ETH zürich

Control and Optimization in Smart Power Grids

INCITE Seminar @ Universitat Politècnica de Catalunya

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Complex Control Systems Group

Energy Science Center

ETH Foundation Zürich

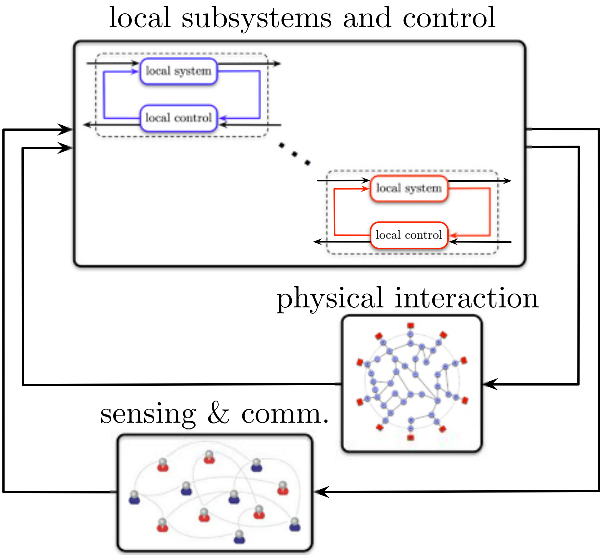
FNSNF
Fonds national suisse
Schweizerischer Nationalfonds
Fondo nazionale svizzero
Swiss National Science Foundation

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

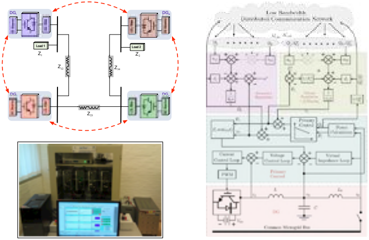
HORIZON 2020

ABB AIT Austrian Institute of Technology

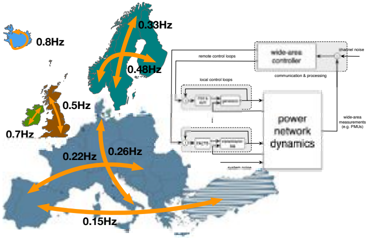
Background: distributed control and optimization



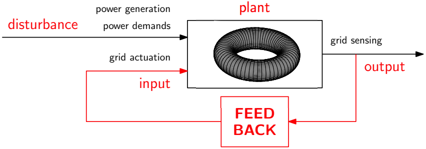
Project samples in power systems



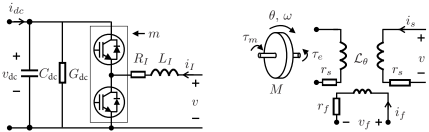
plug-and-play control in microgrids



decentralized wide-area control



feedback online optimization **(now)**



control in low-inertia systems **(later)**

Distributed Control and Optimization in Smart Power Grids

Acknowledgements:

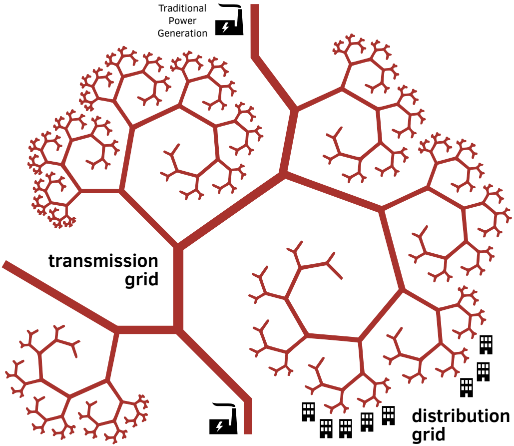
Adrian Hauswirth

Saverio Bolognani

Gabriela Hug

Further project collaborators: A. Zanardi, J. Pázmány, E. Arcari, E. Dall'Anese

How are power systems operated?



- **objective:** deliver power from generators to loads (typically time-varying & uncertain)

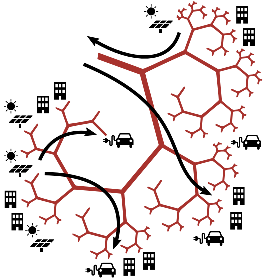
  supply chain without storage

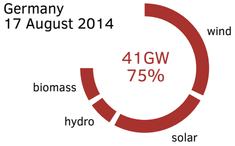
- **physical constraints:** Kirchhoff's and Ohm's laws
- **operational constraints:** thermal and voltage limits, ...
- **specifications:** running costs, reliability, quality of service

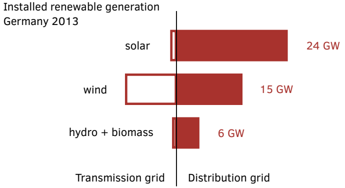
New challenges and opportunities

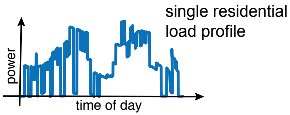
- **fluctuating renewable sources**
  - poor short-range prediction
  - correlated uncertainty

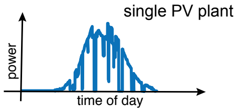
- **distributed microgeneration**
  - conventional and renewable sources
  - congestion (in urban grids)
  - under-/over-voltage (in rural grids)



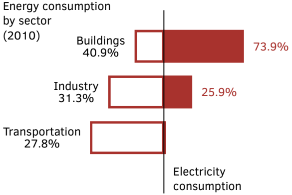


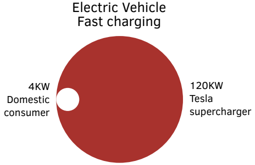






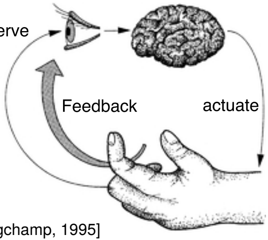
New challenges and opportunities cont'd





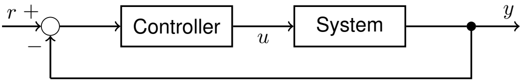
- **electric mobility**
  - flexible demand
  - large peak (power) and total (energy) demand
  - spatio-temporal patterns
- **information and communication technology**
  - inexpensive reliable communication
  - increasingly ubiquitous sensing
- **inverter-based generation**
  - fast actuation
  - control flexibility
  - stability concerns

Recall: feedforward vs. feedback or optimization vs. control



[Longchamp, 1995]

closed-loop $\triangleq$ feedback control



feedback control can achieve

- **no steady-state error**: $r(t) = y(t)$ for $t \to \infty$
- **stability**: bounded output y for bounded input r
- **robustness**: reduce influence of uncertainties & disturbances

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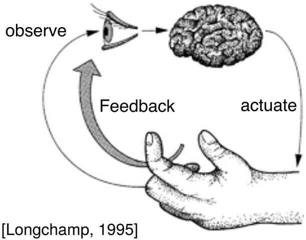

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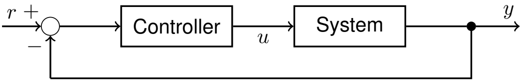
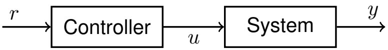
closed-loop ≜ feedback control



open-loop ≜ feedforward optimization



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feedforward optimization can achieve

- transient & asymptotic **optimality**: $\min \int_0^\infty y(t)^2 + u(t)^2 \, dt + \|y(t \to \infty)\|$
- operational **constraints**: $u(t) \in \mathcal{U}$ and $y(t) \in \mathcal{Y}$
- taking into account **forecasts** of reference and disturbance signals

Complementary: feedforward optimization & feedback control

Feedforward optimization

- highly model based
- computationally intensive
- **optimal decision**
- **operational constraints**
- ...

Feedback control

- **model-free (robust) design**
- **fast response**
- suboptimal operation
- unconstrained operation
- ...

