Power Systems Control
from Circuits to Economics

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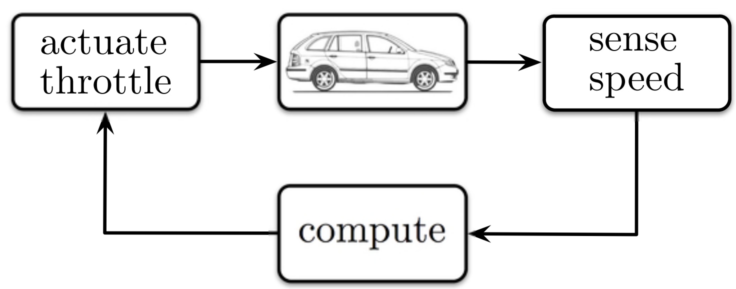
Who are we?

Florian

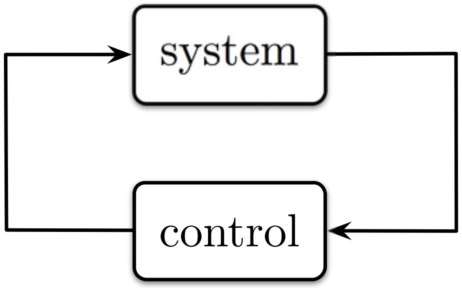
Andrej

why should control engineers or even pure control theorists care about power systems ?

The "simple" control loop



The "simple" control loop

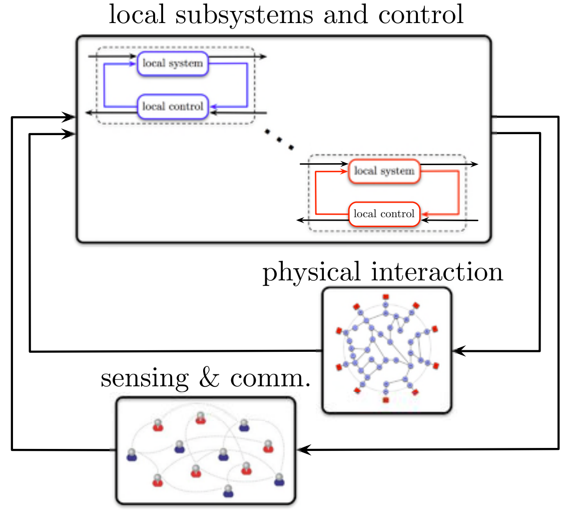


"Simple" control systems are well understood.

"Complexity" can enter this control loop in many ways:

models, disturbances, constraints, uncertainty, optimality,
. . . all of which are embodied in power systems.

More recent focus: "complex" distributed decision making



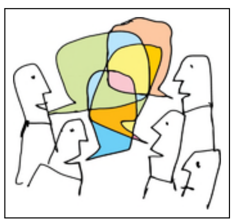
Such distributed systems include **large-scale** physical systems, engineered **multi-agent** systems, & their interconnection in **cyber-physical** systems.

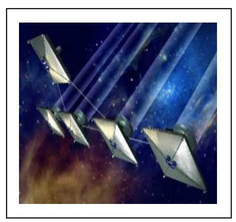
Timely applications of distributed systems control

often the centralized perspective is simply not appropriate


robotic networks


decision making


social networks


sensor networks


self-organization


pervasive computing


traffic networks


smart power grids

what makes power systems (IMHO) so interesting?

My main application of interest – the power grid

NASA Goddard Space Flight Center

- **Electric energy** is critical for our technological civilization
- Energy supply via **power grid**
- **Complexities**: nonlinear, multi-scale, & non-local

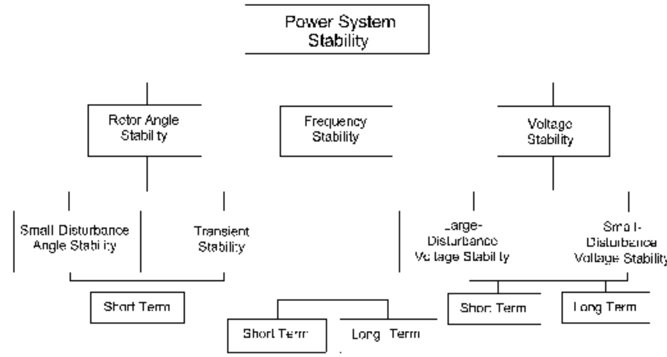
One system with many dynamics & control problems

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 19, NO. 2, MAY 2004 1387

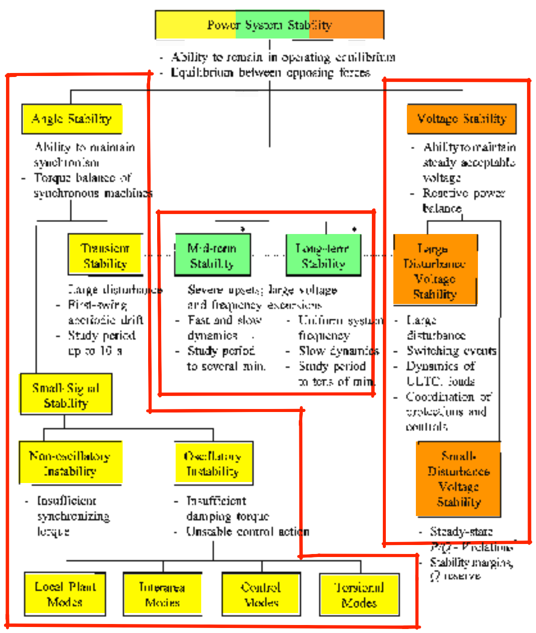
Definition and Classification of Power System Stability

IEEE/CIGRE Joint Task Force on Stability Terms and Definitions

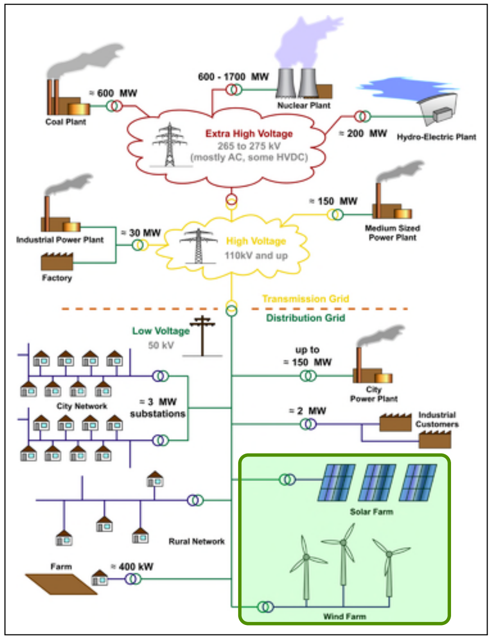
Prabha Kundur (Canada, Convener), John Paserba (USA, Secretary), Venkat Ajjarapu (USA), Göran Andersson (Switzerland), Anjan Bose (USA) , Claudio Canizares (Canada), Nikos Hatziargyriou (Greece), David Hill (Australia), Alex Stankovic (USA), Carson Taylor (USA), Thierry Van Cutsem (Belgium), and *Vijay Vittal (USA)*



Many aspects: spatial/temporal scales, cause & effect, . . .

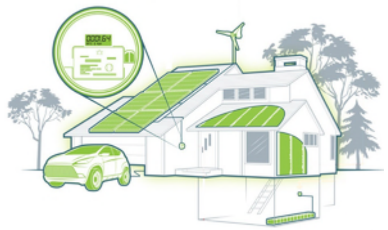


(Conventional) operation of electric power networks



Top-to-bottom operation:

- **purpose** of electric power grid: generate/transmit/distribute
- **operation**: hierarchical & based on bulk generation
- things are changing . . .

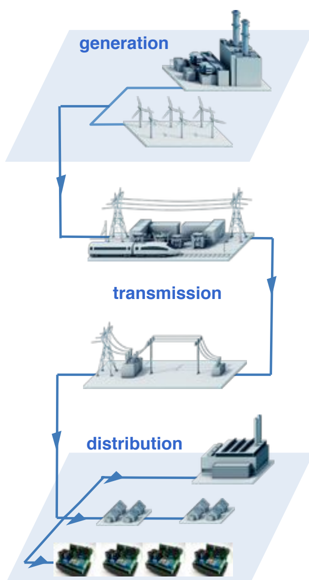


A few (of many) game changers

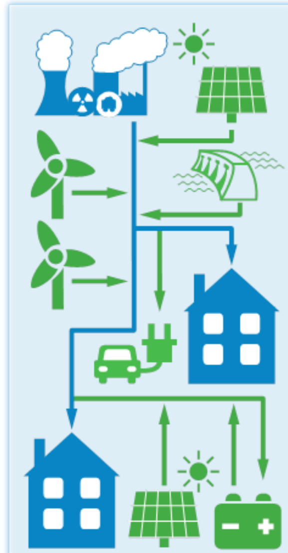
synchronous generator
⇒ power electronics

scaling

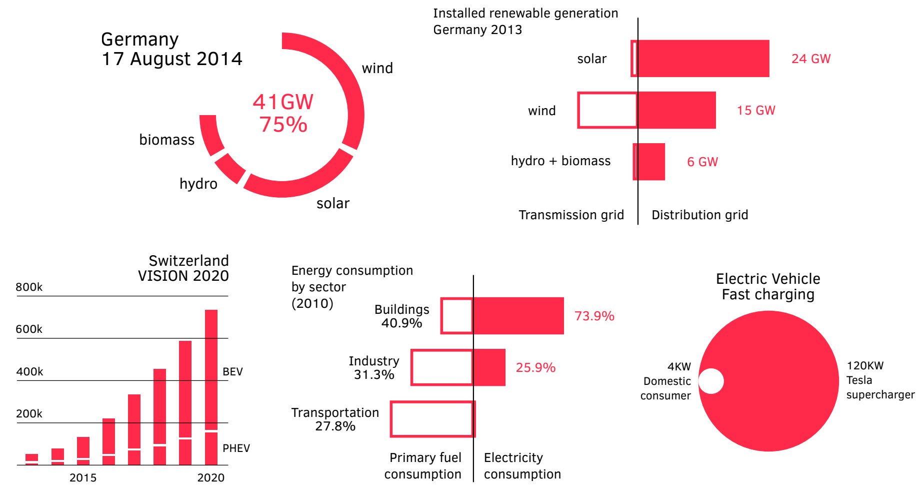
distributed generation



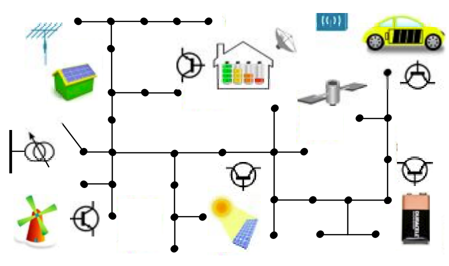
other paradigm shifts



A little bit of drama: examples close to home



Paradigm shifts & new scenarios . . . in a nutshell



1. controllable fossil fuel sources ⇒ stochastic renewable sources
2. centralized bulk generation ⇒ distributed low-voltage generation
3. synchronous generators ⇒ low/no inertia power electronics
4. generation follows load ⇒ controllable load follows generation
5. monopolistic energy markets ⇒ deregulated energy markets
6. centralized top-to-bottom control ⇒ distributed non-hierarchical control
7. human in the loop & heuristics ⇒ "smart" real-time decision making

Challenges & opportunities in tomorrow's power grid

www.offthegridnews.com

☹perational challenges

- more uncertainty & less inertia
- more volatile & faster fluctuations
- deregulation & decentralization

☺pportunities

- re-instrumentation: comm & sensors and actuators throughout grid
- elasticity in storage & demand
- advances in understanding & control of cyber-physical & complex systems

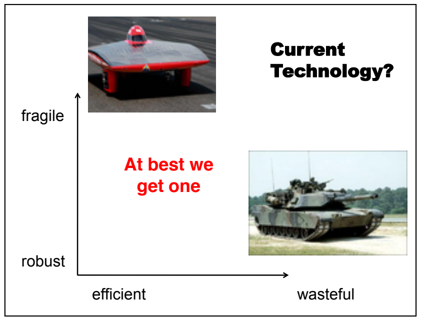
Some profound insights by the giants in the field

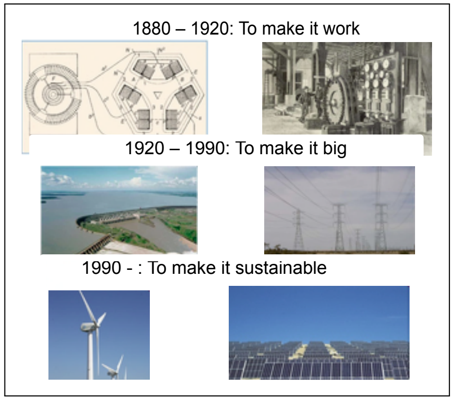
trade-offs & hard limits in control

[J. Doyle, UCSB '12]

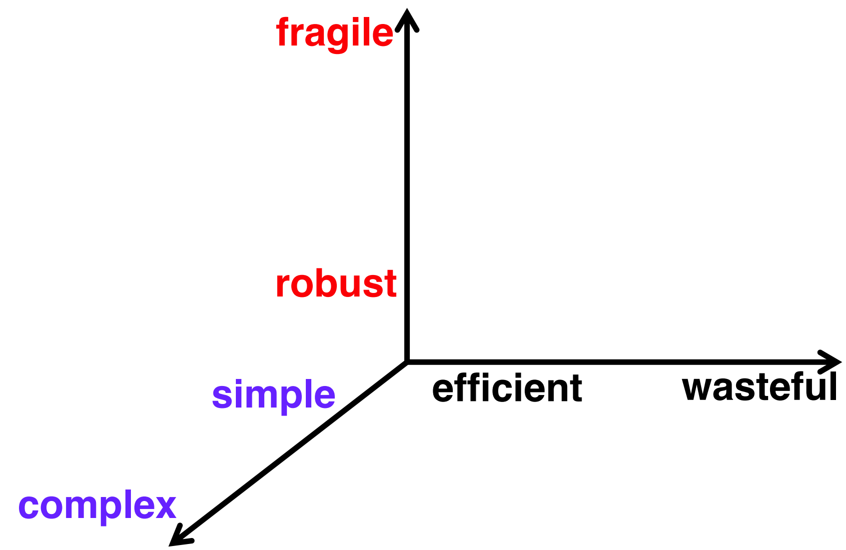
a third challenge in power systems

[G. Andersson, LANL '14]





We should keep John's and Göran's trade-offs in mind



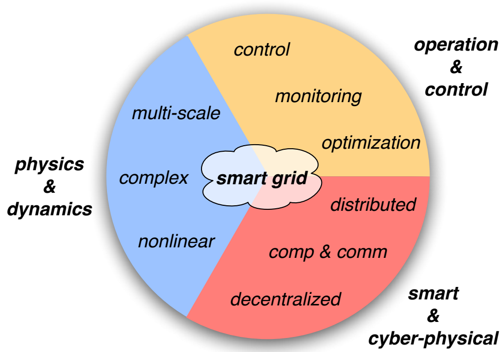
The envisioned power grid

complex, cyber-physical, & "smart"

⇒ smart grid **keywords**

⇒ **interdisciplinary**:
power, control, comm, optim, econ, physics, . . . industry, & society

⇒ **research themes**:
trade-offs in robustness, complexity, & efficiency



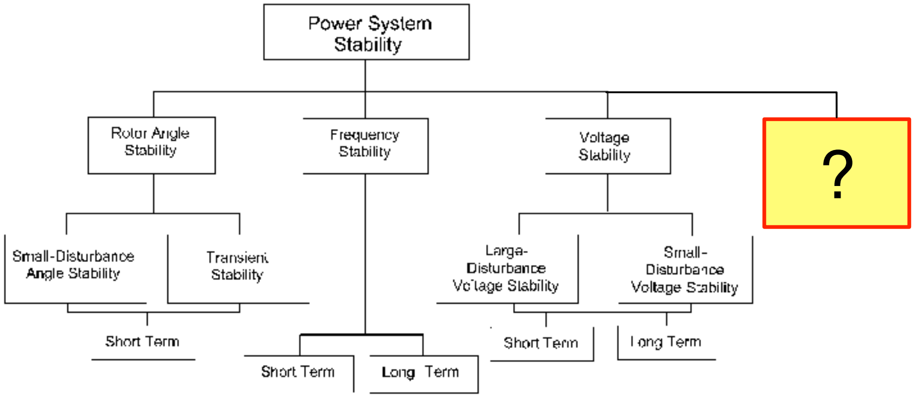
"[It remains] to put some serious science into the idea." — [David Hill, PESGM '12]

Power Systems Control — from Circuits to Economics

Wednesday, February 17, 2016		
10.00 – 11.00	Registration	
11.00 – 11.30	Florian Dörfler	General introduction
11.30 – 12.30	Florian Dörfler	Power System Modeling
12.30 – 14.00	Lunch	
14.00 – 15.00	Florian Dörfler	Power System Stability Control I
15.00 – 15.15	Break	
15.15 – 16.00	Florian Dörfler	Power System Stability Control I
16.00 – 17.30	Exercises	
Thurday, February 18, 2016		
09.00 – 10.15	Florian Dörfler	Power System Stability Control II
10.15 – 10.30	Break	
10.30 – 11.30	Florian Dörfler	Power System Stability Control II
11.30 – 12.30	Exercises	
12.30 – 14.00	Lunch	
14.00 – 15.00	Andrej Jokic	Power System Economics I
15.00 – 15.15	Break	
16.00 – 17.00	Exercises	
19.00	Dinner	
Friday, February 19, 2016		
09.00 – 10.15	Andrej Jokic	Power System Economics II
10.15 – 10.30	Break	
10.30 – 11.30	Andrej Jokic	Power System Economics II
11.30 – 12.30	Exercises	
12.30 – 13.30	Lunch	
13.30 – 14.30	Discussion of future research topics	
14.30	Drinks and closing	

A preview — to be resolved on the last day

The future will hold a new (and very dominant) stability issue



let's start off with a quiz:

what is your background?

why are you interested in power?

what are your expectations?

Power System Stability & Control

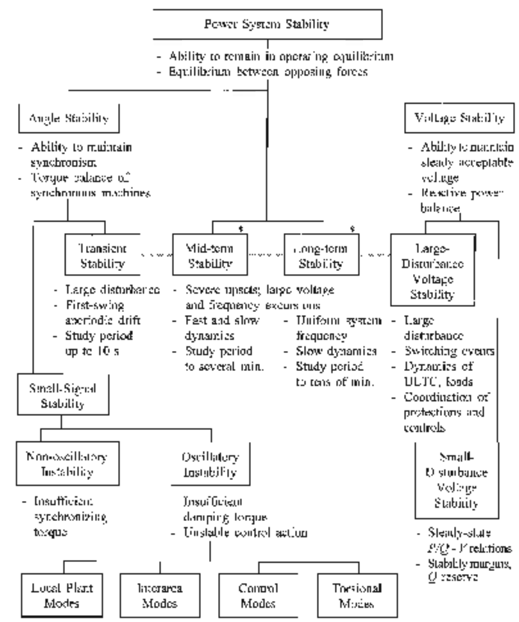
Florian Dörfler Andrej Jokić

disc — dutch institute of systems and control

ETH — Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

University of Zagreb

Power system stability & control: have to choose based on



- what Andrej needs
- what I actually know well
- what is interesting from a network perspective rather than from device perspective
- what is relevant for future (smart) power grids with high renewable penetration
- what gives rise to fun distributed control problems
- what **you** are interested in

Tentative outline

my particular focus is on networks

Disclaimers

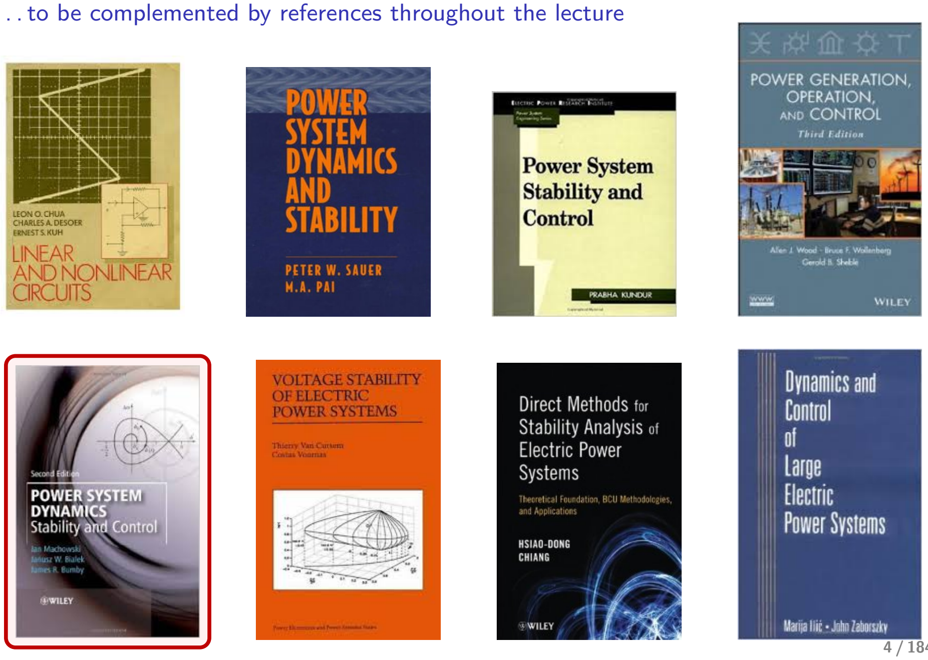
- start off with "boring" modeling before more "sexy" topics
- start off with basic material & before "cutting edge" work
- focus on simple models and physical & math intuition

⇒ cover fundamentals, convey intuition, & give references for the details

Please . . .
- ask me for further reading about any topic,
- and interrupt & correct me anytime.

Many references available ... my personal look-up list

... to be complemented by references throughout the lecture



We will also use the blackboard ...

... respectively, we will outsource the blackboard to the exercises

Outline

You will learn to appreciate the following words of wisdom

"Power system research is all about the art of making the right assumptions."

— [Maria Ilic, Lund LCCC Seminar '14]

Circuit Modeling: Network, Loads, & Devices

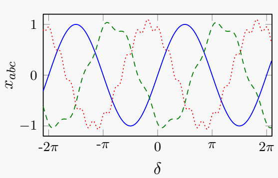
Signal space in three-phase AC circuits

three phase & AC

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = \begin{bmatrix} x_a(t+T) \\ x_b(t+T) \\ x_c(t+T) \end{bmatrix}$$

periodic with 0 average

$$\frac{1}{T}\int_0^T x_i(t)dt = 0$$

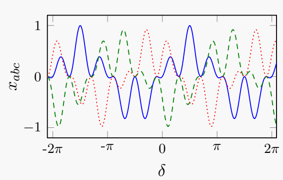


symmetric/balanced

$$= A(t)\begin{bmatrix} \sin(\delta(t)) \\ \sin(\delta(t) - \frac{2\pi}{3}) \\ \sin(\delta(t) + \frac{2\pi}{3}) \end{bmatrix}$$

so that

$$x_a(t)+x_b(t)+x_c(t)=0$$

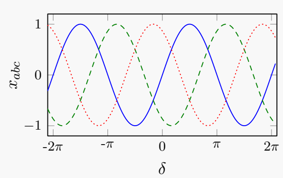


synchronous

$$= A\begin{bmatrix} \sin(\delta_0 + \omega^* t) \\ \sin(\delta_0 + \omega^* t - \frac{2\pi}{3}) \\ \sin(\delta_0 + \omega^* t + \frac{2\pi}{3}) \end{bmatrix}$$

const. freq & amp:

$\Rightarrow$ phasor $Ae^{i(\delta_0 + \omega\ \ t)}$



Park or *dq*0-transformation

$$T(\theta) = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

- is unitary $T(\theta)^{\ \ 1} = T(\theta)^T$ & maps balanced *abc*-signal to

$$x_{dq0} = \begin{bmatrix} x_d(t) \\ x_q(t) \\ x_0(t) \end{bmatrix} = T(\theta)x_{abc}(t) = \sqrt{\frac{3}{2}}A(t)\begin{bmatrix} \sin(\delta(t) - \theta) \\ \cos(\delta(t) - \theta) \\ 0 \end{bmatrix}$$

- $T(\omega\ \ t)$ maps a synchronous signal $x_a(t) = A\sin(\delta_0 + \omega\ \ t)$ to

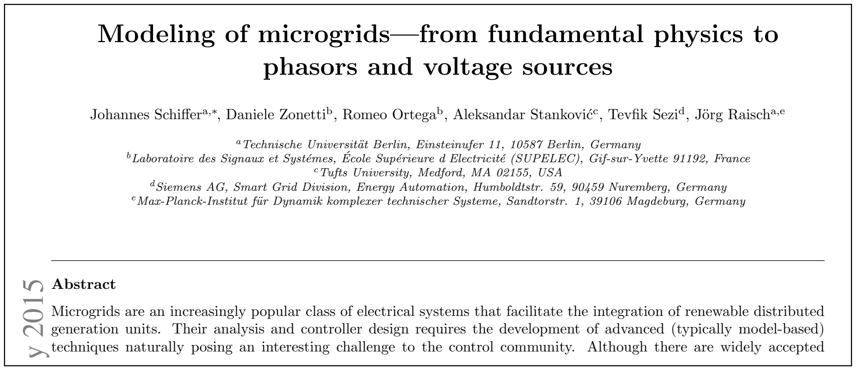
$$x_{dq0} = \begin{bmatrix} x_d(t) \\ x_q(t) \\ x_0(t) \end{bmatrix} = T(\omega\ \ t)x_{abc}(t) = \sqrt{\frac{3}{2}}A\begin{bmatrix} \sin(\delta_0) \\ \cos(\delta_0) \\ 0 \end{bmatrix}$$

- another rotation matrix reduces the signal to *q*-coordinate $\sqrt{3/2} \cdot A$

Long story short ...

We will work with **single-phase phasor signals** $x(t) = Ae^{i(\delta_0+\omega\ t)}$ representing the q-phase of a balanced, synchronous, 3-phase AC circuit.

Everything can be extended ... see, e.g., this control-theoretic tutorial:



Modeling of microgrids—from fundamental physics to phasors and voltage sources

Johannes Schiffer[a,*], Daniele Zonetti[b], Romeo Ortega[b], Aleksandar Stanković[c], Tevfik Sezi[d], Jörg Raisch[a,e]

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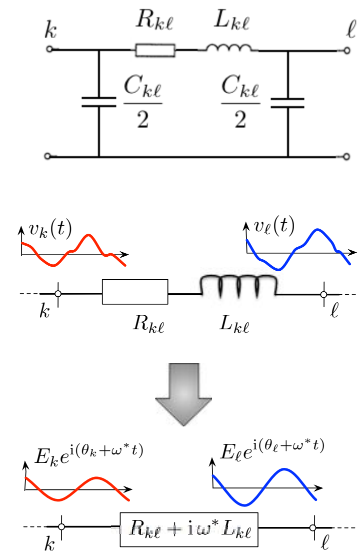
[b] Laboratoire des Signaux et Systémes, École Supérieure d Electricité (SUPELEC), Gif-sur-Yvette 91192, France
[c] Tufts University, Medford, MA 02155, USA
[d] Siemens AG, Smart Grid Division, Energy Automation, Humboldtstr. 59, 90459 Nuremberg, Germany
[e] Max-Planck-Institut für Dynamik komplexer technischer Systeme, Sandtorstr. 1, 39106 Magdeburg, Germany

Abstract

Microgrids are an increasingly popular class of electrical systems that facilitate the integration of renewable distributed generation units. Their analysis and controller design requires the development of advanced (typically model-based) techniques naturally posing an interesting challenge to the control community. Although there are widely accepted

AC circuits in power networks

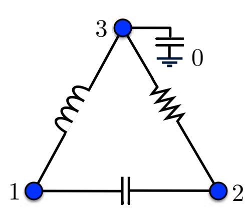
- power network modeled by linear **RLC** circuit, e.g., Π-model for
  - transmission lines (mainly inductive)
  - distribution lines (resistive/inductive)
  - cables (capacitive effects)
- we will work in **single-phase**
- **quasi-stationary modeling**: harmonic waveforms at nominal frequency ω
  - phasor signals: $v_k(t) \approx E_k e^{i(\theta_k+\omega^* t)}$
  - steady-state circuit: $\frac{d}{dt} L_{k\ell} \approx i\,\omega^* L_{k\ell}$



Note: quasi-stationarity assumption can be justified via singular perturbations & modeling can be improved using *dynamic phasors* [A. Stankovic & T. Aydin '00].

AC circuits – graph-theoretic modeling

1. a circuit is a connected & undirected **graph** $G = (\mathcal{V}, \mathcal{E})$
   - $\mathcal{V} = \{1, \dots, n\}$ are the nodes or *buses*
     - buses are partitioned as $\mathcal{V} = \{\text{sources}\} \cup \{\text{loads}\}$
     - the ground is sometimes explicitly modeled as node 0 or $n+1$
   - $\mathcal{E} \subseteq \{\{i,j\} : i,j \in \mathcal{V}\} = \mathcal{V} \times \mathcal{V}$ are the undirected edges or *branches*
     - edges between distinct nodes $\{i,j\}$ are called *lines*
     - edges $\{i,0\}$ connecting node i to ground are called *shunts*



$\mathcal{V} = \{1, 2, 3\}$

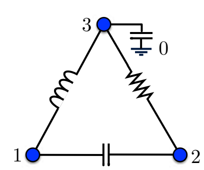
$\mathcal{E} = \{\{1,2\}, \{1,3\}, \{2,3\}, \{3,3\}\}$

AC circuits – the network admittance matrix

2. $Y = [Y_{ij}] \in \mathbb{C}^{n \times n}$ is the **network admittance matrix** with elements

$$Y_{ij} = \begin{cases} -\frac{1}{Z_{ij}} & \text{for off-diagonal elements } i \neq j \\ \frac{1}{Z_{i,\text{shunt}}} + \sum_{j \in \ni i} \frac{1}{Z_{ij}} & \text{for diagonal elements } i = j \end{cases}$$

- impedance = resistance + i · reactance: $Z_{ij} = R_{ij} + i \cdot X_{ij}$
- admittance = conductance + i · susceptance: $\frac{1}{Z_{ij}} = G_{ij} + i \cdot B_{ij}$

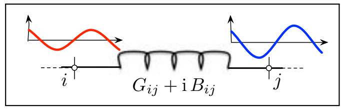


$$Y = \underbrace{\begin{bmatrix} \frac{1}{Z_{12}} + \frac{1}{Z_{13}} & -\frac{1}{Z_{12}} & -\frac{1}{Z_{13}} \\ -\frac{1}{Z_{12}} & \frac{1}{Z_{12}} + \frac{1}{Z_{23}} & -\frac{1}{Z_{23}} \\ -\frac{1}{Z_{13}} & -\frac{1}{Z_{23}} & \frac{1}{Z_{13}} + \frac{1}{Z_{23}} \end{bmatrix}}_{\text{network Laplacian matrix}} + \underbrace{\begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{1}{Z_{3,\text{shunt}}} \end{bmatrix}}_{\text{diag(shunts)}}$$

Note quasi-stationary modeling: $Z_{13} = i\,\omega\ L_{13}$ with nominal frequency ω

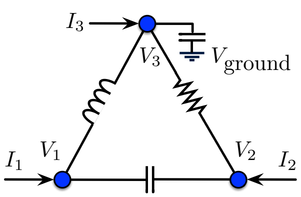
AC circuits – basic variables

3. **basic variables**: voltages & currents
   - on nodes: potentials & current injections
   - on edges: voltages & current flows



4. quasi-stationary **AC phasor coordinates** for harmonic waveforms:
   - e.g., complex voltage $V = E\, e^{\mathrm{i}\,\theta}$ denotes $v(t) = E\cos(\theta + \omega^* t)$

     where $V \in \mathbb{C}$, $E \in \mathbb{R}_{\geq 0}$, $\theta \in \mathbb{S}^1$, $\mathrm{i} = \sqrt{-1}$, and ω^* is nominal frequency



external injections: I_1, I_2, I_3

potentials: V_1, V_2, V_3

reference: $V_{\text{ground}} = 0\text{V}$

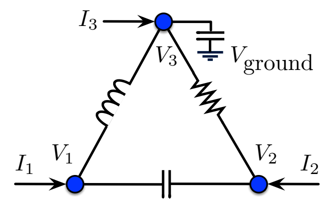
AC circuits – fundamental equations

5. **Ohm's law** at every branch: $I_{i \to j} = \frac{1}{Z_{ij}}(V_i - V_j)$

6. **Kirchhoff's current law** for every bus: $I_i + \sum_j I_{j \to i} = 0$

7. **current balance equations** (treating the ground as node with 0V):

   $I_i = -\sum_j I_{j \to i} = \sum_j \frac{1}{Z_{ij}}(V_i - V_j) = \sum_j Y_{ij} V_j$ or $I = Y \cdot V$

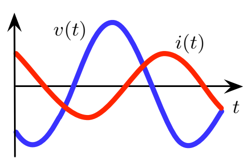


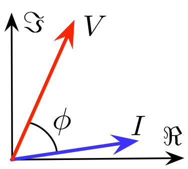
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

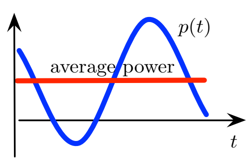
Note: all variables are in per unit (p.u.) system, i.e., normalized wrt base voltage

AC circuits – power

(see also exercises)







- voltage phasor: $V = |V| e^{\mathrm{i}\,\theta_V} \Leftrightarrow v(t) = |V|\cos(\omega\ t + \theta_V)$

  current phasor: $I = |I| e^{\mathrm{i}\,\theta_I} \Leftrightarrow i(t) = |I|\cos(\omega\ t + \theta_I)$

- **instantaneous power**:

$$p(t) = v(t) \cdot i(t) = \frac{1}{2} \cdot |V| \cdot |I| \cdot \cos(\theta_V - \theta_I) + \frac{1}{2} \cdot |V| \cdot |I| \cdot \cos(2\omega\ t + \theta_V + \theta_I)$$

$\Rightarrow$ **active power** (average): $P = \frac{1}{T}\int_0^T v(t) \cdot i(t)\, dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \cos(\phi)$

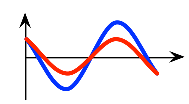
$\Rightarrow$ **reactive power** (0-avg): $Q = \frac{1}{T}\int_0^T v(t) \cdot i(t - \frac{T}{4})\, dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \sin(\phi)$

AC circuits – complex power

(see also exercises)

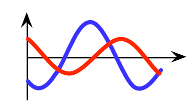
8. **active & reactive power** in AC circuits:
   - active (average) power:

     $$P = \frac{1}{T}\int_0^T v(t) \cdot i(t)\, dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \cos(\phi)$$

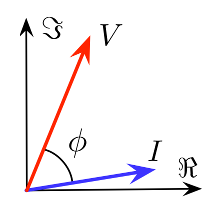


   - reactive (0-average) power:

     $$Q = \frac{1}{T}\int_0^T v(t) \cdot i(t - T/4)\, dt = \frac{1}{2} \cdot |V| \cdot |I| \cdot \sin(\phi)$$



$\Rightarrow$ normalize phasors: $V \mapsto 1/\sqrt{2} \cdot |V| e^{\mathrm{i}\,\theta_V}$



$\Rightarrow$ **complex power:** $S = V \cdot \bar{I} = P + \mathrm{i}Q$

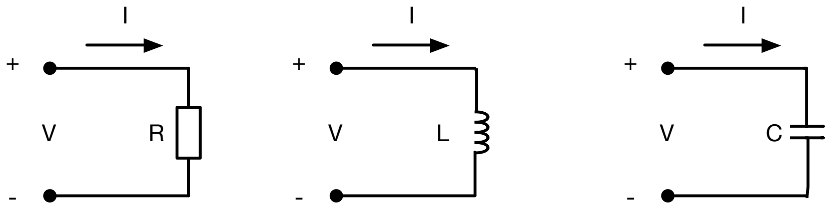
$=$ active power $+\, \mathrm{i}\, \cdot$ reactive power

$\Rightarrow$ $\cos(\phi) = P/|S|$ is power factor

Note: often complex phasors are implicitly normalized $\tilde{V} = 1/\sqrt{2} \cdot E e^{\mathrm{i}\,\theta}$

AC circuits – power dissipated by RLC loads

details in exercises



Power dissipation $S = V \cdot \bar{I} = P + \mathrm{i}Q$ (network sign convention):

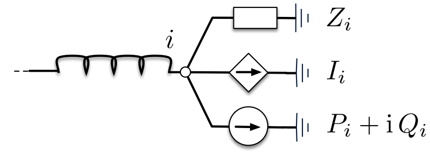
$$S = -\frac{1}{2}|I|^2 R = -\frac{1}{2}\frac{|V|^2}{R} = P < 0$$

$$S = -\frac{1}{2}|I|^2 \cdot \mathrm{i}\omega L = -\mathrm{i}\frac{1}{2}\frac{|V|^2}{\omega L} = Q < 0$$

$$S = \mathrm{i}\frac{1}{2}\frac{|I|^2}{\omega C} = \frac{1}{2}|V|^2 \cdot \mathrm{i}\omega C = Q > 0$$

Static models loads

- aggregated **ZIP load model**:
  constant impedance **Z** +
  constant current **I** +
  constant power **P**



- more general **exponential load model**: power $= const. \cdot (V/V_{\mathrm{ref}})^{const.}$
  (combinations & variations learned from data)
- various dynamic load models for stability studies . . .

"Just use whatever load model fits your mathematics. You will get it wrong anyways." — [Ian Hiskens, lunch @ Zürich '15]

Static models for sources

- most common **static load model** is constant active power demand P and constant reactive power demand Q
- conventional **synchronous generators** are controlled to have constant active power output P and voltage magnitude E
- sources interfaced with **power electronics** are typically controlled to have constant active power P and reactive power Q

⇒ common bus device models

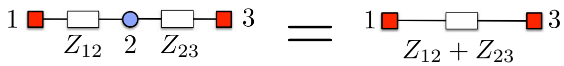
1. **PQ** buses have complex power $S = P + \mathrm{i}Q$ specified
2. **PV** buses have active power P and voltage magnitude E specified
3. **slack buses** have E and θ specified (not really existent)

Kron Reduction of Circuits

Kron reduction

[G. Kron 1939]

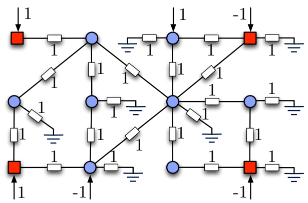
often (almost always) you will encounter Kron-reduced network models



General procedure:

0. convert const. power injections <u>locally</u> to shunt impedances $Z = S/V_{\text{ref}}^2$
1. partition linear current-balance equations via boundary & interior nodes
   (arises naturally, e.g., sources & loads, measurement terminals, etc.)

$$\begin{bmatrix} I_{\text{boundary}} \\ I_{\text{interior}} \end{bmatrix} = \begin{bmatrix} Y_{\text{boundary}} & Y_{\text{bound-int}} \\ Y_{\text{bound-int}}^T & Y_{\text{interior}} \end{bmatrix} \begin{bmatrix} V_{\text{boundary}} \\ V_{\text{interior}} \end{bmatrix}$$

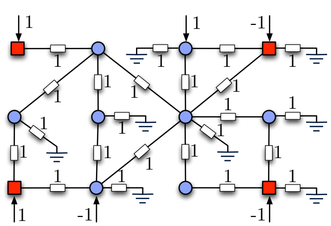


Kron reduction cont'd

on blackboard

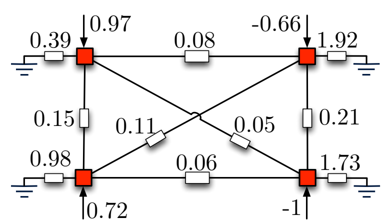
Kron reduction cont'd

2. Gaussian elimination of interior voltages



original circuit

$I = Y \cdot V$



"equivalent" reduced circuit

$I_{\text{red}} = Y_{\text{red}} \cdot V_{\text{boundary}}$

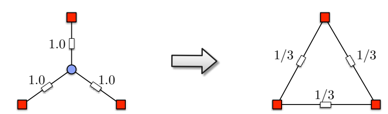
⇒ reduced Y-matrix: $Y_{\text{red}} = Y_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{\ 1} \cdot Y_{\text{bound-int}}^T$

⇒ reduced injections: $I_{\text{red}} = I_{\text{boundary}} - Y_{\text{bound-int}} \cdot Y_{\text{interior}}^{\ 1} \cdot I_{\text{interior}}$

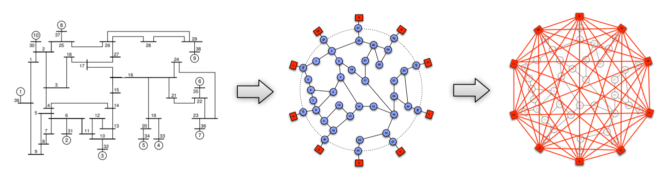
Examples of Kron reduction

algebraic properties are preserved but the network changes significantly

- Star-Δ transformation [A. E. Kennelly 1899, A. Rosen '24]



- Kron reduction of load buses in *IEEE 39 New England power grid*

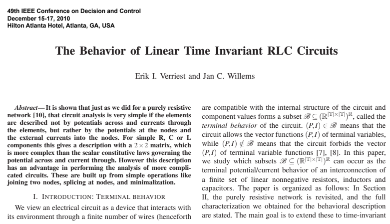


⇒ topology without weights is meaningless!

⇒ shunt resistances (loads) are mapped to line conductances

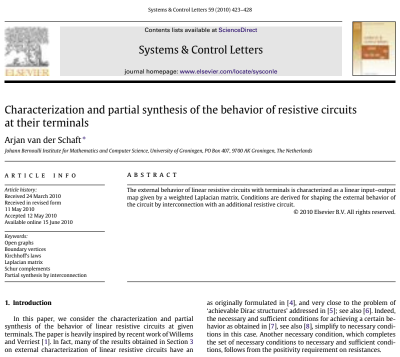
⇒ many properties still open [FD & F. Bullo '13, S. Caliskan & P. Tabuada '14]

Kron reduction – so simple yet still full of mysteries


49th IEEE Conference on Decision and Control
December 15-17, 2010
Hilton Atlanta Hotel, Atlanta, GA, USA

The Behavior of Linear Time Invariant RLC Circuits

Erik I. Verriest and Jan C. Willems


Systems & Control Letters 59 (2010) 423–428

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Systems & Control Letters

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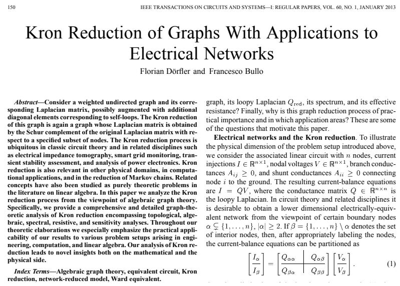
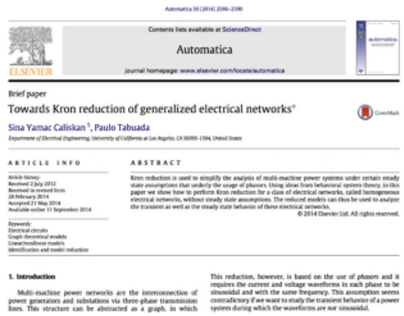
Characterization and partial synthesis of the behavior of resistive circuits at their terminals

Arjan van der Schaft


IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: REGULAR PAPERS, VOL. 60, NO. 1, JANUARY 2013

Kron Reduction of Graphs With Applications to Electrical Networks

Florian Dörfler and Francesco Bullo


Automatica 50 (2014) 2586–2590

Contents lists available at ScienceDirect

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Brief paper

Towards Kron reduction of generalized electrical networks

Sina Yamac Caliskan, Paulo Tabuada

Power Flow Formulations & Approximations

Power balance eqn's: "power injection = Σ power flows"

1. **complex form:** $S_i = V_i \overline{I}_i = \sum_j V_i \overline{Y}_{ij} \overline{V}_j$ or $S = \mathrm{diag}(V)\overline{Y}\,\overline{V}$

   ⇒ purely quadratic and useful for static calculations & optimization

2. **rectangular form:** insert $V = e + if$ and split real & imaginary parts:

   active power: $P_i = \sum_j B_{ij}(e_i f_j - f_i e_j) + G_{ij}(e_i e_j + f_i f_j)$

   reactive power: $Q_i = -\sum_j B_{ij}(e_i e_j + f_i f_j) + G_{ij}(e_i f_j - f_i e_j)$

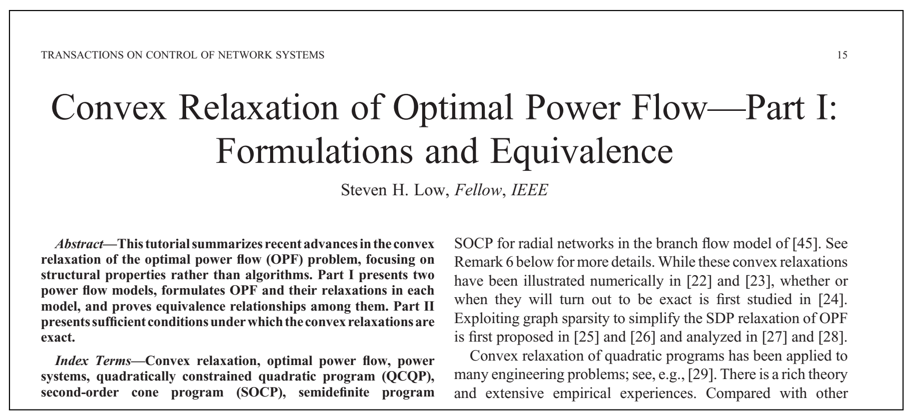
   ⇒ purely quadratic and useful for homotopy methods & QCQPs

⇒ main complexity is quadratic nonlinearity $V_i \overline{V}_j = \begin{bmatrix} e & if \end{bmatrix} \cdot \begin{bmatrix} e & -if \end{bmatrix}^T$

Power balance eqn's – cont'd

3. **matrix form:** define unit-rank p.s.d. Hermitian matrix $W = V \cdot \overline{V}^T$ with components $W_{ij} = V_i \overline{V}_j$, then power flow is $S_i = \sum_j \overline{Y}_{ij} W_{ij}$

   ⇒ linear and useful for relaxations in convex optimization problems


TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS 15

Convex Relaxation of Optimal Power Flow—Part I: Formulations and Equivalence

Steven H. Low, *Fellow, IEEE*

Abstract—**This tutorial summarizes recent advances in the convex relaxation of the optimal power flow (OPF) problem, focusing on structural properties rather than algorithms. Part I presents two power flow models, formulates OPF and their relaxations in each model, and proves equivalence relationships among them. Part II presents sufficient conditions under which the convex relaxations are exact.**

Index Terms—**Convex relaxation, optimal power flow, power systems, quadratically constrained quadratic program (QCQP), second-order cone program (SOCP), semidefinite program**

SOCP for radial networks in the branch flow model of [45]. See Remark 6 below for more details. While these convex relaxations have been illustrated numerically in [22] and [23], whether or when they will turn out to be exact is first studied in [24]. Exploiting graph sparsity to simplify the SDP relaxation of OPF is first proposed in [25] and [26] and analyzed in [27] and [28].

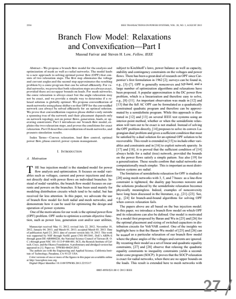
Convex relaxation of quadratic programs has been applied to many engineering problems; see, e.g., [29]. There is a rich theory and extensive empirical experiences. Compared with other

Power balance eqn's – cont'd

4. **branch flow eqn's** parameterized in flow variables [M. Baran & F. Wu '89]:
   - Ohm's law: $V_i - V_j = Z_{ij} I_{i\to j}$
   - branch power flow $i \to j$: $S_{i\to j} = V_i \cdot \overline{I_{i\to j}}$
   - power balance at node i:

$$\underbrace{\sum_{k:\, i\to k} S_{i\to k} + Y_{i,\text{shunt}}|V_i|^2}_{\text{outgoing flows}} = S_i + \underbrace{\sum_{j:\, j\to i} \left(S_{j\to i} - Z_{ij}|I_{i\to j}|^2\right)}_{\text{incoming flows}}$$

- **DistFlow formulation** in terms of square magnitude variables $|V_i|^2$ and $|I_{i\to j}|^2$
  (missing angle variables $\angle V_i$ and $\angle I_{i\to j}$ can sometimes be recovered, e.g., in acyclic case)
- lossless approximation can be solved exactly in acyclic networks (useful for distribution networks)
  [M. Baran & F. Wu '89, M. Farivar, L. Chen, & S. Low '13]


Branch Flow Model: Relaxations and Convexification—Part I
Masoud Farivar and Steven H. Low, Fellow, IEEE

Power balance eqn's – cont'd

5. **polar form:** insert $V = Ee^{i\theta}$ and split real & imaginary parts:

active power:	$P_i =$	$\sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
reactive power:	$Q_i =$	$-\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

$\Rightarrow$ will be our focus these days since ...

- **power system specs** on frequency $\frac{d}{dt}\theta(t)$ and voltage magnitude E
- **dynamics:** generator swing dynamics affect voltage phase angles
  & voltage magnitudes are controlled to be constant
- **physical intuition:** usual operation near flat voltage profile $V_i \approx 1e^{i\phi}$
  which give rise to various insights for analysis & design (later)

Power flow simplifications & approximations

power flow equations are too complex & unwieldy for analysis & large computations

- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

1. **lossless** transmission lines $R_{ij}/X_{ij} = -G_{ij}/B_{ij} \approx 0$

active power:	$P_i =$	$\sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$
reactive power:	$Q_i =$	$-\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$

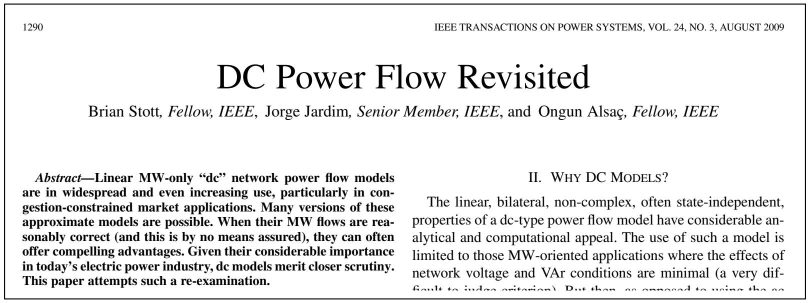
2. **decoupling** near operating point $V_i \approx 1e^{i\phi}$: $\begin{bmatrix} \partial P/\partial\theta & \partial P/\partial E \\ \partial Q/\partial\theta & \partial Q/\partial E \end{bmatrix} \approx \begin{bmatrix} \star & 0 \\ 0 & \star \end{bmatrix}$

active power:	$P_i =$	$\sum_j B_{ij} \sin(\theta_i - \theta_j)$	(function of angles)
reactive power:	$Q_i =$	$-\sum_j B_{ij} E_i E_j$	(function of magnitudes)

Power flow simplifications & approximations cont'd

3. **linearization** for small flows near operating point $V_i \approx 1e^{i\phi}$:

active power:	$P_i =$	$\sum_j B_{ij}(\theta_i - \theta_j)$	known as **DC power flow**
reactive power: :	$Q_i =$	$\sum_j B_{ij}(E_i - E_j)$	(if formulated in p.u. system)


1290 IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 24, NO. 3, AUGUST 2009

DC Power Flow Revisited

Brian Stott, *Fellow, IEEE*, Jorge Jardim, *Senior Member, IEEE*, and Ongun Alsaç, *Fellow, IEEE*

***Abstract*—Linear MW-only "dc" network power flow models are in widespread and even increasing use, particularly in congestion-constrained market applications. Many versions of these approximate models are possible. When their MW flows are reasonably correct (and this is by no means assured), they can often offer compelling advantages. Given their considerable importance in today's electric power industry, dc models merit closer scrutiny. This paper attempts such a re-examination.**

II. Why DC Models?

The linear, bilateral, non-complex, often state-independent, properties of a dc-type power flow model have considerable analytical and computational appeal. The use of such a model is limited to those MW-oriented applications where the effects of network voltage and VAr conditions are minimal (a very dif-

Conclusion on the **most limiting assumption** of DC power flow: $R/X \approx 0$

Power flow simplifications & approximations cont'd

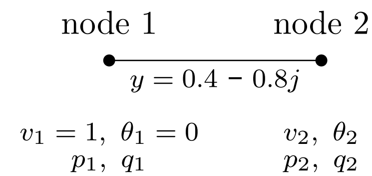
- active power: $P_i \;=\; \sum_j B_{ij}E_iE_j\sin(\theta_i-\theta_j)+G_{ij}E_iE_j\cos(\theta_i-\theta_j)$
- reactive power: $Q_i \;=\; -\sum_j B_{ij}E_iE_j\cos(\theta_i-\theta_j)+G_{ij}E_iE_j\sin(\theta_i-\theta_j)$

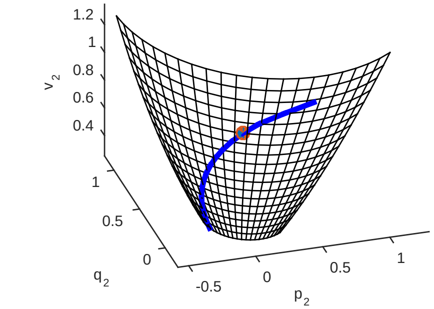
4. Multiple **variations & combinations** of DC power flow
   - power flow transformation for constant R/X ratios (see exercise)
   - linearization & decoupling at arbitrary operating points [D. Deka et al., '15]
   - advanced linearizations especially for reactive power [S. Bolognani & S. Zampieri '12, B. Gentile et al. '14, J. Simpson-Porco et al. '16]
   - linearizations in rectangular coordinates (more accurate for active power) [R. Baldick '13, S. Bolognani & S. Zampieri '15, S. Dhople et al. '15]

"... plenty of heuristics in industry ... especially for approximation of losses."

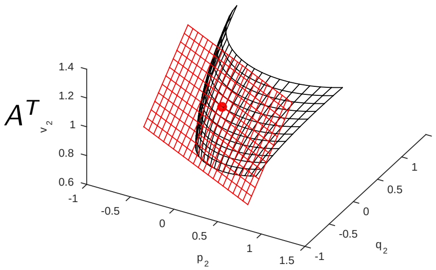
— [Bruce Wollenberg, meeting @ Minneapolis '13]

A unifying geometric perspective [S. Bolognani & F. Dörfler '15]





1. **variables:** all of $x=(E,\theta,p,q)$
2. **power flow manifold:** $F(x)=\mathbb{0}$
3. **normal space** spanned by $\left.\frac{\partial F(x)}{\partial x}\right|_x = A^T$
4. **tangent space** $A(x-x\;)=\mathbb{0}$ is best linear approximant at x
5. **accuracy** depends on curvature $\frac{\partial^2 F(x)}{\partial x^2}$



Closer look at implicit formulae $A(x-x^*)=\mathbb{0}$

$$\left[\underbrace{\left(\langle \mathrm{diag}\,\overline{YE}\;\rangle\right.}_{\text{shunt loads}} + \underbrace{\left.\langle \mathrm{diag}\,E\;\rangle N\langle Y\rangle\right)}_{\text{lossy DC flow}} \cdot \underbrace{\begin{bmatrix}\mathrm{diag}(\cos\theta\;) & -\mathrm{diag}(E\;)\,\mathrm{diag}(\sin\theta\;)\\ \mathrm{diag}(\sin\theta\;) & \mathrm{diag}(E\;)\,\mathrm{diag}(\cos\theta\;)\end{bmatrix}}_{\text{rotation }\times\text{ scaling at operating point}}\right]$$

$$\times \underbrace{\begin{bmatrix}v-v\\ \theta-\theta\end{bmatrix} = \begin{bmatrix}p-p\\ q-q\end{bmatrix}}_{\text{deviation variables}}$$

where $N=\begin{bmatrix}I & 0\\ 0 & -I\end{bmatrix}$ is complex conjugate in real coordinates

and $\langle A\rangle = \begin{bmatrix}\Re(A) & -\Im(A)\\ \Im(A) & \Re(A)\end{bmatrix}$ is complex rotation in real coordinates.

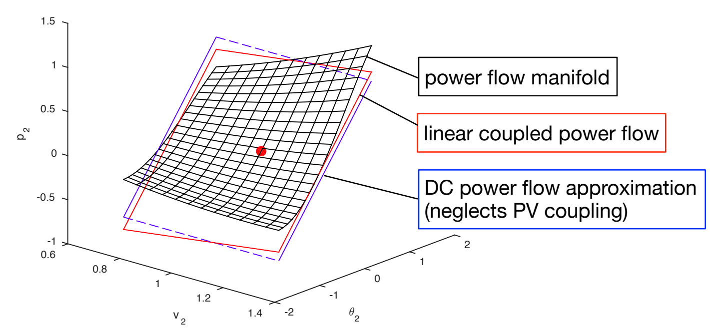
Special cases reveal some old friends I

- **flat-voltage/0-injection point:** $x\;\;=(E\;,\theta\;,P\;,Q\;)=(\mathbb{1},\mathbb{0},\mathbb{0},\mathbb{0})$

$\Rightarrow$ implicit linearization: $\begin{bmatrix}\Re(Y) & -\Im(Y)\\ -\Im(Y) & \Re(Y)\end{bmatrix}\begin{bmatrix}v\\ \theta\end{bmatrix}=\begin{bmatrix}p\\ q\end{bmatrix}$

is **linear coupled power flow** [D. Deka, S. Backhaus, & M. Chertkov, '15]

$\Rightarrow$ $\Re(Y)=\mathbb{0}$ gives **DC power flow:** $-\Im(Y)\theta=P$ and $-\Im(Y)E=Q$



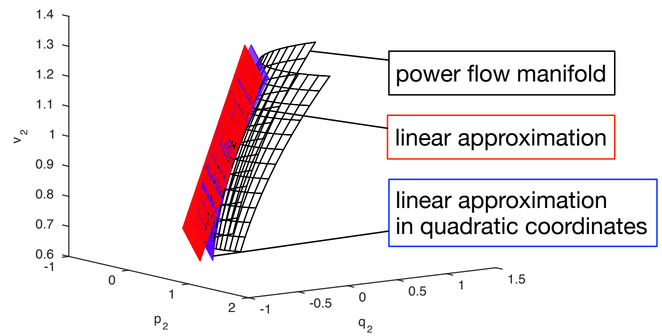
Special cases reveal some old friends II

- **flat-voltage/0-injection point:** $x = (E, \theta, P, Q) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

⇒ rectangular coord. ⇒ **rectangular DC flow** [S. Bolognani & S. Zampieri, '15]

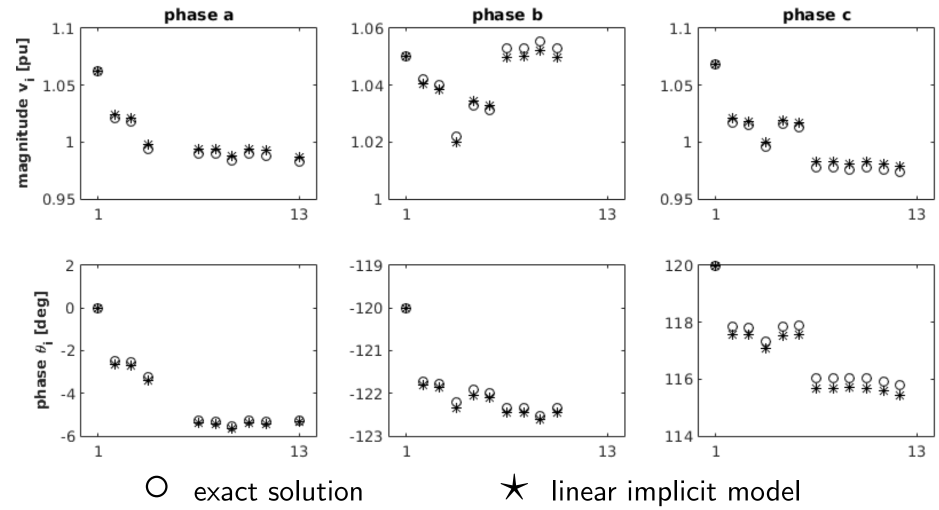
- nonlinear change to **quadratic coordinates** from v_h to v_h^2

⇒ linearization gives (non-radial) **LinDistFlow** [M.E. Baran & F.F. Wu, '88]



Accuracy illustrated with unbalanced three-phase IEEE13

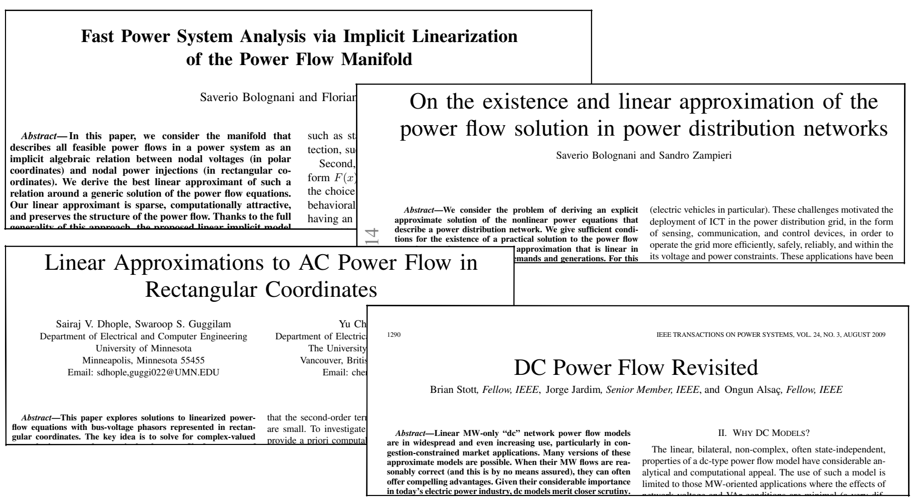
can be extended to three-phase, exponential loads, etc.



