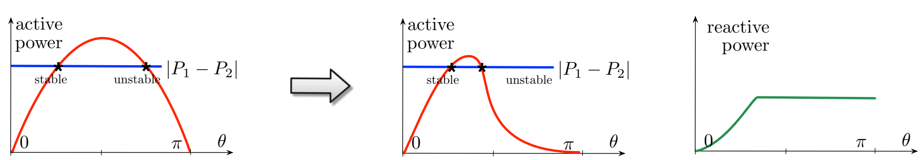


$(D/M) > (D/M)_{\text{critical}}$ $(D/M) = (D/M)_{\text{critical}}$ $(D/M) < (D/M)_{\text{critical}}$

Revisit of the two-node case — cont'd

the story is not complete ... some further effects that we swept under the carpet

- **Voltage reduction**: to maintain a constant voltage, a generator needs to provide reactive power. When encountering the maximum reactive power support, the generator becomes a PQ bus and voltage drops.



- **Load sensitivity**: different behavior depending on load model: resistive, constant power, frequency-dependent, dynamic, power electronics, ...
- **Singularity-issues** for coupled power flows (load voltage collapse)
- **Losses & higher-order dynamics** change stability properties ...

$\Rightarrow$ quickly run into computational approaches

Primer on Lyapunov functions

on blackboard

Hamiltonian analysis of the swing equations

more famously known as "energy function analysis" (on blackboard)

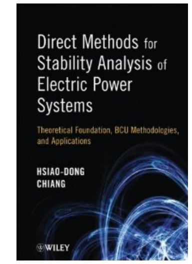
Transient stability in multi-machine power systems

generators:

$$\dot{\theta}_i = \omega_i$$
$$M_i\dot{\omega}_i = -D_i\omega_i + P_i - \sum_j B_{ij}E_iE_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum_j B_{ij}E_iE_j \cos(\theta_i - \theta_j)$$

loads:

$$D_i\dot{\theta}_i = P_i - \sum_j B_{ij}E_iE_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum_j B_{ij}E_iE_j \cos(\theta_i - \theta_j)$$



Challenge (improbable): faster-than-real-time transient stability assessment

Energy function methods for simple lossless models via Lyapunov function

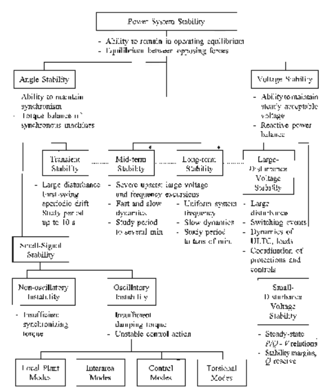
$$V(\omega, \theta, E) = \sum_i \frac{1}{2}M_i\omega_i^2 - \sum_i P_i\theta_i - \sum_i Q_i \log E_i - \sum_{ij} B_{ij}E_iE_j \cos(\theta_i - \theta_j)$$

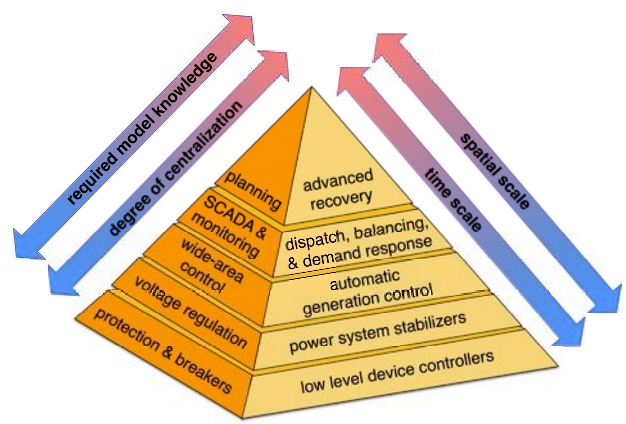
Computational approaches: level sets of energy functions & unstable equilibria, sum-of-squares methods, convex optimization approaches, time-domain simulations, ... (holy grail of power system stability)

Outline

A plethora of control tasks and nested control layers

organized in hierarchy and separated by states & spatial/temporal/centralization scales

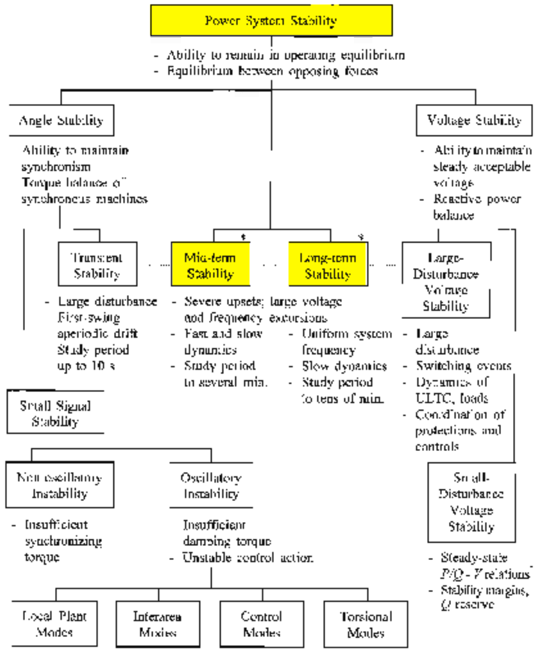




We will focus on frequency control & primary/secondary/tertiary layers.

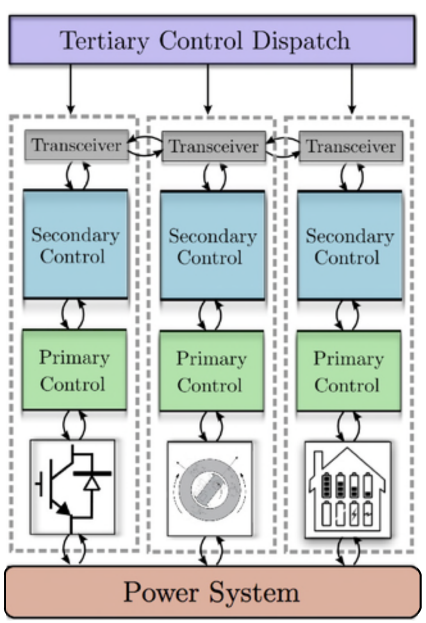
All dynamics & controllers are interacting. Classification & hierarchy are for simplicity.

Where are we on the map?



Objectives

Hierarchical frequency control architecture & objectives



3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (minutes)
   - Goal: maintain operating point in presence of disturbances
   - Strategy: centralized
1. **Primary control** (real-time)
   - Goal: stabilize frequency & share unknown load
   - Strategy: decentralized

Q: Is this layered & hierarchical architecture still appropriate for tomorrow's power system?

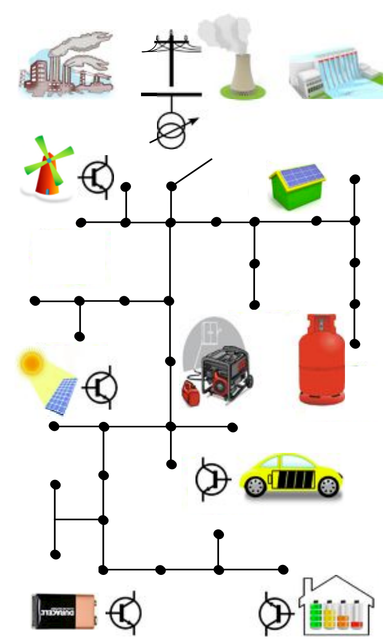
Is this hierarchical control architecture still appropriate?

Some recent developments

- increasing renewable integration & deregulated energy markets
- bulk generation replaced by distributed generation
- synchronous machines replaced by power electronics sources
- low gas prices & substitutions

Some new problem scenarios

- alternative spinning reserves: storage, load control, & DER
- networks of low-inertia & distributed renewable sources
- small-footprint islanded systems



Need to adapt the control hierarchy in tomorrow's grid

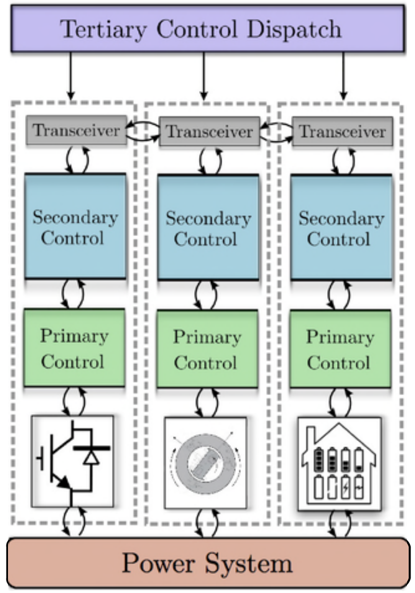
☹ perational challenges

- more uncertainty & less inertia
- more volatile & faster fluctuations
- plug'n'play control: fast, model-free, & without central authority

☺ pportunities

- re-instrumentation: comm & sensors
- more & faster spinning reserves
- advances in control of cyber-physical & complex systems

⇒ **break** vertical & horizontal **hierarchy**

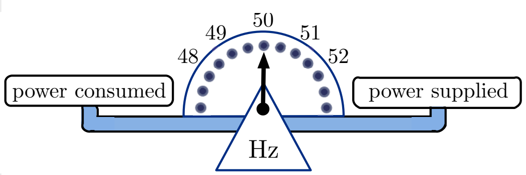


Primary Control

Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$



Conventional wisdom: physics are naturally stable & sync frequency reveals power imbalance

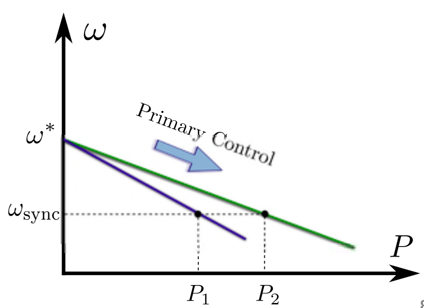
recall: $\omega_{\text{sync}} = \sum_i P_i^*/D_i$

$P/\dot{\theta}$ droop control:

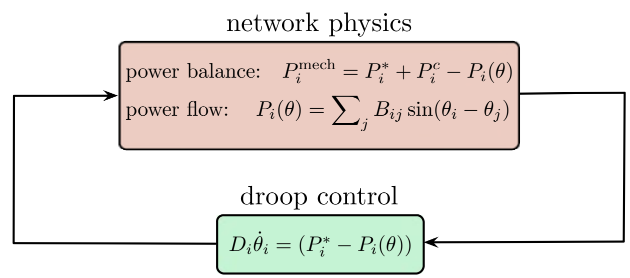
$$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$$

$$\Updownarrow$$

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta)$$



Putting the pieces together...



synchronous machines: $M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

inverter sources & controllable loads: $D_i\dot{\theta}_i = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

passive loads & power-point tracking sources: $0 = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

Closed-loop stability?

Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

$\exists$ unique & exp. stable frequency sync $\iff$ active power flow is feasible

Main **proof ideas** and some **further results**:

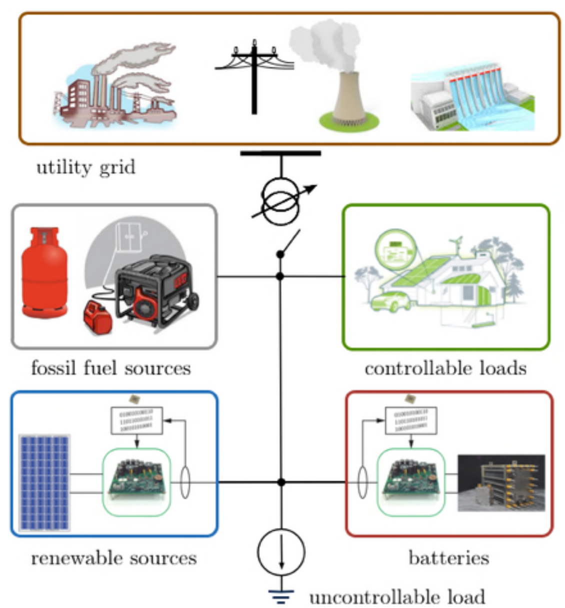
- stability via Jacobian arguments (as before)
- synchronization frequency: ($\propto$ power balance) $$\omega_{\text{sync}} = \omega^* + \frac{\sum_{\text{sources}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{sources}} D_i}$$
- steady-state power injections: (depend on D_i & P_i^*) $$\mathcal{P}_i = \begin{cases} P_i^* & \text{(load } \#i) \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & \text{(source } \#i) \end{cases}$$

power sharing & economic optimality under droop control

(sometimes in tertiary layer)

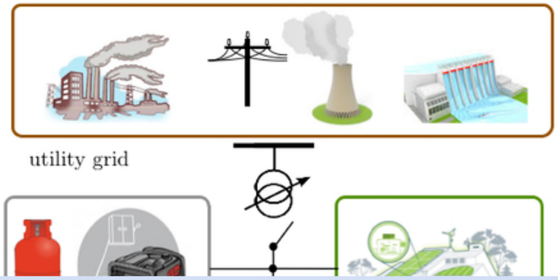
Tertiary control and energy management

an offline resource allocation and scheduling problem



Tertiary control and energy management

an offline resource allocation and scheduling problem



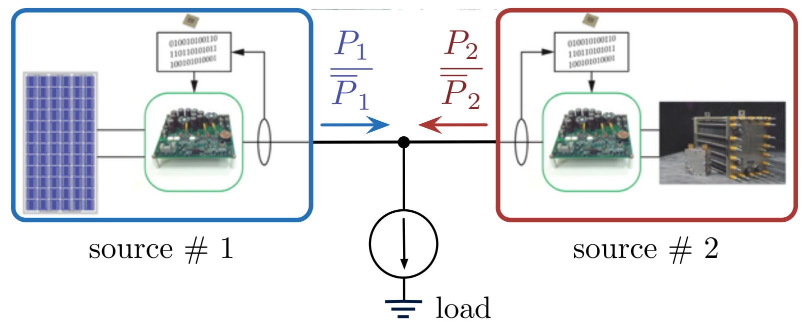
$$\begin{array}{ll} \text{minimize} & \{\text{cost of generation, losses, } \dots\} \\ \text{subject to} & \\ \quad \text{equality constraints:} & \text{power balance equations} \\ \quad \text{inequality constraints:} & \text{flow/injection/voltage constraints} \\ \quad \text{logic constraints:} & \text{commit generators yes/no} \\ & \vdots \end{array}$$

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) / \overline{P}_i = P_j(\theta) / \overline{P}_j$



Analysis of fair proportional load sharing

on blackboard

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) / \overline{P}_i = P_j(\theta) / \overline{P}_j$

Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$\boxed{D_i/\overline{P}_i = D_j/\overline{P}_j \;\; \& \;\; P_i^*/\overline{P}_i = P_j^*/\overline{P}_j}$$

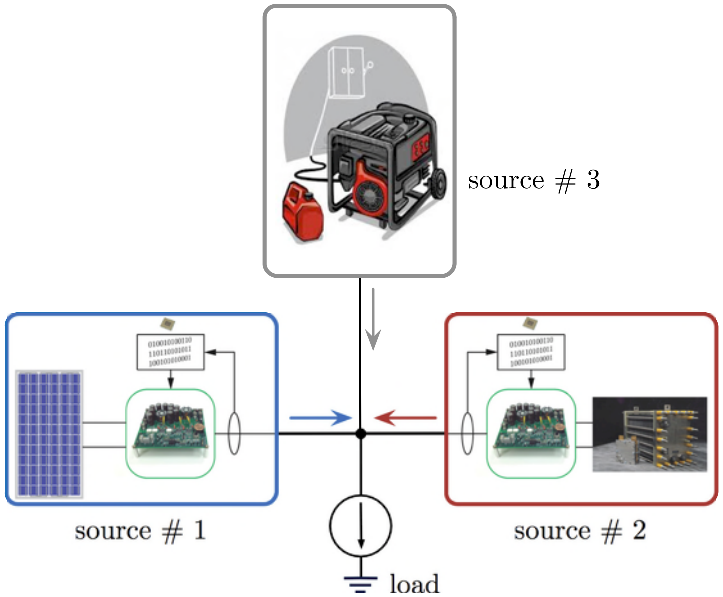
The the following statements hold:

(i) Proportional load sharing: $P_i(\theta) / \overline{P}_i = P_j(\theta) / \overline{P}_j$

(ii) Constraints met: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \overline{P}_j \;\; \Leftrightarrow \;\; P_i(\theta) \in [0, \overline{P}_i]$

Objective I: fair proportional load sharing

proportional load sharing is not always the right objective

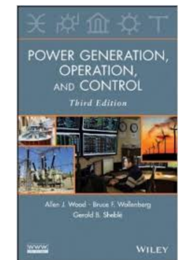


Objective II: optimal power flow = tertiary control

an offline resource allocation/scheduling problem

minimize	{cost of generation, losses, . . . }
subject to	
equality constraints:	power balance equations
inequality constraints:	flow/injection/voltage constraints
logic constraints:	commit generators yes/no
	⋮

Will be discussed more in detail tomorrow.



Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

minimize $_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}}$	$f(u) = \sum_{\text{sources}} \alpha_i u_i^2$
subject to	
source power balance:	$P_i^* + u_i = P_i(\theta)$
load power balance:	$P_i^* = P_i(\theta)$
branch flow constraints:	$\lvert\theta_i - \theta_j\rvert \leq \gamma_{ij} < \pi/2$

An even simpler problem formulation:

minimize $_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}}$	$f(u) = \sum_{\text{sources}} \alpha_i u_i^2$
subject to	
power balance:	$\sum_i P_i^* + \sum_i u_i = 0$

Both are equivalent in the strictly feasible case!

Both are equivalent in the strictly feasible case

. . . and marginal costs are identical: $\alpha_i u_i^* = \alpha_j u_j^*$ (on blackboard)

Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

$$\text{minimize}_{\theta\in\mathbb{T}^n,\, u\in\mathbb{R}^{n_I}} \quad f(u) = \sum\nolimits_{\text{sources}} \alpha_i u_i^2$$

subject to

source power balance: $P_i^* + u_i = P_i(\theta)$

load power balance: $P_i^* = P_i(\theta)$

branch flow constraints: $|\theta_i - \theta_j| \le \gamma_{ij} < \pi/2$

Unconstrained case: identical marginal costs $\alpha_i u_i^* = \alpha_j u_j^*$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

Objective II: decentralized dispatch optimization

Insight: droop-controlled system = decentralized primal/dual algorithm

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

(i) the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer (θ^*, u^*).

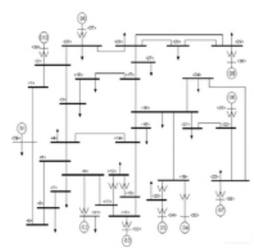
(ii) $\exists$ droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ.

If (i) & (ii) are true, then $\theta_i \sim \theta_i^*$, $u_i^* = -D_i(\omega_{\text{sync}} - \omega^*)$, & $\boxed{D_i\alpha_i = D_j\alpha_j}$.

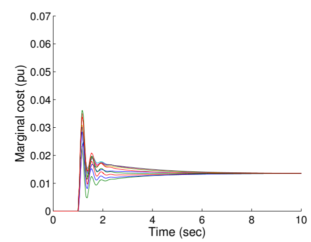
- includes proportional load sharing $\alpha_i \propto 1/\overline{P}_i$
- similar results hold for strictly convex cost & general **constrained** case

Sketch of the main proof ideas

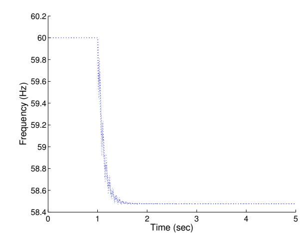
Some quick simulations & extensions



IEEE 39 New England with load step at 1s



$t \to \infty$: convergence to identical marginal costs

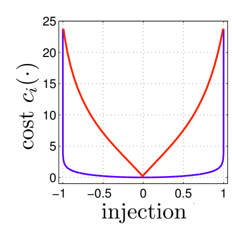


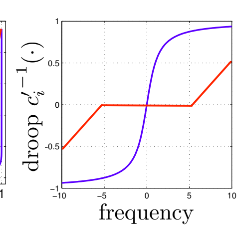
$t \to \infty$: frequency $\propto$ power imbalance

$\Rightarrow$ strictly convex & differentiable cost $f(u) = \sum_{\text{sources}} c_i(u_i)$

$\Rightarrow$ non-linear frequency droop curve $c_i'^{-1}(\dot{\theta}_i) = P_i^* - P_i(\theta)$

$\Rightarrow$ include dead-bands, saturation, etc.

