

(Complex) Dynamics & (Distributed) Control of (Smart) Power Grids

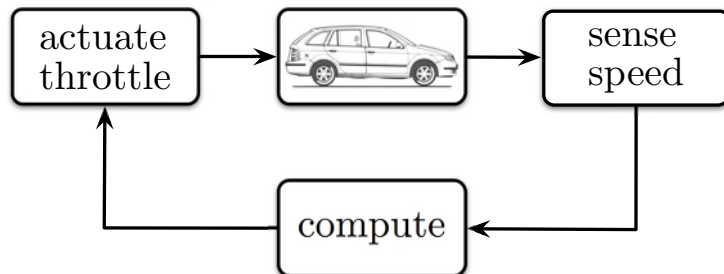
Winter School on Holistic Modeling & Control of Energy Systems

Florian Dörfler

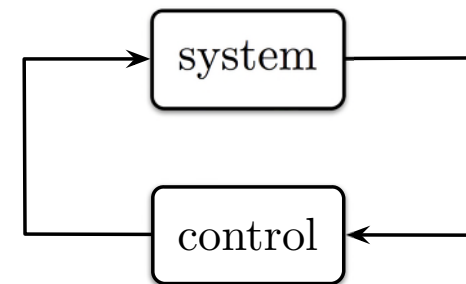


my motivation for studying power systems

My job description @ETH is “Complex Systems Control”



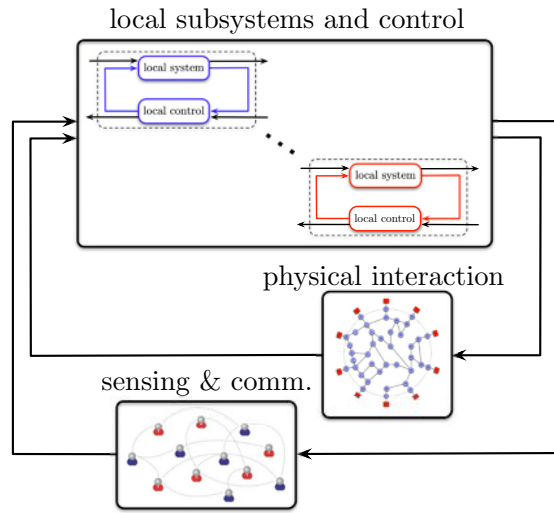
My job description @ETH is “Complex Systems Control”



“Simple” control systems are well understood.

“Complexity” can enter in many ways . . .

A “complex” distributed decision making system



Such distributed systems include **large-scale** physical systems, engineered **multi-agent** systems, & their interconnection in **cyber-physical** systems.

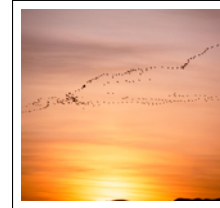
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Timely applications of distributed systems control

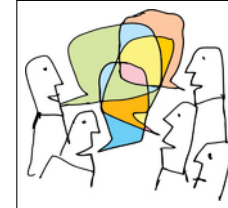
often the centralized perspective is simply not appropriate



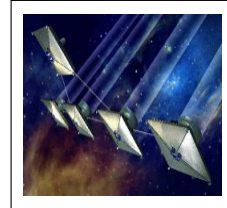
robotic networks



decision making



social networks



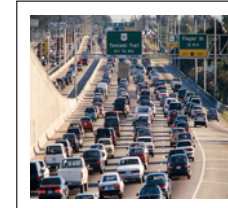
sensor networks



self-organization



pervasive computing



traffic networks



smart power grids

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what makes power systems (IMHO) so interesting?

My main application of interest – the power grid



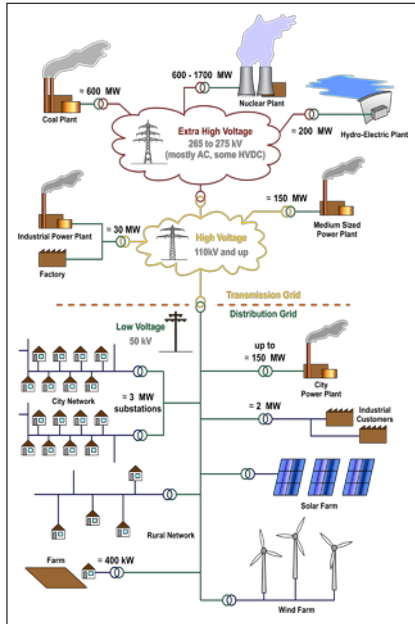
NASA Goddard Space Flight Center



- **Electric energy** is critical for our technological civilization
- Energy supply via **power grid**
- **Complexities**: multiple scales, nonlinear, & non-local

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Paradigm shifts in the operation of power networks

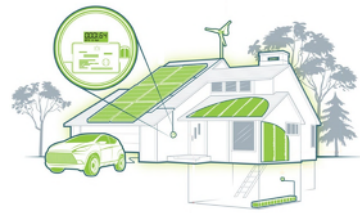


Traditional top to bottom operation:

- ▶ generate/transmit/distribute power
- ▶ hierarchical control & operation

Smart & green power to the people:

- ▶ high renewable penetration
- ▶ distributed generation & deregulation
- ▶ demand response & load control



Why care about power system dynamics & control?



www.offthegridnews.com

- 1 increasing renewables & deregulation
 - 2 growing demand & operation at capacity
- ⇒ increasing volatility & complexity, decreasing robustness margins

Rapid technological and scientific advances:

- 1 re-instrumentation: sensors & actuators
 - 2 complex & cyber-physical systems
- ⇒ cyber-coordination layer for smart grid



⇒ need to understand the **complex** network dynamics & control

One system with many dynamics & control problems

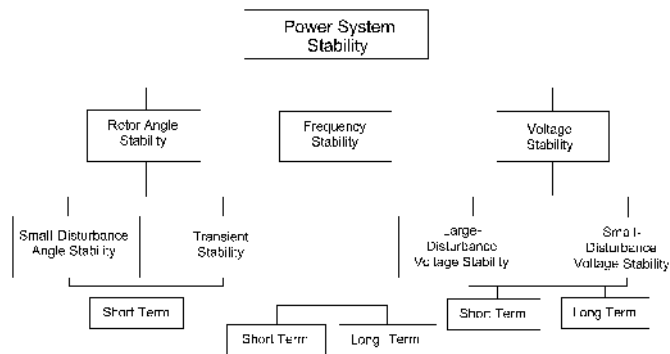
IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 19, NO. 2, MAY 2004

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Definition and Classification of Power System Stability

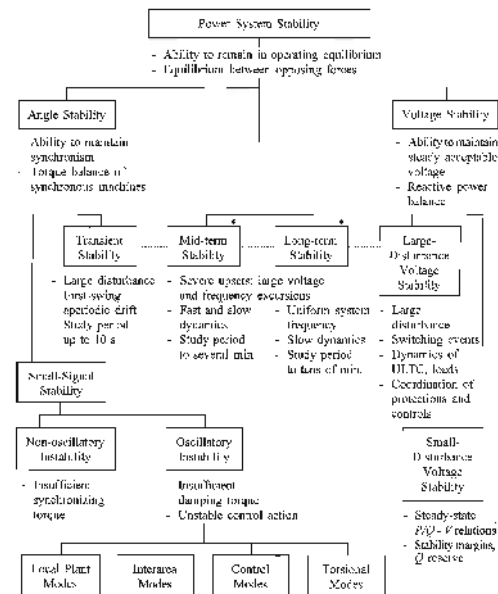
IEEE/CIGRE Joint Task Force on Stability Terms and Definitions

Prabha Kundur (Canada, Convener), John Paserba (USA, Secretary), Venkat Ajjarapu (USA), Göran Andersson (Switzerland), Anjan Bose (USA), Claudio Canizares (Canada), Nikos Hatziargyriou (Greece), David Hill (Australia), Alex Stankovic (USA), Carson Taylor (USA), Thierry Van Cutsem (Belgium), and Vijay Vittal (USA)



We have to make a choice based on . . .

many aspects depending on spatial/temporal/state scales, cause & effect, . . .



- what future speakers need and what will be covered by others
- what I actually know well
- what is interesting from a network perspective rather than from device perspective
- what is relevant for future (smart) power grids with high renewable penetration
- what gives rise to fun distributed control problems
- what **you** are interested in

Tentative outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

my particular focus is on **networks**

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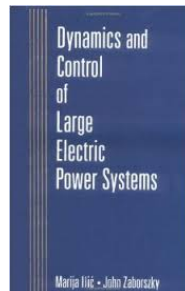
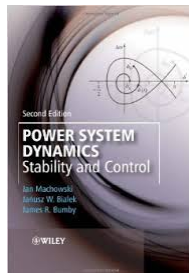
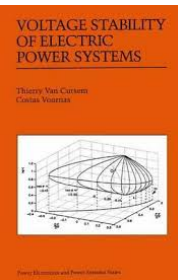
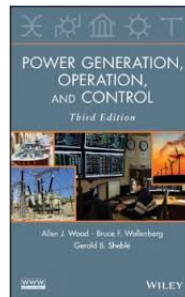
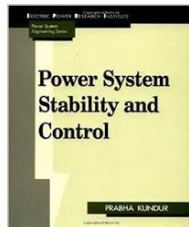
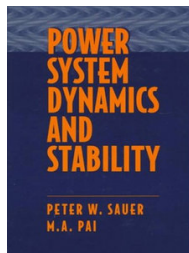
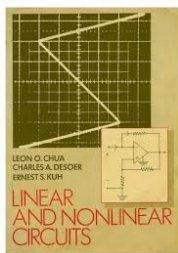
Disclaimers

- start-off with “boring” modeling before we get to more “sexy” topics
 - we will cover mostly basic material & some recent “cutting edge” work
 - we will focus on simple models and developing physical & math intuition
 - we will not go deeply into the math though everything is sound
- ⇒ cover fundamentals, convey intuition, & give references for the details
- notation is mostly “standard” (watch out for sign & p.u. conventions)
 - ask me for further reading about any topic
 - interrupt & correct me anytime

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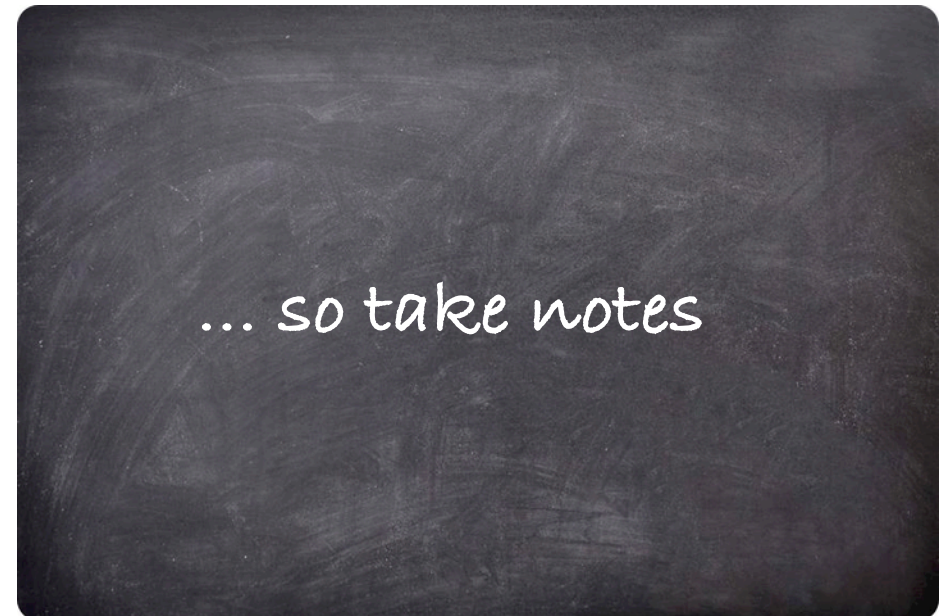
Many references available ... my personal look-up list

... to be complemented by references throughout the lecture



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We will also use the blackboard ...



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let's start off with a quiz:

what is your background?

why are you interested in power?

what are your expectations?

Outline

Introduction

Power Network Modeling

- Circuit Modeling: Network, Loads, & Devices
- Kron Reduction of Circuits
- Power Flow Formulations & Approximations
- Dynamic Network Component Models

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

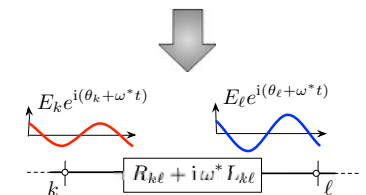
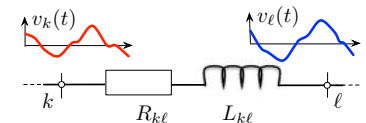
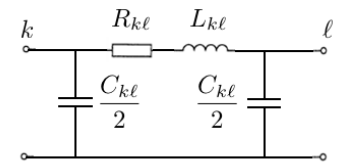
Conclusions

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Circuit Modeling: Network, Loads, & Devices

AC circuits – starting from yesterday's lecture

- power network modeled by linear **RLC** circuit, e.g., Π -model for
 - transmission lines (mainly inductive)
 - distribution lines (resistive/inductive)
 - cables (capacitive effects)
- we will work in **single-phase**, e.g., q -phase of a balanced 3-phase circuit
- quasi-stationary modeling** at time scales of interest: operation at nominal frequency ω^* with harmonic waveforms
 - phasor signals: $v_k(t) \approx E_k e^{i(\theta_k + \omega^* t)}$
 - algebraic circuit: $\frac{d}{dt} L_{k\ell} \approx i\omega^* L_{k\ell}$

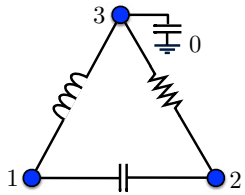


Note: quasi-stationarity assumption can be justified via singular perturbations & modeling can be improved using *dynamic phasors* [A. Stankovic & T. Aydin '00]

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AC circuits – graph-theoretic modeling

- a circuit is a connected & undirected **graph** $G = (\mathcal{V}, \mathcal{E})$
 - $\mathcal{V} = \{1, \dots, n\}$ are the nodes or *buses*
 - buses are partitioned as $\mathcal{V} = \{\text{sources}\} \cup \{\text{loads}\}$
 - the ground is sometimes explicitly modeled as node 0 or $n + 1$
 - $\mathcal{E} \subset \{\{i, j\} : i, j \in \mathcal{V}\} = \mathcal{V} \times \mathcal{V}$ are the undirected edges or *branches*
 - edges between distinct nodes $\{i, j\}$ are the *lines*
 - self-edges $\{i, i\}$ (or edges to ground $\{i, 0\}$) are the *shunts*



$$\mathcal{V} = \{1, 2, 3\}$$

$$\mathcal{E} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{3, 3\}\}$$

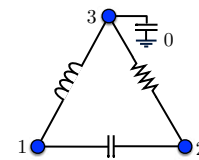
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AC circuits – the network admittance matrix

- $Y = [Y_{ij}] \in \mathbb{C}^{n \times n}$ is the **network admittance matrix** with elements

$$Y_{ij} = \begin{cases} -\frac{1}{Z_{ij}} & \text{for off-diagonal elements } i \neq j \\ \frac{1}{Z_{i,\text{shunt}}} + \sum_{j \neq i} \frac{1}{Z_{ij}} & \text{for diagonal elements } i \neq j \end{cases}$$

- impedance = resistance + $i \cdot$ reactance: $Z_{ij} = R_{ij} + i \cdot X_{ij}$
- admittance = conductance + $i \cdot$ susceptance: $\frac{1}{Z_{ij}} = G_{ij} + i \cdot B_{ij}$



$$Y = \underbrace{\begin{bmatrix} \frac{1}{Z_{12}} + \frac{1}{Z_{13}} & -\frac{1}{Z_{12}} & -\frac{1}{Z_{13}} \\ -\frac{1}{Z_{12}} & \frac{1}{Z_{12}} + \frac{1}{Z_{23}} & -\frac{1}{Z_{23}} \\ -\frac{1}{Z_{13}} & -\frac{1}{Z_{23}} & \frac{1}{Z_{13}} + \frac{1}{Z_{23}} \end{bmatrix}}_{\text{network Laplacian matrix}} + \underbrace{\begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{1}{Z_{3,\text{shunt}}} \end{bmatrix}}_{\text{diag(shunts)}}$$

Note *quasi-stationary* modeling: $Z_{13} = i\omega^* L_{13}$ with nominal frequency ω^*

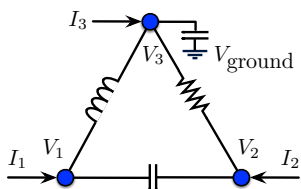
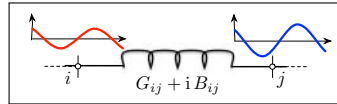
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AC circuits – basic variables

- basic variables**: voltages & currents
 - on nodes: potentials & current injections
 - on edges: voltages & current flows
- quasi-stationary* **AC phasor coordinates** for harmonic waveforms:
 - e.g., complex voltage $V = E e^{i\theta}$ denotes $v(t) = E \cos(\theta + \omega^* t)$

$v(t) = E \cos(\theta + \omega^* t)$

where $V \in \mathbb{C}$, $E \in \mathbb{R}_{\geq 0}$, $\theta \in \mathbb{S}^1$, $i = \sqrt{-1}$, and ω^* is nominal frequency



external injections: I_1, I_2, I_3

potentials: V_1, V_2, V_3

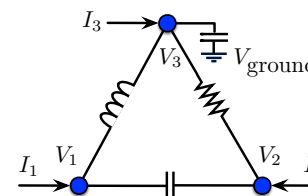
reference: $V_{\text{ground}} = 0V$

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AC circuits – fundamental equations

- Ohm's law** at every branch: $I_{i \rightarrow j} = \frac{1}{Z_{ij}}(V_i - V_j)$
- Kirchhoff's current law** for every bus: $I_i + \sum_j I_{j \rightarrow i} = 0$
- current balance equations** (treating the ground as node with 0V):

$$I_i = -\sum_j I_{j \rightarrow i} = \sum_j \frac{1}{Z_{ij}}(V_i - V_j) = \sum_j Y_{ij} V_j \quad \text{or} \quad \mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$$



$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Note: all variables are in per unit (p.u.) system, i.e., normalized wrt base voltage

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AC circuits – average power and power factor

on blackboard

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AC circuits – complex power

on blackboard

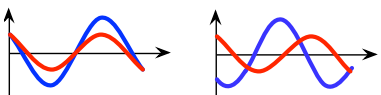
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AC circuits – power dissipated by RLC loads

on blackboard

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AC circuits – complex power summary

• **active & reactive power** in AC circuits: 

- active (average) power:

$$P = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \cos(\phi)$$

- reactive (0-average) power:

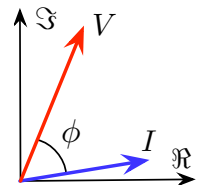
$$Q = \frac{1}{T} \int_0^T v(t) \cdot i(t - T/4) dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \sin(\phi)$$

⇒ normalize voltage & current phasors:

$$V \mapsto 1/\sqrt{2} \cdot E e^{i\theta}$$

⇒ **complex power:** $S = V\bar{I} = P + iQ$

= active power + i · reactive power



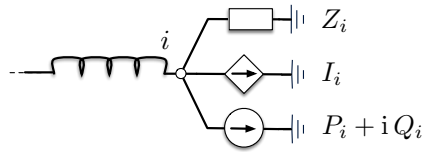
Note: often complex phasors are implicitly normalized $\tilde{V} = 1/\sqrt{2} \cdot E e^{i\theta}$

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Static models for sources & loads

- aggregated **ZIP load model**:

- constant impedance **Z** +
- constant current **I** +
- constant power **P**



- more general **exponential load model**: $\text{power} = \text{const.} \cdot (V/V_{\text{ref}})^{\text{const.}}$
(combinations & variations learned from data)

- conventional **synchronous generators** are typically controlled to have constant active power output P and voltage magnitude E

- sources interfaced with **power electronics** are typically controlled to have constant active power P and reactive power Q

- ⇒ **PQ buses** have complex power $S = P + iQ$ specified
- ⇒ **PV buses** have active power P and voltage magnitude E specified
- ⇒ **slack buses** have E and θ specified (not really existent)

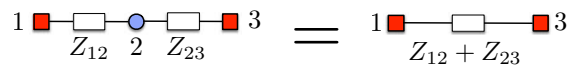
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Kron Reduction of Circuits

Kron reduction

[G. Kron 1939]

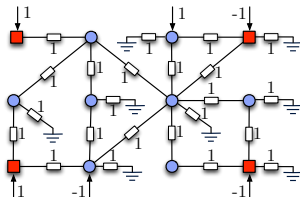
often (almost always) you will encounter Kron-reduced network models



General procedure:

- convert const. power injections locally to shunt impedances $Z = S/V_{\text{ref}}^2$
- partition linear current-balance equations via **boundary** & **interior nodes**:
(arises naturally, e.g., sources & loads, measurement terminals, etc.)

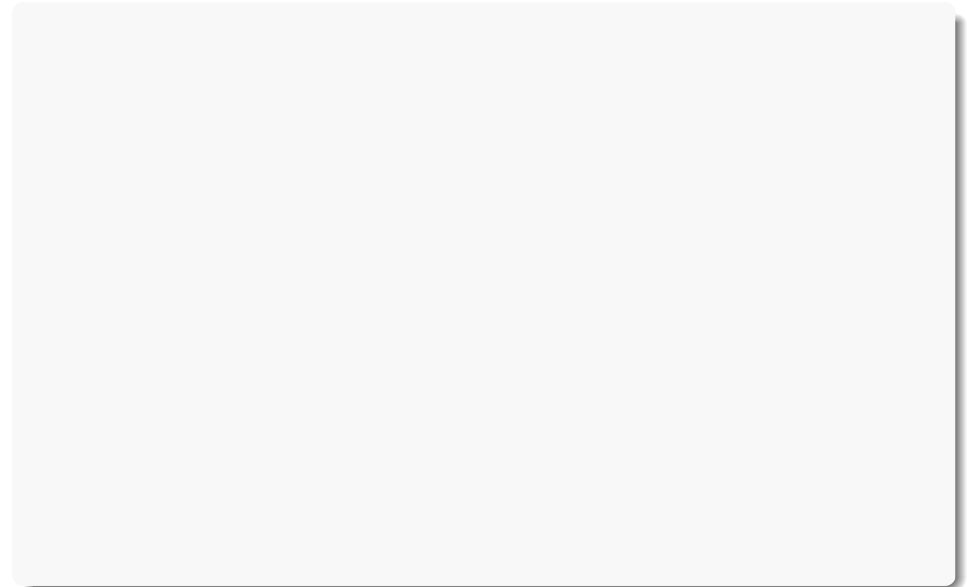
$$\begin{bmatrix} I_{\text{boundary}} \\ I_{\text{interior}} \end{bmatrix} = \begin{bmatrix} Y_{\text{boundary}} & Y_{\text{bound-int}} \\ Y_{\text{bound-int}}^T & Y_{\text{interior}} \end{bmatrix} \begin{bmatrix} V_{\text{boundary}} \\ V_{\text{interior}} \end{bmatrix}$$



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Kron reduction cont'd

on blackboard

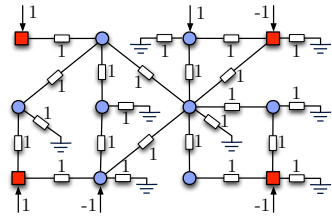


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Kron reduction cont'd

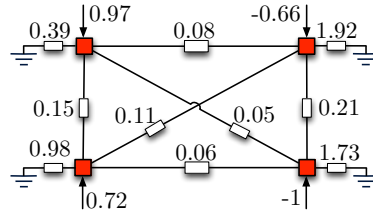
2 Gaussian elimination of interior voltages:

$$V_{\text{interior}} = Y_{\text{interior}}^{-1} \left(I_{\text{interior}} - Y_{\text{bound-int}}^T V_{\text{boundary}} \right)$$



original circuit

$$I = Y \cdot V$$



"equivalent" reduced circuit

$$I_{\text{red}} = Y_{\text{red}} \cdot V_{\text{boundary}}$$

⇒ reduced Y-matrix: $Y_{\text{red}} = Y_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{-1} \cdot Y_{\text{bound-int}}^T$

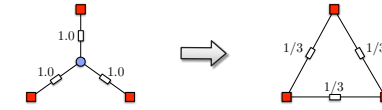
⇒ reduced injections: $I_{\text{red}} = I_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{-1} \cdot I_{\text{interior}}$

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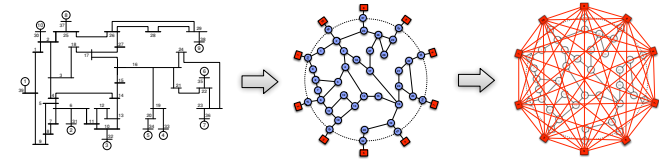
Examples of Kron reduction

algebraic properties are preserved but the network changes significantly

- Star-Δ transformation [A. E. Kennelly 1899, A. Rosen '24]



- Kron reduction of load buses in IEEE 39 New England power grid



⇒ topology without weights is meaningless!

⇒ shunt resistances (loads) are mapped to line conductances

⇒ many properties still open [FD & F. Bullo '13, S. Caliskan & P. Tabuada '14]

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Kron reduction – so simple yet still full of mysteries

49th IEEE Conference on Decision and Control
December 15-17, 2008
Hilton Atlanta Hotel, Atlanta, GA, USA

The Behavior of Linear Time Invariant RLC Circuits

Erik I. Versteeg and Jan C. Willems

Abstract—It is shown that just as we did for a purely resistive network [8], that circuit analysis is very simple if the elements are described not by port-level series and currents through the elements, but rather by the potentials at the nodes and the external currents into the nodes. For single AC, or DC, components this also a description with a 2 × 2 matrix, which is more complex than the scalar conductance laws governing the potential and current through. However, this description flows an advantage in performing the analysis of more complex circuits. These are built by from simple operations like joining two nodes, splitting at nodes, and subcircuits.