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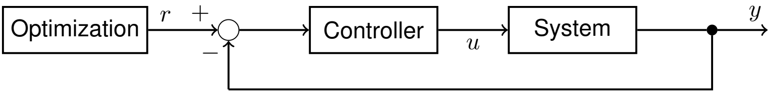
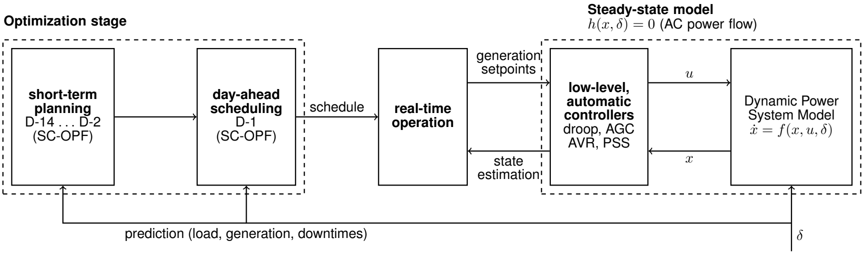
⇒ combine **complementary** operation methods with a **time-scale separation**



offline & feedforward | **real-time & feedback**

Power systems optimization and control architecture



time-scale separation between

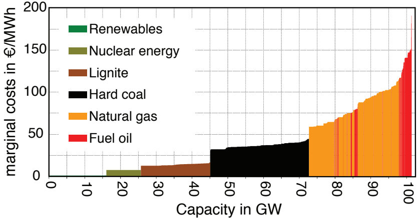
- offline feedforward optimization: SC-OPF, planning, markets, . . .
- real-time feedback control: droop, AGC, AVR, PSS, WAC, . . .

spatial separation: decentralized (PSS) to distributed (WAC) to centralized (OPF)

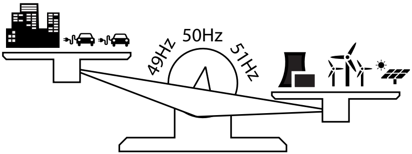
nested and hierarchical operation layers: primary, secondary, tertiary, . . .

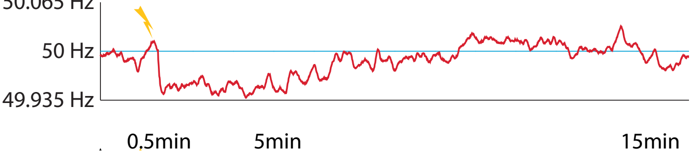
Classic example: balancing

- **optimization phase**
  economic dispatch based on load prediction
- **real-time operation**
  economic re-dispatch, area balancing services
- **local feedback control**
  frequency regulation at the individual generators



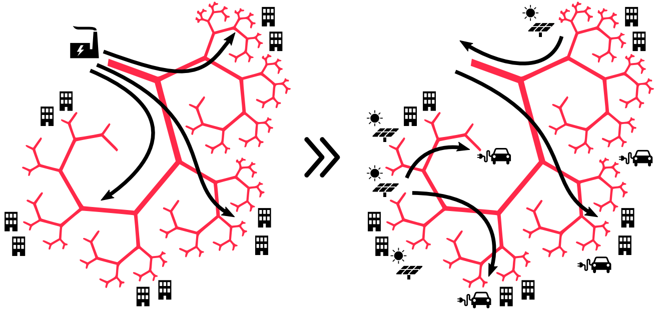
[Elcom/swissgrid, 2010]





[swissgrid, 2010]

Timely recent example: distribution grid congestion



congestion: operation of the grid close or above the physical and operational limits

→ due to simultaneous and uncoordinated distributed generation and demand

→ inefficient, blackouts, curtailment of renewables, bottleneck to electric mobility

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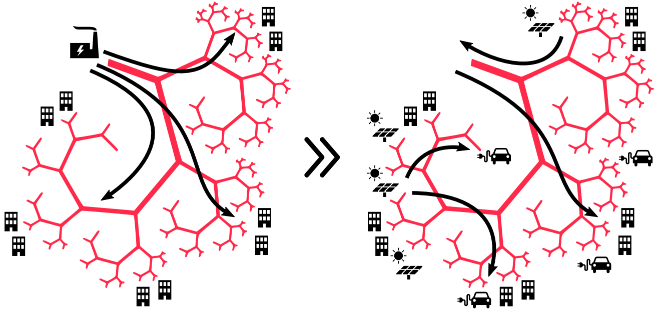

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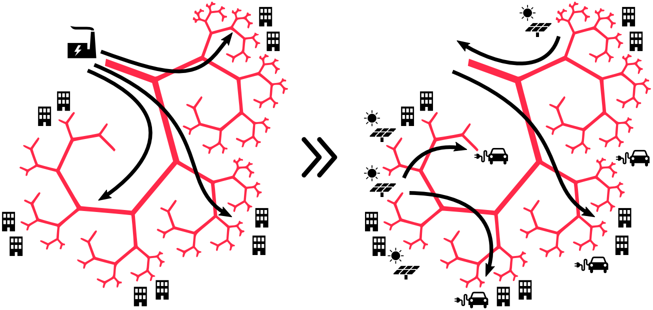

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control & optimization opportunities via ICT, microgeneration, demand response

Ancillary services

- real-time balancing
- frequency control
- economic re-dispatch
- voltage regulation
- voltage collapse prevention
- line congestion relief
- reactive power compensation
- losses minimization

Today: these services are partially automated, implemented independently, online or offline, based on forecasts (or not), and operating on different time/spatial scales.

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Recall new challenges:

- increased variability & uncertainty
- poor short-term prediction

Recall new opportunities:

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- ubiquitous sensing
- reliable communication

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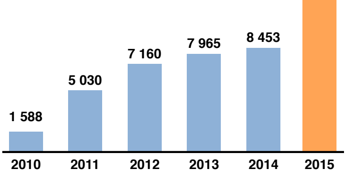
A central paradigm of "smart(er) grids": real-time operation

Future power systems will require faster operation, based on online monitoring and measurement, in order to meet operational specifications in real time.

National & international redispatch

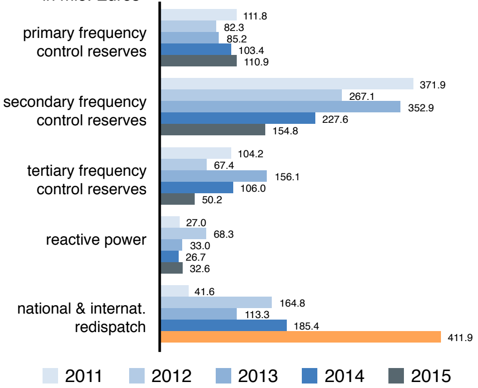
- **unforeseen** congestion or voltage problems
- manually **re-dispatched** on a 15-minute timescale

Redispatch actions in the German transmission grid
in hours



[Bundesnetzagentur, Monitoringbericht 2016]

Cost of ancillary services of German TSOs
in mio. Euros



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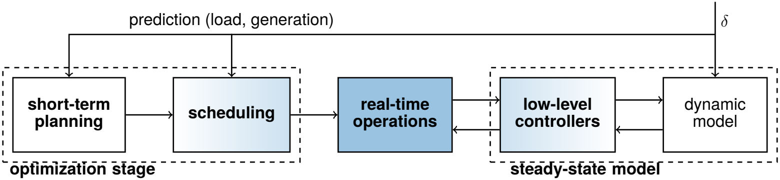
Proposal: online optimization in closed loop



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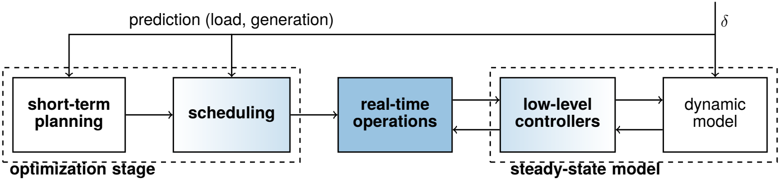

combining optimization & feedback control for real-time operation

- robust (feedback strategy)
- fast response
- steady-state optimality
- satisfaction of operational constraints

disclaimer: no predictive optimization (only for static systems)
focus today on real-time (no distributed) aspects

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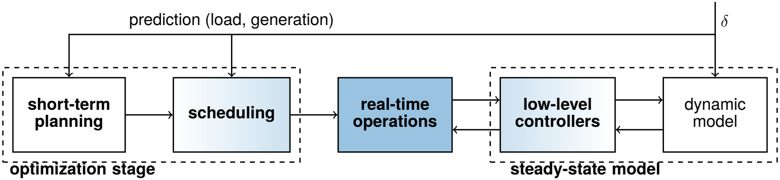

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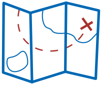
lots of related work: [Bolognani et. al, 2015], [Dall'Anese and Simmonetto, 2016], [Gan and Low, 2016], . . .