Matlab/Octave code @ `https://github.com/saveriob/1ACPF`

Plenty of recent interest in power flow approximations

mainly for the sake of verifying analytic approaches


Fast Power System Analysis via Implicit Linearization of the Power Flow Manifold

Saverio Bolognani and Florian

On the existence and linear approximation of the power flow solution in power distribution networks

Saverio Bolognani and Sandro Zampieri

Linear Approximations to AC Power Flow in Rectangular Coordinates

Sairaj V. Dhople, Swaroop S. Guggilam
Department of Electrical and Computer Engineering
University of Minnesota
Minneapolis, Minnesota 55455
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DC Power Flow Revisited

Brian Stott, *Fellow, IEEE*, Jorge Jardim, *Senior Member, IEEE*, and Ongun Alsaç, *Fellow, IEEE*

Once you try to analyze power flow equations with pen and paper, you will realize ...

"Maybe we should revisit the way we write power flow equations." — [Göran Andersson, Santa Fe Grid Science Workshop '15]

Once you work computationally with data, you will see ...

"The devil introduced the per unit system into power." — [Peter Sauer, ACC '12]

Dynamic Network Component Models

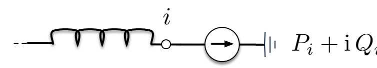
Modeling the "essential" network dynamics

models can be arbitrarily detailed & vary on different time/spatial scales

1. active and reactive **power flow** (e.g., lossless)

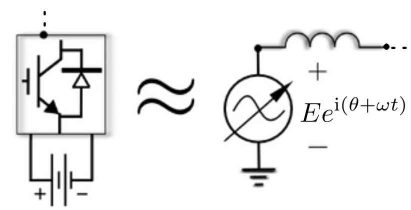
$$P_{i,\text{inj}} = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_{i,\text{inj}} = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

2. passive constant power **loads**



$$P_{i,\text{inj}} = P_i = const.$$
$$Q_{i,\text{inj}} = Q_i = const.$$

3. **inverters**: DC or variable AC sources with power electronics



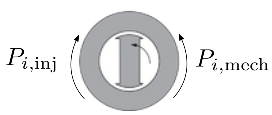
(i) have constant/controllable PQ (max. power-point tracking)

(ii) or mimic generators (more later)

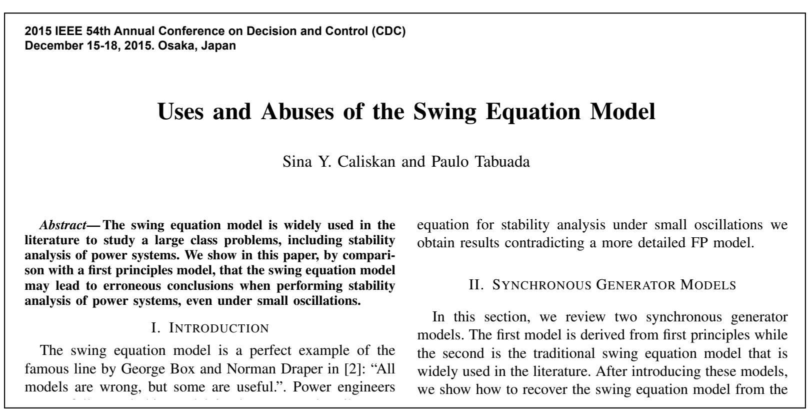
Modeling the "essential" synchronous generator dynamics

4. electromech. **swing dynamics** of synchronous machines

$$M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{i,\text{mech}} - P_{i,\text{inj}}$$
$$E_i = const.$$



(can be derived from first principle model & some (possibly strong) assumptions)



2015 IEEE 54th Annual Conference on Decision and Control (CDC)
December 15-18, 2015. Osaka, Japan

Uses and Abuses of the Swing Equation Model

Sina Y. Caliskan and Paulo Tabuada

Abstract—**The swing equation model is widely used in the literature to study a large class problems, including stability analysis of power systems. We show in this paper, by comparison with a first principles model, that the swing equation model may lead to erroneous conclusions when performing stability analysis of power systems, even under small oscillations.**

I. Introduction

The swing equation model is a perfect example of the famous line by George Box and Norman Draper in [2]: "All models are wrong, but some are useful.". Power engineers

equation for stability analysis under small oscillations we obtain results contradicting a more detailed FP model.

II. Synchronous Generator Models

In this section, we review two synchronous generator models. The first model is derived from first principles while the second is the traditional swing equation model that is widely used in the literature. After introducing these models, we show how to recover the swing equation model from the

Common variations in dynamic network models

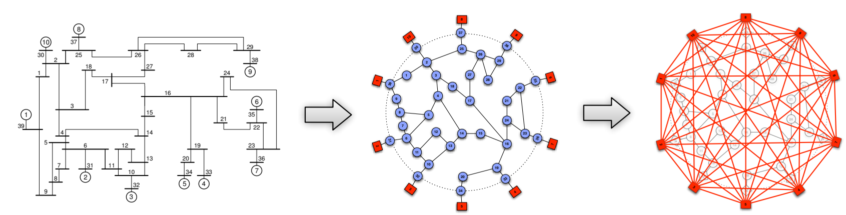
dynamic behavior is very much dependent on load models & generator models

1. frequency/voltage-depend. loads [A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

$$D_i \dot{\theta}_i + P_i = -P_{i,\text{inj}}$$
$$f_i(\dot{V}_i) + Q_i = -Q_{i,\text{inj}}$$

2. network-reduced models after Kron reduction of loads [H. Chiang, F. Wu, & P. Varaiya '94] (very common but poor assumption: $G_{ij} = 0$)

$$M_i \ddot{\theta}_i + D \dot{\theta}_i = P_{i,\text{mech}} - \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) \underbrace{- \sum_j G_{ij} E_i E_j \cos(\theta_i - \theta_j)}_{\text{effect of resistive loads}}$$



Structure-preserving power network model [A. Bergen & D. Hill '81]

without Kron-reduction of load buses

- **generator swing dynamics:**

$$\dot{\theta}_i = \omega_i$$
$$M_i \dot{\omega}_i = -D_i \omega_i + P_i - \sum\nolimits_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

- **frequency-dependent loads:**
  (or inverter-interfaced sources)

$$D_i \dot{\theta}_i = P_i - \sum\nolimits_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

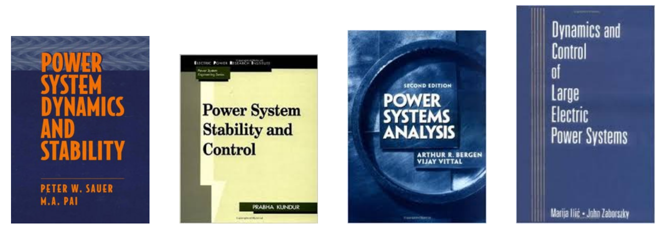
- in **academia:** this "baseline model" is typically further simplified: decoupling, linearization, constant voltages, ...
- in **industry:** much more detailed models used for grid simulations

⇒ **IMHO:** above model captures most interesting network dynamics

Common variations in dynamic network models — cont'd

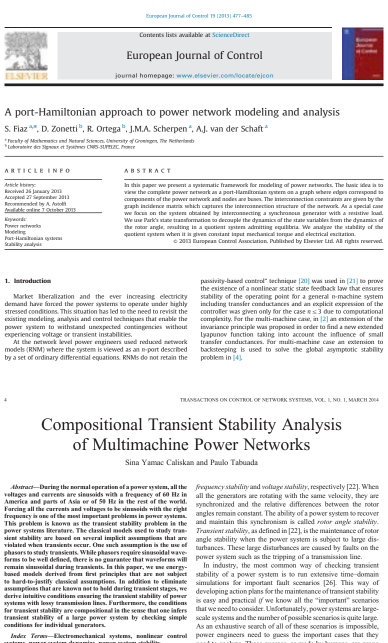
dynamic behavior is very much dependent on load models & generator models

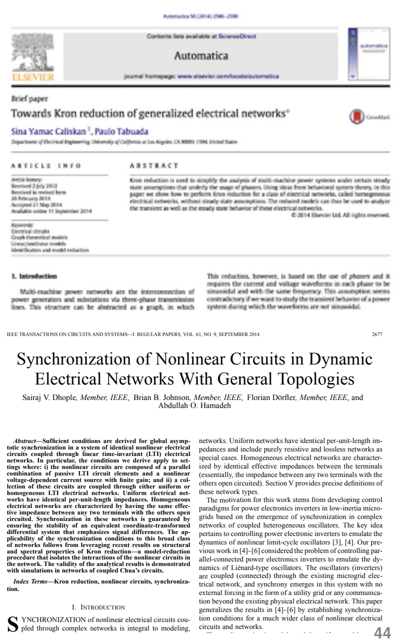
3. higher order generator dynamics [P. Sauer & M. Pai '98] — voltages, controls, magnetics etc. (reduction via singular perturbations)
4. dynamic & detailed load models [D. Karlsson & D. Hill '94] — aggregated dynamic load behavior (e.g., load recovery after voltage step)
5. time-domain models [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12] — passive Port-Hamiltonian models for machines & RLC circuitry



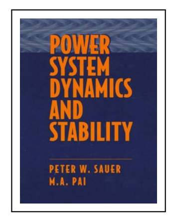
"Power system research is all about the art of making the right assumptions."

Lots of current research activity on time-domain models





On the swing equation ...



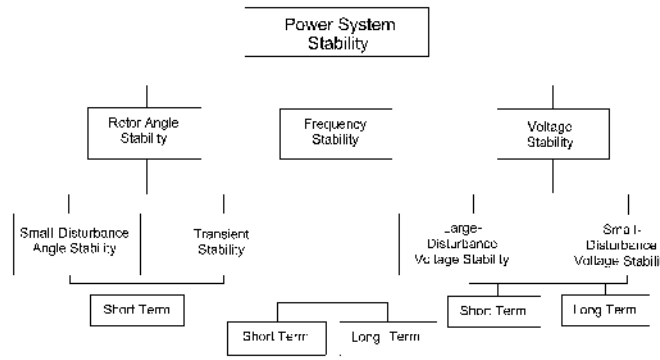
"There is probably more literature on synchronous machines than on any other device in electrical engineering." — [Peter Sauer & M.A. Pai, Power System Dynamics and Stability '98]

"The swing equation model is a perfect example of the famous line [...]: "All models are wrong, but some are useful.""

— [Sina Y. Caliskan and Paulo Tabuada, CDC '15]

Outline

One system with many dynamics & control problems



"From a practical viewpoint, there are four major analytical problems: . . . compute equilibria ... transient stability . . . [inter-area] oscillations . . . voltage collapse. Of course, theoretically they are all aspects of the one overall stability question." — [David Hill, ISCAS '06]

prelims on power flow

Preliminary insights on lossless power flow

power flow equations:

$$P_i = \sum_{j=1}^{n} B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_i = -\sum_{j=1}^{n} B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

$\Rightarrow$ solution space: $\mathbb{T}^n \times \mathbb{R}^n{}_0 = (\mathbb{S}^1 \times \cdots \times \mathbb{S}^1) \times (\mathbb{R}_{\ 0} \times \cdots \times \mathbb{R}_{\ 0})$

rotational symmetry:

if θ is a solution $\Rightarrow$ $\theta + const. \cdot \mathbb{1}_n$ is another solution

$\Rightarrow$ solution space "modulo rotational symmetry": $\mathbb{T}^n \setminus \mathbb{S}^1 \times \mathbb{R}^n{}_0$

index shenanigans:

- active flow $i \to i = B_{ii} E_i E_j \sin(\theta_i - \theta_i) = 0$ ($\Rightarrow$ can drop index i)
- reactive flow $i \to i = -B_{ii} E_i E_j \cos(\theta_i - \theta_i) = -B_{ii} E_i^2$

Preliminary feasibility conditions for lossless power flow

see exercises for details

power flow equations:

$$P_i = \sum_{j=1}^{n} B_{ij} E_i E_j \sin(\theta_i - \theta_j)$$

$$Q_i = -\sum_{j=1}^{n} B_{ij} E_i E_j \cos(\theta_i - \theta_j)$$

necessary feasibility condition I:

$\sum_{i=1}^{n} P_i = 0 \;\Leftarrow\; \exists$ a solution

$\triangleq$ power balance

$\Rightarrow$ typically not true (w/o slack bus) due to unknown load demand

$\Rightarrow$ need to consider dynamics

necessary feasibility condition II:

$\sum_{i=1}^{n} Q_i \geq 0 \;\Leftarrow\; \exists$ a solution

$\triangleq$ reactive power losses

$\Rightarrow$ reactive power must be supplied

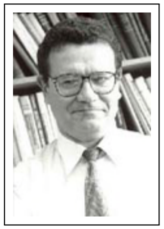
(for inductive grid w/o shunts)

Feasibility power flow is crucial for system operation

Given: network parameters & topology and load & generation profile

Q: "$\exists$ an optimal, stable, and robust synchronous operating point ?"

1. Security analysis [Araposthatis et al. '81, Wu et al. '80 & '82, Ilić '92, ...]
2. Load flow feasibility [Chiang et al. '90, Dobson '92, Lesieutre et al. '99, ...]
3. Optimal generation dispatch [Lavaei et al. '12, Bose et al. '12, ...]
4. Transient stability [Sastry et al. '80, Bergen et al. '81, Hill et al. '86, ...]
5. Inverters in microgrids [Chandorkar et. al. '93, Guerrero et al. '09, Zhong '11,...]
6. Complex networks [Hill et al. '06, Strogatz '01, Arenas et al '08, ...]

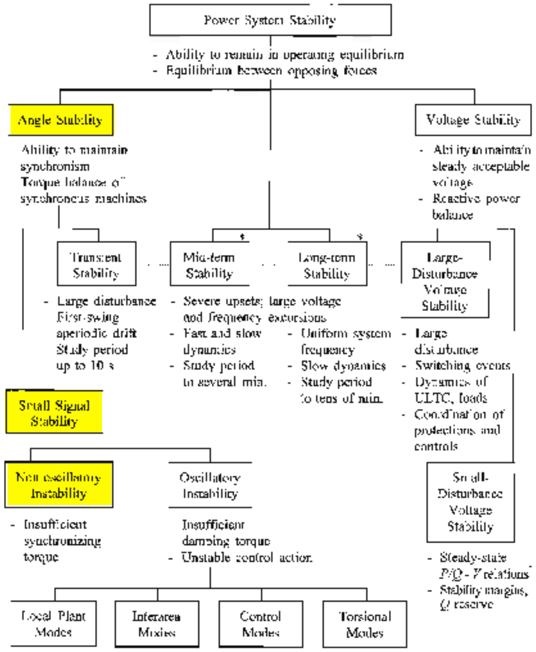


"How do we quantitatively measure feasibility in order to incorporate this attribute in the system design or operation? How do we explicitly describe the region of feasibility in general, and in particular in a large neighborhood around the normal operating injections?"

— [J. Jaris & F. Galiana, IEEE PAS '81]

Decoupled Active Power Flow (Synchronization)

Our first stab at power system stability



Synchronization & feasibility of active power flow

sync is crucial for the functionality and operation of the power grid

- **structure-preserving power network model** [A. Bergen & D. Hill '81]:

synchronous machines: $$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

frequency-dependent loads: $$D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

- **synchronization** = sync'd frequencies & bounded active power flows

$$\dot{\theta}_i = \omega_{\text{sync}} \; \forall\, i \in \mathcal{V} \quad \& \quad |\theta_i - \theta_j| \le \gamma < \pi/2 \; \forall\, \{i,j\} \in \mathcal{E}$$

= active power flow feasibility & security constraints

- **explicit sync frequency:** if sync, then $\omega_{\text{sync}} = \sum_i P_i / \sum_i D_i$

(by summing over all equations)

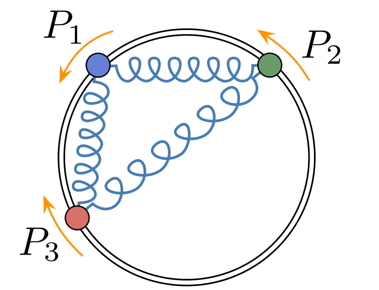
A perspective from coupled oscillators

Mechanical oscillator network

Angles $(\theta_1, \dots, \theta_n)$ evolve on $\mathbb{T}^n$ as

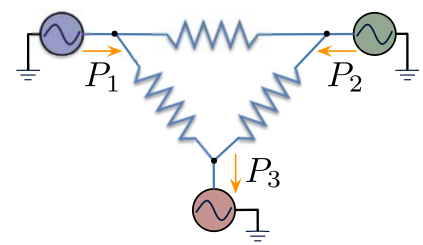
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $P_i \in \mathbb{R}$
- spring constants $B_{ij} \ge 0$



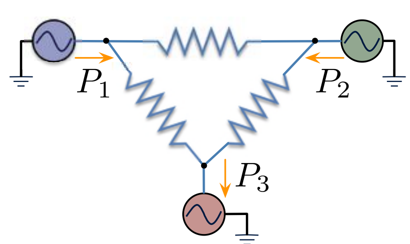
Structure-preserving power network

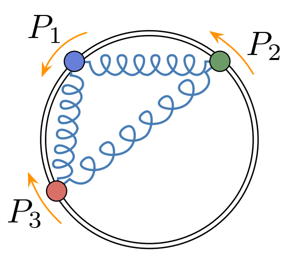
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$
$$D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$



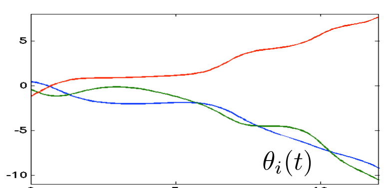
Phenomenology of sync in power networks

- sync is **crucial for AC power grids**

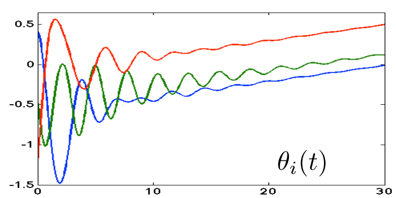




- sync is a **trade-off**



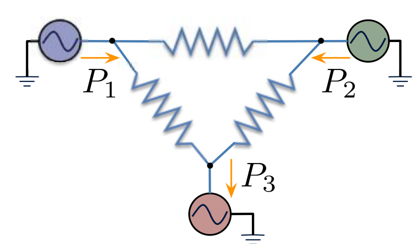
weak coupling & heterogeneous



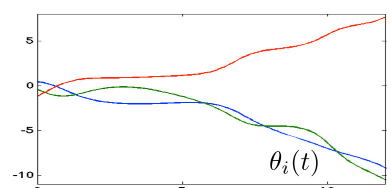
strong coupling & homogeneous

Phenomenology of sync in power networks

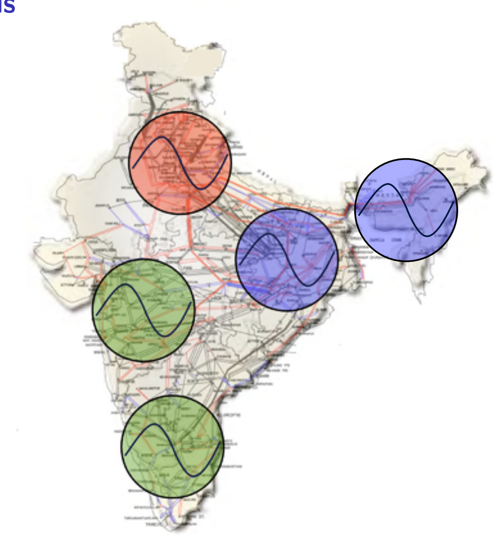
- sync is **crucial for AC power grids**



- sync is a **trade-off**



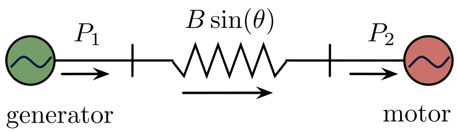
weak coupling & heterogeneous

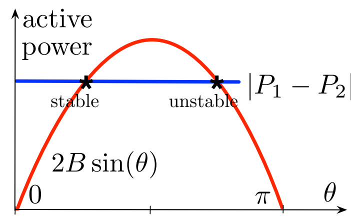


Blackout India July 30/31 2012

Back of the envelope calculations for the two-node case

generator connected to identical motor shows bifurcation at difference angle $\theta = \pi/2$



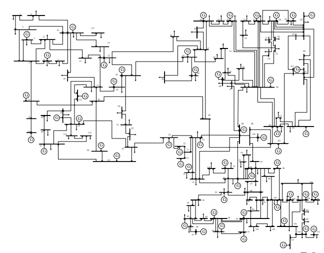


$$M\ddot{\theta} + D\dot{\theta} = P_1 - P_2 - 2B\sin(\theta)$$

$\exists$ stable sync $\Leftrightarrow$ $B > |P_1 - P_2|/2$ $\Leftrightarrow$ "ntwk coupling > heterogeneity"

Q1: Quantitative generalization to a complex & large-scale network?

Q2: What are the particular metrics for coupling and heterogeneity?



Who knows consensus systems?

on blackboard

Primer on algebraic graph theory

for a connected and undirected graph

Laplacian matrix $L =$ "degree matrix" $-$ "adjacency matrix"

$$L = L^T = \begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -B_{i1} & \cdots & \sum_{j=1}^{n} B_{ij} & \cdots & -B_{in} \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix} \geq 0$$

is positive semidefinite with one zero eigenvalue & eigenvector $\mathbb{1}_n$

Notions of connectivity

- spectral: 2nd smallest eigenvalue of L is "algebraic connectivity" $\lambda_2(L)$
- topological: degree $\sum_{j=1}^{n} B_{ij}$ or degree distribution

Notions of heterogeneity

$\|P\|_{E,1} = \max_{\{i,j\}\in E} |P_i - P_j|$, $\quad \|P\|_{E,2} = \left(\sum_{\{i,j\}\in E} |P_i - P_j|^2\right)^{1/2}$

Synchronization in "complex" networks

for a first-order model — all results generalize locally

$$\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

1. **local stability** for equilibria satisfying $|\theta_i - \theta_j| < \pi/2 \; \forall \{i,j\} \in \mathcal{E}$
   (linearization is Laplacian matrix)
2. **necessary sync condition**: $\sum_j B_{ij} \geq |P_i - \omega_{\text{sync}}| \Leftarrow$ sync
   (so that syn'd solution exists)
3. **sufficient sync condition**: $\lambda_2(L) > \|P\|_{E,2} \Rightarrow$ sync
   [FD & F. Bullo '12]

$\Rightarrow$ $\exists$ similar conditions with diff. metrics on coupling & heterogeneity

$\Rightarrow$ **Problem:** sharpest general conditions are conservative

Can we solve the power flow equations exactly?

on blackboard

A nearly exact sync condition

[FD, M. Chertkov, & F. Bullo '13]

1. search equilibrium θ with $|\theta_i - \theta_j| \leq \gamma < \pi/2$ for all $\{i,j\} \in \mathcal{E}$:

$$P_i = \sum\nolimits_j B_{ij} \sin(\theta_i - \theta_j) \qquad (\star)$$

2. consider linear "small-angle" DC approximation of $(\star)$:

$$P_i = \sum\nolimits_j B_{ij}(\delta_i - \delta_j) \quad \Leftrightarrow \quad P = L\delta \qquad (\star\star)$$

unique solution (modulo symmetry) of $(\star\star)$ is $\delta = L^{\dagger} P$

3. solution ansatz for $(\star)$: $\boxed{\theta_i - \theta_j = \arcsin(\delta_i - \delta_j)}$ (**for a tree**)

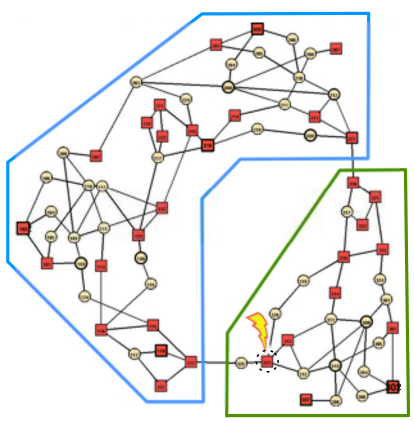
$$P_i = \sum\nolimits_{j=1}^{n} a_{ij} \sin(\theta_i - \theta_j) = \sum\nolimits_{j=1}^{n} a_{ij} \sin\big(\arcsin(\delta_i - \delta_j)\big) = P_i \quad \checkmark$$

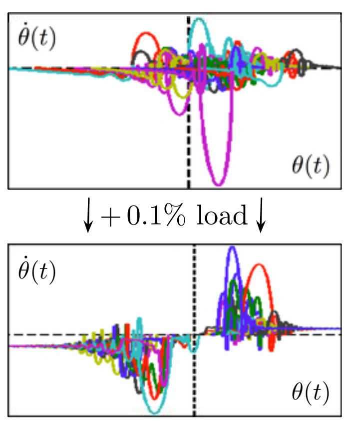
$\Rightarrow$ **Thm:** $\exists\, \theta$ with $|\theta_i - \theta_j| \leq \gamma \;\forall \{i,j\} \in \mathcal{E} \;\Leftrightarrow\; \|L^{\dagger} P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^{\dagger} P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^{\dagger} P)$



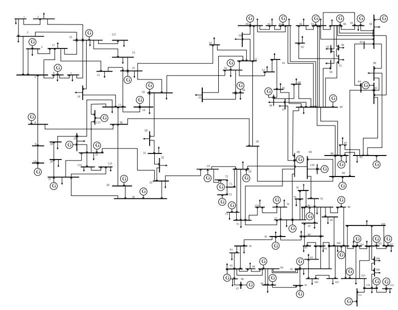


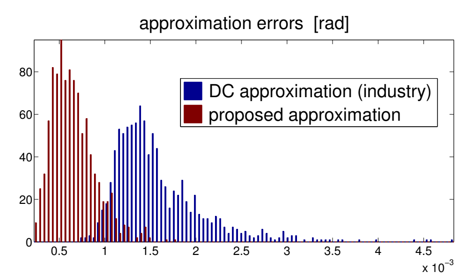
Reliability Test System RTS 96 under two loading conditions

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^{\dagger} P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^{\dagger} P)$





IEEE 118 bus system (Midwest)

Outperforms conventional DC approximation "on average & in the tail".

More on power flow approximations

Randomized power network test cases

with 50 % randomized loads and 33 % randomized generation

Randomized test case (1000 instances)	Numerical worst-case angle differences: $\max_{\{i,j\}\in\mathcal{E}} j\theta_i^* \quad \theta_j^* j$	Analytic prediction of angle differences: $\arcsin(kL^\dagger Pk_{\mathcal{E},\infty})$	Accuracy of condition: $\arcsin(kL^\dagger Pk_{\mathcal{E},\infty})$ $\max_{\{i,j\}\in\mathcal{E}} j\theta_i^* \quad \theta_j^* j$
9 bus system	0.12889 rad	0.12893 rad	4.1218 10^{-5} rad
IEEE 14 bus system	0.16622 rad	0.16650 rad	2.7995 10^{-4} rad
IEEE RTS 24	0.22309 rad	0.22480 rad	1.7089 10^{-3} rad
IEEE 30 bus system	0.16430 rad	0.16456 rad	2.6140 10^{-4} rad
New England 39	0.16821 rad	0.16828 rad	6.6355 10^{-5} rad
IEEE 57 bus system	0.20295 rad	0.22358 rad	2.0630 10^{-2} rad
IEEE RTS 96	0.24593 rad	0.24854 rad	2.6076 10^{-3} rad
IEEE 118 bus system	0.23524 rad	0.23584 rad	5.9959 10^{-4} rad
IEEE 300 bus system	0.43204 rad	0.43257 rad	5.2618 10^{-4} rad
Polish 2383 bus system (winter peak 1999/2000)	0.25144 rad	0.25566 rad	4.2183 10^{-3} rad

Discrete control actions to assure sync

1. (re)dispatch generation subject to **security constraints**:

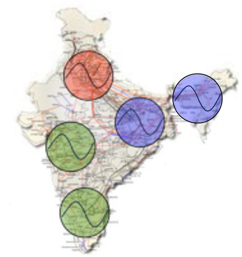
$$\text{find}_{\theta 2\mathbb{T}^n,\, u 2\mathbb{R}^{n_I}} \quad \text{subject to}$$

source power balance: $u_i = P_i(\theta)$

load power balance: $P_i = P_i(\theta)$

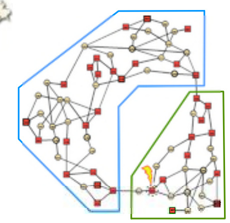
branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

2. **remedial action schemes**: load/production shedding & islanding

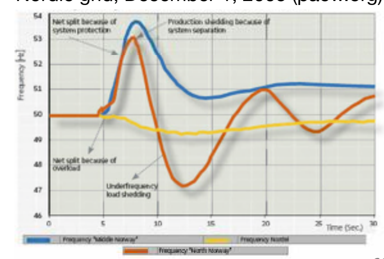


India, July 30/31 2012

RTS 96 example

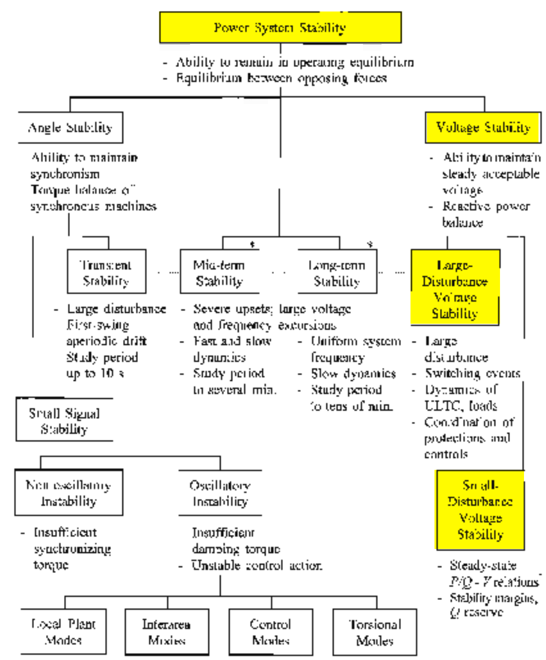


Nordic grid, December 1, 2005 (pacw.org)



Decoupled Reactive Power Flow (Voltage Collapse)

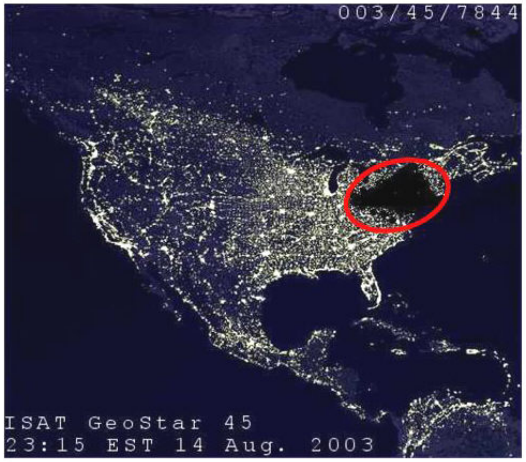
Apparently a different beast



Voltage collapse in power networks

- **voltage instability**: loading > capacity ⇒ voltages drop
  "mainly" a reactive power phenomena
- **recent outages:** Québec '96, Scandinavia '03, Northeast '03, Athens '04

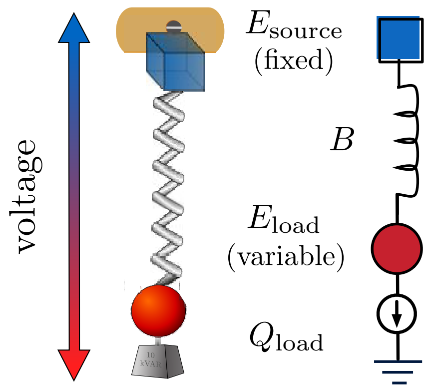
> *"Voltage collapse is still the biggest single threat to the transmission system. It's what keeps me awake at night."*
> – Phil Harris, CEO PJM.

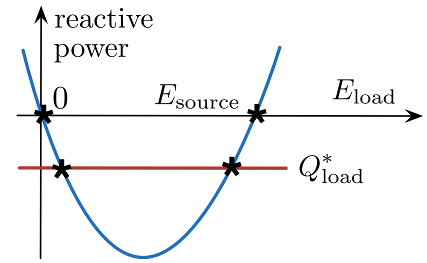


Back of the envelope calculations for the two-node case

source connected to load shows bifurcation at load voltage $E_{\text{load}} = E_{\text{source}}/2$

reactive power balance at load: $Q_{\text{load}} = B\, E_{\text{load}}(E_{\text{load}} - E_{\text{source}})$





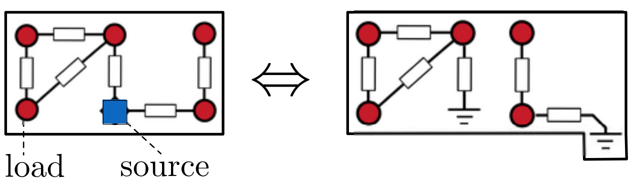
$E_{\text{load}} \in \mathbb{R} \Leftrightarrow Q_{\text{load}} \geq -B\,(E_{\text{source}})^2/4$

∃ high load voltage solution ⇔ (load) < (network)(source voltage)$^2/4$

Preliminary insights when going to "complex" networks

- **sources** with constant voltage magnitudes E_i
- **loads** with constant power demand $Q_i(E) = Q_i$

⇒ WLOG assume that network among loads is connected



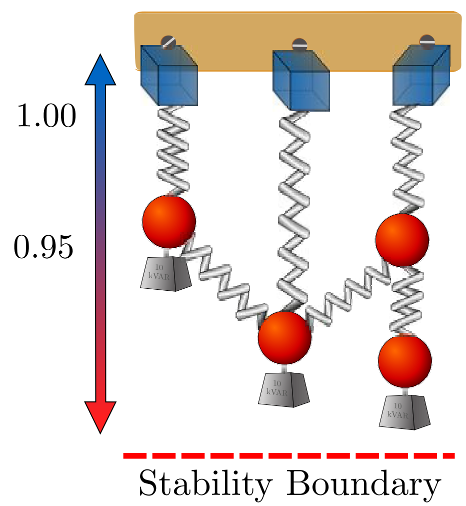
⇒ reactive power balance: $Q_i = -\sum_j B_{ij} E_i E_j$ or $Q = -\,\text{diag}(E) B E$

⇒ necessary feasibility condition: $\sum_{i=1}^{n} Q_i \geq 0 \Leftarrow \exists$ a solution

(by summing all equations and using $-E^T B E \geq 0$)

Intuition extends to complex networks – essential insights

Reactive power balance:
$Q_i = -\sum_j B_{ij} E_i E_j$



Suff. & tight cond' for general case [J. Simpson-Porco, FD, & F. Bullo, '16]:

∃ unique high-voltage solution E_{load}

⇔

$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$

1. nominal (zero load) voltage E_{nom}
   $$0 = -\sum_j B_{ij}\, E_{i,\text{nom}}\, E_{j,\text{nom}}$$
2. coord-trafo to solution guess:
   $$x_i = E_i / E_{i,\text{nom}} - 1$$
3. Picard-Banach iteration $x^+ = f(x)$

Previous condition "$\Delta < 1$" also predicts voltage deviation

for coupled & lossy power flow

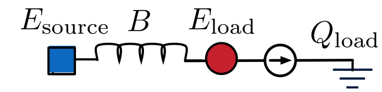
Samples: randomized scenario (50% load and 33% generation variability)

Randomized test case (1000 instances)	**Numerical** Numerical worst-case voltage deviations: $\delta_{\text{exact}} = \max_i \frac{\lvert E_i - E_i^* \rvert}{E_i^*}$	**Theoretical** Analytic prediction of voltage deviations: $\delta_- = (1 - \sqrt{1-\Delta})/2$	**% Error** Accuracy of prediction: $100 \frac{\delta_- - \delta_{\text{exact}}}{\delta_{\text{exact}}}$
9 bus system	5.49 10^{-2}	5.51 10^{-2}	0.366 %
IEEE 14 bus system	2.50 10^{-2}	2.51 10^{-2}	0.200 %
IEEE RTS 24	3.23 10^{-2}	3.24 10^{-2}	0.347 %
IEEE 30 bus system	4.91 10^{-2}	4.95 10^{-2}	0.806 %
New England 39	6.26 10^{-2}	6.30 10^{-2}	0.620 %
IEEE 57 bus system	1.20 10^{-1}	1.24 10^{-2}	3.60 %
IEEE RTS 96	3.43 10^{-2}	3.44 10^{-2}	0.376 %
IEEE 118 bus system	2.60 10^{-2}	2.61 10^{-2}	0.557 %
IEEE 300 bus system	1.05 10^{-1}	1.07 10^{-2}	1.76 %
Polish 2383 bus system (winter peak 1999/2000)	**$3.99 \cdot 10^{-2}$**	**$4.02 \cdot 10^{-2}$**	**0.764 %**

A tight & analytic guarantee: typical prediction error of $\sim 1\%$

More back of the envelope calculations

$$Q_L = B\,E_L(E_L - E_S)$$



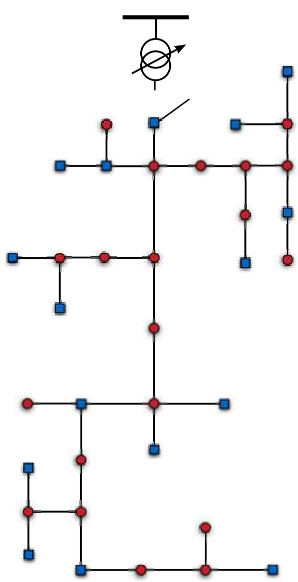
$$\Rightarrow E_L = E_S/2\left(1 + \sqrt{1 + 4Q_L/(BE_S^2)}\right) = \frac{E_S}{2}\left(1 + \sqrt{1 - Q_L/Q_{\text{crit}}}\right)$$

$\Rightarrow$ Taylor exp. for $Q_L/Q_{\text{crit}} \to 0$: $E_L \approx E_S\,(1 + Q_L/Q_{\text{crit}})$

- **general case**: existence & approximation from implicit function thm
  - if all loads Q_i are "sufficiently small" [D. Molzahn, B. Lesieutre, & C. DeMarco '12]
  - if slack bus has "sufficiently large" E_S [S. Bolognani & S. Zampieri '12 & '14]
  - if each source is above a "sufficiently large" E_{source} [B. Gentile et al. '14]
  - if previous existence condition is met [J. Simpson-Porco, FD, & F. Bullo, '16]

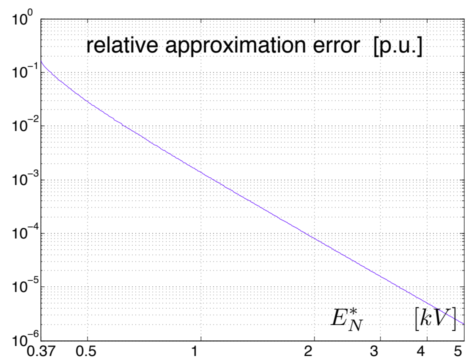
Linear DC approximation extends to complex networks

verification via *IEEE 37 bus distribution system* (SoCal)

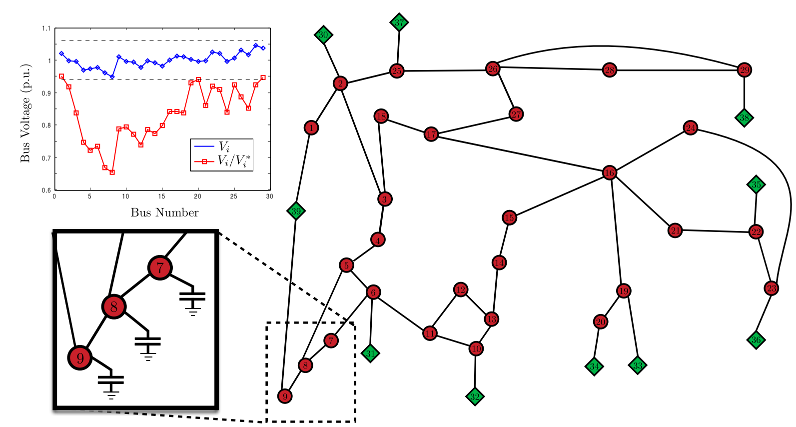


Reactive DC approximation [B. Gentile, J. Simpson-Porco, FD, S. Zampieri, & F. Bullo, '14]:

$$E_L \approx \text{diag}(E_L)\,(\mathbb{1} + Q_{\text{crit}}^{-1} Q_L) + \text{h.o.t.}$$



Discrete control actions for voltage stability



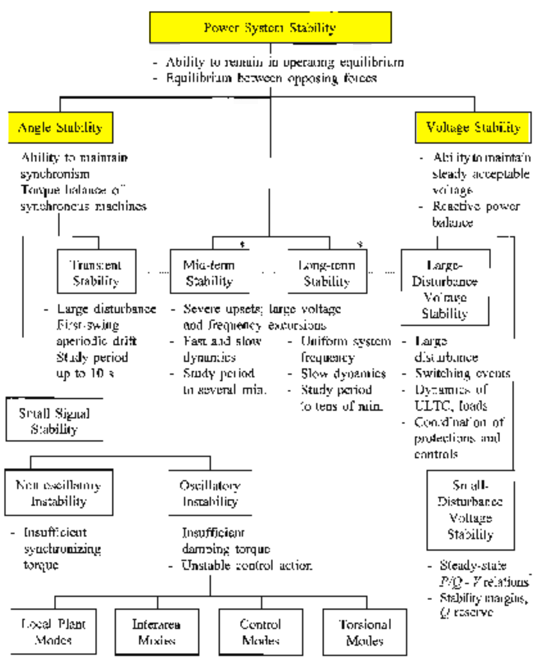
1. shunts support voltage magnitudes, but hide proximity to collapse
   $\Rightarrow$ ratios E_i/E_i^* more useful than per-unit voltages
2. $|Q_{\text{crit},89}^{-1}| > |Q_{\text{crit},87}^{-1}|$ means E_8/E_8^* more sensitive to Q_9 then to Q_7
   $\Longrightarrow$ place SVC at bus 9 to support E_8 & increase stability margin

Coupled & Lossy Power Flow

Coupling matters!

"As systems become more heavily loaded, nonlinearities play an increasingly important role in power system behavior . . . analysis tools should continue to work reliably, even under extreme system conditions . . . the $P - V$ and $Q - \theta$ cross coupling terms become significant." — [Ian Hiskens, Proc. of IEEE '95]

This is not even really on the map

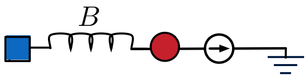


Solving the two-node case

see exercise

Simplest example shows surprisingly complex behavior

- PV source, PQ load, & lossless line

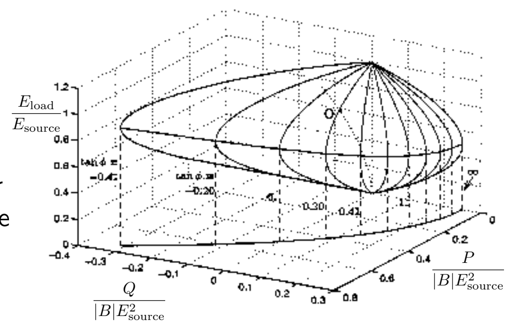


$$P = B\, E_{\text{source}}\, E_{\text{load}} \sin(\theta)$$
$$Q = B\, E_{\text{load}}^2 - B\, E_{\text{source}}\, E_{\text{load}} \cos(\theta)$$

- after eliminating θ, there exists $E_{\text{load}} \in \mathbb{R}_{\;0}$ if and only if

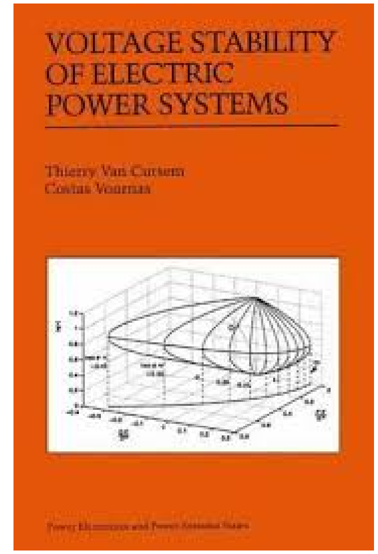
$$P^2 - B\, E_{\text{source}}^2\, Q \le B^2 E_{\text{source}}^4 / 4$$

- Observations:
  1. $P = 0$ case consistent with previous decoupled analysis
  2. $Q = 0$ case delivers 1/2 transfer capacity from decoupled case
  3. intermediate cases $Q = P \tan\phi$ give so-called "nose curves"



Recommended reading to understand a glimpse

at least once in a life-time you should read chapter 2 ...



Coupled & lossy power flow in complex networks

- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

- what makes it so much harder than the previous two node case?
  losses, mixed lines, cycles, PQ-PQ connections, ...
- much theoretic work, qualitative understanding, & numeric approaches:
  - existence of solutions [Thorp, Schulz, & Ilić '86, Wu & Kumagai '82]
  - solution space [Hiskens & Davy '01, Overbye & Klump '96, Van Cutsem '98, ...]
  - distance-to-failure [Venikov '75, Abe & Isono '76, Dobson '89, Andersson & Hill '93, ...]
  - convex relaxation approaches [Molzahn et al. '12, Dvijotham et al. '15]
- little analytic & quantitative understanding beyond the two-node case

"Whoever figures that one out [analysis of $n > 2$ node] wins a noble prize!"

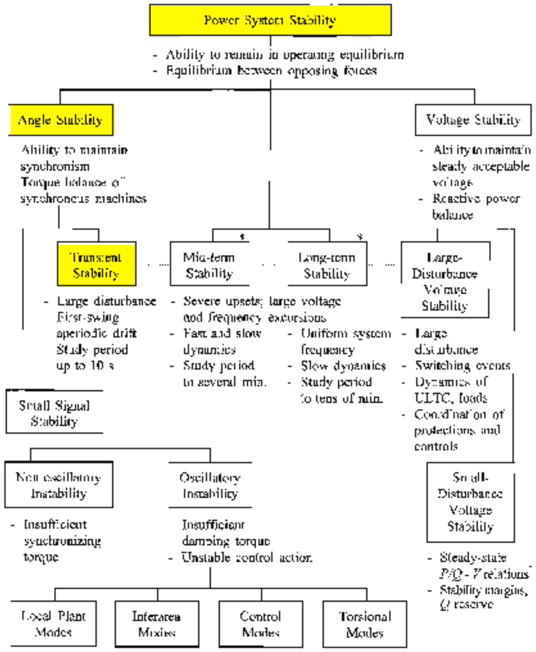
— [Peter Sauer, lunch @ UIUC '13]

Transient Rotor Angle Stability

"The crown jewel of power system stability!"

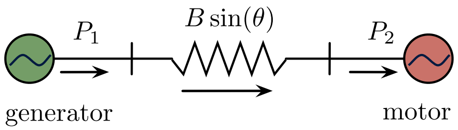
— [Janusz Bialek, skype call '13]

The crown jewel of power system stability

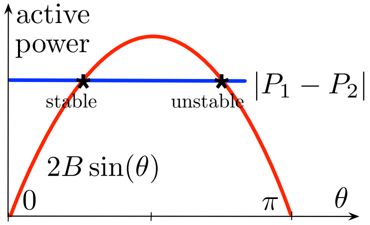


Revisit of the two-node case — the forced pendulum

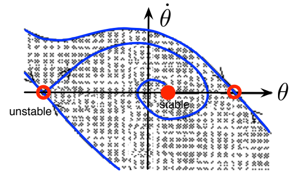
more complex than anticipated



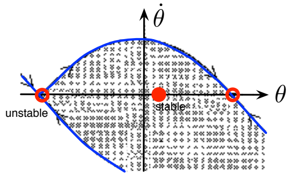
$$M\ddot{\theta} = -D\dot{\theta} + P_1 - P_2 - 2B\sin(\theta)$$



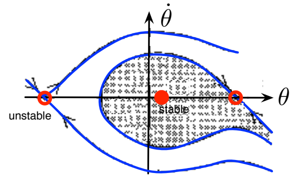
- **Local stability**: $\exists$ local stable solution $\Leftrightarrow$ $B > |P_1 - P_2|/2$
- **Global stability**: depends on gap $B > |P_1 - P_2|/2$ and D/M ratio



$(D/M) > (D/M)_{\text{critical}}$



$(D/M) = (D/M)_{\text{critical}}$

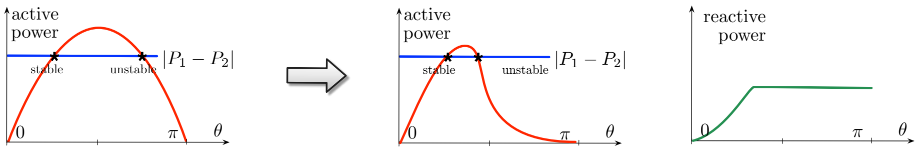


$(D/M) < (D/M)_{\text{critical}}$

Revisit of the two-node case — cont'd

the story is not complete ... some further effects that we swept under the carpet

- **Voltage reduction**: generator needs to provide reactive power for voltage regulation – until saturation, then generator becomes PQ bus



- **Load sensitivity**: different behavior depending on load model: resistive, constant power, frequency-dependent, dynamic, power electronics, ...
- **Singularity-issues** for coupled power flows (load voltage collapse)
- **Losses & higher-order dynamics** change stability properties ...

⇒ quickly run into computational approaches

Primer on Lyapunov functions

on blackboard

Transient stability in multi-machine power systems

generators:

$$\dot{\theta}_i = \omega_i$$
$$M_i\dot{\omega}_i = -D_i\omega_i + P_i - \sum\nolimits_j B_{ij}E_iE_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij}E_iE_j \cos(\theta_i - \theta_j)$$

loads:

$$D_i\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}E_iE_j \sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij}E_iE_j \cos(\theta_i - \theta_j)$$

Challenge (improbable): faster-than-real-time transient stability assessment

Energy function methods for simple lossless models via Lyapunov function

$$V(\omega,\theta,E) = \sum\nolimits_i \frac{1}{2}M_i\omega_i^2 - \sum\nolimits_i P_i\theta_i - \sum\nolimits_i Q_i \log E_i - \sum\nolimits_{ij} B_{ij}E_iE_j\cos(\theta_i - \theta_j)$$

Computational approaches: level sets of energy functions & unstable equilibria, sum-of-squares methods, convex optimization approaches, time-domain simulations, ... (holy grail of power system stability)

Hamiltonian analysis of the swing equations

more famously known as "energy function analysis" (see exercise)

Outline

Brief Introduction

Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

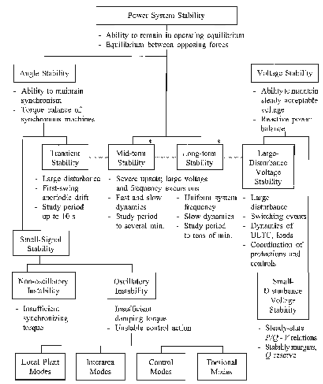
- Primary Control
- Power Sharing
- Secondary control
- Experimental validation
- (Optional material)

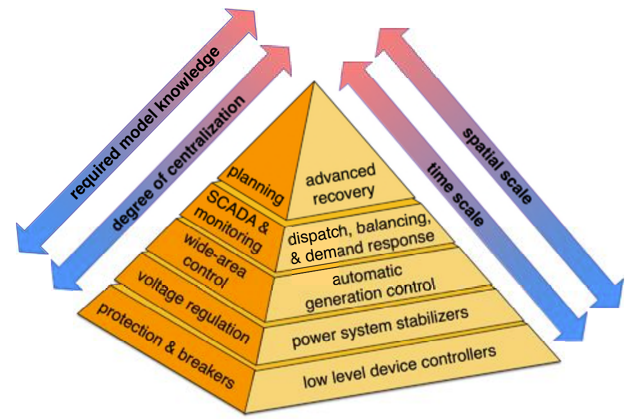
Power System Oscillations

Conclusions

A plethora of control tasks and nested control layers

organized in hierarchy and separated by states & spatial/temporal/centralization scales

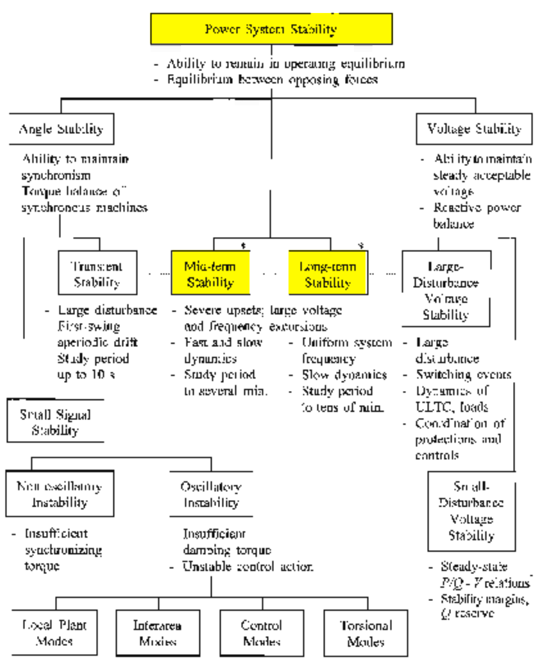




We will focus on frequency control & primary/secondary/tertiary layers.

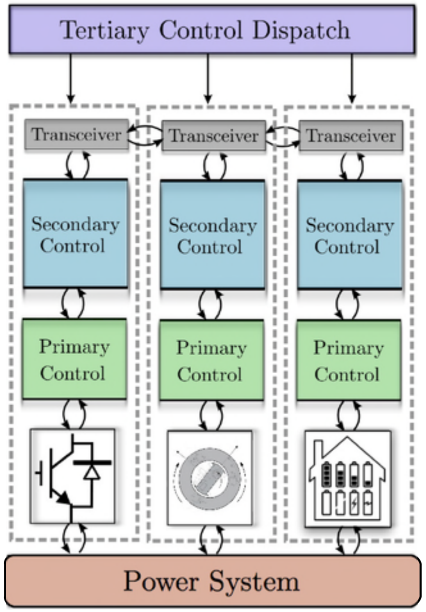
All dynamics & controllers are interacting. Classification & hierarchy are for simplicity.

Where are we on the map?



Objectives

Hierarchical frequency control architecture & objectives



3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (minutes)
   - Goal: maintain operating point in presence of disturbances
   - Strategy: centralized
1. **Primary control** (real-time)
   - Goal: stabilize frequency & share unknown load
   - Strategy: decentralized

Q: Is this layered & hierarchical architecture still appropriate for tomorrow's power system?

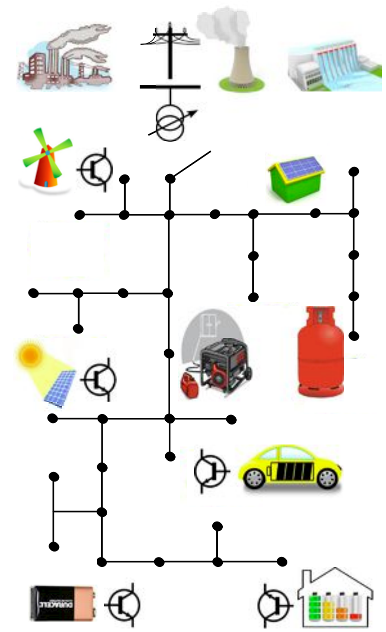
Is this hierarchical control architecture still appropriate?

Some recent developments

- increasing renewable integration & deregulated energy markets
- bulk generation replaced by distributed generation
- synchronous machines replaced by power electronics sources
- low gas prices & substitutions

Some new problem scenarios

- alternative spinning reserves: storage, load control, & DER
- networks of low-inertia & distributed renewable sources
- small-footprint islanded systems



Need to adapt the control hierarchy in tomorrow's grid

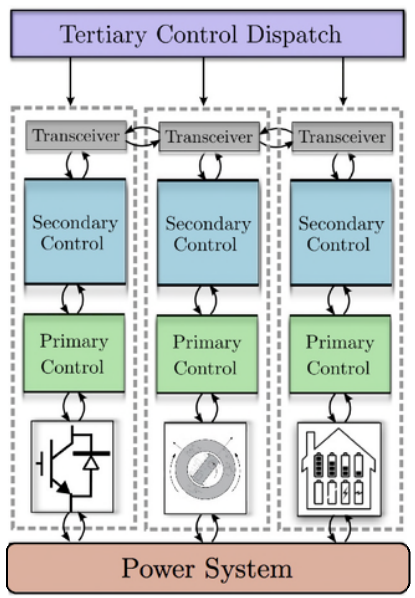
☹perational challenges

- more uncertainty & less inertia
- more volatile & faster fluctuations
- plug'n'play control: fast, model-free, & without central authority

☺pportunities

- re-instrumentation: comm & sensors
- more & faster spinning reserves
- advances in control of cyber-physical & complex systems

⇒ **break** vertical & horizontal **hierarchy**

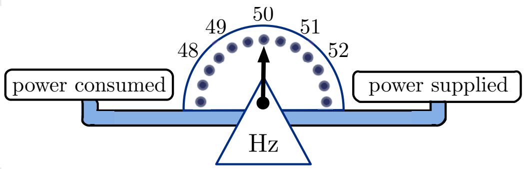


Primary Control

Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$



Conventional wisdom: physics are naturally stable & sync frequency reveals power imbalance

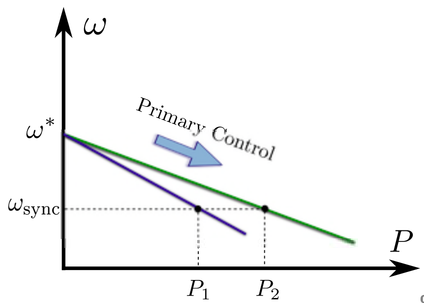
recall: $\omega_{\text{sync}} = \sum_i P_i / D_i$

$P/\dot{\theta}$ droop control:

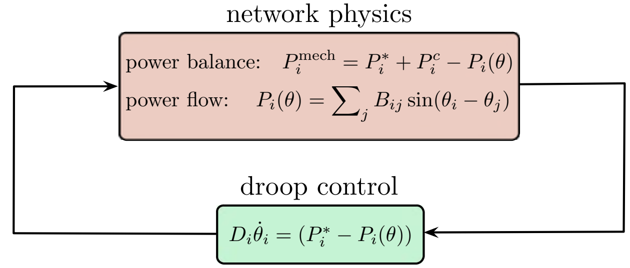
$$(\omega_i - \omega) \propto (P_i - P_i(\theta))$$

$$\Updownarrow$$

$$D_i\dot{\theta}_i = P_i - P_i(\theta)$$



Putting the pieces together...



synchronous machines: $M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

inverter sources & controllable loads: $D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

passive loads & power-point tracking sources: $0 = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

active power flow is feasible $\Longrightarrow$ $\exists$ unique & exp. stable frequency sync

Main **proof ideas** and some **further results**:

- stability via Jacobian & Lyapunov arguments
- synchronization frequency: ($\propto$ power balance) $\quad \omega_{\text{sync}} = \omega + \dfrac{\sum_{\text{sources}} P_i + \sum_{\text{loads}} P_i}{\sum_{\text{sources}} D_i}$
- steady-state power injections: (depend on D_i & P_i) $\quad \mathcal{P}_i = \begin{cases} P_i & \text{(load \#}i\text{)} \\ P_i - D_i(\omega_{\text{sync}} - \omega) & \text{(source \#}i\text{)} \end{cases}$

Closed-loop stability?

