GAME THEORETICAL INFERENCE OF HUMAN BEHAVIOR IN SOCIAL NETWORKS

Symposium on *"Resilience and performance of networked systems"* Zürich, 16.01.2020

[N. Pagan & F. Dörfler, *"Game theoretical inference of human behavior in social networks"*, Nature Communications, 2019]

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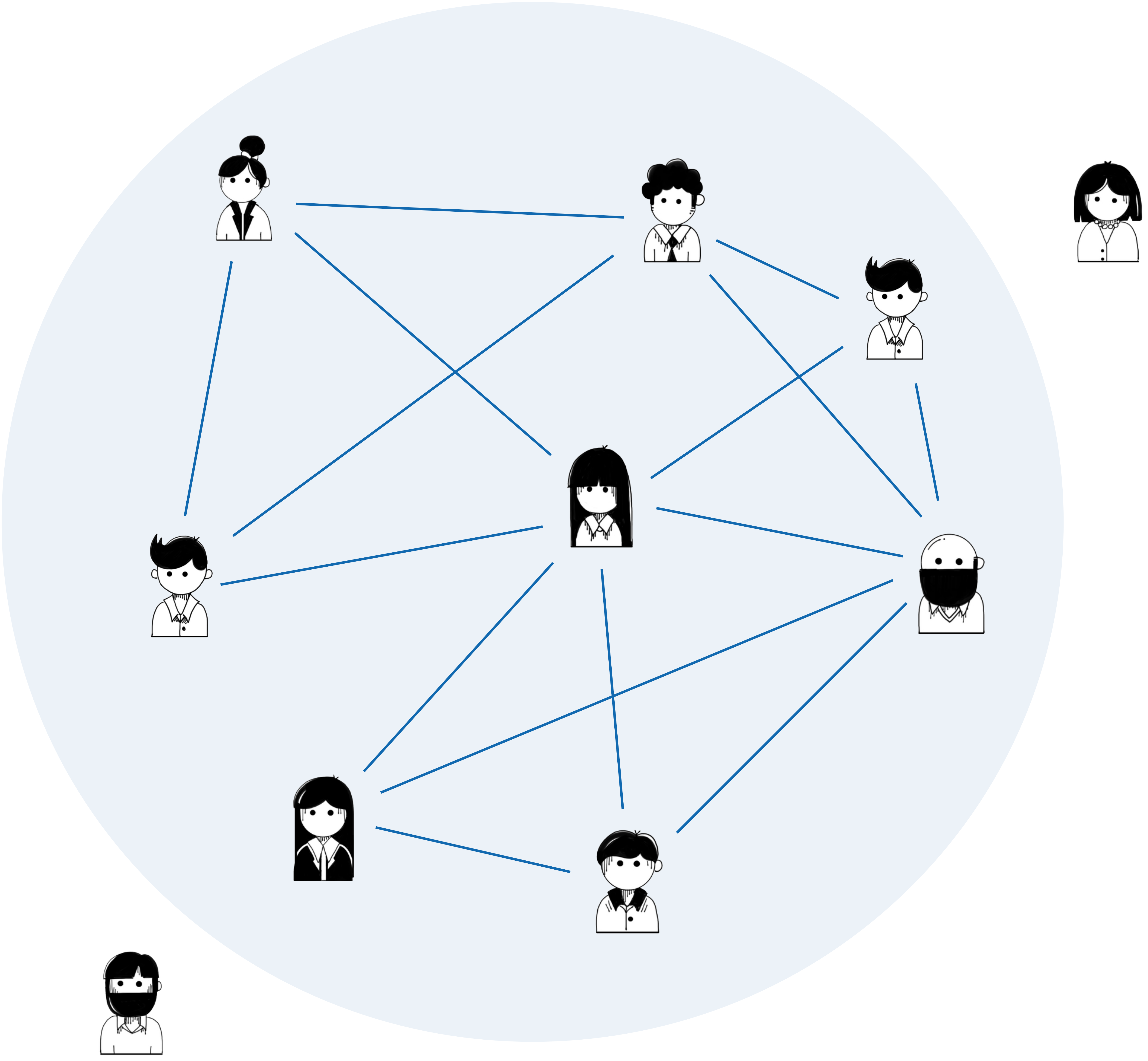
MOTIVATION

How do social networks form?

Individual behavior determines the social network structure.

The social network structure influences individual behavior.

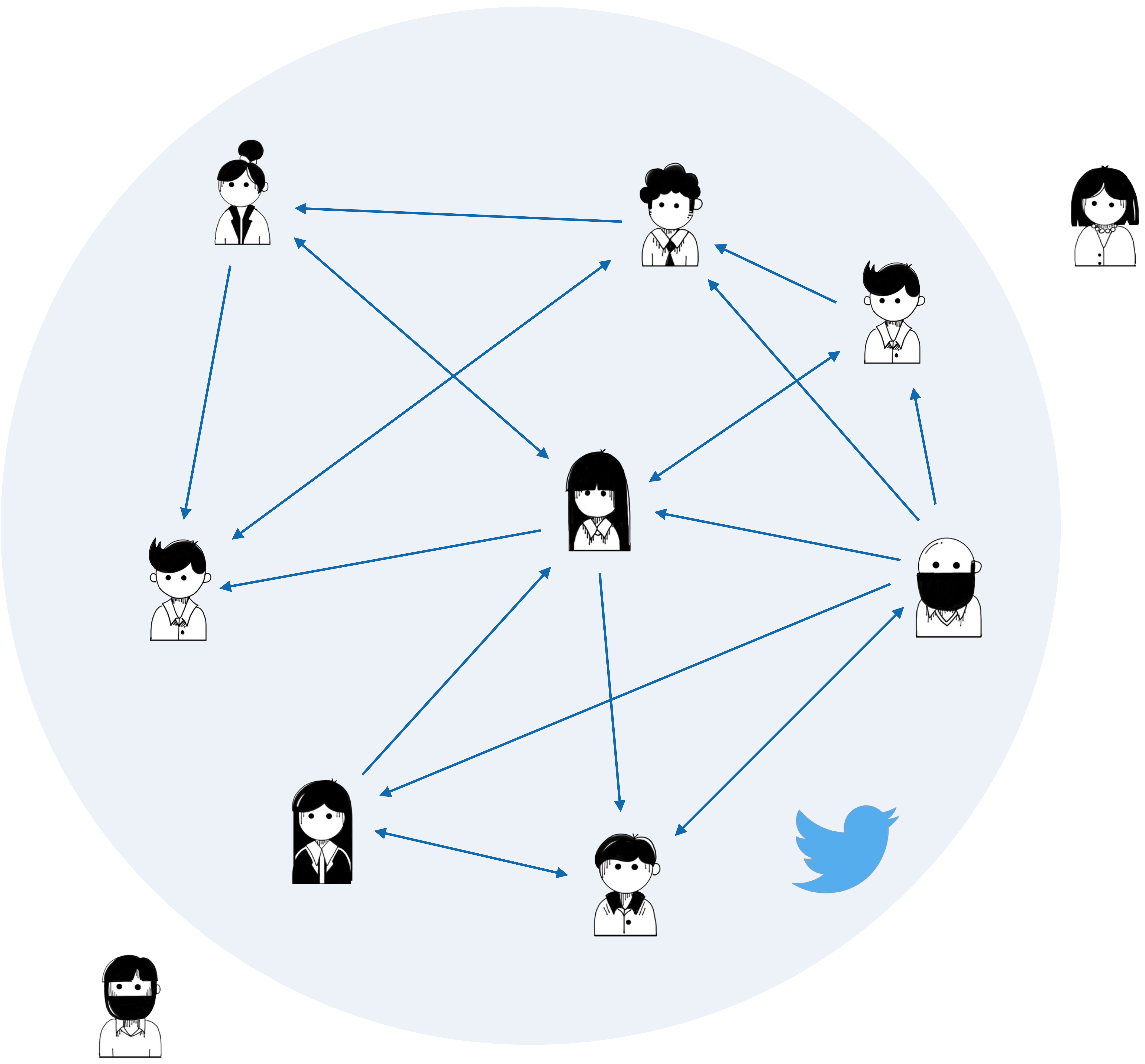


01

OBSERVATIONS

Actors decide with whom they want to interact.

directionality: *followers* ≠ *followees*



02

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Actors decide with whom they want to interact.