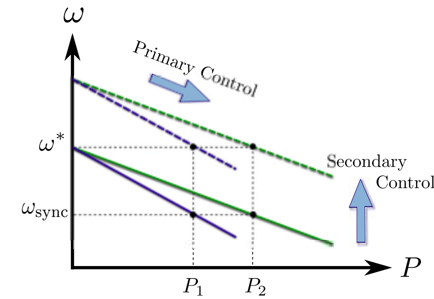




Secondary Control

Secondary frequency control

- **Problem:** steady-state frequency deviation ($\omega_{\text{sync}} \neq \omega^*$)
- **Solution:** integral control of frequency error
- **Basics** of integral control $\boxed{\frac{1}{s}}$:



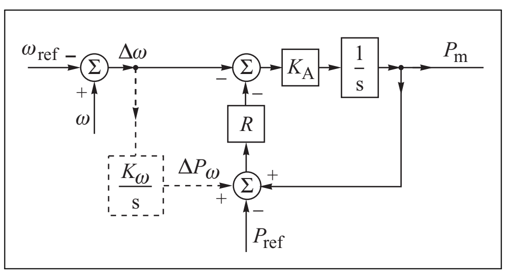
1. discrete time: $u_i(t+1) = u_i(t) + k \cdot \dot\theta_i(t)$ with gain $k > 0$
2. continuous-time: $u_i(t) = k \cdot \int_0^t \dot\theta_i(\tau)\, d\tau$ or $\dot u_i(t) = k \cdot \dot\theta_i(t)$

$\Rightarrow$ $\dot\theta_i(t)$ is zero in (a possibly stable) steady state

$\Rightarrow$ add additional injection $u_i(t)$ to droop control

Decentralized secondary integral frequency control

- $\boxed{\frac{1}{s}}$ add local integral controller to every droop controller
- $\Rightarrow$ stable closed-loop & zero frequency deviation ✓
- $\Rightarrow$ sometimes globally stabilizing [C. Zhao, E. Mallada, & FD, '14] ✓
- ☹ every integrator induces a 1d equilibrium subspace
- ☹ injections live in subspace of dimension # integrators
- ☹ load sharing & economic optimality are lost ...



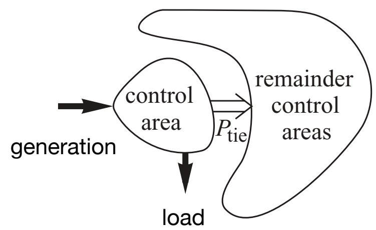
turbine governor integral control loop

Why does decentralized integral control not work?

on blackboard

Automatic generation control (AGC)

- **ACE** area control error =
  { frequency error } +
  { generation - load - tie-line flow }

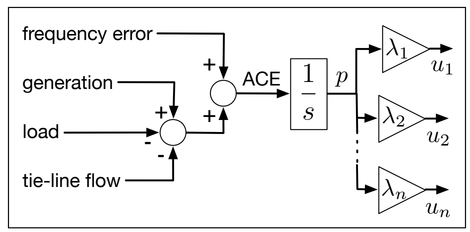


$\frac{1}{s}$ **centralized integral control:**

$$p(t) = \int_0^t \text{ACE}(\tau)\, d\tau$$

- **generation allocation:**
  $u_i(t) = \lambda_i p(t)$, where λ_i is generation participation factor
  (in our case $\lambda_i = 1/\alpha_i$)

⇒ assures identical marginal costs: $\alpha_i u_i = \alpha_j u_j$

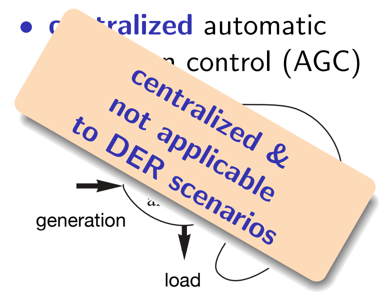


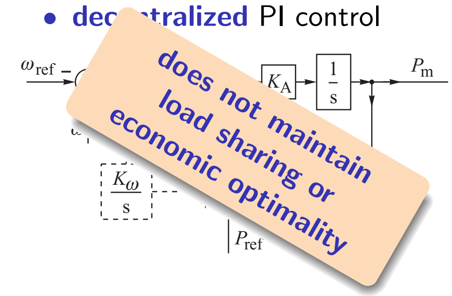
AGC implementation

☺ load sharing & economic optimality are recovered

Drawbacks of conventional secondary frequency control

Interconnected Systems	Isolated Systems
• c… tralized automatic …n control (AGC)	• dec… tralized PI control





Distributed energy ressources require **distributed** (!) secondary control.

An incomplete literature review of a busy field

ntwk with unknown disturbances ∪ integral control ∪ distributed averaging

- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcıa-Domınguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]

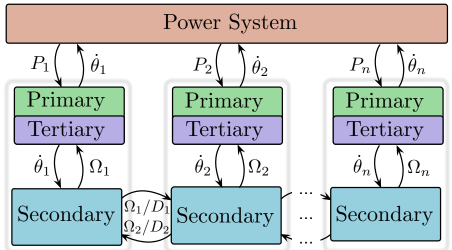
The following idea precedes most references, it's simpler, & it's more robust.

Let's derive a simple distributed control strategy

Distributed Averaging PI (DAPI) control

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot (\alpha_i\Omega_i - \alpha_j\Omega_j)$$



- no tuning & no time-scale separation: $k_i, D_i > 0$
- distributed & modular: connected comm. $\subseteq$ sources
- recovers primary op. cond. (load sharing & opt. dispatch)

$\Rightarrow$ plug'n'play implementation

Theorem: stability of DAPI

[J. Simpson-Porco, FD, & F. Bullo, '12]

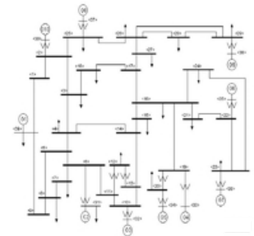
[C. Zhao, E. Mallada, & FD '14]

primary droop controller works

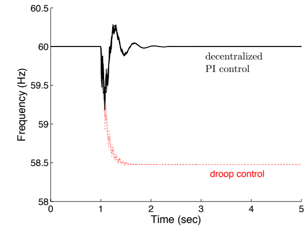
$\Longleftrightarrow$

secondary DAPI controller works

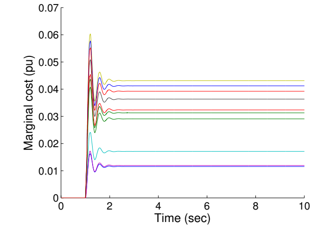
Simulations cont'd



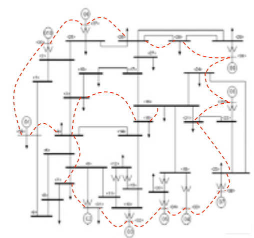
IEEE 39 New England with decentralized PI control



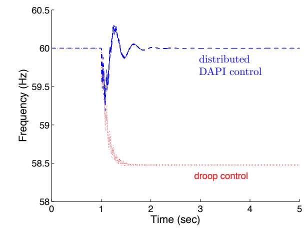
$t \to \infty$: decentralized PI control regulates frequency



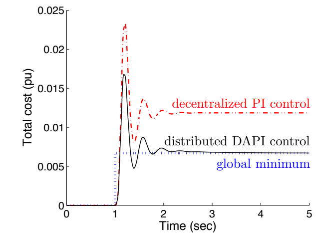
$t \to \infty$: decentralized PI control is not optimal



IEEE 39 New England with distributed DAPI control



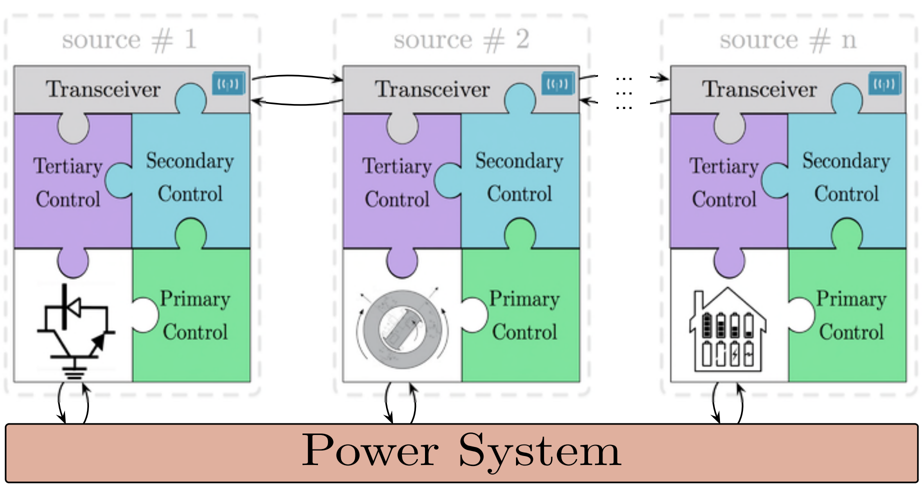
$t \to \infty$: DAPI control regulates frequency



DAPI control minimizes cost with little effort

Plug'n'play architecture

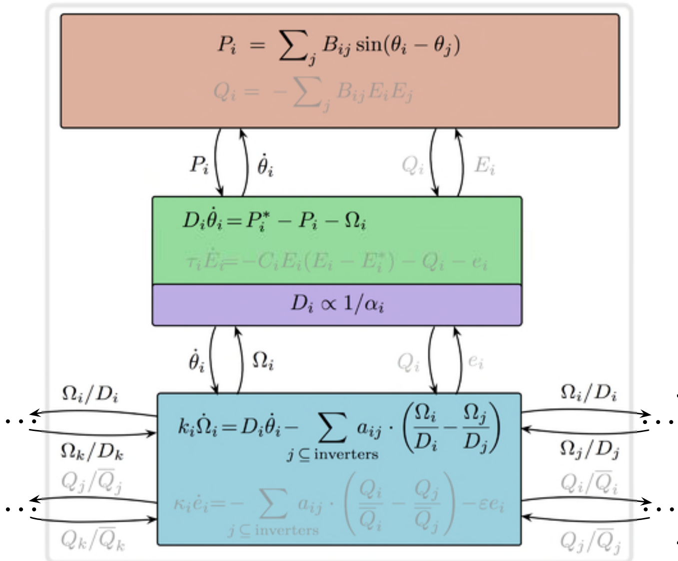
flat hierarchy, distributed, no time-scale separations, & model-free



plug-and-play experiments

Plug'n'play architecture

recap of detailed signal flow (active power only)



Power system: physics & loadflow

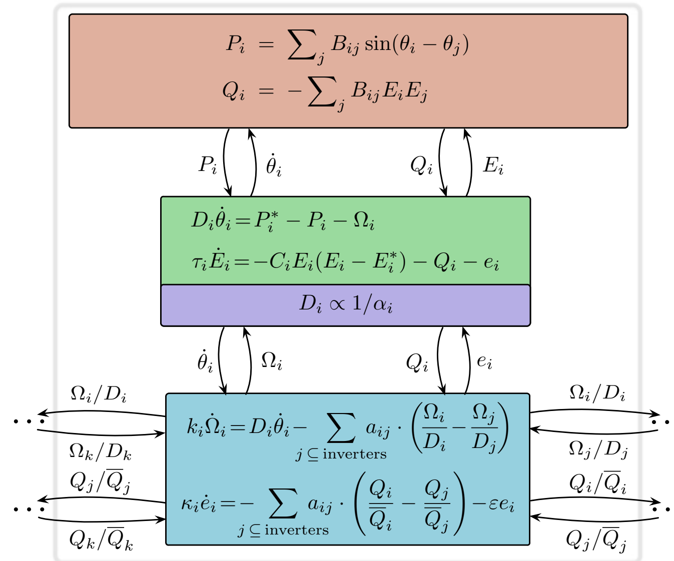
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs $\propto$ 1 /control gains