INDEX TERMS: TERMINAL BEHAVIOR

We view electrical circuit as a device that interacts with its environment through a finite number of wires connected

is compatible with the internal structure of the circuit and component values form a subset $\mathcal{R} \subseteq \mathbb{R}^{(n \times n)}$, called the terminal behavior of the circuit. $(P, I) \in \mathcal{R}$ means that the circuit allows the vector functions (P, I) of terminal variables, while $(P, I) \notin \mathcal{R}$ means that the circuit forbids the vector (P, I) of terminal variables. [1, 8]. In this paper, we study which subsets of $\mathcal{R} \subseteq \mathbb{R}^{(n \times n)}$ can occur as the terminal potential-current behavior of an interconnection of a finite set of linear nonnegative resistors, inductors and capacitors. The paper is organized as follows: In Section II, the purely resistive network is revisited, and the full characterization we obtained for the behavioral description are stated. The main goal is to extend these to time-invariant

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journal homepage: www.elsevier.com/locate/sycon

Characterization and partial synthesis of the behavior of resistive circuits at their terminals

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Circuit reduction
Partial synthesis by interconnection

ABSTRACT

The external behavior of linear resistive circuits with terminals is characterized as a linear map between the set of external currents and voltages. Conditions are derived for shaping the internal behavior of the circuit by interconnection with an additional resistive circuit.

is originally formulated in [4], and very close to the problem of achievable flow structure addressed in [5] (see also [6]). Indeed, the necessary and sufficient conditions for achieving a certain behavior are obtained in [7] (see also [8]), usually in terms of conditions in the case of linear, nonnegative elements, which constitute the set of necessary conditions to necessary and sufficient conditions, follows from the positivity requirement on resistances.

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IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—II: REGULAR PAPERS, VOL. 56, NO. 1, JANUARY 2009

Kron Reduction of Graphs With Applications to Electrical Networks

Florian Dörfler and Francesco Bullo

Abstract—Consider a weighted undirected graph and its corresponding Laplacian matrix, possibly augmented with additional diagonal elements corresponding to self-loops. The Kron reduction of this graph is again a graph whose Laplacian matrix is obtained by the Schur complement of the original Laplacian matrix with respect to a specified subset of nodes. The Kron reduction process is analogous to classic circuit theory and is related to operations such as electrical impedance homomorphism, smart grid monitoring, transient stability assessment, and analysis of power electronics. Kron reduction is also relevant in other physical domains, in computer-aided applications, and in the reduction of Markov chains. Related concepts have also been studied in purely theoretic problems as the literature on these subjects. In this paper we establish the Kron reduction process from the viewpoint of algebraic graph theory. Specifically, we provide a comprehensive and detailed graph-theoretic analysis of Kron reduction encompassing topological, algebraic, spectral, recursive, and combinatorial aspects. Throughout the paper, we especially emphasize the practical applicability of our results to various problems arising in engineering, communications, and power systems. Our analysis of Kron reduction leads to several insights both on the mathematical and the physical level.

Index Terms—Algebraic graph theory, equivalent circuit, Kron reduction, network-reduced model, Ward equivalent.

graph, its loopy Laplacian Q_{aug} , its spectrum, and its effective resistance? Finally, why is the graph reduction process of practical importance and in which application areas? These are some of the questions that motivate this paper.

Electrical networks and the Kron reduction. To illustrate the physical dimension of the problem setup introduced above, we consider the associated linear circuit with n nodes, current injections $f \in \mathbb{R}^n$, and voltages $v \in \mathbb{R}^n$. Branch conductances $A_{ij} > 0$, and shunt conductances $A_{ii} > 0$ connecting node i to the ground. The resulting current-balance equations are $f = Q_{\text{aug}} v$, where the conductance matrix $Q_{\text{aug}} \in \mathbb{R}^{n \times n}$ is the loopy Laplacian. In circuit theory and related disciplines it is desirable to obtain a lower-dimensional electrically equivalent network from the viewpoint of certain boundary nodes $\Phi \subseteq \{1, \dots, n\}$, $|\Phi| \geq 2$. If $\Phi = \{1, \dots, n\}$ (i.e. all nodes) the set of interior nodes, then, after appropriately labeling the nodes, the current-balance equations can be partitioned as

$$\begin{bmatrix} I_{\Phi} \\ I_{\Phi^c} \end{bmatrix} = \begin{bmatrix} Q_{\Phi\Phi} & Q_{\Phi\Phi^c} \\ Q_{\Phi^c\Phi} & Q_{\Phi^c\Phi^c} \end{bmatrix} \begin{bmatrix} v_{\Phi} \\ v_{\Phi^c} \end{bmatrix} \quad (1)$$



Brief paper
Towards Kron reduction of generalized electrical networks^{*}
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1. Introduction

Multi-machine power networks are the interconnection of power generators and substations via three-phase transmission lines. This structure can be abstracted as a graph, in which

Kron reduction is used to simplify the analysis of multi-machine power systems under certain steady state assumptions that underlie the usage of phasors. Doing this via network reduction theory, in this paper we show how to perform Kron reduction for a class of electrical networks, without steady state assumptions. The reduced models can thus be used to analyze the transient as well as the steady state behavior of these electrical networks.

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This reduction, however, is based on the use of phasors and it requires the current and voltage waveforms in each phase to be sinusoidal and with the same frequency. This assumption seems contradictory if we want to study the transient behavior of a power system during which the waveforms are not sinusoidal.

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Power Flow Formulations & Approximations

Power balance eqn's: "power injection = Σ power flows"

① **complex form:** $S_i = V_i \bar{I}_i = \sum_j V_i \bar{Y}_{ij} \bar{V}_j$ or $S = \text{diag}(V) \bar{Y} V$

⇒ purely quadratic and useful for static calculations & optimization

② **rectangular form:** insert $V = e + if$ and split real & imaginary parts:

active power: $P_i = \sum_j B_{ij}(e_i f_j - f_i e_j) + G_{ij}(e_i e_j + f_i f_j)$

reactive power: $Q_i = -\sum_j B_{ij}(e_i e_j + f_i f_j) + G_{ij}(e_i f_j - f_i e_j)$

⇒ purely quadratic and useful for homotopy methods & QCQPs

③ **matrix form:** define unit-rank p.s.d. Hermitian matrix $W = V \cdot \bar{V}^T$ with components $W_{ij} = V_i \bar{V}_j$, then power flow is $S_i = \sum_j \bar{Y}_{ij} W_{ij}$

⇒ linear and useful for relaxations in convex optimization problems

Power balance eqn's – digression

if you're interested in power flow optimization, take a close look at the matrix form

Convex Relaxation of Optimal Power Flow—Part I: Formulations and Equivalence

Steven H. Low, *Fellow, IEEE*

Abstract—This tutorial summarizes recent advances in the convex relaxation of the optimal power flow (OPF) problem, focusing on structural properties rather than algorithms. Part I presents two power flow models, formulates OPF and their relaxations in each model, and proves equivalence relationships among them. Part II presents sufficient conditions under which the convex relaxations are exact.

Index Terms—Convex relaxation, optimal power flow, power systems, quadratically constrained quadratic program (QCQP), second-order cone program (SOCP), semidefinite program (SDP), semidefinite relaxation.

I. INTRODUCTION

FOR our purposes, an optimal power flow (OPF) problem is a mathematical program that seeks to minimize a certain function, such as total power loss, generation cost or user disutility, subject to the Kirchhoff's laws, as well as capacity, stability, and security constraints. OPF is fundamental in power system operations as it underlies many applications such as economic dispatch, unit commitment, state estimation, stability and reliability assessment, volt/var control, demand response, etc.

SOCP for radial networks in the branch flow model of [45]. See Remark 6 below for more details. While these convex relaxations have been illustrated numerically in [22] and [23], whether or when they will turn out to be exact is first studied in [24]. Exploiting graph sparsity to simplify the SDP relaxation of OPF is first proposed in [25] and [26] and analyzed in [27] and [28].

Convex relaxation of quadratic programs has been applied to many engineering problems; see, e.g., [29]. There is a rich theory and extensive empirical experiences. Compared with other approaches, solving OPF through convex relaxation offers several advantages. First, while DC OPF is useful in a wide variety of applications, it is not applicable in other applications; see Remark 10. Second, a solution of DC OPF may not be feasible (may not satisfy the nonlinear power flow equations). In this case, an operator may tighten some constraints in DC OPF and solve again. This may not only reduce efficiency but also relies on heuristics that are hard to scale to larger systems or faster control in the future. Third, when they converge, most nonlinear algorithms compute a local optimal usually without assurance on the quality of the solution. In contrast, a convex relaxation

Power balance eqn's – cont'd

④ **branch flow eqn's** parameterized in flow variables [M. Baran & F. Wu '89]:

• Ohm's law: $V_i - V_j = Z_{ij} I_{ij}$

• branch power flow $i \rightarrow j$: $S_{ij} = V_i \bar{I}_{ij}$

• power balance at node i :

$$\underbrace{\sum_{k: i \rightarrow k} S_{ik} + Y_{i, \text{shunt}} |V_i|^2}_{\text{outgoing flows}} = S_i + \underbrace{\sum_{j: j \rightarrow i} (S_{ji} - Z_{ij} |I_{ij}|^2)}_{\text{incoming flows}}$$

• **DistFlow formulation** (or SOCP relaxation) in terms of square magnitude variables $|V_i|^2$ and $|I_{ij}|^2$

(missing angle variables $\angle V_i$ and $\angle I_{ij}$ can sometimes be recovered, e.g., in acyclic case)

• lossless approximation can be solved exactly in acyclic networks (useful for distribution networks) [M. Baran & F. Wu '89, M. Farivar, L. Chen, & S. Low '13]



Power balance eqn's – cont'd

⑤ **polar form:** insert $V = Ee^{i\theta}$ and split real & imaginary parts:

active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$

reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

⇒ will be our focus today since ...

- **power system specs** on frequency $\frac{d}{dt} \theta(t)$ and voltage magnitude E
- **dynamics:** generator swing dynamics affect voltage phase angles & voltage magnitudes are controlled to be constant
- **physical intuition:** usual operation near flat voltage profile $V_i \approx 1e^{i\phi}$ which will give rise to various insights for analysis & design (later)

Power flow simplifications & approximations

power flow equations are too complex & unwieldy for analysis & large computations

- ▶ active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- ▶ reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

1 lossless transmission lines $R_{ij}/X_{ij} = -G_{ij}/B_{ij} \approx 0$

$$\text{active power: } P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$\text{reactive power: } Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

2 decoupling near operating point $V_i \approx 1e^{i\phi}$: $\begin{bmatrix} \partial P/\partial\theta & \partial P/\partial E \\ \partial Q/\partial\theta & \partial Q/\partial E \end{bmatrix} \approx \begin{bmatrix} * & 0 \\ 0 & * \end{bmatrix}$

$$\text{active power: } P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) \quad (\text{function of angles})$$

$$\text{reactive power: } Q_i = -\sum_j B_{ij} E_i E_j \quad (\text{function of magnitudes})$$

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Power flow simplifications & approximations cont'd

- ▶ active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- ▶ reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

3 linearization for small flows near operating point $V_i \approx 1e^{i\phi}$:

$$\text{active power: } P_i = \sum_j B_{ij}(\theta_i - \theta_j) \quad (\text{known as DC power flow})$$

$$\text{reactive power: } Q_i = \sum_j B_{ij}(E_i - E_j) \quad (\text{formulation in p.u. system})$$

4 Multiple variations & combinations are possible

- linearization & decoupling at arbitrary operating points
- lines with constant R/X ratios [FD, J. Simpson-Porco, & F. Bullo '14]
- advanced linearizations [S. Bolognani & S. Zampieri '12, '15, B. Gentile et al. '14]
- "plenty of heuristics in the hidden stashes of industry" (B. Wollenberg '15)

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DC power flow assumptions are discussed in every book

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IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 24, NO. 3, AUGUST 2009

DC Power Flow Revisited

Brian Stott, *Fellow, IEEE*, Jorge Jardim, *Senior Member, IEEE*, and Ongun Alsac, *Fellow, IEEE*

Abstract—Linear MW-only “dc” network power flow models are in widespread and even increasing use, particularly in congestion-constrained market applications. Many versions of these approximate models are possible. When their MW flows are reasonably correct (and this is by no means assured), they can offer compelling advantages. Given their considerable importance in today’s electric power industry, dc models merit closer scrutiny. This paper attempts such a re-examination.

Index Terms—Congestion revenue rights, contingency analysis, dc power flow, economic dispatch, financial transmission rights, LMP pricing, unit commitment.

I. INTRODUCTION

THIS paper addresses so-called “dc” MW-only power flow modeling, which is of increased interest today because of recent upsurges in its use—mostly in LMP-based market applications where prices are constrained by network congestion. Such applications include real-time security-constrained

II. WHY DC MODELS?

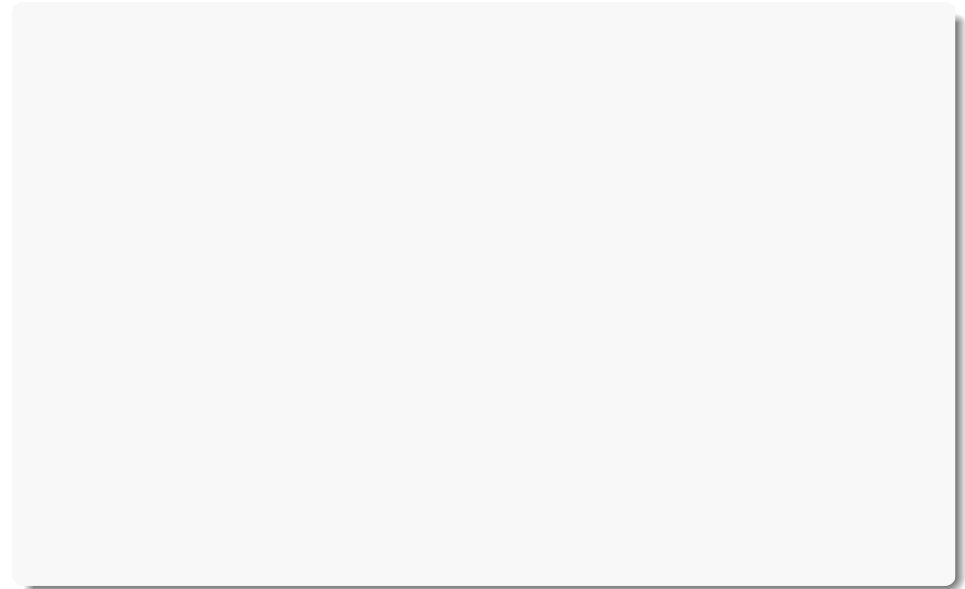
The linear, bilateral, non-complex, often state-independent, properties of a dc-type power flow model have considerable analytical and computational appeal. The use of such a model is limited to those MW-oriented applications where the effects of network voltage and VAR conditions are minimal (a very difficult-to-judge criterion). But then, as opposed to using the ac power flow model, the perceived advantages of a dc model are as follows.

- (a) Its solutions are non-iterative, reliable and unique.
- (b) Its methods and software are relatively simple.
- (c) Its models can be solved and optimized efficiently, particularly in the demanding area of contingency analysis.
- (d) Its network data is minimal and relatively easy to obtain.
- (e) Its linearity fits the economic theory on which much of transmission-oriented market design is based.
- (f) Its approximated MW flows are reasonably accurate, at least for the heavily loaded branches that might constrain system operation.

Conclusion on the **most limiting assumption** of DC power flow: $R/X \approx 0$

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Power flow decoupling for constant (non-zero) R/X ratios typically a much better assumption (on blackboard)



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Advanced approximation method

[S. Bolognani & S. Zampieri '15]

- nonlinear power flow equations in complex form

- power line equations: $YV = I$
- nodal equation: $S_i = V_i \bar{I}_i$
- at least one node regulated at a nominal voltage magnitude E_0

- **no assumption** on topology or X & R and no decoupling
- first order **Taylor's expansion** around $E_0 = \infty$ (or zero loading)
 - 1 existence of flat voltage solution for $E_0 = \infty$
 - 1 Taylor's terms computed via **implicit function theorem**
 - 2 nodal currents: $I_i = \frac{\bar{S}_i}{E_i} = \frac{\bar{S}_i}{E_0} + \frac{c_i(E_0)}{E_0^2}$
 - 3 bus voltages: $YV = \frac{\bar{S}}{E_0} + \frac{c(E_0)}{E_0^2}$
 - 4 $c(E_0)$ bounded in $E_0 \Rightarrow$ **neglect** $\frac{c(E_0)}{E_0^2}$ for large E_0

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Advanced approximation method – cont'd

\Rightarrow **LINEAR** power flow formulation:

$$YV = \frac{\bar{S}}{E_0} + \frac{c(E_0)}{E_0^2}$$

\Rightarrow convenient model for **power distribution grids** with lossy lines.

\Rightarrow explicit **approximation bound**:

$$\text{if } E_0^2 > 4\ell_{\max} \|S\|_{\text{tot}}$$

$$\text{then } \left\| \frac{c(E_0)}{E_0^2} \right\| \leq \frac{4\ell_{\max} \|S\|_{\text{tot}}^2}{E_0^2}$$

test feeder and source code available at
<http://github.com/saveriob/approx-pf>



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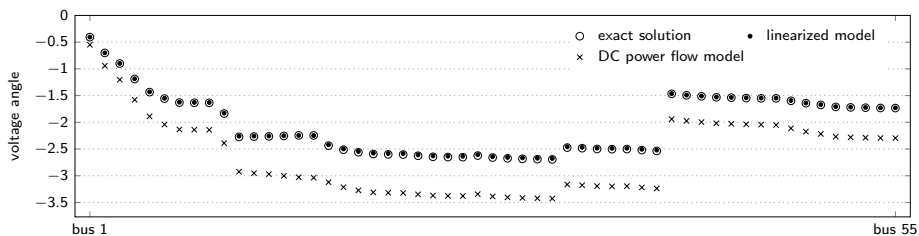
Advanced approximation method – cont'd

- same approximation expressed in **polar coordinates**

- angles: $\theta = \mathbb{1}_n \theta_0 + \frac{1}{E_0^2} \text{Im}(Y^\dagger \bar{S})$
- voltage magnitudes: $E = \mathbb{1}_v E_0 + \frac{1}{E_0} \text{Re}(Y^\dagger \bar{S})$

where Y^\dagger is a pseudoinverse of Y .

- **purely inductive lines** $Y = iB \Rightarrow$ recover **DC power flow model**
- performance evaluation for test feeder:



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Dynamic Network Component Models

Modeling the “essential” network dynamics

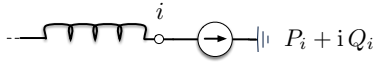
models can be arbitrarily detailed & vary on different time/spatial scales

- ① active and reactive **power flow**
(e.g., lossless)

$$P_{i,\text{inj}} = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_{i,\text{inj}} = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

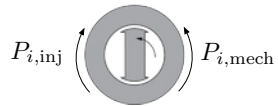
- ② passive constant power **loads**



$$P_{i,\text{inj}} = P_i = \text{const.}$$

$$Q_{i,\text{inj}} = Q_i = \text{const.}$$

- ③ electromech. **swing dynamics**
of synchronous machines



$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{i,\text{mech}} - P_{i,\text{inj}}$$

$$E_i = \text{const.}$$

- ④ **inverters**: DC or variable AC
sources with power electronics

- (i) have constant/controllable PQ
(ii) or mimic generators with $M = 0$

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Common variations in dynamic network models

dynamic behavior is very much dependent on load models & generator models

- ① frequency/voltage-depend. loads
[A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

$$D_i \dot{\theta}_i + P_i = -P_{i,\text{inj}}$$

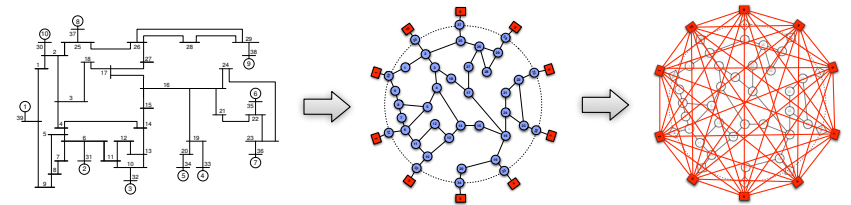
$$f_i(\dot{V}_i) + Q_i = -Q_{i,\text{inj}}$$

- ② network-reduced models after
Kron reduction of loads
[H. Chiang, F. Wu, & P. Varaiya '94]
(very common but poor
assumption: $G_{ij} = 0$)

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{i,\text{mech}}$$

$$- \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$- \underbrace{\sum_j G_{ij} E_i E_j \cos(\theta_i - \theta_j)}_{\text{effect of resistive loads}}$$



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Structure-preserving power network model [A. Bergen & D. Hill '81]

without Kron-reduction of load buses

- **generator swing dynamics:**

$$\dot{\theta}_i = \omega_i$$

$$M_i \dot{\omega}_i = -D_i \omega_i + P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_i = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$
- **frequency-dependent loads:**
(or inverter-interfaced sources)

$$D_i \dot{\theta}_i = P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_i = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

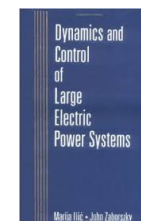
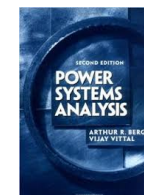
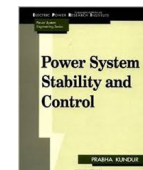
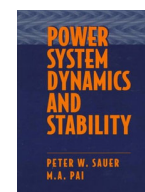
- in **academia**: this “baseline model” is typically further simplified: decoupling, linearization, constant voltages, . . .
 - in **industry**: much more detailed models used for grid simulations
- ⇒ **IMHO**: above model captures most interesting network dynamics

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Common variations in dynamic network models — cont'd

dynamic behavior is very much dependent on load models & generator models

- ③ higher order generator dynamics
[P. Sauer & M. Pai '98] voltages, controls, magnetics etc.
(reduction via singular perturbations)
- ④ dynamic & detailed load models
[D. Karlsson & D. Hill '94] aggregated dynamic load behavior
(e.g., load recovery after voltage step)
- ⑤ time-domain models [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12] passive Port-Hamiltonian models
for machines & RLC circuitry



“Power system research is all about the art of making the right assumptions.”

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Lots of current research activity on time-domain models



A port-Hamiltonian approach to power network modeling and analysis

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1. Introduction
 Market liberalization and the ever increasing electricity demand have forced the power system to operate under highly uncertain conditions. This situation has led to the need to revisit the existing modeling, analysis and control techniques that enable the power system to withstand unexpected contingencies without experiencing voltage or transient instabilities.
 At the network level power engineers used reduced network models (RNM) where the system is viewed as an n -port described by a set of ordinary differential equations. RNM do not retain the

passivity-based control' technique [20] was used in [21] to prove the existence of a stationary state. State feedback that guarantees stability of the operating point for a general n -machine system including transient conditions and a regular representation of the controller was given only for the case $n = 5$. In a computational complexity for the multi-machine case [2]. An extension of the invariance principle was proposed in order to find a new extended Lyapunov function taking into account the influence of small transfer constants. For multi-machine case an extension to backstepping is used to solve the global asymptotic stability problem in [4].



Towards Kron reduction of generalized electrical networks'

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1. Introduction
 Multi-machine power networks are the interconnection of power generators and actuators via three-phase transmission lines. This structure can be abstracted as a graph, in which

This reduction, however, is based on the use of phases and it requires the current and voltage systems in each phase to be sinusoidal and with the same frequency. This assumption is contradictory for use to study the transient behavior of a power system during which the waveforms are not sinusoidal.

IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—REGULAR PAPERS, VOL. 59, NO. 4, SEPTEMBER 2014 3477

Synchronization of Nonlinear Circuits in Dynamic Electrical Networks With General Topologies

Sainy V. Dhople, Member, IEEE, Brian B. Johnson, Member, IEEE, Florian Dörfler, Member, IEEE, and Abdullah O. Hamada

Abstract—Sufficient conditions are derived for global asymptotic synchronization in a system of identical nonlinear electrical circuits coupled through linear time-invariant (TI) electrical networks. In particular, the conditions we derive apply to settings where (i) the nonlinear circuits are composed of a parallel combination of a linear circuit element and a nonlinear element, (ii) the circuits share both their ports and a common set of linear elements, (iii) the nonlinear elements are coupled through either uniform or heterogeneous TI electrical networks. Uniform electrical networks have identical per-unit-length inductors. Heterogeneous electrical networks are characterized by having the same effective differential system that emphasizes signal differences. The applicability of the synchronization conditions to the linear and nonlinear portions of a power system is demonstrated. The applicability of the synchronization conditions to the linear and nonlinear portions of a power system is demonstrated. The applicability of the synchronization conditions to the linear and nonlinear portions of a power system is demonstrated.

Index Terms—Kron reduction, nonlinear circuits, synchronization.

1. INTRODUCTION
 SYNCHRONIZATION of nonlinear electrical circuits coupled through complex networks is integral to modeling,

TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS, VOL. 1, NO. 1, MARCH 2014

Compositional Transient Stability Analysis of Multimachine Power Networks

Sina Yamac Caliskan and Paulo Tabuada

Abstract—During the normal operation of a power system, all the voltage and current are sinusoidal with a frequency of 60 Hz in America and parts of Asia or of 50 Hz in the rest of the world. Forcing all the currents and voltages to be sinusoidal with the right frequency is one of the most important problems in power systems. This problem is known as the transient stability problem in the power system literature. The classical method used to study transient stability is based on several implicit assumptions that are violated during transient events. The first assumption is that all phases in steady transients. While phases require sinusoidal waveforms to be well defined, there is no guarantee that waveforms will remain sinusoidal during transients. In this paper, we use energy-based models derived from first principles that are not subject to hard-to-verify classical assumptions. In addition to classical assumptions that are known not to hold during transient events, we derive sufficient conditions covering the transient stability of power systems with loss transmission lines. Furthermore, the conditions for transient stability are composed in the sense that they inherit transient stability of a large power system by checking simple conditions for individual generators. System by checking simple conditions for individual generators.