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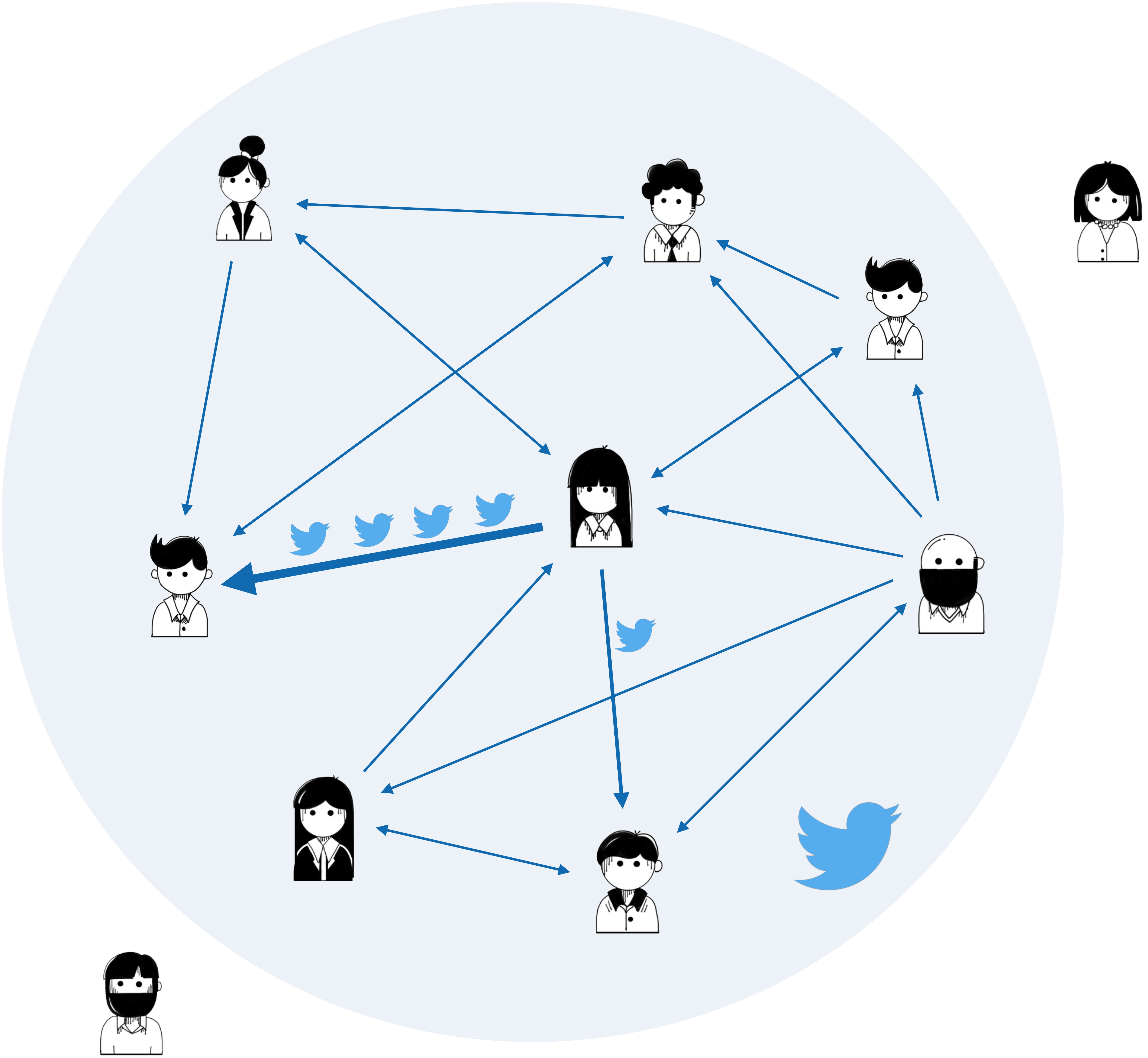
Ties can have different **weights**.



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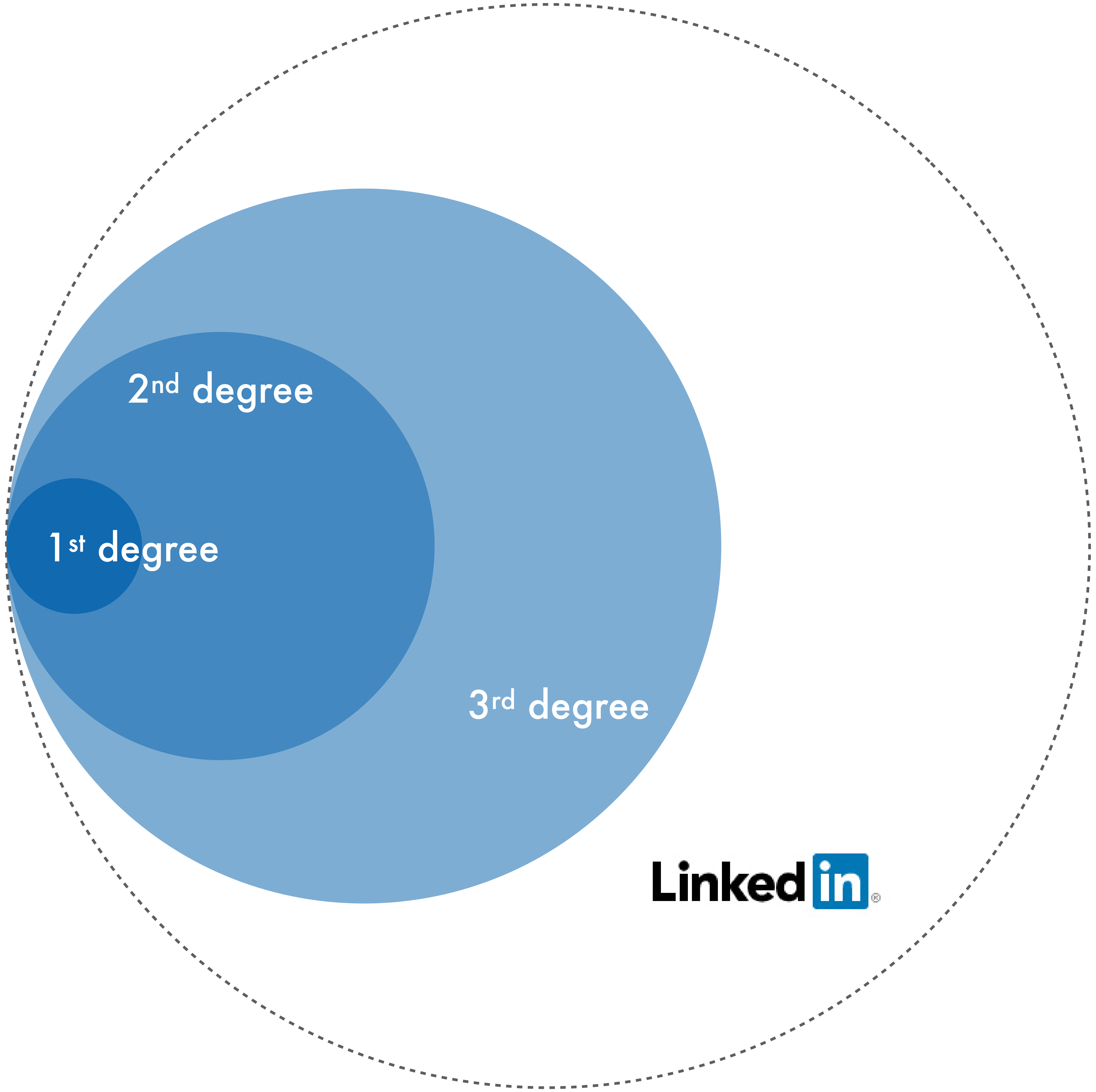
Limited information is available.



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OBSERVATIONS

Actors decide with whom they want to interact.

directionality: *followers ≠ followees*

Ties can have different **weights**.

Limited information is available.

Network positions provide **benefits** to the actors.


Forbes

3,853 views | Sep 11, 2017, 10:09am

Using Social Networks To Advance Your Career

Adi Gaskell Contributor

Shutterstock

"It's not what you know, it's who you know" is one of those phrases

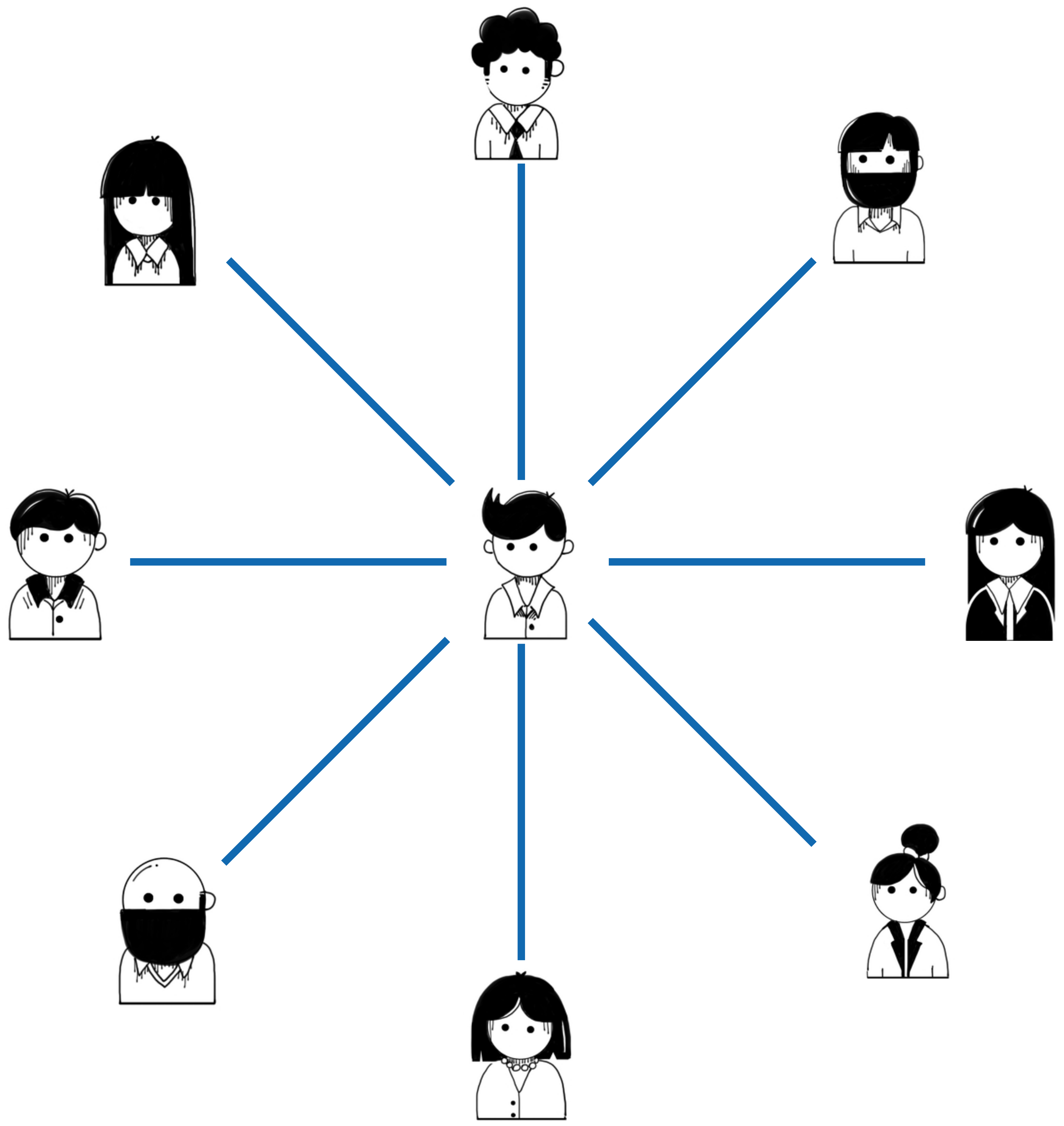
02

SOCIAL NETWORK POSITIONS' BENEFITS

Social Influence

The more people we are connected to, the more we can influence them.