Secondary control: diffusive averaging of injection ratios

Plug'n'play architecture

recap of detailed signal flow (with reactive power)



Power system: physics & loadflow

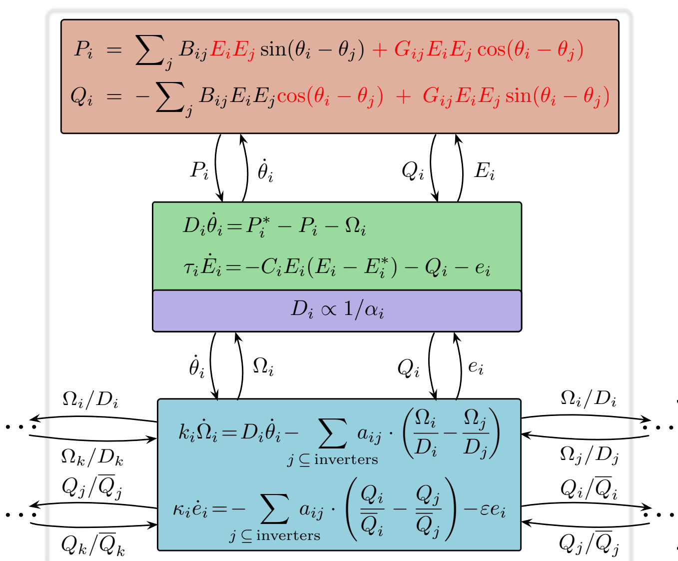
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs $\propto$ 1 /control gains

Secondary control: diffusive averaging of injection ratios

Plug'n'play architecture

experiments also work well in the coupled & lossy case



Power system: physics & loadflow

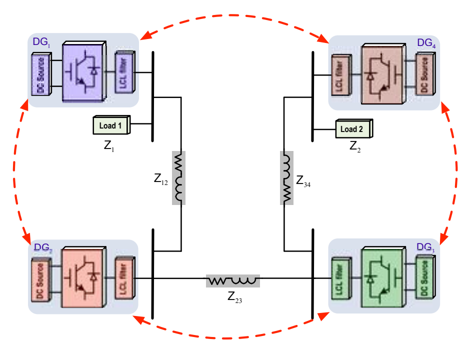
Primary control: mimic oscillators & polyn. symmetry

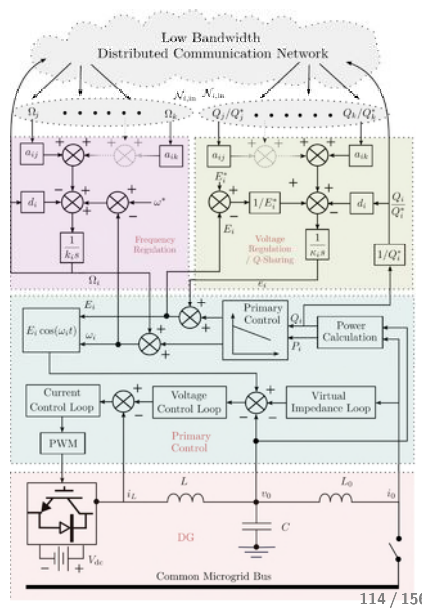
Tertiary control: marginal costs $\propto$ 1 /control gains

Secondary control: diffusive averaging of injection ratios

Experimental validation of control & opt. algorithms

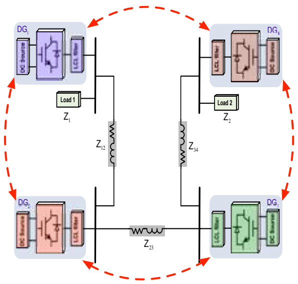
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University

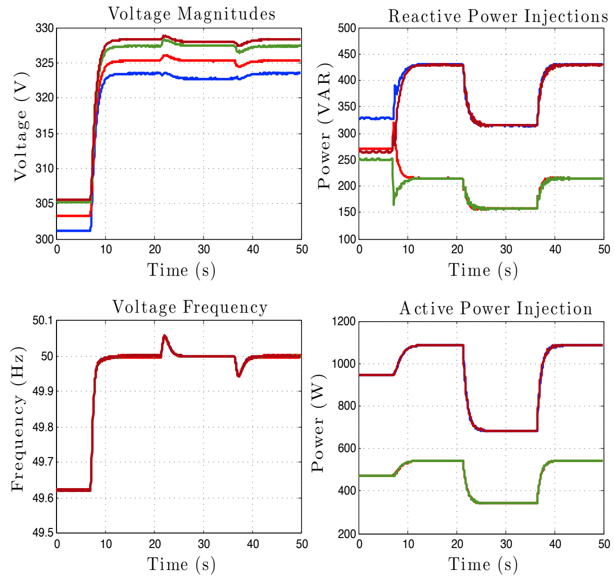




Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing





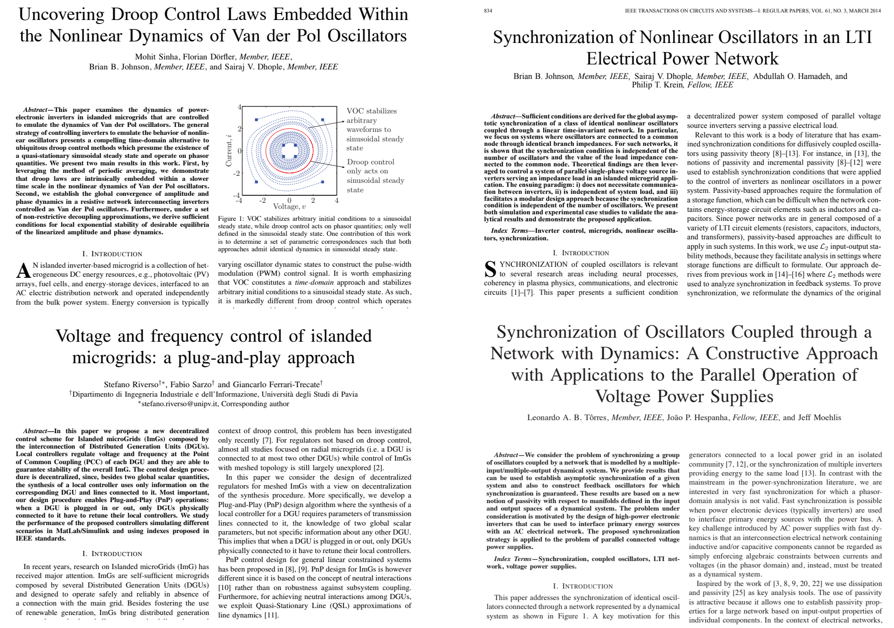
$t \in [0s, 7s]$: primary & tertiary control

$t = 7s$: secondary control activated

$t = 22s$: load # 2 unplugged

$t = 36s$: load # 2 plugged back

There are also many exciting alternatives to droop control



Outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Causes for Oscillations

Slow Coherency Modeling

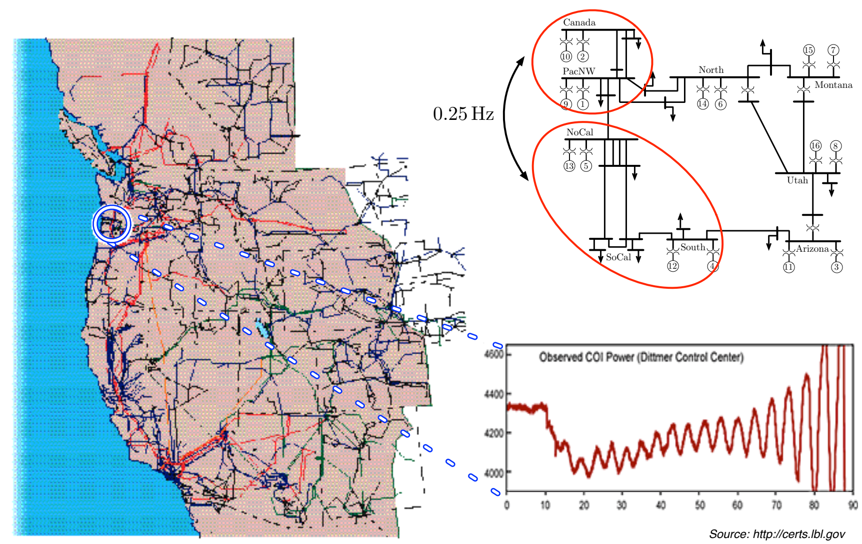
Inter-Area Oscillations & Wide-Area Control

Case Study: IEEE 39 New England Power Grid

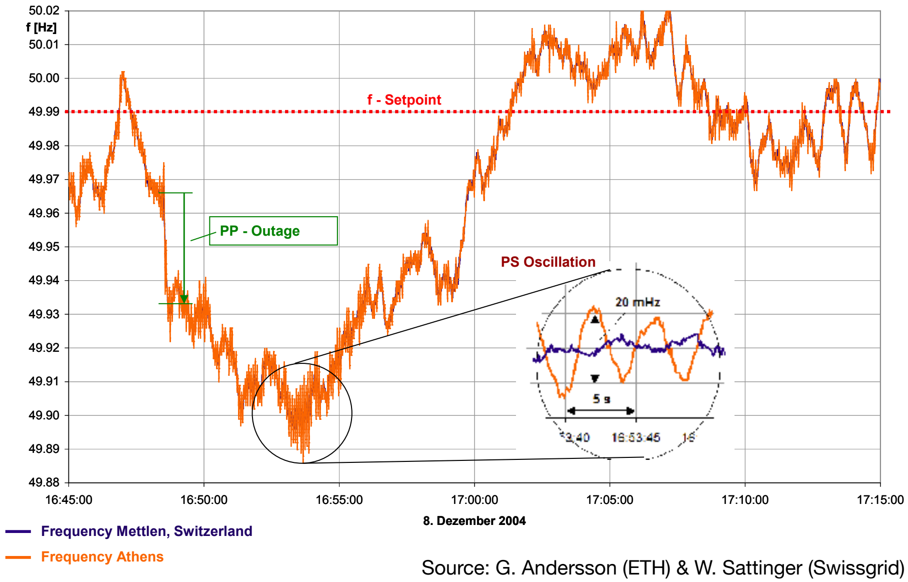
Conclusions

Electro-Mechanical Oscillations in Power Networks

- **Dramatic consequences:** blackout of August 10, 1996, resulted from instability of the 0.25 Hz mode in the Western interconnected system

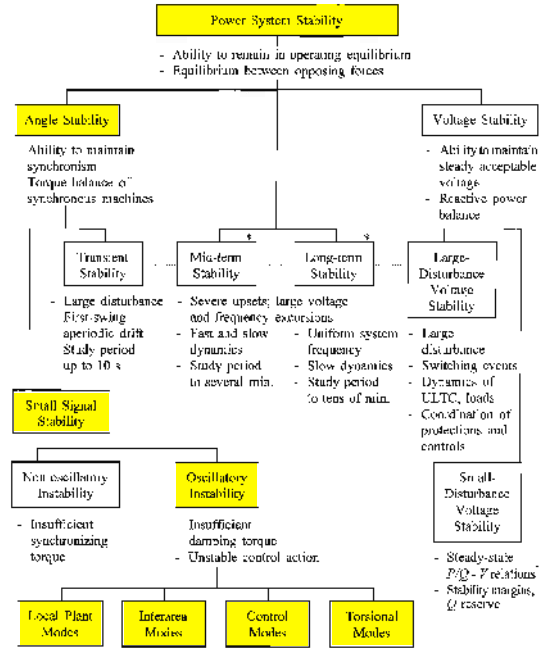


Less dramatic but quite common . . . usually well behaved



Source: G. Andersson (ETH) & W. Sattinger (Swissgrid)

Where are we on the map?