Index Terms—Electromechanical systems, nonlinear control systems, power system dynamics, power system stability.

Outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

- Decoupled Active Power Flow (Synchronization)
- Reactive Power Flow (Voltage Collapse)
- Coupled & Lossy Power Flow
- Transient Rotor Angle Stability

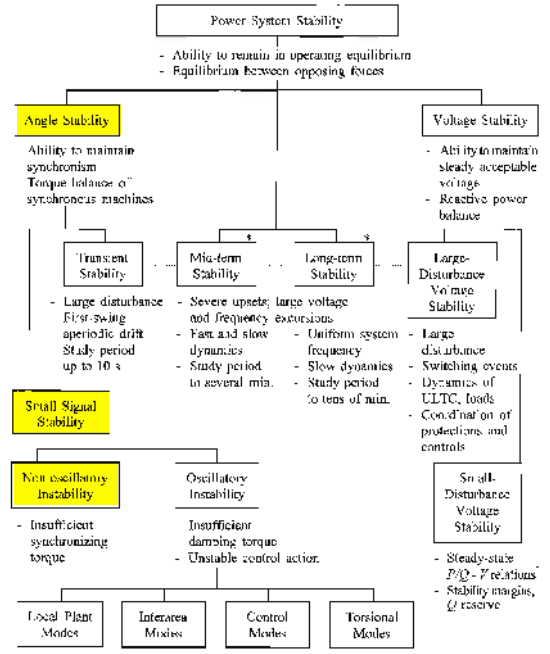
Power System Control Hierarchy

Power System Oscillations

Conclusions

Decoupled Active Power Flow (Synchronization)

Our first stab at power system stability



Preliminary insights on decoupled and lossless power flow

power flow equations:

$$P_i = \sum_{j=1}^n B_{ij} \sin(\theta_i - \theta_j) \Rightarrow \text{solution space: } \mathbb{T}^n = \mathbb{S}^1 \times \dots \times \mathbb{S}^1$$

rotational symmetry:

if θ^* is a solution \Rightarrow solution space: $\mathbb{T}^n \setminus \mathbb{S}^1$
 $\Rightarrow \theta^* + \text{const.} \cdot \mathbb{1}_n$ is another solution

necessary feasibility condition:

$$\sum_{i=1}^n P_i = 0 \Leftarrow \exists \text{ a solution} \Rightarrow \text{typically not true (w/o slack bus) due to unknown load demand}$$

(by summing all equations) \Rightarrow need to consider dynamics

Homework: think about the above conditions in coupled and/or lossy case

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Synchronization & feasibility of active power flow

basic problem setup

- **structure-preserving power network model** [A. Bergen & D. Hill '81]:
 (simple dynamics & decoupled lossless flows capture essential phenomena)

$$\text{synchronous machines: } M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

$$\text{frequency-dependent loads: } D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- **synchronization** = sync'd frequencies & bounded active power flows

$$\dot{\theta}_i = \omega_{\text{sync}} \quad \forall i \in \mathcal{V} \quad \& \quad |\theta_i - \theta_j| \leq \gamma < \pi/2 \quad \forall \{i, j\} \in \mathcal{E}$$

= active power flow feasibility & security constraints

- **sync is crucial** for the functionality and operation of the power grid

- **explicit sync frequency:** if sync, then

$$\omega_{\text{sync}} = \sum_i P_i / \sum_i D_i$$

(by summing over all equations)

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Synchronization & feasibility of active power flow

some key questions

Given: network parameters & topology and load & generation profile

Q: “ \exists an optimal, stable, and robust sync'd operating point?”

- 1 Security analysis [Araposthatis et al. '81, Wu et al. '80 & '82, Ilić '92, ...]
- 2 Load flow feasibility [Chiang et al. '90, Dobson '92, Lesieutre et al. '99, ...]
- 3 Optimal generation dispatch [Lavaei et al. '12, Bose et al. '12, ...]
- 4 Transient stability [Sastry et al. '80, Bergen et al. '81, Hill et al. '86, ...]
- 5 Inverters in microgrids [Chandorkar et al. '93, Guerrero et al. '09, Zhong '11, ...]
- 6 Complex networks [Hill et al. '06, Strogatz '01, Arenas et al '08, ...]

Further reading on sync problem:
 (my perspective)

PNAS **Synchronization in complex oscillator networks and smart grids**
 Florian Dörfler^{a,b,1}, Michael Chertkov^a, and Francesco Bullo^a
^aCenter for Control, Dynamical Systems, and Computation, University of California, Santa Barbara, CA 93106; and ^bCenter for Nonlinear Studies and Theory Division, Los Alamos National Laboratory, Los Alamos, NM 87545
Edited by Steven H. Strogatz, Cornell University, Ithaca, NY, and accepted by the Editorial Board November 14, 2012 (received for review July 16, 2012)
 The emergence of synchronization in a network of coupled oscillators is a fascinating topic in various scientific disciplines. A widely adopted model of a coupled oscillator network is characterized by a population of heterogeneous phase oscillators, a graph descri-

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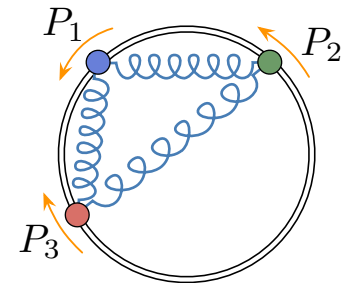
A perspective from coupled oscillators

Mechanical oscillator network

Angles $(\theta_1, \dots, \theta_n)$ evolve on \mathbb{T}^n as

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

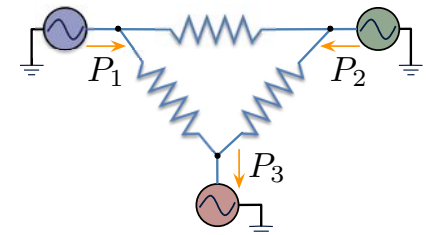
- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $P_i \in \mathbb{R}$
- spring constants $B_{ij} \geq 0$



Structure-preserving power network

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

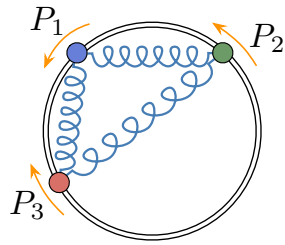
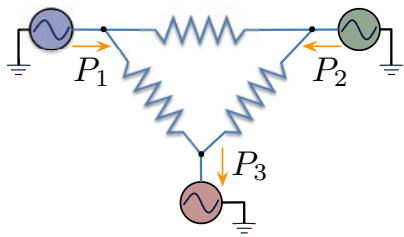
$$D_i \dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$



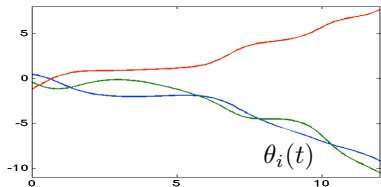
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Phenomenology of sync in power networks

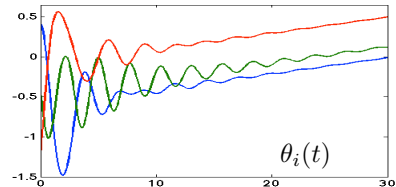
- sync is **crucial for AC power grids**



- sync is a **trade-off**



weak coupling & heterogeneous

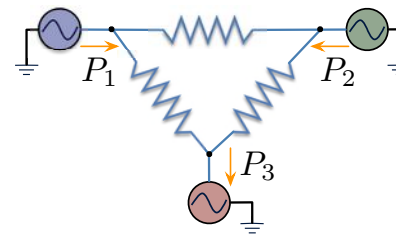


strong coupling & homogeneous

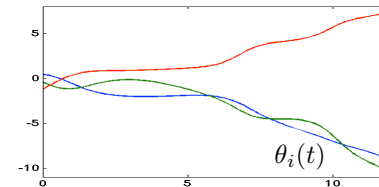
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Phenomenology of sync in power networks

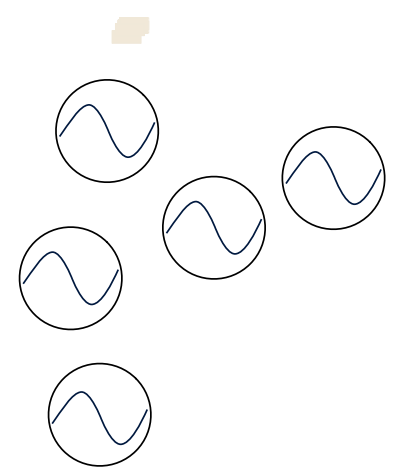
- sync is **crucial for AC power grids**



- sync is a **trade-off**



weak coupling & heterogeneous

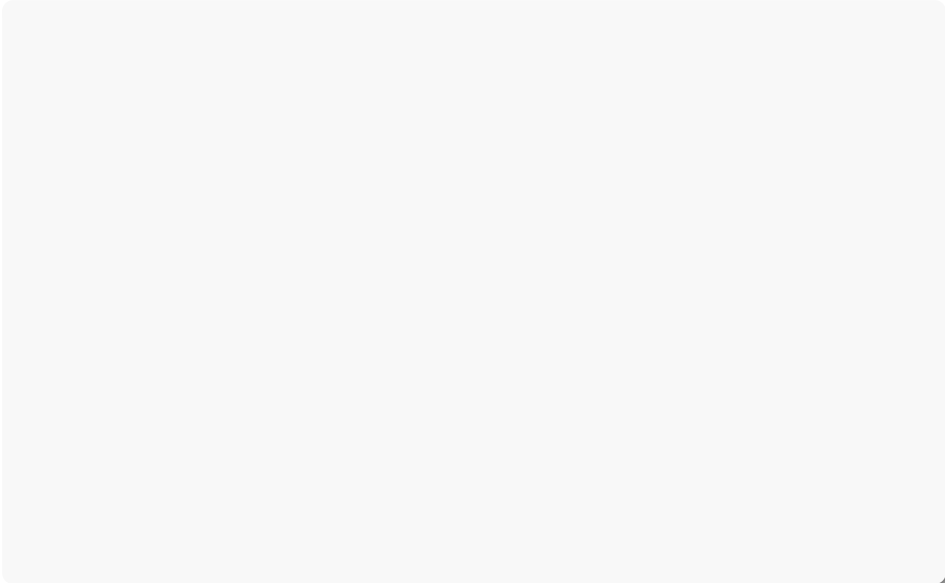


Blackout India July 30/31 2012

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Derivation of a two-bus toy model

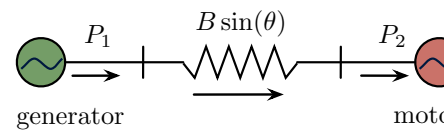
on blackboard



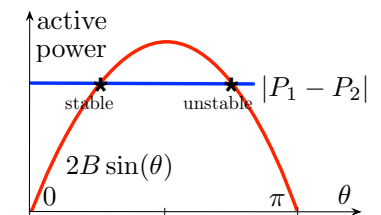
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Back of the envelope calculations for the two-node case

generator connected to identical motor shows bifurcation at difference angle $\theta = \pi/2$



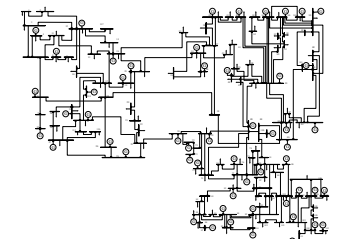
$$M\ddot{\theta} + D\dot{\theta} = P_1 - P_2 - 2B \sin(\theta)$$



\exists stable sync $\Leftrightarrow B > |P_1 - P_2|/2 \Leftrightarrow$ "ntwk coupling > heterogeneity"

Q1: Quantitative generalization to a complex & large-scale network?

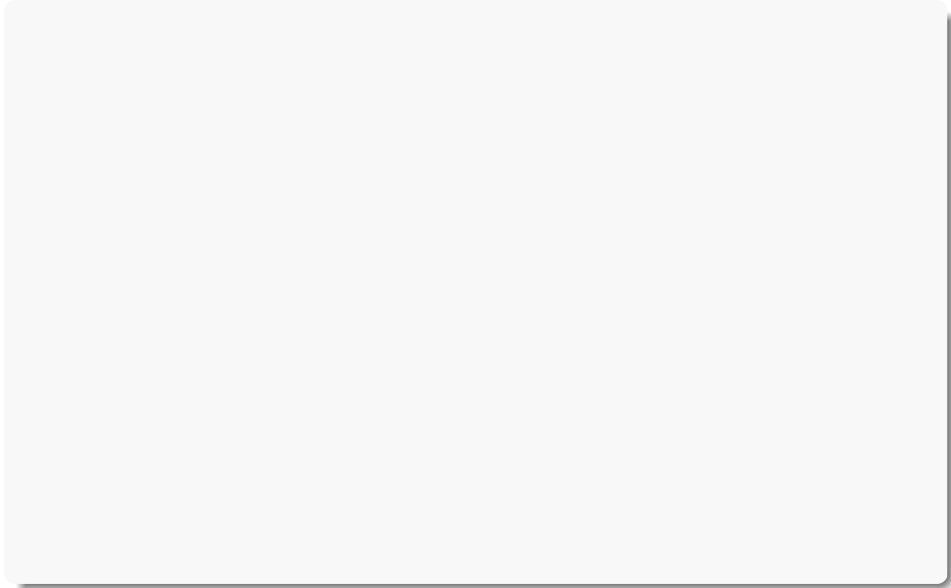
Q2: What are the particular metrics for coupling and heterogeneity?



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Some properties of Laplacian matrices

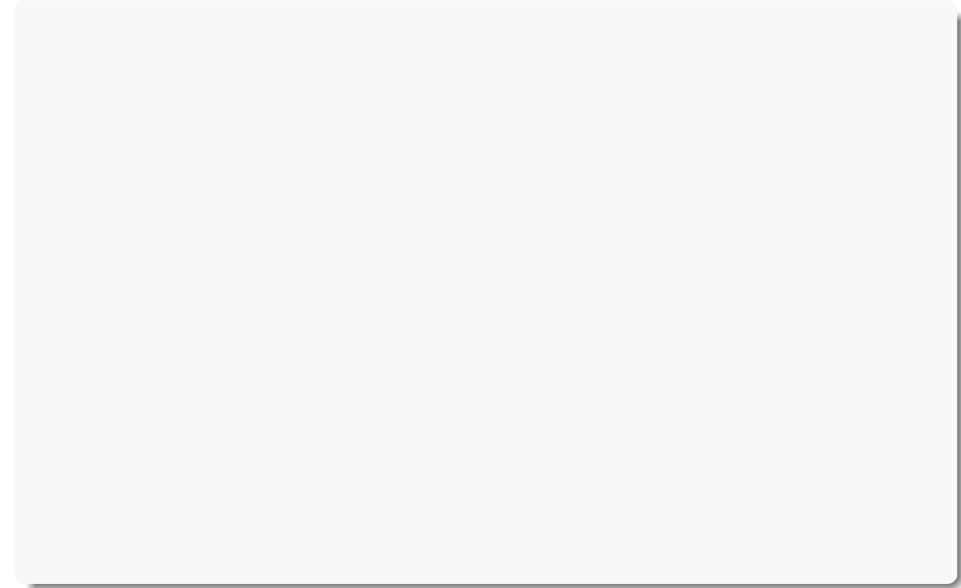
on blackboard



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Who knows consensus systems?

on blackboard



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Primer on algebraic graph theory

for a connected and undirected graph

Laplacian matrix $L =$ “degree matrix” – “adjacency matrix”

$$L = L^T = \begin{bmatrix} \vdots & \ddots & \vdots & \ddots & \vdots \\ -B_{i1} & \cdots & \sum_{j=1}^n B_{ij} & \cdots & -B_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \end{bmatrix} \geq 0$$

is positive semidefinite with one zero eigenvalue & eigenvector $\mathbb{1}_n$

Notions of connectivity

- spectral: 2nd smallest eigenvalue of L is “algebraic connectivity” $\lambda_2(L)$
- topological: degree $\sum_{j=1}^n B_{ij}$ or degree distribution

Notions of heterogeneity

$$\|P\|_{\mathcal{E},\infty} = \max_{\{i,j\} \in \mathcal{E}} |P_i - P_j|, \quad \|P\|_{\mathcal{E},2} = \left(\sum_{\{i,j\} \in \mathcal{E}} |P_i - P_j|^2 \right)^{1/2}$$

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Synchronization in “complex” networks

for a first-order model — all results generalize locally

$$\dot{\theta}_i = P_i - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

- 1 **local stability** for equilibria satisfying
(linearization is Laplacian matrix)

$$|\theta_i^* - \theta_j^*| < \pi/2 \quad \forall \{i,j\} \in \mathcal{E}$$

- 2 **necessary sync condition:**
(so that syn'd solution exists)

$$\sum_j B_{ij} \geq |P_i - \omega_{\text{sync}}| \Leftrightarrow \text{sync}$$

- 3 **sufficient sync condition:**

$$\lambda_2(L) > \|P\|_{\mathcal{E},2} \Rightarrow \text{sync}$$

[FD & F. Bullo '12]

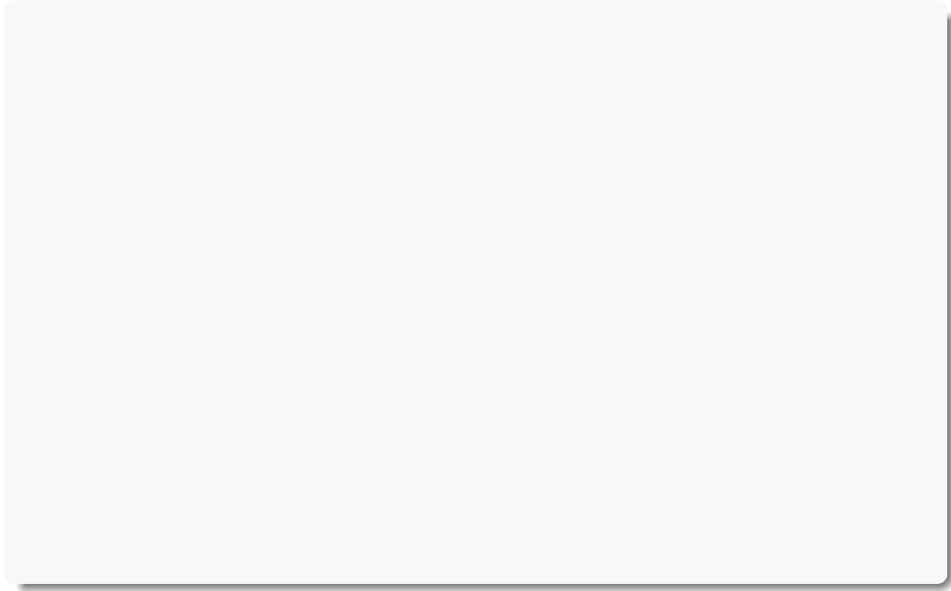
\Rightarrow \exists similar conditions with diff. metrics on coupling & heterogeneity

\Rightarrow **Problem:** sharpest general conditions are conservative

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Can we solve the power flow equations exactly?

on blackboard



A nearly exact sync condition

[FD, M. Chertkov, & F. Bullo '13]

- 1 search equilibrium θ^* with $|\theta_i^* - \theta_j^*| \leq \gamma < \pi/2$ for all $\{i, j\} \in \mathcal{E}$:

$$P_i = \sum_j B_{ij} \sin(\theta_i - \theta_j) \quad (*)$$

- 2 consider linear “small-angle” DC approximation of (*):

$$P_i = \sum_j B_{ij}(\delta_i - \delta_j) \quad \Leftrightarrow \quad P = L\delta \quad (**)$$

unique solution (modulo symmetry) of (***) is $\delta^* = L^\dagger P$

- 3 solution ansatz for (*): $\theta_i^* - \theta_j^* = \arcsin(\delta_i^* - \delta_j^*)$ (for a tree)

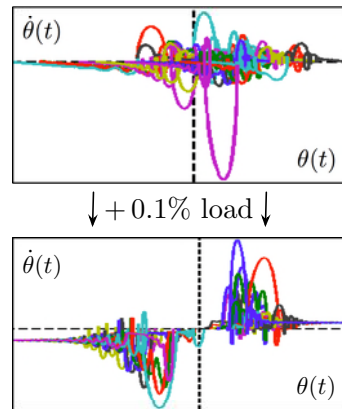
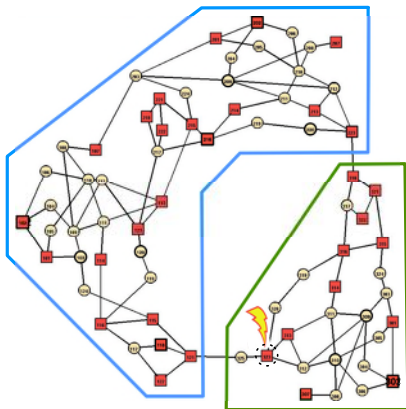
$$P_i = \sum_{j=1}^n a_{ij} \sin(\theta_i - \theta_j) = \sum_{j=1}^n a_{ij} \sin(\arcsin(\delta_i^* - \delta_j^*)) = P_i \quad \checkmark$$

\Rightarrow **Thm:** $\exists \theta^*$ with $|\theta_i^* - \theta_j^*| \leq \gamma \forall \{i, j\} \in \mathcal{E} \Leftrightarrow \|L^\dagger P\|_{\mathcal{E}, \infty} \leq \sin(\gamma)$

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E}, \infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$

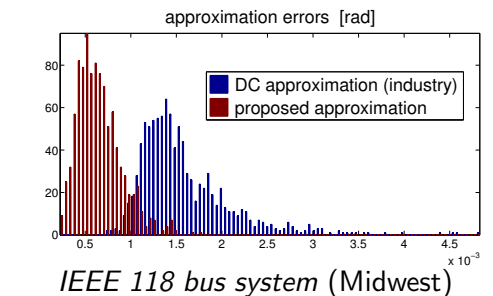
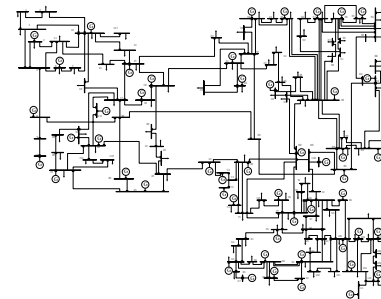


Reliability Test System RTS 96 under two loading conditions

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E}, \infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$



Outperforms conventional DC approximation “on average & in the tail”.

More on power flow approximations

Randomized power network test cases

with 50 % randomized loads and 33 % randomized generation

Randomized test case (1000 instances)	Numerical worst-case angle differences: $\max_{\{i,j\} \in \mathcal{E}} \theta_i^* - \theta_j^* $	Analytic prediction of angle differences: $\arcsin(\ L^\dagger P\ _{\mathcal{E}, \infty})$	Accuracy of condition: $\arcsin(\ L^\dagger P\ _{\mathcal{E}, \infty})$ - $\max_{\{i,j\} \in \mathcal{E}} \theta_i^* - \theta_j^* $
9 bus system	0.12889 rad	0.12893 rad	$4.1218 \cdot 10^{-5}$ rad
IEEE 14 bus system	0.16622 rad	0.16650 rad	$2.7995 \cdot 10^{-4}$ rad
IEEE RTS 24	0.22309 rad	0.22480 rad	$1.7089 \cdot 10^{-3}$ rad
IEEE 30 bus system	0.16430 rad	0.16456 rad	$2.6140 \cdot 10^{-4}$ rad
New England 39	0.16821 rad	0.16828 rad	$6.6355 \cdot 10^{-5}$ rad
IEEE 57 bus system	0.20295 rad	0.22358 rad	$2.0630 \cdot 10^{-2}$ rad
IEEE RTS 96	0.24593 rad	0.24854 rad	$2.6076 \cdot 10^{-3}$ rad
IEEE 118 bus system	0.23524 rad	0.23584 rad	$5.9959 \cdot 10^{-4}$ rad
IEEE 300 bus system	0.43204 rad	0.43257 rad	$5.2618 \cdot 10^{-4}$ rad
Polish 2383 bus system (winter peak 1999/2000)	0.25144 rad	0.25566 rad	$4.2183 \cdot 10^{-3}$ rad

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Discrete control actions to assure sync

1 (re)dispatch generation subject to **security constraints**:

find $\theta \in \mathbb{T}^n, u \in \mathbb{R}^{n_l}$ subject to

source power balance:

$$u_i = P_i(\theta)$$

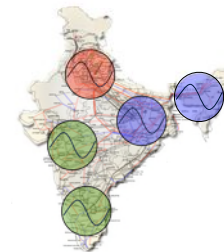
load power balance:

$$P_i = P_i(\theta)$$

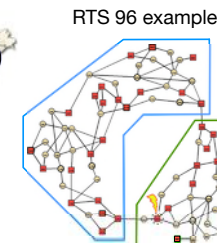
branch flow constraints:

$$|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$$

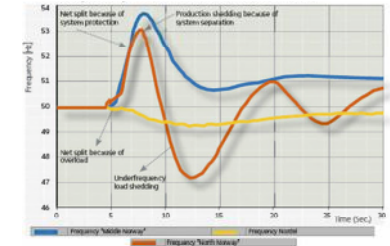
2 remedial action schemes: load/production shedding & islanding



India, July 30/31 2012



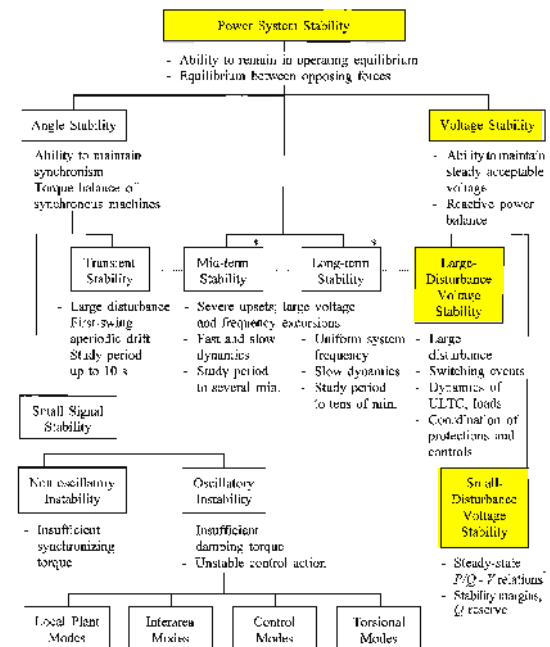
Nordic grid, December 1, 2005 (pacw.org)



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Decoupled Reactive Power Flow (Voltage Collapse)

Apparently a different beast



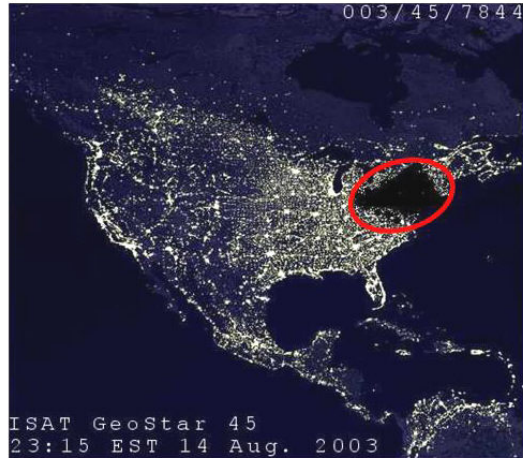
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Voltage collapse in power networks

- **voltage instability:** loading > capacity \Rightarrow voltages drop
"mainly" a reactive power phenomena
- **recent outages:** Québec '96, Scandinavia '03, Northeast '03, Athens '04

"Voltage collapse is still the biggest single threat to the transmission system. It's what keeps me awake at night."