[Robins, G. Doing social network research, 2015]



03

SOCIAL NETWORK POSITIONS' BENEFITS

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The more our friends' friends are our friends, the safer we feel.

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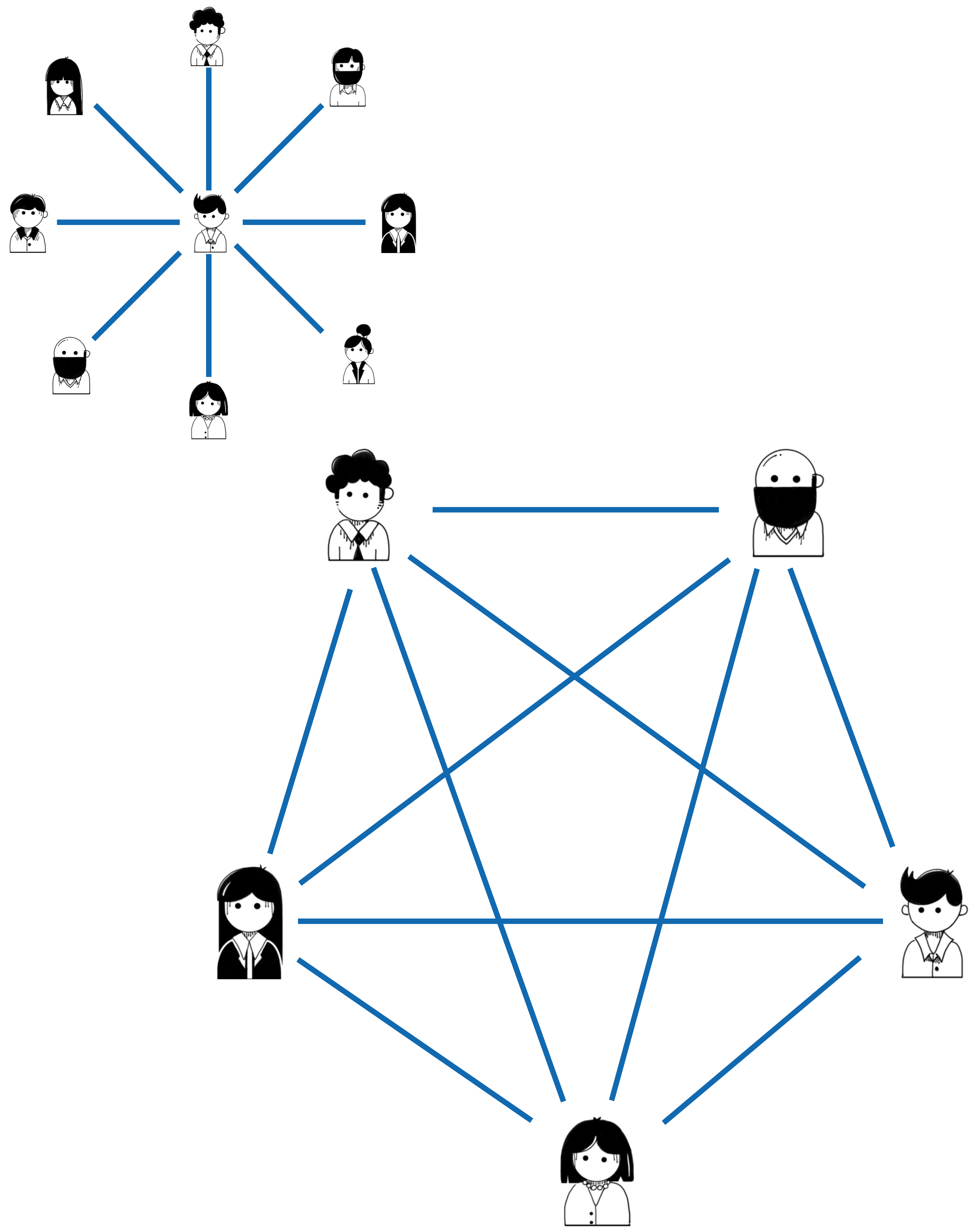

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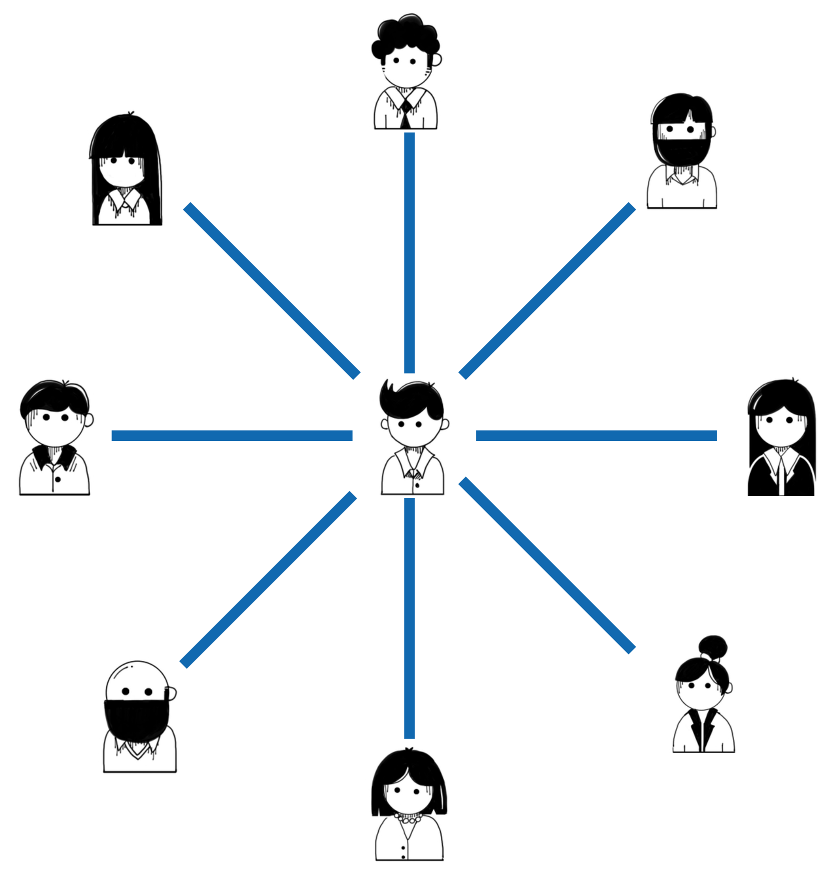
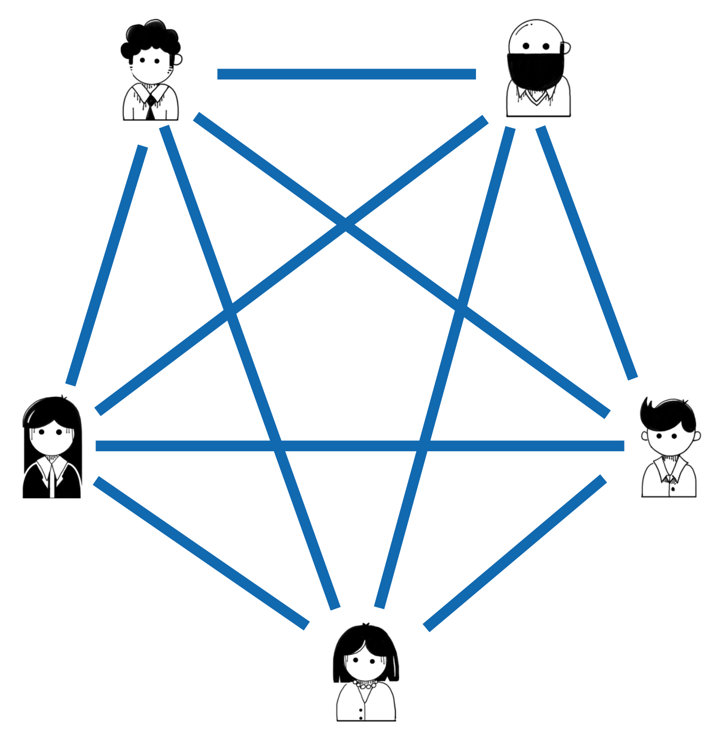
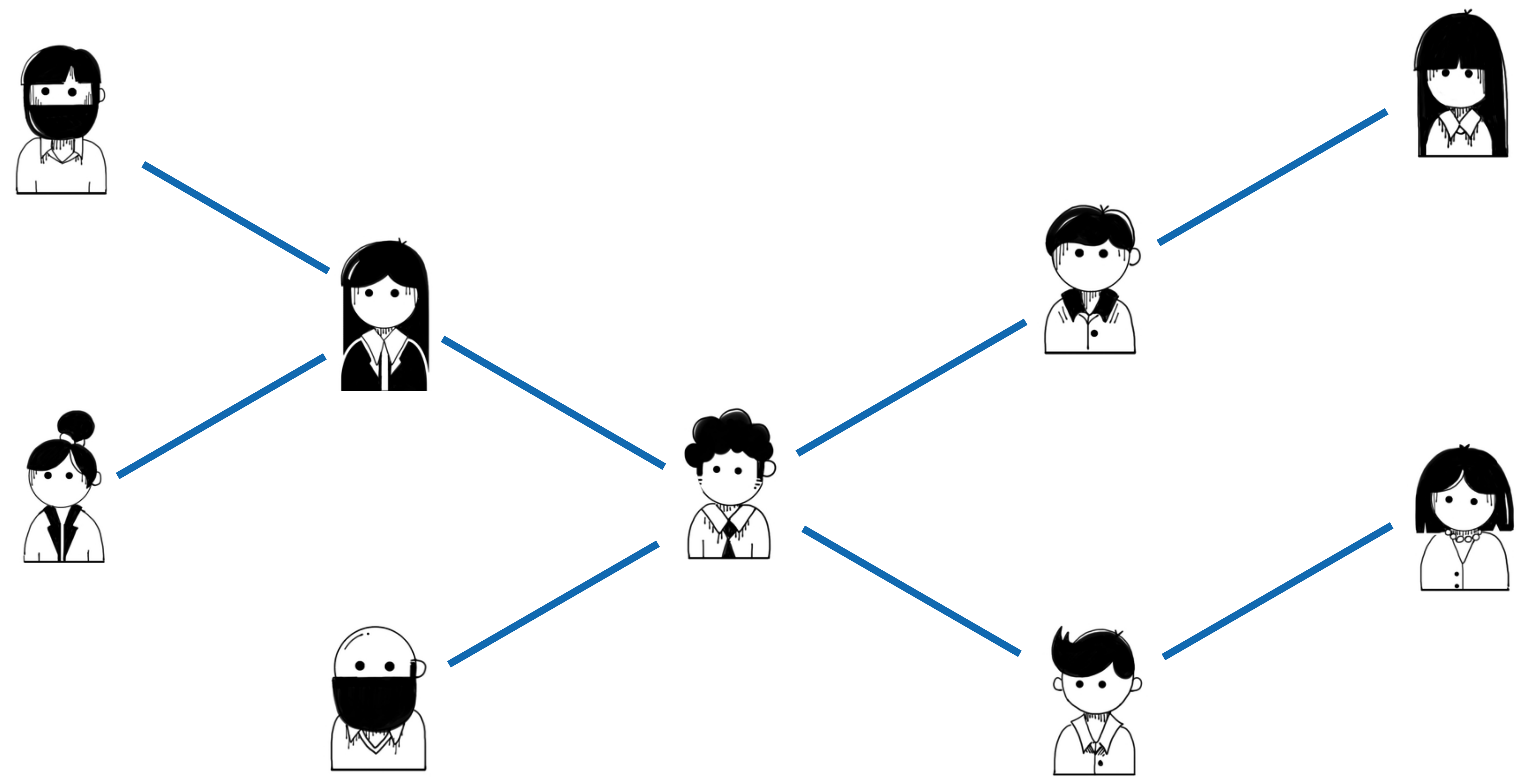
Brokerage