A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems

Daniel K. Molzahn,* *Member, IEEE*, Florian Dörfler,† *Member, IEEE*, Henrik Sandberg,‡ *Member, IEEE*, Steven H. Low,§ *Fellow, IEEE*, Sambuddha Chakrabarti,¶ *Student Member, IEEE*, Ross Baldick,¶ *Fellow, IEEE*, and Javad Lavaei,** *Member, IEEE*



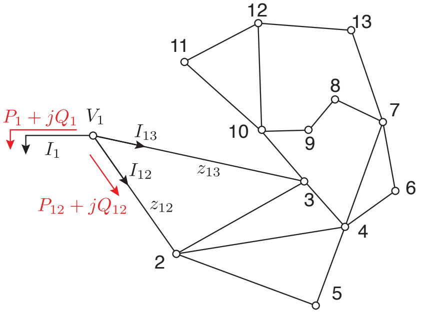
OVERVIEW

1. The power flow manifold, representations, and approximations
2. Projected gradient flow on the power flow manifold
3. Tracking performance and robustness of closed-loop optimization
4. Output feedback and state uncertainty

THE POWER FLOW MANIFOLD, REPRESENTATIONS, AND APPROXIMATIONS

Steady-state AC power flow model

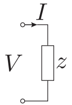
- **quasi-stationary** dynamics $\rightarrow$ complex impedances and voltages
- **sources:** locally controlled $\rightarrow$ buses are *PQ* or *PV* or slack $V\theta$
- **loads:** constant impedance, current, or *PQ* power (today)



V_k nodal voltage
I_k current injection
P_k, Q_k power injections

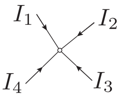
z_{kl} line impedance
I_{kl} line current
P_{kl}, Q_{kl} power flow

Ohm's Law



$V = zI$

Current Law



$0 = I_1 + \ldots + I_k$

AC power $S = P + jQ = VI^*$

AC power flow equations

$$S_k = \sum_{l \in N(k)} \frac{1}{z_{kl}^*} V_k (V_k^* - V_l^*) \quad \forall k \in \mathcal{N}$$

(all variables and parameters are $\mathbb{C}$-valued)

Power flow representations

- **complex form:** $S_k = P_k + jQ_k = \sum_{l \in N(k)} y_{kl}^* V_k \cdot (V_k^* - V_l^*)$ where $y_{kl} = 1/z_{kl}$
  $\rightarrow$ complex-valued quadratic and useful for calculations & optimization

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  $\rightarrow$ this is how power system engineers think: all specs on $|V_k|$ and $\frac{d}{dt}\theta_k$

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  $\rightarrow$ useful in radial networks: equations can be expressed in magnitudes only
- many variations, coordinate changes, convexifications, etc.
  $\rightarrow$ some problems become easier in different coordinates

A brief history of power flow approximations

for computational tractability, analytic studies, & control/optimization design

- **DC power flow**: polar form $\rightarrow \Re(Z) = \mathbb{0}$, $|V| = \mathbb{1}$, and linearization

  B. Stott, J. Jardim, & O. Alsac, DC Power Flow Revisited. *IEEE TPS*, 2009.

  $\rightarrow$ standard (but often poor) approximation for transmission networks

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- **LinDistFlow**: branch flow $\rightarrow$ parameterization $|V|^2$ coordinates and linearization

  M.E. Baran & F.F. Wu, Optimal sizing of capacitors placed on a radial distribution system. *PES*, 1988.

  $\rightarrow$ very useful for voltages in (radial) distribution networks

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  $\rightarrow$ works amazingly well in distribution and transmission

A brief history of power flow approximations

for computational tractability, analytic studies, & control/optimization design

- **DC power flow**: polar form $\rightarrow \Re(Z) = \mathbb{0}$, $|V| = \mathbb{1}$, and linearization

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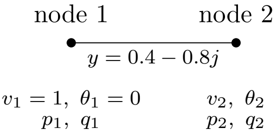
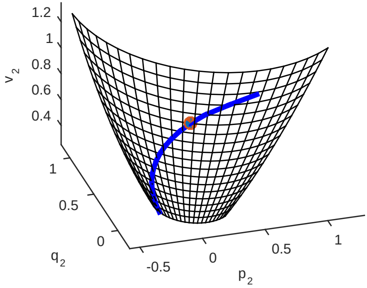
- many variations, extensions, sensitivity and Jacobian methods, etc.

A unifying geometric perspective: the power flow manifold





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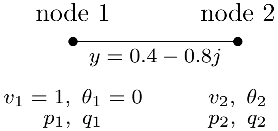

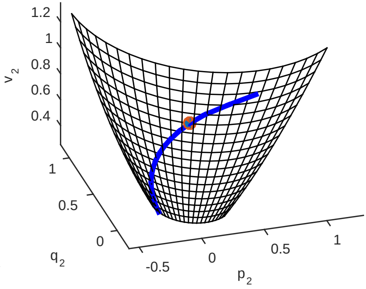

- **variables:** all of $x = (|V|, \theta, P, Q)$
- **power flow manifold:** $\mathcal{M} = \{x : h(x) = \mathbb{0}\}$
  $\rightarrow$ submanifold in $\mathbb{R}^{2n}$ or $\mathbb{R}^{6n}$ (3-phase)

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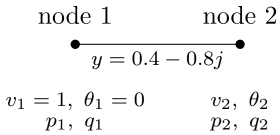

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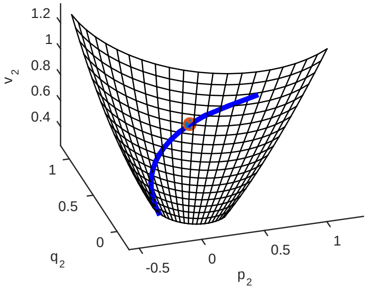

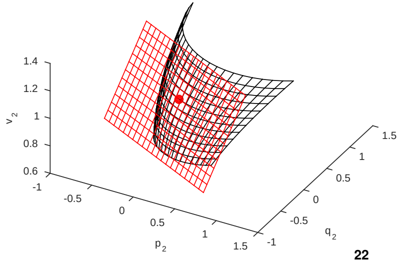

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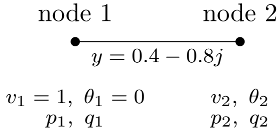