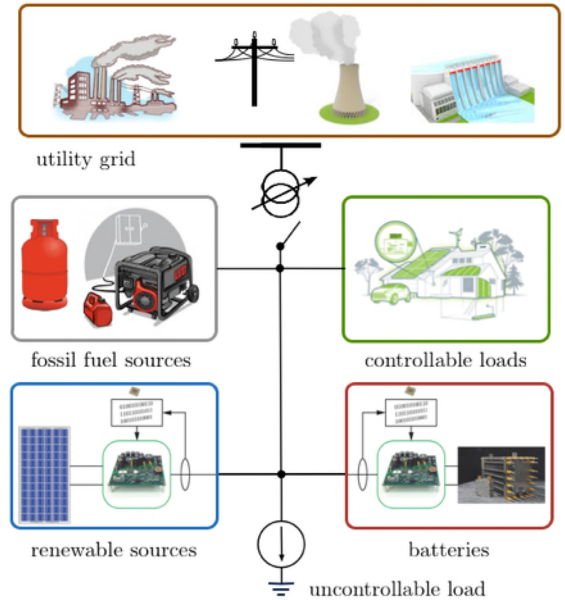
see exercise

power sharing & economic optimality under droop control

(sometimes in tertiary layer)

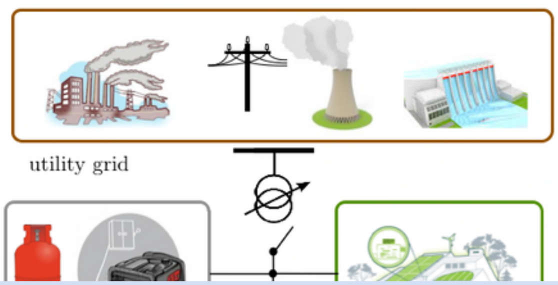
Tertiary control and energy management

an offline resource allocation and scheduling problem



Tertiary control and energy management

an offline resource allocation and scheduling problem



minimize {cost of generation, losses, ...}

subject to

equality constraints: power balance equations

inequality constraints: flow/injection/voltage constraints

logic constraints: commit generators yes/no

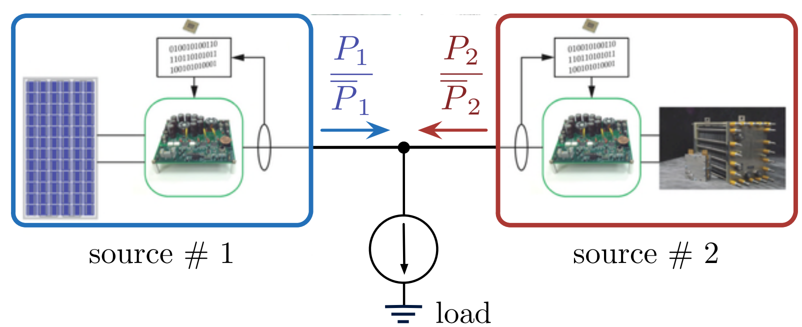
$\vdots$

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left|\sum_{\text{loads}} P_j\right| \leq \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) / \overline{P}_i = P_j(\theta) / \overline{P}_j$



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3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

A little calculation reveals in steady state:

$$\frac{P_i(\theta)}{\overline{P}_i} \overset{!}{=} \frac{P_j(\theta)}{\overline{P}_j} \quad \Rightarrow \quad \frac{P_i - (D_i\omega_{\text{sync}} - \omega\ )}{\overline{P}_i} \overset{!}{=} \frac{P_j - (D_j\omega_{\text{sync}} - \omega\ )}{\overline{P}_i}$$

. . . so choose

$$\frac{P_i}{\overline{P}_i} = \frac{P_j}{\overline{P}_j} \quad \text{and} \quad \frac{D_i}{\overline{P}_i} = \frac{D_j}{\overline{P}_j}$$

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left| \sum_{\text{loads}} P_j \right| \leq \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$\boxed{D_i/\overline{P}_i = D_j/\overline{P}_j \ \ \& \ \ P_i\,/\overline{P}_i = P_j\,/\overline{P}_j}$$

The the following statements hold:

(i) Proportional load sharing: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

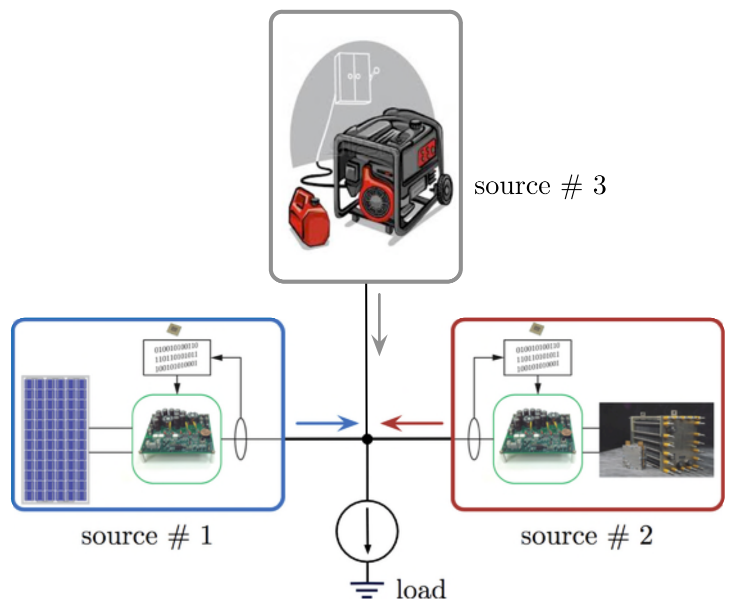
(ii) Constraints met: $0 \leq \left| \sum_{\text{loads}} P_j \right| \leq \sum_{\text{sources}} \overline{P}_j \iff P_i(\theta) \in [0, \overline{P}_i]$

Constraints achieved by fair proportional load sharing

see exercise

Objective I: fair proportional load sharing

proportional load sharing is not always the right objective

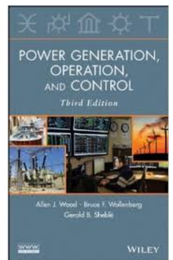


Objective II: optimal power flow = tertiary control

an offline resource allocation/scheduling problem

minimize	{cost of generation, losses, ...}
subject to	
equality constraints:	power balance equations
inequality constraints:	flow/injection/voltage constraints
logic constraints:	commit generators yes/no
	⋮

Will be discussed more in detail by Andrej.



Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

$$\text{minimize}_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}} \quad J(u) = \sum\nolimits_{\text{sources}} \alpha_i u_i^2$$

subject to

source power balance:	$P_i^* + u_i = P_i(\theta)$
load power balance:	$P_i^* = P_i(\theta)$
branch flow constraints:	$\lvert\theta_i - \theta_j\rvert \leq \gamma_{ij} < \pi/2$

A simpler & equivalent (in the strictly feasible case) problem formulation:

$$\text{minimize}_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}} \quad J(u) = \sum\nolimits_{\text{sources}} \alpha_i u_i^2$$

subject to

power balance: $\sum_i P_i^* + \sum_i u_i = 0$

The *abc* of resource allocation

on blackboard

Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

$$\text{minimize}_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}} \quad J(u) = \sum\nolimits_{\text{sources}} \alpha_i u_i^2$$

subject to

source power balance:	$P_i^* + u_i = P_i(\theta)$
load power balance:	$P_i^* = P_i(\theta)$
branch flow constraints:	$\lvert\theta_i - \theta_j\rvert \leq \gamma_{ij} < \pi/2$

Unconstrained case: identical marginal costs $\alpha_i u_i^* = \alpha_j u_j^*$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

Objective II: decentralized dispatch optimization

Insight: droop-controlled system = decentralized optimization algorithm

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

(i) the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer $(\theta\ , u\ )$.

(ii) $\exists$ droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ.

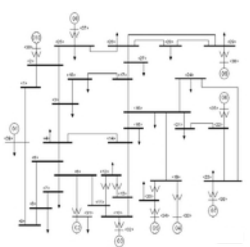
If (i) & (ii) are true, then $\theta_i \sim \theta_i$, $u_i = -D_i(\omega_{\text{sync}} - \omega\ )$, & $\boxed{D_i\alpha_i = D_j\alpha_j}$.

- includes proportional load sharing $\alpha_i \propto 1/\overline{P}_i$
- similar results hold for strictly convex & differentiable cost

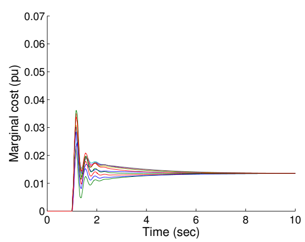
Sketch of the main proof ideas

see exercise

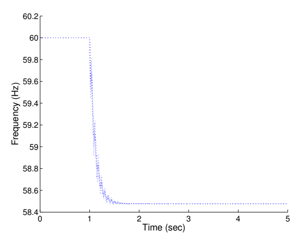
Some quick simulations & extensions



IEEE 39 New England with load step at 1s



$t \to \infty$: convergence to identical marginal costs

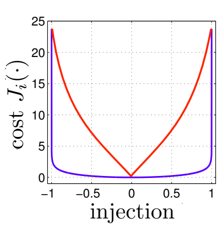


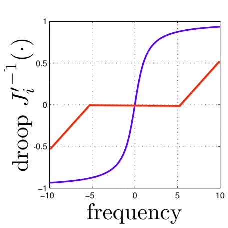
$t \to \infty$: frequency $\propto$ power imbalance

$\Rightarrow$ strictly convex & differentiable cost $J(u) = \sum_{\text{sources}} J_i(u_i)$

$\Rightarrow$ non-linear frequency droop curve $J_i'^{-1}(\dot{\theta}_i) = P_i^* - P_i(\theta)$

$\Rightarrow$ include dead-bands, saturation, etc.

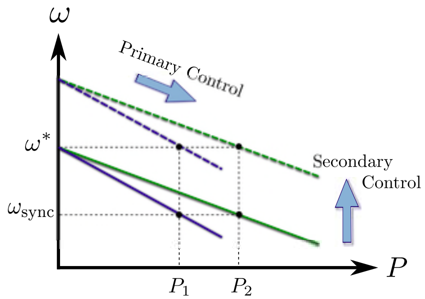




Secondary Control

Secondary frequency control

- **Problem:** steady-state frequency deviation ($\omega_{\text{sync}} \neq \omega^*$)
- **Solution:** integral control of frequency error
- **Basics** of integral control $\boxed{\frac{1}{s}}$:



1. discrete time: $u_i(t+1) = u_i(t) + k \cdot \dot\theta_i(t)$ with gain $k > 0$
2. continuous-time: $u_i(t) = k \cdot \int_0^t \dot\theta_i(\tau)\, d\tau$ or $\dot u_i(t) = k \cdot \dot\theta_i(t)$

$\Rightarrow$ $\dot\theta_i(t)$ is zero in (a possibly stable) steady state

$\Rightarrow$ add additional injection $u_i(t)$ to droop control

Decentralized secondary integral frequency control

$\boxed{\frac{1}{s}}$ add local integral controller to every droop controller

$\Rightarrow$ zero frequency deviation ✓

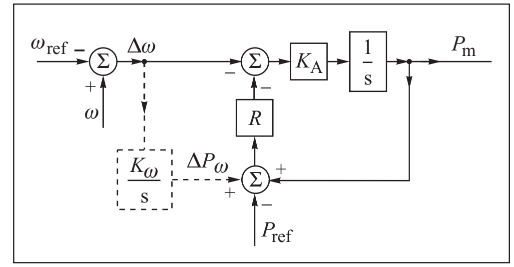
$\Rightarrow$ nominally globally stabilizing [C. Zhao, E. Mallada, & FD, '14] ✓

☹ every integrator induces a 1d equilibrium subspace

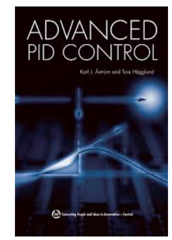
☹ injections live in subspace of dimension # integrators

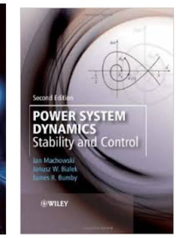
☹ load sharing & economic optimality are lost ...

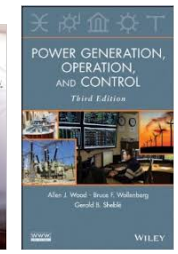
☹ unstable in presence of biased noise [M. Andreasson et al. '14]



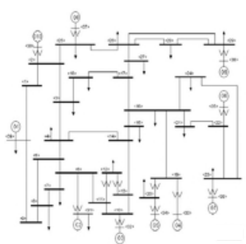
turbine governor integral control loop



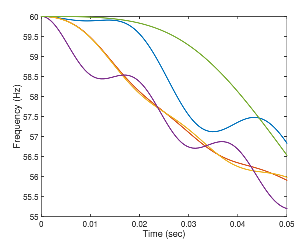




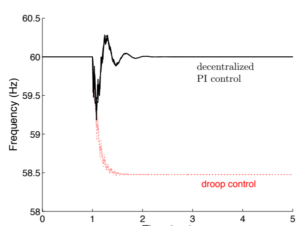
Simulations cont'd



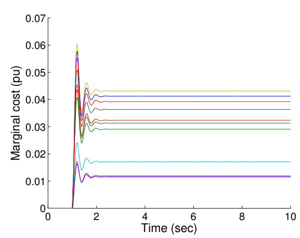
IEEE 39 New England with decentralized PI control



decentralized PI control in presence of biased noise



$t \to \infty$: decentralized PI control regulates frequency



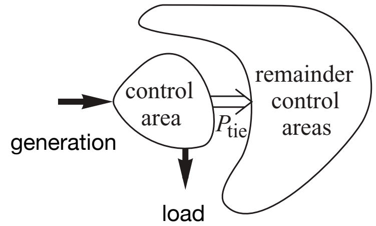
$t \to \infty$: decentralized PI control is not optimal

Why does decentralized integral control not work?

see exercise

Automatic generation control (AGC)

- **ACE** area control error = { frequency error } + { generation - load - tie-line flow }

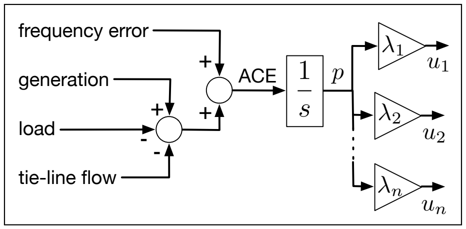


$\frac{1}{s}$ **centralized integral control:**

$$p(t) = \int_0^t \mathrm{ACE}(\tau)\, d\tau$$

- **generation allocation:** $u_i(t) = \lambda_i p(t)$, where λ_i is generation participation factor (in our case $\lambda_i = 1/\alpha_i$)

⇒ assures identical marginal costs: $\alpha_i u_i = \alpha_j u_j$

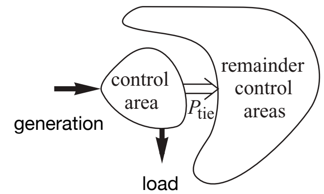


AGC implementation

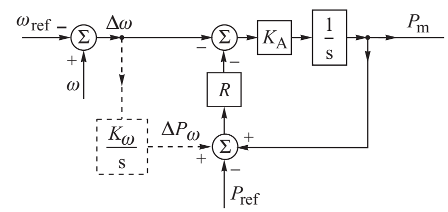
☺ load sharing & economic optimality are recovered

Drawbacks of conventional secondary frequency control

interconnected systems	isolated systems
• **centralized** automatic generation control (AGC)	• **decentralized** PI control



compatible with econ. dispatch
[N. Li, L. Chen, C. Zhao, & S. Low '13]



nominally *globally* stabilizing
[C. Zhao, E. Mallada, & FD, '14]

Drawbacks of conventional secondary frequency control

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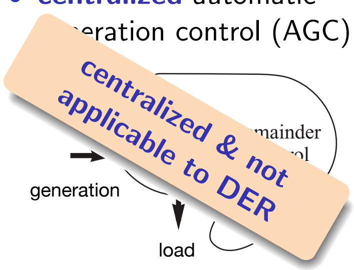

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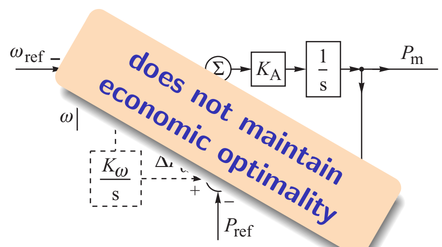


nominally *globally* stabilizing
[C. Zhao, E. Mallada, & FD, '14]

Distributed energy resources require **distributed** (**!**) secondary control.

An incomplete literature review of a busy field

ntwk with unknown disturbances ∪ integral control ∪ distributed averaging

- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcıa-Domınguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]

The following idea precedes most references, it's simpler, & it's more robust.

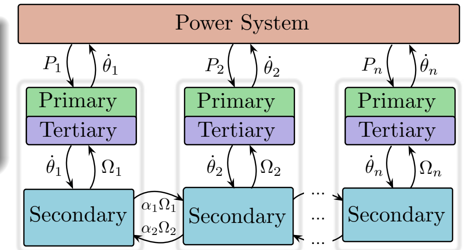
Let's derive a simple distributed control strategy

on blackboard

Distributed Averaging PI (DAPI) control

$$D_i\dot{\theta}_i = P_i - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \,\text{sources}} a_{ij}\cdot(\alpha_i\Omega_i - \alpha_j\Omega_j)$$



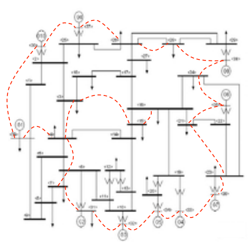
- no tuning & no time-scale separation: $k_i, D_i > 0$
- recovers optimal dispatch
- distributed & modular: connected comm. network
- has seen many extensions [C. de Persis et al., H. Sandberg et al., J. Schiffer et al., M. Zhu et al., ...]

Theorem: stability of DAPI

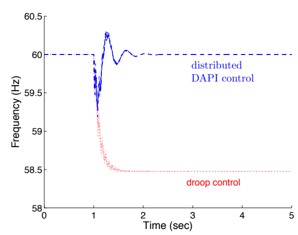
[J. Simpson-Porco, FD, & F. Bullo '12]
[C. Zhao, E. Mallada, & FD '14]

primary droop controller works
$\iff$
secondary DAPI controller works

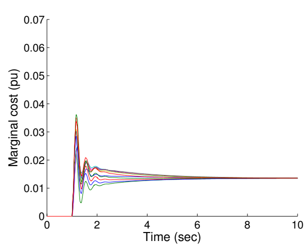
Simulations cont'd



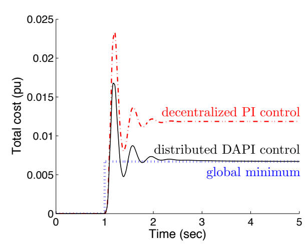
IEEE 39 New England with distributed DAPI control



$t \to \infty$: DAPI control regulates frequency



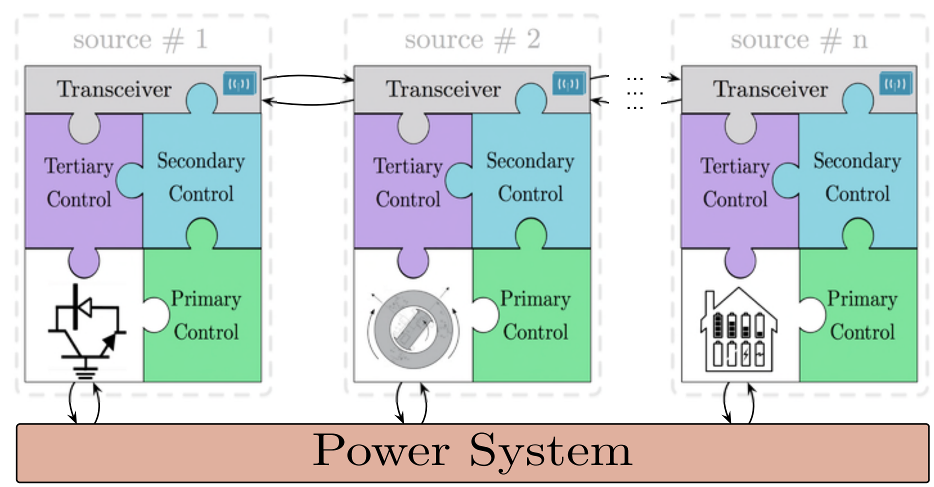
DAPI control synchronizes marginal costs



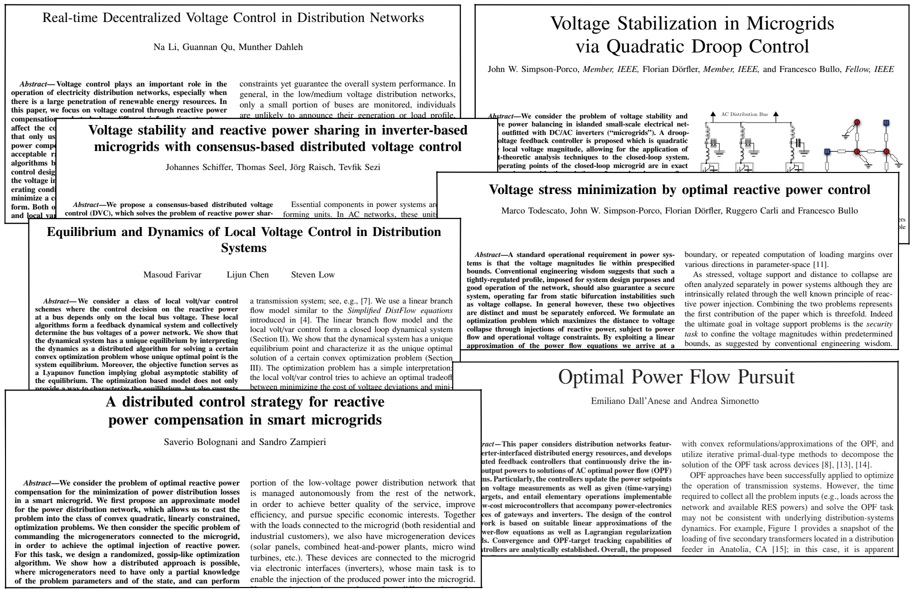
DAPI control minimizes cost with little effort

Plug'n'play architecture

flat hierarchy, distributed, no time-scale separations, & model-free



We can do similar things on the reactive power side


Real-time Decentralized Voltage Control in Distribution Networks
Na Li, Guannan Qu, Munther Dahleh

Voltage Stabilization in Microgrids via Quadratic Droop Control
John W. Simpson-Porco, Member, IEEE, Florian Dörfler, Member, IEEE, and Francesco Bullo, Fellow, IEEE

Voltage stability and reactive power sharing in inverter-based microgrids with consensus-based distributed voltage control
Johannes Schiffer, Thomas Seel, Jörg Raisch, Tevfik Sezi

Voltage stress minimization by optimal reactive power control
Marco Todescato, John W. Simpson-Porco, Florian Dörfler, Ruggero Carli and Francesco Bullo

Equilibrium and Dynamics of Local Voltage Control in Distribution Systems
Masoud Farivar, Lijun Chen, Steven Low

Optimal Power Flow Pursuit
Emiliano Dall'Anese and Andrea Simonetto

A distributed control strategy for reactive power compensation in smart microgrids
Saverio Bolognani and Sandro Zampieri

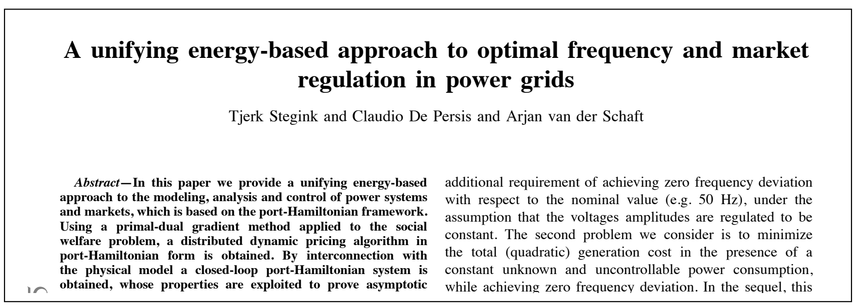
Much recent work on reactive power control

- heuristic linear Q/E droop: $(E_i - E_i) \propto (Q_i - Q_i(E))$
  sometimes with integrator & nonlinearities [J. Simpson-Porco et. al. '16]
- reactive power sharing DAPI [J. Simpson-Porco et. al. '15, J. Schiffer et al. '16]
  $$\kappa_i \dot{e}_i = \sum_{j \text{ sources}} a_{ij} \cdot \left(Q_i/\overline{Q_i} - Q_j/\overline{Q_j}\right) - \varepsilon e_i$$
- voltage regulation [M. Farivar et al. '13]: $\kappa_i \dot{e}_i = E_i - E_i$
- loss minimization: minimize $\sum_{\{i,j\}\in E} B_{ij}(E_i - E_j)^2$ [N. Li et al. '14]
- robustness margins: maximize det (Jacobian) [M. Todescato et al. '16]
- maximize reative reserves s.t. flat voltage profile $E_i \approx 1$ [RTE France]

Main distinction to active power: while each of these objectives is individually feasible, they are also all **mutually exclusive** ...

A great unifying perspective on secondary control

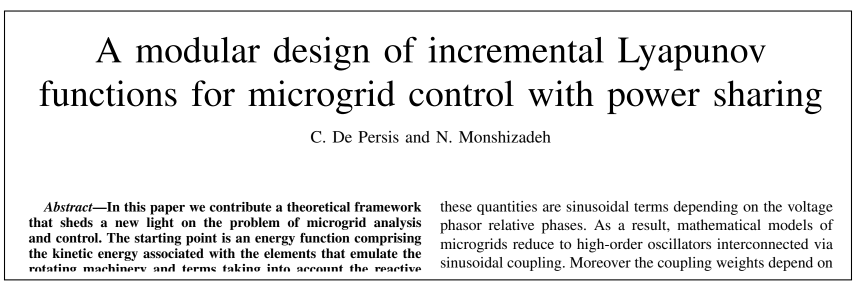
pretty much incorporating everything that we've discussed this far


A unifying energy-based approach to optimal frequency and market regulation in power grids

Tjerk Stegink and Claudio De Persis and Arjan van der Schaft

***Abstract*—In this paper we provide a unifying energy-based approach to the modeling, analysis and control of power systems and markets, which is based on the port-Hamiltonian framework. Using a primal-dual gradient method applied to the social welfare problem, a distributed dynamic pricing algorithm in port-Hamiltonian form is obtained. By interconnection with the physical model a closed-loop port-Hamiltonian system is obtained, whose properties are exploited to prove asymptotic**

additional requirement of achieving zero frequency deviation with respect to the nominal value (e.g. 50 Hz), under the assumption that the voltages amplitudes are regulated to be constant. The second problem we consider is to minimize the total (quadratic) generation cost in the presence of a constant unknown and uncontrollable power consumption, while achieving zero frequency deviation. In the sequel, this


A modular design of incremental Lyapunov functions for microgrid control with power sharing

C. De Persis and N. Monshizadeh

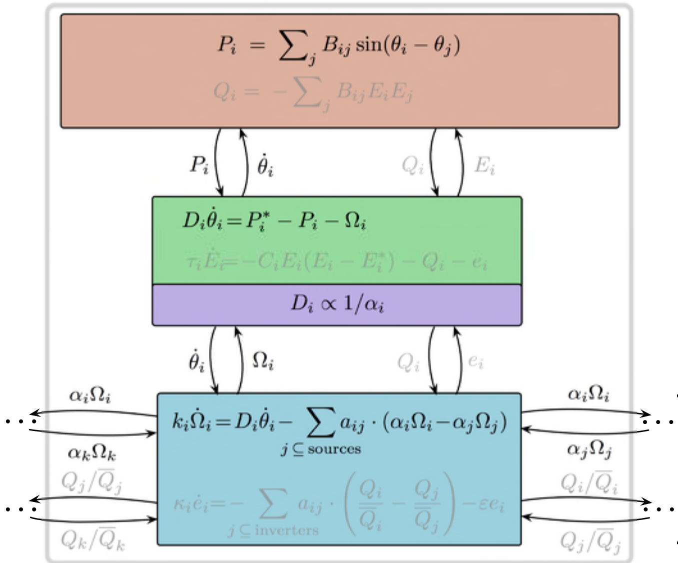
***Abstract*—In this paper we contribute a theoretical framework that sheds a new light on the problem of microgrid analysis and control. The starting point is an energy function comprising the kinetic energy associated with the elements that emulate the rotating machinery and terms taking into account the reactive**

these quantities are sinusoidal terms depending on the voltage phasor relative phases. As a result, mathematical models of microgrids reduce to high-order oscillators interconnected via sinusoidal coupling. Moreover the coupling weights depend on

plug-and-play experiments

Plug'n'play architecture

recap of detailed signal flow (active power only)



Power system:
physics
& loadflow

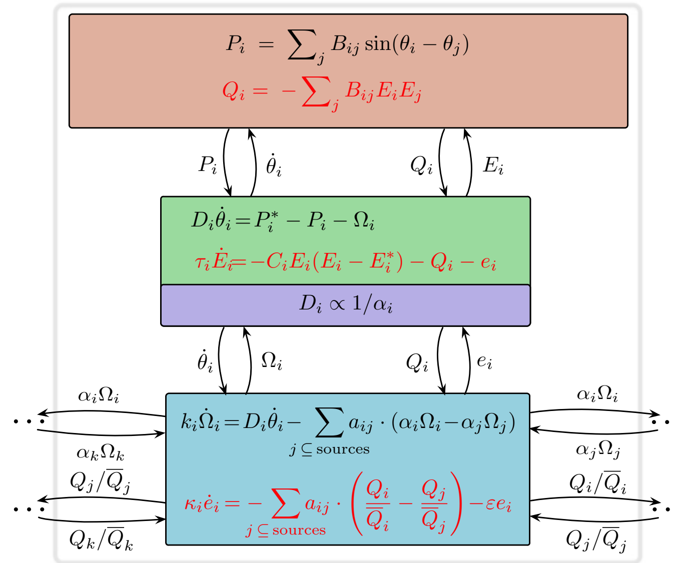
Primary control:
mimic oscillators
& polyn. symmetry

Tertiary control:
marginal costs
∝ 1 /control gains

Secondary control:
diffusive averaging
of optimal injections

Plug'n'play architecture

similar results for decoupled reactive power flow [J. Simpson-Porco, FD, & F. Bullo '13 - '15]



Power system:
physics
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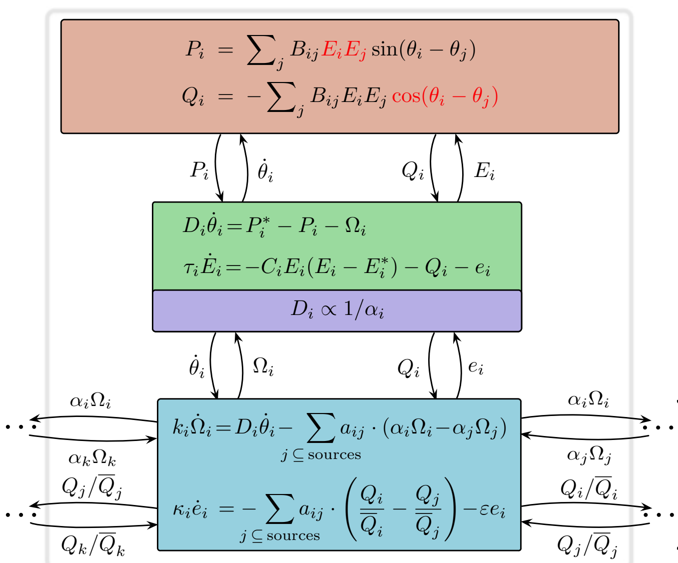
Plug'n'play architecture

can all be proved also in the coupled case [N. Monshizadeh & C. de Persis, '15]



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physics
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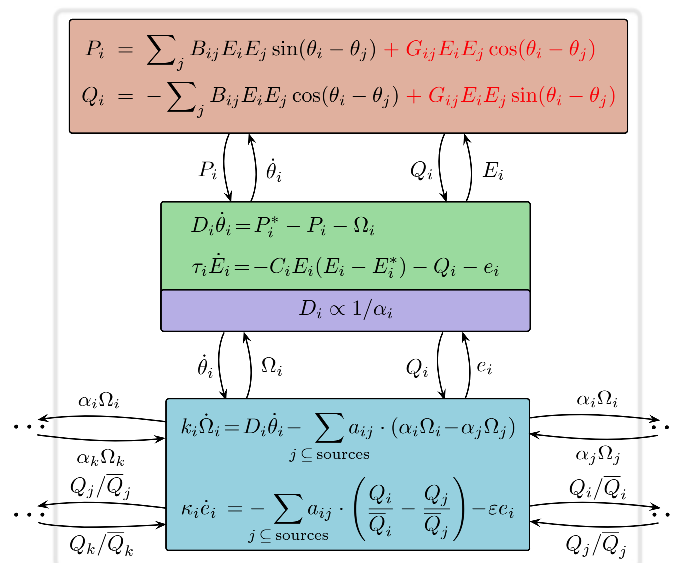
marginal costs
∝ 1 /control gains

Secondary control:
diffusive averaging
of optimal injections

Plug'n'play architecture

experiments also work well in the lossy case



Power system:
physics
& loadflow

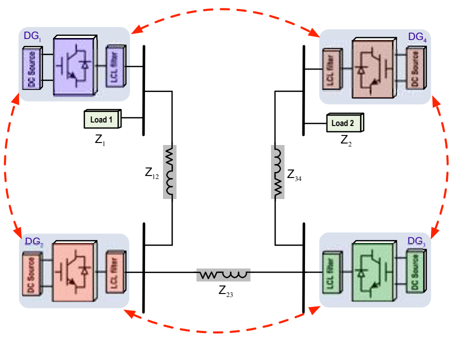
Primary control:
mimic oscillators
& polyn. symmetry

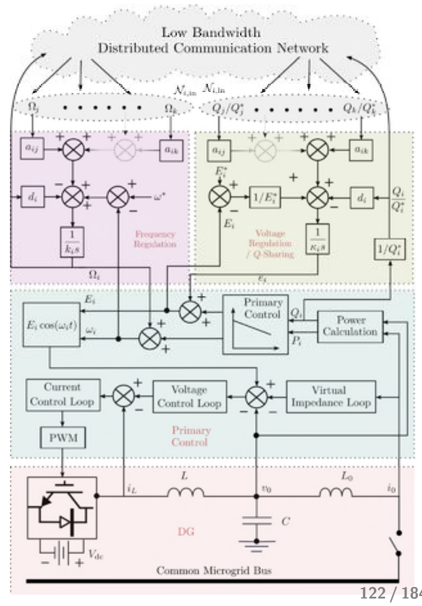
Tertiary control:
marginal costs
∝ 1 /control gains

Secondary control:
diffusive averaging
of optimal injections

Experimental validation of control & opt. algorithms

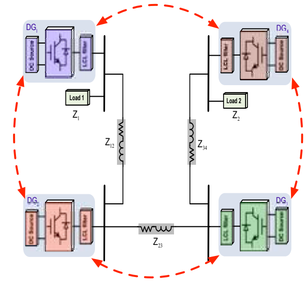
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University

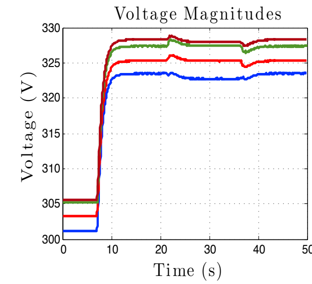


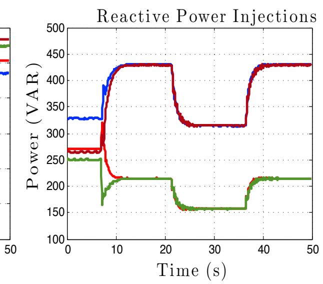


Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing





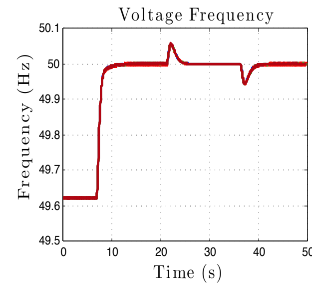


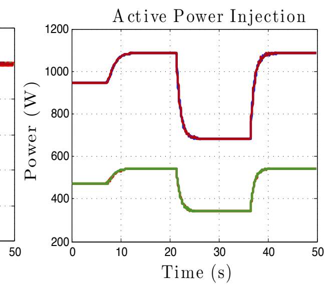
$t \in [0s, 7s]$: primary & tertiary control

$t = 7s$: secondary control activated

$t = 22s$: load # 2 unplugged

$t = 36s$: load # 2 plugged back

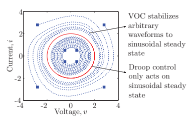




There are also many exciting alternatives to droop control

Uncovering Droop Control Laws Embedded Within the Nonlinear Dynamics of Van der Pol Oscillators

Mohit Sinha, Florian Dörfler, *Member, IEEE*, Brian B. Johnson, *Member, IEEE*, and Sairaj V. Dhople, *Member, IEEE*



Synchronization of Nonlinear Oscillators in an LTI Electrical Power Network

Brian B. Johnson, *Member, IEEE*, Sairaj V. Dhople, *Member, IEEE*, Abdullah O. Hamadeh, and Philip T. Krein, *Fellow, IEEE*

Voltage and frequency control of islanded microgrids: a plug-and-play approach

Stefano Riverso, Fabio Sarzo and Giancarlo Ferrari-Trecate

Synchronization of Oscillators Coupled through a Network with Dynamics: A Constructive Approach with Applications to the Parallel Operation of Voltage Power Supplies

Leonardo A. B. Tôrres, *Member, IEEE*, João P. Hespanha, *Fellow, IEEE*, and Jeff Moehlis

(optional material)

what can we do better?

algorithms, detailed models, cyber-physical aspects, . . .

many groups out there push all these directions heavily

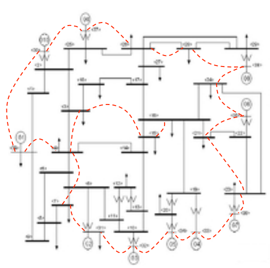
Variation I:

Europe: no centralized dispatch but trade in **energy markets**

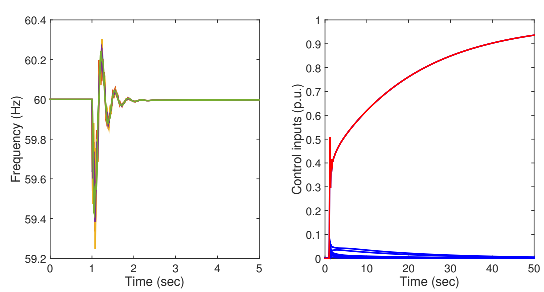
$\Downarrow$

game-theoretic formulation of optimal secondary control

Some strong motivations for game-theoretic perspective



IEEE 39 New England with distributed DAPI control



DAPI control with cheating of generator # 10

A simple (illegal) cheating strategy for generator #10:

1. report wrong injection $u_{10}(t) = 0$ to all neighbors in comm network
2. do not average neighbor values $a_{10,j} = 0$ for all j

$\Rightarrow$ generator #10 alone picks up net load & regulates the frequency

$\Rightarrow$ need an incentive scheme so that everybody plays "best response"

Market formulation of secondary control [FD & S. Grammatico '16]

Competitive spot market:

1. given a prize λ, player i bids
   $u_i^\star = \underset{u_i}{\text{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$
2. market clearing prize $\lambda^\star$ from
   $0 = \sum_i P_i + u_i^\star = \sum_i P_i + J_i'^{-1}(\lambda^\star)$

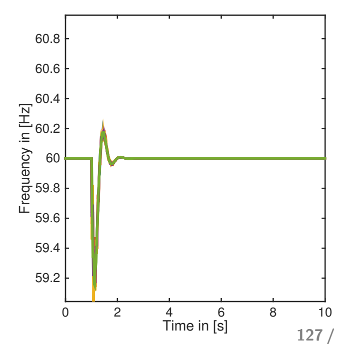
Broadcast controller:

1. convex measurement:
   $k \cdot \dot\lambda(t) = \sum_i C_i \dot\theta_i(t)$
2. local allocation:
   $u_i(t) = J_i'^{-1}(\lambda(t))$

Auction (dual decomposition):

1. $u_i^+ = \underset{u_i}{\text{argmin}} \{J_i(u_i) - \lambda u_i\} = J_i'^{-1}(\lambda)$
2. $\lambda^+ = \lambda - \epsilon \left(\sum_i P_i + u_i^+\right) = \lambda - \epsilon \cdot \omega_{\text{sync}}$

$\Rightarrow$ converges to optimal economic dispatch



Variation II:

VOC: virtual oscillator control

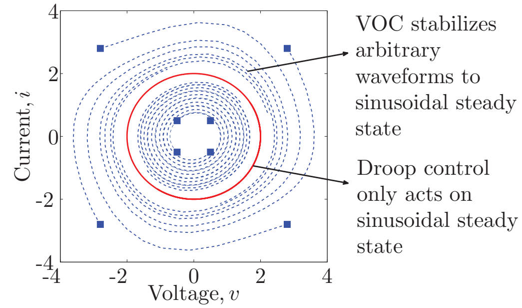
instead of primary droop control

Removing the assumptions of droop control

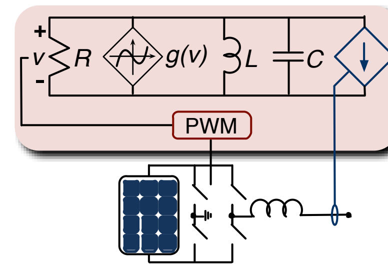
- **idealistic assumptions:** quasi-stationary operation & phasor coordinates

⇒ **future grids:** more power electronics, more renewables, & less inertia

⇒ **Virtual Oscillator Control:** control inverters as limit cycle oscillators
[Torres, Moehlis, & Hespanha '12, Johnson, Dhople, Hamadeh, & Krein '13]

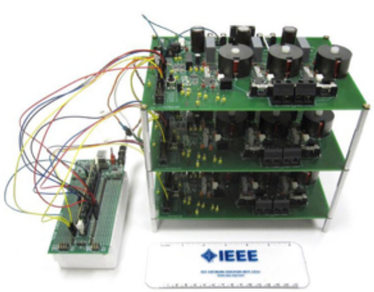


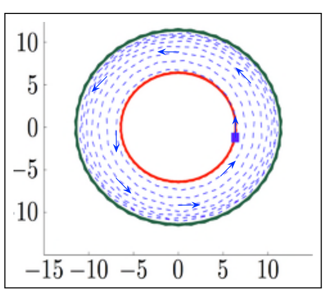
stable sustained oscillations



digitally implemented VOC

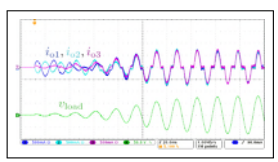
Plug'n'play **V**irtual **O**scillator **C**ontrol (VOC)



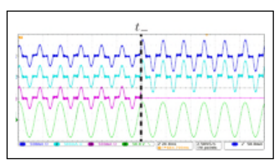


change of setpoint

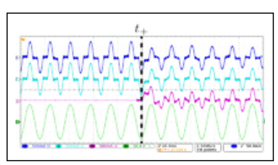
Oscilloscope plots:



emergence of synchrony



removal of inverter

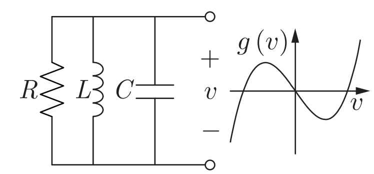


addition of inverter

Crash course on planar limit cycle oscillators

$$L\frac{d}{dt}i = v$$

$$C\frac{d}{dt}v = -Rv - g(v) - i - i_{\text{grid}}$$



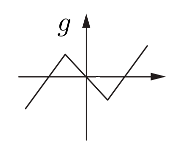
⇒ normalized coordinates

$$\ddot{v} + v + \varepsilon k_1 g'(v)\cdot\dot{v} = \varepsilon k_2 u$$

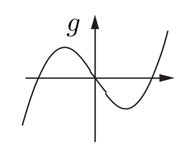
Liénard's limit cycle condition

for virtual oscillator with $u = 0$:

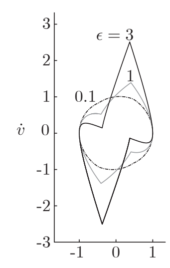
- **if** $\varepsilon = \sqrt{L/C} \to 0$
- ⇒ $\mathcal{O}(\varepsilon)$ close to harmonic oscillator
- **if** damping $g'(v)$ is negative near origin & positive elsewhere
- ⇒ unique & stable limit cycle

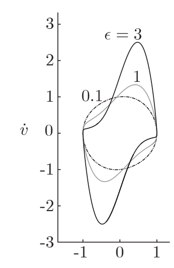


deadzone

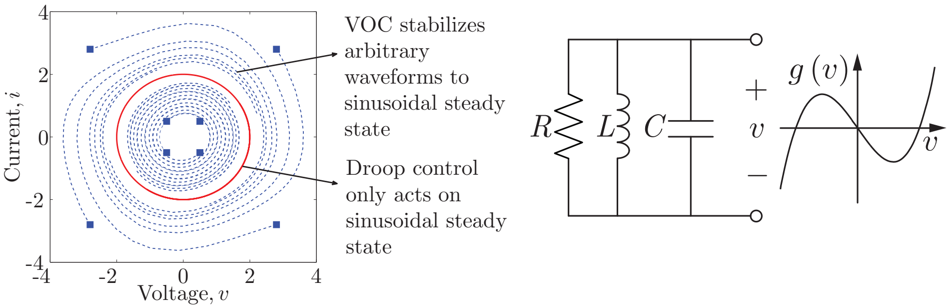


Van der Pol





Backward compatibility to droop [M. Sinha, FD, B. Johnson, & S. Dhople, '14]



⇒ transf. to polar coordinates, averaging, & generalized power definitions

Thm: in vicinity of the limit cycle: **VOC ⊃ droop:**

$$\dot{\theta} = constant \cdot (\text{reactive power})$$

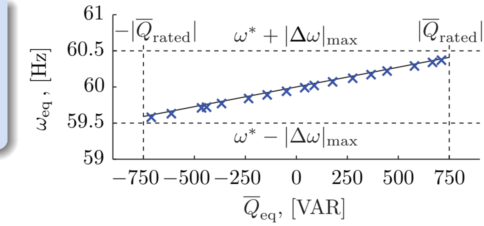
$$r - r \;\; = constant \cdot (P \;\; - \text{active power})$$

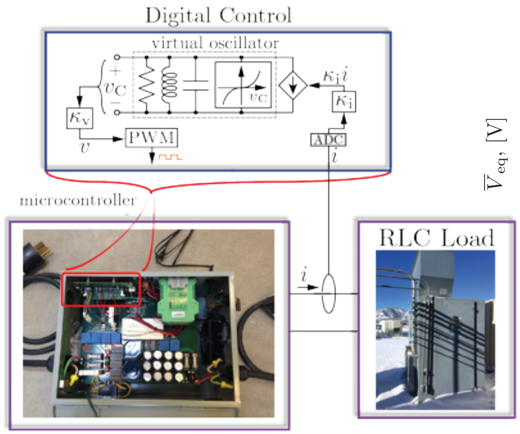
Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

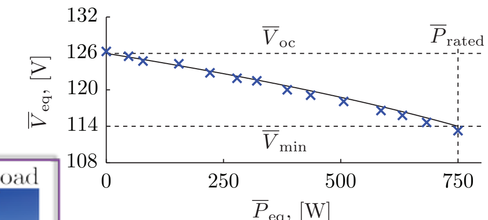
1 VOC ⊃ droop:

$$\dot{\theta} = constant \cdot (\text{reactive power})$$

$$r - r \;\; = constant \cdot (P \;\; - \text{active power})$$



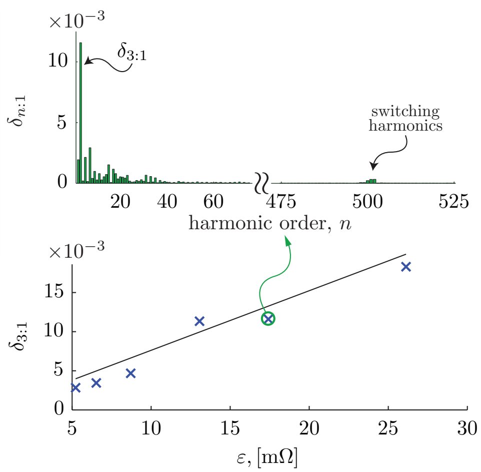




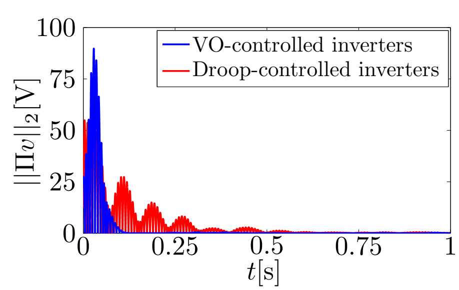
analytic vs. measured droop curves of VOC

Experimental validation [B. Johnson, M. Sinha, N. Ainsworth, FD, & S. Dhople, '15]

1. **VOC ⊃ droop**
2. **VOC $\xrightarrow{\varepsilon \downarrow 0}$ harmonic oscillator** with $\varepsilon/8$ harmonic ratio 3:1
3. **VOC:** faster & better transients than droop-controlled inverters



$\varepsilon/8$ harmonic ratio 3:1

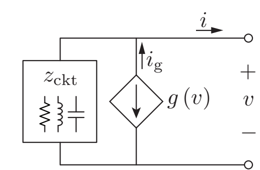


synchronization error: VOC vs. droop

Analysis of VOC system [S. Dhople, B. Johnson, FD, & A. Hamadeh '13]

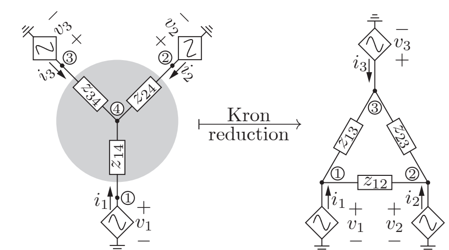
Nonlinear oscillators:

- passive circuit impedance $z_{ckt}(s)$
- active current source $g(v)$



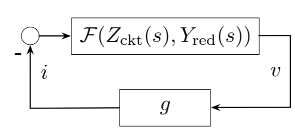
Co-evolving network:

- RLC network & loads are LTI
- Kron reduction: eliminate loads



Stability analysis:

- homogeneity assumption: identical reduced oscillators
- Lure system formulation
- incremental IQC analysis



⇝ sync for strong coupling

Variation III:

can we turn tertiary optimization directly into continuous control?

$\Downarrow$

preview on **online optimization**

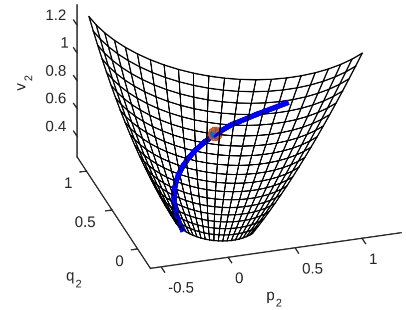
The power flow manifold & linear tangent approximation

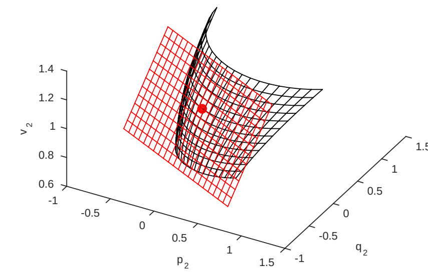
node 1 — node 2

$y = 0.4 - 0.8j$

$v_1 = 1,\ \theta_1 = 0$ $\quad$ $v_2,\ \theta_2$

$p_1,\ q_1$ $\quad$ $p_2,\ q_2$





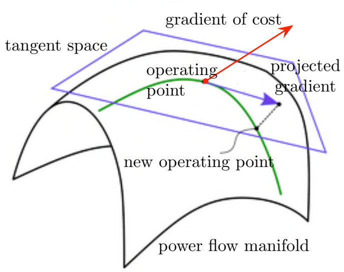
1. **power flow manifold:** $F(x) = \mathbb{0}$
2. **normal space** spanned by $\left.\frac{\partial F(x)}{\partial x}\right|_{x}$
3. **tangent space**: $\left.\frac{\partial F(x)}{\partial x}\right|_{x}^{T}(x - x\ ) = \mathbb{0}$

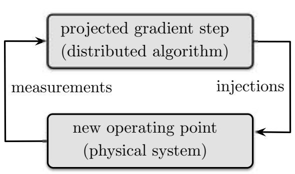
$\Rightarrow$ sparse & implicit model is **structure-preserving** $\rightarrow$ distributed control

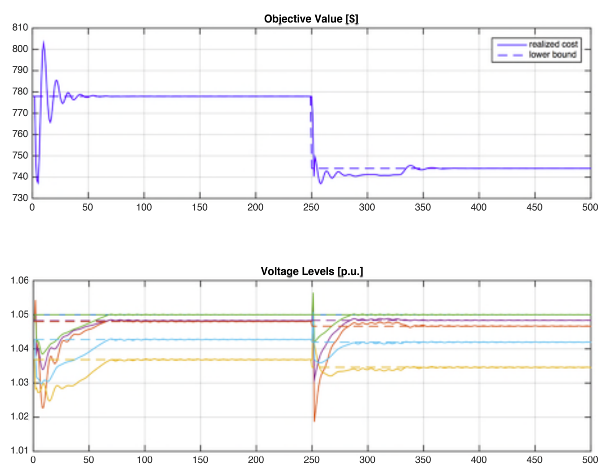
Online optimization on power flow manifold

with Adrian Hauswirth, Saverio Bolognani, & Gabriela Hug

- **manifold optimization** $\rightarrow$ gradient flow on power flow manifold
- **online optimization** $\rightarrow$ controller realizes gradient flow in closed loop



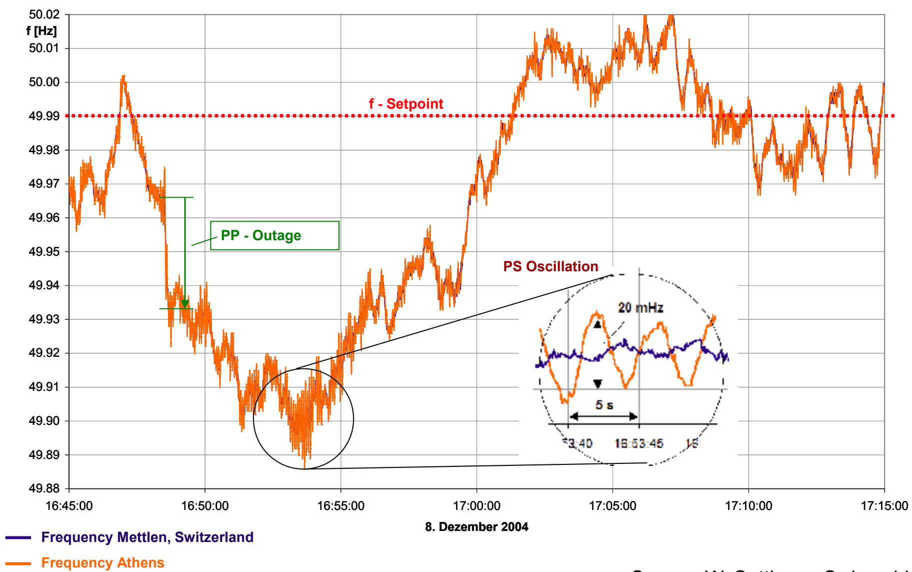




applied to optimal voltage control in IEEE 30 grid

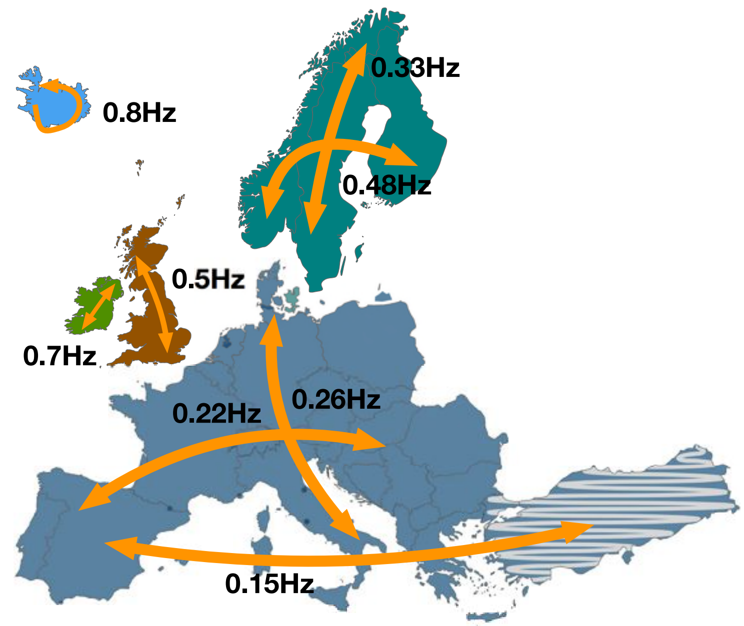
Outline

Frequency time-series reveals inter-area oscillations

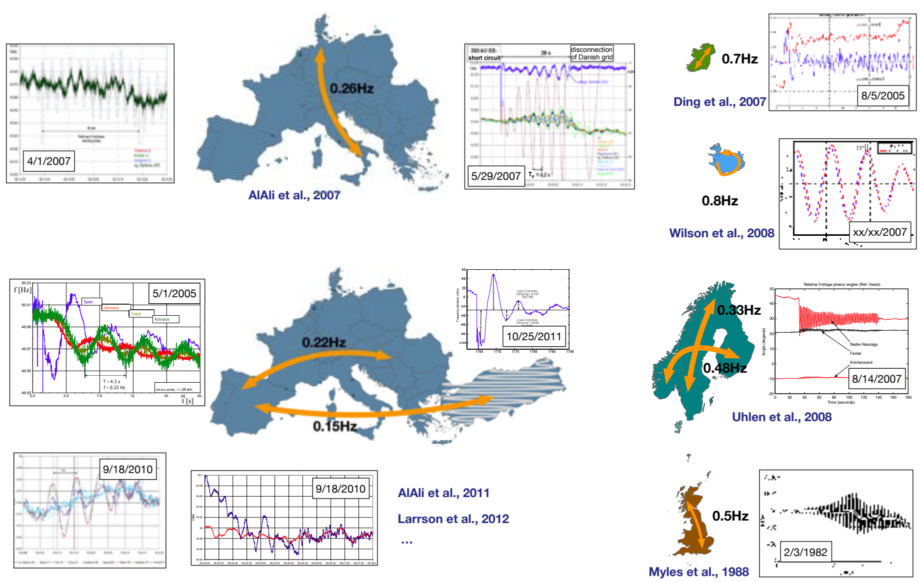


Source: W. Sattinger, Swissgrid

A few typical inter-area oscillations in Europe

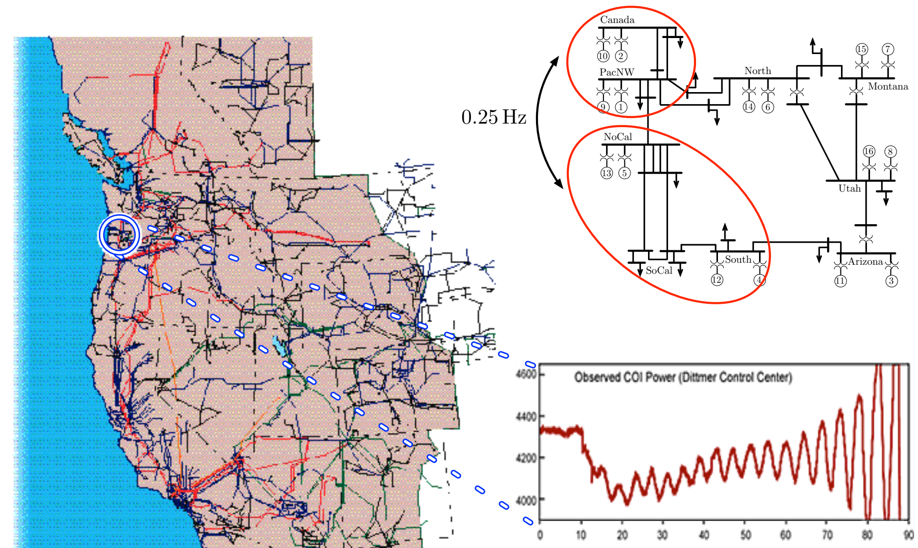


A closer look at some European incidents



Blackout of August 10, 1996

instability of the 0.25 Hz mode in the Western interconnected system



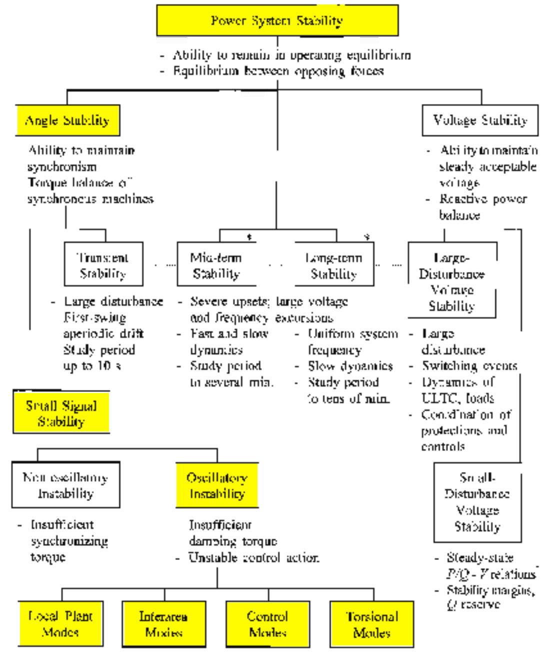
Source: http://certs.lbl.gov

Recent developments putting oscillations in the spotlight

Europe:
- transmission network upgrades & expansion,
- renewable generation in remote locations, &
- deregulated markets ...