The more we are on the path between people, the more we can control.

[Burt, R. S. Structural Hole (Harvard Business School Press, Cambridge, MA, 1992]







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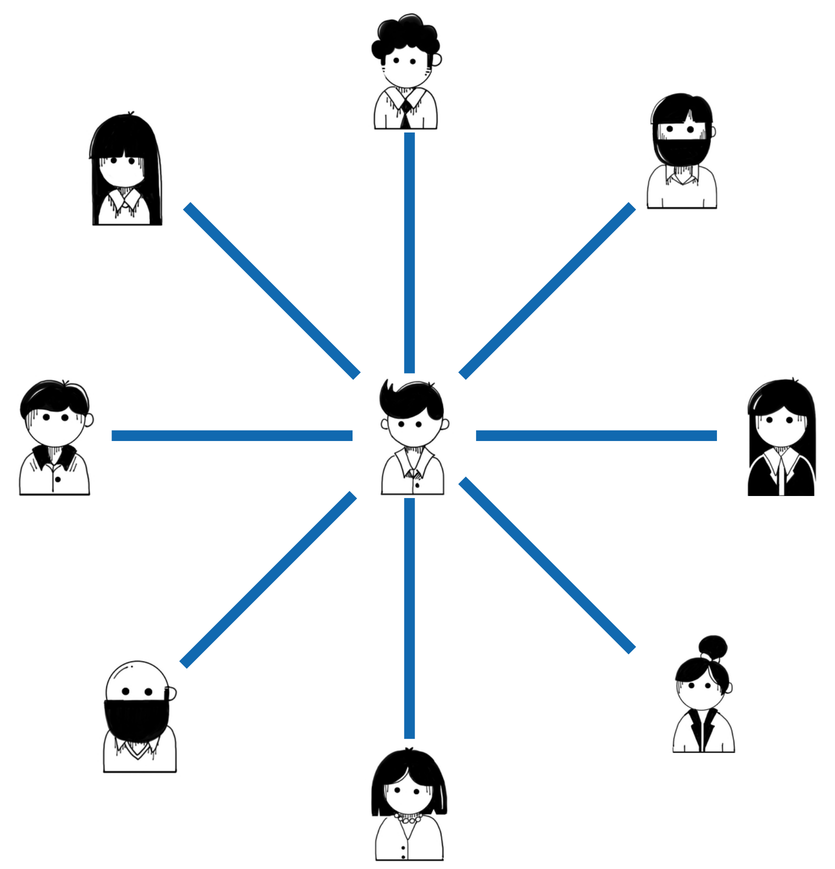
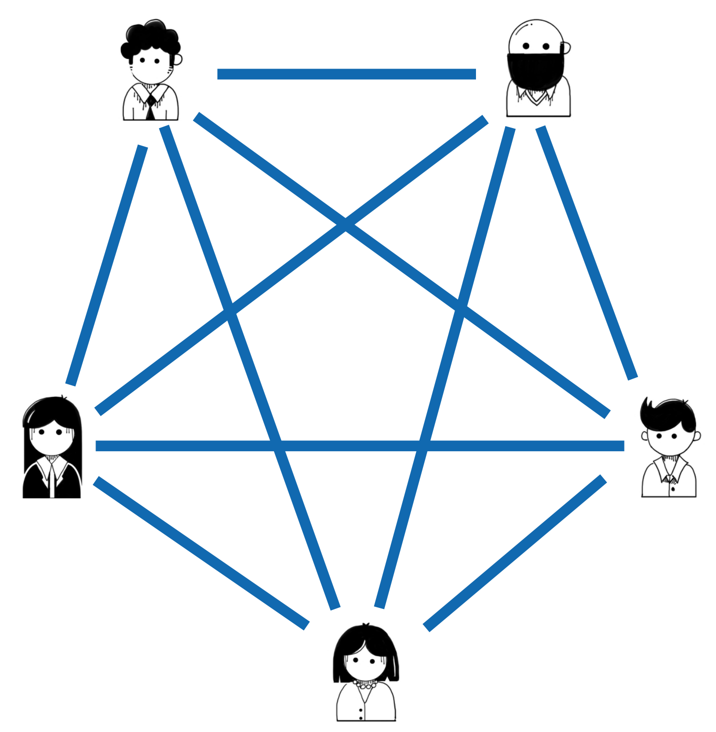
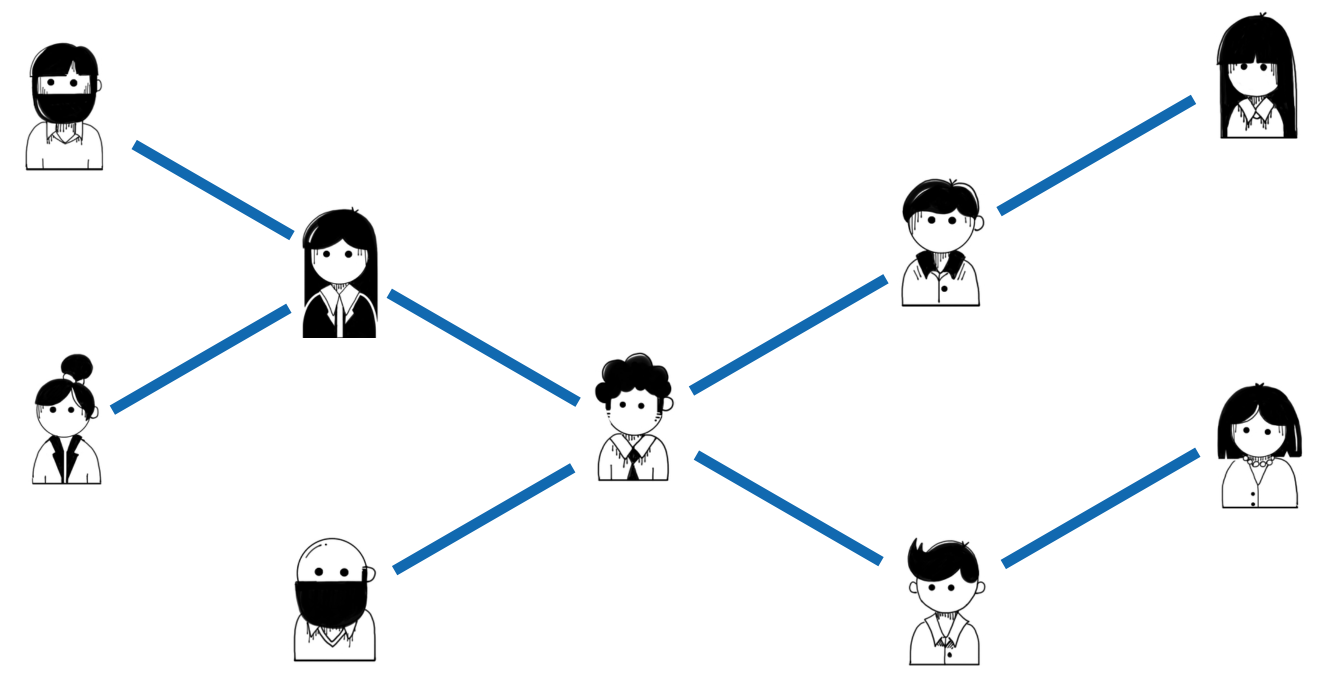