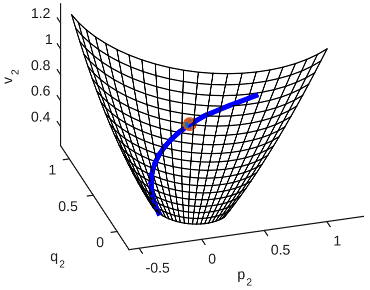

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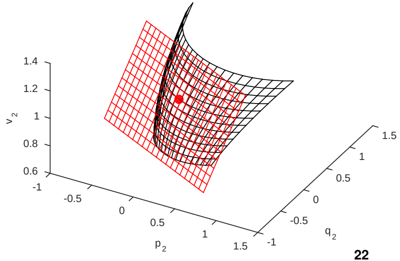
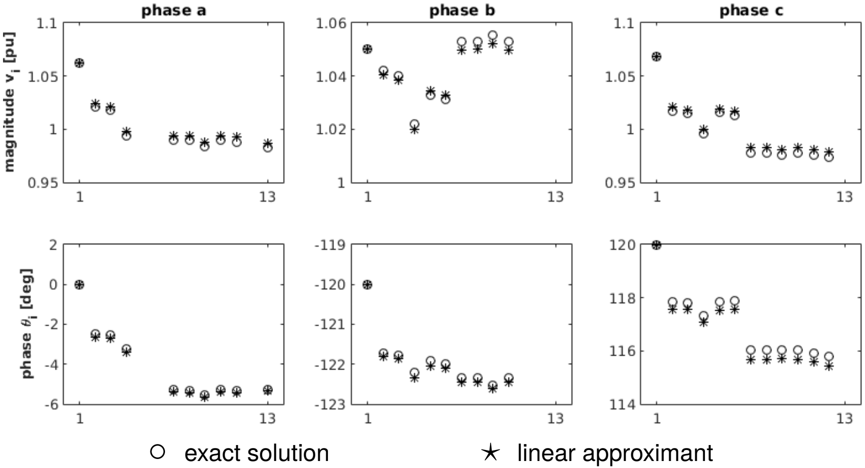
  $\rightarrow$ constant in rectangular coordinates



Accuracy illustrated with unbalanced three-phase IEEE13



Matlab/Octave code @ `https://github.com/saveriob/1ACPF`

Special cases reveal some old friends

- **flat-voltage/0-injection point:** $x^* = (|V|^*, \theta^*, P^*, Q^*) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

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gives **linear coupled power flow** [D. Deka, S. Backhaus, and M. Chertkov, '15]

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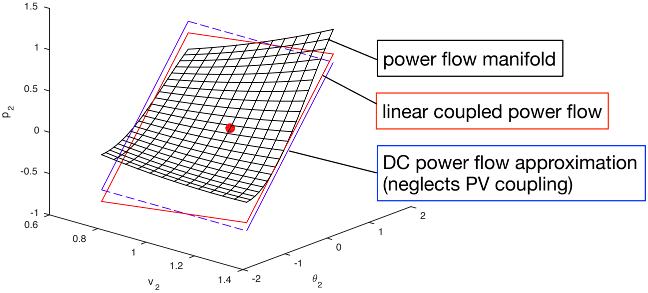

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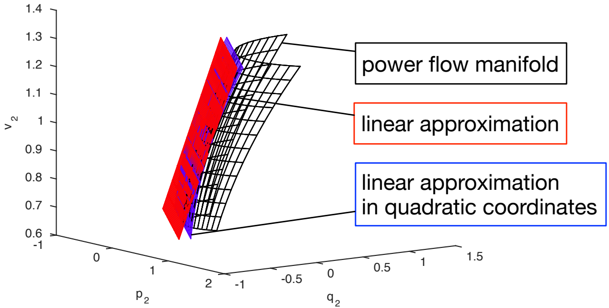


Properties of power flow manifold that we will exploit

- nonlinear power flow is **smooth manifold**

$\rightarrow$ **coordinate-independent** – no singularities

$\rightarrow$ better local linear **approximations**

$\rightarrow$ **methods** for manifold optimization/control

- natural concept for **closed-loop dynamics**

$\rightarrow$ $\mathcal{M}$ is **attractive** for grid dynamics

$\rightarrow$ closed-loop **trajectories** $x(t)$ live on $\mathcal{M}$

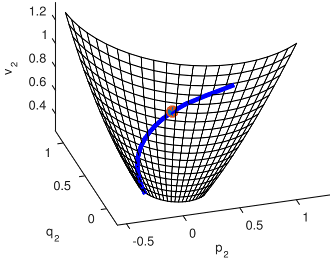
$\rightarrow$ **control** task: steer $\dot{x}(t)$ in tangent space

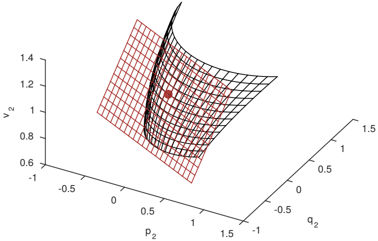
- const.-rank **linearization** $A_{x^*}(x - x^*) = \mathbb{0}$

$\rightarrow$ **implicit** – no input/outputs (no disadvantage)

$\rightarrow$ **sparse** – A_{x^*} has the sparsity of the grid

$\rightarrow$ **structure** – elements of A_{x^*} are local



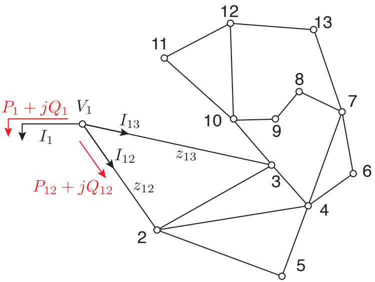


$\rightarrow$ S. Bolognani & F. Dörfler (2015)
"Fast power system analysis via implicit linearization of the power flow manifold"

PROJECTED GRADIENT FLOW ON THE POWER FLOW MANIFOLD

AC power flow model, constraints, and objectives

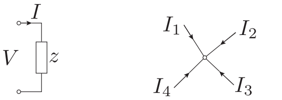
- **model** (physical constraint): $x \in \mathcal{M}$



V_k nodal voltage
I_k current injection
P_k, Q_k power injections

z_{kl} line impedance
I_{kl} line current
P_{kl}, Q_{kl} power flow

Ohm's Law | Current Law



$V = zI$ | $0 = I_1 + \ldots + I_k$

AC power $S = P + jQ = VI^*$

AC power flow equations

$$S_k = \sum_{l \in N(k)} \frac{1}{z_{kl}^*} V_k (V_k^* - V_l^*) \quad \forall k \in \mathcal{N}$$

(all variables and parameters are $\mathbb{C}$-valued)

- **operational constraints:** generation capacity, voltage bands, no congestion
- **objective:** economic dispatch, minimize losses, distance to collapse, etc.
- **control:** state measurements and actuation via generator set-points

Ancillary services as a real-time OPF

Real-time optimal power flow (OPF)

• minimize cost of generation	minimize	$\sum_{k \in \mathcal{N}} \text{cost}_k(P_k^G)$	
• satisfy AC power flow laws	subject to	$P^G + jQ^G = P^L + jQ^L + \text{diag}(V) Y^* V^*$	
• respect generation capacity		$\underline{P}_k \leq P_k^G \leq \overline{P}_k,\ \underline{Q}_k \leq Q_k^G \leq \overline{Q}_k$	$\forall k \in \mathcal{N}$
• no over-/under-voltage		$\underline{V}_k \leq \lvert V_k \rvert \leq \overline{V}_k$	$\forall k \in \mathcal{N}$
• no congestion		$\lvert P_{kl} + jQ_{kl} \rvert \leq \overline{S}_{kl}$	$\forall (k, l) \in \mathcal{E}$

Y admittance matrix, P_k^G, Q_k^G power generation, P_k^L, Q_k^L load, $\{\underline{V}_k, \overline{V}_k, \ldots\}$ nodal limits, $\overline{S}_{kl}$ line flow limit

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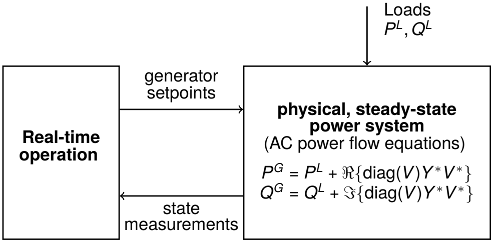