Causes for Oscillations

Swing dynamics = coupled/forced/heterogeneous pendula

- Coarse-grained power network dynamics = generator **swing dynamics**:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum\nolimits_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

- Swing equations **linearized** around an equilibrium $(\theta^*, \dot{\theta}^*, P^*)$:

$$M\ddot{\theta} + D\dot{\theta} + L\theta = P$$

M & $D \in \mathbb{R}^{n \times n}$ diagonal inertia and damping matrices

$L \in \mathbb{R}^{n \times n}$ Laplacian matrix with coupling $a_{ij} = E_i^* E_j^* B_{ij} \cos(\theta_i^* - \theta_j^*)$

$$L = \begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -a_{i1} & \cdots & \sum_{j=1}^{n} a_{ij} & \cdots & -a_{in} \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix}$$

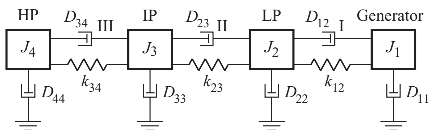
$\Rightarrow$ sparsely **coupled** & **forced** oscillators with **heterogeneous** frequencies

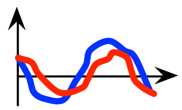
Torsional oscillations in power networks

essentially a (subsynchronous) resonance phenomenon

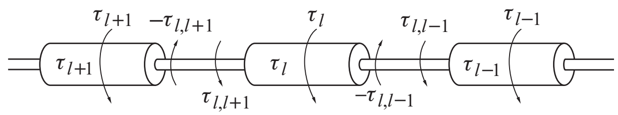
⇒ arise from interplay of

- electrical oscillations
- flexible mechanical shaft models
- generator-turbine coupling





turbine stages | generator | grid

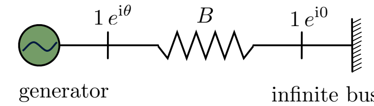


elastic generator shaft as finite-element model

⇒ subsynchronous resonance phenomena often arise in wind turbines

Local oscillations and their control

Automatic Voltage Regulator (AVR):

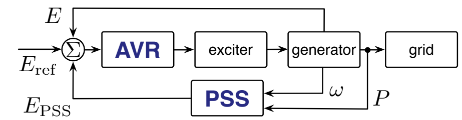


- objective: generator voltage = *const*.

⇒ diminishing damping & sync torque $\frac{\partial P}{\partial \theta}$

⇒ can result in oscillatory instability

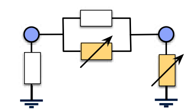
Power System Stabilizer (PSS):



- objective: net damping positive
- typical control design:

→ low-pass → wash-out → lead/lag element → gain →

Flexible AC Transmission Systems (FACTS) or HVDC:



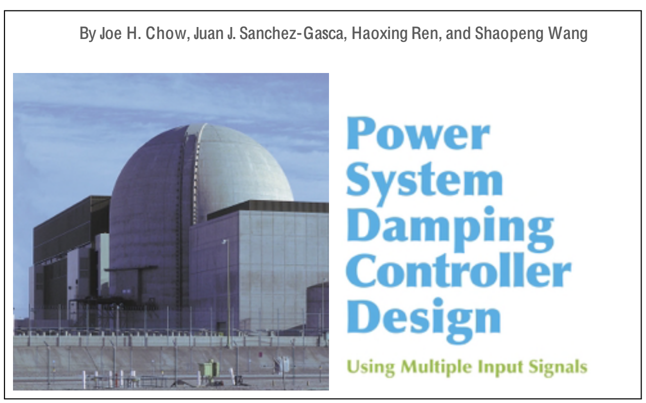
- control by "modulating" transmission line parameters
- either connected in series with a line or as shunt device

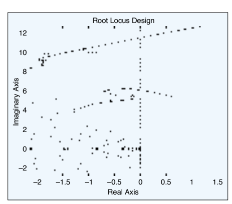
Control-induced oscillations and their control

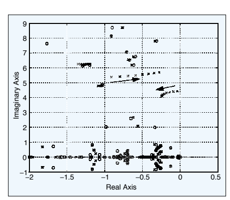
- **short story:** multiple local controllers interact in an adverse way
- **system-theoretic reason:** power system has unstable zeros

⇒ trade-off: high-gain (local stability) vs. low-gain control (avoid zeros)

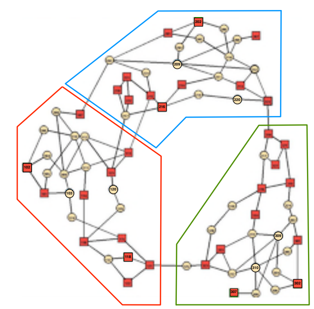
⇒ numerous tuning rules & heuristics for decentralized PSS design



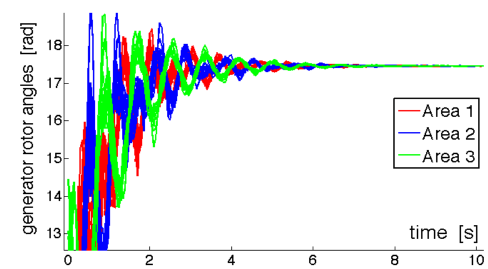




Inter-area oscillations in power networks



RTS 96 power network



swing dynamics

Inter-area oscillations are caused by

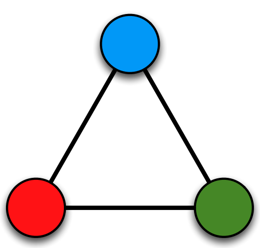
1. **heterogeneity**: fast & slow responses (inertia M_i and damping D_i)
2. **topology**: internally strongly and externally sparsely connected areas
3. **power transfers** between areas: $a_{ij} = B_{ij} E_i^* E_j^* \cos(\theta_i^* - \theta_j^*)$
4. **interaction** of multiple local control loops (e.g., high gain PSSs)

Taxonomy of electro-mechanical oscillations

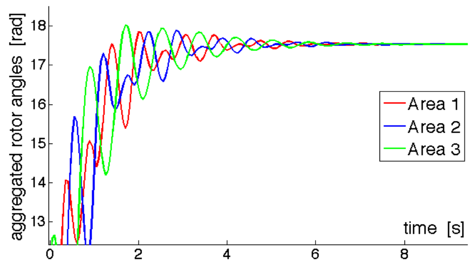
- Synchronous generator = electromech. oscillator ⇒ **local oscillations**:
  - = single generator oscillates relative to the rest of the grid
  - ☹ torsional oscillations induced by mechanical/electrical/flexible coupling
  - ☹ AVR control induces unstable local oscillations
  - ☺ typically damped by local feedback via PSSs
- Power system = complex oscillator network ⇒ **inter-area oscillations**:
  - = groups of generators oscillate relative to each other
  - ☹ poorly tuned local PSSs result in unstable inter-area oscillations
  - ☹ inter-area oscillations are only poorly controllable by local feedback
- Consequences of **recent developments**:
  - ☹ increasing power transfers outpace capacity of transmission system
  - ⟹ ever more lightly damped electromechanical inter-area oscillations
  - ☺ technological opportunities for **wide-area control (WAC)**

Slow Coherency Modeling

Slow coherency and area aggregation



aggregated RTS 96 model



swing dynamics of aggregated model

Aggregate model of lower dimension & with less complexity for

1. analysis and insights into inter-area dynamics [Chow and Kokotovic '85]
2. measurement-based id of equivalent models [Chakrabortty et.al.'10]
3. remedial action schemes [Xu et. al. '11] & wide-area control (later today)

How to find the areas?

a crash course in spectral partitioning

- given: an undirected, connected, & weighted **graph**
- **partition:** $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, and $\mathcal{V}_1, \mathcal{V}_2 \neq \emptyset$
- **cut** is the size of a partition: $J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij}$

⇒ if $x_i = 1$ for $i \in \mathcal{V}_1$ and $x_j = -1$ for $j \in \mathcal{V}_2$, then

$$J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij} = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij}(x_i - x_j)^2 = \frac{1}{2} x^T L x$$

- combinatorial **min-cut** problem: $\text{minimize}_{x \in \{-1,1\}^n \setminus \{-\mathbb{1}_n, \mathbb{1}_n\}} \frac{1}{2} x^T L x$
- **relaxed problem:** $\text{minimize}_{y \in \mathbb{R}^n,\, y \perp \mathbb{1}_n,\, \|y\|_2 = 1} \frac{1}{2} y^T L y$

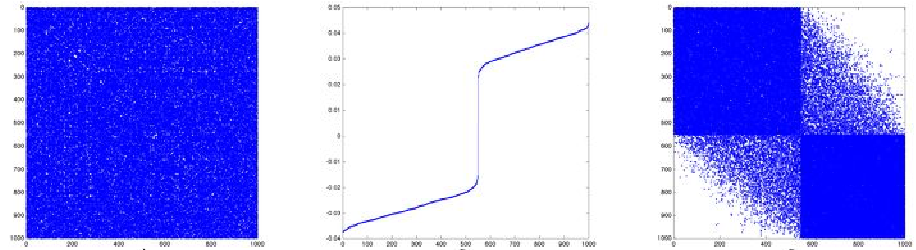
⇒ minimum is *algebraic connectivity* λ_2 and minimizer is *Fiedler vector* v_2

- **heuristic:** $x_i = \text{sign}(y_i)$ ⇒ "spectral partition"

A quick example

```
% choose a graph size
n = 1000;

% randomly assign the nodes to two grous
x = randperm(n);
group_size = 450;
group1 = x(1:group_size);
group2 = x(group_size+1:end);

% assign probabilities of connecting nodes
p_group1 = 0.5;
p_group2 = 0.4;
p_between_groups = 0.1;

% construct adjacency matrix
A(group1, group1) = rand(group_size,group_size) < p_group1;
A(group2, group2) = rand(n-group_size,n-group_size) < p_group2;
A(group1, group2) = rand(group_size, n-group_size) < p_between_groups;
A = triu(A,1); A = A + A';

% can you see the groups?
subplot(1,3,1); spy(A);

% construct Laplacian and its spectrum
L = diag(sum(A))-A;
[V D] = eigs(L, 2, 'SA');

% plot the components of the algebraic connectivity sorted by magnitude
subplot(1,3,2); plot(sort(V(:,2)), '.-');

% partition the matrix accordingly and spot the communities
[ignore p] = sort(V(:,2));
subplot(1,3,3); spy(A(p,p));
```