United states:
- sparse grid with load & generation hubs,
- aging transmission infrastructure, &
- long power transfers ...

Where are we on the map?



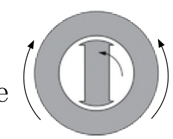
Causes for Oscillations

Why do power systems oscillate ?

power network dynamics ≈ coupled, forced, & heterogeneous pendula

generator **torque balance**:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = \text{mech.} - \text{electr. torque}$$

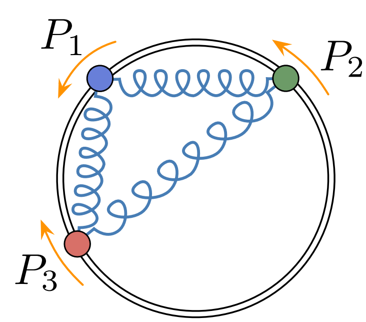


≈ electro-mechanical oscillator

coupled **swing equations**:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

≈ coupled, forced, & heterogeneous pendula



linearized at equilibrium $(\theta\ ,\dot{\theta}\ ,P\ )$:

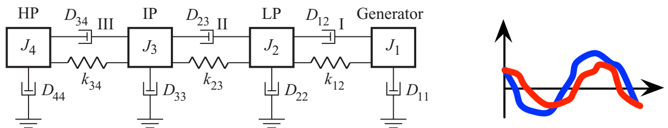
$$M\ddot{\theta} + D\dot{\theta} + L\theta = P$$

where M, D are inertia and damping matrices & L is network Laplacian

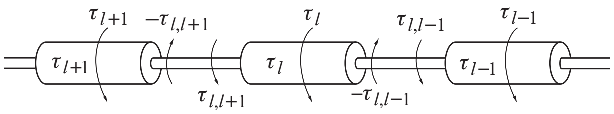
Torsional oscillations in power networks

essentially a (subsynchronous) resonance phenomenon

⇒ arise from interplay of
- electrical oscillations
- flexible mechanical shaft models
- generator-turbine coupling



turbine stages generator grid

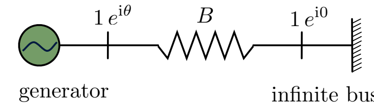


elastic generator shaft as finite-element model

⇒ subsynchronous resonance phenomena often arise in wind turbines

Local oscillations and their control

Automatic Voltage Regulator (AVR):

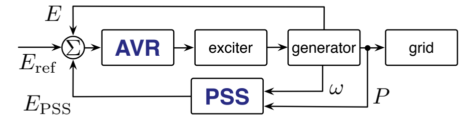


- objective: generator voltage = *const*.

⇒ diminishing damping & sync torque $\frac{\partial P}{\partial \theta}$

⇒ can result in oscillatory instability

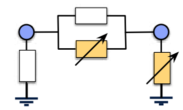
Power System Stabilizer (PSS):



- objective: net damping positive
- typical control design:

→ low-pass → wash-out → lead/lag element → gain →

Flexible AC Transmission Systems (FACTS) or HVDC:



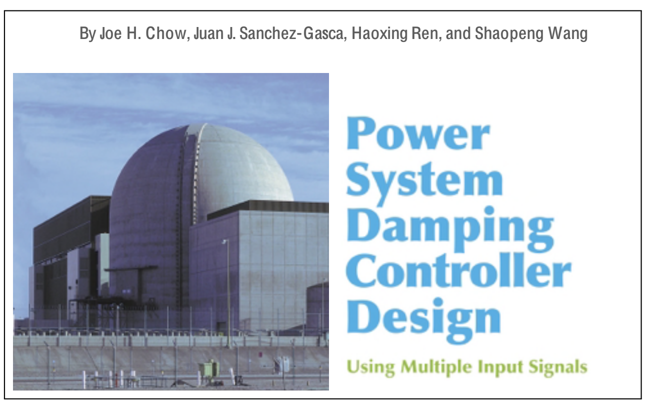
- control by "modulating" transmission line parameters
- either connected in series with a line or as shunt device

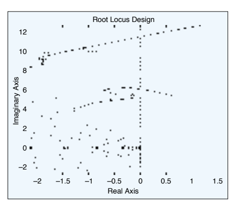
Control-induced oscillations and their control

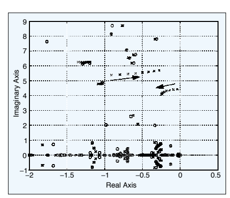
- **short story:** multiple local controllers interact in an adverse way
- **system-theoretic reason:** power system has unstable zeros

⇒ trade-off: high-gain (local stability) vs. low-gain control (avoid zeros)

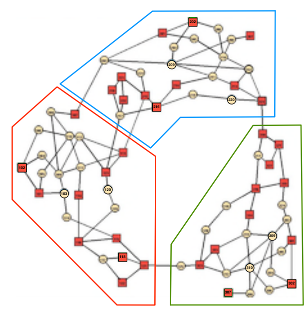
⇒ numerous tuning rules & heuristics for decentralized PSS design



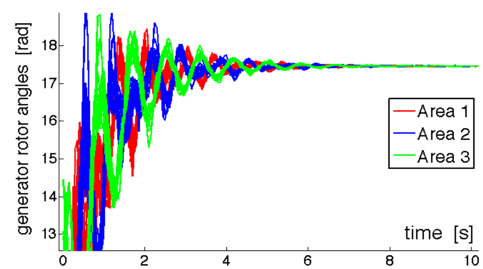




Inter-area oscillations in power networks arise due to



RTS 96 power network



swing dynamics

1. **topology**: modular & clustered
2. **heterogeneity** in responses (inertia M_i & damping D_i)
3. **power transfers** between areas (weaken coupling)
4. **interaction** of multiple local controllers

Taxonomy of electro-mechanical oscillations

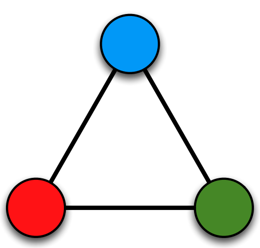
- Synchronous generator = electromech. oscillator ⇒ **local oscillations**:
  - = single generator oscillates relative to the rest of the grid
  - ☹ torsional oscillations induced by mechanical/electrical/flexible coupling
  - ☹ AVR control induces unstable local oscillations
  - ☺ typically damped by local feedback via PSSs

- Power system = complex oscillator network ⇒ **inter-area oscillations**:
  - = groups of generators oscillate relative to each other
  - ☹ poorly tuned local PSSs result in unstable inter-area oscillations
  - ☹ inter-area oscillations are only poorly controllable by local feedback

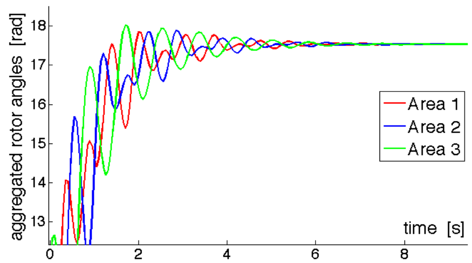
- Consequences of **recent developments**:
  - ☹ increasing power transfers outpace capacity of transmission system
  - ⇒ ever more lightly damped electromechanical inter-area oscillations
  - ☺ technological opportunities for **wide-area control (WAC)**

Slow Coherency Modeling

Slow coherency and area aggregation



aggregated RTS 96 model



swing dynamics of aggregated model

Aggregate model of lower dimension & with less complexity for

1. analysis and insights into inter-area dynamics [Chow and Kokotovic '85]
2. measurement-based id of equivalent models [Chakrabortty et.al.'10]
3. remedial action schemes [Xu et. al. '11] & wide-area control (later today)

How to find the areas?

a crash course in spectral partitioning

- given: an undirected, connected, & weighted **graph**
- **partition:** $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, and $\mathcal{V}_1, \mathcal{V}_2 \neq \emptyset$
- **cut** is the size of a partition: $J = \sum_{i 2 V_1, j 2 V_2} a_{ij}$

⇒ if $x_i = 1$ for $i \in \mathcal{V}_1$ and $x_j = -1$ for $j \in \mathcal{V}_2$, then

$$J = \sum_{i 2 V_1, j 2 V_2} a_{ij} = \frac{1}{2} \sum_{i,j=1}^{n} a_{ij}(x_i - x_j)^2 = \frac{1}{2} x^T L x$$

- combinatorial **min-cut** problem: $\text{minimize}_{x 2 f \ 1,1 g^n n f \ \mathbb{1}_n, \mathbb{1}_n g} \frac{1}{2} x^T L x$
- **relaxed problem:** $\text{minimize}_{y 2 \mathbb{R}^n, \, y ? \mathbb{1}_n, \, k y k_2 = 1} \frac{1}{2} y^T L y$

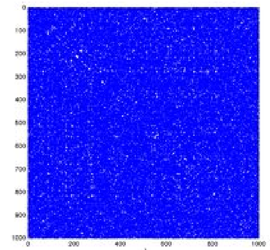
⇒ minimum is *algebraic connectivity* λ_2 and minimizer is *Fiedler vector* v_2

- **heuristic:** $x_i = \text{sign}(y_i)$ ⇒ "spectral partition"

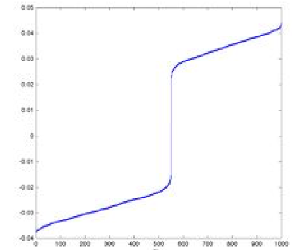
A quick example

```
% choose a graph size
n = 1000;

% randomly assign the nodes to two grous
x = randperm(n);
group_size = 450;
group1 = x(1:group_size);
group2 = x(group_size+1:end);

% assign probabilities of connecting nodes
p_group1 = 0.5;
p_group2 = 0.4;
p_between_groups = 0.1;

% construct adjacency matrix
A(group1, group1) = rand(group_size,group_size) < p_group1;
A(group2, group2) = rand(n-group_size,n-group_size) < p_group2;
A(group1, group2) = rand(group_size, n-group_size) < p_between_groups;
A = triu(A,1); A = A + A';

% can you see the groups?
subplot(1,3,1); spy(A);

% construct Laplacian and its spectrum
L = diag(sum(A))-A;
[V D] = eigs(L, 2, 'SA');

% plot the components of the algebraic connectivity sorted by magnitude
subplot(1,3,2); plot(sort(V(:,2)), '.-');

% partition the matrix accordingly and spot the communities
[ignore p] = sort(V(:,2));
subplot(1,3,3); spy(A(p,p));
```

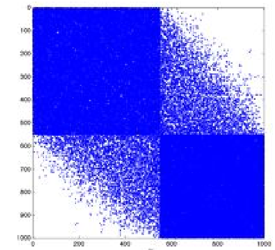

A quick example – cont'd



adjacency matrix



Fiedler vector v_2



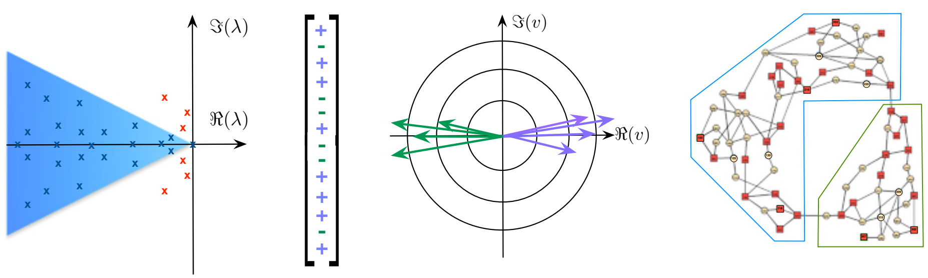
re-arranged adj. matrix

Classical power system partitioning ≈ spectral partitioning

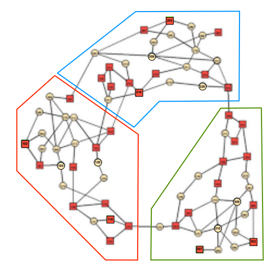
1. construct a linear model $\dot{x} = Ax$
2. recall solution via eigenvalues λ_i and left/right eigenvectors w_i and v_i:

$$x(t) = \sum_i v_i e^{\lambda_i t} \cdot w_i^T x_0 = \sum_i \{\text{mode } \#i\} \cdot \{\text{contribution from } x_0\}$$

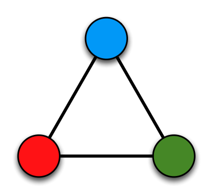
3. look at poorly damped complex conjugate mode pairs
4. look at angle & frequency components of eigenvectors
5. group the generators according to their polarity in eigenvectors



Setup in slow coherency



original model



aggregated model

- network r given **areas** (from spectral partition [Chow et al. '85 & '13])
- small **sparsity parameter**:

$$\delta = \frac{\max_\alpha(\Sigma \text{ external connections in area } \alpha)}{\min_\alpha(\Sigma \text{ internal connections in area } \alpha)}$$

- **inter-area dynamics** by center of inertia:

$$y_\alpha = \frac{\sum_{i 2 \alpha} M_i \theta_i}{\sum_{i 2 \alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

- **intra-area dynamics** by area differences:

$$z_i^\alpha{}_1 = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

Linear transformation & time-scale separation

Swing equation $\implies$ singular perturbation standard form

$$M\ddot{\theta} + D\dot{\theta} + L\theta = 0 \quad \implies \quad \left\{ \frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \cdot^{\cdot^{\cdot}} \\ \cdots & A & \cdots \\ \cdot^{\cdot^{\cdot}} & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \right.$$

Slow motion given by center of inertia:

$$y_\alpha = \frac{\sum_{i2\alpha} M_i \theta_i}{\sum_{i2\alpha} M_i}, \quad \alpha \in \{1, \dots, r\}$$

Fast motion given by intra-area differences:

$$z_i^\alpha{}_1 = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1, \dots, r\}$$

Slow time scale: $t_s = \delta \cdot t \cdot$ "max internal area degree"

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \cdot^{\cdot^{\cdot}} \\ \cdots & A & \cdots \\ \cdot^{\cdot^{\cdot}} & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\, \ddot{\varphi} + D_a\, \dot{\varphi} + L_{\text{red}}\, \varphi = 0$$

Properties of aggregated model [D. Romeres, FD, & F. Bullo, '13]

1. $M_a = \begin{bmatrix} \ddots & & \\ & \sum_{i2\alpha} M_i & \\ & & \ddots \end{bmatrix}$ and $D_a = \begin{bmatrix} \ddots & & \\ & \sum_{i2\alpha} D_i & \\ & & \ddots \end{bmatrix}$
2. L_{red} = "inter-area Laplacian" + "intra-area contributions"
   = positive semidefinite Laplacian with possibly negative weights

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \cdot^{\cdot^{\cdot}} \\ \cdots & A & \cdots \\ \cdot^{\cdot^{\cdot}} & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\, \ddot{\varphi} + D_a\, \dot{\varphi} + L_{\text{red}}\, \varphi = 0$$

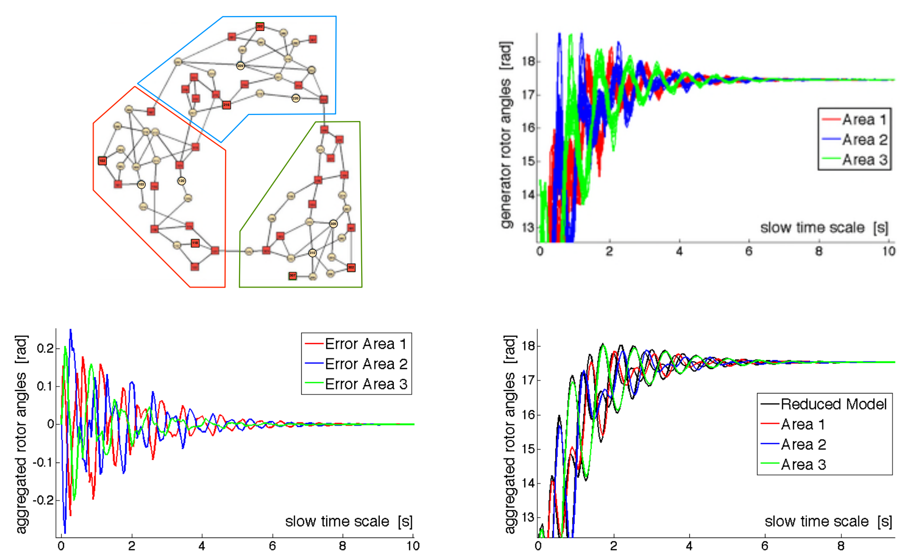
Singular perturbation approximation [D. Romeres, FD, & F. Bullo, '13]

There exist δ sufficiently small such that for $\delta \leq \delta$ and for all $t > 0$:

$$\begin{bmatrix} y(t_s) \\ \dot{y}(t_s) \end{bmatrix} = \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}), \quad \begin{bmatrix} z(t_s) \\ \dot{z}(t_s) \end{bmatrix} = \tilde{A} \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta}).$$

center of inertia $\approx$ solution of aggregated swing equation

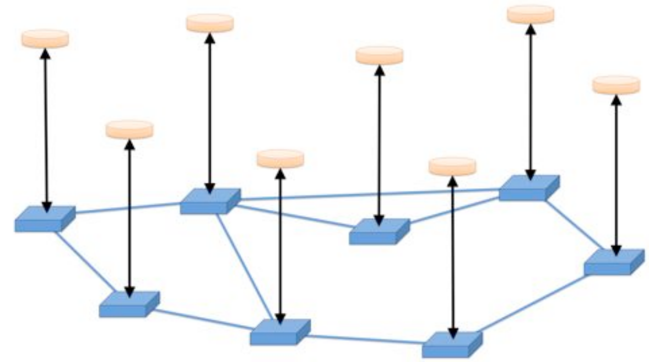
RTS 96 swing dynamics revisited



Inter-Area Oscillations & Wide-Area Control

Remedies against electro-mechanical oscillations

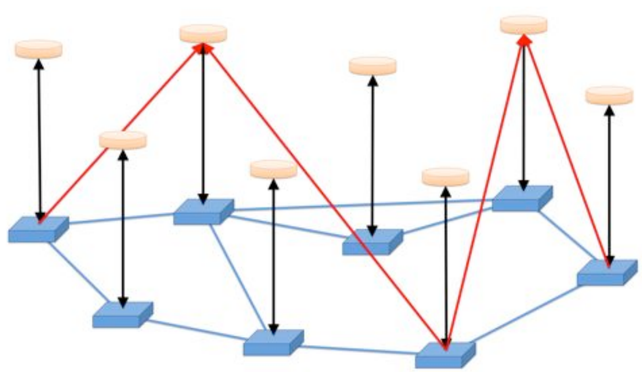
conventional control



- blue layer: interconnected generators
- fully decentralized control implemented locally
  - ☺ effective against local oscillations
  - ☹ ineffective against inter-area oscillations

Remedies against electro-mechanical oscillations

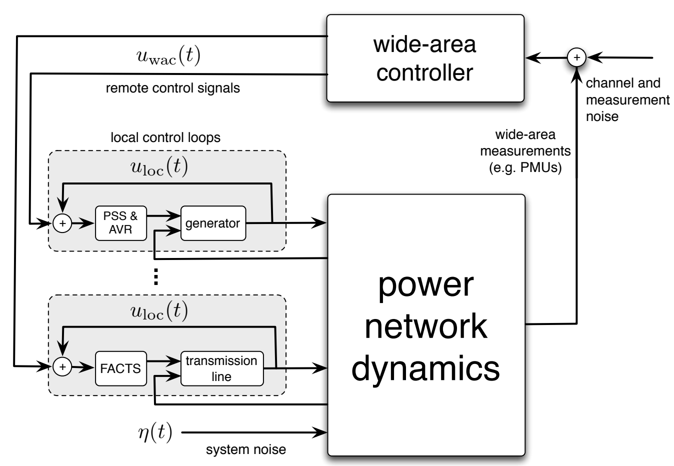
wide-area control (WAC)



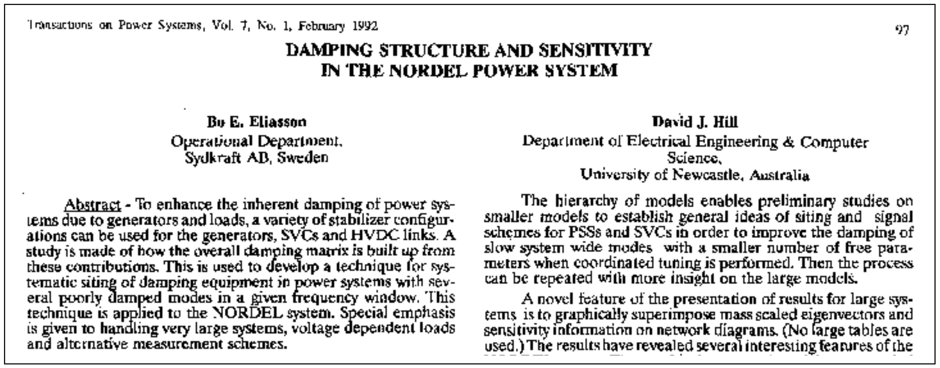
- blue layer: interconnected generators
- fully decentralized control implemented locally
- distributed wide-area control using remote signals

Setup in wide-area control

1. remote control signals & remote measurements (e.g., PMUs)
2. excitation (PSS & AVR) and power electronics (FACTS) actuators
3. communication backbone network



Debated: do we need **distributed** wide-area control or can we get away with **fully decentralized** control?



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DAMPING STRUCTURE AND SENSITIVITY IN THE NORDEL POWER SYSTEM

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Sydkraft AB, Sweden

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Abstract - To enhance the inherent damping of power systems due to generators and loads, a variety of stabilizer configurations can be used for the generators, SVCs and HVDC links. A study is made of how the overall damping matrix is built up from these contributions. This is used to develop a technique for systematic siting of damping equipment in power systems with several poorly damped modes in a given frequency window. This technique is applied to the NORDEL system. Special emphasis is given to handling very large systems, voltage dependent loads and alternative measurement schemes.

The hierarchy of models enables preliminary studies on smaller models to establish general ideas of siting and signal schemes for PSSs and SVCs in order to improve the damping of slow system wide modes with a smaller number of free parameters when coordinated tuning is performed. Then the process can be repeated with more insight on the large models.

A novel feature of the presentation of results for large systems is to graphically superimpose mass scaled eigenvectors and sensitivity information on network diagrams. (No large tables are used.) The results have revealed several interesting features of the

"The above reasoning implies that if observability is small, so is also controllability. The benefits of remote signals for power system damping should thus be marginal." [follow-up comments by G. Andersson & T. Smed, '92]

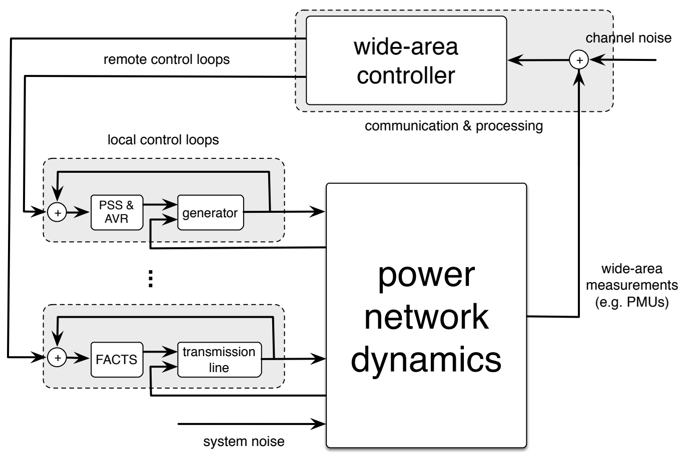
conventional analysis & wide-area control

(based on spectral methods)

I will be a little provocative . . .

Canonical setup in wide-area control

local actuators, remote measurements, & communication backbone



⇒ **problem I:** signal selection (sensors & actuators)

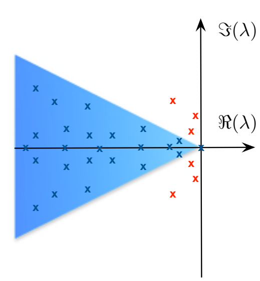
⇒ **problem II:** WAC design (subject to control signals)

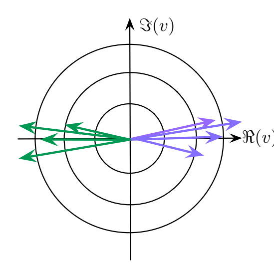
Recall: spectral analysis reveals critical modes & areas

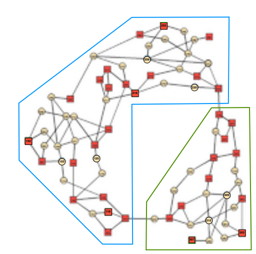
1. recall solution of $\dot{x} = Ax$: $x(t) = \sum_i \underbrace{v_i e^{\lambda_i t}}_{\text{mode } \#i} \cdot \underbrace{w_i^T x_0}_{\text{contribution from } x_0}$

2. analyze eigenvectors & participation factors of weakly damped modes

3. spectral partitioning reveals coherent groups in eigenvectors polarities







Which sensors and actuators ?

1. **Linear control system:** $\dot{x} = Ax + Bu \;\;,\;\; y = Cx$
   - B with column b_j = control location $\#j$
   - C with row c_j^T = sensor location $\#j$
   - A: eigenvalues λ_i and orthonormal right & left eigenvectors v_i & w_i^*
2. **Diagonalization:** $x = Vz = \begin{bmatrix} v_1 & \dots & v_n \end{bmatrix} z \;\;,\;\; z = Wx = \begin{bmatrix} w_1 & \dots & w_n \end{bmatrix}^* x$

$$\Rightarrow \dot{z} = \underbrace{\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}}_{=WAV} z + \underbrace{\begin{bmatrix} & \vdots & \\ \dots & w_i^* b_j & \dots \\ & \vdots & \end{bmatrix}}_{=WB} u \;\;,\;\; y = \underbrace{\begin{bmatrix} & \vdots & \\ \dots & c_i^* v_j & \dots \\ & \vdots & \end{bmatrix}}_{=CV} uz$$

3. Controllability of mode i by input $j \triangleq \cos\left(\angle(w_i\,,\,b_j)\right) = \frac{w_i\, b_j}{\|w_i\|\|b_j\|}$
4. Observability of mode i by sensor $j \triangleq \cos\left(\angle(c_i\,,\,v_j)\right) = \frac{c_i\, v_j}{\|c_i\|\|v_j\|}$

Modal signal selection metrics

Assessment of Two Methods to Select Wide-Area Signals for Power System Damping Control

Annissa Heniche, *Member, IEEE*, and Innocent Kamwa, *Fellow, IEEE*

***Abstract*—In this paper, two different approaches are applied to the Hydro-Québec network in order to select the most effective signals to damp inter-area oscillations. The damping is obtained by static var compensator (SVC) and synchronous condenser (SC) modulation. The robustness analysis, the simulations, and statistical results show, unambiguously, that in the case of wide-area signals, the geometric approach is more reliable and useful than the residues approach. In fact, this study shows that the best robustness and performances are always obtained with the stabilizer configuration using the signals recommended by the geometric approach. In addition, the results confirm that wide-area control is more effective than local control for damping inter-area oscillations.**

the results concern only the Hydro-Québec network, it is important to notice that a statistical analysis was realized. This analysis allowed the test of the two approaches using 1140 different configurations of the network.

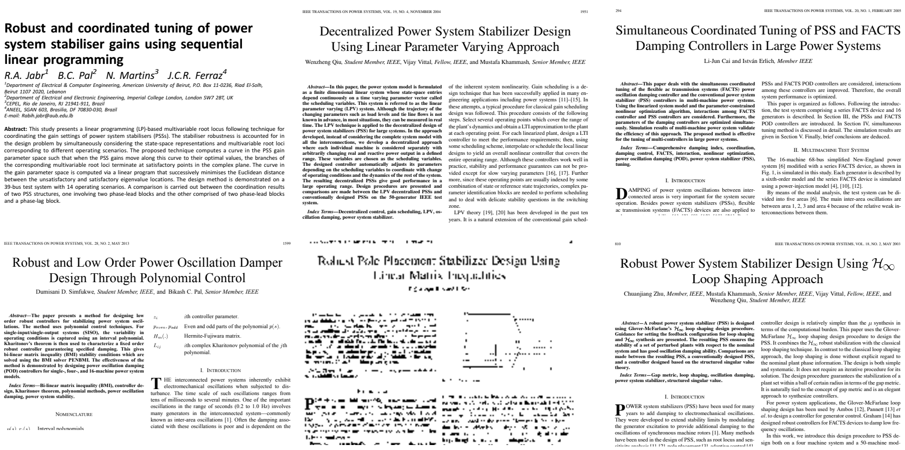
The aims of this paper are on one hand to show that the two measures do not provide the same conclusion in terms of control loop selection and on the other hand to demonstrate the efficiency and reliability of one measure in comparison to the other. To do that, the two measures were applied in order to select the most effective control loops for damping the 0.6-Hz inter-area

1. **geometric criteria** [H.M.A. Hamdan & A.M.A. Hamdan '87]:
   - e.g., modal controllability: effect of control input $\#j$ on mode $\#i$
2. **frequency criteria** [M. Tarokh '92]: modal residues of transfer function

$\Rightarrow$ suboptimal procedures with many requirements: (i) identification of critical modes, (ii) sensor/actuator catalog, (iii) **combinatorial** evaluation

Decentralized WAC control design . . .

- . . . subject to structural constraints is **tough**
- . . . usually handled with **suboptimal heuristics** in MIMO case



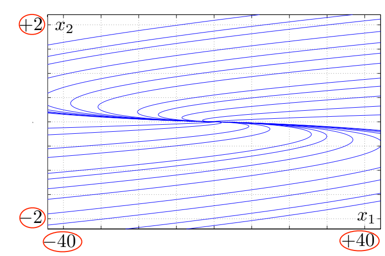
$\Rightarrow$ signal selection is combinatorial & control design is suboptimal

Challenges in wide-area control

1. **signal selection** is combinatorial
2. **decentralized control** is suboptimal
3. **identification** of critical modes is somewhat *ad hoc*

What information is contained in the spectrum of a *non-normal* matrix?

Example: $\dot{x} = \begin{bmatrix} -1 & 10^2 \\ 0 & -1 \end{bmatrix} x$



Today [X. Wu, FD, & M. Jovanovic '15]:

$\Rightarrow$ performance metric: variance amplification of stochastic system

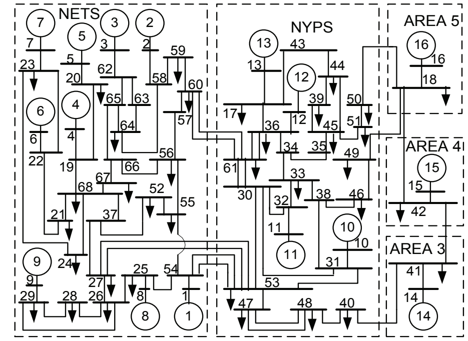
$\Rightarrow$ simultaneously optimize performance & control architecture

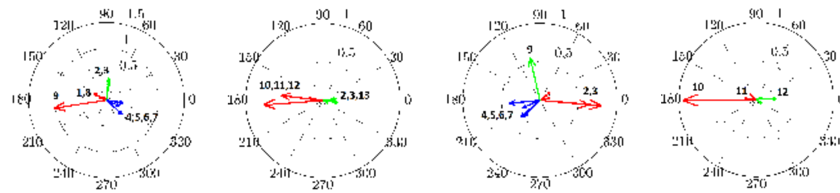
$\Rightarrow$ fully decentralized & nearly optimal controller

running case study: New England – New York

Case study: New England – New York test system

- **model features** (242 states):
  - sub-transient generator models [Singh et. al. '14]
  - open loop is unstable
  - exciters & tuned PSSs
- frequency & damping ratios of dominant **inter-area modes**





1.1Hz @ 3.8% | 1.3Hz @ 4.2% | 1.1Hz @ 4.7% | 1.3Hz @ 4.9%

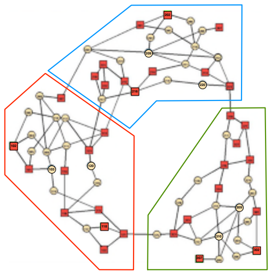
variance amplification as performance metric

$$\int_0^\infty x(t)^T Q\, x(t)\, dt$$

Primer on $\mathcal{H}_2$ - **norms**

Slow coherency performance objectives

- recall **sources** for inter-area oscillations:



  - linearized **swing equation**: $M\ddot{\theta} + D\dot{\theta} + L\theta = P$
  - **mechanical energy**: $\frac{1}{2}\,\dot{\theta}M\dot{\theta} + \frac{1}{2}\,\theta^T L\theta$
  - **heterogeneities** in topology, power transfers, & machine responses (inertia & damp)

$\Rightarrow$ performance **objective** = energy of homogeneous network:

$$x^T Q\, x \;=\; \dot{\theta}^T M\,\dot{\theta} \;+\; \theta^T\big(I_n - (1/n)\cdot \mathbb{1}_{n\times n}\big)\,\theta$$

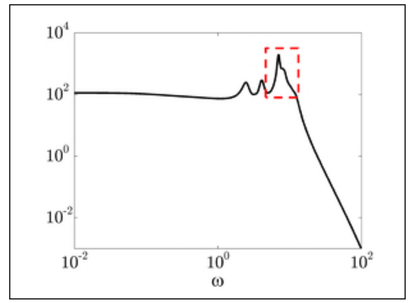
- other choices possible: center of inertia, inter-area differences, etc.

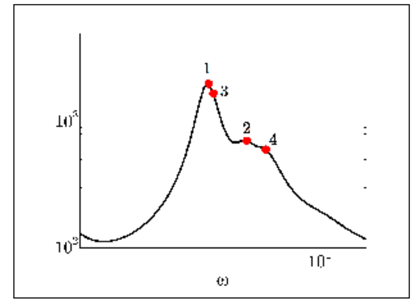
Input-output analysis in $\mathcal{H}_2$ - metric

- linear system with **white noise input**: $\dot{x} = Ax + B_1\eta$
- energy of homogeneous network as **performance output**: $z = Q^{1/2}x$
- **steady-state variance** of the output is given by the $\mathcal{H}_2$-norm

$$\|G\|_{H_2}^2 \;:=\; \lim_{t\to\infty} \mathbb{E}\left(x(t)^T\, Q\, x(t)\right) \;=\; \frac{1}{2\pi}\int_{-\infty}^{\infty} \|G(\mathrm{j}\,\omega)\|_{\mathrm{HS}}^2\, \mathrm{d}\omega$$

- **power spectral density** $\|G(\mathrm{j}\,\omega)\|_{\mathrm{HS}}^2$ reveals NE-NY inter-area modes





$\mathcal{H}_2$ - norms for consensus-like systems

see exercise

sparsity-promoting optimal control

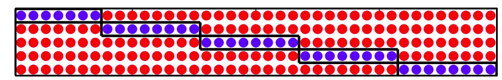
Primer on Linear Quadratic Control (LQR)

Optimal linear quadratic regulator (LQR)

- **model:** linearized ODE dynamics $\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2 u(t)$
- **control:** memoryless linear state feedback $u = -Kx(t)$
- **optimal centralized control** with quadratic $\mathcal{H}_2$ - performance index:

$$\begin{aligned} \text{minimize} \quad & J(K) \triangleq \lim_{t \to \infty} \mathbb{E}\left\{x(t)^T Q x(t) + u(t)^T R u(t)\right\} \\ \text{subject to} \quad & \\ \text{linear dynamics:} \quad & \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2 u(t), \\ \text{linear control:} \quad & u(t) = -Kx(t), \\ \text{stability:} \quad & (A - B_2 K) \text{ Hurwitz.} \end{aligned}$$

(no structural constraints on K)



Sparsity-promoting optimal LQR

[Lin, Fardad, & Jovanović, '13]

simultaneously optimize performance & architecture

$$\begin{aligned} \text{minimize} \quad & \lim_{t \to \infty} \mathbb{E}\left\{x(t)^T Q x(t) + u(t)^T R u(t)\right\} + \gamma \cdot \text{card}(K) \\ \text{subject to} \quad & \\ \text{linear dynamics:} \quad & \dot{x}(t) = Ax(t) + B_1\eta(t) + B_2 u(t), \\ \text{linear control:} \quad & u(t) = -Kx(t), \\ \text{stability:} \quad & (A - B_2 K) \text{ Hurwitz.} \end{aligned}$$

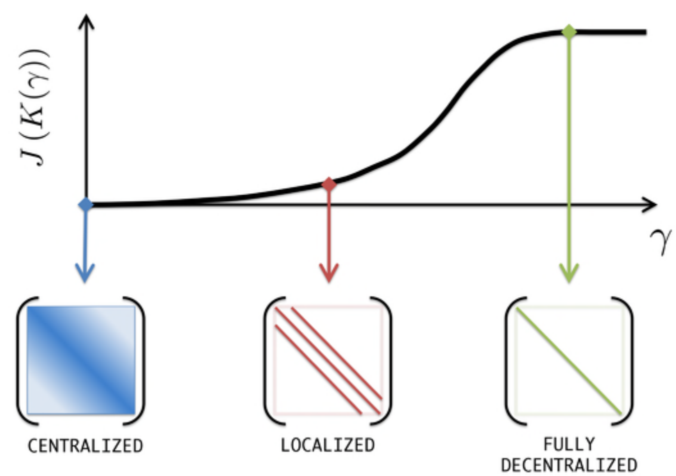
$\Rightarrow$ for $\gamma = 0$: standard optimal control (typically not sparse)

$\Rightarrow$ for $\gamma > 0$: sparsity is promoted (problem is combinatorial)

$\Rightarrow$ card(K) convexified by weighted ℓ_1-norm $\sum_{i,j} w_{ij}|K_{ij}|$

Parameterized family of feedback gains

$$K(\gamma) = \arg\min_K \left(J(K) + \gamma \cdot \sum_{i,j} w_{ij}|K_{ij}|\right)$$



Algorithmic approach in an nutshell (detailed in back-up slides)

1. **Algebraic formulation** via Gramian and Lyapunov equation
2. **Non-convexity** in K: use homotopy path in γ & ADMM

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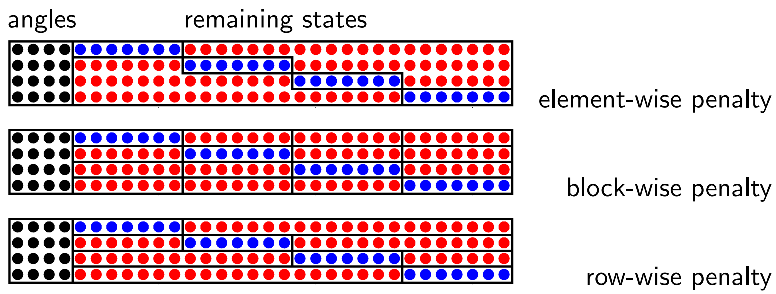
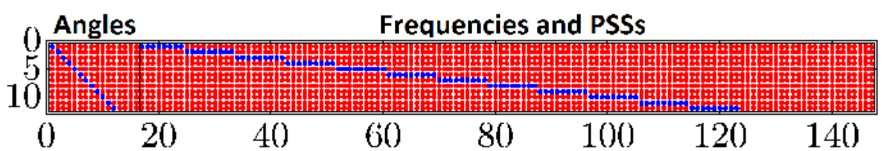
3. **Rotational symmetry:** remove absolute angle by COI transformation
4. **Block/row-sparsity**-promoting optimal control

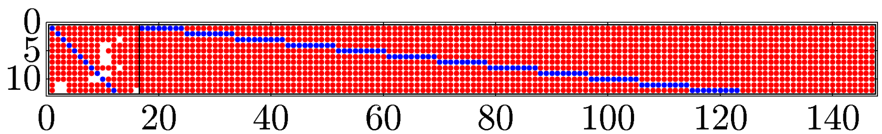


sparsity-promoting control of inter-area oscillations

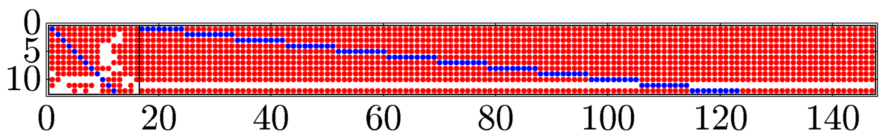
Sparsity-promoting control architecture



$\gamma = 0$, **card** $(K) = 1764$

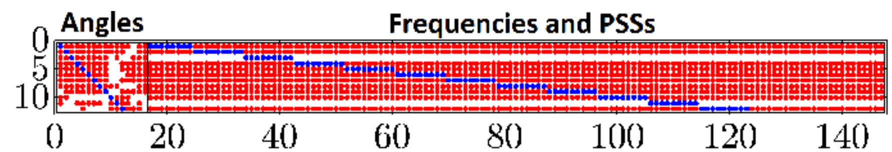


$\gamma = 10^{-4}$, **card** $(K) = 1746$

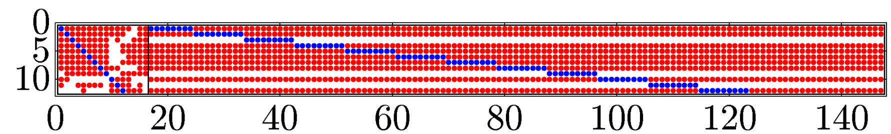


$\gamma = 0.00015$, **card** $(K) = 1603$

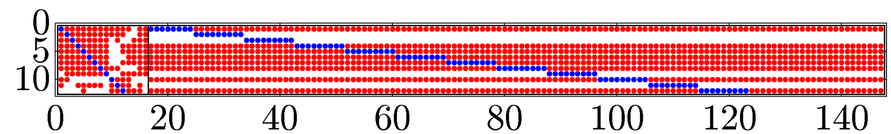
Sparsity-promoting control architecture



$\gamma = 0.00023$, **card** $(K) = 1475$

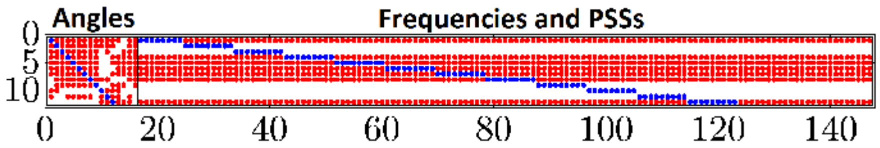


$\gamma = 0.00031$, **card** $(K) = 1353$

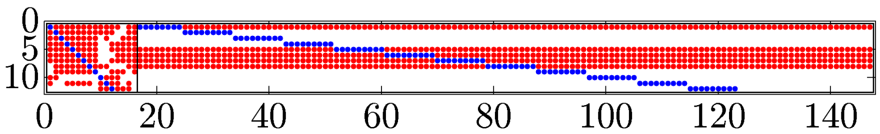


$\gamma = 0.00041$, **card** $(K) = 1231$

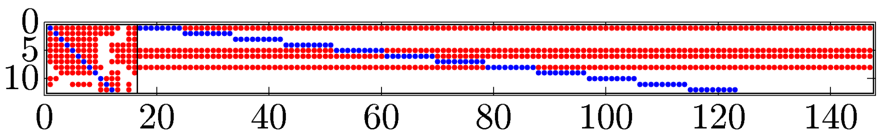
Sparsity-promoting control architecture



$\gamma = 0.00047$, **card** $(K) = 1106$

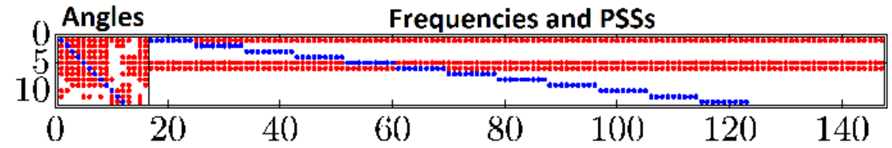


$\gamma = 0.00054$, **card** $(K) = 862$

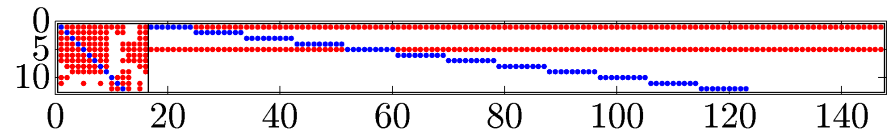


$\gamma = 0.00063$, **card** $(K) = 733$

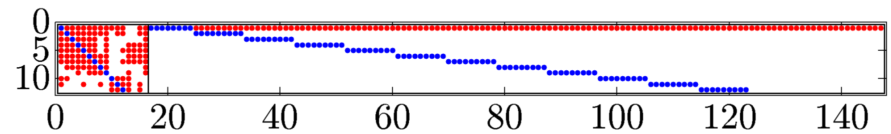
Sparsity-promoting control architecture



$\gamma = 0.00095$, **card** $(K) = 609$

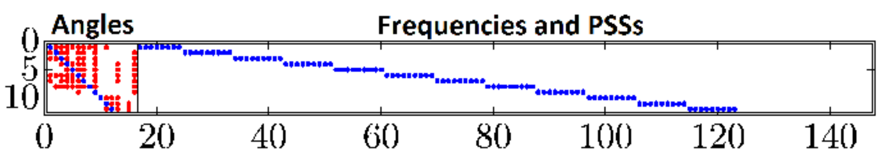


$\gamma = 0.0011$, **card** $(K) = 484$

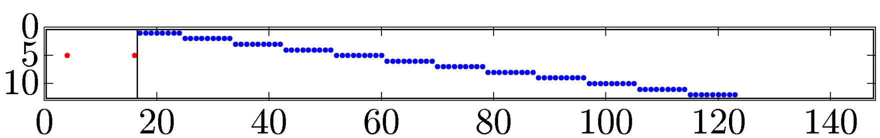


$\gamma = 0.0015$, **card** $(K) = 353$

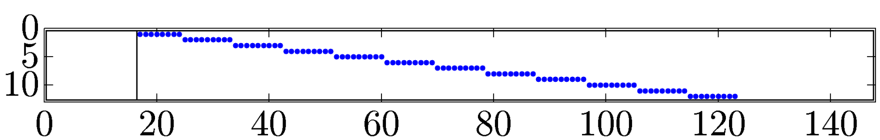
Sparsity-promoting control architecture



$\gamma = 0.0060$, **card** $(K) = 191$



$\gamma = 0.0655$, **card** $(K) = 109$

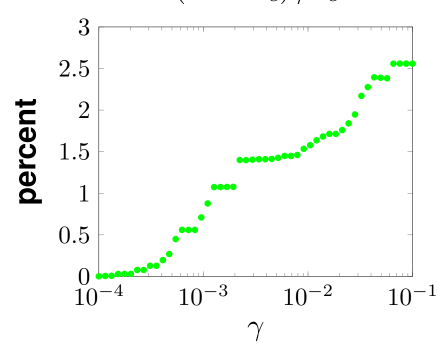


$\gamma = 0.1$, **card** $(K) = 107$

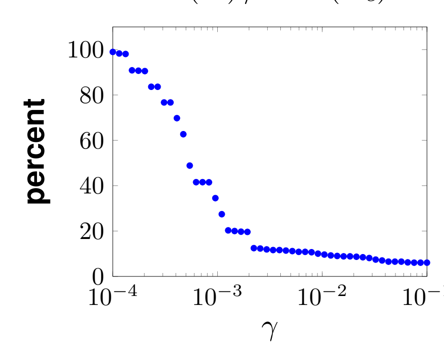
Performance vs. sparsity

$Q =$ energy of homogeneous network , $R = I_n$, $\gamma \in [10^{-4}, 0.1]$

$(J - J_c)/J_c$



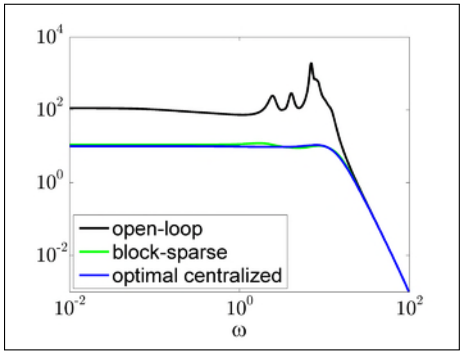
card (K) / **card** (K_c)



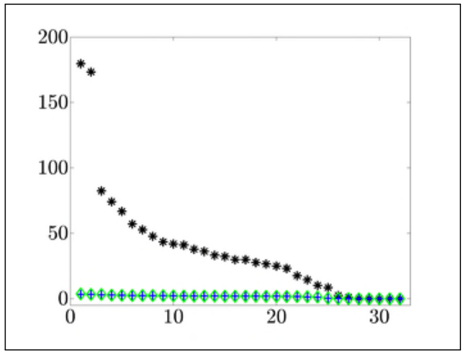
$$\gamma = 0.1 \Rightarrow \begin{cases} 2.6\,\% & \text{relative performance loss} \\ 6.1\,\% & \text{non-zero elements in } K \end{cases}$$

$\Rightarrow$ fully decentralized control is nearly optimal **!**

Performance comparison of different approaches



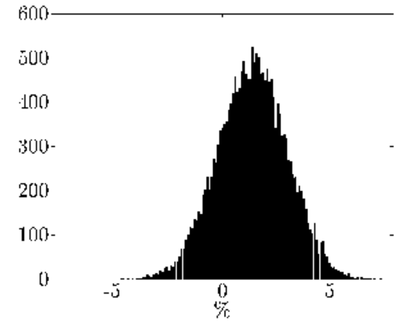
power spectral density



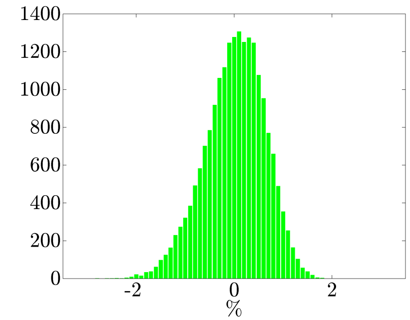
spectrum of covariance matrix

Robustness: optimal control reduces sensitivity

nominal controller applied to 20,000 operating points with ±20% randomized loading



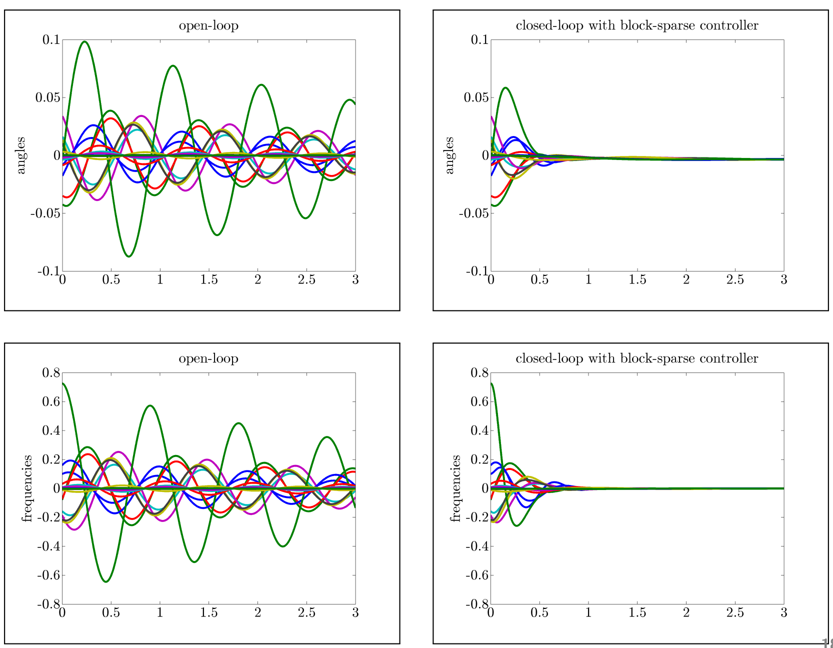
open-loop system



block-sparse controller

⇒ optimal (decentralized) control reduces sensitivity

Eye candy: time-domain simulations



Outline

Looking for data, toolboxes, & test cases

- **Matpower (static)** for (optimal) power flow & static models
  `http://www.pserc.cornell.edu//matpower/`
- **Matpower (dynamic)** with generator models
  `http://www.kios.ucy.ac.cy`
- **Power System Toolbox** for dynamics & North American models
  `http://www.eps.ee.kth.se/personal/vanfretti/pst/Power_System_Toolbox_Webpage/PST.html`
- **IEEE Task Force PES PSDPC SCS**: New York, Brazil, Australian grids etc.; `http://www.sel.eesc.usp.br/ieee/`
- **ObjectStab** for Modelica for dynamics & models
  `https://github.com/modelica-3rdparty/ObjectStab`
- **More freeware:** MatDyn, PSAT, THYME, Dome, . . .
  `http://ewh.ieee.org/cmte/psace/CAMS_taskforce/`
- **Other:** many test cases in papers, reports, task forces, . . .

Conclusions

Brief Introduction

Power Network Modeling

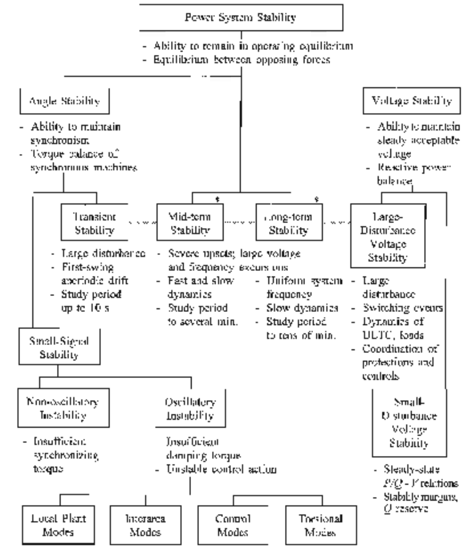
Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions

Obviously, there is a lot more . . .



I hope I could give you a little insight into a few interesting problems.

final words of wisdom

Power system economics

Market-based operation: formulations, basic principles, problems and benefits
Spatial dimension of energy trading and power balancing
Ancillary services and real-time control

Andrej Jokić
Control Systems group
Faculty of Mechanical Engineering and Naval Architecture
University of Zagreb

smart grids ?

hidden technology

invisible hand of market

important (for the “smart“ part): get the fundamentals right and well

Outline

Deregulation

Unifying approach: optimization

In general terms, problems of a power system on global level can be summarized as follows

i) Economical efficiency subject to: Global *energy* balance + Transmission system security constraints

ii) Economical efficiency subject to: Accumulation of sufficient amount of ancillary service + Transmission system security constraints

iii) Economical and dynamical efficiency, subject to: Global *power* balance + Robust stability

ECONOMY versus **RELIABILITY**

- Formulation of **PROBLEMS**: structured, time-varying optimization problems
- **SOLUTIONS:**
  - not only algorithms that give solution (as desired output), but also:
  - efficient, robust (optimally account for trade-offs), scalable and flexible control and operational architecture (who does what and when? relations?)
  - long term benefits of markets due to different solution architecture compared to regulated system

Positioning in time scale

Market commodities

- Energy markets: commodity is energy [MWh]
- Ancillary services markets (power balancing): commodity is energy (options) and sometimes capacity (placed on disposal over some time) [MWh]

Observation: Commodities are defined over time intervals (necessary to quantify energy)

Program time unit (PTU)

Program time unit (PTU): a market trading period (5min to 1h) for forward and real-time markets.
Some markets trade with over longer intervals (days, months,...)

Positioning in time scale

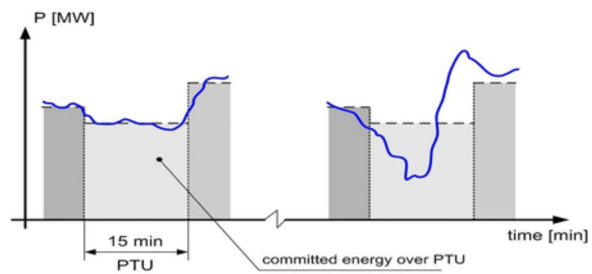
Market commodities

- Energy markets: commodity is energy [MWh]
- Ancillary services markets (power balancing): commodity is energy (options) and sometimes capacity (placed on disposal over some time) [MWh]

Positioning in time scale

Power versus energy

- Ancillary services: provision of power (real-time), trading in energy/capacity
- Congestion: constraints on power flows (real-time), trading in energy



Positioning in time scale

Power versus energy

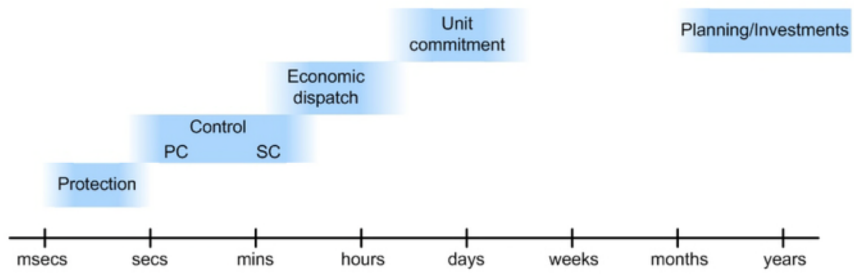
- Ancillary services: provision of power (real-time), trading in energy/capacity
- Congestion: constraints on power flows (real-time), trading in energy

Economy(energy), Control(power)

- Interplay between power and energy → coupling economy and physics/engineering (control)
- Increased uncertainties (renewables, decentralization) both in power and energy → tighter coupling economy, physics/control → requires design for efficiency and robustness

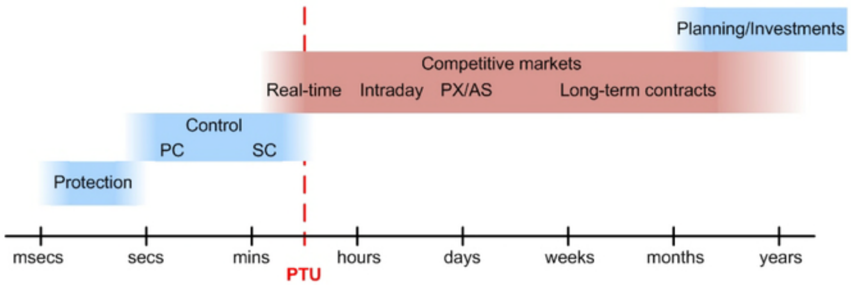
Out of scope in this talk: investments, legislation, details of regulation, political aspects

Positioning in time scale



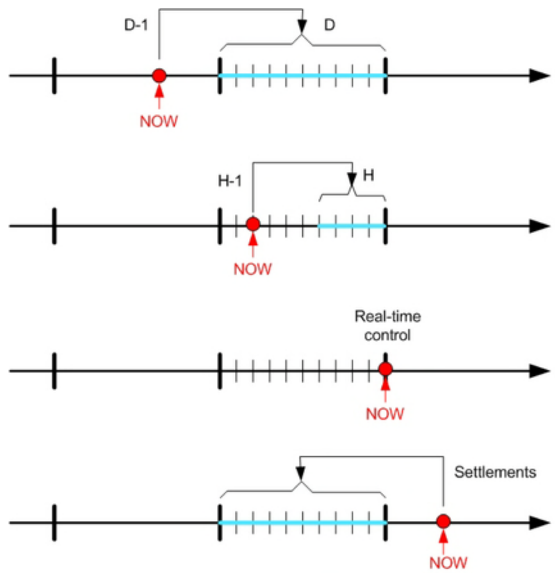
Traditional power system

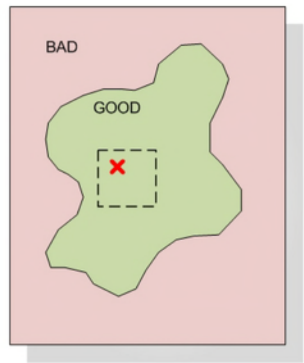
Positioning in time scale



Market based power system

Actions in time





Conditions for deregulation

Natural monopoly

- Economy of scale: Efficiency(100 MW plant) > Efficiency(10 MW plant) > Efficiency(1 MW plant)
- Large generating companies: one owner of many plants → cheaper production due to hiring of specialists, sharing parts and repair crews...