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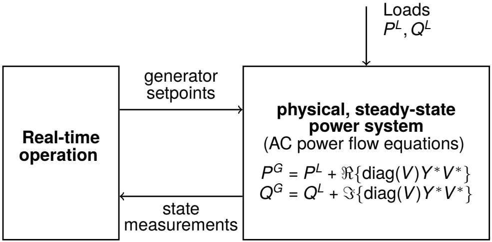
A **control problem** with **challenging specifications** on the closed-loop system:

1. its trajectory $x(t)$ must satisfy the constraints at all times
2. it must converge to $x^\star$, the solution of the AC OPF



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Prototype of real-time OPF

$$\begin{aligned} \text{minimize} \quad & \phi(x) \\ \text{subject to} \quad & x \in \mathcal{K} = \mathcal{M} \cap \mathcal{X} \end{aligned}$$

$x = \begin{bmatrix} |V| & \theta & P & Q \end{bmatrix}$ grid state

$\phi : \mathbb{R}^n \to \mathbb{R}$ objective function

$\mathcal{M} \subset \mathbb{R}^n$ AC power flow equations

$\mathcal{X} \subset \mathbb{R}^n$ operational constraints

Unconstrained optimization on the power flow manifold

- **geometric objects:**

manifold $\mathcal{M} = \{x : h(x) = \mathbb{0}\}$

objective $\phi : \mathcal{M} \to \mathbb{R}$

tangent space $T_x\mathcal{M} = \ker h(x)$

Riemann metric $g : T_x\mathcal{M} \times T_x\mathcal{M} \to \mathbb{R}$

(degree of freedom)

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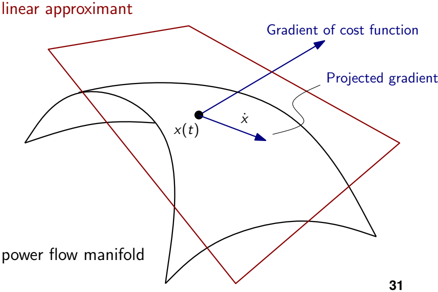
- continuous-time **gradient descent** on $\mathcal{M}$:

  1. $\text{grad}\,\phi(x)$: **gradient** of cost function (& soft constraints) in ambient space
  2. $\Pi_x\text{grad}\,\phi(x)$: **projection** of gradient on the linear approximant $T_x\mathcal{M}$
  3. **flow** on manifold: $\dot{x} = -\gamma\,\Pi_x\text{grad}\,\phi(x)$



Constraints: projected dynamical systems for feasibility

Operational constraints

Per specification, the trajectories need to satisfy operational constraints at all times.

$$x(t) \in \mathcal{K} = \mathcal{M} \cap \mathcal{X}$$

where

$\mathcal{M}$ power flow manifold

$\mathcal{X}$ operational constraints

$\rightarrow \dot{x}(t)$ must belong to a **feasible cone**, subset of the tangent space of $\mathcal{M}$

precisely: $\dot{x}(t) \in T_x\mathcal{K} \subset T_x\mathcal{M}$,

the inward tangent cone at x

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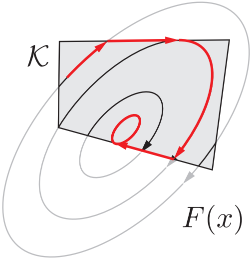

$F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ vector field, $\mathcal{K} \subset \mathbb{R}^n$ closed domain

Projected dynamical systems:

$$\dot{x} = \Pi_{\mathcal{K}}\left(x, F(x)\right)$$

where

$$\Pi_{\mathcal{K}}(x, F(x)) \in \arg\min_{v \in T_x\mathcal{K}} \|F(x) - v\|_g$$

Projected gradient descent on the power flow manifold

$$\dot{x} = \Pi_{\mathcal{K}} \left(x, -\text{grad}\, \phi(x) \right), \quad x(0) = x_0$$

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- Are solution trajectories (asymptotically) stable?
- Do solution trajectories converge to a minimizer of ϕ ?

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Corollary (simplified)

Let $x : [0, \infty) \to \mathcal{K}$ be a (Carathéodory-)solution of the initial value problem

$$\dot{x} = \Pi_{\mathcal{K}}(x, -\operatorname{grad}\phi(x)), \qquad x(0) = x_0 \; .$$

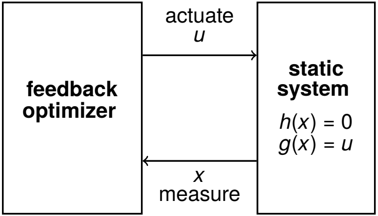
If ϕ has compact level sets on $\mathcal{K}$, $x(t)$ will converge to a critical point $x^\star$ of ϕ on $\mathcal{K}$. Furthermore, if $x^\star$ is asymptotically stable then it is a local minimizer of ϕ on $\mathcal{K}$.

$\to$ Hauswirth, Bolognani, Hug, & Dörfler (2016)
"Projected gradient descent on Riemanniann manifolds with applications to online power system optimization"

How to induce the projected gradient flow

Controlled system

$$\begin{aligned} \text{minimize}_{u,x} \quad & \phi(x) \\ \text{subject to} \quad & x \in \mathcal{K} \\ & g(x) = u \end{aligned}$$



- the **state** x is uniquely determined by
  - the algebraic model $h(x) = 0$ describing the power flow equations
  - an algebraic input constraint $g(x) = u$

How to induce the projected gradient flow

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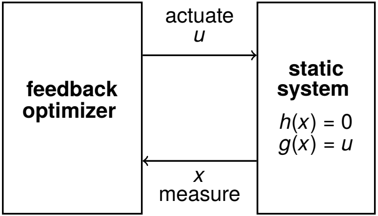

- the **state** x is uniquely determined by
  - the algebraic model $h(x) = 0$ describing the power flow equations
  - an algebraic input constraint $g(x) = u$
- **steady state:** the closed-loop system converges to the solution of the OPF
- **closed-loop trajectory** remains in $\mathcal{K}$ at all times

→ no need to solve the optimization problem numerically

→ no need to solve any power flow equation

From projected gradient flow to discrete-time feedback control

partition: $x = \begin{bmatrix} x_{\text{exo}} \\ x_{\text{endo}} \end{bmatrix}$

exogenous variables:

inputs/disturbances

(e.g., reactive injection Q_k)

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determined by the physics

(e.g., voltage V_k)

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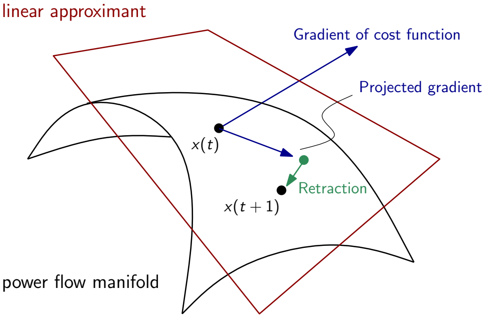
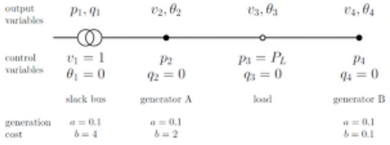


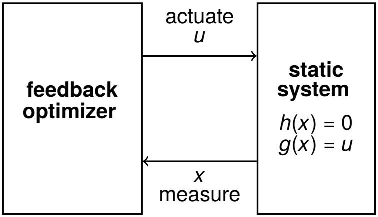
1. compute continuous **feasible descent direction**: $d^t = \Pi_{\mathcal{K}}\left(x, -\text{grad}\,\phi(x(t))\right)$
2. Euler integration step to **compute new set-points**: $\tilde{x}(t+1) = x(t) + \alpha \cdot d^t$
3. **actuate exogeneous variables** (inputs) based on $\tilde{x}_{\text{endo}}(t+1)$