A quick example – cont'd



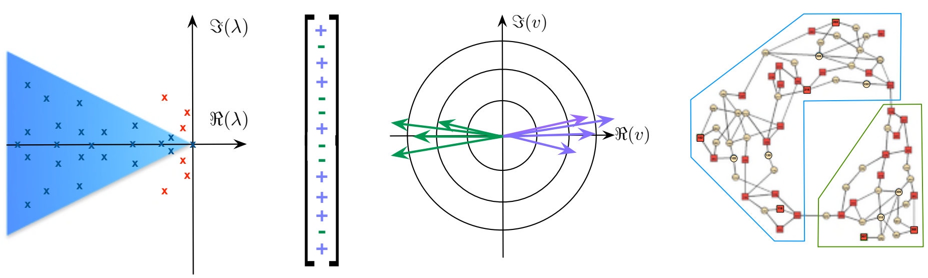
adjacency matrix   Fiedler vector v_2 re-arranged adj. matrix

Classical power system partitioning ≈ spectral partitioning

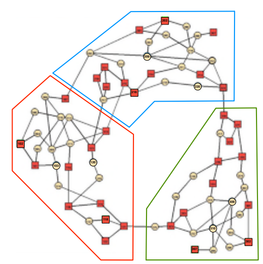
1. construct a linear model $\dot{x} = Ax$ (via, e.g., Power Systems Toolbox)
2. recall solution via eigenvalues λ_i and left/right eigenvectors w_i and v_i:

$$x(t) = \sum_i v_i e^{\lambda_i t} \cdot w_i^T x_0 = \sum_i \{\text{mode } \#i\} \cdot \{\text{contribution from } x_0\}$$

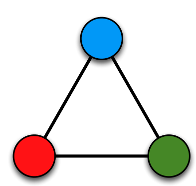
3. look at poorly damped complex conjugate mode pairs
4. look at angle & frequency components of eigenvectors
5. group the generators according to their polarity in eigenvectors



Setup in slow coherency



original model



aggregated model

- r given areas (from spectral partition [Chow et al. '85 & '13])
- small **sparsity parameter**:

$$\delta = \frac{\max_\alpha(\Sigma \text{ external connections in area } \alpha)}{\min_\alpha(\Sigma \text{ internal connections in area } \alpha)}$$

- **inter-area dynamics** by center of inertia:

$$y_\alpha = \frac{\sum_{i\in\alpha} M_i \theta_i}{\sum_{i\in\alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

- **intra-area dynamics** by area differences:

$$z^\alpha_{i-1} = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

Linear transformation & time-scale separation

Swing equation $\Longrightarrow$ singular perturbation standard form

$$M\ddot{\theta} + D\dot{\theta} + L\theta = 0 \quad \Longrightarrow \quad \left\{ \frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \right.$$

Slow motion given by center of inertia:

$$y_\alpha = \frac{\sum_{i\in\alpha} M_i \theta_i}{\sum_{i\in\alpha} M_i}, \quad \alpha \in \{1,\dots,r\}$$

Fast motion given by intra-area differences:

$$z^\alpha_{i-1} = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\},\ \alpha \in \{1,\dots,r\}$$

Slow time scale: $t_s = \delta \cdot t \cdot$ "max internal area degree"

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\,\ddot{\varphi} + D_a\,\dot{\varphi} + L_{\mathrm{red}}\,\varphi = 0$$

Properties of aggregated model [D. Romeres, FD, & F. Bullo, '13]

1. $M_a = \begin{bmatrix} \ddots & & \\ & \sum_{i\in\alpha} M_i & \\ & & \ddots \end{bmatrix}$ and $D_a = \begin{bmatrix} \ddots & & \\ & \sum_{i\in\alpha} D_i & \\ & & \ddots \end{bmatrix}$
2. L_{red} = "inter-area Laplacian" + "intra-area contributions"
   = positive semidefinite Laplacian with possibly negative weights

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\,\ddot{\varphi} + D_a\,\dot{\varphi} + L_{\mathrm{red}}\,\varphi = 0$$

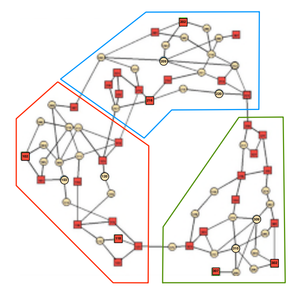
Singular perturbation approximation [D. Romeres, FD, & F. Bullo, '13]

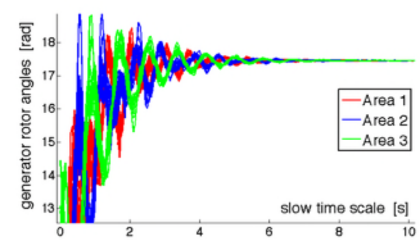
There exist δ^* sufficiently small such that for $\delta \leq \delta^*$ and for all $t > 0$:

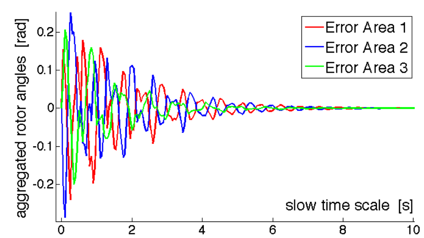
$$\begin{bmatrix} y(t_s) \\ \dot{y}(t_s) \end{bmatrix} = \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}), \quad \begin{bmatrix} z(t_s) \\ \dot{z}(t_s) \end{bmatrix} = \tilde{A} \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}).$$

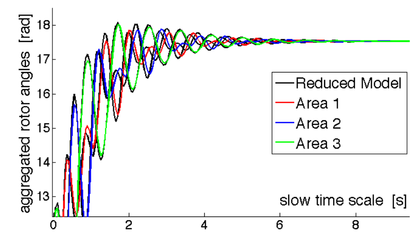
center of inertia $\approx$ solution of aggregated swing equation

RTS 96 swing dynamics revisited







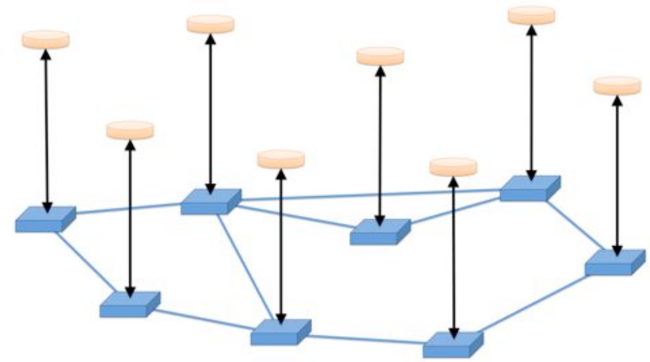


Inter-Area Oscillations & Wide-Area Control

Remedies against electro-mechanical oscillations

conventional control

- Blue layer: interconnected generators

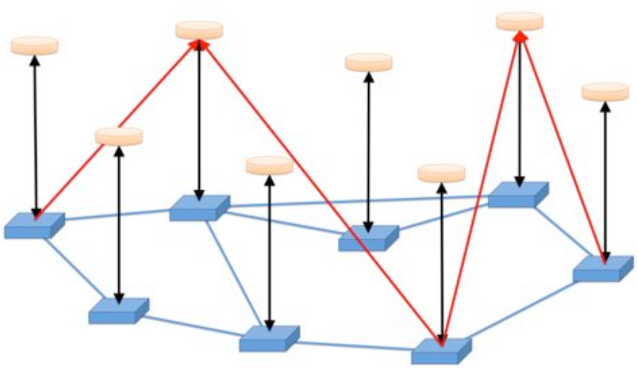


- Fully decentralized control implemented via PSS, HVDC, or FACTS:
  - ☺ effective against local oscillations
  - ☹ ineffective against inter-area oscillations

Remedies against electro-mechanical oscillations

wide-area control

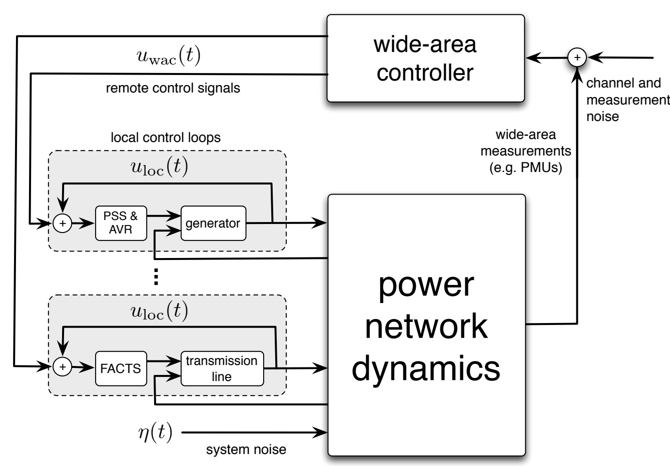
- Blue layer: interconnected generators



- Fully decentralized control
- Distributed wide-area control requires identification of sparse control architecture: actuators, measurements, & communication channels

Setup in Wide-Area Control

1. remote control signals & remote measurements (e.g., PMUs)
2. excitation (PSS & AVR) and power electronics (FACTS) actuators
3. communication backbone network



Modal signal selection metrics [H.M.A. Hamdan & A.M.A. Hamdan '87]

1. **Linear control system:** $\dot{x} = Ax + Bu \;,\; y = Cx$
   - B with column b_j = control location $\#j$
   - C with row c_j^T = sensor location $\#j$
   - A: eigenvalues λ_i and orthonormal right & left eigenvectors v_i & w_i^*

2. **Diagonalization:** $x = Vz = \begin{bmatrix} v_1 & \dots & v_n \end{bmatrix} z \;,\; z = Wx = \begin{bmatrix} w_1 & \dots & w_n \end{bmatrix}^* x$

$$\Rightarrow \dot{z} = \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{=WAV} z + \underbrace{\begin{bmatrix} & \vdots & \\ \dots & w_i^* b_j & \dots \\ & \vdots & \end{bmatrix}}_{=WB} u \;,\; y = \underbrace{\begin{bmatrix} & \vdots & \\ \dots & c_i^* v_j & \dots \\ & \vdots & \end{bmatrix}}_{=CV} uz$$

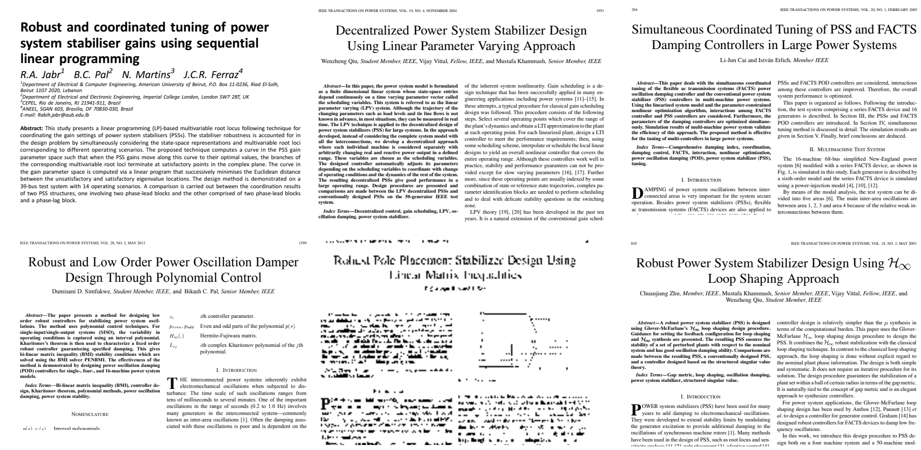
3. Controllability of mode i by input $j \triangleq \cos\left(\angle(w_i \,,\, b_j)\right) = \frac{w_i^* b_j}{\|w_i\|\|b_j\|}$

4. Observability of mode i by sensor $j \triangleq \cos\left(\angle(c_i \,,\, v_j)\right) = \frac{c_i^* v_j}{\|c_i\|\|v_j\|}$

Alternatives based on modal residues of transfer function [M. Tarokh '92].