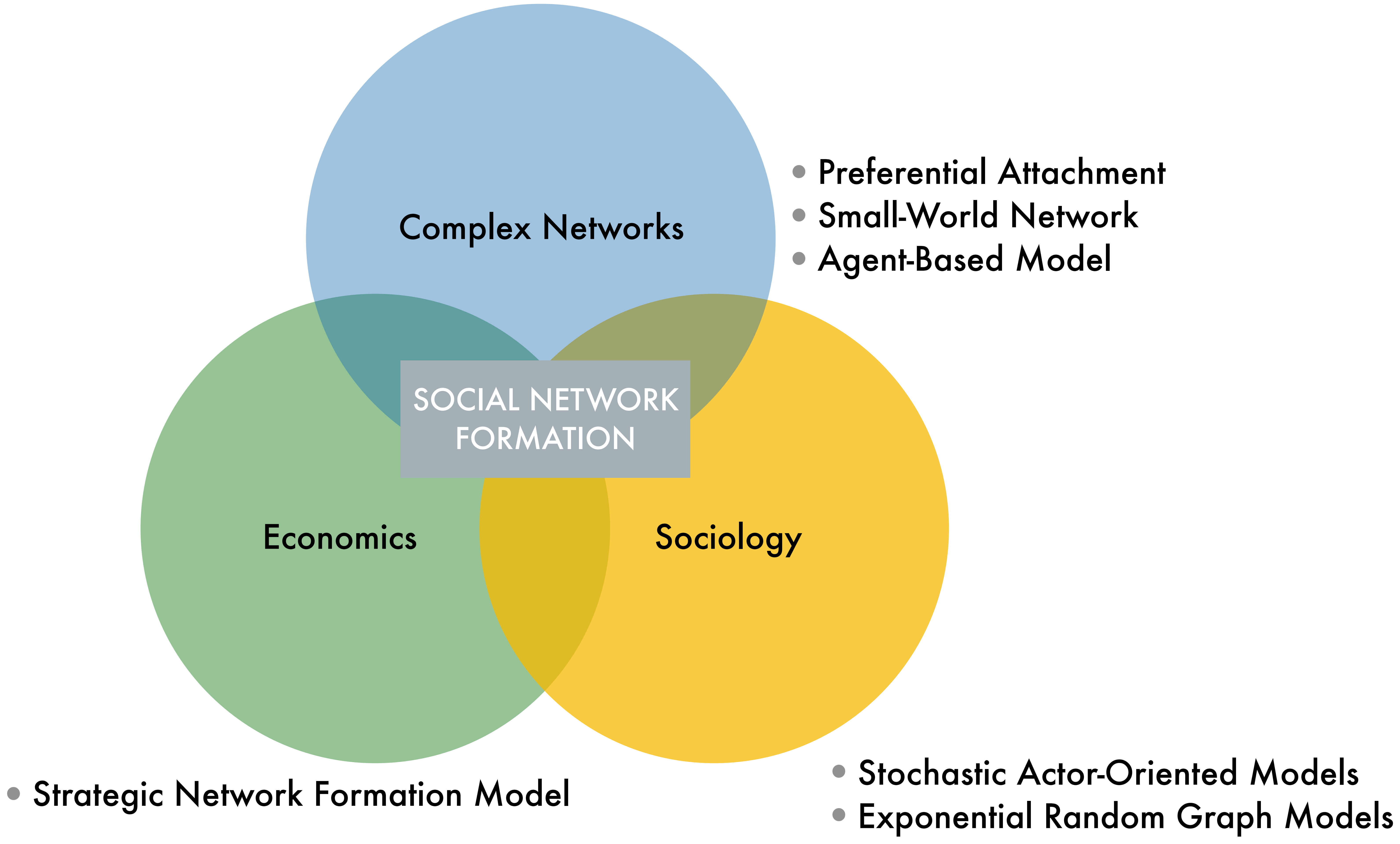
Degree Centrality



Clustering Coefficient



Betweenness Centrality


Complex Networks
Preferential Attachment
Small-World Network
Agent-Based Model
SOCIAL NETWORK FORMATION
Economics
Sociology
Strategic Network Formation Model
Stochastic Actor-Oriented Models
Exponential Random Graph Models

SOCIAL NETWORK FORMATION MODEL

Directed weighted network $\mathscr{G}$ with $\mathscr{N} = \{1,\ldots,N\}$ agents.

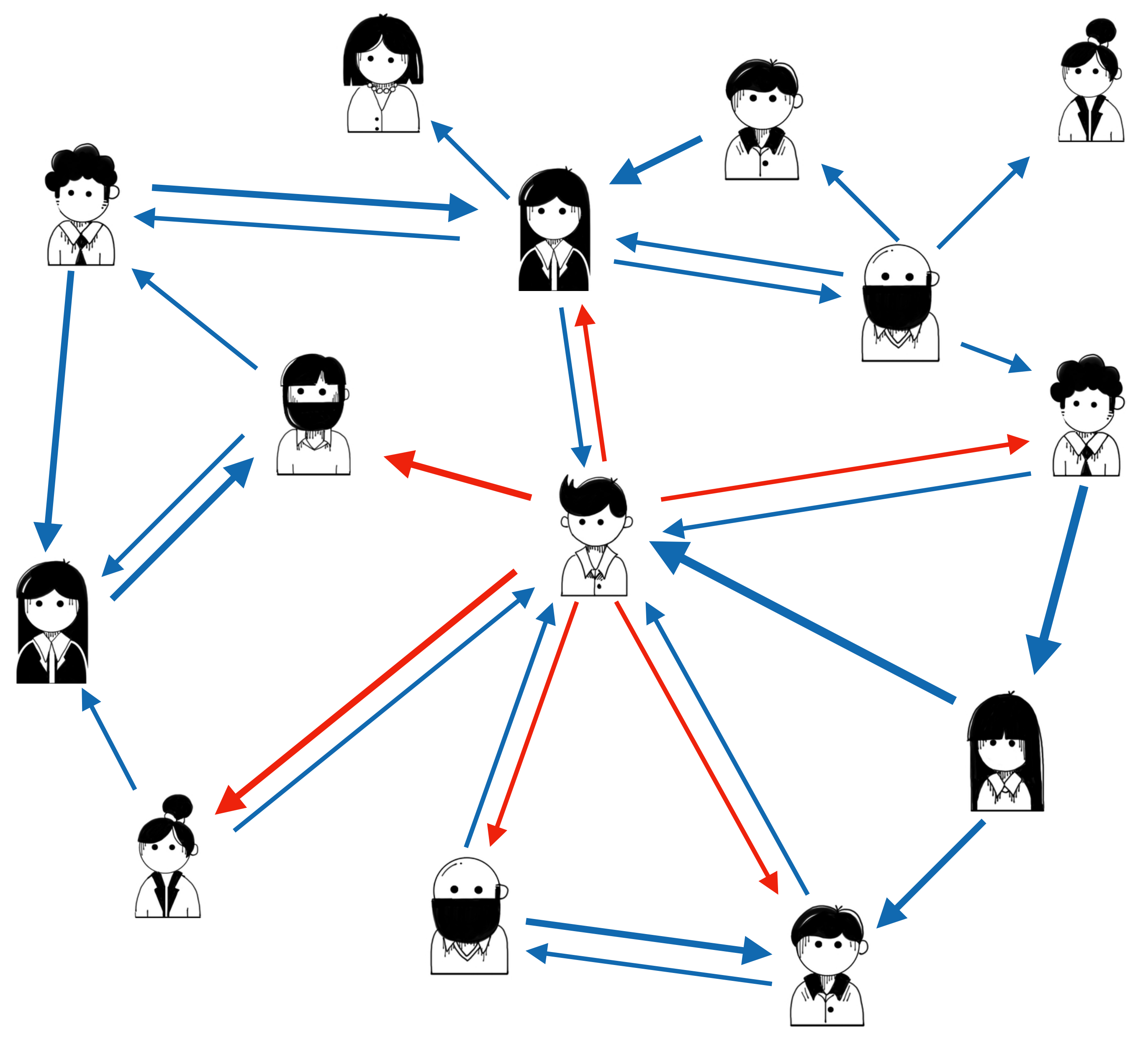
The weight $a_{ij} \in [0,1]$ quantifies the importance of the friendship among i and j from i's point of view.