Conditions for successful deregulation

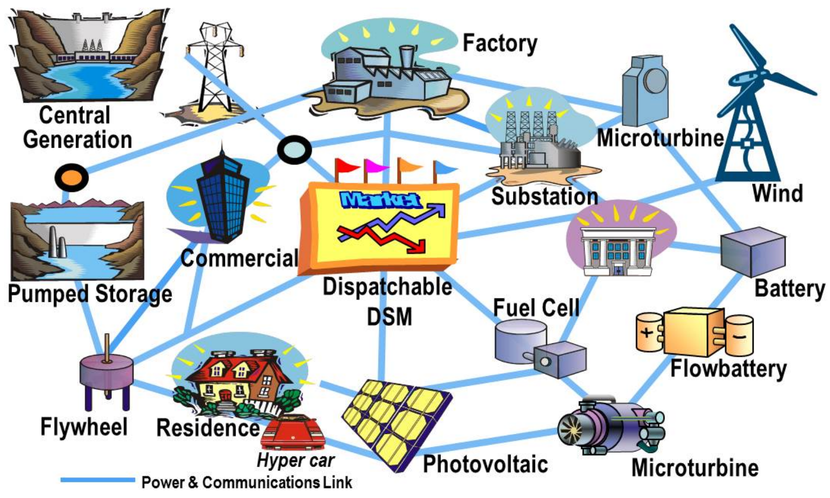
Lack of natural monopoly, or the conditions of natural monopoly should hold only weakly.

... if monopolist can produce power at significantly lower cost than the best competitive market, then regulation makes little sense.

Emerging playground for competition

More efficient low power plants (cheap gas turbines); renewable generation; smaller size distributed generation distributed on all levels in the system; price elastic demand,...

Conditions for deregulation



Maximizing social welfare

Energy market

- Production cost function: $C_i(p_i)$
- Consumption benefit function: $B_j(d_j)$

Social welfare maximization (isolated system)

$$\min_{p_1,\ldots,p_n,d_1,\ldots,d_m} \quad \sum_{i=1}^{n} C_i(p_i) - \sum_{j=1}^{m} B_j(d_j) \qquad (= \max \text{ social welfare})$$

subject to

$$p_i \in \mathcal{P}_i, \quad i = 1,\ldots,n \qquad \text{(local production constraints)}$$
$$d_j \in \mathcal{D}_j, \quad j = 1,\ldots,m \qquad \text{(local consumption constraints)}$$
$$\sum_{i=1}^{n} p_i = \sum_{j=1}^{m} d_j \qquad \text{(balance supply and demand)}$$

example local constraints: $\mathcal{P}_i := \{p \mid \underline{p}_i \leq p \leq \overline{p}_i\}, \quad \mathcal{D}_j := \{d \mid \underline{d}_j \leq d \leq \overline{d}_j\}$

Intermezzo: Lagrange duality

Optimization problem

$$\min_x \{ f(x) \mid g(x) \leq 0, \; h(x) = 0 \}$$

where $h : \mathbb{R}^n \to \mathbb{R}^m \quad g : \mathbb{R}^n \to \mathbb{R}^p$

Lower bounds

Let x be feasible point ($g(x) \leq 0$, $h(x) = 0$). For arbitrary $\lambda \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^p$ with $\mu \geq 0$ we have

$$L(x,\lambda,\mu) := f(x) + \lambda^\top h(x) + \mu^\top g(x) \leq f(x).$$

After infimization we have

$$\ell(\lambda,\mu) := \inf_x L(x,\lambda,\mu) \leq \inf_{\{x \mid g(x)\leq 0,\, h(x)=0\}} f(x)$$

Since λ and $\mu \geq 0$ were arbitrary we conclude

$$\sup_{\{\lambda,\mu \mid \mu \geq 0\}} \ell(\lambda,\mu) \quad \leq \quad \inf_{\{x \mid g(x)\leq 0,\, h(x)=0\}} f(x)$$

Intermezzo: Lagrange duality

Terminology and observations

- Lagrange function: $L(x, \lambda, \mu) := f(x) + \lambda^\top h(x) + \mu^\top g(x)$
- Lagrange dual cost: $\ell(\lambda, \mu) := \inf_x L(x, \lambda, \mu)$
- Lagrange dual problem: $d_{opt} = \sup_{\{\lambda,\mu \,|\, \mu \geq 0\}} \ell(\lambda, \mu)$
- Primal problem: $p_{opt} = \inf_{\{x \,|\, g(x) \leq 0,\, h(x)=0\}} f(x)$

Dual problem is **concave maximization** problem. Constraints are often simpler than in primal problem.

Weak duality (lower bounds)

Dual optimal value (d_{opt}) $\leq$ Primal optimal value (p_{opt})

Weak duality is always true.

Intermezzo: Lagrange duality

Lagrange Duality Theorem

Weak duality always holds: $d_{opt} \leq p_{opt}$

Let primal problem be **convex** with satisfied **Slater's constraint qualification**. Then strong duality holds: $d_{opt} = p_{opt}$.

Strong duality in compact form

$$\max_{\{\lambda,\mu \,|\, \mu \geq 0\}} \left(\inf_x f(x) + \lambda^\top h(x) + \mu^\top g(x) \right) = \inf_{\{x \,|\, g(x) \leq 0,\, h(x)=0\}} f(x)$$

Slater's constraint qualification

Define sets $\mathcal{I}_n, \mathcal{I}_a$: $i \in \mathcal{I}_n$ if $g_i(\cdot)$ is nonlinear; $i \in \mathcal{I}_a$ if $g_i(\cdot)$ is affine.
Slater CQ: the set

$$\{x \mid h(x) = 0,\ g_i(x) < 0 \text{ for } i \in \mathcal{I}_n,\ g_i(x) \leq 0 \text{ for } i \in \mathcal{I}_a, \}$$

is nonempty.

Maximizing social welfare via dual problem

Energy market

Primal

$$\min_{p_i \in \mathcal{P}_i, d_j \in \mathcal{D}_j} \sum_{i=1}^{n} C_i(p_i) - \sum_{j=1}^{m} B_j(d_j)$$

$$\text{subject to} \quad \sum_{i=1}^{n} p_i = \sum_{j=1}^{m} d_j$$

Dual

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda)$$

where

$$\ell(\lambda) = \min_{p_i \in \mathcal{P}_i, d_j \in \mathcal{D}_j} \sum_{i=1}^{n} C_i(p_i) - \sum_{j=1}^{m} B_j(d_j) + \lambda \Big(\sum_{j=1}^{m} d_j - \sum_{i=1}^{n} p_i \Big)$$

Assumption: convexity. $C_i(\cdot)$ convex functions, $B_j(\cdot)$ concave fun., $\mathcal{P}_i, \mathcal{D}_j$ convex sets.

Maximizing social welfare via dual problem

Energy market

Dual

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda)$$

where

$$\ell(\lambda) = \min_{p_i \in \mathcal{P}_i, d_j \in \mathcal{D}_j} \sum_{i=1}^{n} C_i(p_i) - \sum_{j=1}^{m} B_j(d_j) + \lambda \Big(\sum_{j=1}^{m} d_j - \sum_{i=1}^{n} p_i \Big)$$

Observation 1: Lagrange dual cost function $\ell(\lambda)$ is decomposable (for a fixed λ, can be decomposed into $n + m$ separate minimization problems)

Observation 2: $\max_{\lambda \in \mathbb{R}} \ell(\lambda)$ is attained when $\sum_{j=1}^{m} d_j = \sum_{i=1}^{n} p_i$ ((sub)gradient of $\ell(\lambda)$ is zero).

Maximizing social welfare via dual problem

Energy market

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda)$$

Supplier's *local* minimizations

$$\min_{\mathcal{P}_1} \; C_1(p_1) - \lambda p_1$$
$$\min_{\mathcal{P}_2} \; C_2(p_2) - \lambda p_2$$
$$\vdots$$
$$\min_{\mathcal{P}_n} \; C_n(p_n) - \lambda p_n$$

Demand's *local* minimizations

$$\min_{\mathcal{D}_1} \; \lambda d_1 - B_1(d_1)$$
$$\min_{\mathcal{D}_2} \; \lambda d_2 - B_1(d_2)$$
$$\vdots$$
$$\min_{\mathcal{D}_m} \; \lambda d_m - B_1(d_m)$$

Maximizing social welfare via dual problem

Energy market

Market operator

$$\max_{\lambda \in \mathbb{R}} \ell(\lambda) \quad \Leftrightarrow \quad \text{determine } \lambda : \sum_{j=1}^{m} d_j^\star = \sum_{i=1}^{n} p_i^\star$$

Rational behaviour of market players (max its own benefits)

Supplier's *local* minimizations

$$p_1^\star = \text{argmin}_{p_1 \in \mathcal{P}_1} \; C_1(p_1) - \lambda p_1$$
$$p_2^\star = \text{argmin}_{p_2 \in \mathcal{P}_2} \; C_2(p_2) - \lambda p_2$$
$$\vdots$$
$$p_n^\star = \text{argmin}_{p_n \in \mathcal{P}_n} \; C_n(p_n) - \lambda p_n$$

Demand's *local* minimizations

$$d_1^\star = \text{argmin}_{d_1 \in \mathcal{D}_1} \; \lambda d_1 - B_1(d_1)$$
$$d_2^\star = \text{argmin}_{d_2 \in \mathcal{D}_2} \; \lambda d_2 - B_1(d_2)$$
$$\vdots$$
$$d_m^\star = \text{argmin}_{d_m \in \mathcal{D}_m} \; \lambda d_m - B_1(d_m)$$

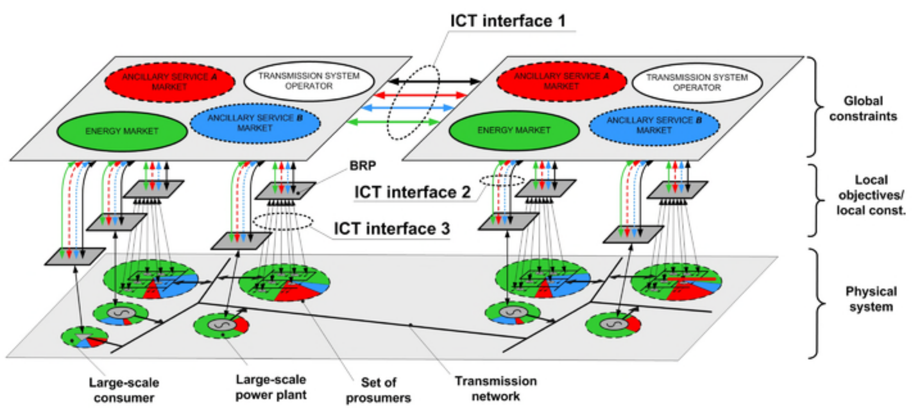
λ^* which solves the above problem is the (market clearing) price

Market based operation

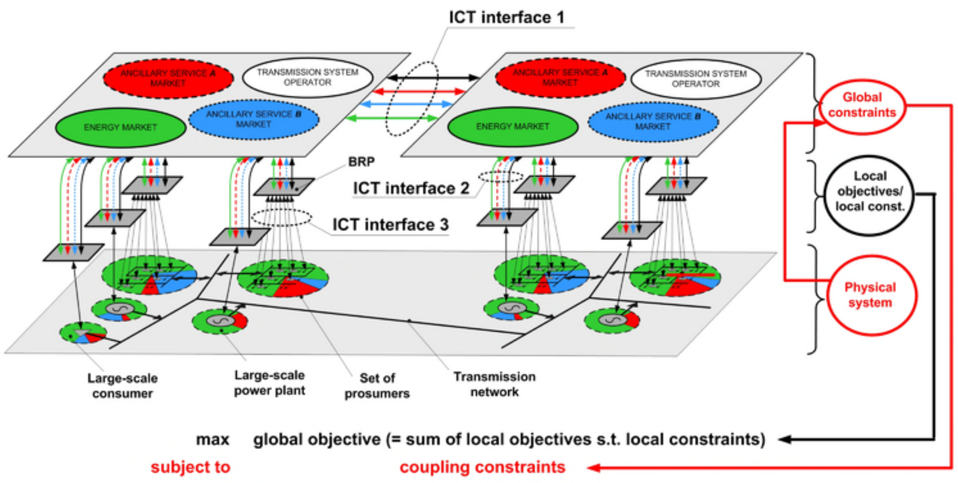
Some observations/remarks

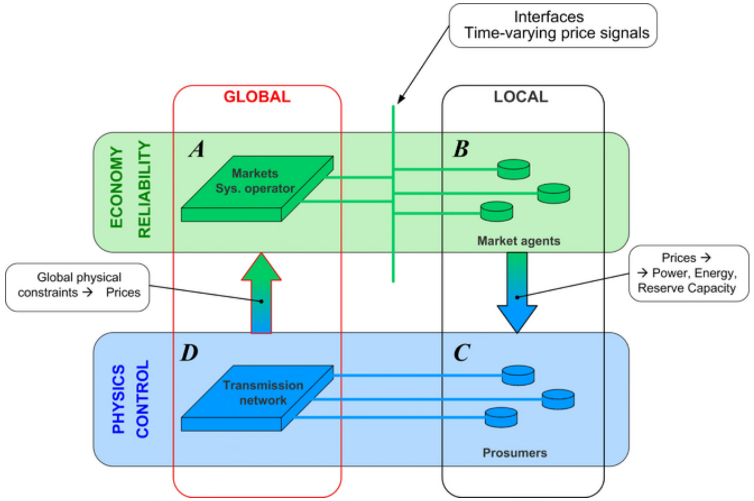
- change from regulated and single utility owned and operated system to the market based system can be seen as shift from explicitly solving **primal** problem to explicitly solving **dual** problem
- Lagrange dual (and "complementarity problems"): suitable as manipulates with both **physical** (**primal**) variables and **economy** related variables - prices (**dual**)
- generic approach: assign prices to **global** constraints (i.e. power balance) and use them to coordinate **local** behaviours to meet the **global** constraints
- By shifting to solving dual problem we have introduced different **solution architecture**: *i)* new players: market operators, competing market agents; *ii)* we have defined who does what; *iii)* we have introduced prices and bids as protocols for coordination among players.
- Large-scale complex systems: rely on **protocols**, **modularity** and **architecture** (Internet: TCP/IP; power system: 50 Hz is a "protocol"; money / bid format;... a bit wider view: passivity in control as a protocol...)

Market based operation



Market based operation





Time varying price signals as

- Protocols and defining ingredients of uniform interfaces in communication between producers, consumers, market and system operators
- Signals for coordination and time synchronization of local behaviours to achieve global goals

Market based operation

Supplier: $p_i^\star = \operatorname{argmin}_{p_i \in \mathcal{P}_i} \quad C_i(p_i) - \lambda p_i$
Consumer: $d_j^\star = \operatorname{argmin}_{d_j \in \mathcal{D}_j} \quad \lambda d_1 - B_j(d_j)$

Suppose λ is given such that $p_i^\star \in$ interior of $\mathcal{P}_i$, $d_j^\star \in$ interior of $\mathcal{D}_j$, then we have

$$\frac{\mathrm{d}C_i(p_i^\star)}{\mathrm{d}p_i} = \lambda$$
$$\frac{\mathrm{d}B_j(d_j^\star)}{\mathrm{d}d_j} = \lambda$$

i.e., social welfare is maximized when all *prosumers* (producers/consumers) adjust their prosumption levels so that marginal cost/benefit functions are equal to the price.

Market clearing problem

Bids from marginal costs/benefits

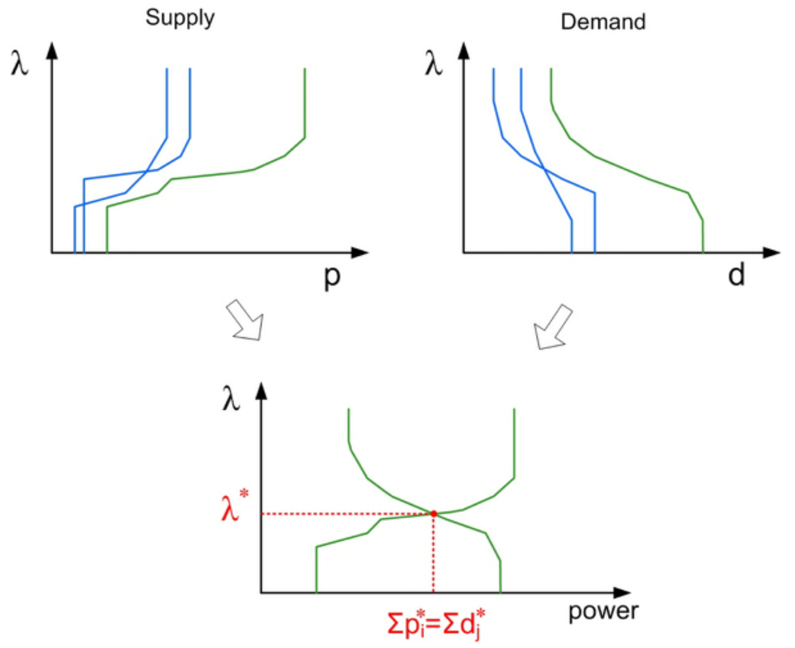
$$\frac{\mathrm{d}C_i(p_i)}{\mathrm{d}p_i} = \lambda \quad \Leftrightarrow \quad p_i = \gamma_i^p(\lambda) \quad \Leftrightarrow \quad \lambda = \beta_i^p(p_i)$$
$$\frac{\mathrm{d}B_j(d_j)}{\mathrm{d}d_j} = \lambda \quad \Leftrightarrow \quad d_j = \gamma_j^d(\lambda) \quad \Leftrightarrow \quad \lambda = \beta_j^d(d_i)$$

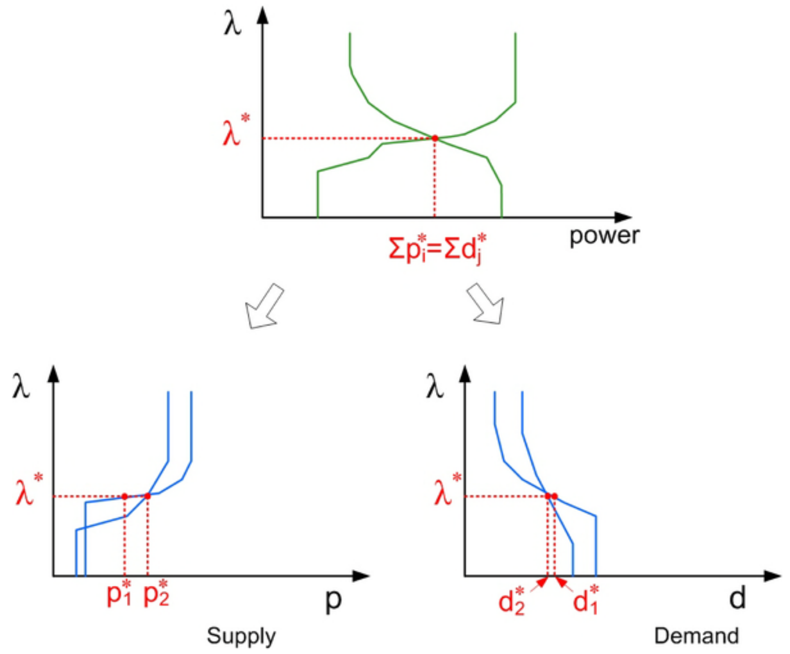
Market clearing problem in practice

Find the **market clearing price** $\lambda^\star$ at intersection of the aggregated supply bid curve $\tilde{\gamma}^p(\lambda) := \sum_i \gamma_i^p(\lambda)$ with the aggregated demand bid curve $\tilde{\gamma}^d(\lambda) := \sum_j \gamma_j^d(\lambda)$:

$$\sum_{i=1}^{n} p_i^\star = \sum_{i=1}^{n} \gamma_i^p(\lambda^\star) = \tilde{\gamma}^p(\lambda^\star) = \tilde{\gamma}^d(\lambda^\star) = \sum_{j=1}^{m} \gamma_j^d(\lambda^\star) = \sum_{i=1}^{m} d_i^\star$$

Remark: extension to cases when assumptions $p_i^\star \in$ interior of $\mathcal{P}_i$, $d_j^\star \in$ interior of $\mathcal{D}_j$ are not valid are straightforward. Easy to include constraints in the bids.

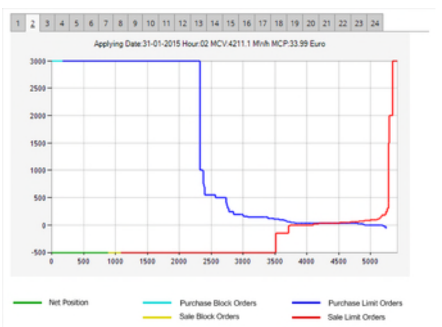


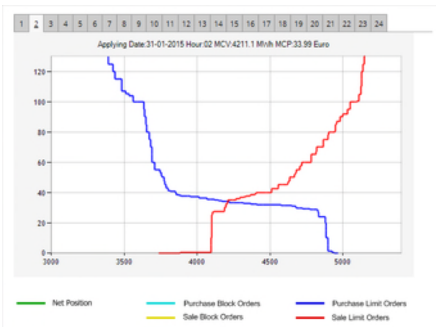


Market clearing: example

APX, aggregated bids

30. January 2015, 2 a.m.



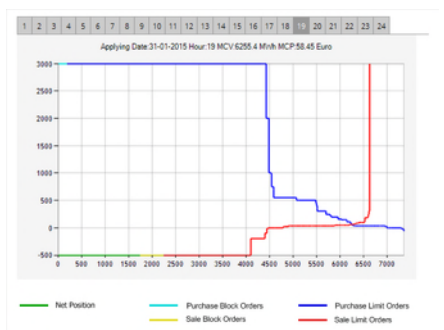


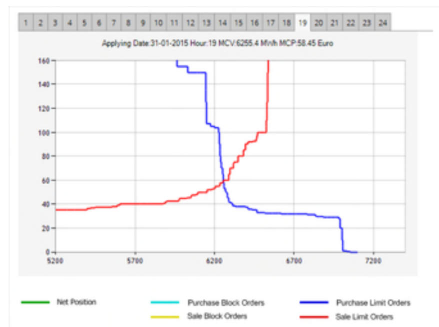
In some markets (e.g., APX) block bids are possible (bids for more trading periods; convenient to account for start-up costs. Origin of nonconvexity.)

Market clearing: example

APX, aggregated bids

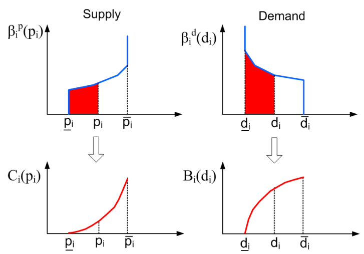
30. January 2015, 7 p.m.





In some markets (e.g., APX) block bids are possible (bids for more trading periods; convenient to account for start-up costs. Origin of nonconvexity.)

Market clearing problem



Terminology: "all supply bids smaller than some price are accepted

Exercise 1. Prove the following:

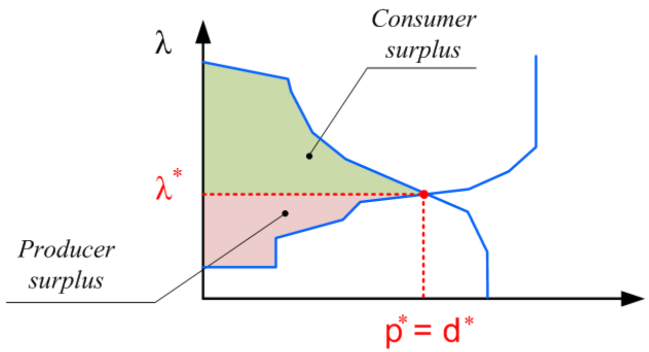
Non-decreasing $\beta_i^p(\cdot) \quad \Rightarrow \quad C_i(\cdot)$ is convex
Non-increasing $\beta_i^d(\cdot) \quad \Rightarrow \quad B_i(\cdot)$ is concave

$$C_i(p_i) = \int_{\underline{p}_i}^{p_i} \beta_i^p(\xi)\mathrm{d}\xi, \quad B_i(d_i) = \int_{\underline{d}_i}^{d_i} \beta_i^d(\xi)\mathrm{d}\xi$$

Market operators require bids to be non-decreasing/non-increasing (irrespective of true marginal costs/benefits).

Exercise 2.
Let the bids be piecewise constant (non-decreasing for supply, non-increasing for demand). Formulate market clearing problem as an optimization problem (primal).

Maximizing social welfare via dual problem



Consumer surplus

$CS(d_i) := B_i(d_i) - \lambda d_i$

Producer surplus

$PS(p_i) := \lambda d_i - C_i(p_i)$

Remarks:
In fact graphical interpretation of solving dual problem.
Maximized areas (surpluses) = optimal value of Lagrange multiplier (price).

In practice it is often told that all the bids till Market clearing volume / Market clearing price are accepted.

Balance responsible party

Balance responsible party (BRP)

- BRP is a legal entity that is capable and allowed to trade on energy and ancillary service markets.
- BRP is defined by specification of its responsibilities (operational rules) and interfaces with other subsystems in the operational architecture of the overall system.

By defining the interfaces and responsibilities, we are in fact defining the BRPs as crucial building blocks (modules) of the system.

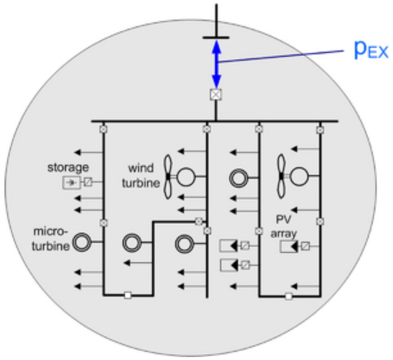
- Responsible for own production and load prediction;
- Responsible for behavior in markets (e.g. market power misuses);
- Responsible for behavior in power system (e.g. responsibility to react on real-time SC signal from TSO);
- Can pay bills;

Bidding

Basics of bidding

BRPs portfolio: • m generators $\{C_i(p_i), \underline{p}_i, \overline{p}_i\}_{i=1,\dots,m}$; • n controllable loads $\{B_i(d_i), \underline{d}_i, \overline{d}_i\}$; • aggregated price inelastic power injection q

How could the BRP bid for its aggregated prosumption p_{EX}? $\beta_{BRP}(p_{EX}) = ?$



Bidding

Basics of bidding

Approach I

$$\min_{\{p_i\},\{d_j\},p_{EX}} \sum_{i=1}^{m} C_i(p_i) - \sum_{j=1}^{n} B_j(d_j) - \lambda p_{EX}$$

$$\text{subject to } \sum_{i=1}^{m} p_i - \sum_{j=1}^{n} d_j + q = p_{EX}$$

$$\underline{p}_i \le p_i \le \overline{p}_i, \quad i = 1,\dots,m$$

$$\underline{d}_j \le d_j \le \overline{d}_j \; j = 1,\dots,n$$

λ as parameter, calculate p_{EX}

Approach II

$$\min_{\{p_i\},\{d_j\}} \sum_{i=1}^{m} C_i(p_i) - \sum_{j=1}^{n} B_j(d_j)$$

$$\text{subject to } \sum_{i=1}^{m} p_i - \sum_{j=1}^{n} d_j + q = p_{EX} \; ♣$$

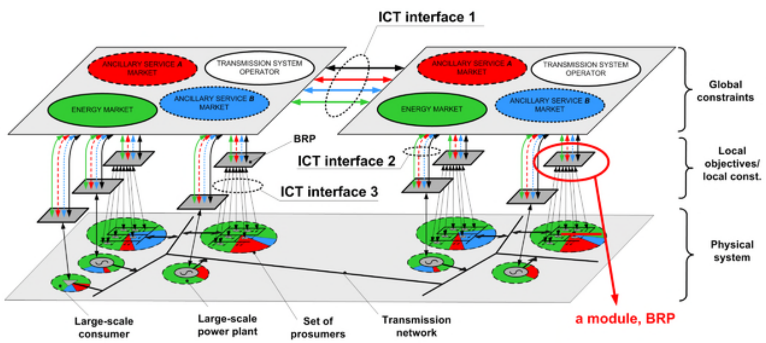
$$\underline{p}_i \le p_i \le \overline{p}_i \; i = 1,\dots,m$$

$$\underline{d}_j \le d_j \le \overline{d}_j \; j = 1,\dots,n$$

p_{EX} as parameter, Lagrange multiplier to ♣ as price

Exercise 3: Show equivalence between Approach I and Approach II.

Balance responsible party



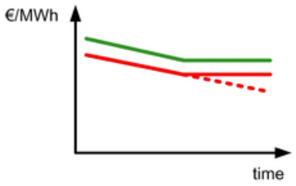
- All market participants interact with markets through a BRP, or are a BRP themselves.
- BRP as a **module** (building block)
- Heterogeneity, local "issues".... all "hidden" behind the interface ("Interface 2")
- Example: bids are requested to be increasing functions (CONVEXITY) - simple and "smart" way to deal with complexity
- Later on: BRP will have to internally "decouple" services to comply with protocols

Outline

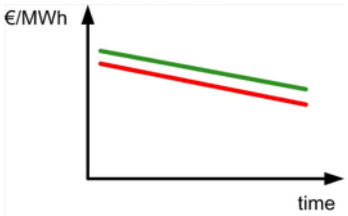
Benefits of market-based (price-based) operation

In mathematical terms we reached (via dual) the same solution (as primal).
Why deregulation?

a) price-cap regulation



b) cost of service regulation



Benefits of market-based (price-based) operation

In mathematical terms we reached (via dual) the same solution (as primal).
Why deregulation?

Competitive markets simultaneously

- hold prices down to marginal cost
- minimize cost

Regulation can do one or the other, but not both.

Perfect competition

Adam Smith (*"Wealth of Nations"*):

- perfectly competitive market $\implies$ economic efficiency
- "invisible hand of market" (*Solution architecture matters*)

Perfect competition (conditions)

- large number of generators (market agents)
- each agent act competitively (attempts to maximize its profits)
- price taking agents
- good information (market prices are publicly known)
- well-behaved costs

Well-behaved costs = convexity. Important for existence of equilibrium.
Difficulties: start up costs

Competitive equilibrium

A market condition in which supply equals demand and traders are price takers.

Particularities of markets in power systems

Problems with electrical energy as commodity

- **No buffering.** Cannot be efficiently stored in large quantities. Consumed as produced $\rightarrow$ fast changing production costs.
- **No free routing.** Other transportation systems have free choices among alternative paths between source and destination. Power transmission system: power flows governed by physical laws.

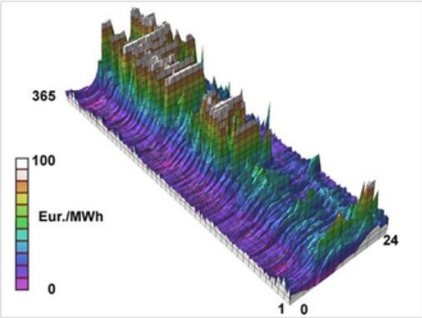
Demand-side flaws

- Lack of metering and real-time billing. Customers disconnected from market (do not respond to real-time fluctuations in price/cost of supply)
- Lack of real-time control of power flow to specific customers. Ability of load to take power from the grid without prior contract with a generator.

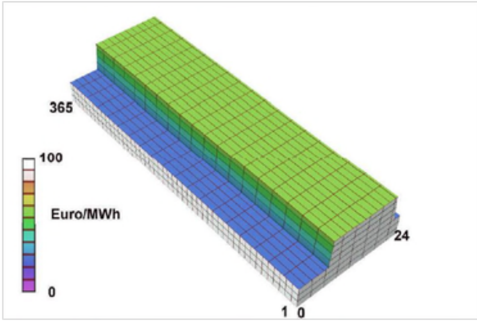
Consequences: necessity of an **independent system operator** as supplier in real-time, responsible for balancing;
necessity of well designed **market architecture**

Prices

Demand-side flaws



Yearly market prices (APX)



Prices for consumers

Benefits of market-based (price-based) operation

Some expected benefits:

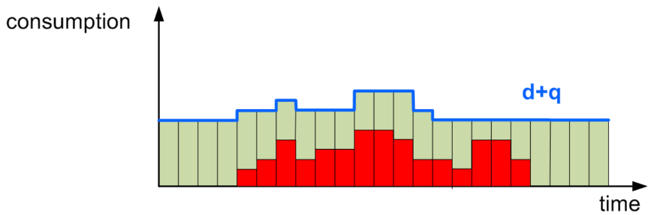
- large benefits expected to come from demand side (price-elastic consumers in "smart grids") when exposed to real-time prices (smart meters)
- $\rightarrow$ lower demand when generation is most costly
- $\rightarrow$ in long run: less generators to be built, reduced production costs

Load factor

$$\text{load factor} = \frac{\text{average demand}}{\text{peak demand}}$$

Real-time pricing reduces load factor (but in the most general case does not achieve load factor of 1).

Benefits of market-based (price-based) operation



$p(k)$=controllable power production at time k
$q(k)$=uncontrollable load or negated uncontrollable power
$d(k)$=controllable load
$C(p)$=cost function for producing at power level p
$B(d)$=benefit function of consuming at power level d
Energy constrained load: $\sum_{k=1}^{N} d(k) = E_N$
(with $B(d) = const.$, the goal of consumption profile $d(1), \dots, d(N)$ is to *shift the load* to minimize payments while satisfying energy production over the time horizon)

Benefits of market-based (price-based) operation

Example

Social welfare maximization ($\equiv$ market solution under perfect competition)

$$\min_{\{p(k),d(k)\}_{k=1,\dots,N}} \sum_{k=1}^{N} \Big(C(p(k)) - B(d(k)) \Big)$$
$$\text{subject to} \quad p(k) = d(k) + q(k), \quad k = 1, \dots, N$$
$$\sum_{k=1}^{N} d(k) = E_N$$

- With $C(\cdot), B(\cdot)$ strictly convex/concave and q is not constant in time, power factor is necessarily smaller than 1.
- With $B(\cdot) \equiv 0$, load shifting leads to power factor 1 even with $q \neq \mathbf{1}c$

Exercise 4: Prove the above statements.

Benefits of market-based (price-based) operation

Example

Social welfare maximization ($\equiv$ market solution under perfect competition)

$$\begin{aligned} \min_{\{p(k),d(k)\}_{k=1,\dots,N}} \quad & \sum_{k=1}^{N} \Big(C(p(k)) - B(d(k)) \Big) \\ \text{subject to} \quad & p(k) = d(k) + q(k), \quad k = 1,\dots,N \\ & \sum_{k=1}^{N} d(k) = E_N \end{aligned}$$

Constant power profiles

($q = 0$) Let $C_i(\cdot)$ be strictly convex function ($B_i(\cdot)$ strictly concave function). Then optimal power production (consumption) profile to produce (consume) certain amount of *energy* over some PTU is a *constant production (consumption) profile*.

...observation in favour of dealing with real-time power balancing and congestion.

Benefits of market-based (price-based) operation

Load shifting (load factor improvement) caused by pricing is in some cases self-limiting

still ...

(+) changing load factor from 60% to 80% gives 25% reduction in needed generation capacity.

but...

(-) with more loads as baseload, reduction of for peaking generators: fixed costs reduction of $\approx$ 12% (peaking generators cost roughly half of an average generator costs per installed megawatt). Overall reduction in **cost** of supply relatively low (several percent). [Stoft "Power system economics"]

but ...

(+) price-elastic demand side reduces conditions for **market power**

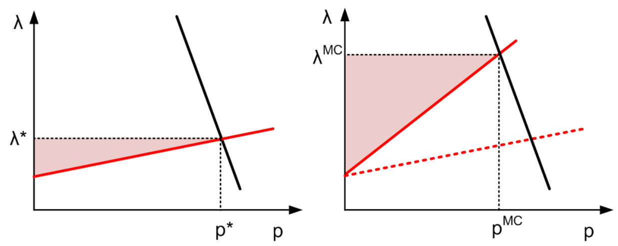
Outline

Market power

Market power

The ability to alter *profitably* prices away from competitive levels.

"profitably": important in definition. Some baseload plant (e.g. nuclear power plant) can influence the system when needed, even if it looses money by exercising this influence (e.g. by shutting down).



(λ^{MC}, p^{MC}) = monopolistic equilibrium
$(\lambda^{\star}, p^{\star})$ = competitive equilibrium

$$\max \ \lambda^{MC}(\beta(p))\, p^{MC}(\beta(p)) - C\big(p^{MC}(\beta(p))\big)$$

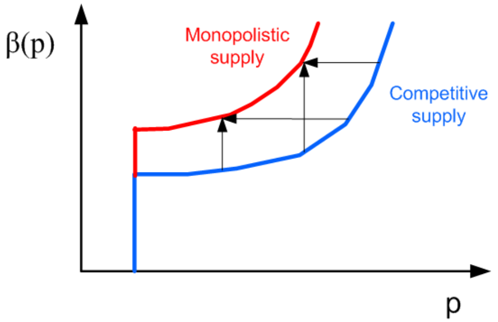
Market power

Market power

- on supply side: monopoly power. result: price higher than competitive
- on demand side: monopsony power. result: price lower than competitive

Exercising monopoly power

- quantity withholding (reducing output)
- financial withholding (raising the price for output)

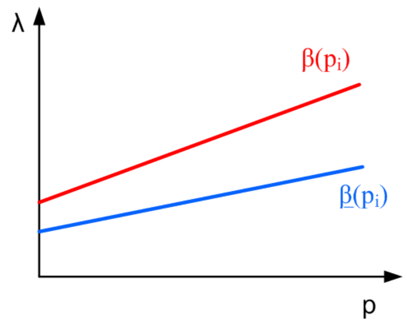


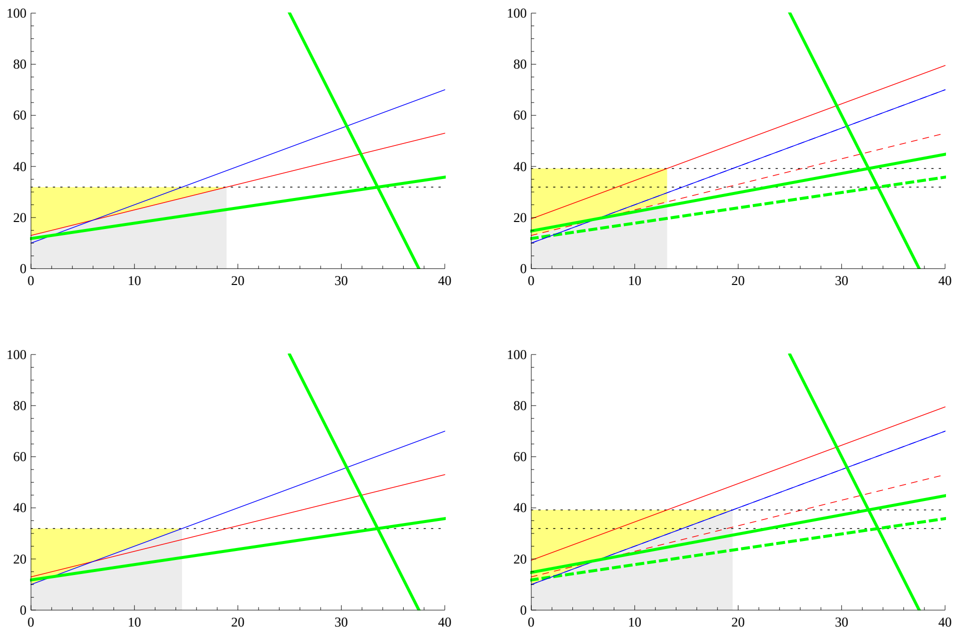
Market power

Example

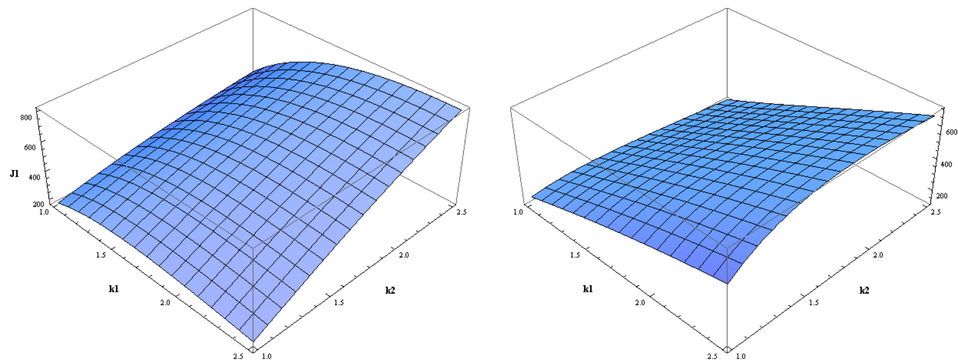
Incremental costs of a supplier: $a_i p_i + b_i$, with $a_i > 0$

Strategy: selecting $k_i \geq 0$ for the bid $\beta_i(p_i) = k_i \underline{\beta}(p_i) = k_i a_i p_i + k_i b_i$





Market power



Market power

Competitive equilibrium (Walrasian equilibrium)

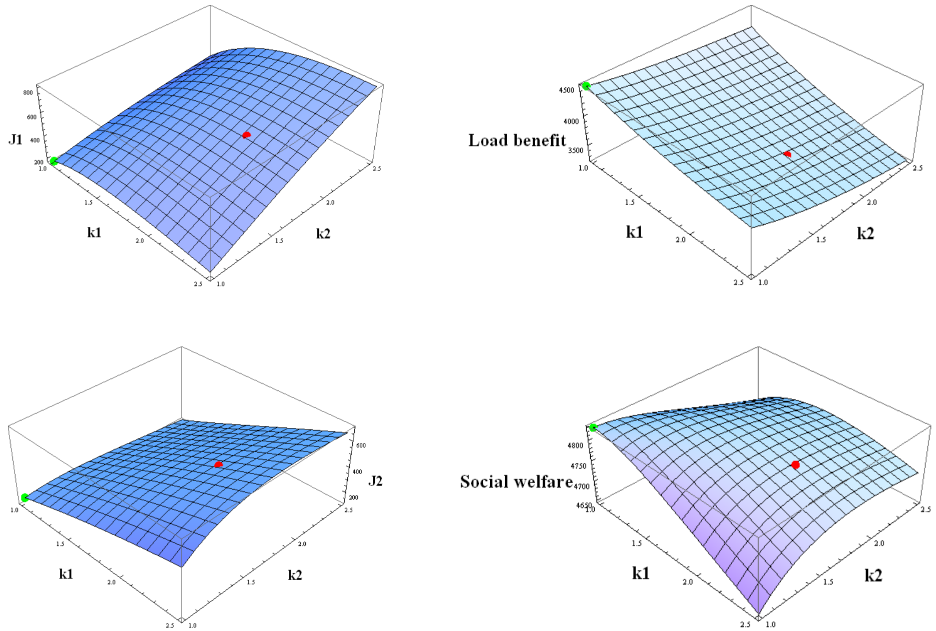
A market condition in which supply equals demand and traders are price takers.

Nash equilibrium

None of the players can increase its benefits by changing its own strategy, provided that other players continue with their strategies.

Strategy S_i of a player i (algorithm for playing in the market)
$J_i(s_1, \ldots, s_n)$: benefits of player i, as outcome of all strategies

$$\forall i, s_i \in S_i : J_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*) \geq J_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*)$$



green dot ← perfect competition; red dot ← Nash equilibrium

Market power

Elasticity of demand (e)

With aggregated demand $D := \sum_i d_i$ and price λ

$$e = -\frac{\Delta D}{D} \Big/ \frac{\Delta \lambda}{\lambda} \qquad \rightarrow \qquad e = -\frac{\mathrm{d}D}{\mathrm{d}\lambda}\frac{\lambda}{D}$$

Market share

$$s_i = \frac{p_i}{\sum_i p_i}$$

Lerner index for Cournot oligopoly (group of uncoordinated suppliers)

$$L_x = \frac{s}{e}$$

For monopoly: $s = 1, L_x = 1/e$.

Summary/illustration of problems

including time couplings

- Forward time BRP bidding over finite horizon of N PTUs.
- Similar formulation: internal BRP re-scheduling / real-time (MPC type) control over one or several PTUs

$\mathbf{p_i} := (p_i(1), \ldots, p_i(N)), \quad \mathbf{d_i} := (d_i(1), \ldots, d_i(N))$
$q(k)$ = (predicted) uncontrollable prosumption at k-th PTU for the considered BRP

BRP's problem with time couplings (example)

$$\min_{\{p_i\},\{d_j\}} \quad \sum_{k=1}^{N} \Big(\sum_i C_i(p_i(k)) - \sum_j B_j(d_j(k)) \Big) - \lambda(k) p_{EX}(k)$$

$$\text{subject to} \quad \sum_i p_i(k) - \sum_j d_j(k) + q(k) = p_{EX}(k)$$

$$p_i(k) \in \mathcal{P}_i(\mathbf{p_i(k)}), \quad d_j(k) \in \mathcal{D}_j(\mathbf{d_j(k)}) \quad (dynamics, constraints)$$

Summary/illustration of problems

including time couplings

$$\min_{\{p_i\},\{d_j\}} \sum_{k=1}^{N} \Big(\sum_i C_i(p_i(k)) - \sum_j B_j(d_j(k)) \Big) - \lambda(k) p_{EX}(k)$$

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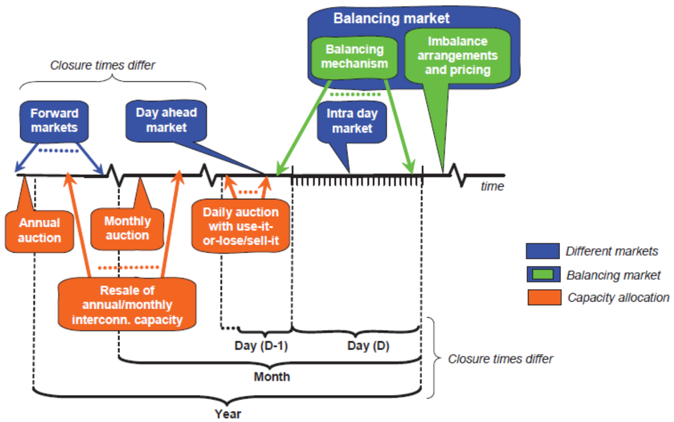
$$p_i(k) \in \mathcal{P}_i(\mathbf{p_i(k)}), \quad d_j(k) \in \mathcal{D}_j(\mathbf{d_j(k)}) \quad (dynamics, constraints)$$

General philosophy: keep market operator's job simple and transparent; let BRPs cope with their problems

- Market operator services for time couplings: block bids, intra-day market
- Similarity with hierarchical/distributed (dual decomposition based) MPC
- Iterations replaced with bids (functions relating primal-dual variables)
- Complexity: largely on the BRP's side, behind the "market interface", behind bid
- Market power, game theory: $\lambda(k, p_{EX}(k))$

Market architecture

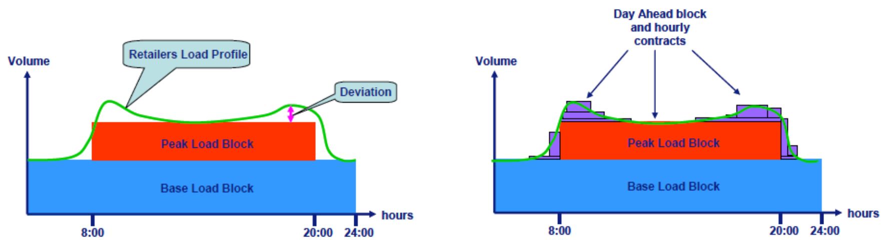
Architecture = functionality allocation: "who does what?", "how are the subsystems interrelated and connected?"



Forward time markets (Bilateral markets; "Over the counter (OTC) trade": reducing risks
Day ahead market: adapting to $D-1$ state/prediction. competition; liquidity
Intraday markets: adaptation to $H-1$ state/prediction (some similarity with MPC)
Balancing market: reflecting true physical transactions

Market architecture

"Submarkets"



(a) The base and peak load contracted on OTC (b) Realized power

The base and peak load on energy markets

Market architecture

Market types

Two basic ways to arrange trades between buyers and sellers

- bilateral (trade directly)
- mediated (over intermediary)

Arrangement	Type of Market				
Bilateral:	Search	Bulletin Board	Brokered		
Mediated:			Dealer	Exchange	Pool

Less organized ⟷ More centralized

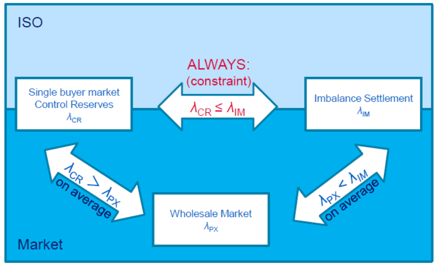
- Currently there is no consensus on the best list of submarkets from which to construct an entire power market.
- Design of market architecture must consider market structure in which it is embedded.
- Market structure = properties of the market closely tied to technology and ownership.

Market architecture

Linkages

- implicit (e.g., prices on forward markets (longer term) try to approximate expected spot prices (short term))
- explicit

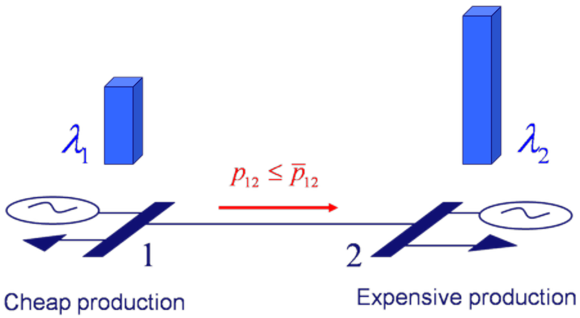
Implicit linkages are important part of market architecture (e.g., they create incentives for certain business opportunities.)



Relations between prices on different markets (TenneT NL)

Outline

Congestion management

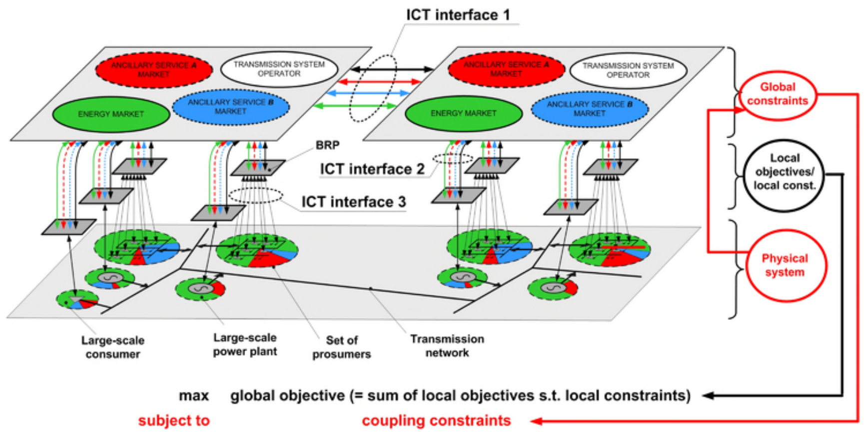


Line flow limits:

- physical: thermal limits, stability limits
- contingency limits (robustness): physical limits following contingency

Congestion is a problem on more time-scales (day-ahead, real-time).

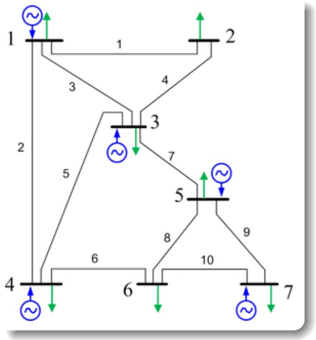
Congestion management



Traditional system: vertically integrated utility with full knowledge and control.
Market-based system. Responsible party: Transmission system operator (TSO).
Transmission system used in different way than planned. One of the toughest problems in market-based operation. Several solution architectures in practice

Recall: power flow equations (DC)

Transmission system: connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$



DC power flow model:

$$p_{ij} = b_{ij}(\theta_i - \theta_j) = -p_{ji}$$

b_{ij} = susceptance of line $\epsilon_{ij} \in \mathcal{E}$,
θ_i = voltage phase angle at node (bus) $v_i \in \mathcal{V}$.

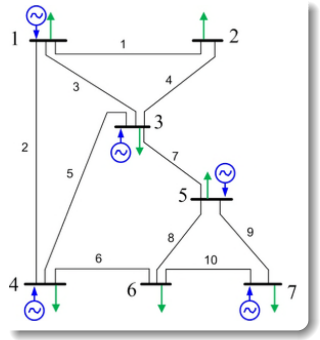
Node v_i with neighbouring nodes $\mathcal{N}_i$, power balance:
$p_i = \sum_{j \in \mathcal{N}_i} p_{ij}$

p_i = node aggregated controllable power injection

- $p_i < 0$ consumption
- $p_i > 0$ production

Recall: power flow equations (DC)

Transmission system: connected undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$



$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} = \begin{pmatrix} b_{\mathcal{N}_1} & -b_{12} & \dots & -b_{1n} \\ -b_{12} & b_{\mathcal{N}_2} & \dots & -b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -b_{1n} & -b_{2n} & \dots & b_{\mathcal{N}_n} \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{pmatrix}$$

with $b_{\mathcal{N}_i} := \sum_{j \in \mathcal{N}_i} b_{ij}$

Power flow equations

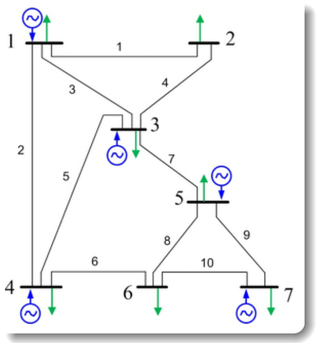
$$p = B\theta$$

Remark: $B^\top = B, \quad B\mathbf{1}_n = 0.$

Line flow limits

$$L\theta \leq \bar{e}_{\mathcal{E}}$$

Power Transfer Distribution Factors (PTDF)



Power Transfer Distribution Factors (PTDF)

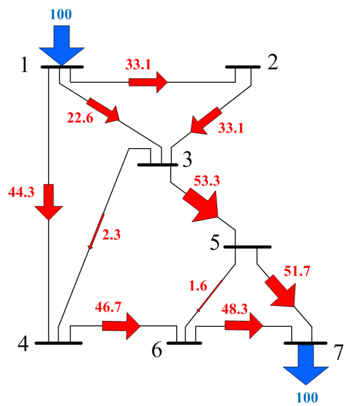
PTDF (*of a line with respect to a transaction*) is the coefficient of the linear relationship between the amount of transaction and the flow on the line.

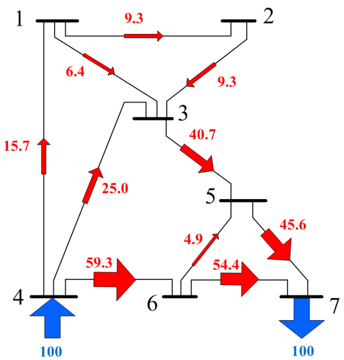
A transaction = specific amount of power injected at one (specified) node and removed at another (specified) node.

PTDF is the fraction of the amount of a transaction from one node to the other that flows over a given transmission line.

Power Transfer Distribution Factors (PTDF)

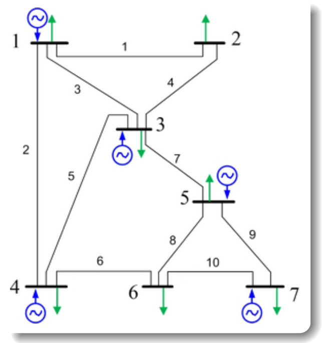
Example.





↓ No free routing.
(↑ Frequency as global variable.)

Power Transfer Distribution Factors (PTDF)



Set $\theta_1 = 0$. With abbreviations
$\tilde{p} := \begin{pmatrix} p_2 & \dots & p_n \end{pmatrix}^\top$, $\tilde{\theta} := \begin{pmatrix} \theta_2 & \dots & \theta_n \end{pmatrix}^\top$:

$$\underbrace{\begin{pmatrix} p_1 \\ \tilde{p} \end{pmatrix}}_{p} = \underbrace{\begin{pmatrix} \tilde{B}_{11} & \tilde{B}_{21}^\top \\ \tilde{B}_{21} & \tilde{B}_{22} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} 0 \\ \tilde{\theta} \end{pmatrix}}_{\theta}$$

$$\underbrace{\begin{pmatrix} \theta_1 \\ \tilde{\theta} \end{pmatrix}}_{\theta} = \underbrace{\begin{pmatrix} 0 & \mathbf{0}_{\mathbf{n-1}}^\top \\ \mathbf{0}_{\mathbf{n}} & \tilde{B}_{22}^{-1} \end{pmatrix}}_{F} \underbrace{\begin{pmatrix} p_1 \\ \tilde{p} \end{pmatrix}}_{p}$$

$\psi_{ij,mn}$ the fraction of transaction from node m to node n, which flows over line ij.

$$\psi_{ij,mn} = b_{ij}(F_{im} - F_{in} - F_{jm} + F_{jn})$$

Optimal power flow problem

p_i = node aggregated controllable power injection with assigned economic objective function $J_i(p_i)$:

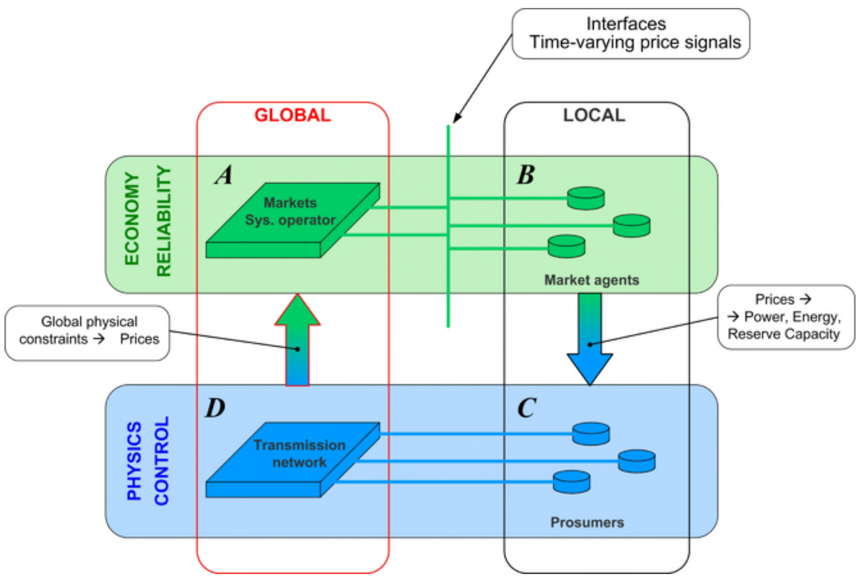
- $p_i < 0$, net consumption, $J_i(p_i) = -B_i(p_i)$
- $p_i > 0$, net production, $J_i(p_i) = C_i(p_i)$

q_i = uncontrollable, price inelastic, nodal power injection (net consumption: $q_i < 0$, net production : $q_i > 0$).

Optimal power flow problem (OPF)

$$\min_{p,\theta} \quad \sum_{i=1}^{n} J_i(p_i)$$
$$\text{subject to} \quad p + q - B\theta = 0$$
$$\underline{p} \leq p \leq \overline{p}$$
$$L\theta \leq \overline{e}_{\mathcal{E}}$$

Market-based solution?



Outline

1. Market-based operation: benefits, problems and basic principles
   - Basic principles
   - Benefits of deregulation
   - Market power
2. Congestion management
   - Basic notions
   - Congestion management approaches
   - Using full AC model
3. Markets for ancillary services
   - Market commodities
   - Actions on power time scale
   - Actions on energy time scale
   - Aggregation and spatial dimension of ancillary services
4. Distributed, real-time, price-based control
5. Conclusions

Congestion management approaches

Allocation methods

- Nodal pricing (Locational marginal pricing)
- Zonal pricing:
  - Market splitting
  - Flow-based coupling
- Explicit auctioning
- ...other.. (uniform pricing with congestion relief,...)