Decentralized WAC control design ...

- ...subject to structural constraints is **tough**
- ...usually handled with suboptimal **heuristics** in MIMO case



$\Rightarrow$ signal selection is combinatorial & control design is suboptimal

Challenges in wide-area control

- **Objectives:** wide-area control should achieve
  1. optimal closed-loop performance
  2. low control complexity (comm, measurements, & actuation)
- **Problem:** objectives are conflicting
  1. design (optimal) centralized control $\Rightarrow$ identify control architecture
     - ☹ complete state info & measurements
     - ☹ high communication complexity
  2. identify measurements & control architecture $\Rightarrow$ design control
     - ☹ decentralized (optimal) control is hard
     - ☹ combinatorial criteria for control channels

Today: simultaneously optimize closed-loop performance
& identify sparse control architecture

Primer on **L**inear **Q**uadratic **C**ontrol (**LQR**)

Optimal wide-area damping control

- **Model:** linearized ODE dynamics $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t)$
- **Control:** memoryless linear state feedback $u = -Kx(t)$
- **Optimal centralized control** with quadratic performance index:

$$\begin{aligned} \text{minimize} \quad & J(K) \triangleq \lim_{t\to\infty} \mathcal{E}\left\{x(t)^TQx(t) + u(t)^TRu(t)\right\} \\ \text{subject to} \quad & \\ \text{linear dynamics:} \quad & \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ \text{linear control:} \quad & u(t) = -Kx(t), \\ \text{stability:} \quad & (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

(no structural constraints on K)

Sparsity-promoting optimal wide-area damping control

- **Sparsity-promoting optimal control** [Lin, Fardad, & Jovanović '13]: simultaneously optimize control performance & control architecture

$$\begin{aligned} \text{minimize} \quad & \lim_{t\to\infty} \mathcal{E}\left\{x(t)^TQx(t) + u(t)^TRu(t)\right\} + \gamma\, \text{card}(K) \\ \text{subject to} \quad & \\ \text{linear dynamics:} \quad & \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ \text{linear control:} \quad & u(t) = -Kx(t), \\ \text{stability:} \quad & (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

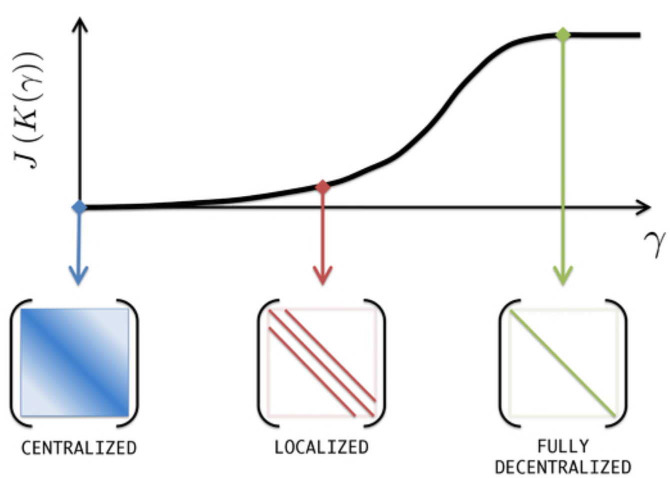
$\Rightarrow$ for $\gamma = 0$: standard optimal control (typically not sparse)

$\Rightarrow$ for $\gamma > 0$: sparsity is promoted (problem is combinatorial)

$\Rightarrow$ card(K) approximated by weighted ℓ_1-norm $\sum_{i,j} w_{ij}|K_{ij}|$

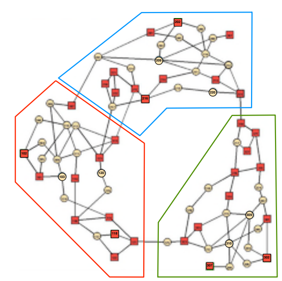
Parameterized family of feedback gains

$$K(\gamma) = \arg\min_K \left(J(K) + \gamma \cdot \sum_{i,j} w_{ij}|K_{ij}|\right)$$



Slow coherency performance objectives

- recall **sources** for inter-area oscillations:



- linearized **swing equation**: $M\ddot{\theta} + D\dot{\theta} + L\theta = P$
- **mechanical energy**: $\frac{1}{2}\dot{\theta}M\dot{\theta} + \frac{1}{2}\theta^TL\theta$
- **heterogeneities** in topology, power transfers, & machine responses (inertia & damp)

$\Rightarrow$ performance **objectives** = energy of homogeneous network:

$$x^TQx = \frac{1}{2}\dot{\theta}^T \underbrace{M_{\text{uniform}}}_{I_n} \dot{\theta} + \frac{1}{2}\theta^T \underbrace{L_{\text{uniform}}}_{I_n - (1/n)\cdot\mathbb{1}_{n\times n}} \theta$$

- other choices possible: center of inertia, inter-area differences, etc.

Algorithmic approach to sparsity-promoting control

1. Equivalent formulation via **observability Gramian** P:

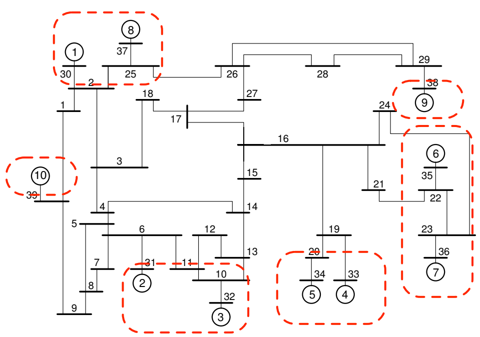
$$\begin{array}{ll} \text{minimize} & J_\gamma(K) \triangleq \text{trace}\left(B_1^T P B_1\right) + \gamma \sum_{i,j} w_{ij}\,|K_{ij}| \\ \text{subject to} & (A - B_2 K)^T P + P(A - B_2 K) \\ & \qquad = -(Q + K^T R K)\,; \end{array}$$

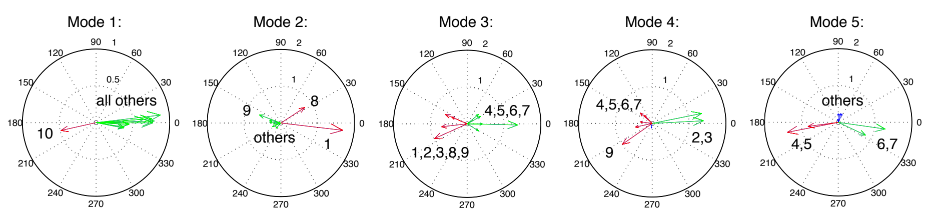
2. **Warm-start** at optimal centralized $\mathcal{H}_2$ controller with $\gamma = 0$
3. **Homotopy path:** continuously increase γ until the desired value γ_{des}
4. **ADMM:** iterative solution for each value of $\gamma \in [0,\, \gamma_{\text{des}}]$
5. **Update weights:** update w_{ij} in each ADMM step: $w_{ij} \mapsto \frac{1}{|K_{ij}| + \varepsilon}$
6. **Polishing:** structured optimization with desired sparsity pattern

Case Study: IEEE 39 New England Power Grid

Case study: IEEE 39 New England power grid

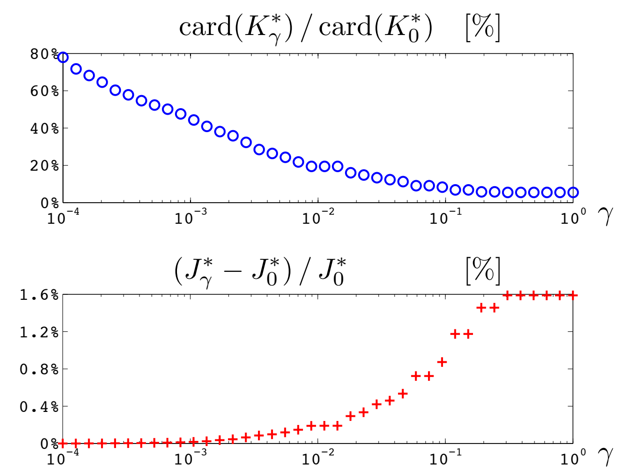
- **Model features**:
  - sub-transient generator models [Athay et. al. '79]
  - exciters & carefully tuned PSS data [Jabr et. al. '09]
- dominant **inter-area modes** of New England grid with PSSs