A typical **action** of agent i :

$$a_i = \left[a_{i1}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{iN}\right] \in \mathscr{A} = [0,1]^{N-1}$$

Every agent i is endowed with a **payoff function** V_i and is looking for

$$a_i^\star \in \arg\max_{a_i \in \mathscr{A}} V_i(a_i, \mathbf{a}_{-i})$$



04

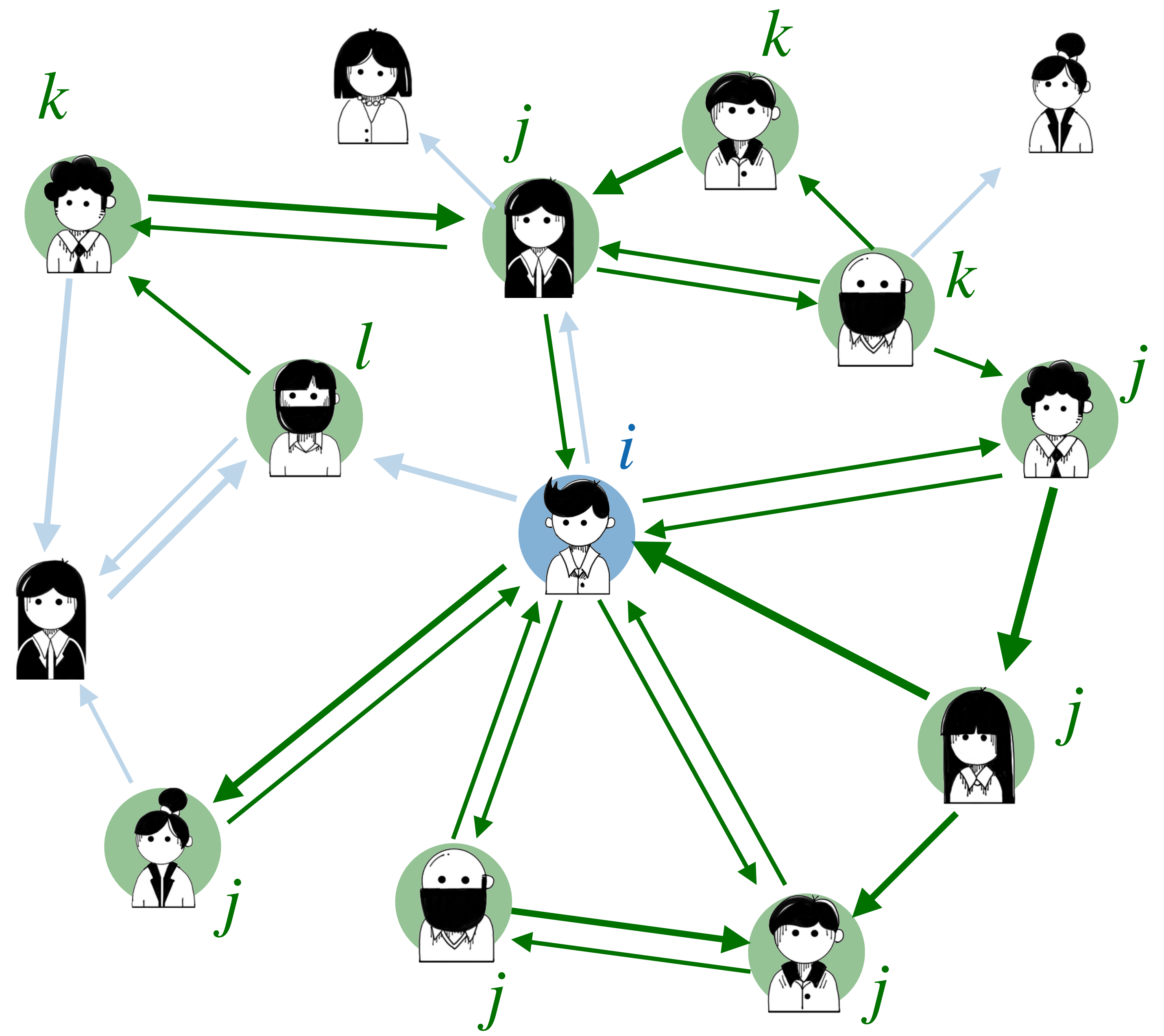
PAYOFF FUNCTION

$$V_i(a_i, \mathbf{a}_{-i}) = t_i(a_i, \mathbf{a}_{-i})$$

Social influence on friends

$$t_i(a_i, \mathbf{a}_{-i}) = \sum_{j \neq i} a_{ji} + \underbrace{\delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}} + \underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji}}_{\text{paths of length 3}}$$

where $\delta_i \in [0,1]$ [Jackson, M. O. & Wolinsky, A. A strategic model of social & economic networks. J. Econom. Theory 71, 44–74 (1996)]



PAYOFF FUNCTION

$$V_i(a_i, \mathbf{a}_{-i}) = t_i(a_i, \mathbf{a}_{-i}) + u_i(a_i, \mathbf{a}_{-i})$$

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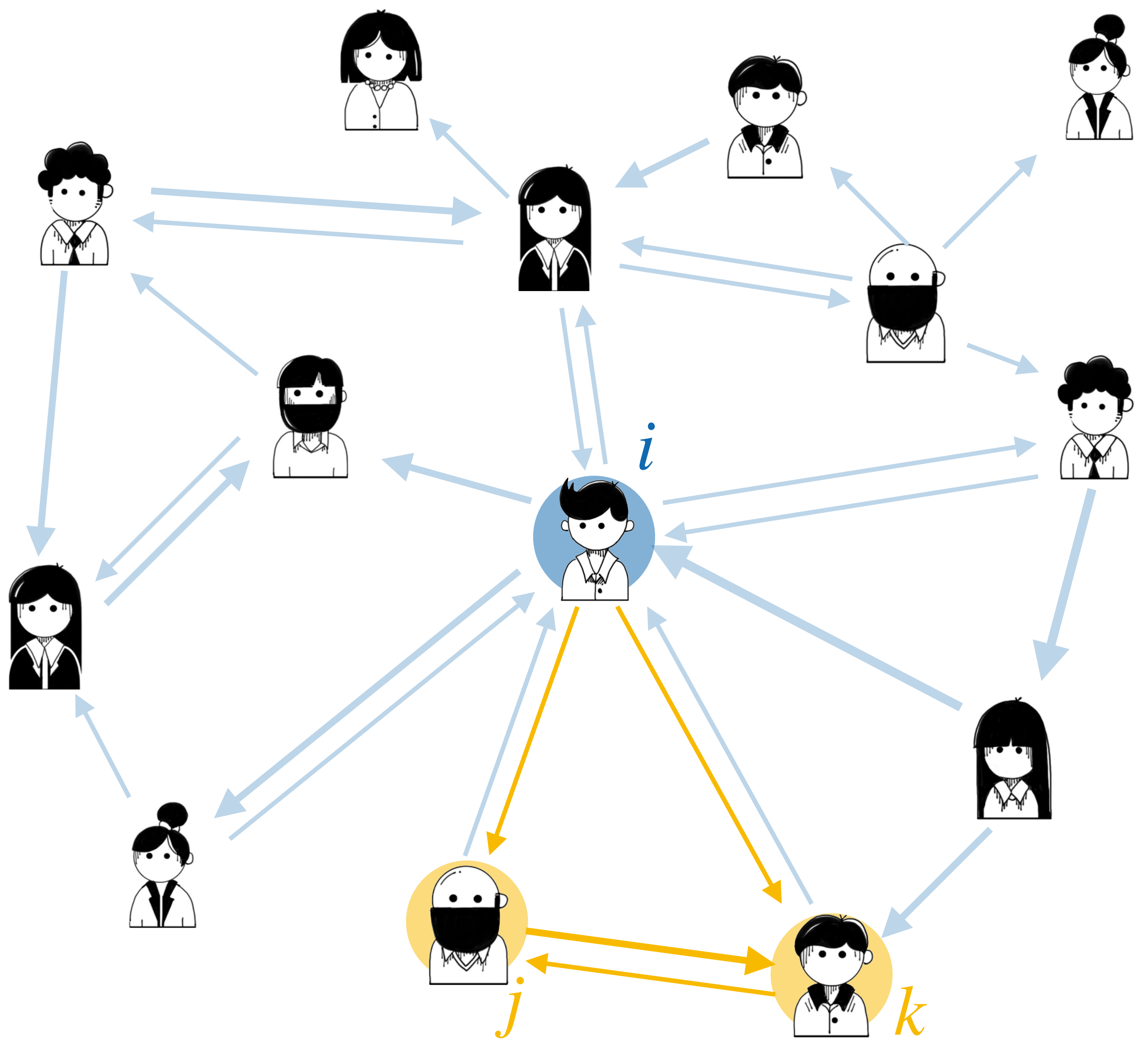
Clustering coefficient

$$u_i(a_i, \mathbf{a}_{-i}) = \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i,j} a_{ik} a_{kj} \right)$$

[Burger, M. J. & Buskens, V. Social context and network formation: an experimental study. Social Networks 31, 63–75 (2009)]



PAYOFF FUNCTION

$$V_i(a_i, \mathbf{a}_{-i}) = t_i(a_i, \mathbf{a}_{-i}) + u_i(a_i, \mathbf{a}_{-i}) - c_i(a_i)$$

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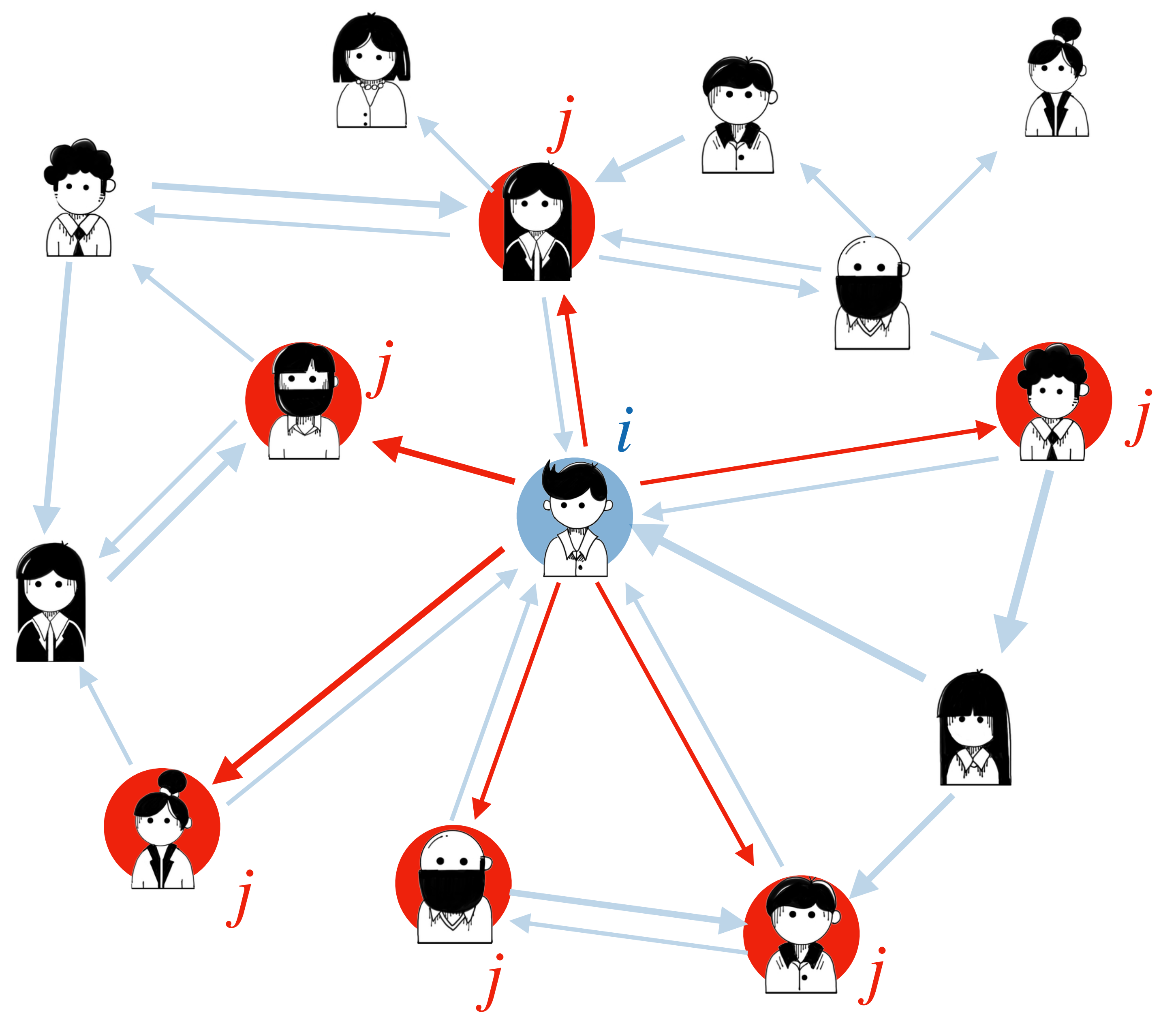
Cost