Alleviation methods

- Generation dispatching
- Buy-back countertrade

Congestion management approaches

- common: maintaining security; different: impact on market economy
- Why such diversity? previous market developments (history) and conservative engineering, national politics and economic developments, strategic approach to market players, specific topologies, generation portfolios, policy, young filed (?)...
- Congestion management is depended on the energy market architecture

Nodal pricing

Given: bids $\beta(p) := \begin{pmatrix}\beta_1(p_1) & \dots & \beta_n(p_n)\end{pmatrix}^\top$. Deduced: prosumption limits $\{\underline{p}_i, \overline{p}_i\}$, $\underline{p} < \overline{p}$, cost functions $J_i(p_i) := \int_{\underline{p}_i}^{p_i} \beta_i(\xi)d\xi$ for $p_i \geq 0$ and $J_i(p_i) := \int_{p_i}^{\overline{p}_i} \beta_i(\xi)d\xi$ for $p_i < 0$

Optimal pricing problem

with $\lambda = \begin{pmatrix}\lambda_1 & \dots & \lambda_n\end{pmatrix}^\top$

$$\min_{p,\theta,\lambda} \sum_{i=1}^{n} J_i(p_i) \quad \text{(max welfare)}$$

subject to

$$\beta(p) = \lambda$$
$$p - B\theta = 0$$
$$L\theta \leq \overline{e}_\mathcal{E}$$

OPF problem

$$\min_{p,\theta} \sum_{i=1}^{n} J_i(p_i)$$

subject to

$$p - B\theta = 0 \quad ♣$$
$$\underline{p} \leq p \leq \overline{p}$$
$$L\theta \leq \overline{e}_\mathcal{E}$$

Proposition

Vector of optimal dual variables related to the constraint (♣) in the dual to OPF problem is the vector of optimal nodal prices.

Intermezzo: Lagrange duality, KKT conditions

$f : \mathbb{R}^n \to \mathbb{R}, \quad h : \mathbb{R}^n \to \mathbb{R}^m, \quad g : \mathbb{R}^n \to \mathbb{R}^p$

$$\begin{aligned} \min_x \quad & f(x) \\ \text{subject to} \quad & h(x) = 0 \\ & g(x) \leq 0 \end{aligned}$$

Lagrange function

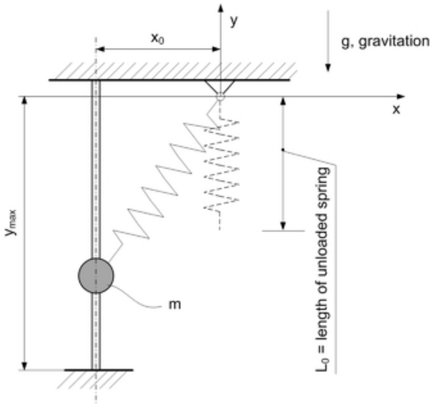
$$L(x, \lambda, \mu) := f(x) + \lambda^\top h(x) + \mu^\top g(x)$$

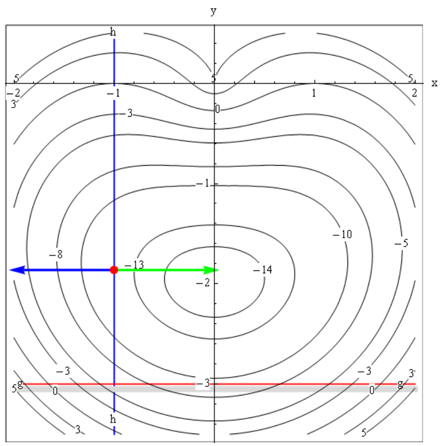
KKT optimality conditions

$$\nabla f(x) + \sum_{i=1}^{m} \lambda_i \nabla h_i(x) + \sum_{i=1}^{p} \mu_i \nabla g_i(g) = 0$$
$$h(x) = 0$$
$$0 \leq -g(x) \perp \mu \geq 0$$

Intermezzo: Lagrange duality, KKT conditions

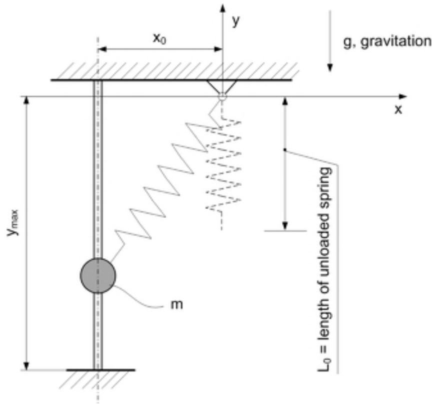
Illustrative example

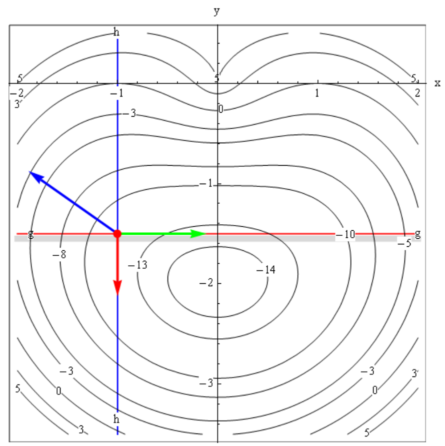




Intermezzo: Lagrange duality, KKT conditions

Illustrative example





Nodal pricing

KKT conditions (after "including back" the limits $\{\underline{p}_i, \overline{p}_i\}$ into the bids $\beta_i(p_i)$)

OPF problem

$$\min_{p,\theta} \sum_{i=1}^{n} J_i(p_i)$$
$$\text{subject to } p - B\theta = 0$$
$$\underline{p} \le p \le \overline{p}$$
$$L\theta \le \overline{e}_{\mathcal{E}}$$

KKT conditions

$$\beta(p^\star) - \lambda^\star = 0$$
$$p^\star - B\theta^\star = 0$$
$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star \ge 0$$

Singe price in case of no congestion

$$-L\theta^\star + \overline{e}_{\mathcal{E}} < 0 \quad \Longrightarrow \quad \mu^\star = 0 \quad \Longrightarrow \quad B\lambda^\star = 0 \quad \Longrightarrow \quad \lambda^\star = \mathbf{1}_n\hat{\lambda},\ \hat{\lambda} \in \mathbb{R}$$

In case of singe congested line, optimal nodal price in general have different value for each node. $(B\lambda^\star = -L^\top \mu^\star)$

Nodal pricing

Accounting for contingencies

OPF problem with contingencies

$$\min_{p,\theta} \sum_{i=1}^{n} J_i(p_i)$$
$$\text{subject to } p - B\theta = 0$$
$$p - B_c\theta_c = 0$$
$$\underline{p} \le p \le \overline{p}$$
$$L\theta \le \overline{e}_{\mathcal{E}}$$
$$L_c\theta_c \le \overline{e}_c$$

KKT conditions

$$\beta(p^\star) - \underbrace{(\lambda_n^\star + \lambda_c^\star)}_{\lambda^\star} = 0$$
$$p^\star - B\theta^\star = 0$$
$$p^\star - B\theta_c^\star = 0$$
$$B\lambda_n^\star + L^\top \mu_n^\star = 0$$
$$B_c\lambda_c^\star + L_c^\top \mu_c^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star \ge 0$$
$$0 \le (-L_c\theta_c^\star + \overline{e}_c) \quad \perp \quad \mu_c^\star \ge 0$$

Accounting for overloads when a singe circuit is out: "N-1 criteria.

Usually post contingency flow limits are higher than nominal $(\overline{e}_{\mathcal{E}} \le \overline{e}_c)$

Nodal pricing

Congestion revenue (collected by the market operator): $-(p^\star)^\top \lambda^\star$

Congestion revenue (merchandise surplus) is nonnegative

With losses neglected (DC), it always hold that

$$-(p^\star)^\top \lambda^\star \geq 0.$$

In case of at least one line congested (line flow constraint active), we have

$$-(p^\star)^\top \lambda^\star > 0.$$

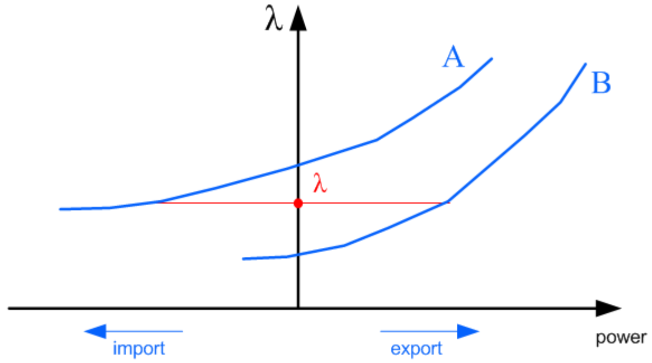
With $p = p_g + p_d$ where $p_g \geq 0$ are generator injections and $p_d \leq 0$ load, we have

$$-(p^\star)^\top \lambda^\star \geq 0 \quad \Longrightarrow \quad (\lambda^\star)^\top |p_d| - (\lambda^\star)^\top |p_g| \geq 0 \quad \text{(market operator profits)}$$

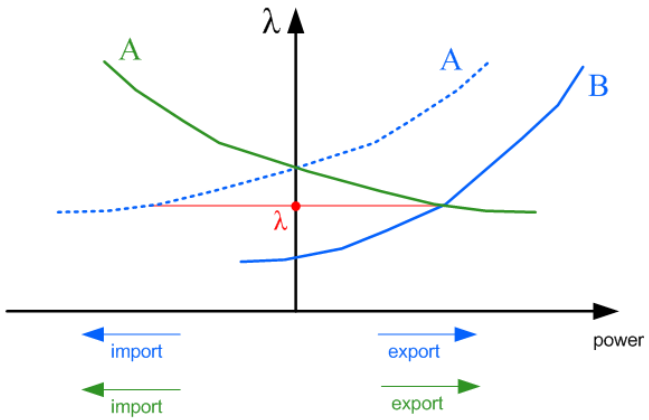
where $|\cdot|$ is elementwise applied absolute value on the vector.

Exercise 5: prove that congestion revenue is always nonnegative
(Hint: multiply optimality condition $B\lambda^\star + L^\top \mu^\star = 0$ from left with $(\theta^\star)^\top$.)

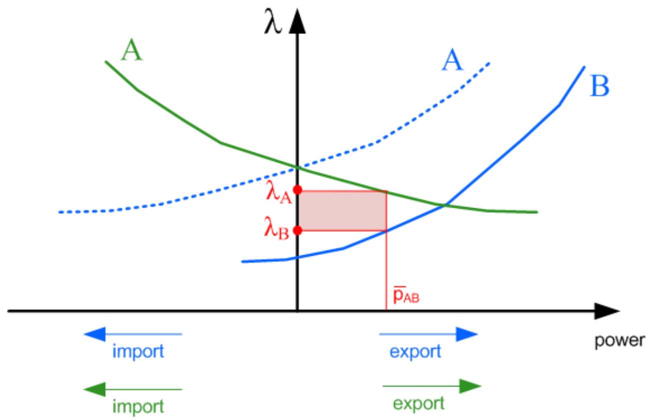
Nodal pricing



Nodal pricing



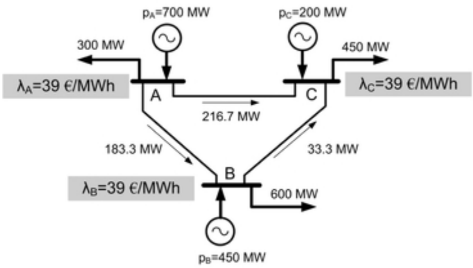
Nodal pricing

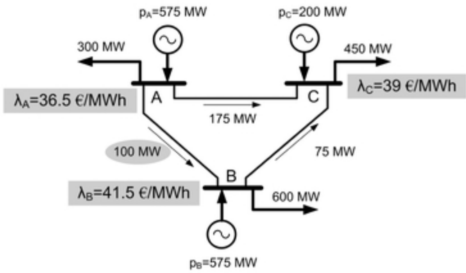


Nodal pricing

Example I

Exercise 6: Solve the nodal pricing problem from the figure.

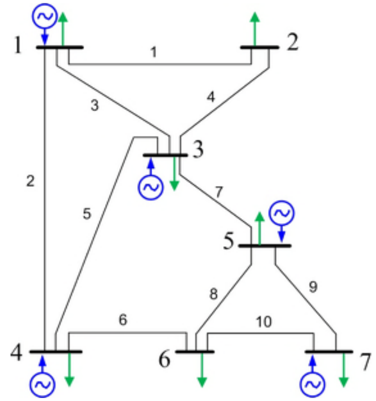


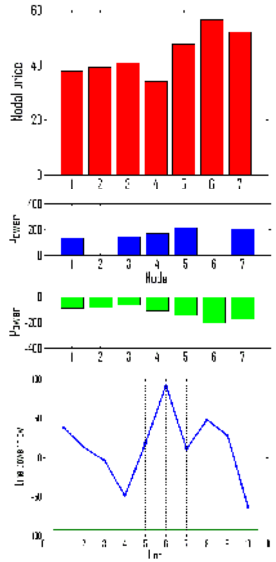


- The bids (incremental costs): $\beta_A(p_A) = 25 + 0.02p_A$, $\beta_B(p_B) = 30 + 0.02p_B$, $\beta_C(p_C) = 35 + 0.02p_C$
- Load is price inelastic.
- Line flow limits: only line $A - B$ has a limit on power flow, which is set to 100MW.
- All three lines are identical

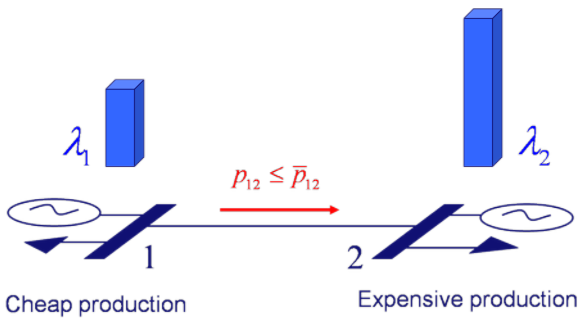
Nodal pricing

Example II





Congestion and market power



- Bid lower then incremental cost in one location to induce congestion and profit by exercising market power in other location.
- Positive side of market power due to congestion or number of generators: larger prices "invite" new players/investments.
- Market power due to exploration of holes in market rules or exploitation of conflict of interest: no useful economic signals

Transmission rights

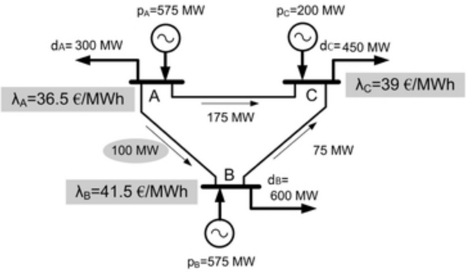
Transmission is scarce.

There is an extra money (congestion rent).

↓

Organize market for transmission rights. Use extra money to control financial risks of congestion induced price variations.

Transmission rights



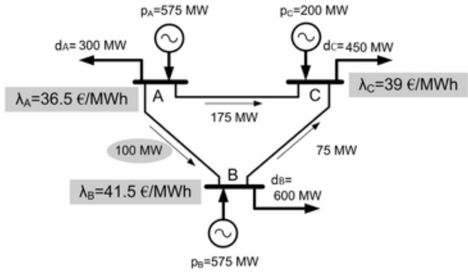
CR = congestion rent

$$CR = \lambda_A(d_A - p_A) + \lambda_B(d_B - p_B) + \lambda_C(d_C - p_C)$$
$$= p_{AB}(\lambda_B - \lambda_A) + p_{BC}(\lambda_B - \lambda_C) + p_{AC}(\lambda_C - \lambda_A)$$
$$= 750$$

Example a)

- d_B has contract for 150MW from p_A.
- Physically max transaction from A to B = 150MW (2/3 of transaction flows across line AB and 1/3 across path $AC - CB$).
- p_B buys 150MW of its power at locational price of node A: pays $d_B * \lambda_B$ but gets compensated (paid by generator in A) in amount $150 * (\lambda_B - \lambda_A) = 750$.
- Market operator compensates generator at A for $750 = CR$

Transmission rights



CR = congestion rent

$$CR = \lambda_A(d_A - p_A) + \lambda_B(d_B - p_B) + \lambda_C(d_C - p_C)$$
$$= p_{AB}(\lambda_B - \lambda_A) + p_{BC}(\lambda_B - \lambda_C) + p_{AC}(\lambda_C - \lambda_A)$$
$$= 750$$

Example b)

- d_C has contract for 300MW from p_A.
- Physically max transaction from A to C = 300MW (1/3 of transaction flows across path $AB - BC$ and 2/3 across line AC).
- p_C buys 300MW of its power at locational price of node A: pays $d_C * \lambda_C$ but gets compensated (paid by generator in A) in amount $300 * (\lambda_C - \lambda_A) = 750$.
- Market operator compensates generator at A for $750 = CR$

Transmission rights

Optimal nodal prices are competitive prices. → Well designed markets with perfect competition will find the same set of prices as calculated via Lagrange multipliers.

So, using optimization (duality) is a "shortcut". However...

- One might purchase a transmission right to protect itself against locational price swings due to congestion (congestion implies more local balancing → local conditions are more volatile than global (no aggregation) → volatility of locational prices)
- Owning a transmission right protects loads from market power exercise of local producers
- Market operator might have losses if contracted transmission rights are in excess of transmission capacity across a congested interface (sell according to worst case contingency)
- With limited amount of transmission rights, not all loads are protected from market power in case of congestion

Zonal pricing (market splitting)

Given: bids $\beta(p) := \big(\beta_1(p_1) \quad \dots \quad \beta_n(p_n)\big)^\top$
Deduced: cost functions $J_i(p_i)$

Optimal pricing problem

with $\lambda = \big(\mathbf{1_{n1}}^\top \lambda_{\mathcal{Z}_1} \quad \dots \quad \mathbf{1_{nK}}^\top \lambda_{\mathcal{Z}_K}\big)^\top$

$$\min_{p,\theta,\lambda} \sum_{i=1}^{n} J_i(p_i) \quad \text{(max welfare)}$$

subject to

$$\beta(p) = \lambda$$
$$p - B\theta = 0$$
$$L\theta \leq \bar{e}_{\mathcal{E}}$$

Different types of bids - different class of optimization problem:

i) QP for $\{\beta_i(p_i)\}_{i=1,\dots,n}$ affine with no saturation

ii) MILP for $\{\beta_i(p_i)\}_{i=1,\dots,n}$ piecewise constant (often in current practice)

iii) MIQP $\{\beta_i(p_i)\}_{i=1,\dots,n}$ affine with saturations

No simple characterization via duality, except for (i).

$\lambda_{\mathcal{Z}_i}$ zonal price for n_i nodes in zone i (zone $\mathcal{Z}_i$).
First n_1 nodes in zone $\mathcal{Z}_2$, then next n_2 nodes in zone $\mathcal{Z}_2$,...

Zonal pricing (market splitting)

Given: bids $\beta(p) := \big(\beta_1(p_1) \quad \dots \quad \beta_n(p_n)\big)^\top$
Deduced: cost functions $J_i(p_i)$

Optimal pricing problem

with $\lambda = \big(\mathbf{1_{n1}}^\top \lambda_{\mathcal{Z}_1} \quad \dots \quad \mathbf{1_{nK}}^\top \lambda_{\mathcal{Z}_K}\big)^\top$

$$\min_{p,\theta,\lambda} \sum_{i=1}^{n} J_i(p_i) \quad \text{(max welfare)}$$

subject to

$$\beta(p) = \lambda$$
$$p - B\theta = 0 \quad \clubsuit$$
$$L\theta - \bar{e}_{\mathcal{E}} \leq 0 \quad \spadesuit$$

Zonal prices for affine bids (case (i))

$\gamma_i(\cdot) = \beta_i^{-1}(\cdot)$

$\tilde{\mu}$ opt. Lagrange multiplier for $\spadesuit$
$\tilde{\lambda}$ opt. Lagrange multiplier for $\clubsuit$ ("auxiliary nodal prices", note that $B\tilde{\lambda} + L^\top \tilde{\mu} = 0$)

$$\sum_{j \in \mathcal{Z}_i} (\tilde{\lambda}_j - \lambda_{\mathcal{Z}_i})\gamma_j'(\lambda_{\mathcal{Z}_i}) = 0, \quad i = 1, \dots, K$$

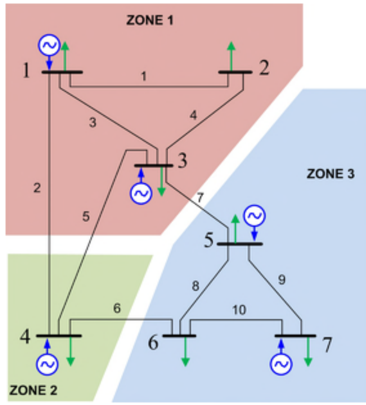
where $\gamma_j'(\cdot)$ is derivative of $\gamma_j(\cdot)$.

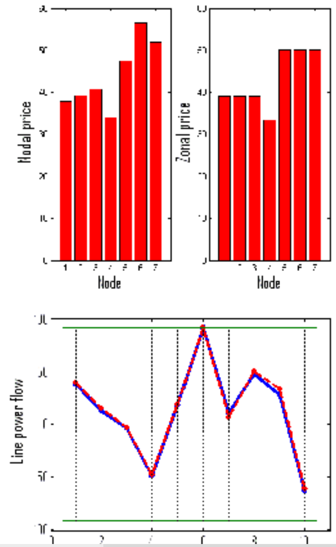
In case of affine bids, zonal prices can be calculated as averaged sum of auxiliary nodal prices, where the weights are derived from the bids.

Zonal pricing (market splitting)

Example

Exercise 7 (on next slide)





INTERMEZZO: Exercise 7

Exercise 7

For network with topology on previous slide calculate: nodal prices, zonal prices, PTDFs for transactions of choice, ...

line i-j	x_{ij}	flow limit
1-2	0.0576	100
1-4	0.092	100
1-3	0.17	100
2-3	0.0586	100
3-4	0.1008	100
4-6	0.072	100
3-5	0.0625	100
3-5	0.161	100
3-5	0.085	100
3-5	0.0856	100

node i	a_i	b_i	load
1	0.13	1.73	88
2	-	-	87
3	0.13	1.86	64
4	0.09	2.13	110
5	0.10	2.39	147
6	-	-	203
7	0.12	2.53	172

Cost function of generator at node i:
$C_i(p_i) = a_i p_i^2 + b_i p_i$

Zonal pricing (flow-based market coupling)

CWE FB market coupling

CWE = Central Western Europe
NWE = North-West Europe
The market coupling evolved from market splitting.
In EU, price zones already exist (national networks).
Goal: coupling of price zones (pan-EU market).

- Available Transfer Capacity (ATC) based market coupling: in 2010 for NWE
- Flow-based market coupling: parallel run and testing for CWE region
  - estimated increase in day-head market welfare: 95M Euro / year (report 9 May 2014)

Zonal pricing (flow-based market coupling)

CWE FB market coupling

Market coupling

- matching orders on several power exchanges (market operators)
- implicit (transfer) capacity allocation mechanism
- market prices and net positions of the connected markets simultaneously determined
- goal: efficient and safe usage of transmission system under coupled markets

Zonal pricing (flow-based market coupling)

CWE FB market coupling

$e_{\mathcal{C}} \in \mathbb{R}^T$ vector power flows in T congestion critical lines
$e_{\mathcal{C}}^{ref} \in \mathbb{R}^T$ vector of predicted (reference) line power flows in congestion critical lines
$p_{\mathcal{Z}_i} \in \mathbb{R}$ aggregated prosumption in zone i
$p_{\mathcal{Z}_i}^{ref} \in \mathbb{R}$ predicted aggregated prosumption in zone i
$\Psi \in \mathbb{R}^{T\times K}$ matrix of "zonal" Power Transfer Distribution Factors (PTDF)
$p_{\mathcal{Z}} := \left(p_{\mathcal{Z}_1}, \ldots, p_{\mathcal{Z}_K}\right)^\top$, $p_{\mathcal{Z}}^{ref} := \left(p_{\mathcal{Z}_1}^{ref}, \ldots, p_{\mathcal{Z}_K}^{ref}\right)^\top$

$$e_{\mathcal{C}} = e_{\mathcal{C}}^{ref} + \Psi(p_{\mathcal{Z}} - p_{\mathcal{Z}}^{ref})$$

Generation Shift Key (GSK)

$$\Psi = \tilde{\Psi}\underbrace{\mathrm{diag}(M_1, \ldots, M_K)}_{M}$$

$M_i \in \mathbb{R}^{R_i}$ = Generation Shift Key (GSK) = mapping from aggregated zone power variation (scalar value) into variations of R_i nodal "market active" power injections in that zone.
$\tilde{\Psi} \in \mathbb{R}^{T\times(R_1+\ldots+R_K)}$ = matrix of "standard" PTDF factors

From aggregated zonal bids $\beta_{\mathcal{Z}_i}(p_{\mathcal{Z}_i})$ deduce objective functions $J_i(p_{\mathcal{Z}_i})$.
$p_{\mathcal{Z}} := \left(p_{\mathcal{Z}_1}, \ldots, p_{\mathcal{Z}_K}\right)^\top$, $p_{\mathcal{Z}_i} \in \mathbb{R}$ (not sign restricted, possible net import and net export)
$\lambda_{\mathcal{Z}} := \left(\lambda_{\mathcal{Z}_1}, \ldots, \lambda_{\mathcal{Z}_K}\right)^\top$, $\lambda_{\mathcal{Z}_i} \in \mathbb{R}$, $s_{\mathcal{C}}$ is vector of reliability margins

Market coupling problem

$$\begin{aligned} \min_{p_{\mathcal{Z}},\lambda_{\mathcal{Z}}} \quad & \sum_{i=1}^{K} J_{\mathcal{Z}_i}(p_{\mathcal{Z}_i}) \\ \text{subject to} \quad & \beta_{\mathcal{Z}}(p_{\mathcal{Z}}) = \lambda_{\mathcal{Z}} \\ & \sum_{i=1}^{K} p_{\mathcal{Z}_i} = 0 \\ & \underbrace{e_{\mathcal{C}}^{ref} + \boxed{\tilde{\Psi}M(p_{\mathcal{Z}} - p_{\mathcal{Z}}^{ref})}}_{e_{\mathcal{C}}} + s_{\mathcal{C}} - \bar{e}_{\mathcal{C}} \le 0 \end{aligned}$$

Market coupling problem ♣

$$\begin{aligned} \min_{p_{\mathcal{Z}},\theta,\lambda_{\mathcal{Z}}} \quad & \sum_{i=1}^{K} J_{\mathcal{Z}_i}(p_{\mathcal{Z}_i}) \\ \text{subject to} \quad & \beta_{\mathcal{Z}}(p_{\mathcal{Z}}) = \lambda_{\mathcal{Z}} \\ & \boxed{Mp_{\mathcal{Z}}} - B\theta = 0 \\ & \underbrace{e_{\mathcal{C}}^{ref} + L\theta}_{e_{\mathcal{C}}} + s_{\mathcal{C}} - \bar{e}_{\mathcal{C}} \le 0 \end{aligned}$$

boxed parts = relaxation of difficult part for zonal pricing (origin of nonconvexity).

citation:"*...due to convexity pre-requisite of the flow based domain, the GSK must be linear...*"
There is more structure in ♣ formulation (possible to exploit).

Zonal pricing (flow-based market coupling)

CWE FB market coupling

Remarks

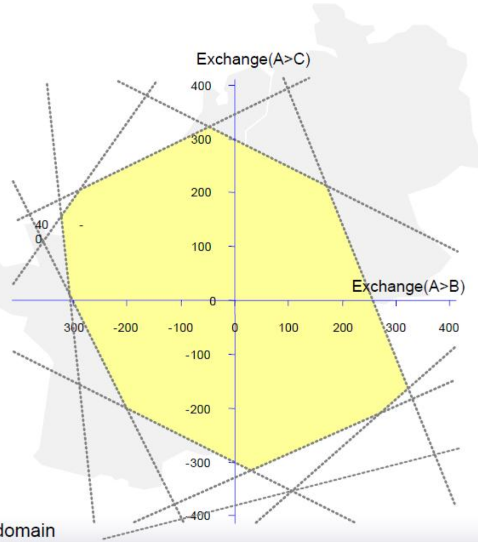
- "a critical branch is considered to be significantly impacted by CWE cross border trade, if its maximum CWE zone-to-zone PTDF is larger then 5%"
- regularly updated (D-2 days) detailed transmission system model and parameters estimation in detailed model used for PTDF calculation
- regular cooperation of all TSO's in gathering data
- reliability margins $s_{\mathcal{C}}$: to capture uncertainties, among others from GSK approximation

Zonal pricing (flow-based market coupling)

CWE FB market coupling

Numbers are for illustration only

Monitored Lines	Outage scenario	Margin left (MW)	Influence of exchange on lines (PTDF)		
			A→B	A→C	B→C
Line 1	No outage	150	1%	10%	3%
	Outage 1	120	5%	20%	1%
	Outage 2	100	6%	25%	1%
Line 2	No outage	150	-2%	0	5%
	Outage 3	100	-12%	0	10 %
Line 3	No outage				
	Outage 4				

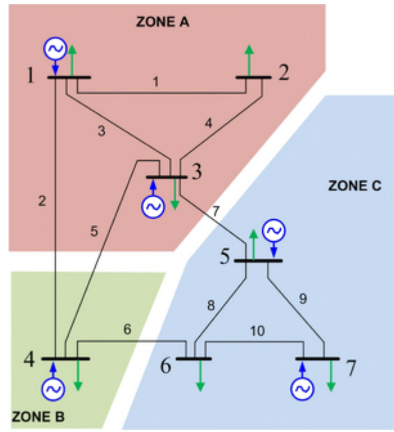


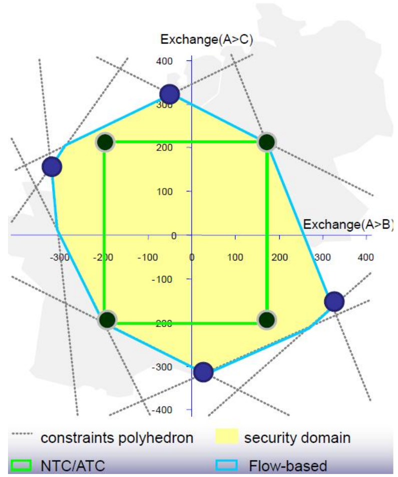
...... Constraints

Security of Supply domain

Zonal pricing (flow-based market coupling)

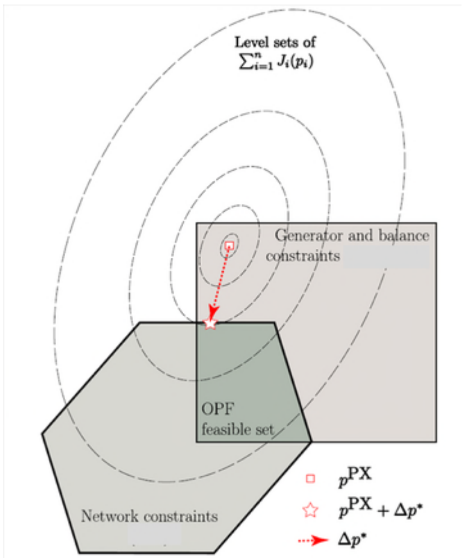
CWE FB market coupling





Alleviation methods

Illustration of optimal redispatch



1) Clear energy market ignoring (internal) line flow limits $\rightarrow (p^{PX}, \theta^{PX})$
2) Redispatch if a line flow limit violated

$$\min_{\Delta p, \Delta\theta} \sum_i J_i(\Delta p_i)$$
$$\text{subject to } \Delta p - B\Delta\theta = 0$$
$$L(\theta^{PX} + \Delta\theta) \leq \bar{e}_{\mathcal{E}}$$

3) Based on $\Delta p^{\star}$, the TSO pays $J_i(\Delta p_i)$ to i-th prosumer

Outline

Convexification of OPF

Bus injection model

$\mathbf{v_k}, \mathbf{i_k}, \mathbf{s_k}$ = voltage, current, power (all complex) at node k
$\mathbf{Y}$ admittance matrix
e_k column vector with 1 in the k-th entry, zero elsewhere

$\mathbf{s_k} = p_k + iq_k$

$\mathbf{s_k} = \mathbf{v_k}\mathbf{i_k}^* = (e_k^\top \mathbf{v})(e_k^\top \mathbf{Y}\mathbf{v})^* = \text{tr } (\mathbf{Y}^* e_k e_k^\top)\mathbf{v}\mathbf{v}^*$

with $\mathbf{Y_k} = e_k e_k^\top \mathbf{Y}$, $\quad \Phi_k := \frac{1}{2}(\mathbf{Y_k}^* + \mathbf{Y_k})$, $\quad \Psi_k := \frac{1}{2i}(\mathbf{Y_k}^* - \mathbf{Y_k})$, $\quad J_k := e_k e_k^\top$

$$p_k = \text{tr } \Phi_k \mathbf{v}\mathbf{v}^*$$
$$q_k = \text{tr } \Psi_k \mathbf{v}\mathbf{v}^*$$
$$|\mathbf{v_k}|^2 = \text{tr } J_k \mathbf{v}\mathbf{v}^*$$

Convexification of OPF

OPF problem (QCQP)

$$\min_{\mathbf{v}} \sum_k \text{tr } C_k \mathbf{v}\mathbf{v}^*$$

subjet to

$$\underline{p}_k \leq \text{tr } \Phi_k \mathbf{v}\mathbf{v}^* \leq \overline{p}_k$$
$$\underline{q}_k \leq \text{tr } \Psi_k \mathbf{v}\mathbf{v}^* \leq \overline{q}_k$$
$$\underline{\mathbf{v}}_\mathbf{k}^2 \leq \text{tr } J_k \mathbf{v}\mathbf{v}^* \leq \overline{\mathbf{v}}_\mathbf{k}^2$$

SDP formulation of the OPF problem

$$\min_{\mathbf{v}} \sum_k \text{tr } C_k W$$

subjet to

$$\underline{p}_k \leq \text{tr } \Phi_k W \leq \overline{p}_k$$
$$\underline{q}_k \leq \text{tr } \Psi_k W \leq \overline{q}_k$$
$$\underline{\mathbf{v}}_\mathbf{k}^2 \leq \text{tr } J_k W \leq \overline{\mathbf{v}}_\mathbf{k}^2$$
$$W \succeq 0$$
$$\text{rank}(W) = 1$$

SDP relaxation of the OPF problem

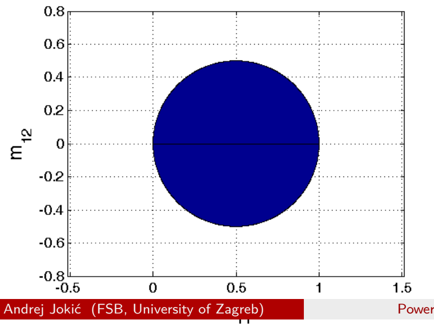
Omit the constraint $\text{rank}(W) = 1$

Convexification of OPF

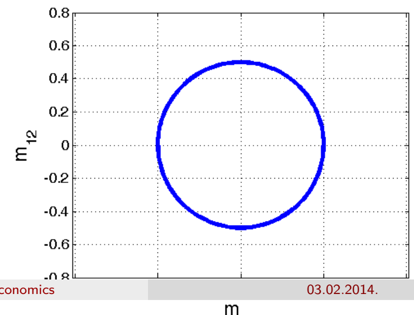
Example. Rank constraint as origin of nonconvexity.

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix}$$

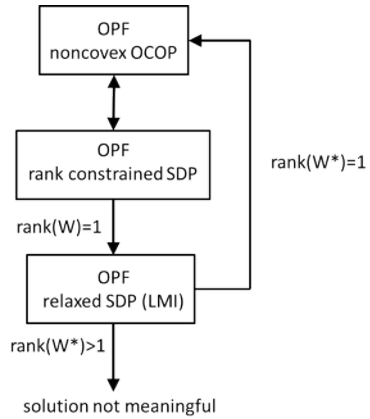
$$M \succeq 0$$
$$\text{trace}(M) = 1$$

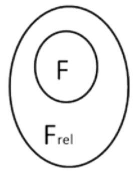


$$M \succeq 0$$
$$\text{trace}(M) = 1$$
$$\text{rank}(M) = 1$$



Convex relaxation of OPF





- Radial networks: ∃ (mild) sufficient conditions for exactness of relaxation
- Branch flow model: radial net → exact
- Mesh networks: convexification via phase shifters
- When exact: strong duality

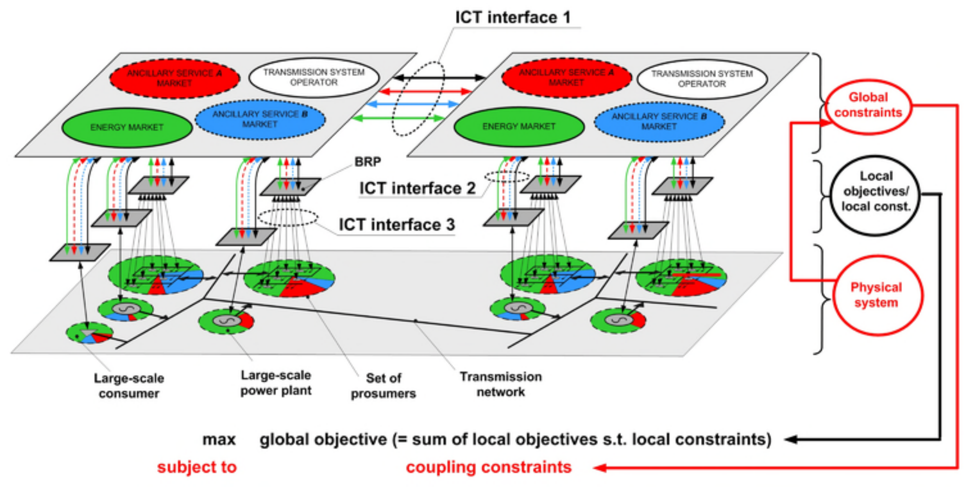
Convex relaxation of OPF

Mesh network

OPF Problem	SDP Relaxation of OPF
Minimize $\sum_{k\in\mathcal{G}} f_k(P_{G_k})$ over $\mathbf{P}_G, \mathbf{Q}_G, \mathbf{V}$	Minimize $\sum_{k\in\mathcal{G}} f_k(P_{G_k})$ over $\mathbf{P}_G, \mathbf{Q}_G, \mathbf{W} \in \mathbb{H}_+^n$
Subject to:	Subject to:
1- A capacity constraint for each line $(l,m)\in\mathcal{L}$	1- A convexified capacity constraint for each line
2- The following constraints for each bus $k\in\mathcal{N}$:	2- The following constraints for each bus $k\in\mathcal{N}$:
$P_{G_k} - P_{D_k} = \sum_{l\in\mathcal{N}(k)} \mathrm{Re}\{V_k(V_k^* - V_l^*)y_{kl}^*\}$ (1a)	$P_{G_k} - P_{D_k} = \sum_{l\in\mathcal{N}(k)} \mathrm{Re}\{(W_{kk} - W_{kl})y_{kl}^*\}$ (2a)
$Q_{G_k} - Q_{D_k} = \sum_{l\in\mathcal{N}(k)} \mathrm{Im}\{V_k(V_k^* - V_l^*)y_{kl}^*\}$ (1b)	$Q_{G_k} - Q_{D_k} = \sum_{l\in\mathcal{N}(k)} \mathrm{Im}\{(W_{kk} - W_{kl})y_{kl}^*\}$ (2b)
$P_k^{\min} \le P_{G_k} \le P_k^{\max}$ (1c)	$P_k^{\min} \le P_{G_k} \le P_k^{\max}$ (2c)
$Q_k^{\min} \le Q_{G_k} \le Q_k^{\max}$ (1d)	$Q_k^{\min} \le Q_{G_k} \le Q_k^{\max}$ (2d)
$V_k^{\min} \le \lvert V_k\rvert \le V_k^{\max}$ (1e)	$(V_k^{\min})^2 \le W_{kk} \le (V_k^{\max})^2$ (2e)

Capacity constraint for line $(l,m)\in\mathcal{L}$	Convexified capacity constraint for line $(l,m)\in\mathcal{L}$
$\lvert\theta_{lm}\rvert = \lvert\measuredangle V_l - \measuredangle V_m\rvert \le \theta_{lm}^{\max}$ (3a)	$\mathrm{Im}\{W_{lm}\} \le \mathrm{Re}\{W_{lm}\}\tan(\theta_{lm}^{\max})$ (4a)
$\lvert P_{lm}\rvert = \lvert\mathrm{Re}\{V_l(V_l^* - V_m^*)y_{lm}^*\}\rvert \le P_{lm}^{\max}$ (3b)	$\mathrm{Re}\{(W_{ll} - W_{lm})y_{lm}^*\} \le P_{lm}^{\max}$ (4b)
$\lvert S_{lm}\rvert = \lvert V_l(V_l^* - V_m^*)y_{lm}^*\rvert \le S_{lm}^{\max}$ (3c)	$\lvert(W_{ll} - W_{lm})y_{lm}^*\rvert \le S_{lm}^{\max}$ (4c)
$\lvert V_l - V_m\rvert \le \Delta V_{lm}^{\max}$ (3d)	$W_{ll} + W_{mm} - W_{lm} - W_{ml} \le (\Delta V_{lm}^{\max})^2$ (4d)

Solution architecture: Some challenges and potentials



Solution architecture: Some challenges and potentials

- do not use PTDF - easier to decompose on Interface 1
- Keeping voltage phase angles preserves the structure
- Interface 1 in reality replaced with higher hierachical level, not reflecting toplogy of the system
- Both interface 1 and 2 require parts of variables of the power flow
- Interface 3 currently hardly exists - large potentials
- Full AC with uncertainties - robust solutions, conservatism? Stohastic settings...

Outline

Ancillary services (AS)

Regulated system: AS bundled with energy
Deregulated system: unbundling of AS, creation of competitive markets for AS

Ancillary services

- Real power balancing
- Voltage support (voltage stability)
- Network congestion relief (transmission security)
- Economic dispatch
- Financial trade enforcement
- Black start

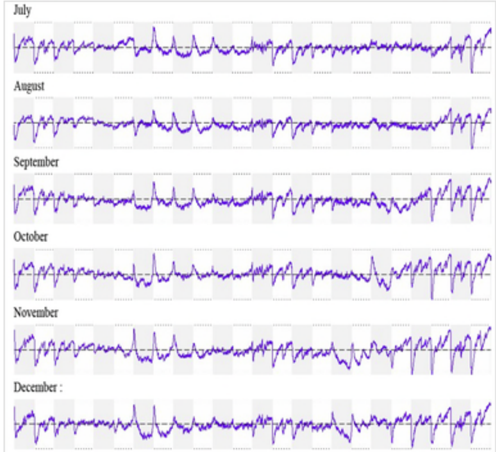


Figure 6.1. Monthly frequency profiles for 2007
Scaling is +/- 50 mHz (vertical) and 24 hours (horizontal)

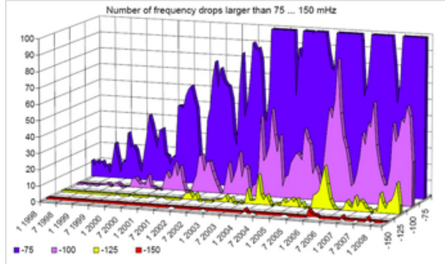
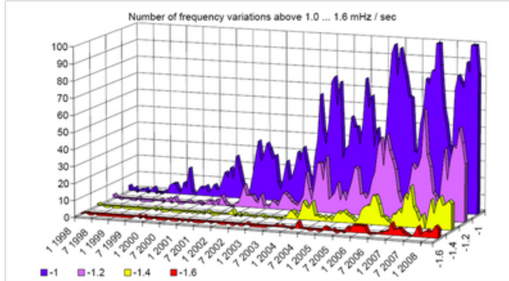
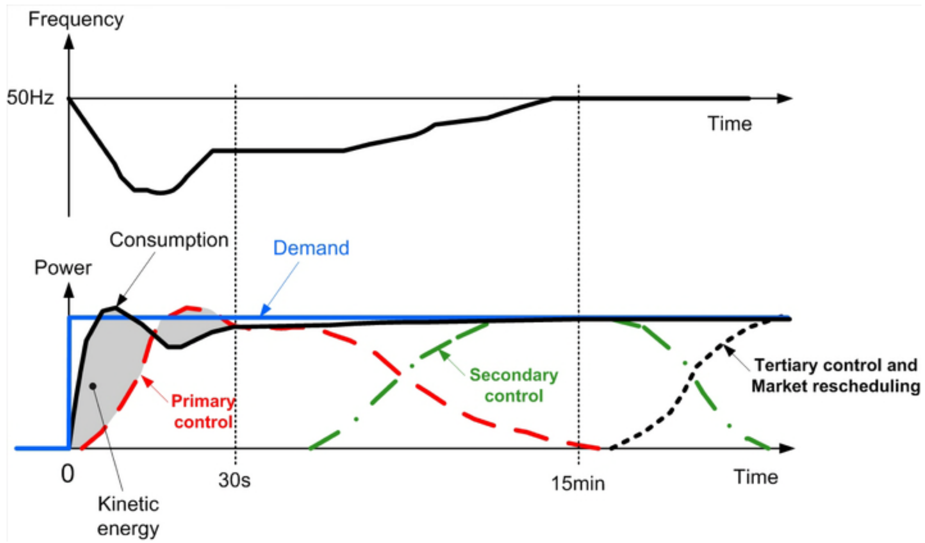


Figure 3.4: Evolution of number of 75, 100, 125, 150 mHz frequency drops



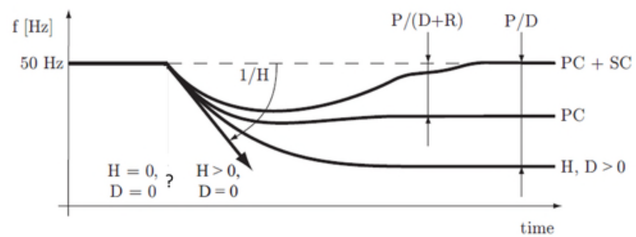
Power balancing ancillary services



Commodities

Related AS commodities

- Inertia: not a commodity.
- Primary control (PC) commodities: capacity (usually mapped into control gain (droop). (Control gain as market commodity!)
- Secondary control (SC) commodities: activated energy; allocated capacity (various arrangements)
- Tertiary control commodities: capacity and energy

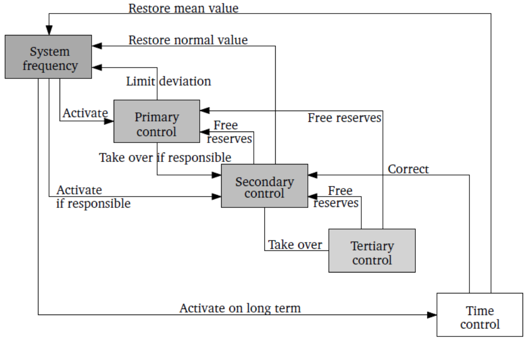


Some questions:
Can one benefit from investing in flywheel?
What about inertia in future?

Category.	Function	Reserves
FCR	contain frequency deviations	primary reserves, FCR
FRR	restore nominal frequency	secondary reserves LFC, AR, FADR tertiary reserves
RR	replace used FCR and FRR	tertiary reserves, FADR

ENTSO

FCR = Frequency containment reserves (local, automatic, activation time 30s)
FRR = Frequency restoration reserves (central, automatic or manual, 30s to 15 min)
RR = Replacement reserves (several min to 1 h)



Continental Europe synchronous system

- primary reserve
- secondary reserve
- tertiary reserve

Category.	Function	Reserves
FCR	contain frequency deviations	primary reserves, FCR
FRR	restore nominal frequency	secondary reserves LFC, AR, FADR tertiary reserves
RR	replace used FCR and FRR	tertiary reserves, FADR

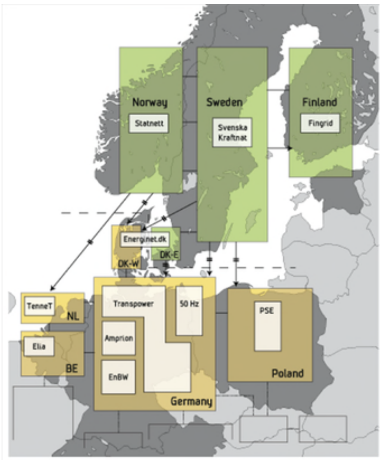
ENTSO

FCR = Frequency containment reserves (local, automatic, activation time 30s)
FRR = Frequency restoration reserves (central, automatic or manual, 30s to 15 min)
RR = Replacement reserves (several min to 1 h)

Nordic synchronous system

FCNR = Frequency controlled normal reserve (automatic, instantaneous; with rapid change to 49.9/50.1 Hz, up/down regulation within 2-3 min)
FCDR = Frequency controlled disturbance reserve (automatic, instantaneous; with rapid change to 49.5 Hz, up regulation within 2-3 min)
AR = Automatic reserves
FADR = Fast active disturbance reserve (manual, 15 min)

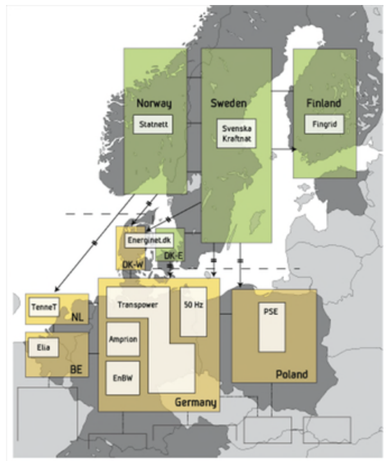
Service objectives and commodities



		DE	NL	BE	DK-W
Primary	capacity	weekly pay-as-bid	mandatory -	4-yearly bilateral	daily marginal
	energy	unpaid -	unpaid -	unpaid -	unpaid -
Secondary	capacity	weekly pay-as-bid	annually bilateral	2-yearly pay-as-bid	monthly pay-as-bid
	energy	weekly average	daily marginal	daily pay-as-bid	daily spot-based
Tertiary	capacity	daily pay-as-bid	unpaid -	4-yearly bilateral	daily marginal
	energy	daily average	daily marginal	daily mixed	daily marginal

Balancing services in continental Europe synchronous system (yellow TSOs in the Fig.) [source: S. Jaehnert, PhD thesis]
Remark: from 2014 in TenneT PC capacity is commodity.

Service objectives and commodities



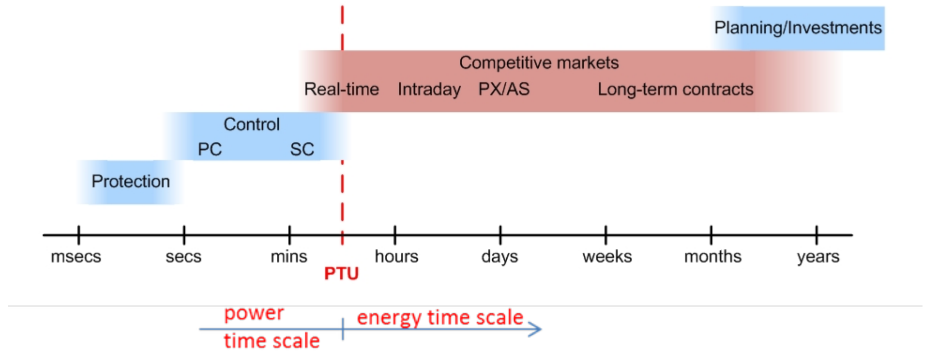
		NO	SE	FI	DK-E
FCR	capacity	yearly / daily marginal	weekly / hourly pay-as-bid	yearly / daily pay-as-bid	daily pay-as-bid
	energy	unpaid -	unpaid -	unpaid -	unpaid -
AR	capacity energy	to be decided			
FADR	capacity	yearly / weekly marginal	yearly bilateral	yearly pay-as-bid	daily pay-as-bid
	energy	hourly marginal			

Balancing services in Nordic synchronous system (green TSOs in the Fig.)

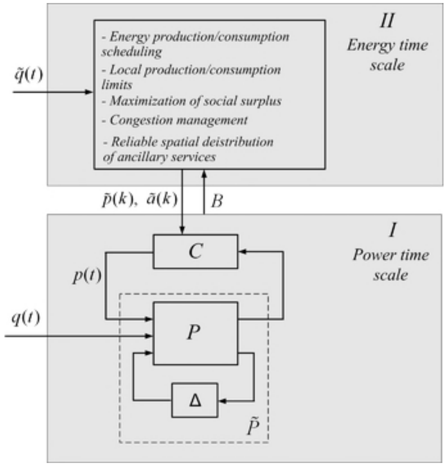
Sync. Area	Process	Product	Activation	Local/Central	Dynamic/ Static	Full Activation Time
BALTIC	Frequency Containment	Primary Reserve	Auto	Local	D	30 s
Cyprus	Frequency Containment	Primary Reserve	Auto	Local	D	20 s
Iceland	Frequency Containment	Primary Control Reserve	Auto	Local	D	variable
Ireland	Frequency Containment	Primary operating reserve	Auto	Local	D/S	5 s
Ireland	Frequency Containment	Secondary operating reserve	Auto	Local	D/S	15 s
NORDIC	Frequency Containment	FNR (FCR N)	Auto	Local	D	120 s -180 s
NORDIC	Frequency Containment	FDR (FCR D)	Auto	Local	D	30 s
RG CE	Frequency Containment	Primary Control Reserve	Auto	Local	D	30 s
UK	Frequency Containment	Frequency response dynamic	Auto	Local	D	Primary 10 s / Secondary 30 s
UK	Frequency Containment	Frequency response static	Auto	Local	S	variable
BALTIC	Frequency Restoration	Secondary emergency reserve	Manual	Central	S	15 Min
Cyprus	Frequency Restoration	Secondary Control Reserve	Auto/Manual	Local/Central	D/S	5 Min
Iceland	Frequency Restoration	Regulating power	Manual	Central	S	10 Min
Ireland	Frequency Restoration	Tertiary operational reserve 1	Auto/Manual	Local/Central	D/S	90 s
Ireland	Frequency Restoration	Tertiary operational reserve 2	Manual	Central	S	5 Min
Ireland	Frequency Restoration	Replacement reserves	Manual	Central	S	20 Min
NORDIC	Frequency Restoration	Regulating power	Manual	Central	S	15 Min
RG CE	Frequency Restoration	Secondary Control Reserve	Auto	Central	D	≤ 15 Min
RG CE	Frequency Restoration	Direct activated Tertiary Control Reserve	Manual	Central	S	≤ 15 Min
UK	Frequency Restoration	Various Products	Manual	D/S	N/A	variable
BALTIC	Replacement	Tertiary (cold) reserve	Manual	Central	S	12 h
Cyprus	Replacement	Replacement reserves	Manual	Central	S	20 min
Iceland	Replacement	Regulating power	Manual	Central	S	10 Min
Ireland	Replacement	Replacement reserves	Manual	Central	S	20 Min
NORDIC	Replacement	Regulating power	Manual	Central	S	15 Min
RG CE	Replacement	Schedule activated Tertiary Control Reserve	Manual	Central	S	individual
RG CE	Replacement	Direct activated Tertiary Control Reserve	Manual	Central	S	individual
UK	Replacement	Various Products but the main one is Short Term Operating Reserve (STOR)	Manual	D/S	N/A	from 20 min to 4 h

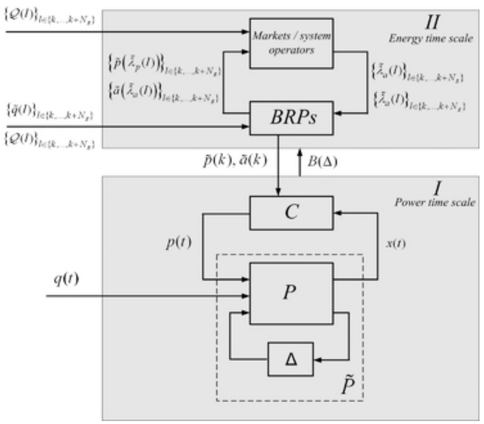
entsoe

Power balancing ancillary services in time scale



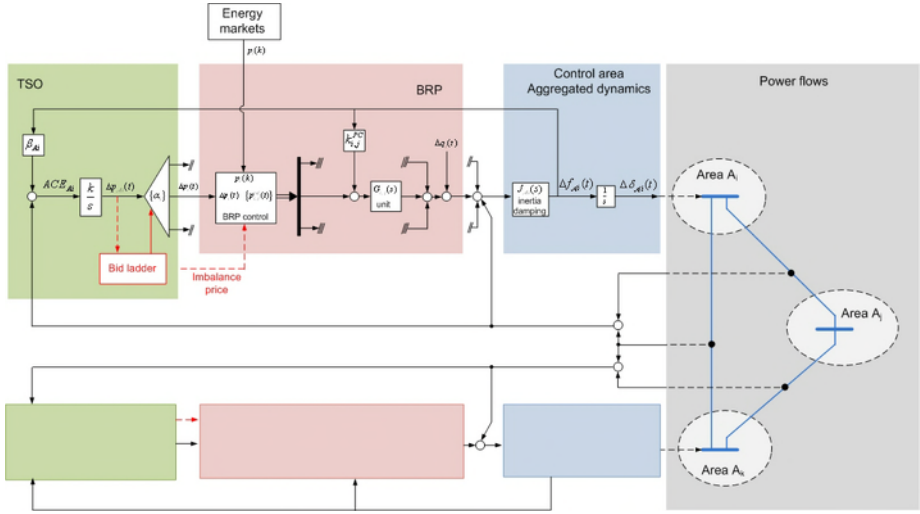
TSO is responsible for balancing within the PTU
BRP is responsible for their balance over whole PTU





Outline

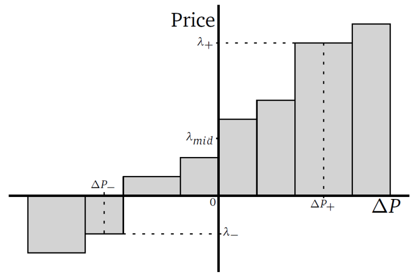
AS provision



AS provision

Primary control

- Sold capacity (market commodity) mapped into PC control gain (local droop)

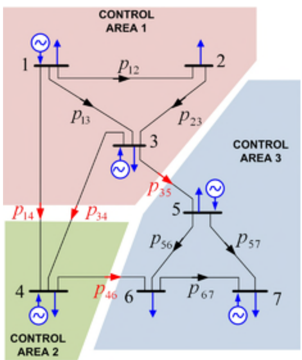


Secondary control

- ACE is matched with bidding ladder every 4 seconds
- Bid ladder changes every PTU (changing parameters in SC loop)

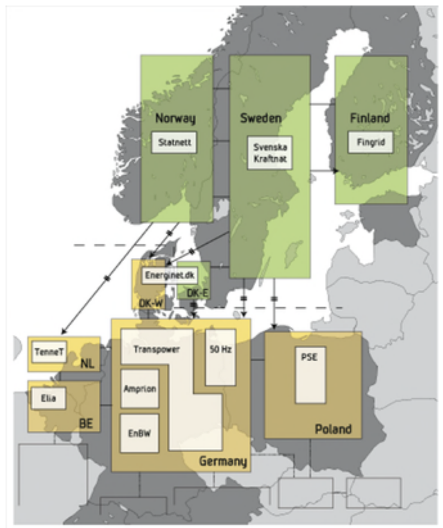
AS provision

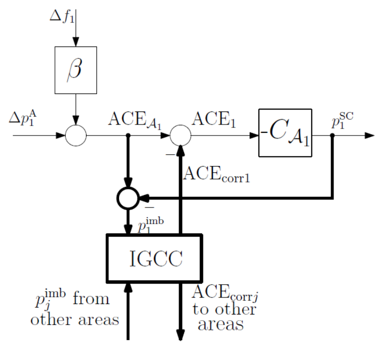
Exercise 8: show that $ACE_i = 0, \forall i \rightarrow \Delta f = 0$ total power exchanges among control areas as at scheduled values. Hint: write down the equations for a simple example (e.g. in the figure), and generalize.



$$\text{ACE}_1 = \beta_1 \Delta f_1 + \Delta p_1^{ex}, \quad p_1^{ex} = \Delta p_{14} + \Delta p_{34} + \Delta p_{35}$$

Inter Control Area Cooperation (IGCC)

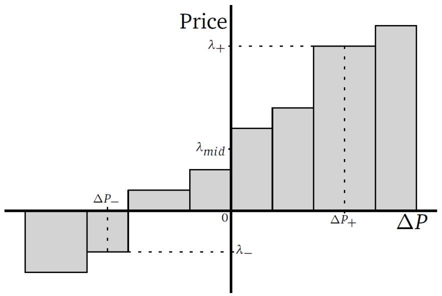




Outline

Imbalance settlement

Example of TenneT NL



BSP (Balance Service Provider) = BRP asked for active contribution

other BRPs: contribute on their own (passive contribution)

λ_p = penalty/incentive price

			BSP			BRP		
			Short	0	Long	Short	0	Long
Situation	-1	(long)	$-(\lambda_-)$	0	n.a.	$-(\lambda_- + \lambda_p)$	0	$\lambda_- - \lambda_p$
	0		n.a.	0	n.a.	$-(\lambda_{mid} + \lambda_p)$	0	$\lambda_{mid} - \lambda_p$
	1	(short)	n.a.	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_+ - \lambda_p$
	2	(both)	$-(\lambda_-)$	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_- - \lambda_p$

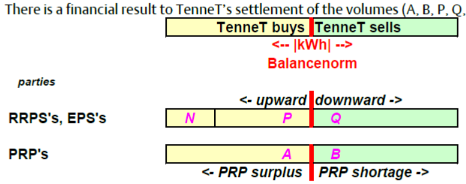
Imbalance settlement

Example of TenneT NL

state	meaning	occurrence
1	no imbalance in whole PTU	0.14%
-1	the system is long (surplus), requested only negative options	51.77%
0	the system is short (deficit), requested only positive options	38.25%
0	the system has been both long and short within PTU	9.85%

			BSP			BRP		
			Short	0	Long	Short	0	Long
Situation	-1	(long)	$-(\lambda_-)$	0	n.a.	$-(\lambda_- + \lambda_p)$	0	$\lambda_- - \lambda_p$
	0		n.a.	0	n.a.	$-(\lambda_{mid} + \lambda_p)$	0	$\lambda_{mid} - \lambda_p$
	1	(short)	n.a.	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_+ - \lambda_p$
	2	(both)	$-(\lambda_-)$	0	λ_+	$-(\lambda_+ + \lambda_p)$	0	$\lambda_- - \lambda_p$

There is a financial result to TenneT's settlement of the volumes (A, B, P, Q, N) at the designated prices.