– Phil Harris, CEO PJM.



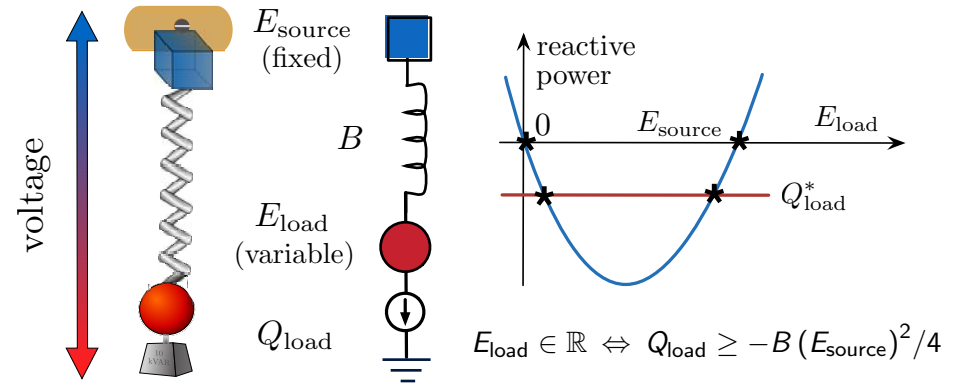
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Back of the envelope calculations for the two-node case

source connected to load shows bifurcation at load voltage $E_{load} = E_{source}/2$

reactive power balance at load:

$$Q_{load} = B E_{load}(E_{load} - E_{source})$$



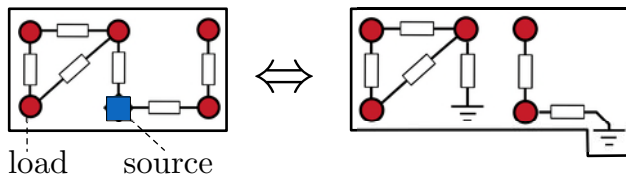
$$\exists \text{ high load voltage solution} \Leftrightarrow (\text{load}) < (\text{network})(\text{source voltage})^2/4$$

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Preliminary insights when going to "complex" networks

- **sources** with constant voltage magnitudes E_i
- **loads** with constant power demand $Q_i(E) = Q_i$

\Rightarrow WLOG assume that network among loads is connected



\Rightarrow reactive power balance: $Q_i = -\sum_j B_{ij} E_i E_j$ or $Q = -\text{diag}(E) B E$

\Rightarrow necessary feasibility condition: $\sum_{i=1}^n Q_i \geq 0 \Leftrightarrow \exists$ a solution

(by summing all equations and using $-E^T B E \geq 0$)

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Intuition extends to complex networks – essential insights

Reactive power balance:

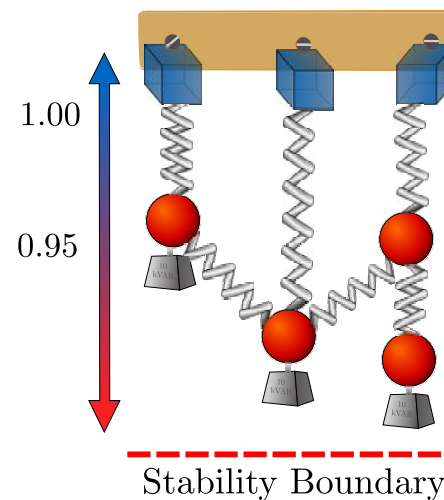
$$Q_i = -\sum_j B_{ij} E_i E_j$$

Suff. & tight cond' for general case [J. Simpson-Porco, FD, & F. Bullo, '14]:

\exists unique high-voltage solution E_{load}

\Leftrightarrow

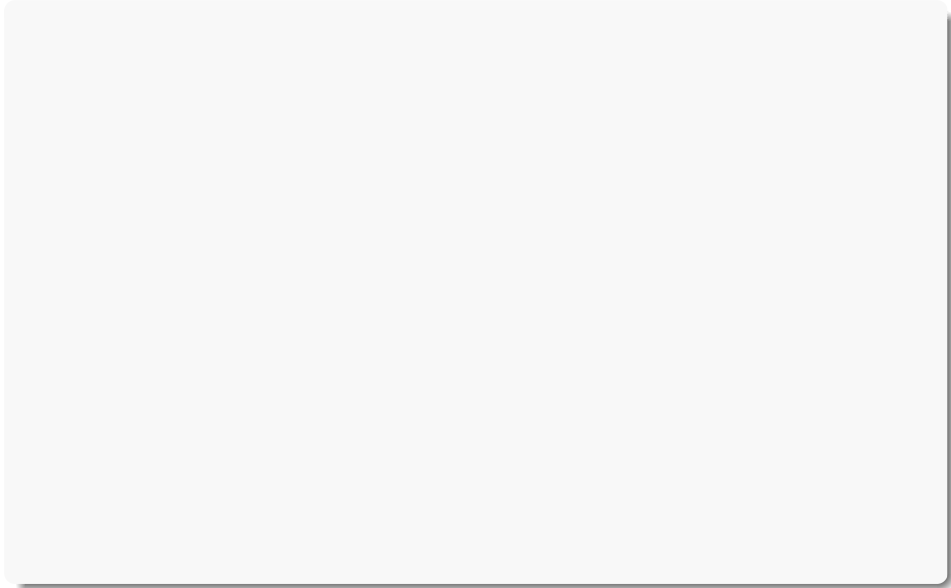
$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$



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Intuition extends to complex networks – details

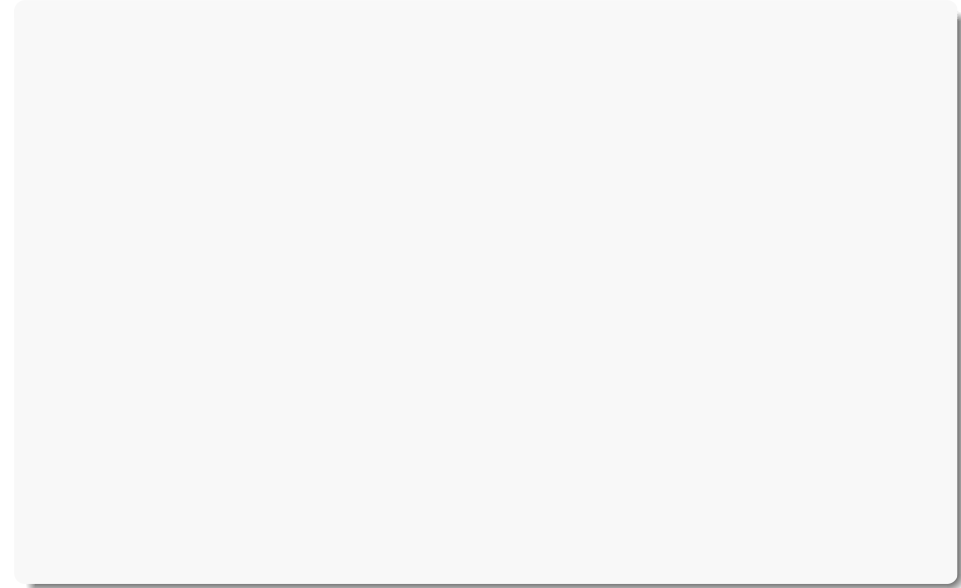
on blackboard



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Intuition extends to complex networks – details cont'd

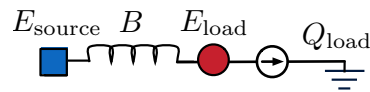
on blackboard



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More back of the envelope calculations

$$Q_L = B E_L (E_L - E_S)$$



$$\text{Exact soln: } E_L = \frac{E_S}{2} \left(1 + \sqrt{1 + 4Q_L / (B E_S^2)} \right) = \frac{E_S}{2} \left(1 + \sqrt{1 - \frac{Q_L}{Q_{\text{crit}}}} \right)$$

⇒ Taylor exp. for $\frac{Q_L}{Q_{\text{crit}}} \rightarrow 0$:

$$E_L \approx E_S \left(1 - \frac{1}{4} \frac{Q_L}{Q_{\text{crit}}} \right)$$

• **General case:** existence & approximation from implicit function thm

- if all loads Q_i are “sufficiently small” [D. Molzahn, B. Lesieutre, & C. DeMarco '12]
- if slack bus has “sufficiently large” E_S [S. Bolognani & S. Zampieri '12 & '14]
- if each source is above a “sufficiently large” E_{source} [B. Gentile et al. '14]
- if previous existence condition is met [J. Simpson-Porco, FD, & F. Bullo, '14]

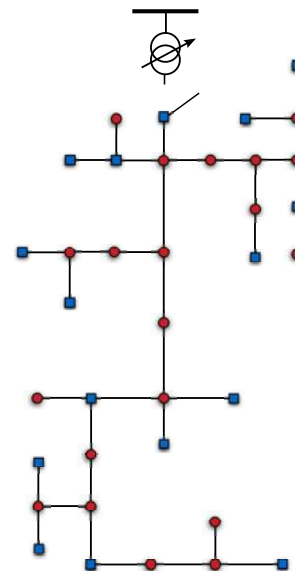
⇒ 1st order approximation:

$$E_L \approx \text{diag}(E_L^*) \left(\mathbb{1} - \frac{1}{4} Q_{\text{crit}}^{-1} Q_L \right)$$

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Linear DC approximation extends to complex networks

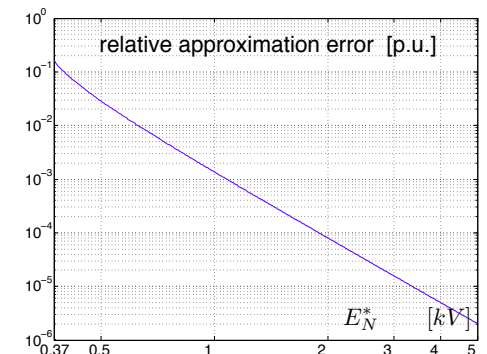
verification via IEEE 37 bus distribution system (SoCal)



Reactive DC approximation [B. Gentile,

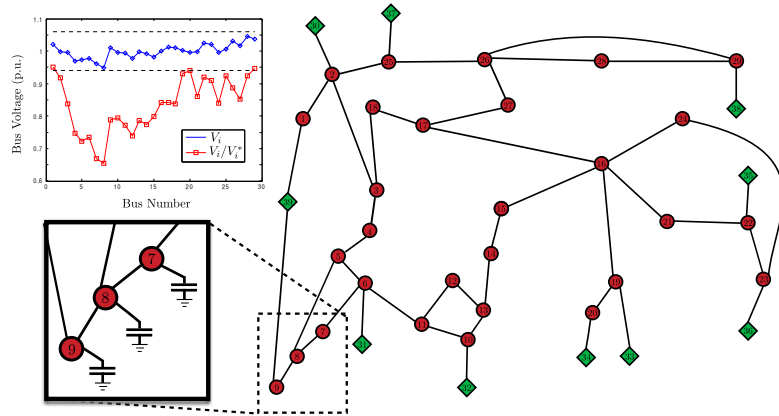
J. Simpson-Porco, FD, S. Zampieri, & F. Bullo, '14]:

$$E_L \approx \text{diag}(E_L^*) \left(\mathbb{1} + \frac{1}{4} Q_{\text{crit}}^{-1} Q_L \right) + \text{h.o.t.}$$



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Discrete control actions for voltage stability

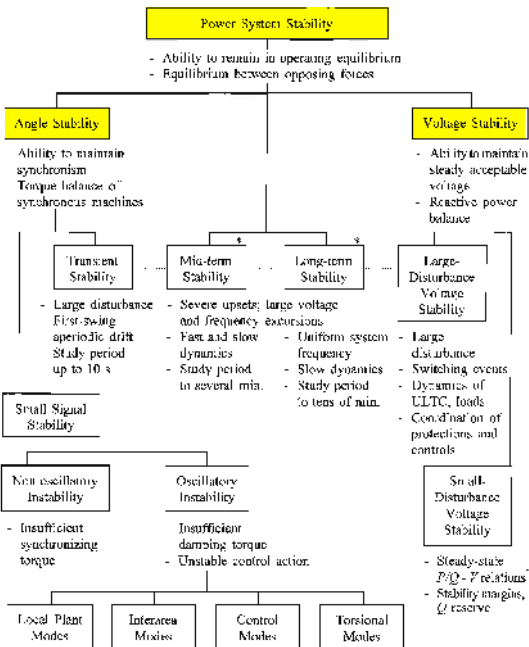


- 1 shunts support voltage magnitudes, but hide proximity to collapse
 \Rightarrow ratios E_i/E_i^* more useful than per-unit voltages
- 2 $|Q_{crit,89}^{-1}| > |Q_{crit,87}^{-1}|$ means E_8/E_8^* more sensitive to Q_9 than to Q_7
 \Rightarrow place SVC at bus 9 to support E_8 & increase stability margin.

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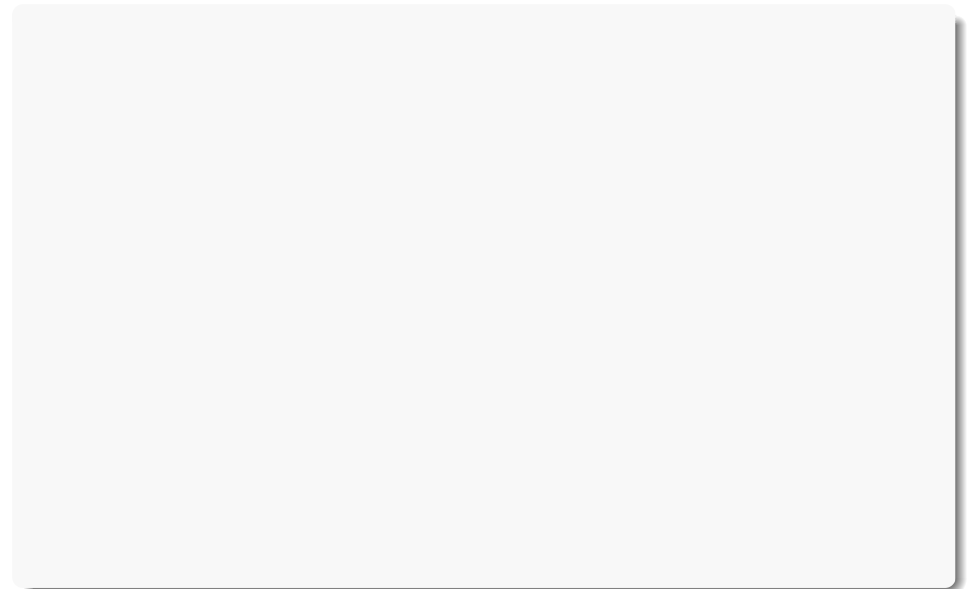
Coupled & Lossy Power Flow

This is not even really on the map



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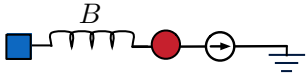
Solving the two-node case on blackboard



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Simplest example shows surprisingly complex behavior

- PV source, PQ load, & lossless line



$$P = B E_{\text{source}} E_{\text{load}} \sin(\theta)$$

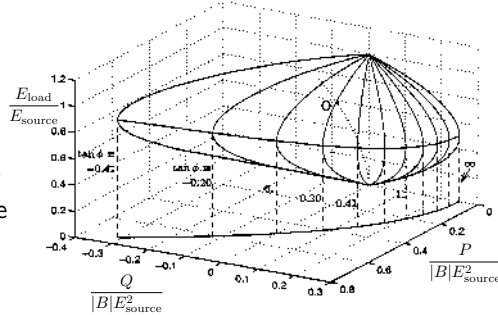
$$Q = B E_{\text{load}}^2 - B E_{\text{source}} E_{\text{load}} \cos(\theta)$$

- after eliminating θ , there exists $E_{\text{load}} \in \mathbb{R}_{\geq 0}$ if and only if

$$P^2 - B E_{\text{source}}^2 Q \leq B^2 E_{\text{source}}^4 / 4$$

- Observations:

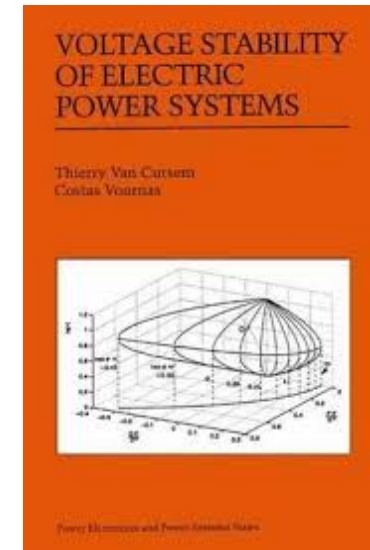
- 1 $P = 0$ case consistent with previous decoupled analysis
- 2 $Q = 0$ case delivers 1/2 transfer capacity from decoupled case
- 3 intermediate cases $Q = P \tan \phi$ give so-called “nose curves”



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Recommended reading to understand a glimpse

at least once in a life-time you should read chapter 2 ...



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Coupled & lossy power flow in complex networks

- ▶ active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- ▶ reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

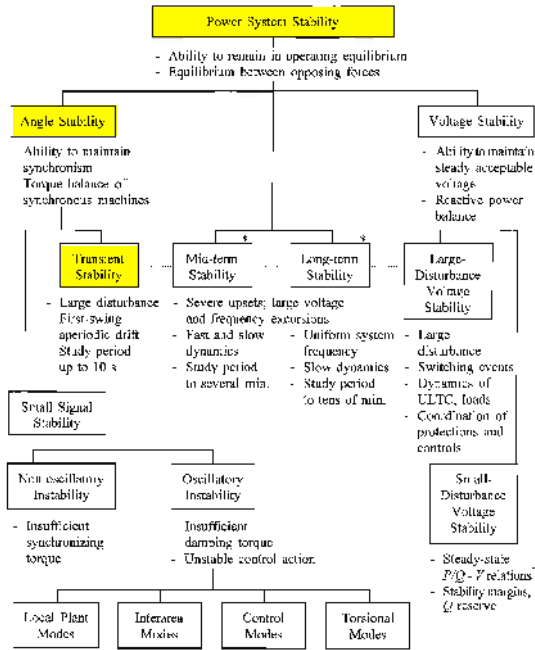
- what makes it so much harder than the previous two node case?
 - losses, mixed lines, cycles, PQ-PQ connections, ...
- much theoretic work, qualitative understanding, & numeric approaches:
 - existence of solutions [Thorp, Schulz, & Ilić '86, Wu & Kumagai '82]
 - solution space [Hiskens & Davy '01, Overbye & Klump '96, Van Cutsem '98, ...]
 - distance-to-failure [Venikov '75, Abe & Isono '76, Dobson '89, Andersson & Hill '93, ...]
 - convex relaxation approaches [Molzahn, Lesieutre, & DeMarco '12]
- little analytic & quantitative understanding beyond the two-node case

“Whoever figures that one out wins a noble prize!” Pete Sauer

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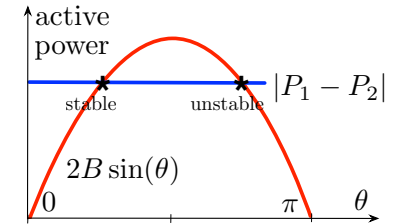
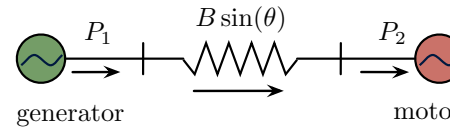
Transient Rotor Angle Stability

The crown jewel of power system stability



Revisit of the two-node case — the forced pendulum

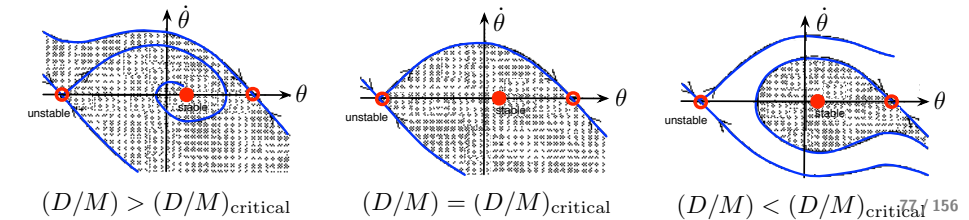
more complex than anticipated



$$\dot{\theta} = \omega$$

$$M\dot{\omega} = -D\omega + P_1 - P_2 - 2B \sin(\theta)$$

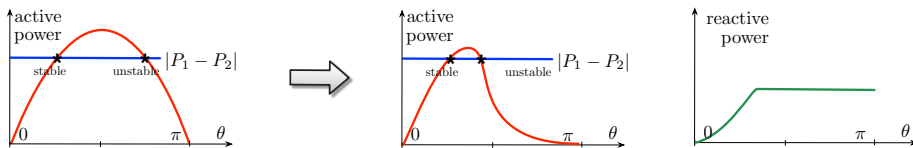
- **Local stability:** \exists local stable solution $\Leftrightarrow B > |P_1 - P_2|/2$
- **Global stability:** depends on gap $B > |P_1 - P_2|/2$ and D/M ratio



Revisit of the two-node case — cont'd

the story is not complete ... some further effects that we swept under the carpet

- **Voltage reduction:** to maintain a constant voltage, a generator needs to provide reactive power. When encountering the maximum reactive power support, the generator becomes a PQ bus and voltage drops.

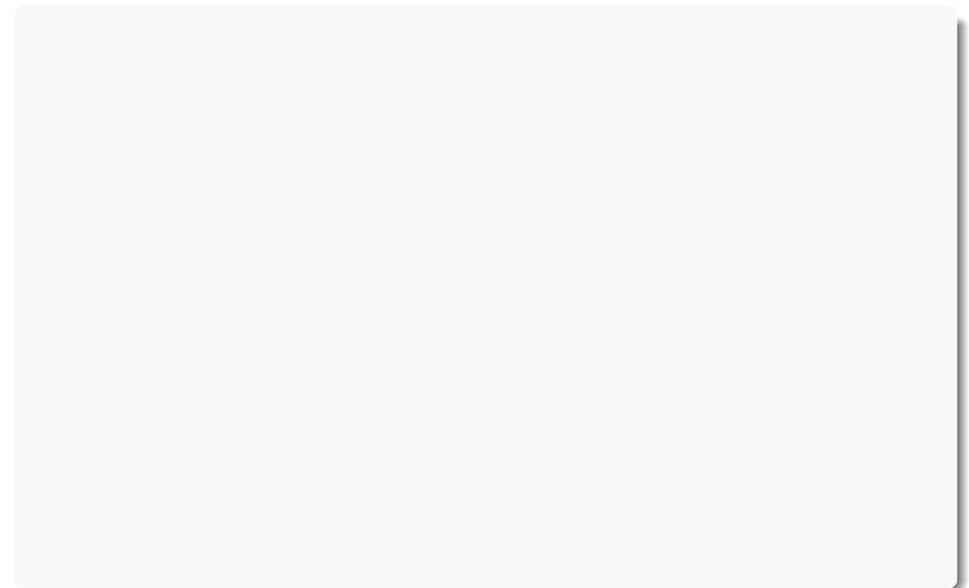


- **Load sensitivity:** different behavior depending on load model: resistive, constant power, frequency-dependent, dynamic, power electronics, ...
- **Singularity-issues** for coupled power flows (load voltage collapse)
- **Losses & higher-order dynamics** change stability properties ...