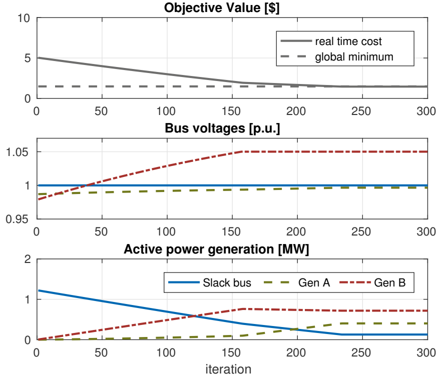
   (note: x_{exo} will be updated accordingly since $h(x) = 0$ holds implicitly by physics)
4. **retraction step** $x(t+1) = R_{x(t)}(\tilde{x}(t+1)) \quad \Rightarrow x(t+1) \in \mathcal{M}$

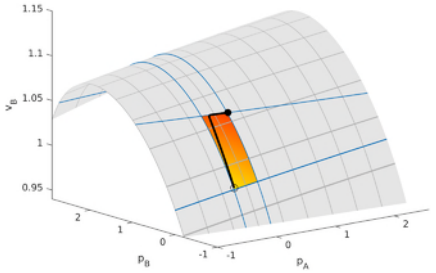
   (note: carried out by physics since $\mathcal{M}$ is attractive / use AC PF solver in simulations)

Simple illustrative case study









TRACKING PERFORMANCE AND ROBUSTNESS OF CLOSED-LOOP OPTIMIZATION

The tracking problem

- the power system state is also affected by **exogeneous inputs w_t**

→ because of these inputs, the state could leave the feasible region $\mathcal{K}$

→ outside of $\mathcal{K}$, the projected gradient flow is not defined

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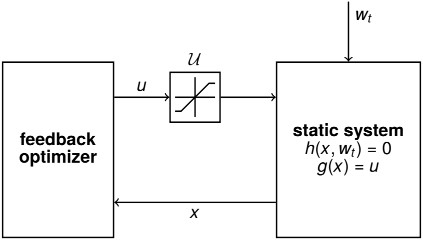
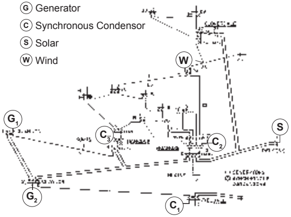


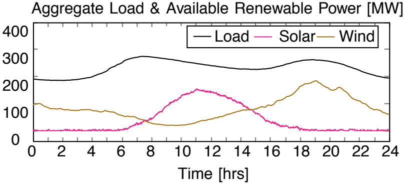
constraints satisfaction for non-controllable variables:

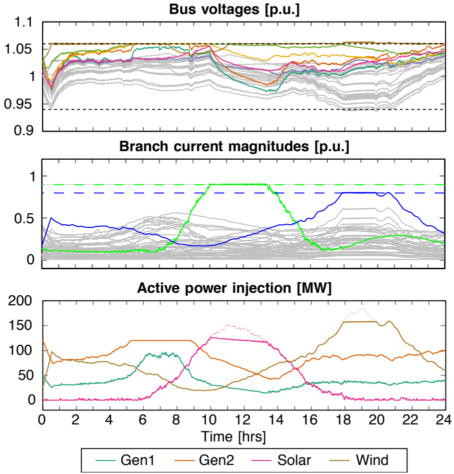
- $\mathcal{K}$ accounts only for **hard constraints** on controllable variables u (e.g., generation limits)
- gradient projection becomes **input saturation** (saturated proportional feedback control)
- **soft constraints** included via **penalty functions** in ϕ (e.g., thermal and voltage limits)

→ alternative method (not discussed today) is **dualization** (i.e., integral control)

Tracking performance



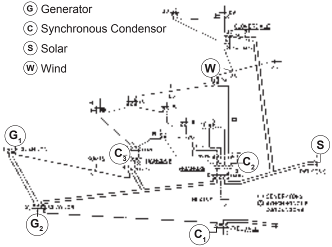




controller: penalty + saturation

→ Hauswirth, Bolognani, Dörfler, & Hug (2017)
“Online Optimization in Closed Loop on the Power Flow Manifold”

Tracking performance



Comparison

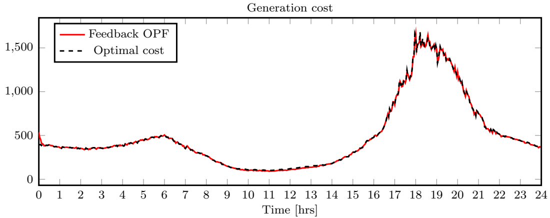
- closed-loop feedback trajectory
- benchmark: feedforward OPF

(solution of an ideal OPF without computation delay)

- practically **exact tracking**

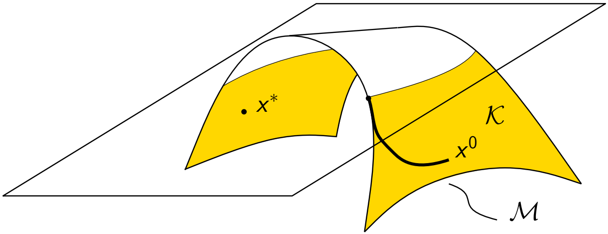
\+ **trajectory feasibility**

\+ **robustness to model mismatch**



Trajectory feasibility

The feasible region $\mathcal{K} = \mathcal{M} \cap \mathcal{X}$ often has **disconnected components**.



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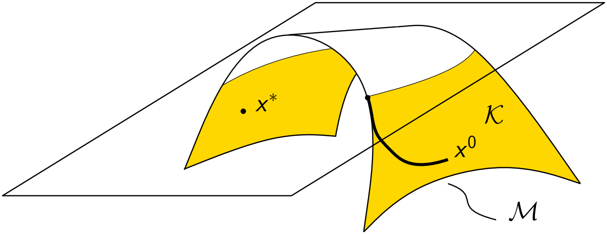
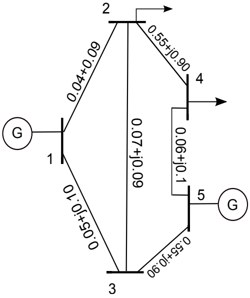


- **feedback (gradient descent)**
  - $\rightarrow$ the closed-loop trajectory $x(t)$ is guaranteed to be **feasible**
  - $\rightarrow$ convergence of $x(t)$ to a **local minimum** is guaranteed

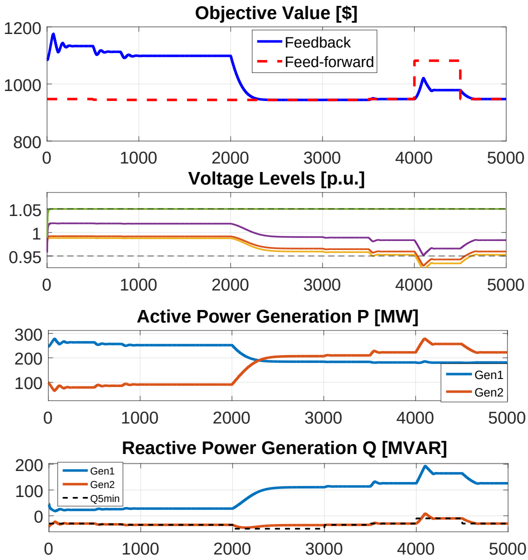
- **feedforward (OPF)**
  - – optimizer $x^\star = \arg\min_{x \in \mathcal{K}} \phi(x)$ can be in different **disconnected component**
  - $\rightarrow$ no feasible trajectory exists: $x_0 \rightarrow x^\star$ must **violate constraints**