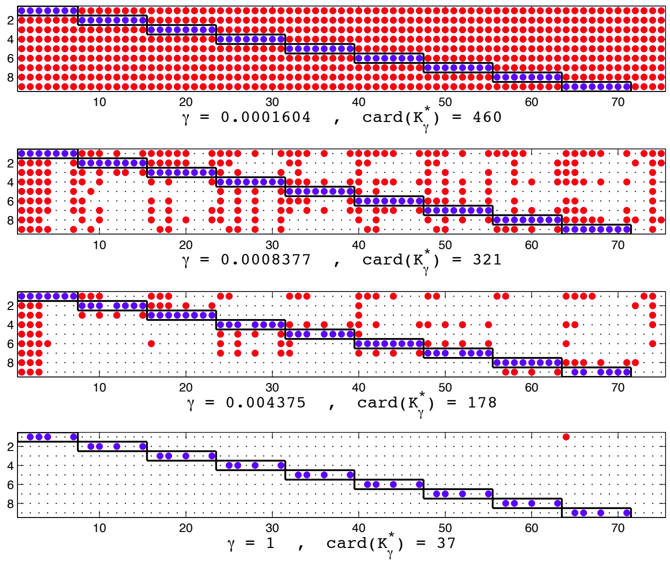
Performance vs. sparsity

Q = energy of homogeneous network , $R = I_n$, $\gamma \in [10^{-4}, 10^{0}]$



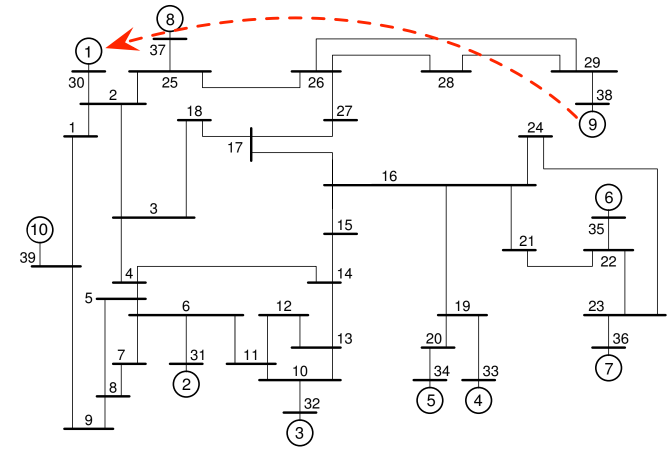
for $\gamma = 1 \Longrightarrow \begin{cases} 1.6\,\% & \text{relative performance loss} \\ 5.5\,\% & \text{non-zero elements in } K \end{cases}$

Control architecture & signal exchange network



For $\gamma = 1$: local decentralized optimal control $+\ K^{*}_{19}\,\theta_9(t)$

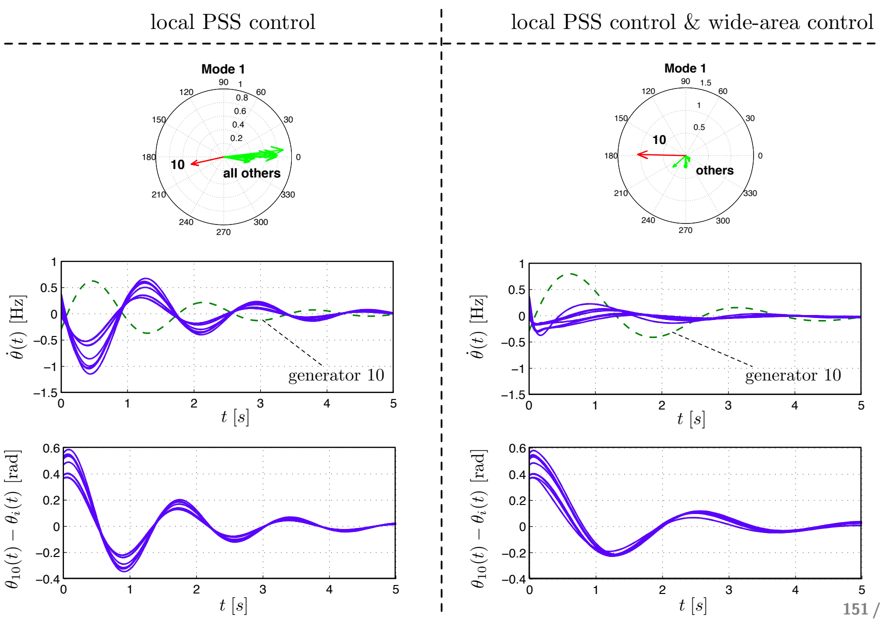
Sparse & nearly optimal wide-area control architecture



single wide-area control link $\Longrightarrow$ nearly centralized performance

Closed-loop performance for $\gamma = 1$

modes $\#2, 3, \ldots$ are strongly damped and mode $\#1$ is distorted



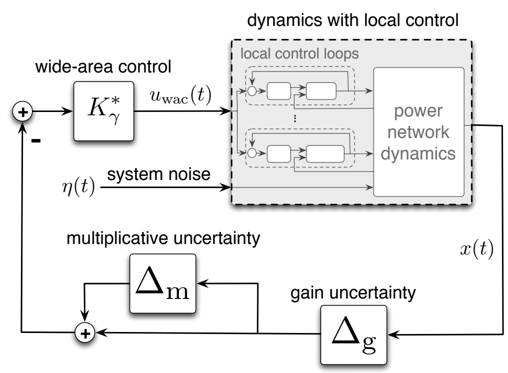
Robustness achieved by sparsity-promoting control

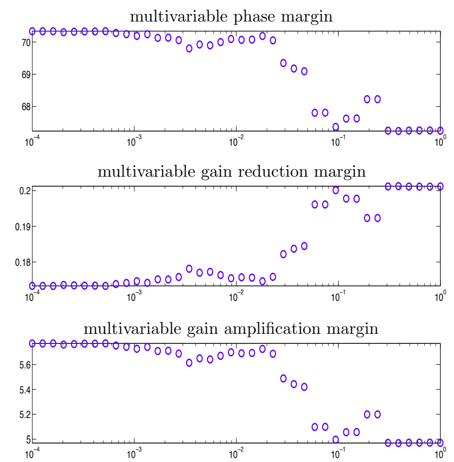
1. tuning of PSSs & operating cond

$\Rightarrow$ gain margins

2. time delays, multiple SCADA rates, & comm uncertainties

$\Rightarrow$ phase margins & input uncertainties



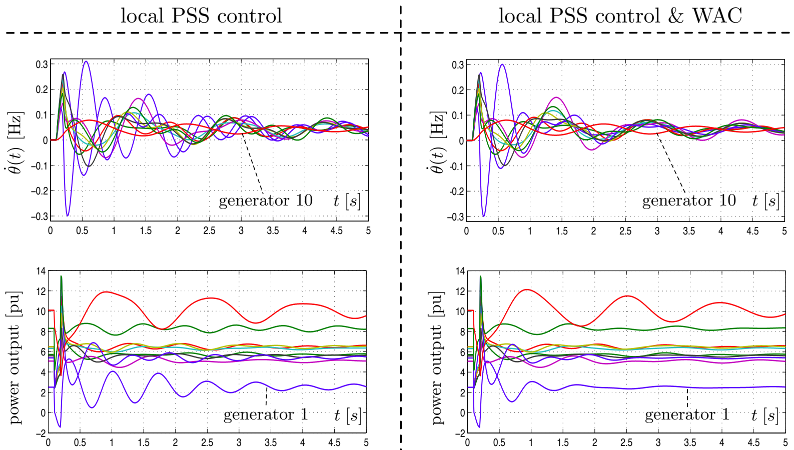


Additionally: sparsity pattern is not sensitive to operating point

Sparsity identification & control by alternative means

- identified WAC channel: $\theta_9(t)$ needs to be communicated to AVR #1

⇒ proportional feedback $u_1(t) = -K_{19}^* \left(\theta_1(t) - \theta_9(t)\right)$ applied to nonlinear DAE system without local optimal decentralized control



You can also get rid of communication entirely . . .

Analysis and Design Trade-Offs for Power Network Inter-Area Oscillations

Xiaofan Wu, Florian Dörfler, and Mihailo R. Jovanović

Abstract— **Conventional analysis and control approaches to inter-area oscillations in bulk power systems are based on a modal perspective. Typically, inter-area oscillations are identified from spatial profiles of poorly damped modes, and they are damped using carefully tuned decentralized controllers. To improve upon the limitations of conventional decentralized strategies, recent efforts aim at distributed wide-area control which involves the communication of remote signals. Here, we introduce a novel approach to the analysis and control of inter-area oscillations. Our framework is based on a stochastically driven system with performance outputs chosen such that the $\mathcal{H}_2$ norm is associated with incoherent inter-area oscillations. We show that an analysis of the output covariance matrix offers new insights relative to modal approaches. Next, we leverage the recently proposed sparsity-promoting optimal control approach to design controllers that use relative angle measurements and simultaneously optimize the closed-loop performance and the control architecture. For the IEEE 39 New England model, we investigate performance trade-offs of different control architectures and show that optimal retuning of decentralized control strategies can effectively guard against inter-areas oscillations.**

damped via decentralized controllers, whose gains are carefully tuned according to root locus criteria [7]–[9].

To improve upon the limitations of decentralized controllers, recent research efforts aim at distributed wide-area control strategies that involve the communication of remote signals, see the surveys [10], [11] and the excellent articles in [12]. The wide-area control signals are typically chosen to maximize modal observability metrics [13], [14], and the control design methods range from root locus criteria to robust and optimal control approaches [15]–[17].

Here, we investigate a novel approach to the analysis and control of inter-area oscillations. Our unifying analysis and control framework is based on a stochastically driven power system model with performance outputs inspired by slow coherency theory [18], [19]. We analyze inter-area oscillations by means of the $\mathcal{H}_2$ norm of this system, as in recent related approaches for interconnected oscillator networks and multi-machine power systems [20]–[22]. We show that an analysis of power spectral density and variance amplification

Outline

Looking for data, toolboxes, & test cases

- **Matpower** for (optimal) power flow & static models
  `http://www.pserc.cornell.edu//matpower/`
- **Power System Toolbox** for dynamics & North American models
  `http://www.eps.ee.kth.se/personal/vanfretti/pst/Power_System_Toolbox_Webpage/PST.html`
- **IEEE Task Force PES PSDPC SCS**: New York, Brazil, Australian grids etc.; `http://www.sel.eesc.usp.br/ieee/`
- **ObjectStab** for Modelica for dynamics & models
  `https://github.com/modelica-3rdparty/ObjectStab`
- **More freeware:** MatDyn, PSAT, THYME, Dome, . . .
  `http://ewh.ieee.org/cmte/psace/CAMS_taskforce/`
- **Other:** many test cases in papers, reports, task forces, . . .

Conclusions

Introduction

Power Network Modeling

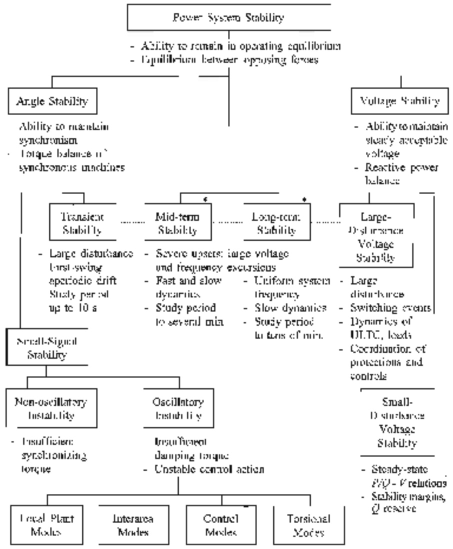
Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

Obviously, there is a lot more . . .



I hope I could give you a little insight into a few interesting problems.

final words of wisdom