⇒ quickly run into computational approaches

Primer on Lyapunov functions

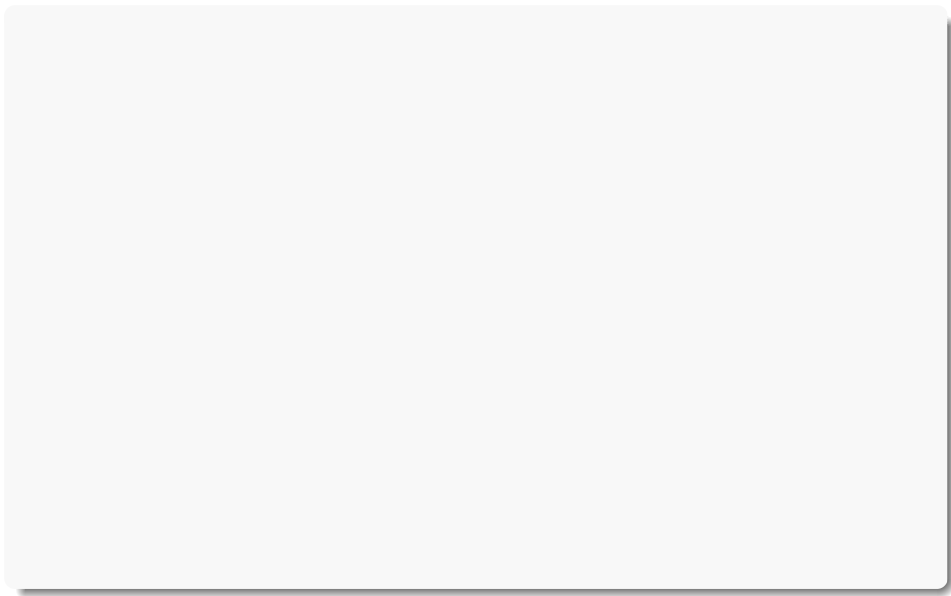
on blackboard



Hamiltonian analysis of the swing equations

more famously known as "energy function analysis"

(on blackboard)



Transient stability in multi-machine power systems

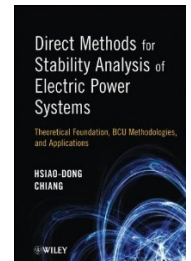
$$\dot{\theta}_i = \omega_i$$

generators: $M_i \dot{\omega}_i = -D_i \omega_i + P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$

$$Q_i = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

$$D_i \dot{\theta}_i = P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

loads: $Q_i = - \sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$



Challenge (improbable): faster-than-real-time transient stability assessment

Energy function methods for simple lossless models via Lyapunov function

$$V(\omega, \theta, E) = \sum_i \frac{1}{2} M_i \omega_i^2 - \sum_i P_i \theta_i - \sum_i Q_i \log E_i - \sum_{ij} B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

Computational approaches: level sets of energy functions & unstable equilibria, sum-of-squares methods, convex optimization approaches, time-domain simulations, . . . (holy grail of power system stability)

Outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

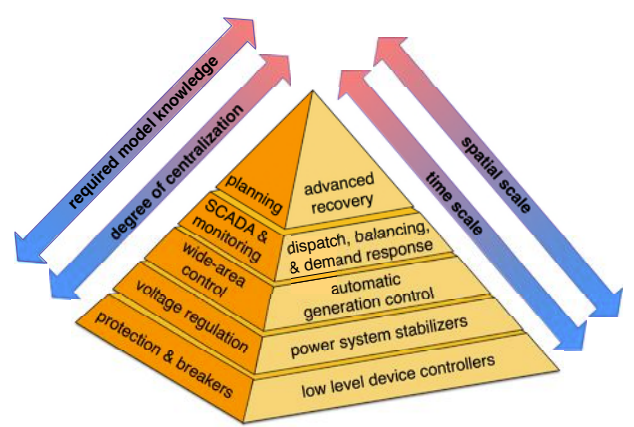
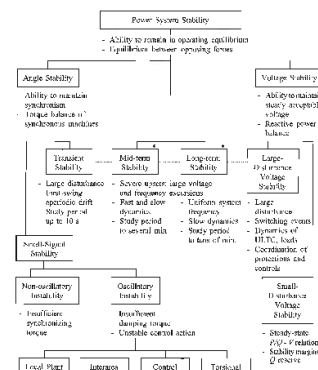
Power System Control Hierarchy

- Primary Control
- Power Sharing
- Secondary control
- Experimental validation

Power System Oscillations

Conclusions

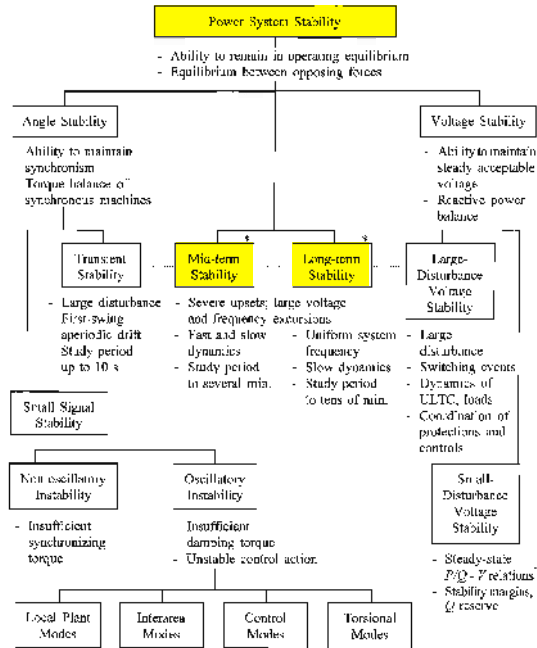
A plethora of control tasks and nested control layers organized in hierarchy and separated by states & spatial/temporal/centralization scales



We will focus on frequency control & primary/secondary/tertiary layers.

All dynamics & controllers are interacting. Classification & hierarchy are for simplicity.

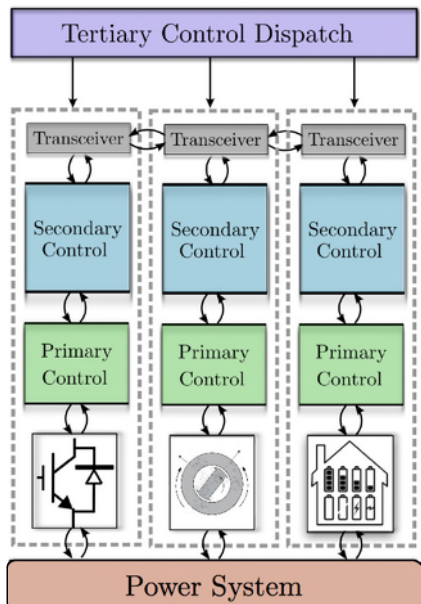
Where are we on the map?



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Objectives

Hierarchical frequency control architecture & objectives



3. **Tertiary control** (offline)
 - Goal: optimize operation
 - Strategy: centralized & forecast
2. **Secondary control** (minutes)
 - Goal: maintain operating point in presence of disturbances
 - Strategy: centralized
1. **Primary control** (real-time)
 - Goal: stabilize frequency & share unknown load
 - Strategy: decentralized

Q: Is this layered & hierarchical architecture still appropriate for tomorrow's power system?

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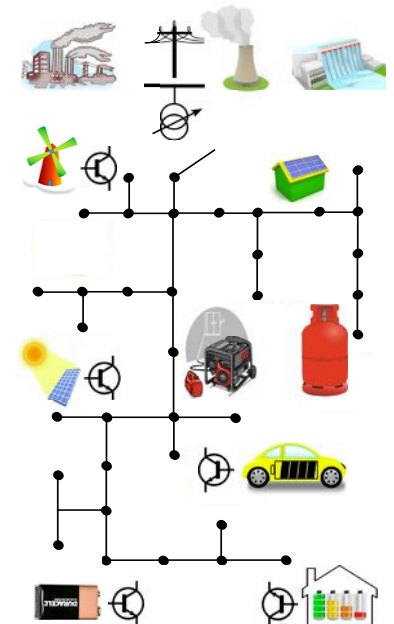
Is this hierarchical control architecture still appropriate?

Some recent developments

- ▶ increasing renewable integration & deregulated energy markets
- ▶ bulk generation replaced by distributed generation
- ▶ synchronous machines replaced by power electronics sources
- ▶ low gas prices & substitutions

Some new problem scenarios

- ▶ alternative spinning reserves: storage, load control, & DER
- ▶ networks of low-inertia & distributed renewable sources
- ▶ small-footprint islanded systems



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Need to adapt the control hierarchy in tomorrow's grid

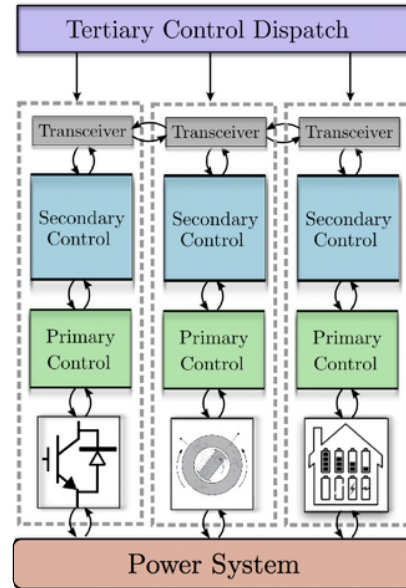
Operational challenges

- ▶ more uncertainty & less inertia
- ▶ more volatile & faster fluctuations
- ▶ plug'n'play control: fast, model-free, & without central authority

Opportunities

- ▶ re-instrumentation: comm & sensors
- ▶ more & faster spinning reserves
- ▶ advances in control of cyber-physical & complex systems

⇒ break vertical & horizontal hierarchy



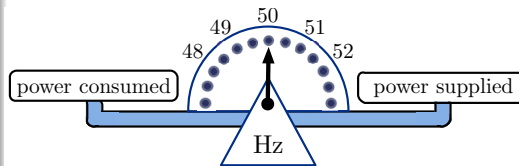
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Primary Control

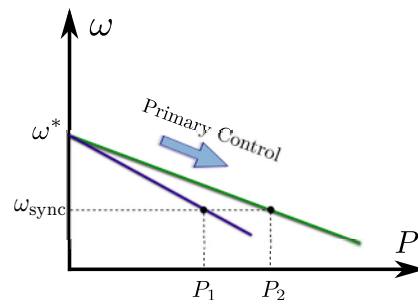
Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$



recall: $\omega_{\text{sync}} = \sum_i P_i^* / D_i$



$P/\dot{\theta}$ droop control:

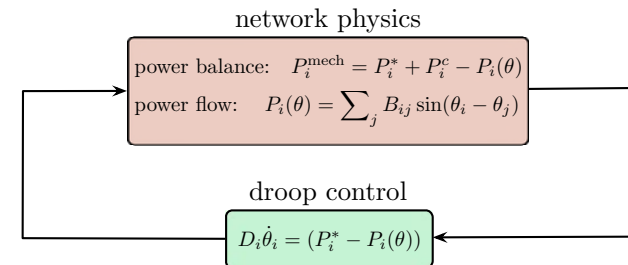
$$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$$

⇕

$$D_i \dot{\theta}_i = P_i^* - P_i(\theta)$$

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Putting the pieces together...



synchronous machines:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

inverter sources & controllable loads:

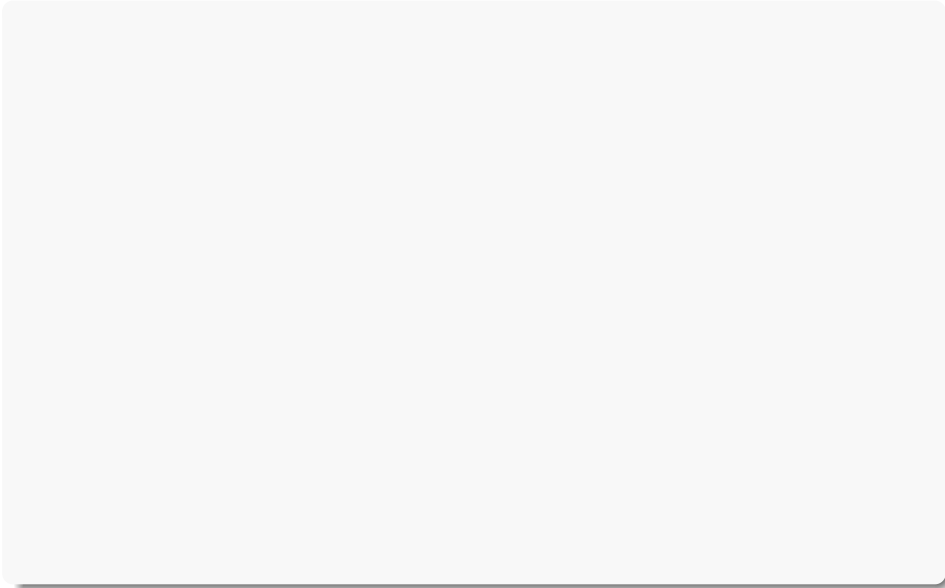
$$D_i \dot{\theta}_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

passive loads & power-point tracking sources:

$$0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$$

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Closed-loop stability?



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Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

\exists unique & exp. stable frequency sync \iff active power flow is feasible

Main **proof ideas** and some **further results**:

- stability via Jacobian arguments (as before)

- synchronization frequency:
(\propto power balance)
$$\omega_{\text{sync}} = \omega^* + \frac{\sum_{\text{sources}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{sources}} D_i}$$

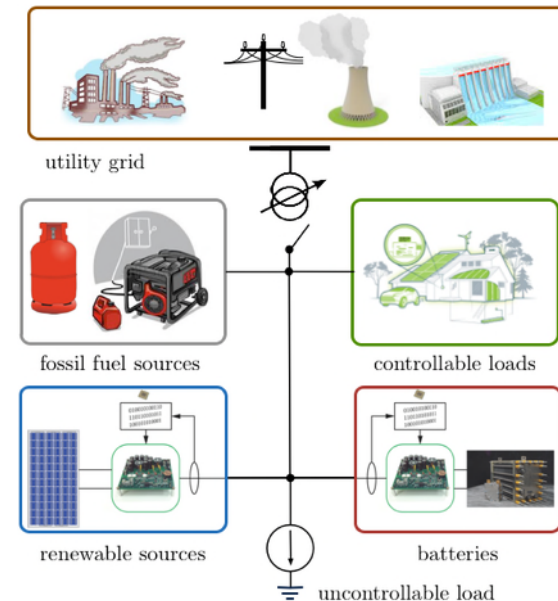
- steady-state power injections:
(depend on D_i & P_i^*)
$$P_i = \begin{cases} P_i^* & (\text{load } \#i) \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & (\text{source } \#i) \end{cases}$$

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**power sharing &
economic optimality
under droop control**
(sometimes in tertiary layer)

Tertiary control and energy management

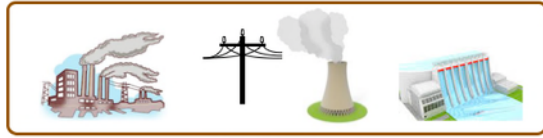
an offline resource allocation and scheduling problem



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Tertiary control and energy management

an offline resource allocation and scheduling problem



utility grid



minimize {cost of generation, losses, ... }

subject to

equality constraints: power balance equations

inequality constraints: flow/injection/voltage constraints

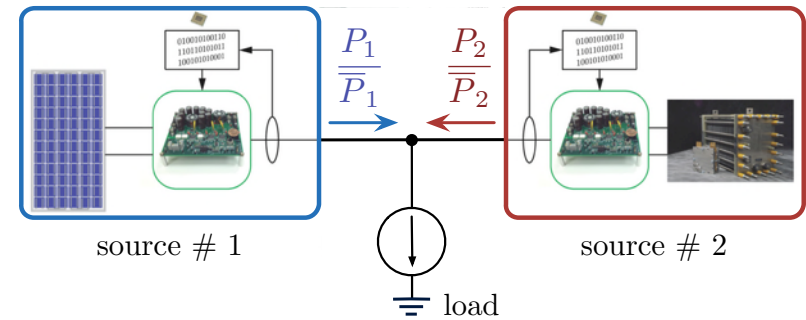
logic constraints: commit generators yes/no

⋮

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Objective I: decentralized proportional load sharing

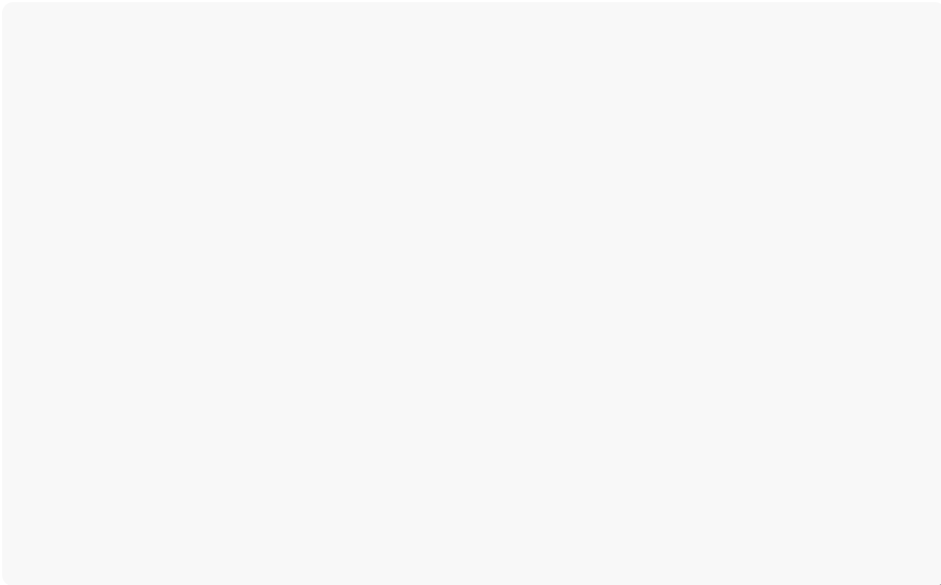
- 1) Sources have **injection constraints**: $P_i(\theta) \in [0, \bar{P}_i]$
- 2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \bar{P}_j$
- 3) **Fairness**: load should be shared proportionally: $P_i(\theta) / \bar{P}_i = P_j(\theta) / \bar{P}_j$



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Analysis of fair proportional load sharing

on blackboard



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Objective I: decentralized proportional load sharing

- 1) Sources have **injection constraints**: $P_i(\theta) \in [0, \bar{P}_i]$
- 2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \bar{P}_j$
- 3) **Fairness**: load should be shared proportionally: $P_i(\theta) / \bar{P}_i = P_j(\theta) / \bar{P}_j$

Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$D_i / \bar{P}_i = D_j / \bar{P}_j \quad \& \quad P_i^* / \bar{P}_i = P_j^* / \bar{P}_j$$

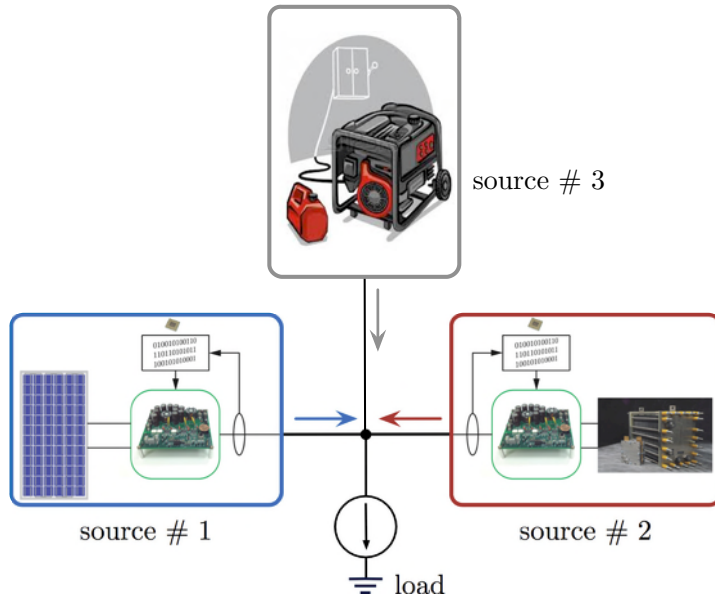
The the following statements hold:

- (i) Proportional load sharing: $P_i(\theta) / \bar{P}_i = P_j(\theta) / \bar{P}_j$
- (ii) Constraints met: $0 \leq \left| \sum_{\text{loads}} P_j^* \right| \leq \sum_{\text{sources}} \bar{P}_j \Leftrightarrow P_i(\theta) \in [0, \bar{P}_i]$

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Objective I: fair proportional load sharing

proportional load sharing is not always the right objective



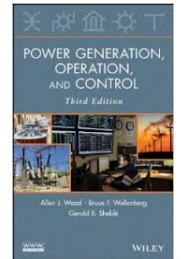
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Objective II: optimal power flow = tertiary control

an offline resource allocation/scheduling problem

minimize {cost of generation, losses, ... }
 subject to
 equality constraints: power balance equations
 inequality constraints: flow/injection/voltage constraints
 logic constraints: commit generators yes/no
 ⋮

Will be discussed more in detail tomorrow.



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Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

minimize $\theta \in \mathbb{T}^n, u \in \mathbb{R}^{n_l}$ $f(u) = \sum_{\text{sources}} \alpha_i u_i^2$
 subject to
 source power balance: $P_i^* + u_i = P_i(\theta)$
 load power balance: $P_i^* = P_i(\theta)$
 branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

An even simpler problem formulation:

minimize $\theta \in \mathbb{T}^n, u \in \mathbb{R}^{n_l}$ $f(u) = \sum_{\text{sources}} \alpha_i u_i^2$
 subject to
 power balance: $\sum_i P_i^* + \sum_i u_i = 0$

Both are equivalent in the strictly feasible case!

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Both are equivalent in the strictly feasible case

... and marginal costs are identical: $\alpha_i u_i^* = \alpha_j u_j^*$ (on blackboard)

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Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

$$\text{minimize } \theta \in \mathbb{T}^n, u \in \mathbb{R}^n \quad f(u) = \sum_{\text{sources}} \alpha_i u_i^2$$

subject to

$$\text{source power balance:} \quad P_i^* + u_i = P_i(\theta)$$

$$\text{load power balance:} \quad P_i^* = P_i(\theta)$$

$$\text{branch flow constraints:} \quad |\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$$

Unconstrained case: identical marginal costs $\alpha_i u_i^* = \alpha_j u_j^*$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

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Objective II: decentralized dispatch optimization

Insight: droop-controlled system = decentralized primal/dual algorithm

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

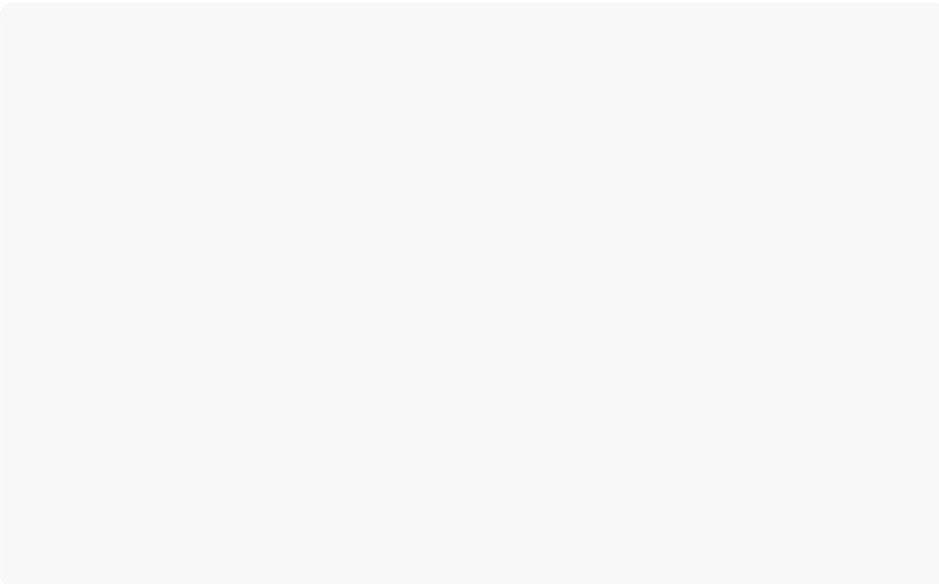
- the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer (θ^*, u^*) .
- \exists droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ .

If (i) & (ii) are true, then $\theta_i \sim \theta_i^*$, $u_i^* = -D_i(\omega_{\text{sync}} - \omega^*)$, & $D_i \alpha_i = D_j \alpha_j$.

- includes proportional load sharing $\alpha_i \propto 1/\bar{P}_i$
- similar results hold for strictly convex cost & general **constrained** case

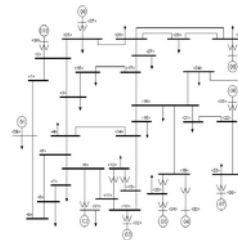
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Sketch of the main proof ideas

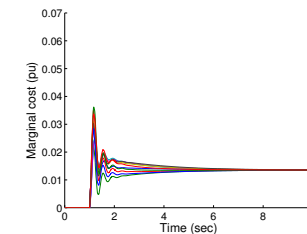


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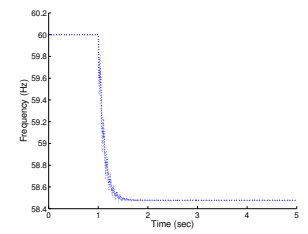
Some quick simulations & extensions



IEEE 39 New England
with load step at 1s



$t \rightarrow \infty$: convergence to
identical marginal costs



$t \rightarrow \infty$: frequency
 \propto power imbalance

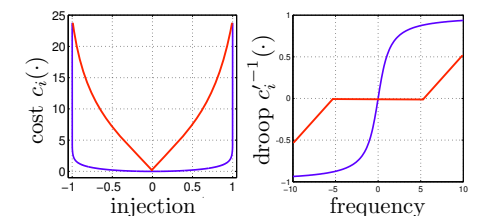
\Rightarrow strictly convex & differentiable cost

$$f(u) = \sum_{\text{sources}} c_i(u_i)$$

\Rightarrow non-linear frequency droop curve

$$c_i'^{-1}(\hat{\theta}_i) = P_i^* - P_i(\theta)$$

\Rightarrow include dead-bands, saturation, etc.