Illustration of trajectory feasibility

5-bus example known to have two disconnected feasible regions:

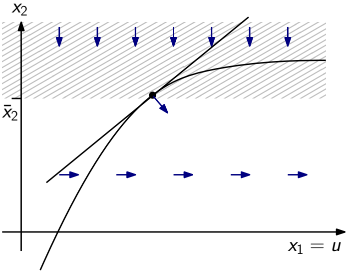


- [0s,2000s]: separate feasible regions
- [2000s,3000s]: loosen limits on reactive power $\underline{Q}_2$ → regions merge
- [4000s,5000s]: tighten limits on $\underline{Q}_2$ → vanishing feasible region



Robustness to model mismatch

Intuition in 2D case: cost on x_1, soft penalty for constraint $x_2 \leq \bar{x}_2$, actuation on x_1



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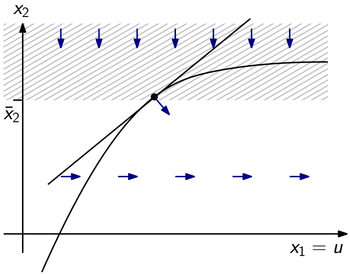

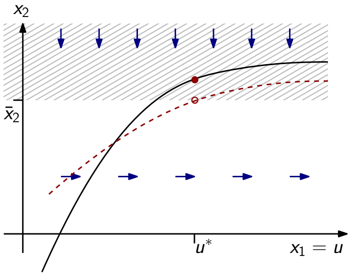

↑ **feedforward (OPF)**
model-based approach: model mismatch directly affects the decision $u^\star$

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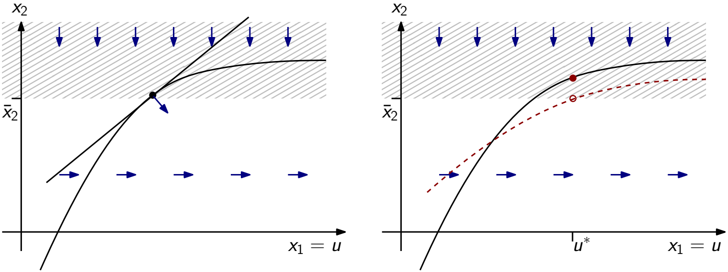
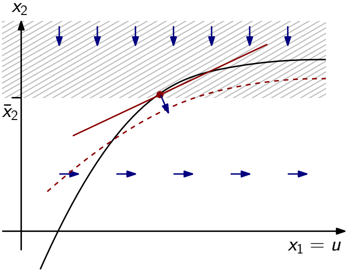


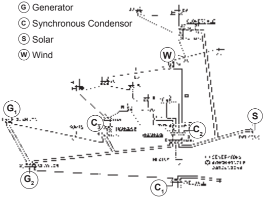


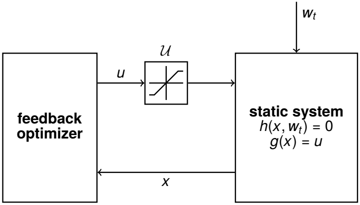
↑ **feedforward (OPF)**
model-based approach: model mismatch directly affects the decision $u^\star$

← **feedback (gradient descent)**
grad ϕ is orthogonal to the tangent plane

Illustration of robustness to model mismatch

IEEE 30-bus test system





controller:

- saturation of generation constraints
- penalty for operational constraints

	no automatic re-dispatch			feedback optimization		
model uncertainty	feasible ?	$f - f^*$	$\|v - v^*\|$	feasible ?	$f - f^*$	$\|v - v^*\|$
loads $\pm$40%	no	94.6	0.03	yes	0.0	0.0
line params $\pm$20%	yes	0.19	0.01	yes	0.01	0.003
2 line failures	no	-0.12	0.06	yes	0.19	0.007

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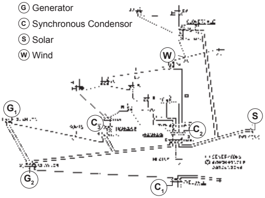

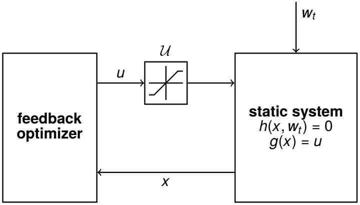

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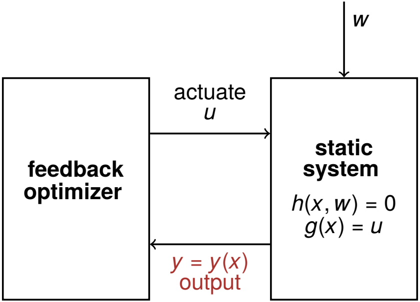
on-going work: observations can be made mathematically rigorous and quantified

OUTPUT FEEDBACK AND STATE UNCERTAINTY

Use real-time output measurements to reduce uncertainty



How to project the trajectory to $\mathcal{K} = \mathcal{M} \cap \mathcal{X}$ when the state is **partially known**?

- **power flow manifold $\mathcal{M}$:** attractive manifold + robustness ✓
- **operational constraints $\mathcal{X}$:** how to deal with state uncertainty ?

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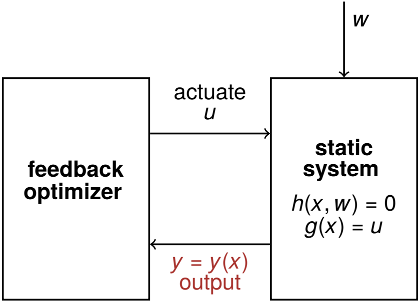

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Chance constraints

generally non-convex set of all u such that $\quad \mathbb{P}\left[x \in \mathcal{X}_w \,|\, y(x) = y\right] \geq 1 - \epsilon$

where w is random and $\epsilon \in (0, 1)$ is probability of constrained violation

Scenario approach to chance-constrained optimization

- **chance constraint:** $\mathbb{P}[x \in \mathcal{X}_w] \geq 1 - \epsilon$ where w is random and $\epsilon \in (0, 1)$

$\rightarrow$ often **intractable** for complex (possibly unknown) distributions/constraints

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→ # samples to **approximate** chance constraint depends on n, ε, and accuracy

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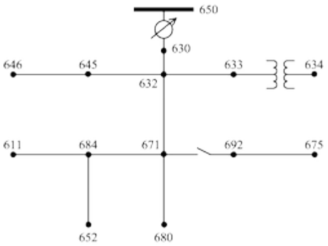
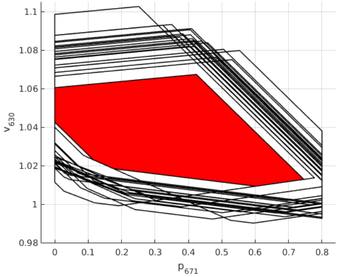


IEEE 13 grid with random demand and actuation (microgenerators & tap changers)



feasible region with scenario approach

Scenario approach with real-time measurements

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$$\mathbb{P}_w[x \in \mathcal{X}_w] \geq 1 - \epsilon \;\to\; x \in \mathcal{X}_{w^{(i)}},\; i \in \{1, \ldots, N\}$$

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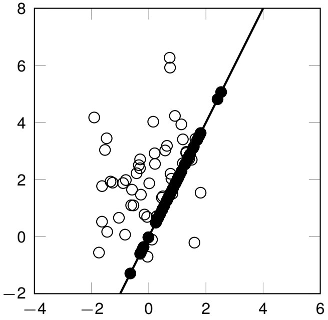
  $\to$ classic scenario-based approach



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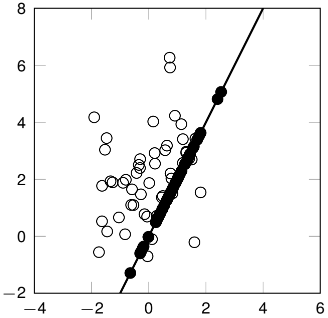