$$c_i(a_i) = \sum_{j \neq i} a_{ij}$$



05

PAYOFF FUNCTION

$$V_i(a_i, \mathbf{a}_{-i} | \theta_i) = \alpha_i \, t_i(a_i, \mathbf{a}_{-i}) + \beta_i \, u_i(a_i, \mathbf{a}_{-i}) - \gamma_i \, c_i(a_i)$$

$$\theta_i = \{\alpha_i, \beta_i, \gamma_i\}$$

$$\alpha_i \geq 0, \beta_i \in \mathbb{R}, \gamma_i > 0$$

Social influence on friends

$$t_i(a_i, \mathbf{a}_{-i}) = \sum_{j \neq i} a_{ji} + \underbrace{\delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}} + \underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji}}_{\text{paths of length 3}}$$

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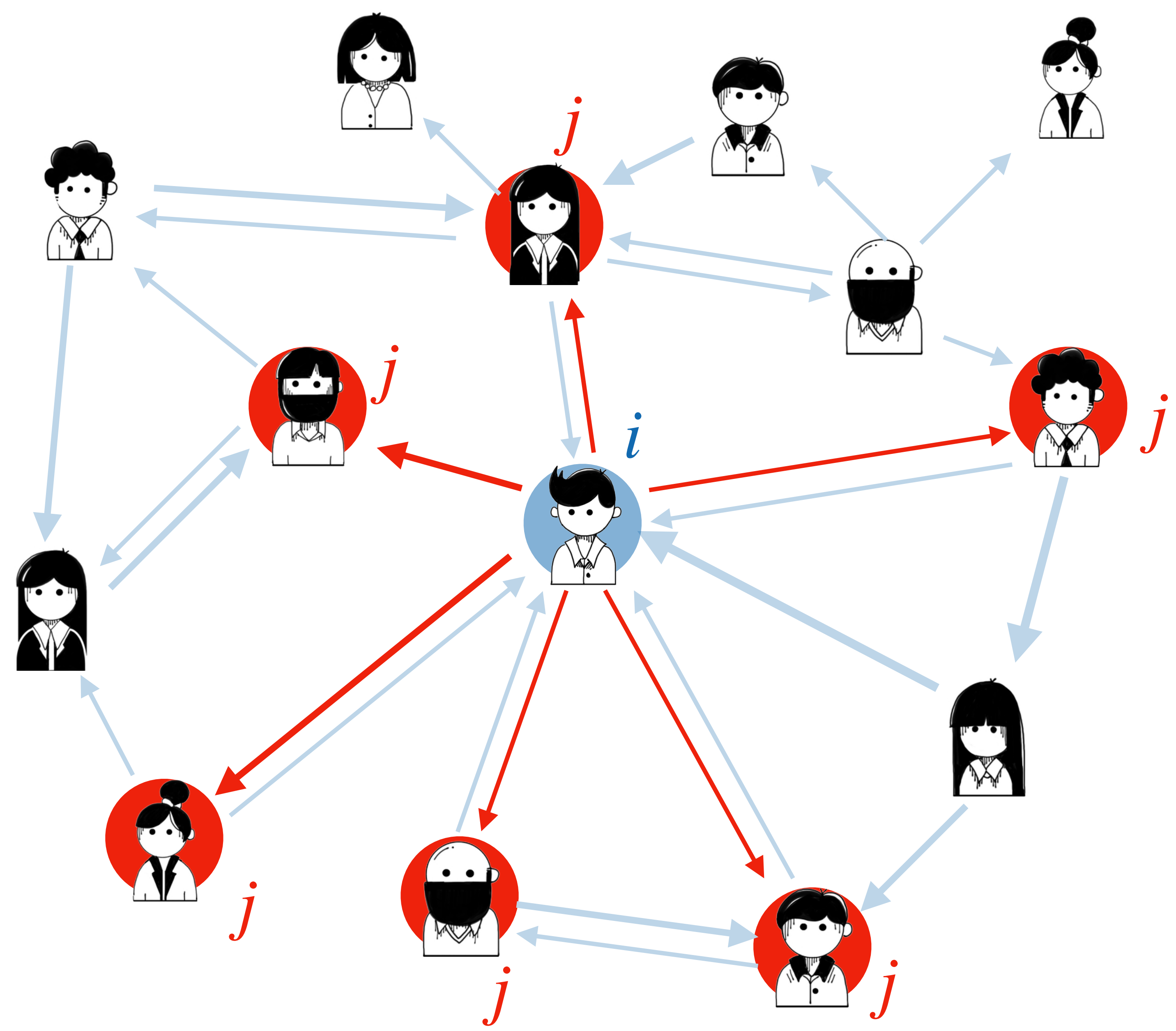


05

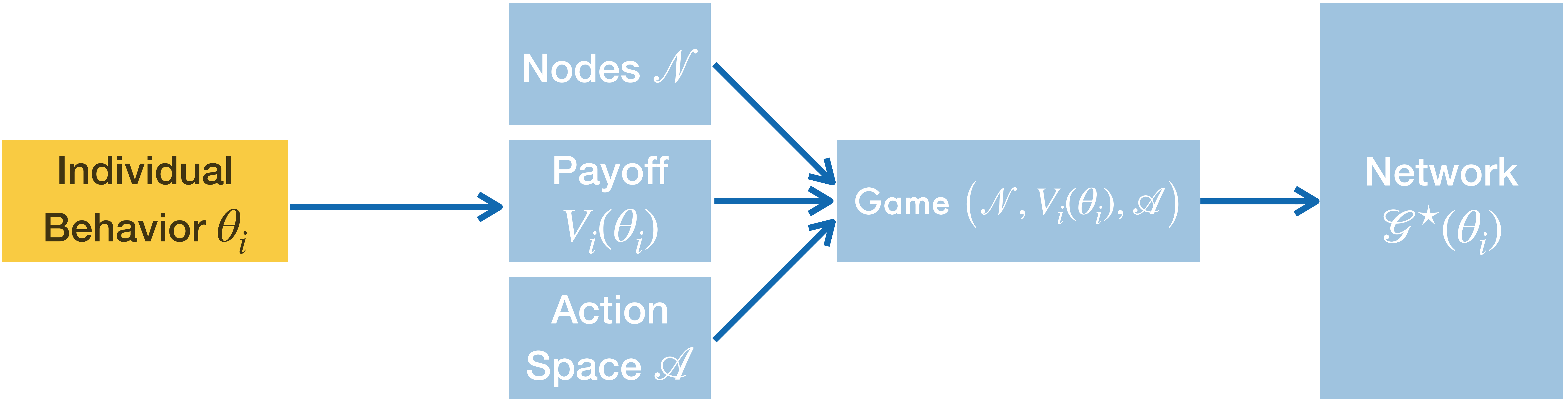
NASH EQUILIBRIUM

Definition.