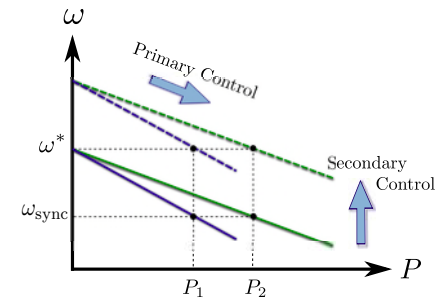


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Secondary Control

Secondary frequency control

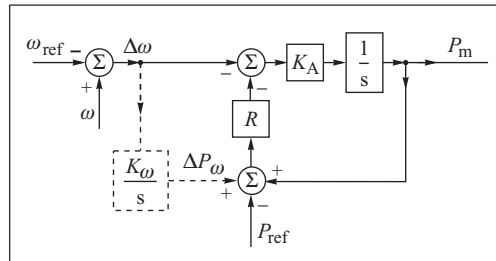
- **Problem:** steady-state frequency deviation ($\omega_{\text{sync}} \neq \omega^*$)
- **Solution:** integral control of frequency error
- **Basics** of integral control $\left[\frac{1}{s}\right]$:



- 1 discrete time: $u_i(t+1) = u_i(t) + k \cdot \dot{\theta}_i(t)$ with gain $k > 0$
 - 2 continuous-time: $u_i(t) = k \cdot \int_0^t \dot{\theta}_i(\tau) d\tau$ or $\dot{u}_i(t) = k \cdot \dot{\theta}_i(t)$
- ⇒ $\dot{\theta}_i(t)$ is zero in (a possibly stable) steady state
- ⇒ add additional injection $u_i(t)$ to droop control

Decentralized secondary integral frequency control

- $\left[\frac{1}{s}\right]$ add local integral controller to every droop controller
- ⇒ stable closed-loop & zero frequency deviation ✓
- ⇒ sometimes globally stabilizing [C. Zhao, E. Mallada, & FD, '14] ✓

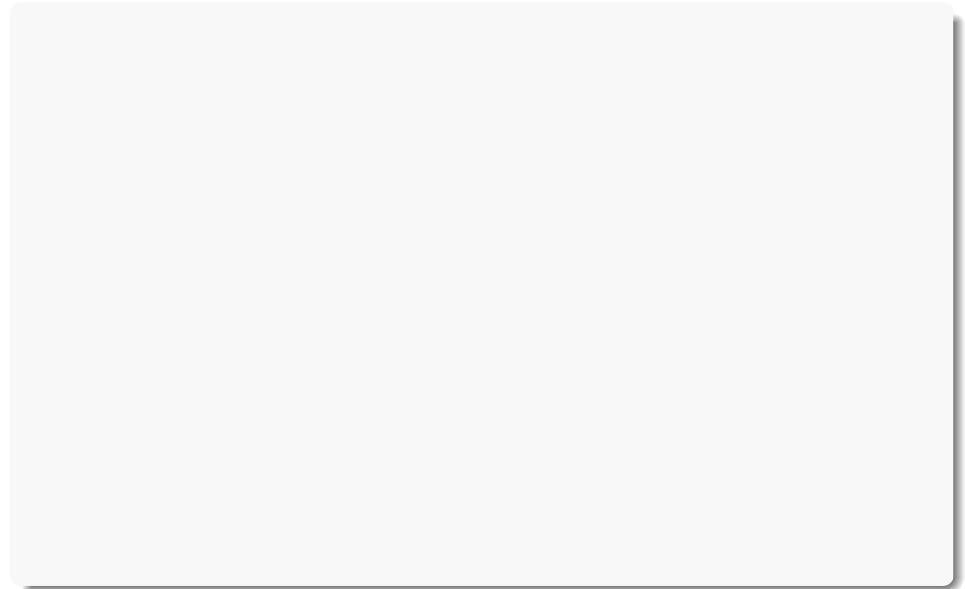


turbine governor integral control loop



Why does decentralized integral control not work?

on blackboard



- ☹ every integrator induces a 1d equilibrium subspace
- ☹ injections live in subspace of dimension \neq integrators
- ☹ load sharing & economic optimality are lost ...

Automatic generation control (AGC)

- **ACE** area control error =
 $\{ \text{frequency error} \} +$
 $\{ \text{generation - load - tie-line flow} \}$

$\frac{1}{s}$

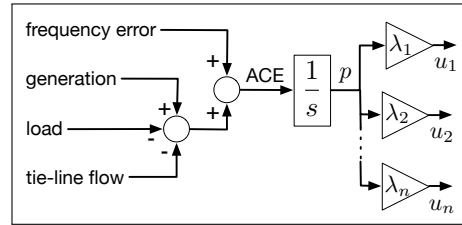
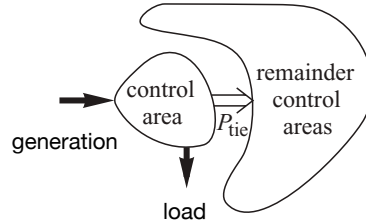
centralized integral control:

$$p(t) = \int_0^t \text{ACE}(\tau) d\tau$$

- **generation allocation:**
 $u_i(t) = \lambda_i p(t)$, where λ_i is
generation participation factor
(in our case $\lambda_i = 1/\alpha_i$)

⇒ assures identical marginal
costs: $\alpha_i u_i = \alpha_j u_j$

- 😊 load sharing & economic
optimality are recovered



AGC implementation

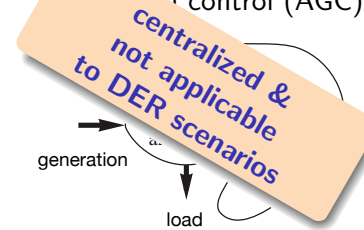
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Drawbacks of conventional secondary frequency control

Interconnected Systems

Isolated Systems

- centralized automatic
generation control (AGC)



- decentralized PI control
-

Distributed energy resources require **distributed (!)** secondary control.

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An incomplete literature review of a busy field

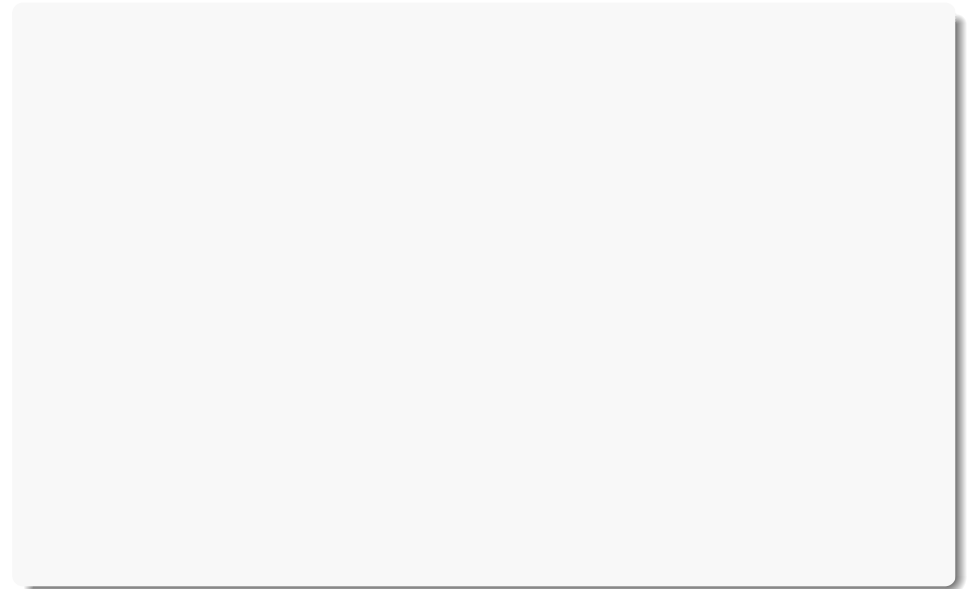
ntwk with unknown disturbances \cup integral control \cup distributed averaging

- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcia-Dominguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]

The following idea precedes most references, it's simpler, & it's more robust.

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Let's derive a simple distributed control strategy

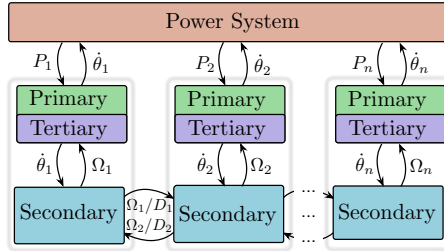


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Distributed Averaging PI (DAPI) control

$$D_i \dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i \dot{\Omega}_i = D_i \dot{\theta}_i - \sum_{j \in \text{sources}} a_{ij} \cdot (\alpha_i \Omega_i - \alpha_j \Omega_j)$$

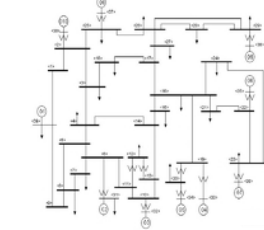


Theorem: stability of DAPI
 [J. Simpson-Porco, FD, & F. Bullo, '12]
 [C. Zhao, E. Mallada, & FD '14]

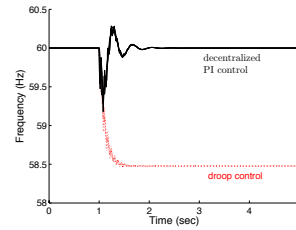
primary droop controller works \iff secondary DAPI controller works

- no tuning & no time-scale separation: $k_i, D_i > 0$
 - distributed & modular: connected comm. \subseteq sources
 - recovers primary op. cond. (load sharing & opt. dispatch)
- \Rightarrow plug'n'play implementation

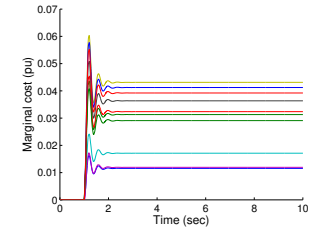
Simulations cont'd



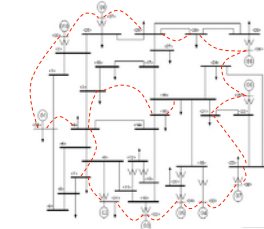
IEEE 39 New England with decentralized PI control



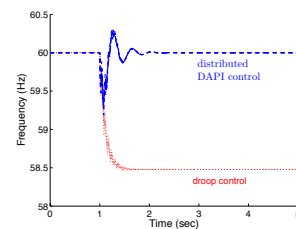
$t \rightarrow \infty$: decentralized PI control regulates frequency



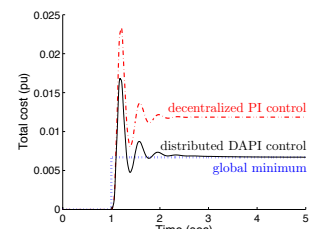
$t \rightarrow \infty$: decentralized PI control is not optimal



IEEE 39 New England with distributed DAPI control



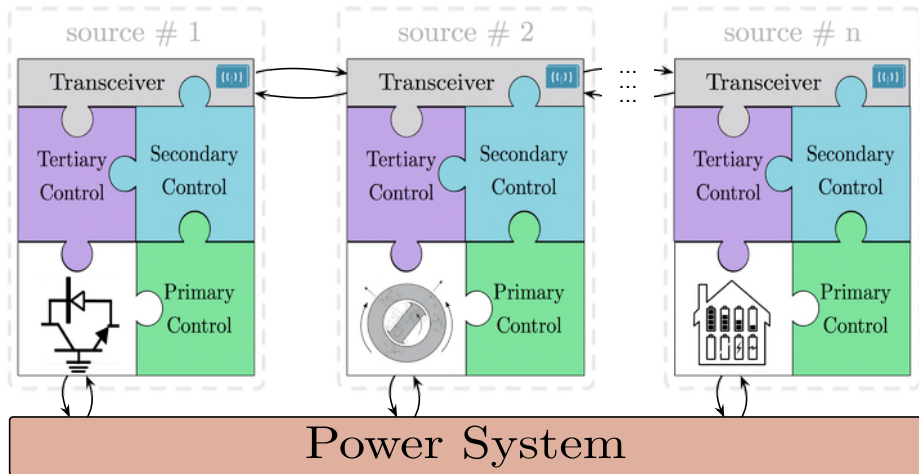
$t \rightarrow \infty$: DAPI control regulates frequency



DAPI control minimizes cost with little effort

Plug'n'play architecture

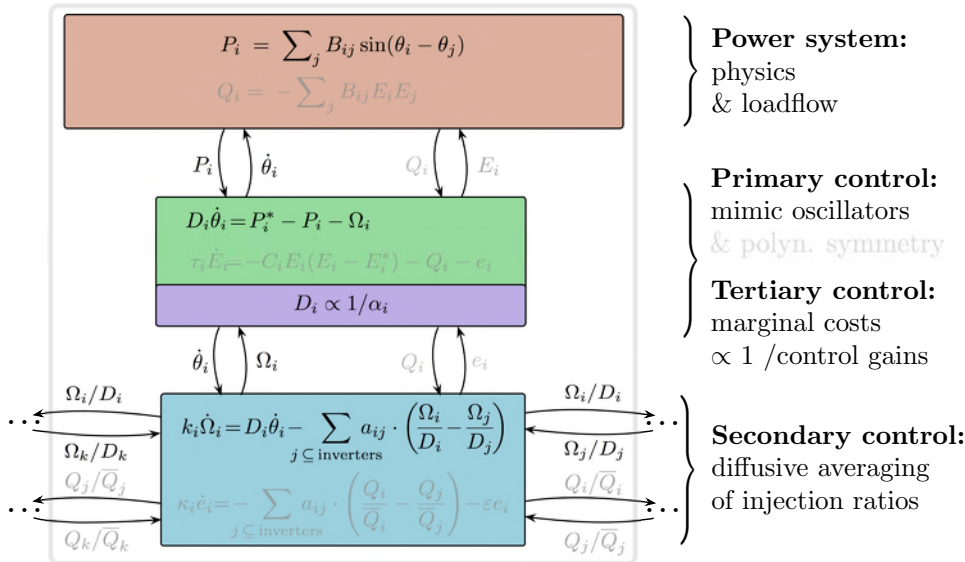
flat hierarchy, distributed, no time-scale separations, & model-free



plug-and-play experiments

Plug'n'play architecture

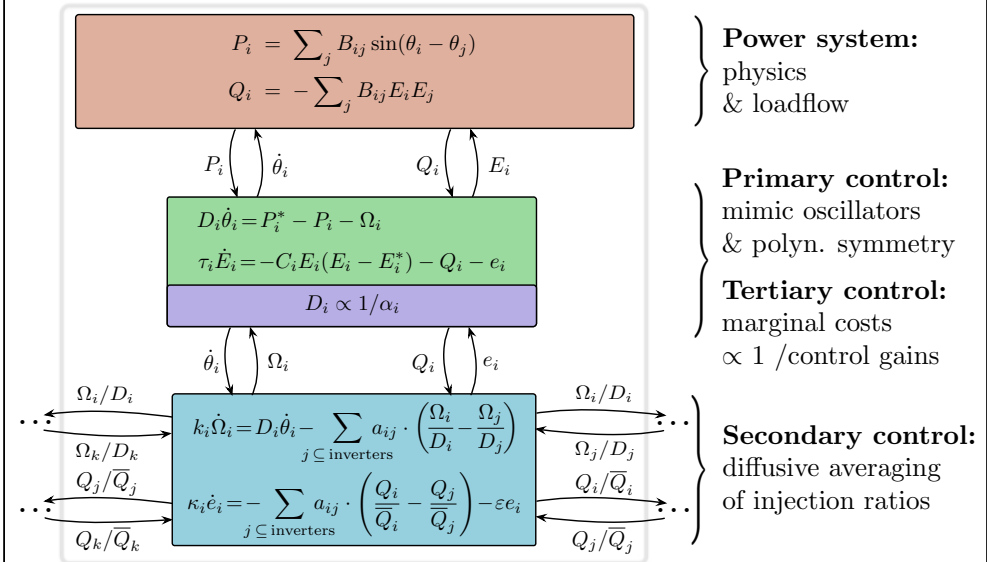
recap of detailed signal flow (active power only)



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Plug'n'play architecture

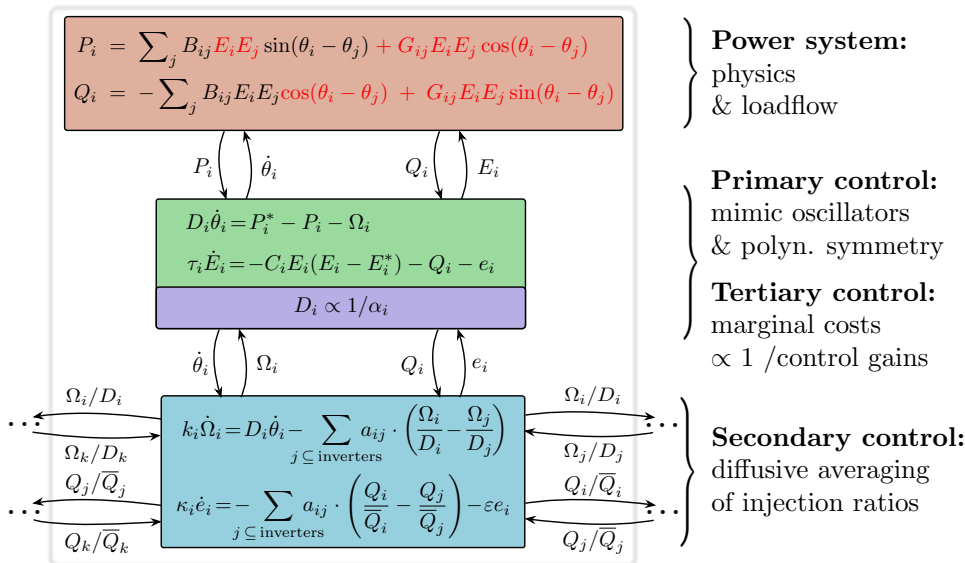
recap of detailed signal flow (with reactive power)



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Plug'n'play architecture

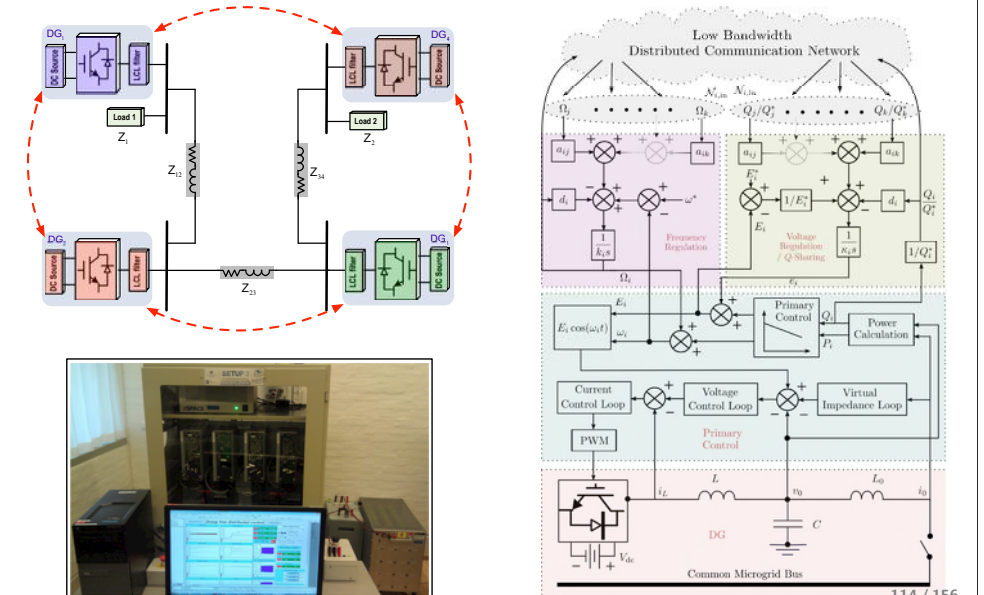
experiments also work well in the coupled & lossy case



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Experimental validation of control & opt. algorithms

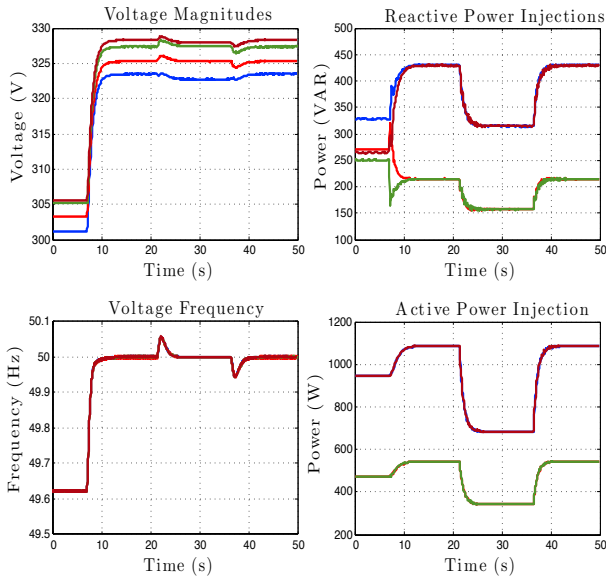
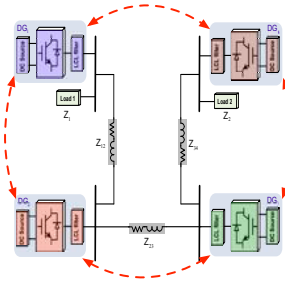
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University



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Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing



$t \in [0s, 7s]$: primary & tertiary control
 $t = 7s$: secondary control activated
 $t = 22s$: load # 2 unplugged
 $t = 36s$: load # 2 plugged back

Outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Causes for Oscillations

Slow Coherency Modeling

Inter-Area Oscillations & Wide-Area Control

Case Study: IEEE 39 New England Power Grid

Conclusions

There are also many exciting alternatives to droop control

Uncovering Droop Control Laws Embedded Within the Nonlinear Dynamics of Van der Pol Oscillators

Mohit Sinha, Florian Dörfler, Member, IEEE, Brian B. Johnson, Member, IEEE, and Saijy V. Dhoople, Member, IEEE

Abstract—This paper examines the dynamics of power electronic inverters in islanded microgrids that are controlled to emulate the dynamics of Van der Pol oscillators. The general strategy of controlling inverters to emulate the behavior of nonlinear oscillators presents a compelling time-domain alternative to ubiquitous droop control methods which presume the existence of a quasi-steady-state sinusoidal steady state and operate on phase quantities. We present two main results in this work. First, by leveraging the method of periodic averaging, we demonstrate that droop laws are intrinsically embedded within a broader class of nonlinear Van der Pol oscillators. Second, we establish the global convergence of amplitude and phase dynamics in a nonlinear autonomous network of inverters controlled as Van der Pol oscillators. Furthermore, under a set of nonrestrictive decoupling approximations, we derive sufficient conditions for local asymptotic stability of desirable equilibria of the linearized amplitude and phase dynamics.

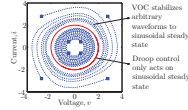


Figure 1. VCC stabilizes arbitrary initial conditions in a sinusoidal steady state, while droop control acts on phase quantities, only well defined in the sinusoidal steady state. The condition of the work is to determine a set of parabolic correspondences such that both approaches admit identical dynamics in sinusoidal steady state.

I. INTRODUCTION
 A n islanded inverter-based microgrid is a collection of fast-response DC energy resources, e.g., photovoltaic (PV) arrays, fuel cells, and energy-storage devices, interfaced to an AC electric distribution network and operated independently from the bulk power system. Energy conversion is typically

Voltage and frequency control of islanded microgrids: a plug-and-play approach

Stefano Rivera¹, Fabio Sarzo¹ and Giancarlo Ferrari-Trecate¹

¹Dipartimento di Ingegneria Industriale e dell'Informazione, Università degli Studi di Pavia, info@rivera.unipv.it, Corresponding author

Abstract—In this paper we propose a new decentralized control scheme for islanded microgrids (MGs) composed by the interconnection of Distributed Generation Units (DGUs). Local controllers regulate voltage and frequency at the Point of Common Coupling (PCC) of each DGU and they are able to guarantee stability of the overall MG. The control design procedure is decentralized, since, besides the global scalar quantities, the synthesis of a local controller uses only information on the corresponding DGU and lines connected to it. Most importantly, our design procedure enables Plug-and-Play (PnP) operations, where a DGU is plugged in or out, only DGUs physically connected to it have to re-tune their local controllers. We study the performance of the proposed controllers simulating different scenarios in Matlab/Simulink and using indices proposed in IEEE standards.

I. INTRODUCTION
 In recent years, research on islanded microgrids (MGs) has received major attention. MGs are self-sufficient microgrids composed by several Distributed Generation Units (DGUs) and designed to operate safely and reliably in absence of a connection with the main grid. Besides fostering the use of renewable generation, MGs help distributed generation

control of droop control, this problem has been investigated only recently [7]. For regulators not based on droop control, almost all studies focused on radial microgrids (i.e. a DGU is connected to at most two other DGUs) while control of MGs with meshed topology is still largely unexplored [2]. In this paper we consider the design of decentralized regulators for meshed MGs with a view on decentralization of the synthesis procedure. More specifically, we develop a Plug-and-Play (PnP) design algorithm where the synthesis of a local controller for a DGU requires parameters of transmission lines connected to it, the knowledge of two global scalar quantities, but no specific information about any other DGUs physically connected to it have to re-tune their local controllers. PnP control design for general linear constrained systems has been proposed in [8], [9]. PnP design for MGs is however different since it is based on the concept of neutral interactions [10] rather than on robustness against subsystem coupling. Furthermore, for achieving neutral interactions among DGUs, we exploit Quasi-Stationary Lines (QSL) approximations of line dynamics [11].