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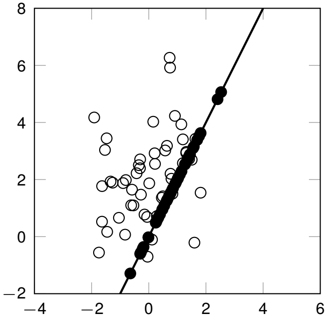

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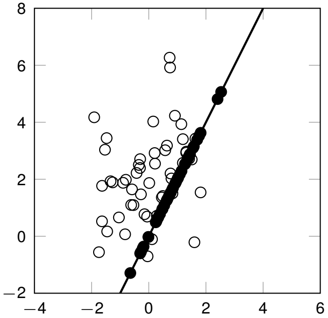

- **re-sampling solution:** scenario approach based on conditional distribution

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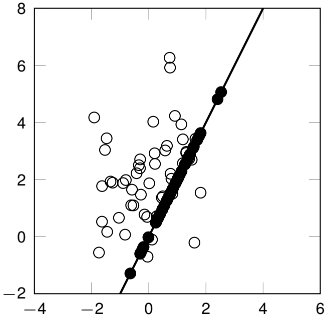

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- today: **online computation** of posterior distribution after measurement

Linear case

- **linear grid model**

  $x = Au + Bw$

- **polytopic constraints**

  $Cx \leq z$

- **linear measurement**

  $y = Hw$

Linear case

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Approximate conditioning

affine transformation:

$$\hat{w}_y = w + K(y - Hw)$$

where $K = \Sigma H^\top \left(H \Sigma H^\top\right)^{-1}$

$\rightarrow$ **projection** of uncertainty in the subspace $\{y = Hw\}$

$\rightarrow$ **Gaussian case:** recovers the conditional distribution

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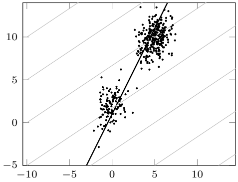
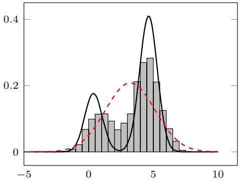
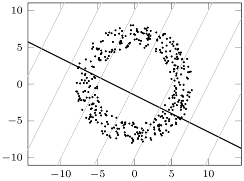
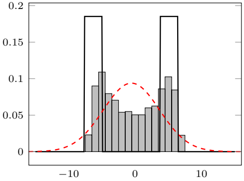
Bimodal distribution	Mean	Variance	Skewness	Kurtosis
True posterior	3.35	4.23	-0.74	2.00
Gaussian approximation	3.20	3.57	0	3
Affine transformation	3.20	3.57	-0.54	2.35





Annular distribution	Mean	Variance	Skewness	Kurtosis
True posterior	-0.6	32.9	0	1.08
Gaussian approximation	-0.6	17.8	0	3
Affine transformation	-0.6	17.8	0	1.60





Affine transformation of the feasible region

transformation: the feasible polytope $Cx \leq z$ can be rewritten as

$$C\underbrace{(Au + B\hat{w}_y)}_{x|y=Hw} \leq z \quad \approx \quad C\Big(Au + B(w + K(y - Hw)\Big) \leq z$$

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transformation: the feasible polytope $Cx \leq z$ can be rewritten as

$$C\underbrace{(Au + B\hat{w}_y)}_{x|y=Hw} \leq z \quad \approx \quad C\Big(Au + B(w + K(y - Hw)\Big) \leq z$$

scenario approach: replace w with finitely many historical samples $w^{(i)}$

$$\bigcap_{i=1}^{N} C\Big(Au + B(w^{(i)} + K(y - Hw^{(i)})\Big) \leq z \quad \rightarrow \quad \text{polytope } \hat{\mathcal{U}} \text{ in } u \text{ and } y$$

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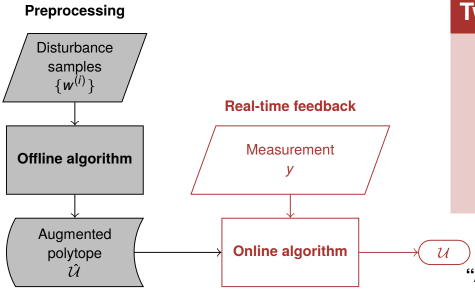


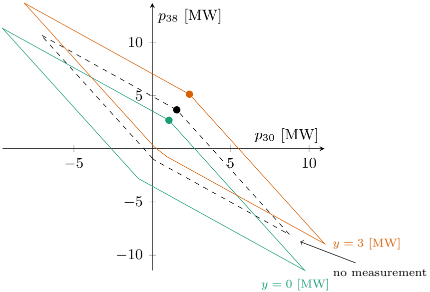
Two-phase algorithm

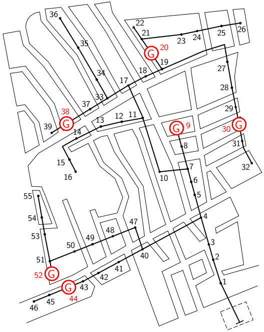
- **offline:** construct a feasible region $\hat{\mathcal{U}}(y)$ parametrized in y
- **online:** compute the conditional feasible polytope $\mathcal{U} = \hat{\mathcal{U}}(y_{\text{measured}})$

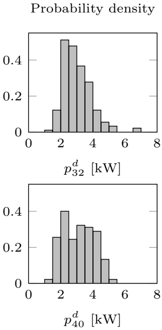
→ Bolognani, Arcari, & Dörfler (2017)
"A fast method for real-time chance-constrained decision with application to power systems"

Example: IEEE 123-bus test system

- **scalar measurement**
  total demand
- **operational constraint**
  overvoltage limits
- **actuation**
  distributed microgenerators
- **samples**
  metered demand of 1200 households







Computation time

Offline	Compute Σ and K	
	Construct augmented polytope $\hat{\mathcal{U}}$	
	Compute minimal representation of $\hat{\mathcal{U}}$	
	Total offline computation time	*55 min*
Online	Slice $\hat{\mathcal{U}}$ at $y = y^{\text{meas}}$ to obtain $\mathcal{U}$	
	Total online computation time	*1.8 ms*

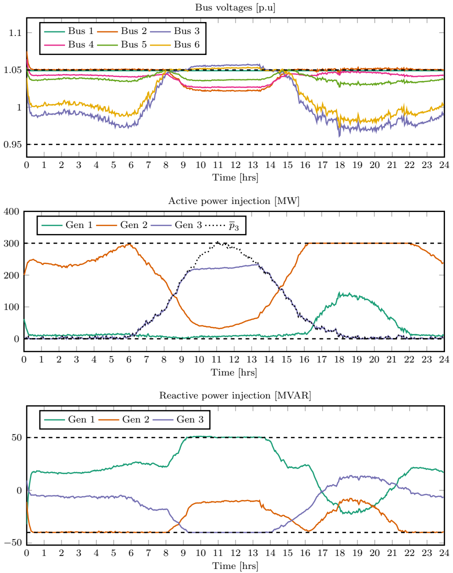
Memory footprint

Offline	Augmented polytope $\hat{\mathcal{U}}$	48620 constraints
Online	Minimal representation of $\hat{\mathcal{U}}$	12 constraints

CONCLUSIONS

Summary and conclusions

- **control perspective on real-time power system operation**
  - feedback control on manifolds
  - steady-state optimality
  - feasibility at all times
- **robustness and performance**
  - real-time constrained tracking
  - robust to model uncertainty
  - chance constraints
- **ongoing and future work**
  - quantify robustness margins
  - saddle-flows on manifolds for primal-dual optimization
  - distributed control approach
  - include primary frequency control
  - online scenario-based approach



Thanks !

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