The basic formula that applies to the financial result is:

(Q * Pdo + B * Pshort) - (N * Pem + P * Pup + A * Psurp)

Or:

B * Pshort – A * Psurp + Q * Pdown – P * Pup – N * Pem

Elaborated per regulation state, this becomes:

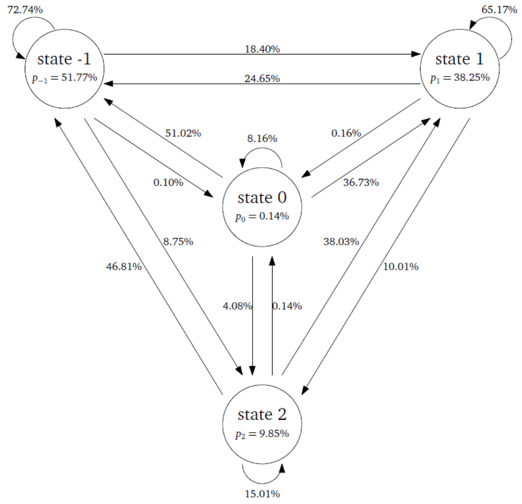
reg. state: 0	B * (Pmid + ic)	- A * (Pmid - ic)		
-1	B * (Pdo + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	
+1	B * (Pup + ic)	- A * (Pup - ic)	+ Q * Pdo - P * Pup	
2	B * (Pup + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	
-1, em	B * (Pdo + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	- N * Pem
+1, em	B * (max(Pem, Pup) + ic)	- A * (max(Pem, Pup) - ic)	+ Q * Pdo - P * Pup	- N * Pem
2, em	B * (max(Pem, Pup) + ic)	- A * (Pdo - ic)	+ Q * Pdo - P * Pup	- N * Pem

Where Pem > Pup, and after a bit of reshuffling this becomes:

reg. state: 0	(B - A) * Pmid			+ (A + B) * ic
-1	(B - A + Q) * Pdo	- P * Pup		+ (A + B) * ic
+1	(B - A - P) * Pup	+ Q * Pdo		+ (A + B) * ic
2	((A + B) - (P + Q)) * (Pup - Pdo)/2			+ (A + B) * ic
-1, em	(B - A + Q) * Pdo	- (P + N) * Pem	+ P * (Pem-Pup)	+ (A + B) * ic
+1, em	(B - A - P - N) * Pem	+ Q * Pdo	+ P * (Pem-Pup)	+ (A + B) * ic
2, em	((A + B) - (P + N + Q)) * (Pem - Pdo)/2		+ P * (Pem-Pup)	+ (A + B) * ic

Risk of bidding less or equal than the risk of not bidding

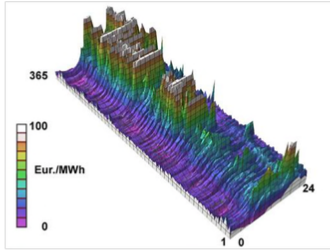
Risk of requested action less or equal than risk of unrequested actions



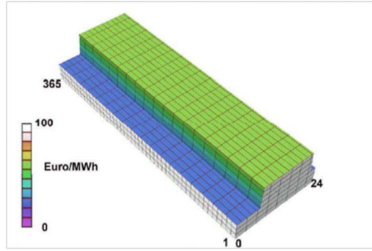
The last info I have:

"Afraid" to announce current situation in real time (delay of one PTU), and close the loop

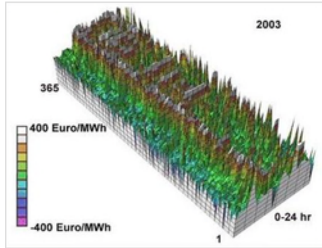
Prices



Day ahead market prices (APX)



Prices for consumers



Balancing prices (TenneT)

Bidding

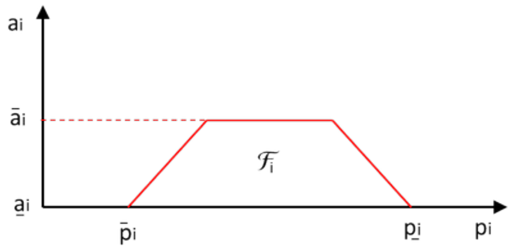
$$\min_{\{p_i\},\{a_i\}} \sum_i C_i(p_i)$$

subject to

$$(p_i, a_i) \in \mathcal{F}_i$$

$$\sum_i p_i - d_{int} = P_{ex} \quad (\lambda_P)$$

$$\sum_i a_i - a_{int} = A_{ex} \quad (\lambda_A)$$



a_i AS allocated capacity at unit i

p_i power production from unit i

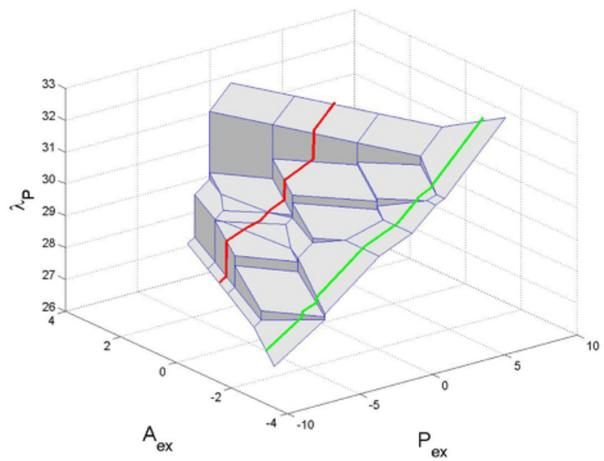
d_{int} internal BRP demand

a_{int} internal BRP's request for local AS capacity

Most often: sequential clearing of markets

Bidding

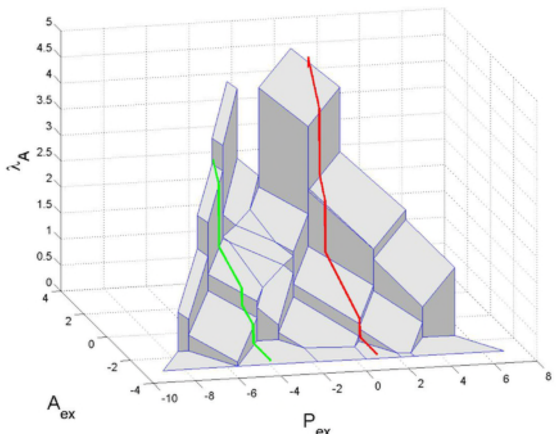
"Behind the interface"; inside BRP



$$\beta(P_{ex}, A_{ex}) \quad \rightarrow \quad \tilde{\beta}(P_{ex})$$

Bidding

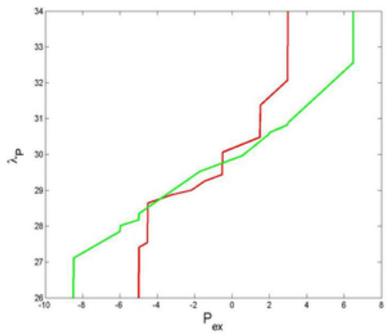
"Behind the interface"; inside BRP

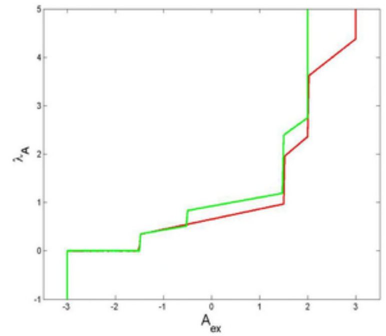


$$\beta(P_{ex}, A_{ex}) \quad \rightarrow \quad \tilde{\beta}(A_{ex})$$

Bidding

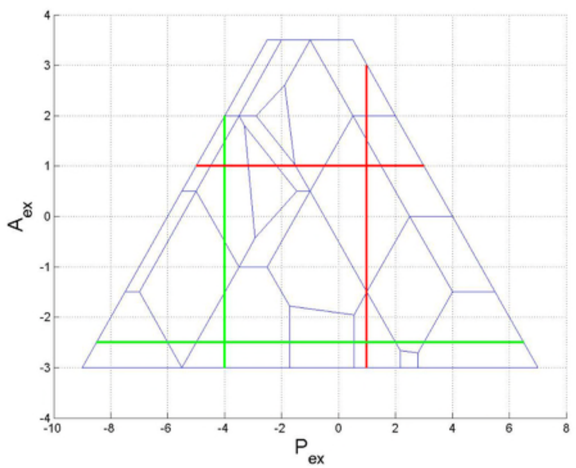
"for the outside world"





$\tilde{\beta}(P_{ex})$ $\tilde{\beta}(A_{ex})$

Bidding



Bidding

$$\min \ \ell + \frac{1}{1-\beta}\left(\sum_{s=1}^{L} \pi_s^{AS+}[f_s - \ell]^+ + \sum_{s=L+1}^{2L} \pi_{s-L}^{AS-}[f_i - \ell]^+\right) \quad (6.4a)$$

$$\text{s.t.} \ f_s = \sum_{j=1}^{n} \frac{C_j u_{sj}}{M_j\left(a_{2,j}\left(\frac{u_{sj}}{TP_{max,j}}\right)^2 + a_{1,j}\frac{u_{sj}}{TP_{max,j}} + a_{0,j}\right)} + [\lambda_{imb,s} x_{imb,s}]^- $$
$$- \lambda_p^{PX} x_p^{PX} - \lambda_s^{AS+} x_{up,s}^{AS}, \ s = 1, \dots, L, \quad (6.4b)$$

$$f_s = \sum_{j=1}^{n} \frac{C_j u_{sj}}{M_j\left(a_{2,j}\left(\frac{u_{sj}}{TP_{max,j}}\right)^2 + a_{1,j}\frac{u_{sj}}{TP_{max,j}} + a_{0,j}\right)} + |\lambda_{imb,s} x_{imb,s}|$$
$$- \lambda_p^{PX} x_p^{PX} + \lambda_{s-L}^{AS-} x_{do,s-L}^{AS}, \ s = L+1, \dots, 2L, \quad (6.4c)$$

$$\underline{u_j} \le u_{sj} \le \overline{u_j}, \ j = 1, \dots, n, \ s = 1, \dots, 2L, \quad (6.4d)$$

$$\sum_{j=1}^{n} u_{sj} - x_p^{PX} - x_{up,s}^{AS} = x_{imb,s}, \ s = 1, \dots, L, \quad (6.4e)$$

$$\sum_{j=1}^{n} u_{sj} - x_p^{PX} + x_{do,s-L}^{AS} = x_{imb,s}, \ s = L+1, \dots, 2L, \quad (6.4f)$$

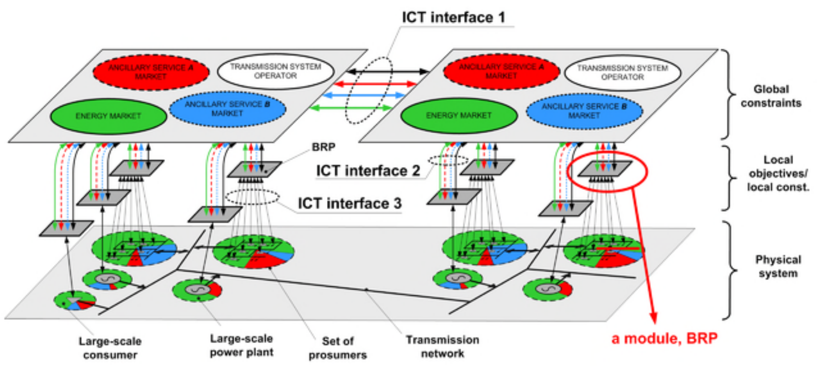
$$x_{do,s}^{AS} \le x_p^{PX}, \ s = 1, \dots, L, \quad (6.4g)$$

$$x_p^{PX} \ge 0, \quad (6.4h)$$

$$x_{up,s}^{AS} \ge 0, \ s = 1, \dots, L, \quad (6.4i)$$

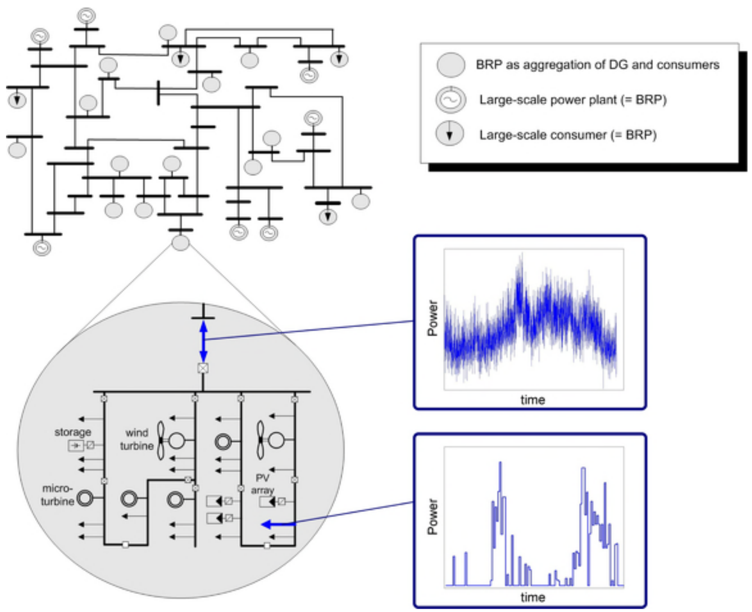
$$x_{do,s}^{AS} \ge 0, \ s = 1, \dots, L. \quad (6.4j)$$

Bids as well defined protocol



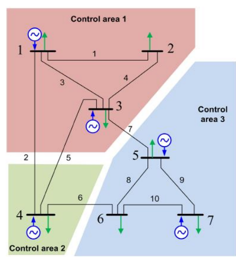
- All that matters are interfaces and protocols on them
- Heterogeneity, local complexities.... all "hidden" behind the interface (*Interface 2*)
- *Interface 2* requires decoupling of coupled problems (e.g. no 2D bids are allowed): enforcing manageable simplicity on the higher level

Outline

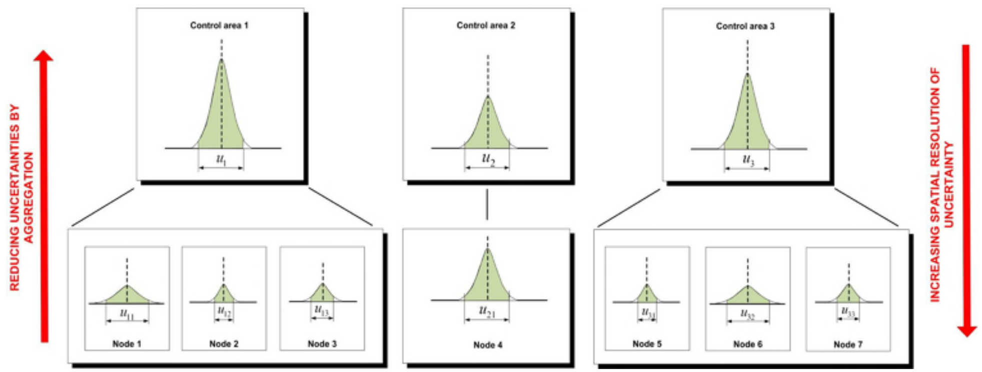


What is the added value of aggregation? Can the rest of network do a better job than my neighbour?

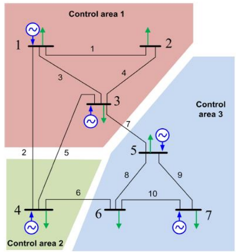
Spatial resolution of uncertainty



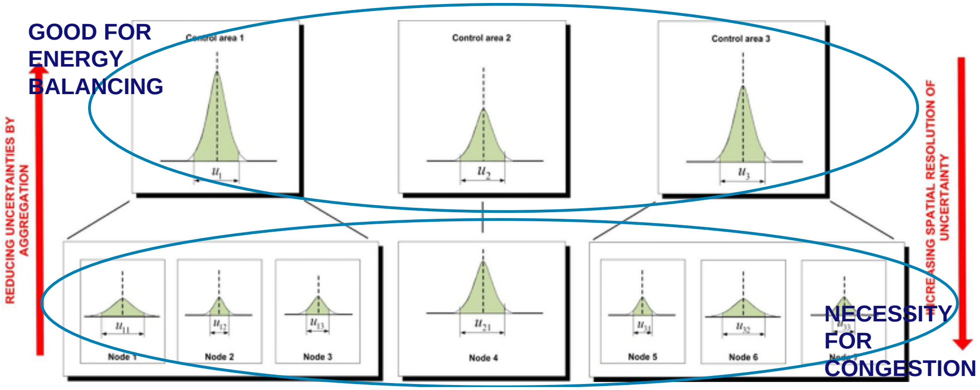
Spatial distribution of uncertainties is crucial in defining uncertainties in power flows



Spatial resolution of uncertainty

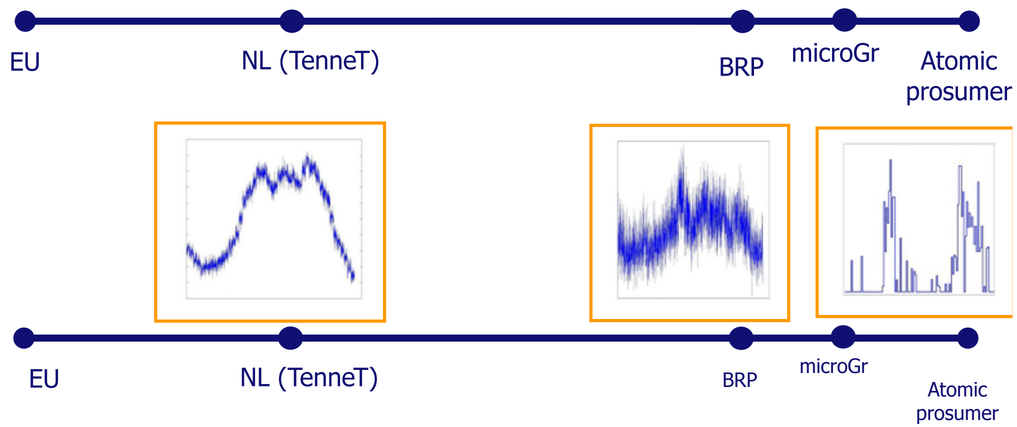


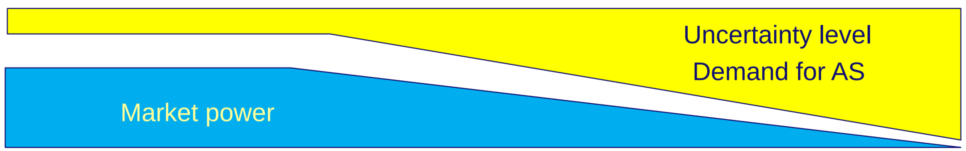
Spatial distribution of uncertainties is crucial in defining uncertainties in power flows



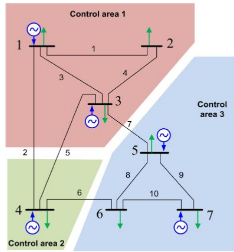
Spatial information

$$\int \Delta f(t) + \beta \Delta P_{NL}(t) \qquad \Delta E(15\text{min}) = \int \Delta P(t)$$

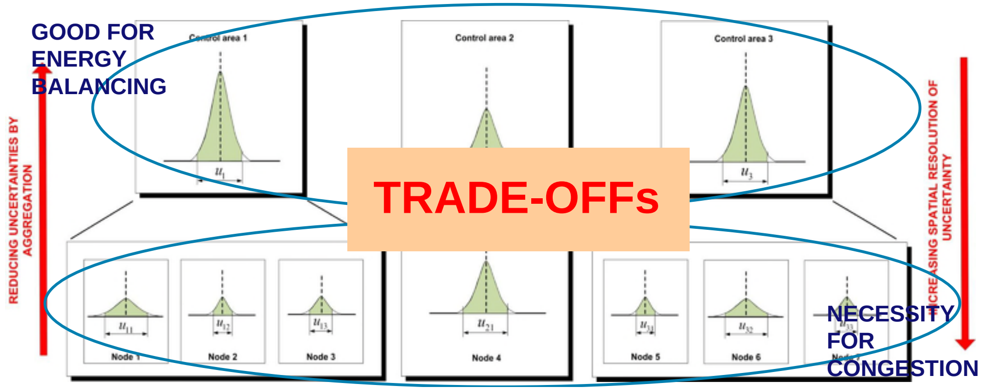




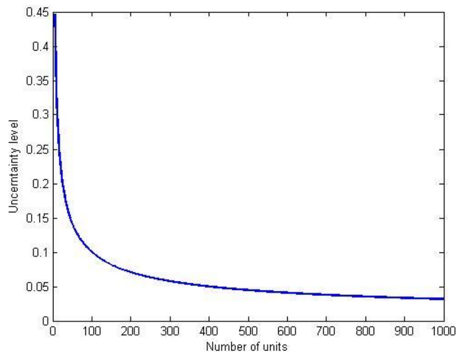
Spatial resolution of uncertainty

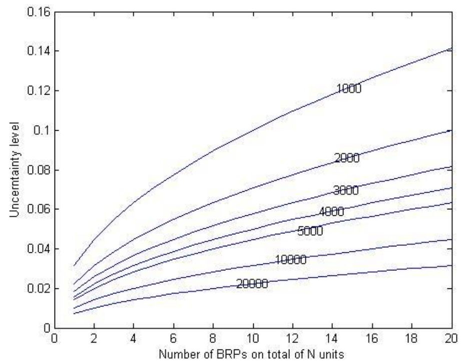


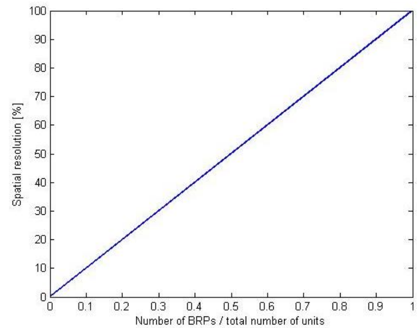
Spatial distribution of uncertainties is crucial in defining uncertainties in power flows



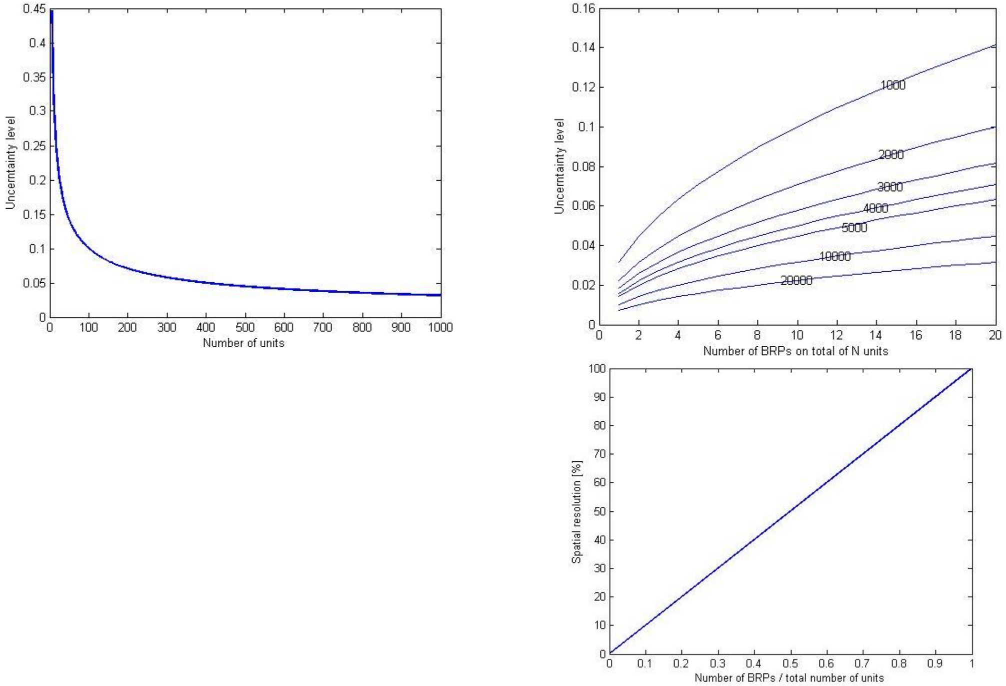
Trade-offs



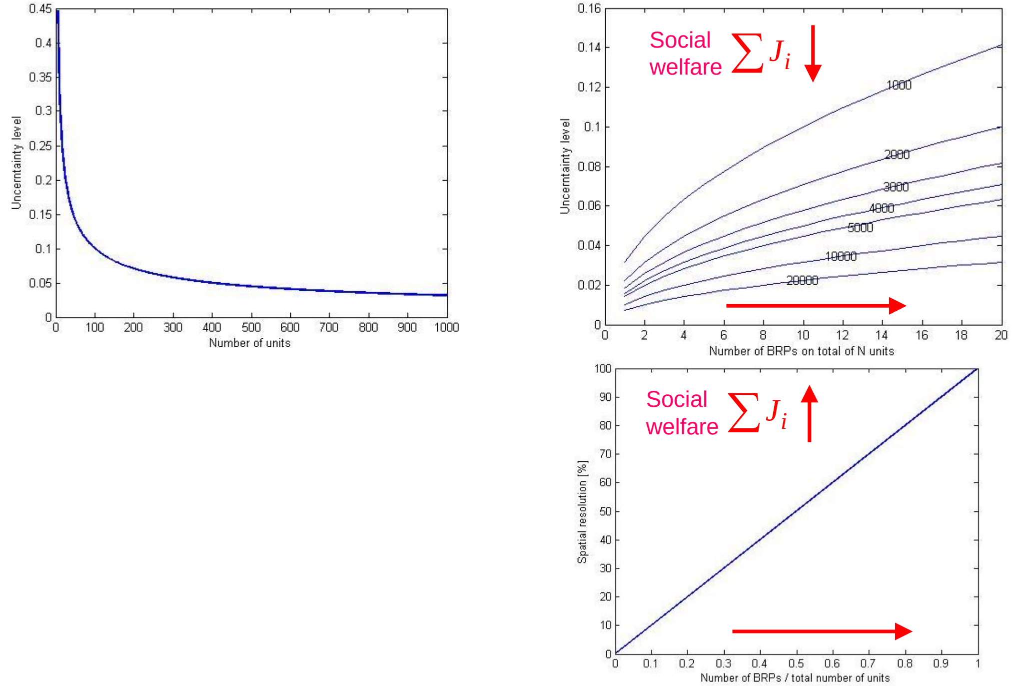




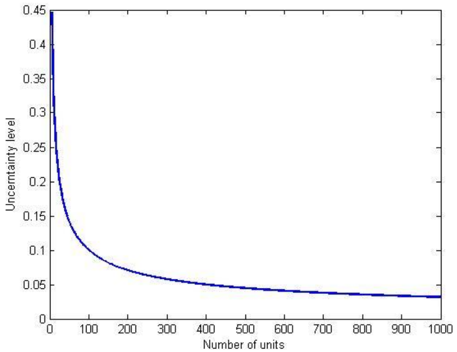
Trade-offs

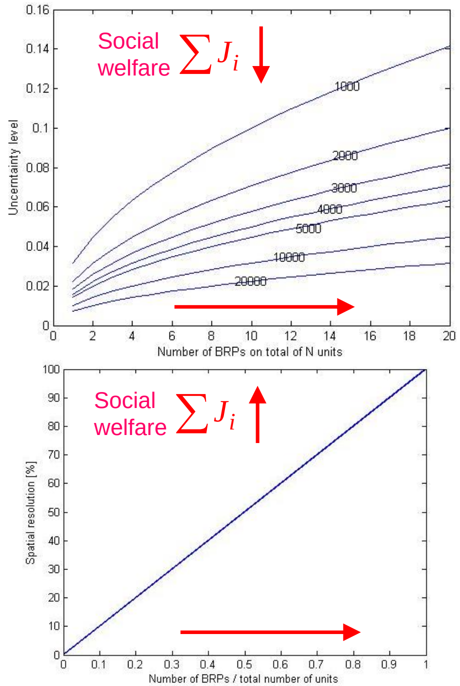


Trade-offs



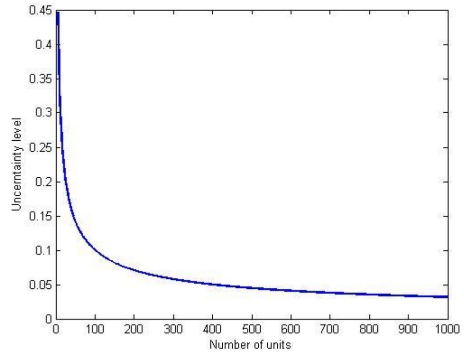
Trade-offs

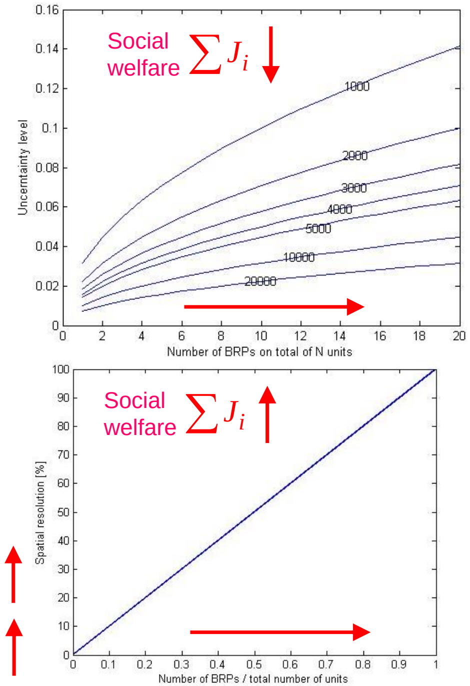




Number of BRP's ↑ $\Rightarrow$ Market power ↓ $\Rightarrow$ Social welfare $\sum J_i$ ↓

Trade-offs

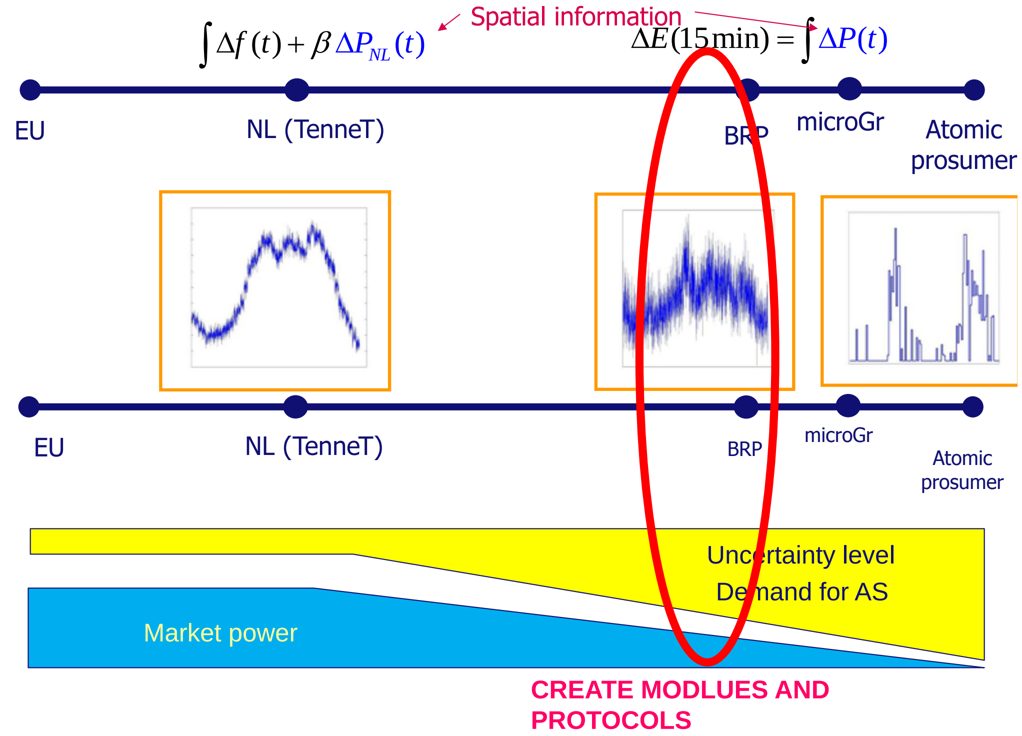




Number of BRP's ↑ $\Rightarrow$ Market power ↓ $\Rightarrow$ Social welfare $\sum J_i$ ↓

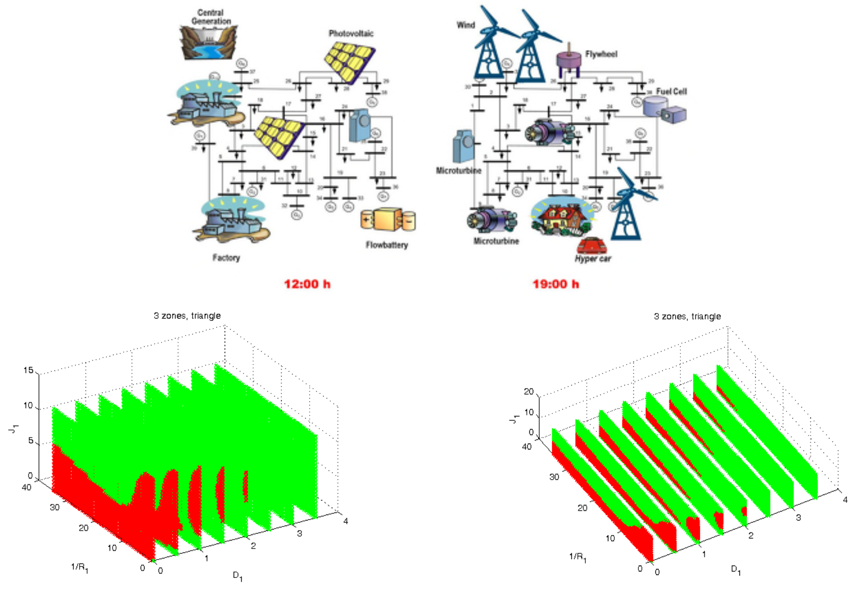
Duration of trading interval ↓ $\Rightarrow$ Unicerntaity level ↓ $\Rightarrow$ Social welfare $\sum J_i$ ↑

$\Rightarrow$ Coupling economy-physics ↑



CHALLENGE

Accumulating /adapting proper amount of gains (AS) for time-varying system



CHALLENGE

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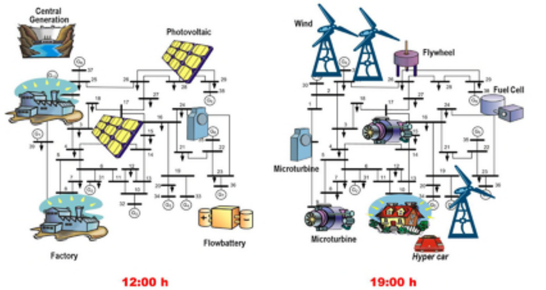
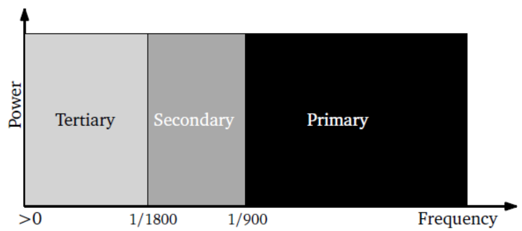
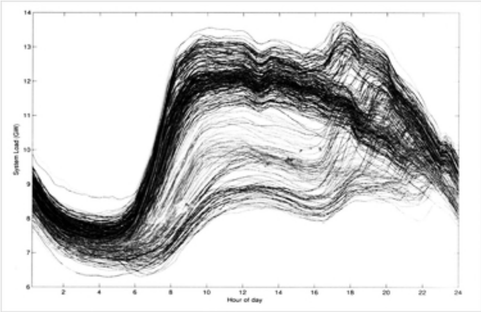
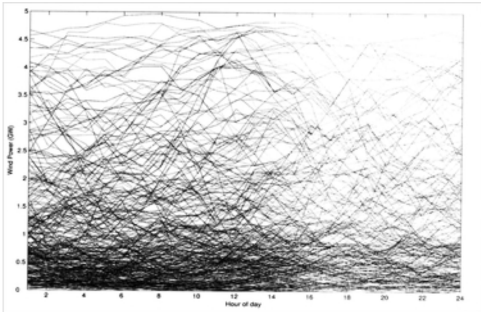




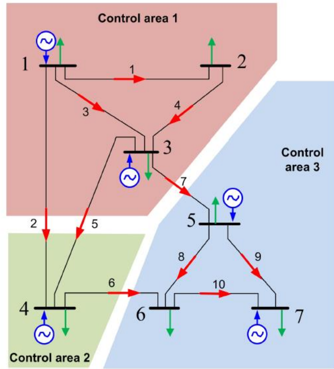
Outline

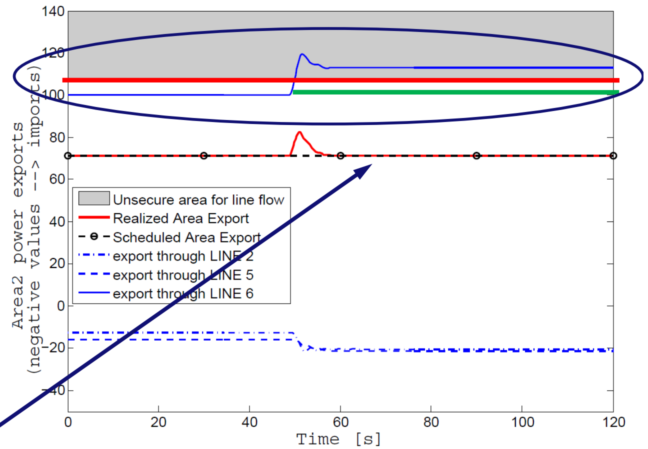




NOW **FUTURE**

- Increased uncertainties $\rightarrow$ Tight coupling economy (markets), physics and RT control
- Uncertain spatial distribution of uncertainties $\rightarrow$ uncertain power flows
- In today's systems efficiency largely relies on repetitiveness
- Put economic optimization in closed loop; care of congestion constraints





In current system, reliability is accounted for in "aggregated" form here

RELIABILITY MARGIN

Size of reliability margin: reliability vs. efficiency trade-off

Economically optimal working point is often on the border of feasible region

Distributed, real-time, price-based control

Optimal nodal pricing problem

$$\min_{\lambda,\delta} \sum_{i=1}^{n} J_i(\gamma_i(\lambda_i))$$

$$\text{subject to} \quad \gamma(\lambda) - B\delta + \hat{p} = 0,$$
$$b_{ij}(\delta_i - \delta_j) \leq \overline{p}_{ij}, \ \forall (i,j \in I(N_i)),$$

Distributed, real-time, price-based control

Optimal power flow problem

$$\min_{p,\delta} \sum_{i} J_i(p_i)$$

$$\text{subject to} \quad p - B\delta + \hat{p} = 0,$$
$$L\delta \leq \overline{e}_c,$$
$$\underline{p} \leq p \leq \overline{p},$$

KKT conditions

$$p - B\delta + \hat{p} = 0,$$
$$B\lambda + L^{\top}\mu = 0,$$
$$\nabla J(p) - \lambda + \nu^{+} - \nu^{-} = 0,$$
$$0 \leq \ (-L\delta + \overline{e}_c) \ \perp \ \mu \ \geq 0,$$
$$0 \leq \ (-p + \overline{p}) \ \perp \ \nu^{+} \ \geq 0,$$
$$0 \leq \ (p + \underline{p}) \ \perp \ \nu^{-} \ \geq 0,$$

Distributed, real-time, price-based control

$\Delta p_L = L\delta - \bar{e}_c$

Nodal pricing controller

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$
$$0 \le \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \ge \ 0,$$
$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

$$\begin{aligned} p - B\delta + \hat{p} &= 0, \\ B\lambda + L^\top \mu &= 0, \\ \nabla J(p) - \lambda + \nu^+ - \nu^- &= 0, \\ 0 \le \ (-L\delta + \bar{e}_c) \ \perp \ \mu \ &\ge 0, \\ 0 \le \ (-p + \overline{p}) \ \perp \ \nu^+ \ &\ge 0, \\ 0 \le \ (p + \underline{p}) \ \perp \ \nu^- \ &\ge 0, \end{aligned}$$

$$B\lambda + L^\top \mu + \Delta f^\star \mathbf{1} = 0,$$
$$\mathbf{1}^\top \begin{pmatrix} B & L^\top \end{pmatrix} = 0 \implies \mathbf{1} \notin \operatorname{Im} \begin{pmatrix} B & L^\top \end{pmatrix},$$
$$\implies \Delta f = 0, \ B\lambda + L^\top \mu = 0$$

Distributed, real-time, price-based control

$\Delta p_L = L\delta - \bar{e}_c$

Nodal pricing controller

$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$
$$0 \le \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \ge \ 0,$$
$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

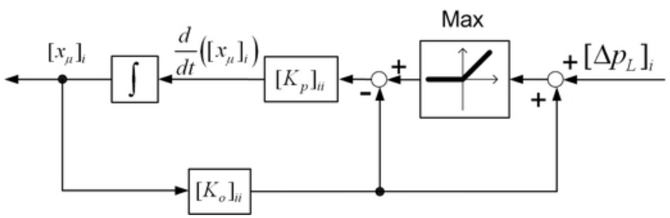
- no knowledge of cost/benefit functions of producers/consumers required
- required no knowledge of actual power injections
- required: B and L
- preserves the structure of B and L

Distributed, real-time, price-based control

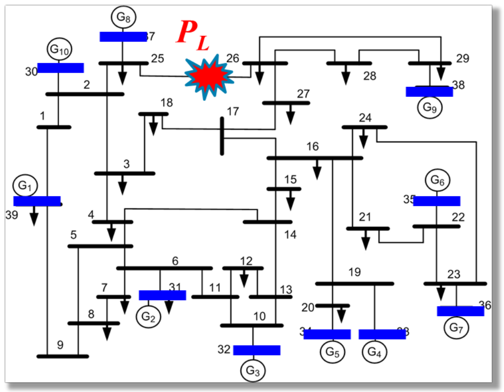
$\Delta p_L = L\delta - \bar{e}_c$

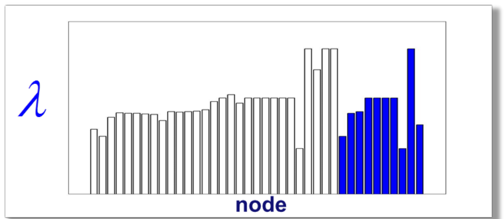
Nodal pricing controller

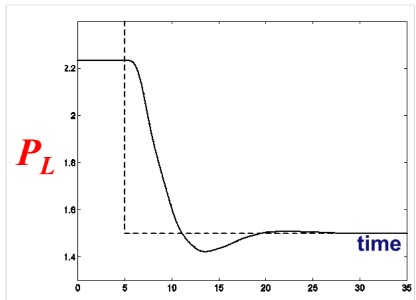
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix} + \begin{pmatrix} -K_f & 0 \\ 0 & K_p \end{pmatrix} \begin{pmatrix} \Delta f \\ \Delta p_L + w \end{pmatrix},$$
$$0 \le \ w \ \perp \ K_o x_\mu + \Delta p_L + w \ \ge \ 0,$$
$$\lambda = \begin{pmatrix} I_n & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \end{pmatrix},$$

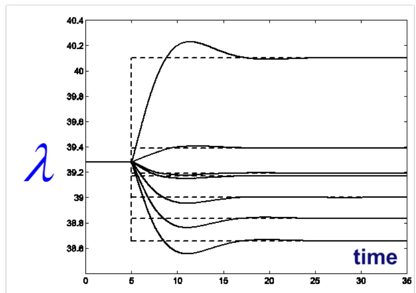


max-based complementarity integrator



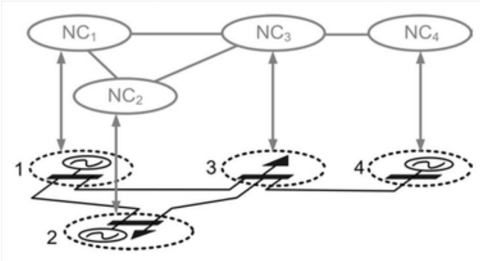






Distributed, real-time, price-based control

REAL-TIME MARKET AND CONGESTION CONTROL



$B\lambda + L^\top \mu = 0$, λ prices for local balance, μ prices for not overloanding the lines

$$\left(\begin{array}{cccc|cc} b_{12,13} & -b_{12} & -b_{13} & 0 & b_{12} & b_{13}\\ -b_{12} & b_{12,23} & -b_{23} & 0 & -b_{12} & 0\\ -b_{13} & -b_{23} & b_{13,23,34} & -b_{34} & 0 & -b_{13}\\ 0 & 0 & -b_{34} & b_{34} & 0 & 0 \end{array}\right)\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\\ \lambda_4\\ \hline \mu_{12}\\ \mu_{13}\end{pmatrix} = 0,$$

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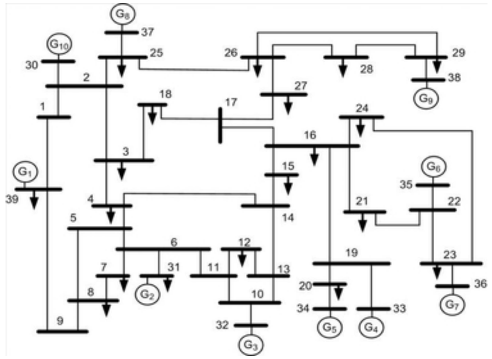
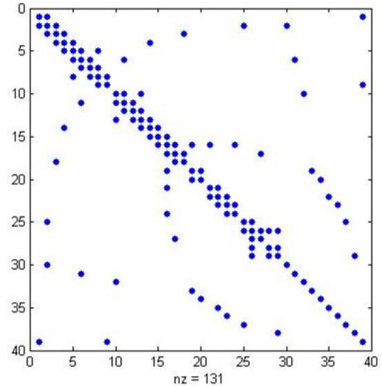
REAL-TIME MARKET AND CONGESTION CONTROL

$$B\lambda + L^\top \mu = 0$$





Distributed, real-time, price-based control

SEPARATING BALANCING PRICING FROM CONGESTION PRICING

$$B = \begin{pmatrix} * & * \\ * & B_\Delta \end{pmatrix} \qquad L = \begin{pmatrix} * & L \end{pmatrix}$$

Modified price-based controller

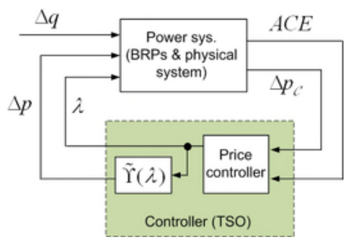
$$\begin{pmatrix}\dot{x}_{\lambda_0}\\ \dot{x}_{\Delta\lambda}\\ \dot{x}_\mu\end{pmatrix} = \begin{pmatrix} 0 & 0 & 0\\ 0 & -K_\Delta B_\Delta & -K_\Delta L_\Delta^\top\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_{\lambda_0}\\ x_{\Delta\lambda}\\ x_\mu\end{pmatrix} + \begin{pmatrix} -k_f \mathbf{1}_n^\top & 0\\ 0 & 0\\ 0 & K_p\end{pmatrix}\begin{pmatrix}\Delta f\\ \Delta p_L + w\end{pmatrix},$$

$$0 \leq w \perp K_o x_\mu + \Delta p_L + w \geq 0,$$

$$\lambda = \begin{pmatrix} 1 & 0 & 0\\ \mathbf{1}_{n-1} & I_{n-1} & 0\end{pmatrix}\begin{pmatrix}x_{\lambda_0}\\ x_{\Delta\lambda}\\ x_\mu\end{pmatrix},$$

Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



Optimality conditions

$$\beta(p^\star) - \lambda^\star = 0$$
$$p^\star - B\theta^\star = 0$$
$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \leq (-L\theta^\star + \overline{e}_\mathcal{E}) \quad \perp \quad \mu^\star \geq 0$$

Real-time nodal price based SC controller (each control area balanced separately)

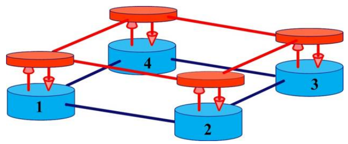
$$\begin{pmatrix}\dot{x}_\lambda\\ \dot{x}_\mu\\ \dot{x}_\sigma\end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_\lambda\\ x_\mu\\ x_\sigma\end{pmatrix} + \begin{pmatrix} 0 & 0\\ 0 & K_\mu\\ -K_\sigma & 0\end{pmatrix}\begin{pmatrix}ACE\\ \Delta p_C\end{pmatrix} + \begin{pmatrix}0\\ K_\mu w\\ 0\end{pmatrix},$$

$$0 \leq w \perp K_0 x_\mu + \Delta p_C + w \geq 0,$$

$$\lambda = \begin{pmatrix} \boxed{I} & 0 & E\end{pmatrix}\begin{pmatrix}x_\lambda\\ x_\mu\\ x_\sigma\end{pmatrix}, \qquad \Delta p = \tilde{\Upsilon}(\lambda)$$

Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



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$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix},$$

$$0 \le w \perp K_0 x_\mu + \Delta p_{\mathcal{C}} + w \ge 0,$$

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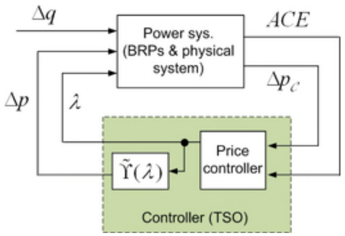
Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



Optimality conditions

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$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star \ge 0$$

Real-time zonal price based SC controller (each control area balanced separately)

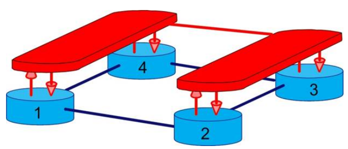
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix}$$

$$0 \le w \perp K_0 x_\mu + \Delta p_{\mathcal{C}} + w \ge 0$$

$$\lambda_{\mathcal{Z}} = \begin{pmatrix} \boxed{F(\cdot)} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \Upsilon(\lambda_{\mathcal{Z}})$$

Distributed, real-time, price-based control

PROVISION OF ANCILLARY SERVICES



Optimality conditions

$$\beta(p^\star) - \lambda^\star = 0$$
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$$B\lambda^\star + L^\top \mu^\star = 0$$
$$0 \le (-L\theta^\star + \overline{e}_{\mathcal{E}}) \quad \perp \quad \mu^\star \ge 0$$

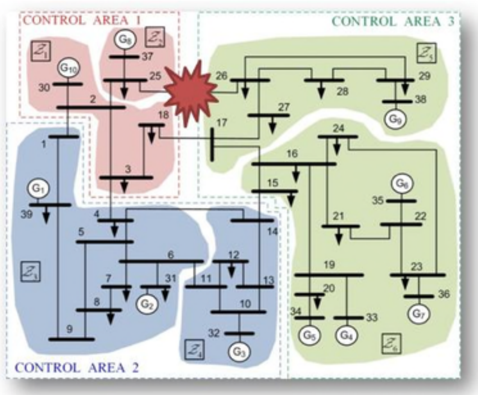
Real-time zonal price based SC controller (each control area balanced separately)

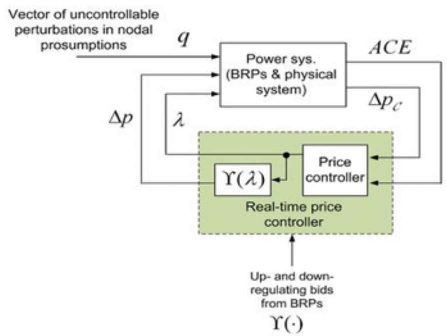
$$\begin{pmatrix} \dot{x}_\lambda \\ \dot{x}_\mu \\ \dot{x}_\sigma \end{pmatrix} = \begin{pmatrix} -K_\lambda B & -K_\lambda L^\top & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & K_\mu \\ -K_\sigma & 0 \end{pmatrix} \begin{pmatrix} ACE \\ \Delta p_{\mathcal{C}} \end{pmatrix} + \begin{pmatrix} 0 \\ K_\mu w \\ 0 \end{pmatrix}$$

$$0 \le w \perp K_0 x_\mu + \Delta p_{\mathcal{C}} + w \ge 0$$

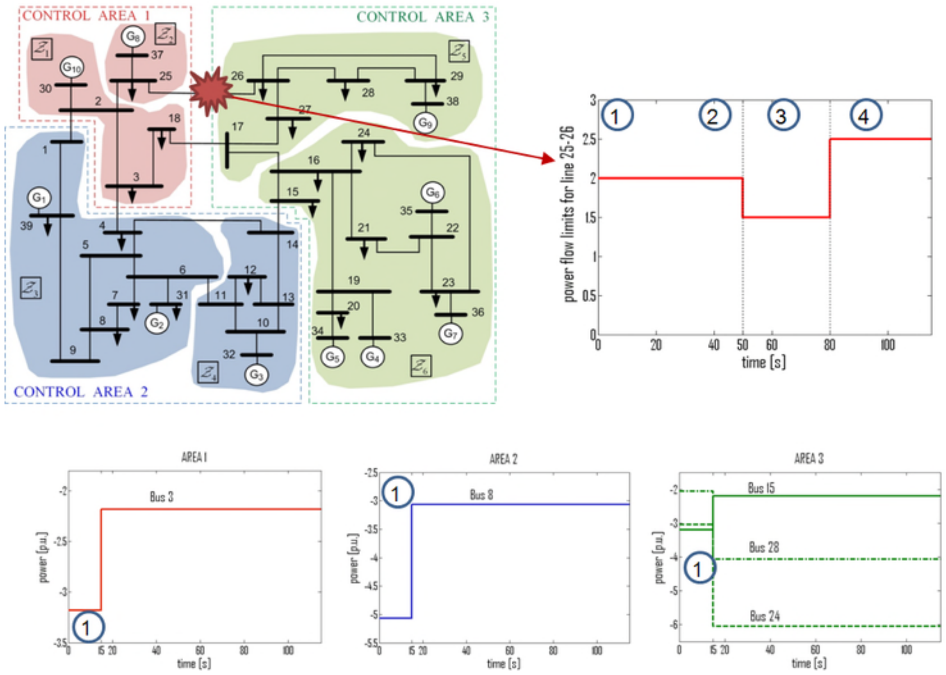
$$\lambda_{\mathcal{Z}} = \begin{pmatrix} \boxed{F(\cdot)} & 0 & E \end{pmatrix} \begin{pmatrix} x_\lambda \\ x_\mu \\ x_\sigma \end{pmatrix}, \qquad \Delta p = \Upsilon(\lambda_{\mathcal{Z}})$$

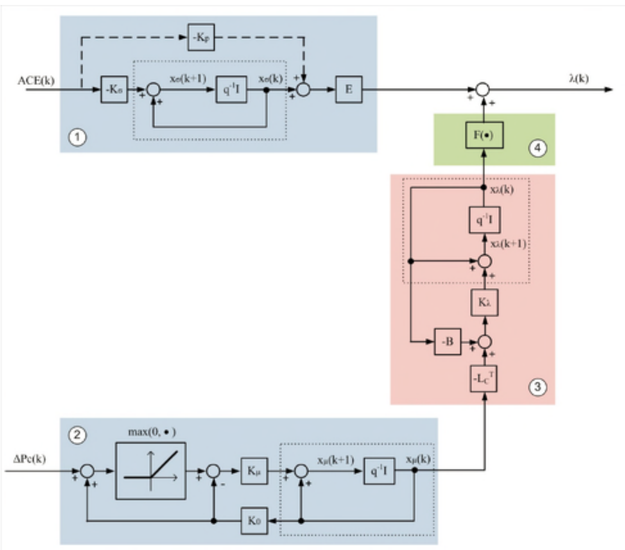
Distributed, real-time, price-based congestion control