Synchronization of Nonlinear Oscillators in an LTI Electrical Power Network

Brian B. Johnson, Member, IEEE, Saijy V. Dhoople, Member, IEEE, Abdallah O. Hamadeh, and Philip T. Koenig, Fellow, IEEE

Abstract—Sufficient conditions are derived for the global asymptotic synchronization of a class of identical nonlinear oscillators coupled through a linear time-invariant network. In particular, we focus on systems where oscillators are connected to a common node through identical branch impedances. For such networks, it is shown that the synchronization condition is independent of the number of oscillators and the value of the load impedance connected to the common node. Theoretical findings are then leveraged to control a system of parallel single-phase voltage source inverters serving an impedance load in an islanded microgrid application. The resulting primary energy sources with the power line. A key challenge introduced by AC power supplies with fast dynamics is that an interconnection electrical network containing inductive and/or capacitive components cannot be regarded as simply enforcing algebraic constraints between currents and voltages in the phase domain and, instead, must be treated as a dynamical system.

I. INTRODUCTION
 Synchronization of coupled oscillators is relevant to several research areas including neural processes, coherence in plasma physics, communications, and electronic circuits [1]–[7]. This paper presents a sufficient condition

a decentralized power system composed of parallel voltage source inverters serving a passive electrical load. Relevant to this work is a body of literature that has examined synchronization conditions for diffusively coupled oscillators using passivity theory [8]–[13]. For instance, in [13], the notion of passivity and incremental passivity [9]–[12] were used to establish synchronization conditions that were applied to the control of inverters as nonlinear oscillators in a power system. Passivity-based approaches require the formulation of a storage function, which can be difficult when the network contains energy-storage circuit elements such as inductors and capacitors. Since power networks are in general composed of a variety of LTI circuit elements (resistors, capacitors, inductors, and transformers), passivity-based approaches are difficult to apply in such systems. In this work, we use C_2 input-output stability methods, because they facilitate analysis in settings where storage functions are difficult to formulate. Our approach derives from previous work in [14]–[16] where C_2 methods were used to analyze synchronization in feedback systems. To prove synchronization, we reformulate the dynamics of the original

Synchronization of Oscillators Coupled through a Network with Dynamics: A Constructive Approach with Applications to the Parallel Operation of Voltage Power Supplies

Leonardo A. B. Torres, Member, IEEE, João P. Hespahan, Fellow, IEEE, and Jeff Meelis

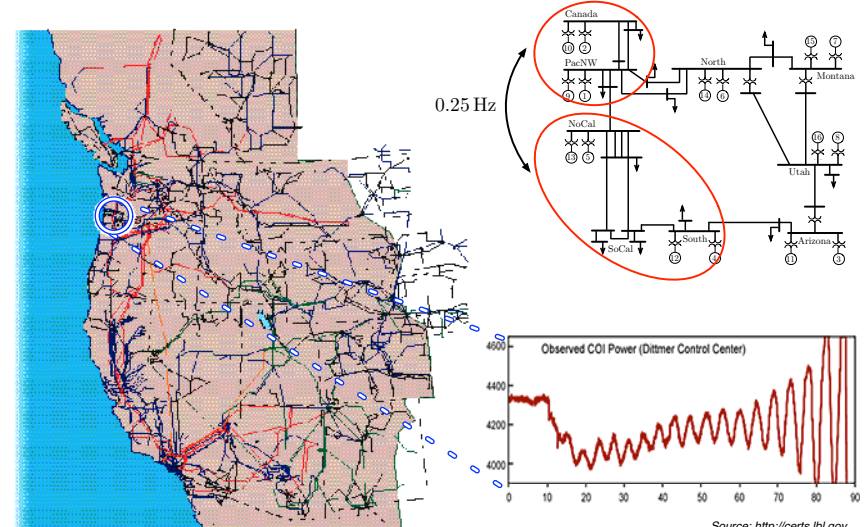
Abstract—We consider the problem of synchronizing a group of oscillators coupled by a network that is modeled by a multi-input-multi-output dynamical system. We provide results that can be used to establish asymptotic synchronization of a given system and also to construct feedback controllers for which synchronization is guaranteed. These results are based on a new notion of passivity with respect to measurable defined in the input and output spaces of a dynamical system. The problem under consideration is motivated by the design of high-power electronic inverters that can be used to interface primary energy sources with an AC electrical distribution network. The proposed synchronization strategy is applied to the problem of parallel connected voltage power supplies.

I. INTRODUCTION
 This paper addresses the synchronization of identical oscillators connected through a network represented by a dynamical system as shown in Figure 1. A key motivation for this

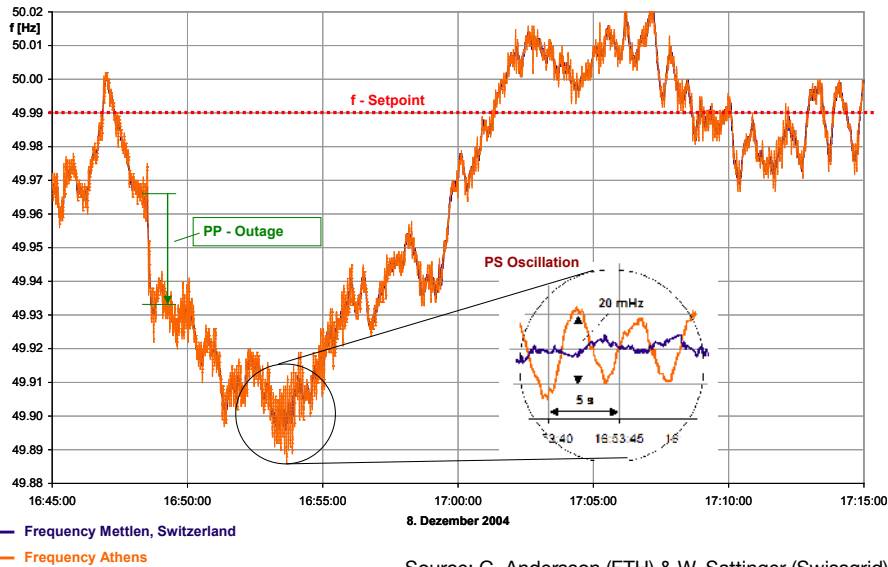
generators connected to a local power grid in an isolated community [7], [12], or the synchronization of multiple inverters providing energy to the same load [13]. In contrast with the mainstream in the power-synchronization literature, we are interested in very fast synchronization for which a phase-domain analysis is not valid. Fast synchronization is possible when power electronic devices (typically inverters) are used to interface primary energy sources with the power line. A key challenge introduced by AC power supplies with fast dynamics is that an interconnection electrical network containing inductive and/or capacitive components cannot be regarded as simply enforcing algebraic constraints between currents and voltages in the phase domain and, instead, must be treated as a dynamical system. Inspired by the work of [1, 8, 9, 20, 22] we use dissipation and passivity [25] as key analysis tools. The use of passivity is attractive because it allows one to establish passivity properties for a large network based on input-output properties of individual components. In the context of electrical networks,

Electro-Mechanical Oscillations in Power Networks

- **Dramatic consequences:** blackout of August 10, 1996, resulted from instability of the 0.25 Hz mode in the Western interconnected system

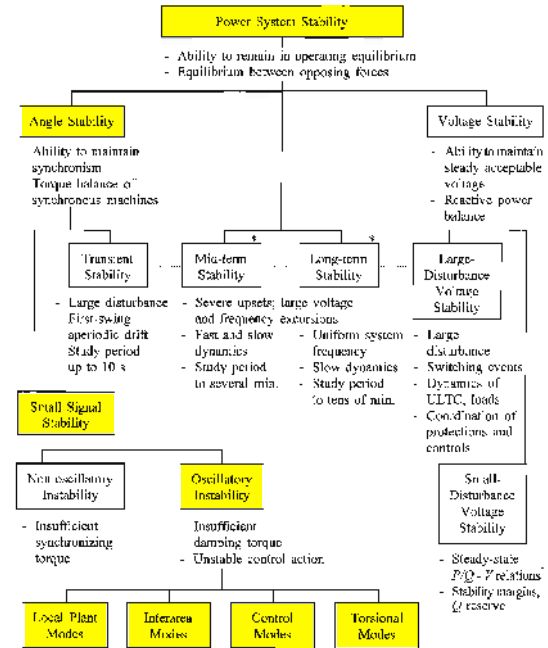


Less dramatic but quite common ... usually well behaved



Source: G. Andersson (ETH) & W. Sattinger (Swissgrid)

Where are we on the map?



Causes for Oscillations

Swing dynamics = coupled/forced/heterogeneous pendula

- Coarse-grained power network dynamics = generator **swing dynamics**:

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

- Swing equations **linearized** around an equilibrium $(\theta^*, \dot{\theta}^*, P^*)$:

$$M \ddot{\theta} + D \dot{\theta} + L \theta = P$$

M & $D \in \mathbb{R}^{n \times n}$ diagonal inertia and damping matrices

$L \in \mathbb{R}^{n \times n}$ Laplacian matrix with coupling $a_{ij} = E_i^* E_j^* B_{ij} \cos(\theta_i^* - \theta_j^*)$

$$L = \begin{bmatrix} \vdots & \ddots & \vdots & \ddots & \vdots \\ -a_{i1} & \cdots & \sum_{j=1}^n a_{ij} & \cdots & -a_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \end{bmatrix}$$

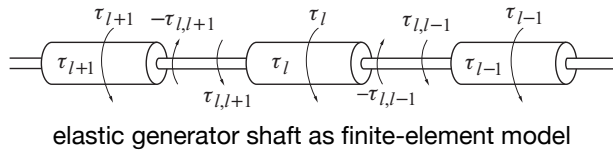
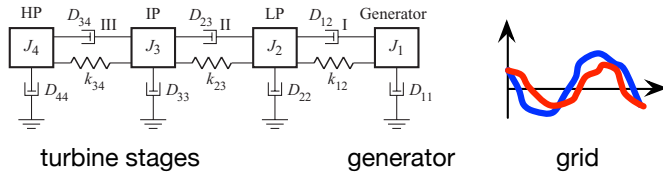
⇒ sparsely **coupled** & **forced** oscillators with **heterogeneous** frequencies

Torsional oscillations in power networks

essentially a (subsynchronous) resonance phenomenon

⇒ arise from interplay of

- electrical oscillations
- flexible mechanical shaft models
- generator-turbine coupling



⇒ subsynchronous resonance phenomena often arise in wind turbines 121 / 156

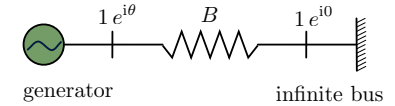
Local oscillations and their control

Automatic Voltage Regulator (AVR):

- objective: generator voltage = *const.*

⇒ diminishing damping & sync torque $\frac{\partial P}{\partial \theta}$

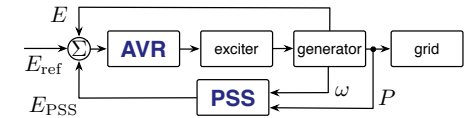
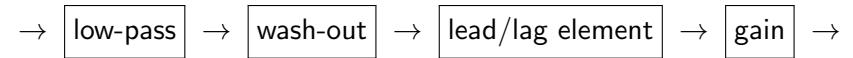
⇒ can result in oscillatory instability



Power System Stabilizer (PSS):

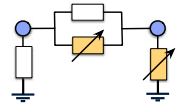
- objective: net damping positive

- typical control design:



Flexible AC Transmission Systems (FACTS) or HVDC:

- control by “modulating” transmission line parameters
- either connected in series with a line or as shunt device



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Control-induced oscillations and their control

- **short story:** multiple local controllers interact in an adverse way

- **system-theoretic reason:** power system has unstable zeros

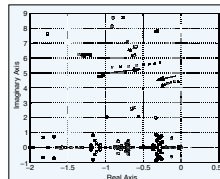
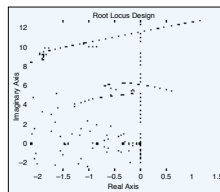
⇒ trade-off: high-gain (local stability) vs. low-gain control (avoid zeros)

⇒ numerous tuning rules & heuristics for decentralized PSS design

By Joe H. Chow, Juan J. Sanchez-Gasca, Haoxing Ren, and Shaopeng Wang

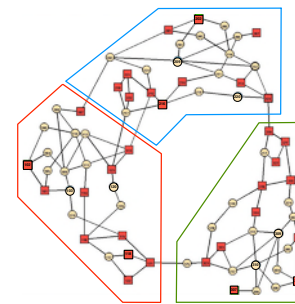


Power System Damping Controller Design
Using Multiple Input Signals

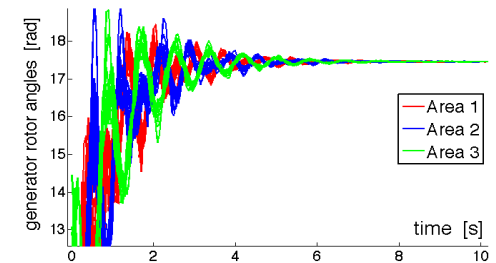


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Inter-area oscillations in power networks



RTS 96 power network



swing dynamics

Inter-area oscillations are caused by

- 1 **heterogeneity:** fast & slow responses (inertia M_i and damping D_i)
- 2 **topology:** internally strongly and externally sparsely connected areas
- 3 **power transfers** between areas: $a_{ij} = B_{ij} E_i^* E_j^* \cos(\theta_i^* - \theta_j^*)$
- 4 **interaction** of multiple local control loops (e.g., high gain PSSs)

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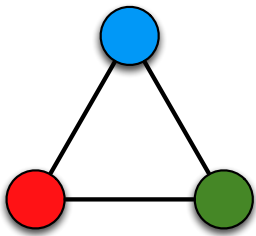
Taxonomy of electro-mechanical oscillations

- Synchronous generator = electromech. oscillator \Rightarrow **local oscillations**:
 - = single generator oscillates relative to the rest of the grid
 - ☹ torsional oscillations induced by mechanical/electrical/flexible coupling
 - ☹ AVR control induces unstable local oscillations
 - ☺ typically damped by local feedback via PSSs
- Power system = complex oscillator network \Rightarrow **inter-area oscillations**:
 - = groups of generators oscillate relative to each other
 - ☹ poorly tuned local PSSs result in unstable inter-area oscillations
 - ☹ inter-area oscillations are only poorly controllable by local feedback
- Consequences of **recent developments**:
 - ☹ increasing power transfers outpace capacity of transmission system
 - \Rightarrow ever more lightly damped electromechanical inter-area oscillations
 - ☹ technological opportunities for **wide-area control (WAC)**

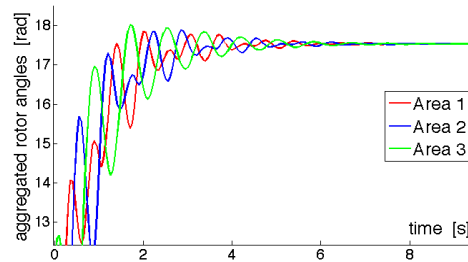
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Slow Coherency Modeling

Slow coherency and area aggregation



aggregated RTS 96 model



swing dynamics of aggregated model

Aggregate model of lower dimension & with less complexity for

- 1 analysis and insights into inter-area dynamics [Chow and Kokotovic '85]
- 2 measurement-based id of equivalent models [Chakraborty et al. '10]
- 3 remedial action schemes [Xu et al. '11] & wide-area control (later today)

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How to find the areas?

a crash course in spectral partitioning

- given: an undirected, connected, & weighted **graph**
- **partition**: $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, and $\mathcal{V}_1, \mathcal{V}_2 \neq \emptyset$

- **cut** is the size of a partition: $J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij}$
- \Rightarrow if $x_i = 1$ for $i \in \mathcal{V}_1$ and $x_j = -1$ for $j \in \mathcal{V}_2$, then

$$J = \sum_{i \in \mathcal{V}_1, j \in \mathcal{V}_2} a_{ij} = \frac{1}{2} \sum_{i,j=1}^n a_{ij} (x_i - x_j)^2 = \frac{1}{2} x^T L x$$

- combinatorial **min-cut** problem: minimize $x \in \{-1, 1\}^n \setminus \{-\mathbf{1}_n, \mathbf{1}_n\}$ $\frac{1}{2} x^T L x$
- **relaxed problem**: minimize $y \in \mathbb{R}^n, y \perp \mathbf{1}_n, \|y\|_2 = 1$ $\frac{1}{2} y^T L y$

\Rightarrow minimum is *algebraic connectivity* λ_2 and minimizer is *Fiedler vector* v_2

- **heuristic**: $x_i = \text{sign}(y_i) \Rightarrow$ "spectral partition"