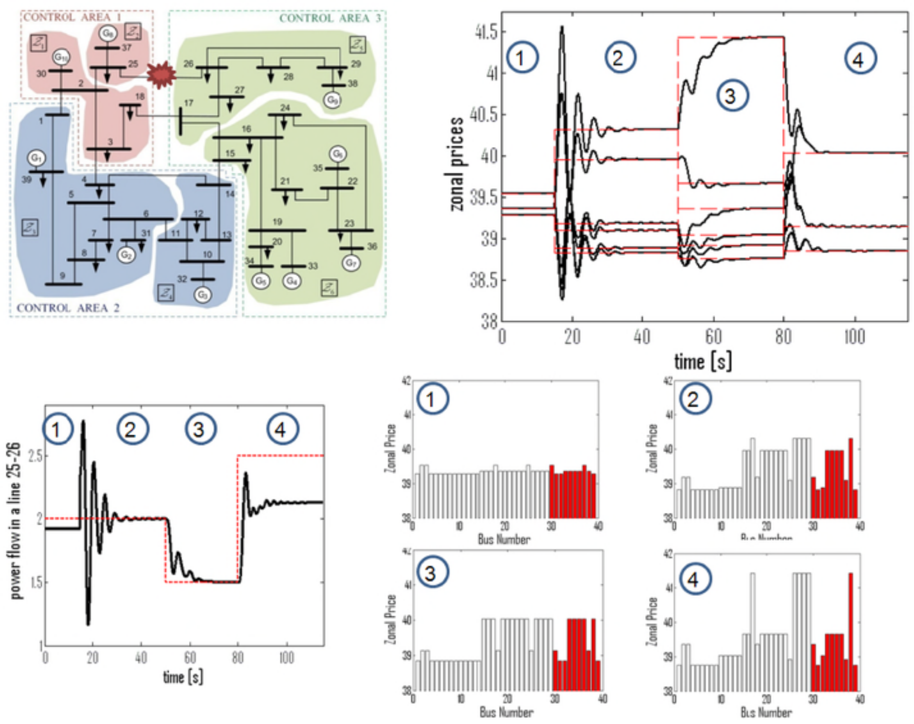




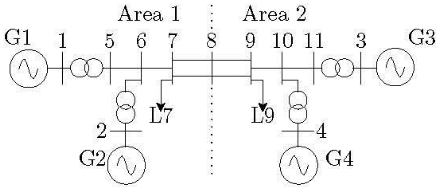
EXAMPLE

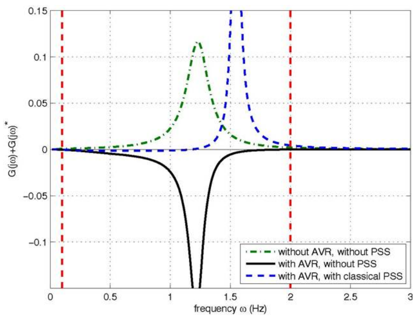


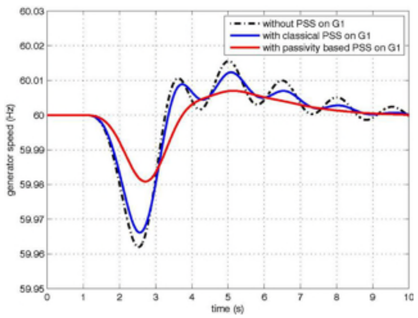


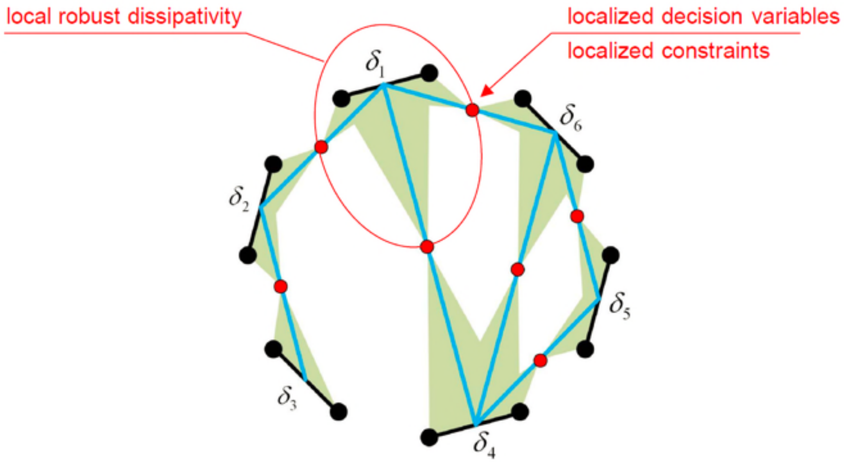


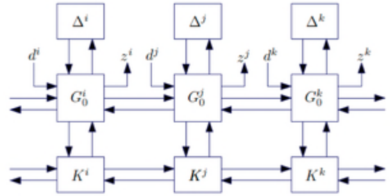
More on real-time distributed control

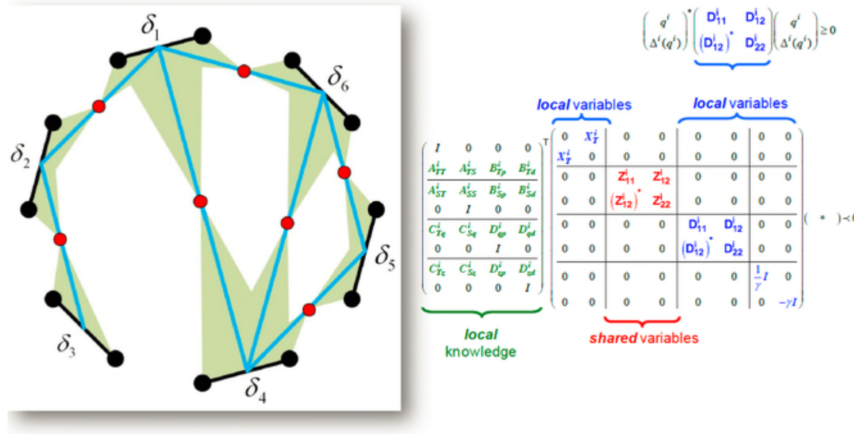




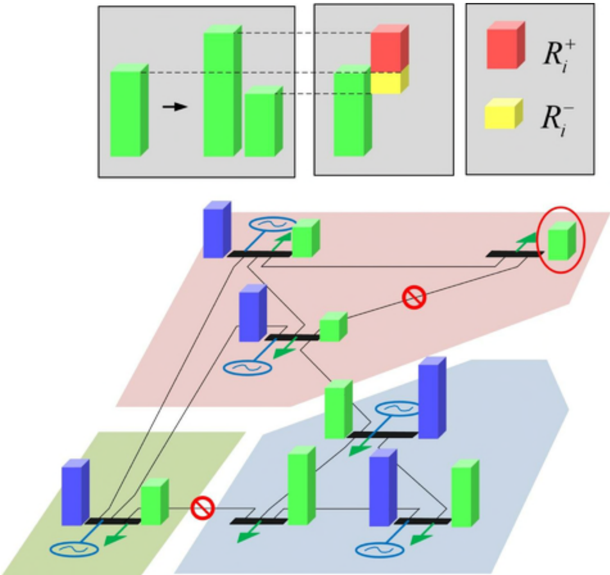


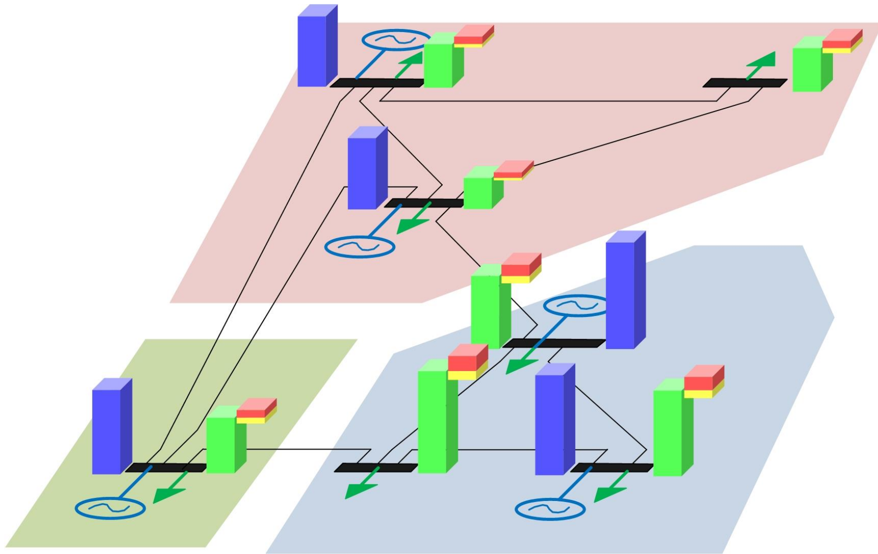


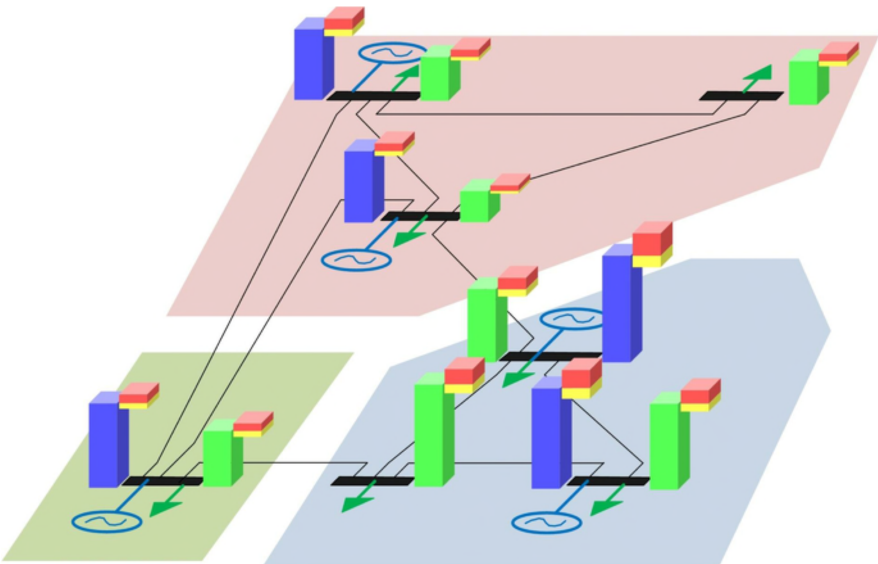


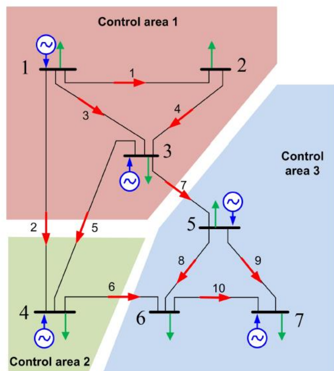


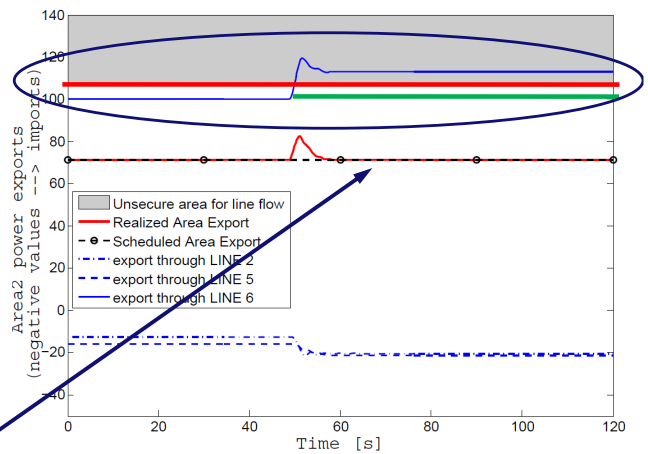
Market-based robust spatial distribution of ancillary services











In current system, reliability is accounted for in "aggregated" form here

RELIABILITY MARGIN

Size of reliability margin: reliability vs. efficiency trade-off

Economically optimal working point is often on the border of feasible region

Problem definition

Robust congestion constraints

The participation function

$$f(t) = \gamma(\tilde{a}^+(k), \tilde{a}^-(k), q(t))$$

$\tilde{a}^+(k)$ = purchased and allocated up-regulating AS
$\tilde{a}^-(k)$ = purchased and allocated down-regulating AS
$\tilde{a}^+(k)$ and $\tilde{a}^-(k)$ are **vectors defining spatial distribution of AS**

Uncertainty model

$$q(t) \in \tilde{\mathcal{Q}}(k) = \{ q \mid q = \tilde{R}(k)w,\ w \in \tilde{\mathcal{W}}(k) \subset \mathbb{R}^m \}$$
$$\tilde{\mathcal{W}}(k) = \mathrm{conv}\{\tilde{w}_1(k), \ldots, \tilde{w}_T(k)\}, \qquad 0 \in \tilde{\mathcal{W}}(k)$$

Robust congestion constraints

$$L\delta \le \Delta\tilde{l}(k) \qquad \text{for all} \quad \delta \in \tilde{\mathcal{D}}(k) \text{ where}$$
$$\tilde{\mathcal{D}}(k) := \{\delta \mid \begin{array}{l} \tilde{R}(k)w + \gamma\left(\tilde{a}^+(k), \tilde{a}^-(k), \tilde{R}(k)w\right) = B\delta, \\ w \in \tilde{\mathcal{W}}(k) \end{array} \}$$

The participation function $f(t) = \gamma(\tilde{a}^+(k), \tilde{a}^-(k), q(t))$

- **structure**: defined by the real-time secondary control scheme
- **parameters**: defined by $\tilde{a}^+(k), \tilde{a}^-(k)$ = the AS market clearing results

Example

Participation vectors:

$$\tilde{\alpha}^+(k) := \tilde{a}^+(k)\frac{1}{\sum_i \tilde{a}_i^+(k)}, \quad \tilde{\alpha}^-(k) := \tilde{a}^-(k)\frac{1}{\sum_i \tilde{a}_i^-(k)}$$

Real-time SC controller of a area:

$$f_{\mathcal{A}_i}(t) = \begin{cases} -\tilde{\alpha}^+_{\mathcal{A}_i} k_I \int ACE_i(t)dt & \text{for } \int ACE_i(t)dt \le 0 \\ -\tilde{\alpha}^-_{\mathcal{A}_i} k_I \int ACE_i(t)dt & \text{for } \int ACE_i(t)dt > 0 \end{cases}$$

The participation function

$$f(t) = \gamma(\tilde{a}^+(k), \tilde{a}^-(k), q(t)) = -\tilde{\alpha}^+(k)\min(\mathbf{1}^\top q(t), 0) + \tilde{\alpha}^-(k)\max(\mathbf{1}^\top q(t), 0)$$

AS market clearing problem

For a time instant k on energy time scale

Input

- AS bids: $\beta_i^+(a_i^+, k)$, $\beta_i^-(a_i^-, k)$ $\quad\to\quad$ deduce objective functions
- Uncertainties (spatial distribution): $\mathcal{Q}(k)$

Market clearing problem (optimal spatial distribution of AS)

$$\min_{a^+, a^-, \{\delta_t\}_{t\in\{1,\ldots,T\}}} \sum_{i=1}^N \left(J_i^+(a_i^+) + J_i^-(a_i^-)\right), \qquad \text{(max socail welfare)}$$

subject to

$$\boxed{\gamma(a^+(k), a^-(k), q_t) + q_t = B\delta_t,\ t = 1, \ldots, T} \qquad \textbf{(spatial info.)}$$
$$L\delta_t \le \Delta l, \quad t = 1, \ldots, T \qquad \text{(robust congestion constraints)}$$
$$\sum_i a_i^+ = r^+ \qquad \text{(required AS+ accomulation)}$$
$$\sum_i a_i^- = r^- \qquad \text{(required AS- accomulation)}$$

Nodal prices solution

Lagrangian

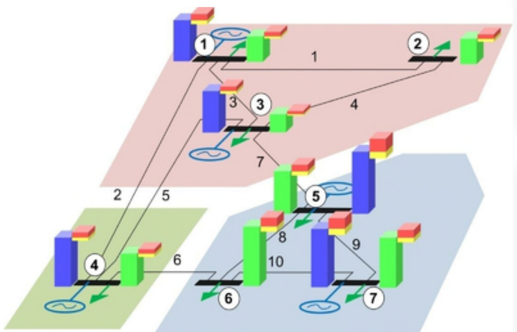
$$\mathcal{L} = \sum_{i=1}^N \left(J_i^+(a_i^+) + J_i^-(a_i^-)\right)$$
$$+ \sum_{t=1}^T \mu_t^\top (L\delta_t - \Delta l) + \sum_{t=1}^T \tau_t^\top \left(\gamma(a^+(k), a^-(k), q_t) + q_t - B\delta_t\right)$$
$$+ (\sigma^+)^\top \left(\sum_i a_i^+ - r^+\right) + (\sigma^-)^\top \left(\sum_i a_i^- - r^-\right)$$

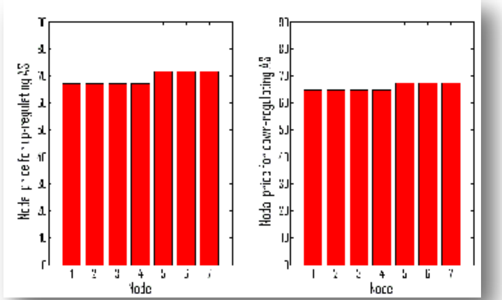
Optimal AS nodal prices

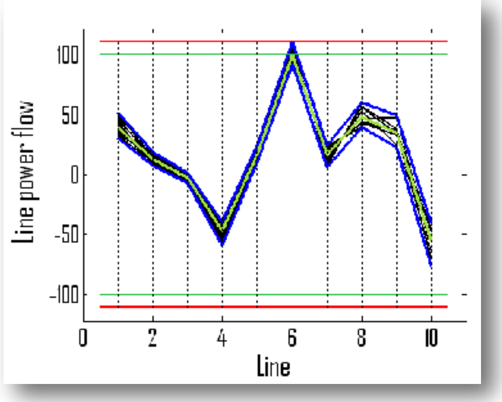
$\overline{q}^+ := \min(\{\mathbf{1}^\top q_t\}_{t=1,\ldots,T}, 0)$, $\overline{q}^- := \max(\{\mathbf{1}^\top q_t\}_{t=1,\ldots,T}, 0)$, $z_t^+ := \mathbf{1}\frac{\overline{q}^+}{r^+}$, $\quad z_t^- := \mathbf{1}\frac{\overline{q}^-}{r^-}$

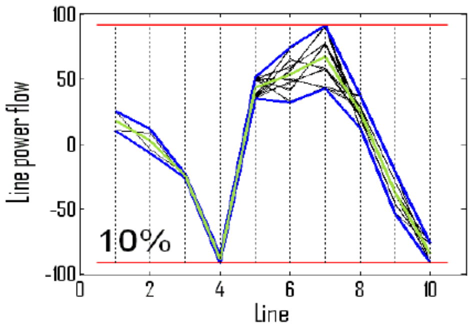
$$\lambda^+ = -\mathbf{1}\tilde{\sigma}^+ + \sum_{t=1}^T \tilde{\tau}_t \circ z_t^+, \quad \lambda^- = -\mathbf{1}\tilde{\sigma}^- + \sum_{t=1}^T \tilde{\tau}_t \circ z_t^-$$

Robustly optimal AS spatial distribution: $\beta^+(a^+) = \lambda^+$, $\quad \beta^-(a^-) = \lambda^-$.

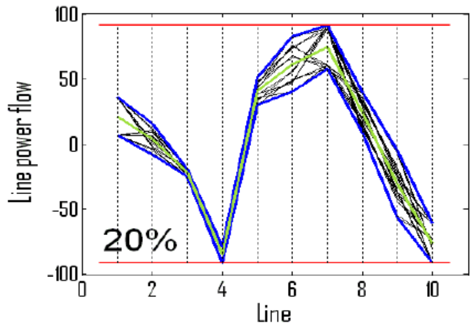




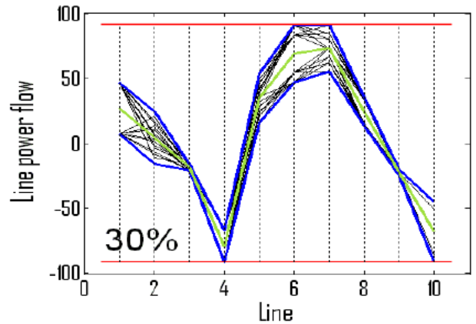




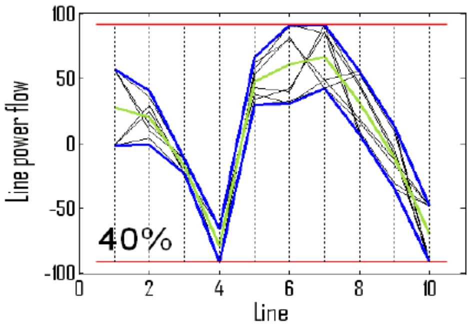
(a) Power flows for 10% uncertainty level.



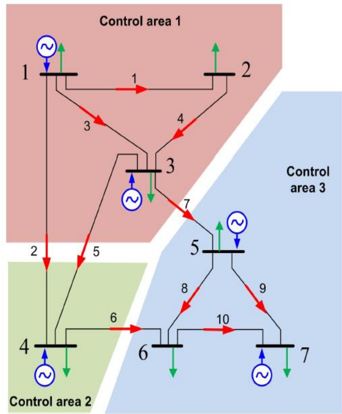
(b) Power flows for 20% uncertainty level.

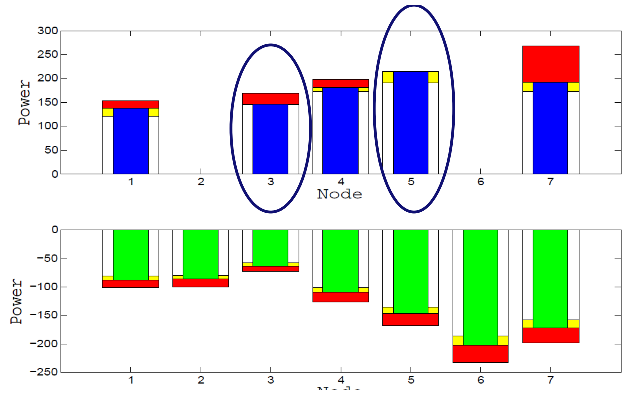


(c) Power flows for 30% uncertainty level.



(d) Power flows for 40% uncertainty level.

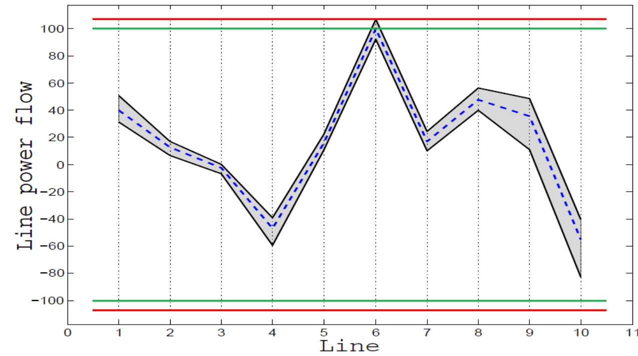




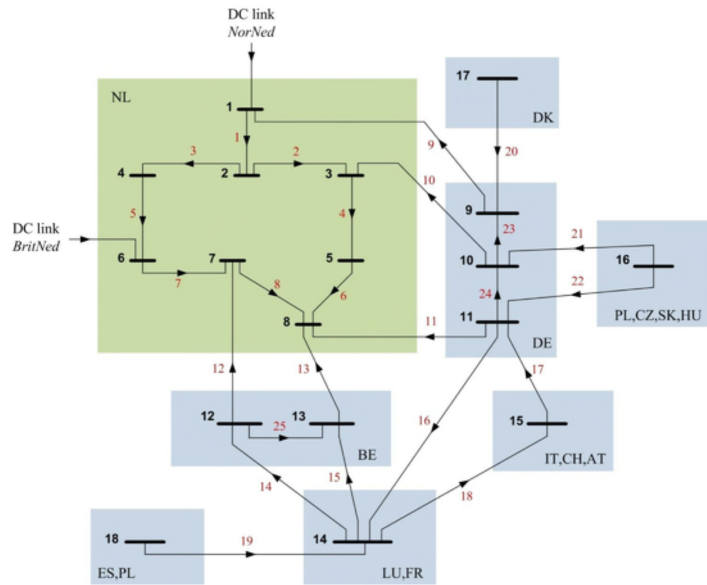
Spatial distribution of AS:
Shaping the "uncertainty tube"→

Get reliability for best costs

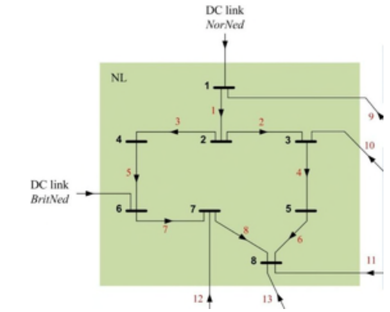
Possible to include optimal cooperation between control areas

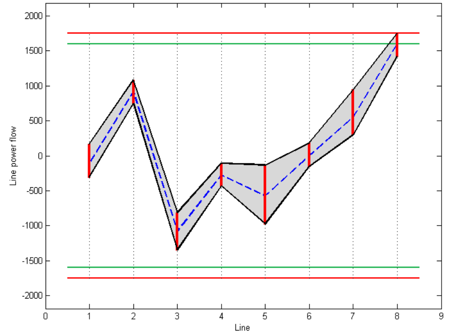


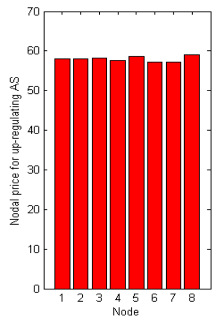
The E-Price benchmark model

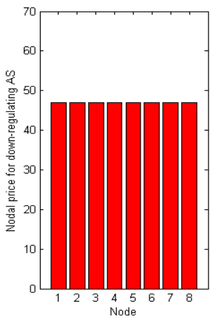


Locational prices for ancillary services









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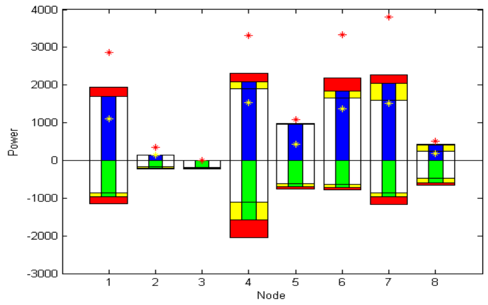
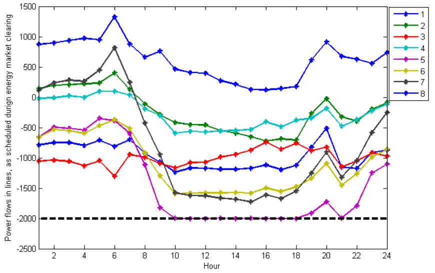
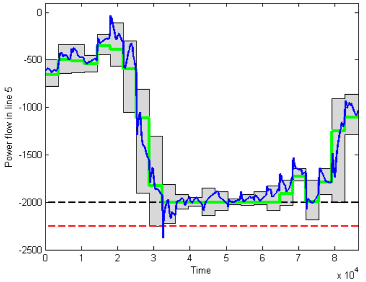
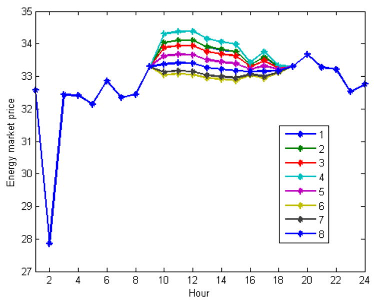
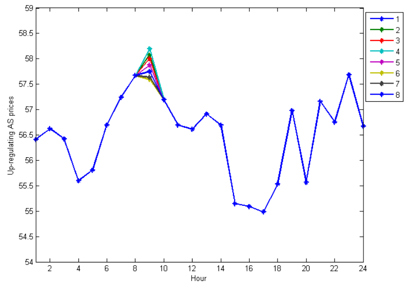


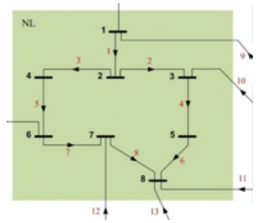


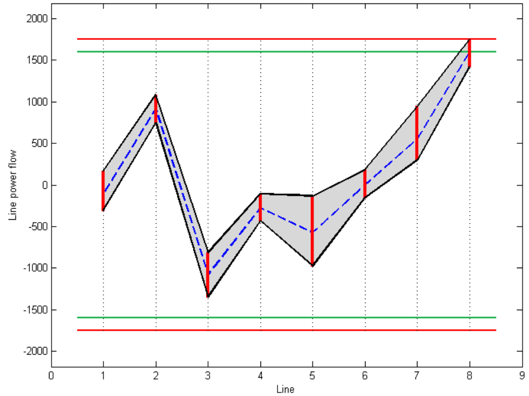


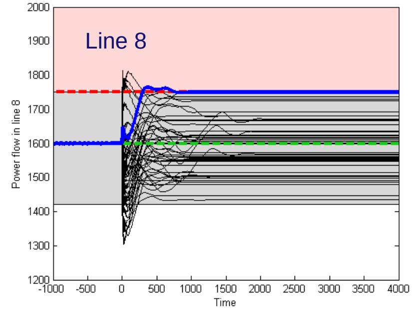


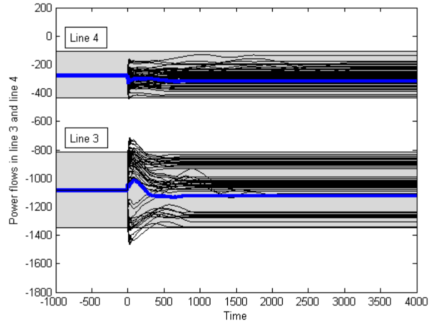


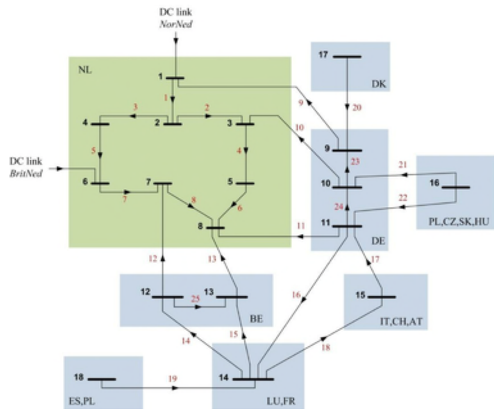
Optimized uncertainty in line power flows

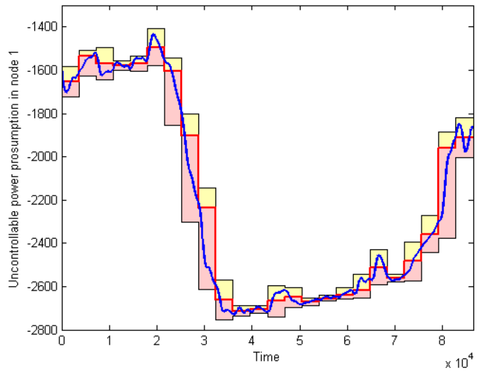


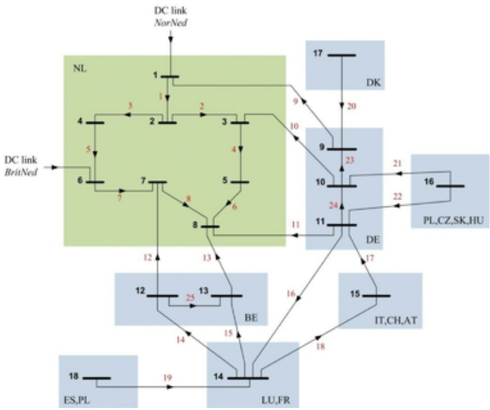


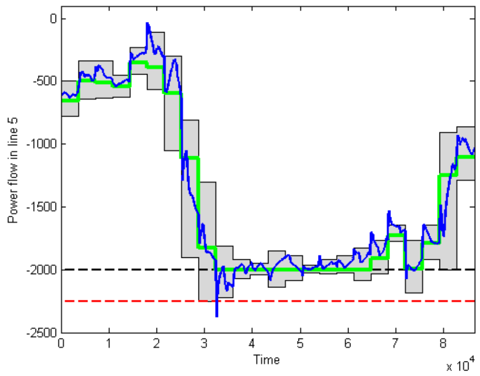


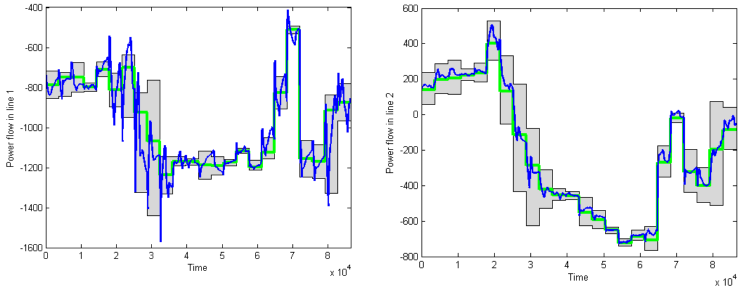


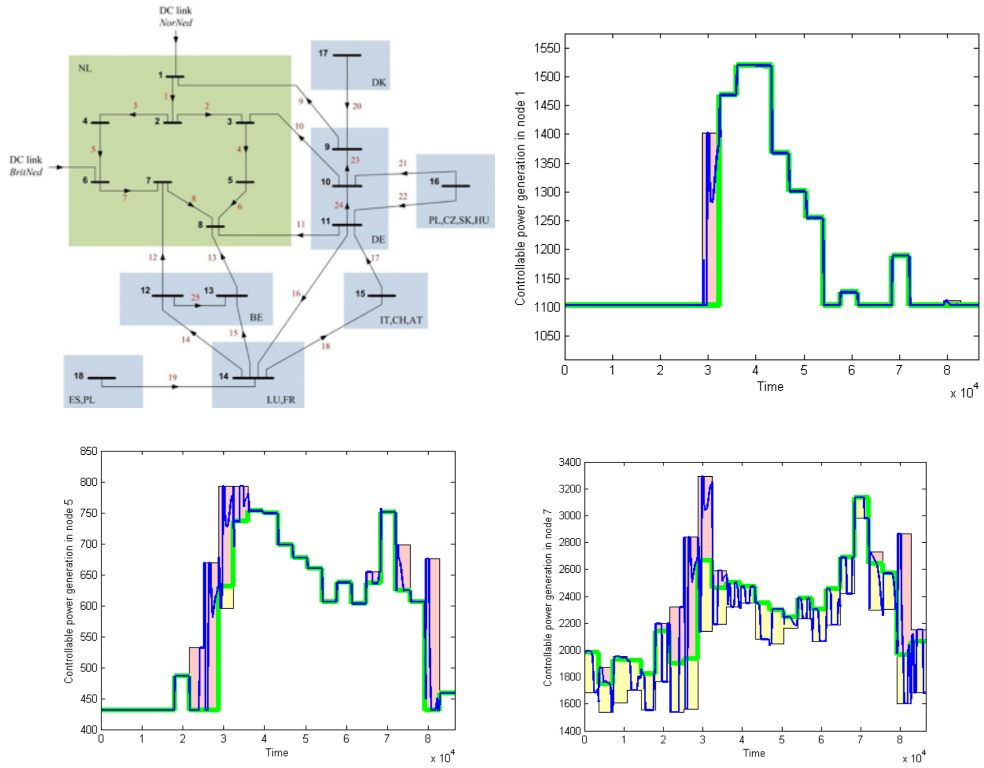


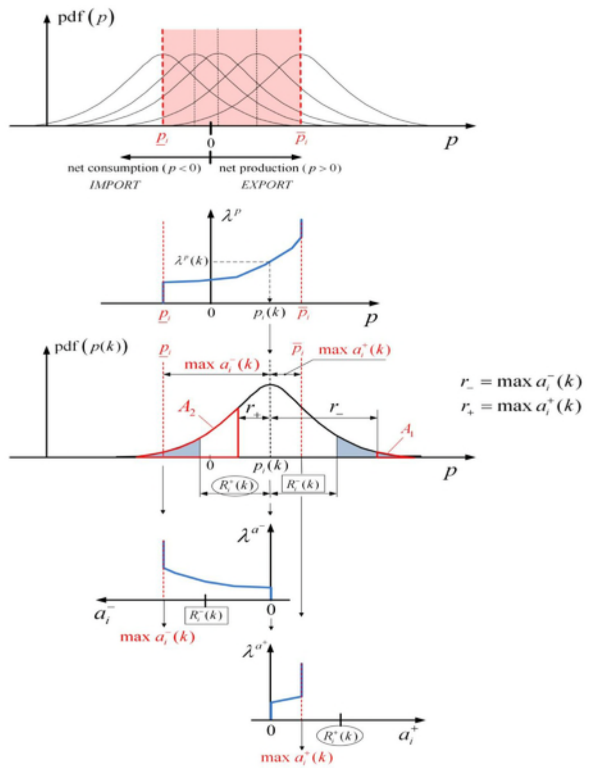




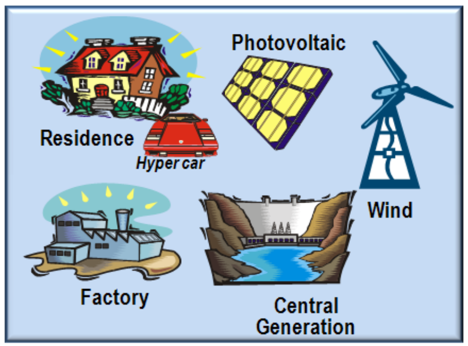


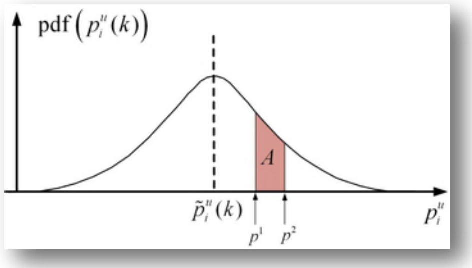


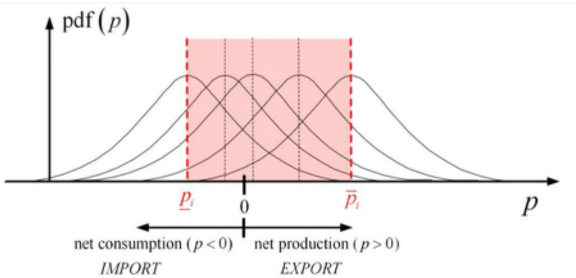




Double sided Ancillary Services (AS) markets







- Employ controllable prosumers in its own portfolio for keeping up the contracted prosumption level
- Buy/sell options on double-sided AS markets

Conclusions and messages

- Today's robustness: partly due to conservative engineering
- Future: increased complexity. Robustness (fragility?), efficiency, scalability?
- Exploit the networking! (often neglected in research)
- smart? better understood, explained: hidden (technology), invisible (hand of market)
- think in terms of modules (plug and play), protocols and architecture
- Optimization (duality!): holistic approach to market (and control)
- Huge area for important research (exciting parallel research in control systems field)

www.e-price-project.eu

www.fsb.hr/ConDis

Power Systems Control
Discussion of **Future** Research Topics

Florian Dörfler   Andrej Jokić

dutch institute of systems and control

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

University of Zagreb

We talked about a whole range of topics

"Power Systems Control – from Circuits to Economics"

All these topics have been expensively studied in the past, and they remain important in the future — possibly with a different emphasis:

- increasing uncertainty in generation
- deregulated markets & pricing schemes
- more and more power electronics sources
- new technologies for sensing/comm/actuation
- new elasticity in demand and batteries
- advances in distributed control & optimization
- . . .

Other very important topics that we did not touch upon

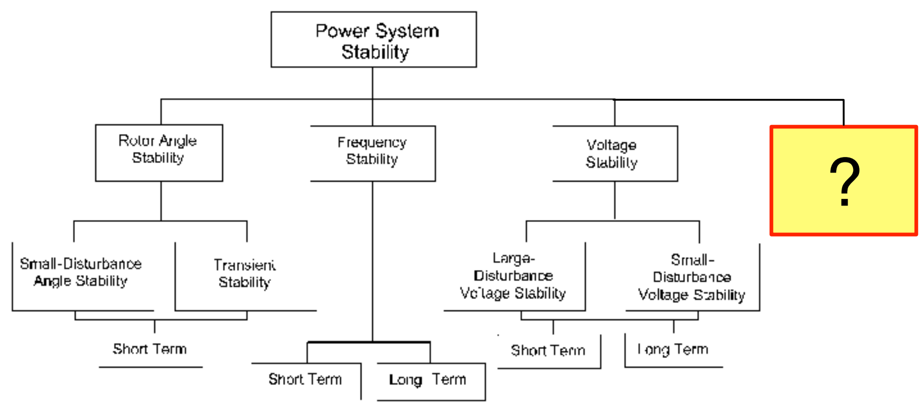
- **wide-area estimation:** PMUs, load identification, etc.
- **DC components** in HVDC transmission, microgrids, etc.
- **power system optimization** using latest start of the art tools
- role of **battery storage** for balancing
- **load control & demand response**
  (vehicle charging, thermostatically-controlled loads, etc.)

"There are more papers on electric vehicles than there are electric vehicles out there."

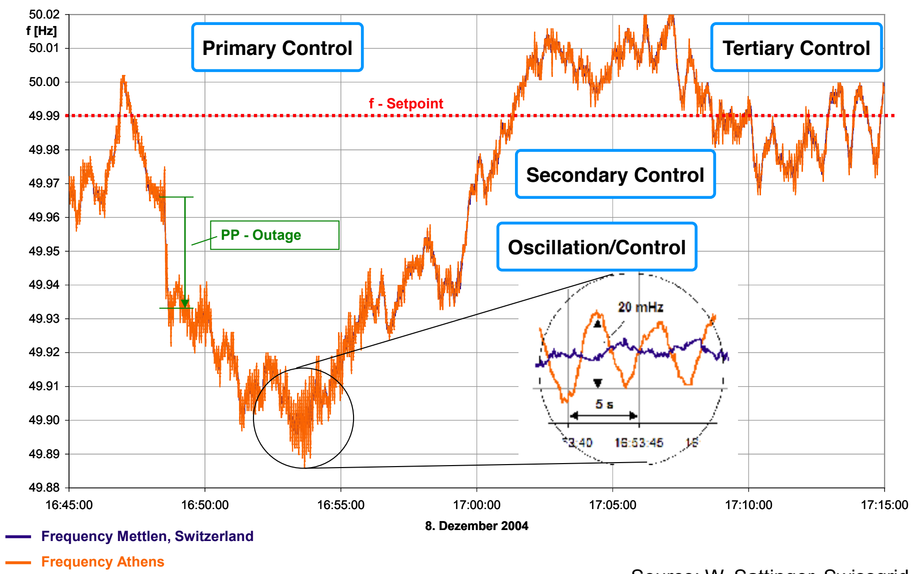
— [Alejandro Garcia-Domingiez, Allerton '15]

Remember? — to be resolved on the last day

the very near future (actually today) holds a new (and very dominant) stability issue



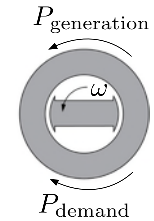
A little summary of almost everything we talked about



Source: W. Sattinger, Swissgrid

System operation centered around synchronous generators

At the beginning was Tesla with the **synchronous machine**:



$$M \frac{d}{dt}\omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

change of kinetic energy = instantaneous power balance

The **AC power grid** has been designed around synchronous machines.

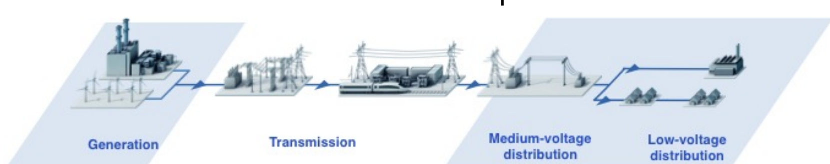
All of **power system operation** has been designed around them as well.

Recently: increasing renewables = retiring synchronous machines

Recall: a few (of many) game changers

synchronous generator | new workhorse | scaling

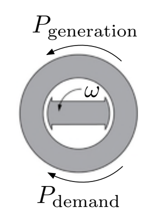
location & distributed implementation



Almost all operational problems can principally be resolved . . . **but one (?)**

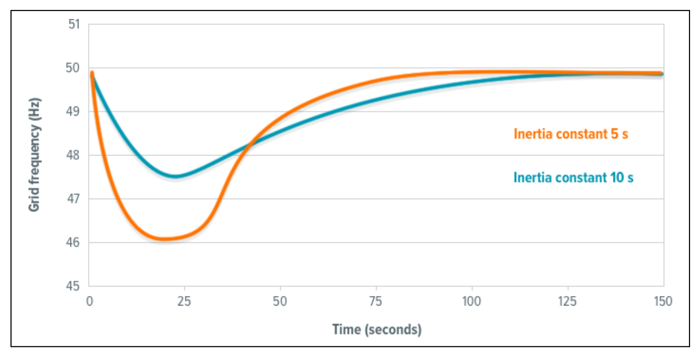
Fundamental challenge: operation of low-inertia systems

We slowly loose our giant electromechanical low-pass filter:

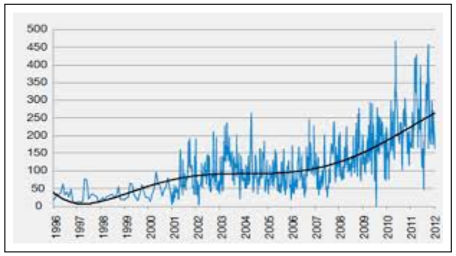


$$\mathbf{M} \frac{d}{dt}\omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$

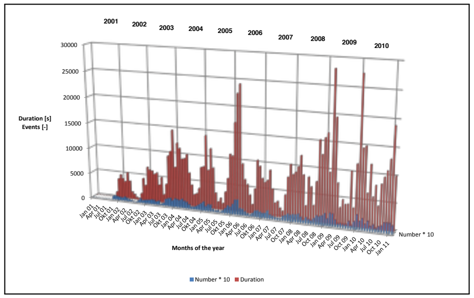
change of kinetic energy = instantaneous power balance



Low-inertia stability = true # 1 problem with renewables



frequency violations in Nordic grid (source: ENTSO-E)

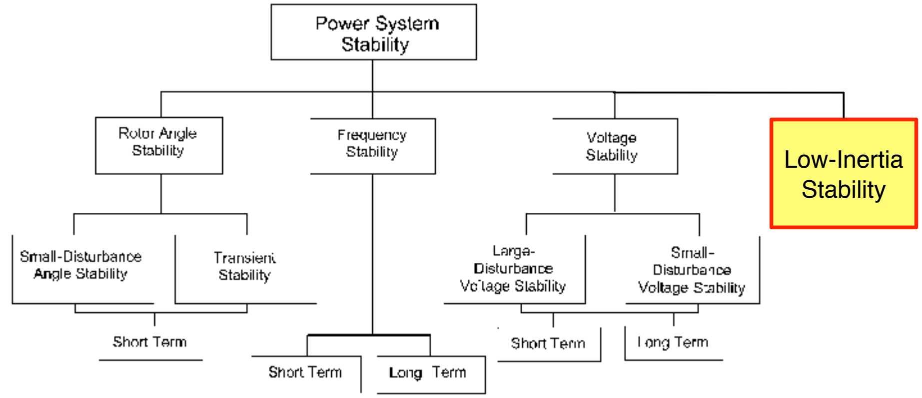


same in Switzerland (source: Swissgrid)

inertia is shrinking, time-varying, & localized, . . . & increasing disturbances

Solutions in sight: none really . . . other than **emulating virtual inertia** through fly-wheels, batteries, super caps, HVDC, demand-response, . . .

Resolution — the dominant future stability issue



Virtual inertia emulation


Improvement of Transient Response in Microgrids Using Virtual Inertia

Nimish Soni, Student Member, IEEE, Suryanarayana Doolla, Member, IEEE, and Mukul C. Chandorkar, Member, IEEE

Implementing Virtual Inertia in DFIG-Based Wind Power Generation

...mmadreza Fakhari Moghaddam Arani, Student Member, IEEE, and Ehab F. El-Saadany, Senior Member, IEEE

Virtual synchronous generators: A survey and new perspectives

Hassan Bevrani, Toshifumi Ise, Yushi Miura

Dynamic Frequency Control Support: a Virtual Inertia Provided by Distributed Energy Storage to Isolated Power Systems

...authier Delille, Member, IEEE, Bruno François, Senior Member, IEEE, and Gilles Malarange

Inertia Emulation Control Strategy for VSC-HVDC Transmission Systems

Jiebei Zhu, Campbell D. Booth, Grain P. Adam, Andrew J. Roscoe, and Chris G. Bright

Grid Tied Converter with Virtual Kinetic Storage

M.P.N van Wesenbeeck, S.W.H. de Haan, Senior member, IEEE, P. Varela and K. Visscher

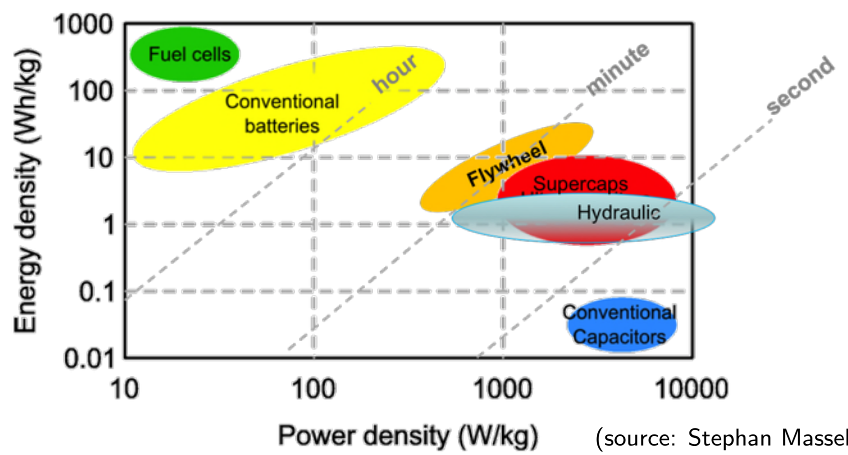
$$M \frac{d}{dt}\omega(t) = P_{\text{generation}}(t) - P_{\text{demand}}(t)$$ . . . essentially **D-control**

☺ decentralized & plug-and-play (passive mechanical loop)

☹ suboptimal, wasteful in control effort, & need for new actuators

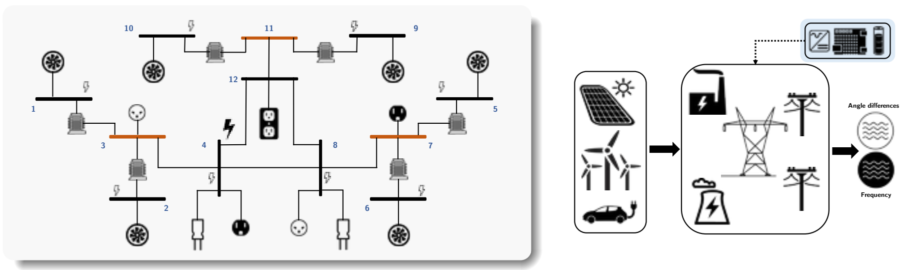
Classification & choice of actuators

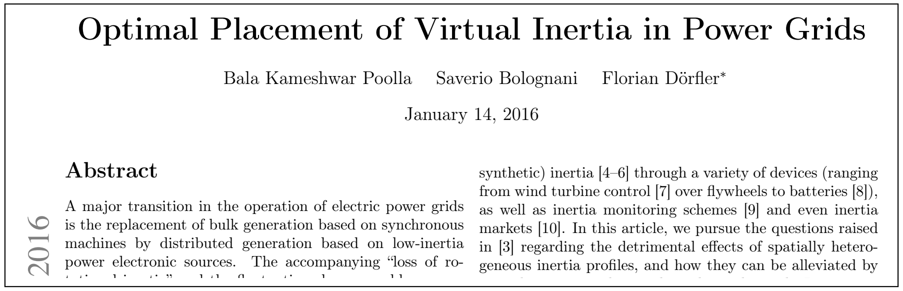
Feasibility: what are the key actuators to emulate inertia or other transient control approaches? (how) can this be realized in large?



(source: Stephan Masselis)

It actually matters **where** you emulate inertia!




Optimal Placement of Virtual Inertia in Power Grids

Bala Kameshwar Poolla Saverio Bolognani Florian Dörfler*

January 14, 2016

2016

Abstract

A major transition in the operation of electric power grids is the replacement of bulk generation based on synchronous machines by distributed generation based on low-inertia power electronic sources. The accompanying "loss of ro-

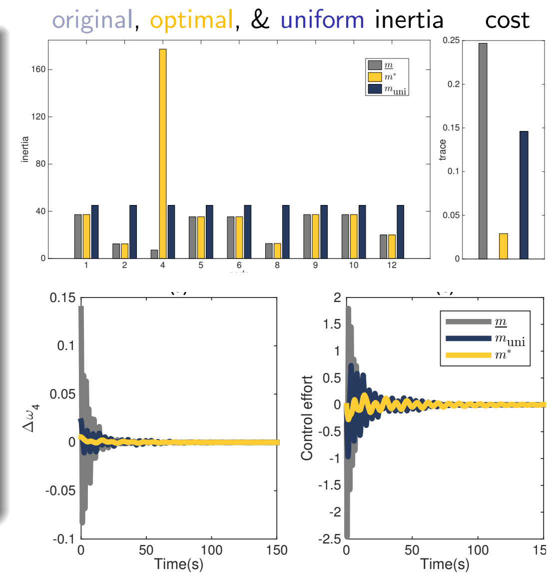
synthetic) inertia [4–6] through a variety of devices (ranging from wind turbine control [7] over flywheels to batteries [8]), as well as inertia monitoring schemes [9] and even inertia markets [10]. In this article, we pursue the questions raised in [3] regarding the detrimental effects of spatially heterogeneous inertia profiles, and how they can be alleviated by

Heuristics outperformed by $\mathcal{H}_2$ - optimal allocation

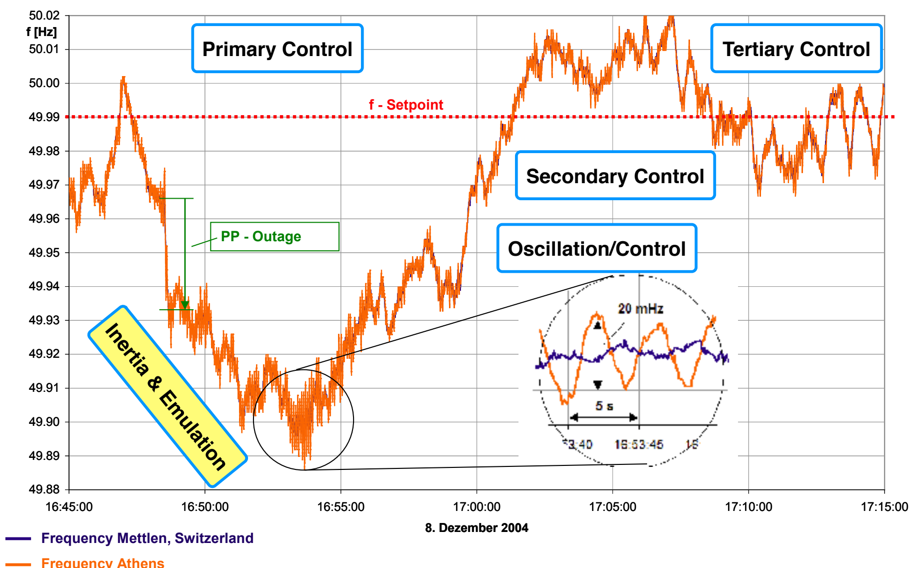
Scenario: disturbance at #4

- locally optimal solution **outperforms heuristic** uniform allocation
- optimal allocation ≈ **matches disturbance**
- inertia emulation at all undisturbed nodes is actually **detrimental**

⇒ **location** of disturbance & inertia emulation matters



An updated summary of almost everything we talked about



Source: W. Sattinger, Swissgrid

A control perspective of almost everything we talked about

Classic power electronics control: **emulate generator physics & control**

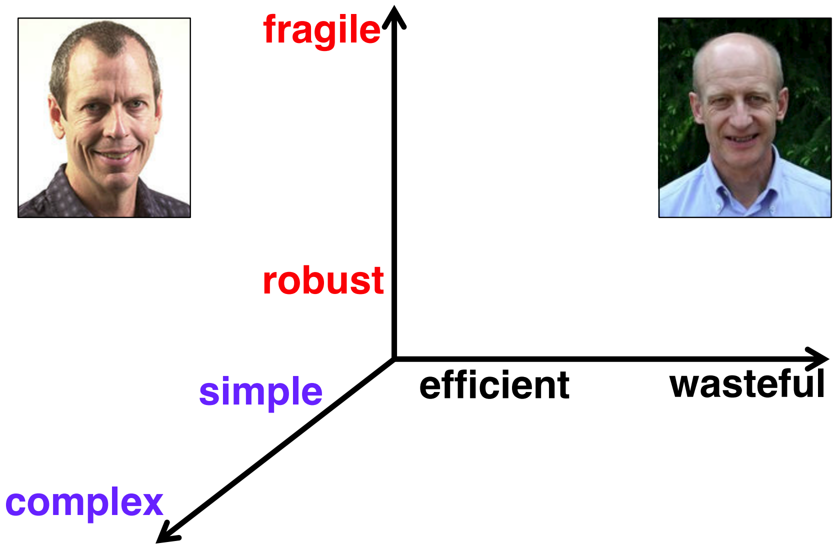
$$\underbrace{M\dot{\omega}(t)}_{\text{(virtual) inertia}} = \underbrace{P_{\text{mech}}}_{\text{tertiary control}} - \underbrace{D\omega(t)}_{\text{primary control}} - \underbrace{\int_0^t \omega(\tau)\, d\tau}_{\text{secondary control}} - P_{\text{elec}}$$

Essentially all **PID + setpoint control** (simple, robust, & scalable)

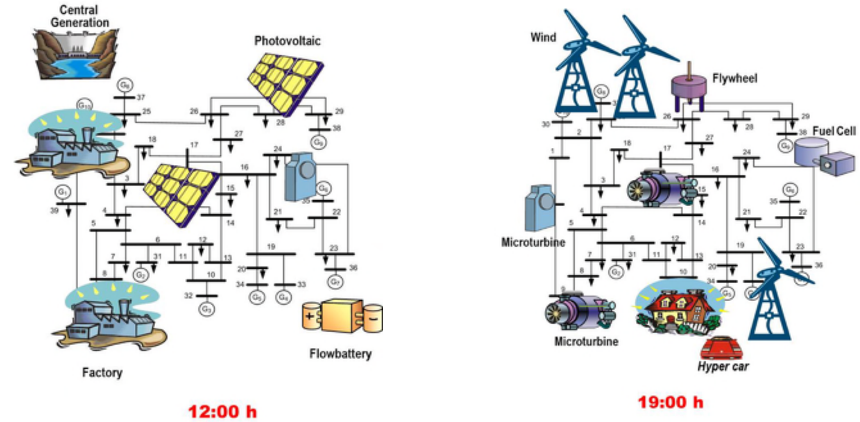
$$\underbrace{M\dot{\omega}(t)}_{\text{D}} = \underbrace{P}_{\text{set-point}} - \underbrace{D\omega(t)}_{\text{P}} - \underbrace{\int_0^t \omega(\tau)\, d\tau}_{\text{I}} - P_{\text{elec}}$$

Control engineers should be able to do better ...

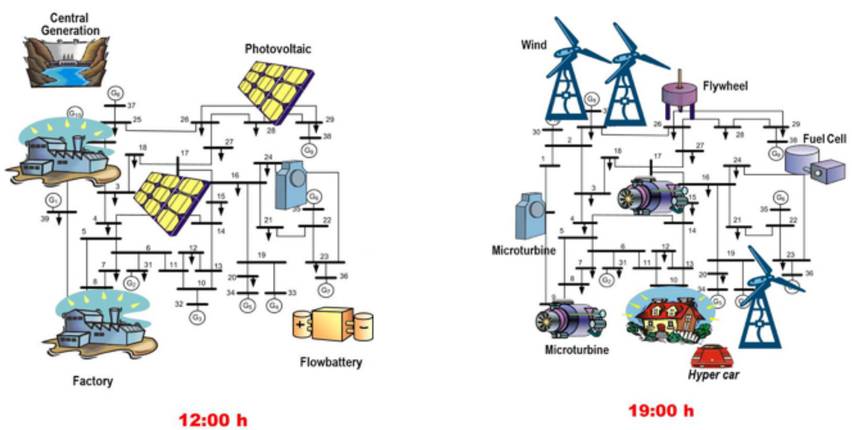
When searching for solutions remember John and Göran



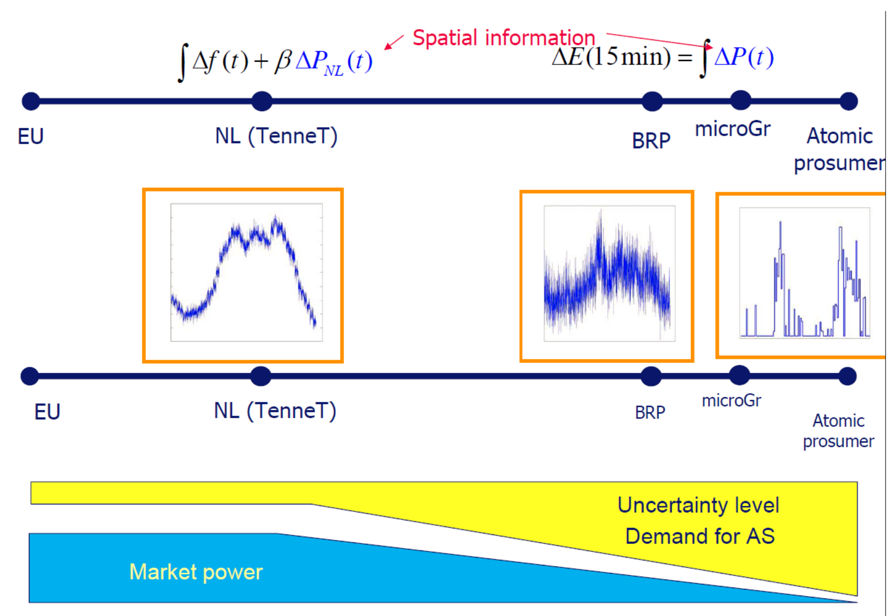
The business case



- Who and how keeps track of system-wide inertia level and its spatial distribution? How to schedule / monitor / bring it “online” / bill?
- Inertia as market commodity? Or obligation? Who buys? Single sided market? Double sided markets for balancing? (Why should I buy a flywheel or install more complex control on my wind turbine?)



- from predictability and **repetitiveness** to uncertainty
- Power flow volatility. Trade-off: spatial resolution versus aggregation of uncertainties. Challenge: Exploit the networking! (old idea, currently often neglected in research). How to manage uncertainity on global (EU) level?



From macroscopic to "atomic" world and back

- There is a benefit from aggregation: BRPs as building blocks on macro-scale with good incentives. Good incentives for atomic end-users?
- Challenge: Economical incentives and built-in feedbacks for "good level of" localisation of "desirable macroscopic properties" (inertia, controllable primary and secondary power). "Good level" $\leftarrow$ exploit the networking by mastering and controlling inherent trade-offs
- Challenge: Solution architecture is crucial ("hidden" and "invisible": local incentives form global behaviour), together with well defined modules as open systems with well defined protocols and distributed information / algorithms.

the end