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A quick example

```

% choose a graph size
n = 1000;

% randomly assign the nodes to two groups
x = randperm(n);
group_size = 450;
group1 = x(1:group_size);
group2 = x(group_size+1:end);

% assign probabilities of connecting nodes
p_group1 = 0.5;
p_group2 = 0.4;
p_between_groups = 0.1;

% construct adjacency matrix
A(group1, group1) = rand(group_size,group_size) < p_group1;
A(group2, group2) = rand(n-group_size,n-group_size) < p_group2;
A(group1, group2) = rand(group_size, n-group_size) < p_between_groups;
A = triu(A,1); A = A + A';

% can you see the groups?
subplot(1,3,1); spy(A);

% construct Laplacian and its spectrum
L = diag(sum(A))-A;
[V D] = eigs(L, 2, 'SA');

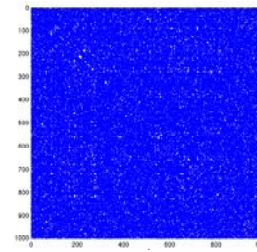
% plot the components of the algebraic connectivity sorted by magnitude
subplot(1,3,2); plot(sort(V(:,2)), '-');

% partition the matrix accordingly and spot the communities
[ignore p] = sort(V(:,2));
subplot(1,3,3); spy(A(p,p));

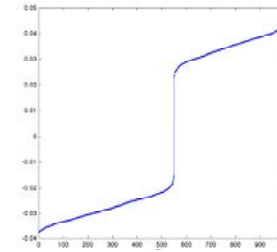
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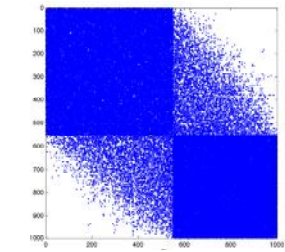
A quick example – cont'd



adjacency matrix



Fiedler vector v_2



re-arranged adj. matrix

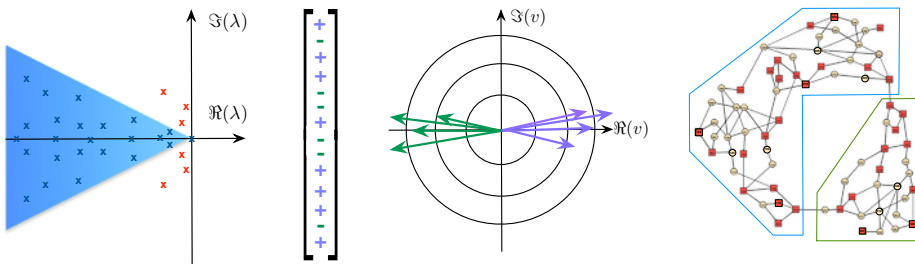
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Classical power system partitioning \approx spectral partitioning

- 1 construct a linear model $\dot{x} = Ax$ (via, e.g., Power Systems Toolbox)
- 2 recall solution via eigenvalues λ_i and left/right eigenvectors w_i and v_i :

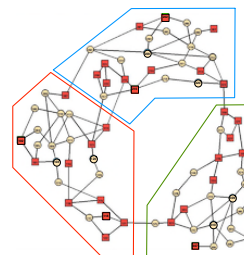
$$x(t) = \sum_i v_i e^{\lambda_i t} \cdot w_i^T x_0 = \sum_i \{\text{mode \#}i\} \cdot \{\text{contribution from } x_0\}$$

- 3 look at poorly damped complex conjugate mode pairs
- 4 look at angle & frequency components of eigenvectors
- 5 group the generators according to their polarity in eigenvectors

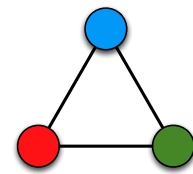


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Setup in slow coherency



original model



aggregated model

- r given areas
(from spectral partition [Chow et al. '85 & '13])

- small **sparsity parameter**:

$$\delta = \frac{\max_{\alpha} (\sum \text{external connections in area } \alpha)}{\min_{\alpha} (\sum \text{internal connections in area } \alpha)}$$

- **inter-area dynamics** by center of inertia:

$$y_{\alpha} = \frac{\sum_{i \in \alpha} M_i \theta_i}{\sum_{i \in \alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

- **intra-area dynamics** by area differences:

$$z_{i-1}^{\alpha} = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

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Linear transformation & time-scale separation

Swing equation \implies singular perturbation standard form

$$M\ddot{\theta} + D\dot{\theta} + L\theta = 0 \implies \begin{cases} \frac{d}{dt_s} \begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta} z \\ \sqrt{\delta} \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & A & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \end{cases}$$

Slow motion given by center of inertia:

$$y_\alpha = \frac{\sum_{i \in \alpha} M_i \theta_i}{\sum_{i \in \alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

Fast motion given by intra-area differences:

$$z_{i-1}^\alpha = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

Slow time scale: $t_s = \delta \cdot t$ "max internal area degree"

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Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s} \begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta} z \\ \sqrt{\delta} \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & A & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a \ddot{\varphi} + D_a \dot{\varphi} + L_{\text{red}} \varphi = 0$$

Properties of aggregated model

[D. Romeres, FD, & F. Bullo, '13]

- $M_a = \begin{bmatrix} \ddots & & \\ & \sum_{i \in \alpha} M_i & \\ & & \ddots \end{bmatrix}$ and $D_a = \begin{bmatrix} \ddots & & \\ & \sum_{i \in \alpha} D_i & \\ & & \ddots \end{bmatrix}$
- $L_{\text{red}} =$ "inter-area Laplacian" + "intra-area contributions"
= positive semidefinite Laplacian with possibly negative weights

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Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s} \begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta} z \\ \sqrt{\delta} \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \ddots \\ \cdots & A & \cdots \\ \ddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a \ddot{\varphi} + D_a \dot{\varphi} + L_{\text{red}} \varphi = 0$$

Singular perturbation approximation

[D. Romeres, FD, & F. Bullo, '13]

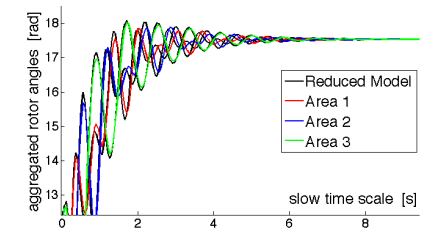
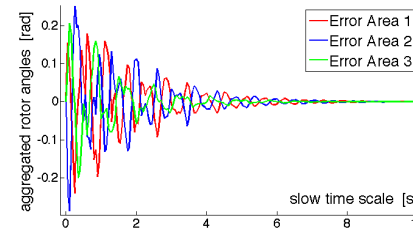
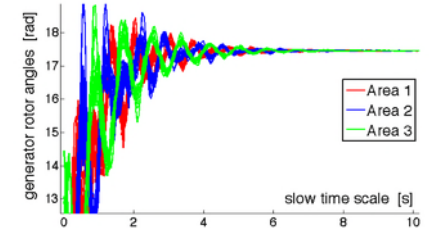
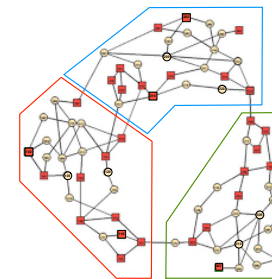
There exist δ^* sufficiently small such that for $\delta \leq \delta^*$ and for all $t > 0$:

$$\begin{bmatrix} y(t_s) \\ \dot{y}(t_s) \end{bmatrix} = \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}), \quad \begin{bmatrix} z(t_s) \\ \dot{z}(t_s) \end{bmatrix} = \tilde{A} \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}).$$

center of inertia \approx solution of aggregated swing equation

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RTS 96 swing dynamics revisited



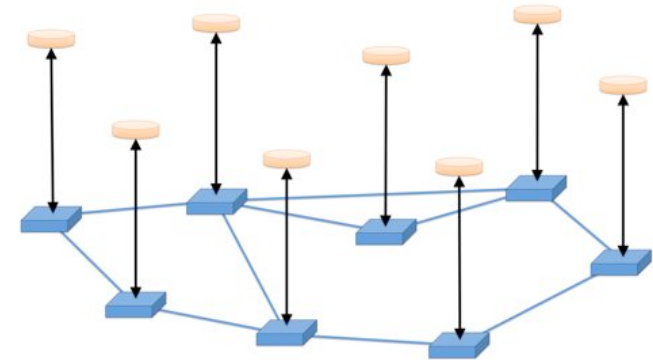
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Inter-Area Oscillations & Wide-Area Control

Remedies against electro-mechanical oscillations

conventional control

- **Blue layer:** interconnected generators



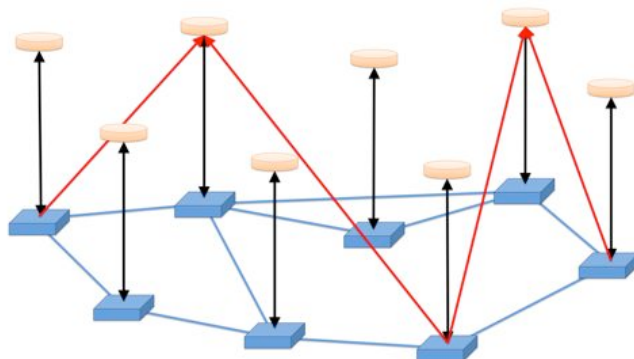
- **Fully decentralized control** implemented via PSS, HVDC, or FACTS:
 - ☺ effective against local oscillations
 - ☹ ineffective against inter-area oscillations

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Remedies against electro-mechanical oscillations

wide-area control

- **Blue layer:** interconnected generators

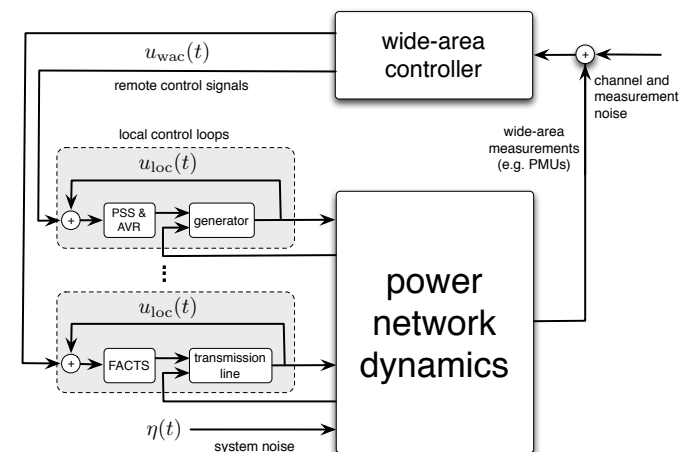


- **Fully decentralized control**
- **Distributed wide-area control** requires identification of sparse control architecture: actuators, measurements, & communication channels

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Setup in Wide-Area Control

- 1 remote control signals & remote measurements (e.g., PMUs)
- 2 excitation (PSS & AVR) and power electronics (FACTS) actuators
- 3 communication backbone network



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Modal signal selection metrics [H.M.A. Hamdan & A.M.A. Hamdan '87]

- 1 **Linear control system:** $\dot{x} = Ax + Bu$, $y = Cx$
 - B with column b_j = control location # j
 - C with row c_j^T = sensor location # j
 - A : eigenvalues λ_i and orthonormal right & left eigenvectors v_i & w_i^*

- 2 **Diagonalization:** $x = Vz = [v_1 \dots v_n] z$, $z = Wx = [w_1 \dots w_n]^* x$

$$\Rightarrow \dot{z} = \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{=WAV} z + \underbrace{\begin{bmatrix} \dots & w_i^* b_j & \dots \\ \vdots & \vdots & \vdots \\ \dots & w_i^* b_j & \dots \end{bmatrix}}_{=WB} u, \quad y = \underbrace{\begin{bmatrix} \vdots & \vdots & \vdots \\ \dots & c_i^* v_j & \dots \\ \vdots & \vdots & \vdots \end{bmatrix}}_{=CV} uz$$

- 3 Controllability of mode i by input $j \triangleq \cos(\angle(w_i, b_j)) = \frac{w_i^* b_j}{\|w_i\| \|b_j\|}$
- 4 Observability of mode i by sensor $j \triangleq \cos(\angle(c_i, v_j)) = \frac{c_i^* v_j}{\|c_i\| \|v_j\|}$

Alternatives based on modal residues of transfer function [M. Tarokh '92]. 138 / 156

Challenges in wide-area control

- **Objectives:** wide-area control should achieve
 - 1 optimal closed-loop performance
 - 2 low control complexity (comm, measurements, & actuation)
- **Problem:** objectives are conflicting
 - 1 design (optimal) centralized control \Rightarrow identify control architecture
 - ☹ complete state info & measurements
 - ☹ high communication complexity
 - 2 identify measurements & control architecture \Rightarrow design control
 - ☹ decentralized (optimal) control is hard
 - ☹ combinatorial criteria for control channels

Today: simultaneously optimize closed-loop performance & identify sparse control architecture

Decentralized WAC control design ...

- ... subject to structural constraints is **tough**
- ... usually handled with suboptimal **heuristics** in MIMO case

Robust and coordinated tuning of power system stabiliser gains using sequential linear programming

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Abstract: This paper presents a method for designing the stabiliser gains for a power system. The method is based on the sequential linear programming (SLP) technique. The SLP technique is used to iteratively tune the stabiliser gains. The method is applied to a power system with two phase lead blocks and a phase lag block.

Decentralized Power System Stabilizer Design Using Linear Parameter Varying Approach

Wenhang Qiu, Student Member IEEE, Yijun Yan, Fellow IEEE, and Mustafa Khannous, Senior Member IEEE

Abstract: This paper proposes the power system model is formulated as a linear parameter varying (LPV) system. The LPV model is used to design the stabilizer gains. The method is applied to a power system with two phase lead blocks and a phase lag block.

Robust and Low Order Power Oscillation Damper Design Through Polynomial Control

Dhanraj D. Senthilvelu, Student Member IEEE, and Bikash C. Pal, Senior Member IEEE

Abstract: This paper presents a method for designing the stabiliser gains for a power system. The method is based on the polynomial control technique. The method is applied to a power system with two phase lead blocks and a phase lag block.

Robust Pole Placement: Stabilizer Design Using Linear Matrix Inequalities

Wenhang Qiu, Student Member IEEE, Yijun Yan, Fellow IEEE, and Mustafa Khannous, Senior Member IEEE

Abstract: This paper proposes the power system model is formulated as a linear parameter varying (LPV) system. The LPV model is used to design the stabilizer gains. The method is applied to a power system with two phase lead blocks and a phase lag block.

Robust Power System Stabilizer Design Using H_∞ Loop Shaping Approach

Changshun Zhu, Member IEEE, Mustafa Khannous, Senior Member IEEE, Yijun Yan, Fellow IEEE, and Wenhang Qiu, Student Member IEEE

Abstract: This paper proposes the power system model is formulated as a linear parameter varying (LPV) system. The LPV model is used to design the stabilizer gains. The method is applied to a power system with two phase lead blocks and a phase lag block.

Simultaneous Coordinated Tuning of PSS and FACTS Damping Controllers in Large Power Systems

Jin Guo and Ibrahim Eker, Member IEEE

Abstract: This paper proposes the power system model is formulated as a linear parameter varying (LPV) system. The LPV model is used to design the stabilizer gains. The method is applied to a power system with two phase lead blocks and a phase lag block.

\Rightarrow signal selection is combinatorial & control design is suboptimal

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Primer on Linear Quadratic Control (LQR)

Linear Quadratic Control (LQR) is a method for designing a controller for a linear system. The controller is designed to minimize a quadratic cost function. The cost function is defined as the integral of a quadratic function of the state and control variables over an infinite time horizon.

The LQR problem is to find a control law $u = -Kx$ that minimizes the cost function $J = \int_0^\infty (x^T Q x + u^T R u) dt$, where Q and R are symmetric positive definite matrices. The optimal control law is given by $K = R^{-1} B^T P$, where P is the solution to the Riccati equation $A^T P + P A - P B R^{-1} B^T P + Q = 0$.

The Riccati equation is a nonlinear matrix equation. It can be solved using numerical methods. The solution P is a symmetric positive definite matrix. The control law $u = -Kx$ is a linear feedback control law. It is optimal in the sense that it minimizes the cost function for all initial conditions.

LQR is a powerful tool for designing controllers for linear systems. It is widely used in control engineering. It is a special case of the more general Linear Quadratic Gaussian (LQG) control.

Optimal wide-area damping control

- **Model:** linearized ODE dynamics $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t)$
- **Control:** memoryless linear state feedback $u = -Kx(t)$
- **Optimal centralized control** with quadratic performance index:

$$\begin{aligned} &\text{minimize } J(K) \triangleq \lim_{t \rightarrow \infty} \mathcal{E} \left\{ x(t)^T Q x(t) + u(t)^T R u(t) \right\} \\ &\text{subject to} \\ &\text{linear dynamics: } \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ &\text{linear control: } u(t) = -Kx(t), \\ &\text{stability: } (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

(no structural constraints on K)

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Sparsity-promoting optimal wide-area damping control

- **Sparsity-promoting optimal control** [Lin, Fardad, & Jovanović '13]: simultaneously optimize control performance & control architecture

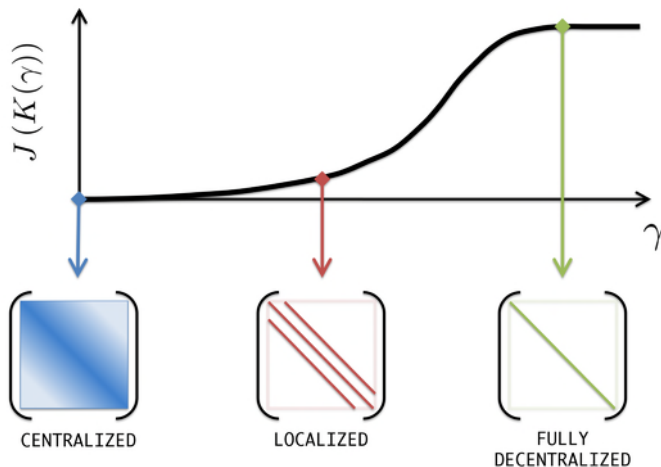
$$\begin{aligned} &\text{minimize } \lim_{t \rightarrow \infty} \mathcal{E} \left\{ x(t)^T Q x(t) + u(t)^T R u(t) \right\} + \gamma \text{card}(K) \\ &\text{subject to} \\ &\text{linear dynamics: } \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t), \\ &\text{linear control: } u(t) = -Kx(t), \\ &\text{stability: } (A - B_2K) \text{ Hurwitz.} \end{aligned}$$

- ⇒ for $\gamma = 0$: standard optimal control (typically not sparse)
- ⇒ for $\gamma > 0$: sparsity is promoted (problem is combinatorial)
- ⇒ $\text{card}(K)$ approximated by weighted ℓ_1 -norm $\sum_{i,j} w_{ij} |K_{ij}|$

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Parameterized family of feedback gains

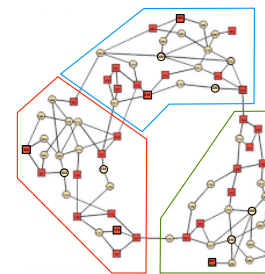
$$K(\gamma) = \arg \min_K \left(J(K) + \gamma \cdot \sum_{i,j} w_{ij} |K_{ij}| \right)$$



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Slow coherency performance objectives

- recall **sources** for inter-area oscillations:



- linearized **swing equation**: $M\ddot{\theta} + D\dot{\theta} + L\theta = P$

- **mechanical energy**: $\frac{1}{2} \dot{\theta}^T M \dot{\theta} + \frac{1}{2} \theta^T L \theta$

- **heterogeneities** in topology, power transfers, & machine responses (inertia & damp)

⇒ performance **objectives** = energy of homogeneous network:

$$x^T Q x = \frac{1}{2} \dot{\theta}^T \underbrace{M_{\text{uniform}}}_{I_n} \dot{\theta} + \frac{1}{2} \theta^T \underbrace{L_{\text{uniform}}}_{I_n - (1/n) \cdot \mathbf{1}_{n \times n}} \theta$$

- other choices possible: center of inertia, inter-area differences, etc.

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Algorithmic approach to sparsity-promoting control

- 1 Equivalent formulation via **observability Gramian** P :

$$\begin{aligned} & \text{minimize } J_\gamma(K) \triangleq \text{trace}(B_1^T P B_1) + \gamma \sum_{i,j} w_{ij} |K_{ij}| \\ & \text{subject to } (A - B_2 K)^T P + P(A - B_2 K) \\ & \quad \quad \quad = -(Q + K^T R K); \end{aligned}$$

- 2 **Warm-start** at optimal centralized \mathcal{H}_2 controller with $\gamma = 0$
- 3 **Homotopy path**: continuously increase γ until the desired value γ_{des}
- 4 **ADMM**: iterative solution for each value of $\gamma \in [0, \gamma_{\text{des}}]$
- 5 **Update weights**: update w_{ij} in each ADMM step: $w_{ij} \mapsto \frac{1}{|K_{ij}| + \epsilon}$
- 6 **Polishing**: structured optimization with desired sparsity pattern

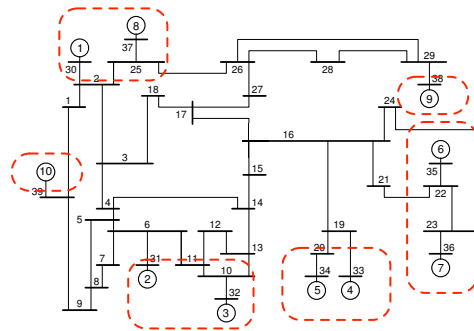
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Case Study: IEEE 39 New England Power Grid

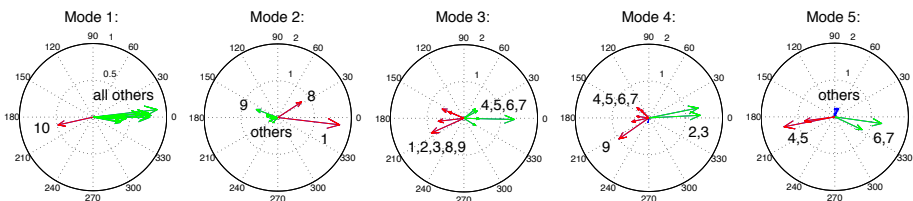
Case study: IEEE 39 New England power grid

Model features:

- sub-transient generator models [Athay et. al. '79]
- exciters & carefully tuned PSS data [Jabr et. al. '09]



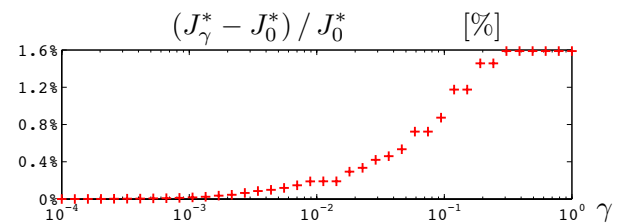
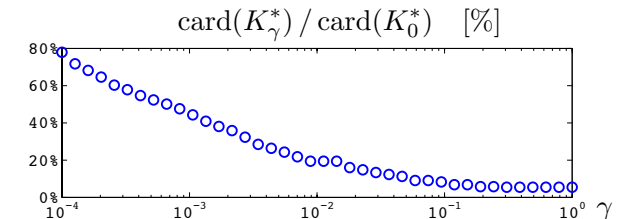
- dominant **inter-area modes** of New England grid with PSSs



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Performance vs. sparsity

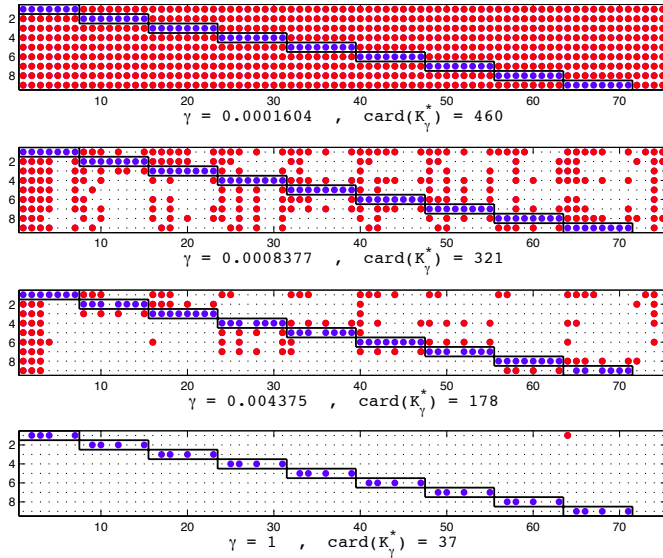
Q = energy of homogeneous network, $R = I_n$, $\gamma \in [10^{-4}, 10^0]$



for $\gamma = 1 \Rightarrow \begin{cases} 1.6\% & \text{relative performance loss} \\ 5.5\% & \text{non-zero elements in } K \end{cases}$

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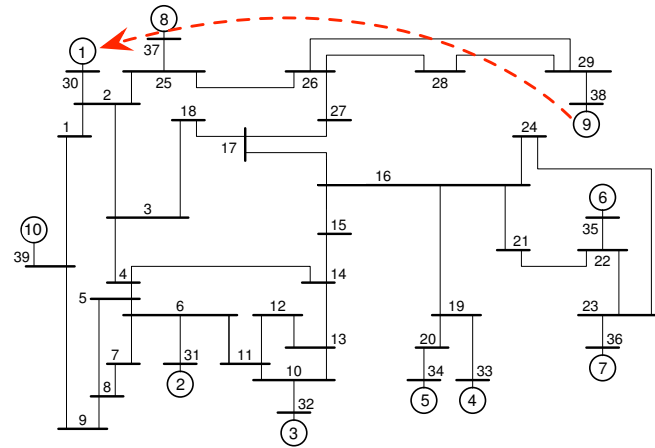
Control architecture & signal exchange network



For $\gamma = 1$: local decentralized optimal control + $K_{19}^* \theta_9(t)$

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Sparse & nearly optimal wide-area control architecture

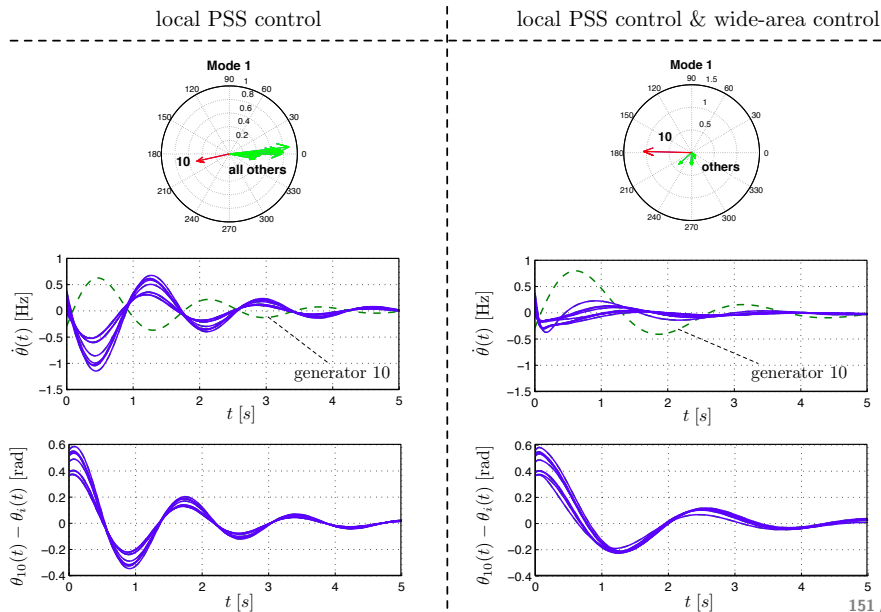


single wide-area control link \implies nearly centralized performance

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Closed-loop performance for $\gamma = 1$

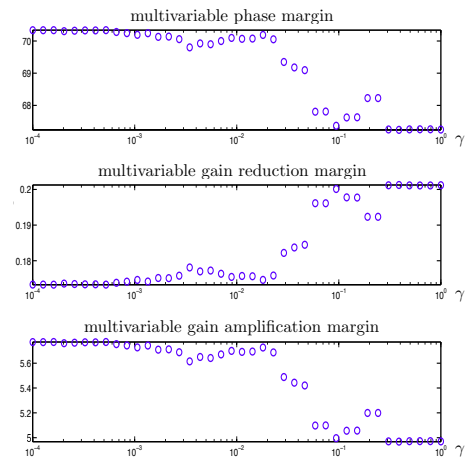
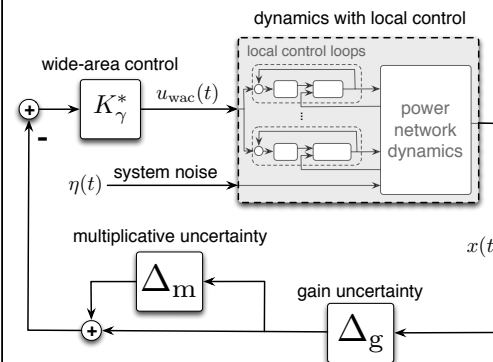
modes #2,3,... are strongly damped and mode #1 is distorted



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Robustness achieved by sparsity-promoting control

- tuning of PSSs & operating cond
 \implies gain margins
- time delays, multiple SCADA rates, & comm uncertainties
 \implies phase margins & input uncertainties



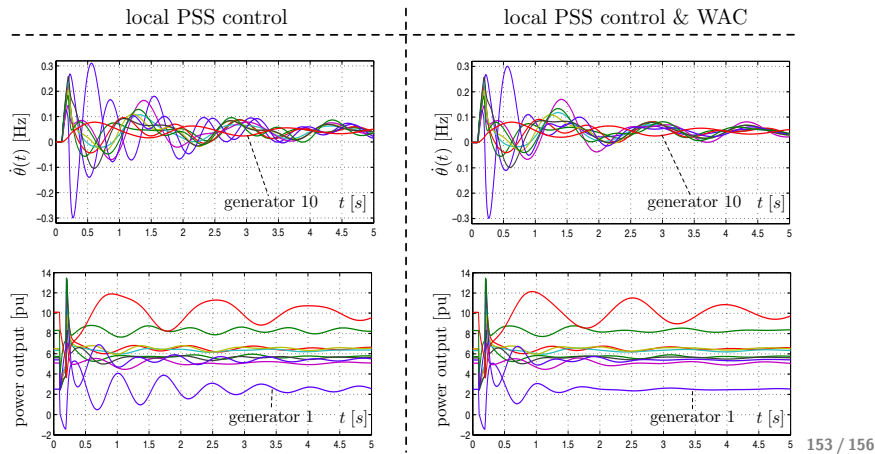
Additionally: sparsity pattern is not sensitive to operating point

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Sparsity identification & control by alternative means

- identified WAC channel: $\theta_9(t)$ needs to be communicated to AVR #1

⇒ proportional feedback $u_1(t) = -K_{19}^* (\theta_1(t) - \theta_9(t))$ applied to nonlinear DAE system without local optimal decentralized control



You can also get rid of communication entirely ...

Analysis and Design Trade-Offs for Power Network Inter-Area Oscillations

Xiaofan Wu, Florian Dörfler, and Mihailo R. Jovanović

Abstract—Conventional analysis and control approaches to inter-area oscillations in bulk power systems are based on a modal perspective. Typically, inter-area oscillations are identified from spatial profiles of poorly damped modes, and they are damped using carefully tuned decentralized controllers. To improve upon the limitations of conventional decentralized strategies, recent efforts aim at distributed wide-area control which involves the communication of remote signals. Here, we introduce a novel approach to the analysis and control of inter-area oscillations. Our framework is based on a stochastically driven system with performance outputs chosen such that the \mathcal{H}_2 norm is associated with incoherent inter-area oscillations. We show that an analysis of the output covariance matrix offers new insights relative to modal approaches. Next, we leverage the recently proposed sparsity-promoting optimal control approach to design controllers that use relative angle measurements and simultaneously optimize the closed-loop performance and the control architecture. For the IEEE 39 New England model, we investigate performance trade-offs of different control architectures and show that optimal retuning of decentralized control strategies can effectively guard against inter-areas oscillations.

damped via decentralized controllers, whose gains are carefully tuned according to root locus criteria [7]–[9].

To improve upon the limitations of decentralized controllers, recent research efforts aim at distributed wide-area control strategies that involve the communication of remote signals, see the surveys [10], [11] and the excellent articles in [12]. The wide-area control signals are typically chosen to maximize modal observability metrics [13], [14], and the control design methods range from root locus criteria to robust and optimal control approaches [15]–[17].

Here, we investigate a novel approach to the analysis and control of inter-area oscillations. Our unifying analysis and control framework is based on a stochastically driven power system model with performance outputs inspired by slow coherency theory [18], [19]. We analyze inter-area oscillations by means of the \mathcal{H}_2 norm of this system, as in recent related approaches for interconnected oscillator networks and multi-machine power systems [20]–[22]. We show that an analysis of power spectral density and variance amplification

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Outline

Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

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Looking for data, toolboxes, & test cases

- **Matpower** for (optimal) power flow & static models
<http://www.pserc.cornell.edu/matpower/>
- **Power System Toolbox** for dynamics & North American models
http://www.eps.ee.kth.se/personal/vanfretti/pst/Power_System_Toolbox_Webpage/PST.html
- **IEEE Task Force PES PSDPC SCS**: New York, Brazil, Australian grids etc.; <http://www.sel.eesc.usp.br/ieee/>
- **ObjectStab** for Modelica for dynamics & models
<https://github.com/modelica-3rdparty/ObjectStab>
- **More freeware**: MatDyn, PSAT, THYME, Dome, ...
http://ewh.ieee.org/cmt/pspace/CAMS_taskforce/
- **Other**: many test cases in papers, reports, task forces, ...

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Conclusions

Introduction

Power Network Modeling

Feasibility, Security, & Stability

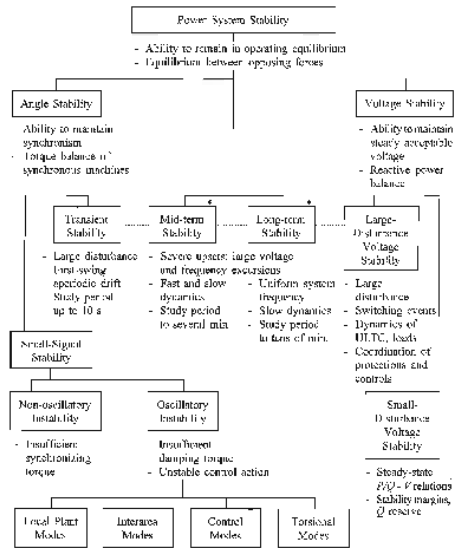
Power System Control Hierarchy

Power System Oscillations

Conclusions

Obviously, there is a lot more ...

I hope I could give you a little insight into a few interesting problems.



final words of wisdom