The network $\mathscr{G}^\star$ is a **Nash Equilibrium** if for all agents i:

$$V_i\left(a_i, \mathbf{a_{-i}}^\star | \theta_i\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star | \theta_i\right), \forall a_i \in \mathscr{A}$$

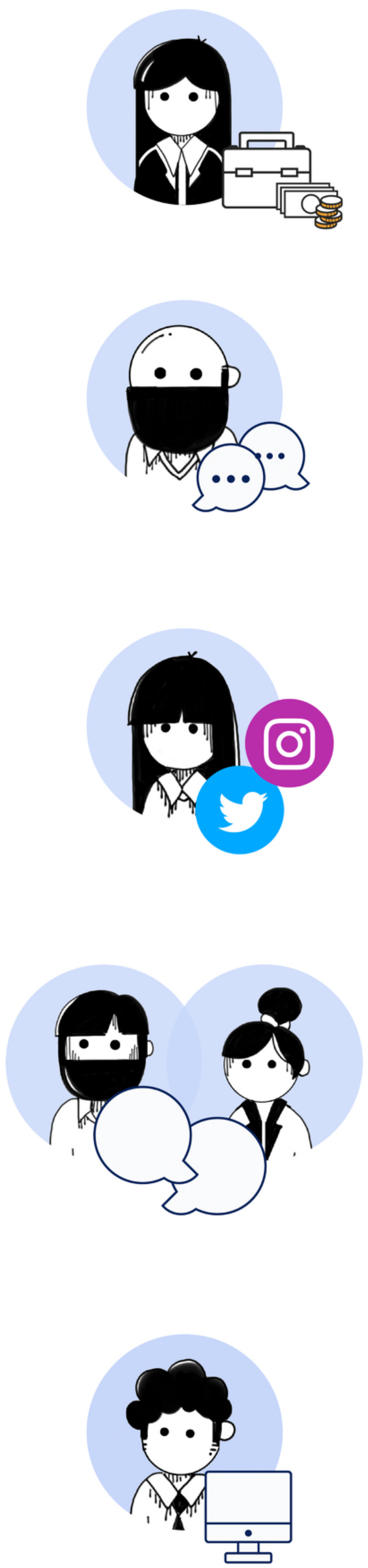
$$\implies \quad \forall i,\ a_i^\star \in \arg\max_{a_i \in \mathscr{A}} V_i\left(a_i, \mathbf{a}_{-i}^\star | \theta_i\right)$$



06

Question: Given θ_i, which $\mathscr{G}^\star$ is in equilibrium ?

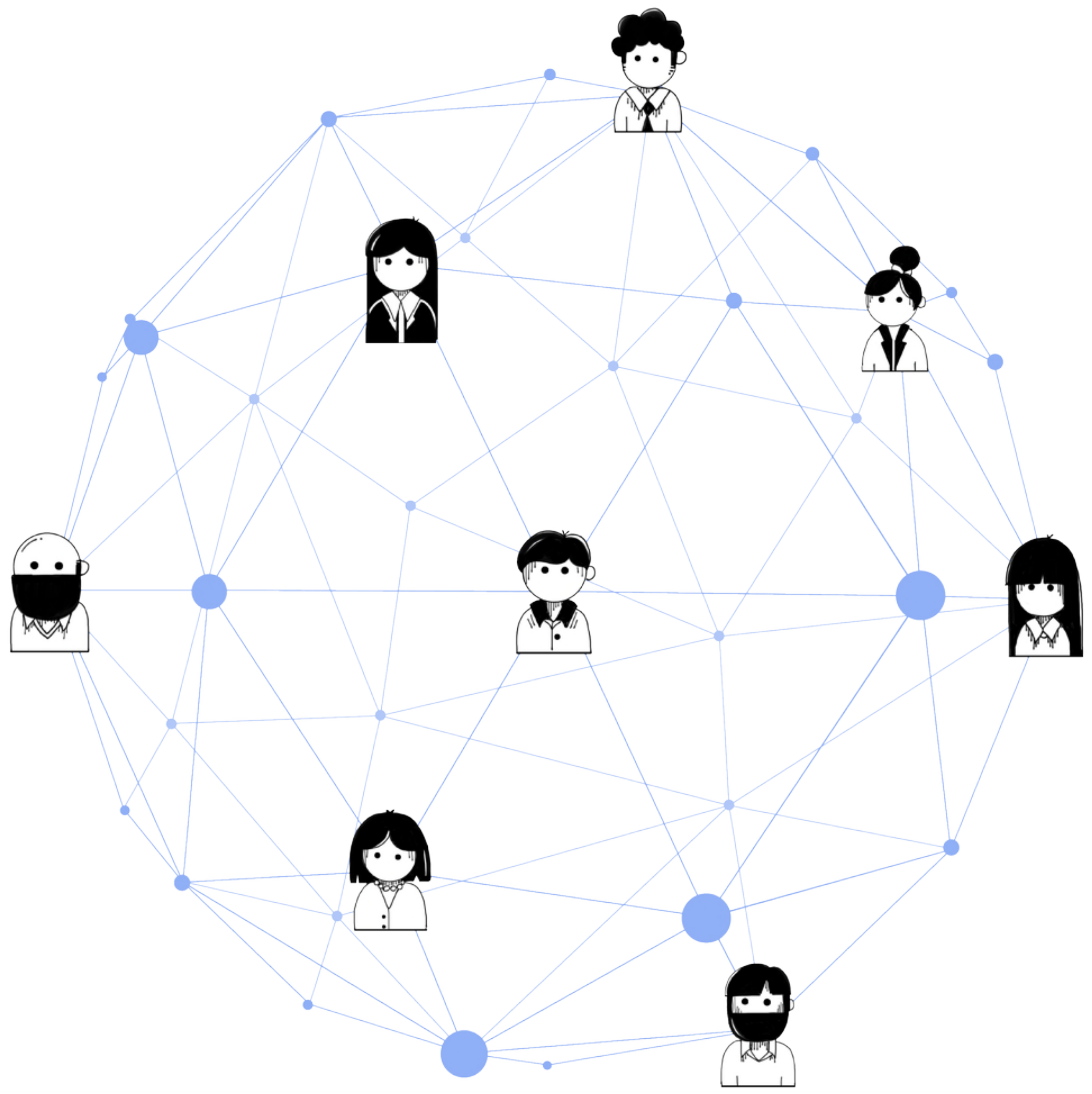
INDIVIDUAL BEHAVIOR θ_i



DETERMINE

$$\forall i,\ a_i^\star \in \arg\max_{a_i \in \mathscr{A}} V_i\left(a_i, \mathbf{a}_{-i}^\star \,|\, \theta_i\right)$$

STRATEGIC NETWORK FORMATION MODEL



SOCIAL NETWORK STRUCTURE $\mathscr{G}^\star\left(\theta_i\right)$

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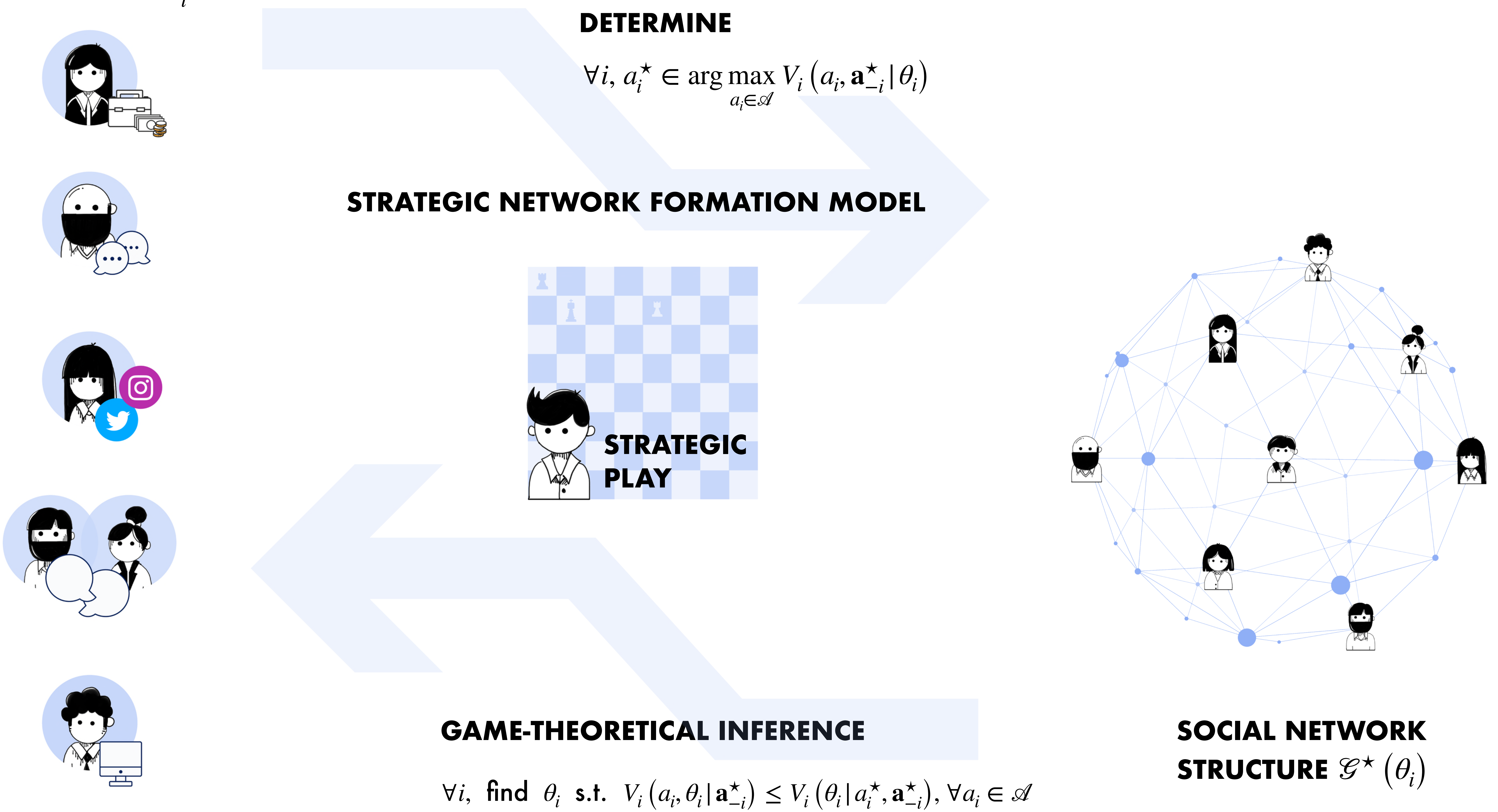


HOMOGENEOUS RATIONAL AGENTS

$$V_i(a_i, \mathbf{a}_{-i} \mid \theta) = \frac{\alpha}{\gamma}\, t_i(a_i, \mathbf{a}_{-i}) + \frac{\beta}{\gamma}\, u_i(a_i, \mathbf{a}_{-i}) - c_i(a_i)$$

$$\theta = \{\alpha, \beta, \gamma\}$$

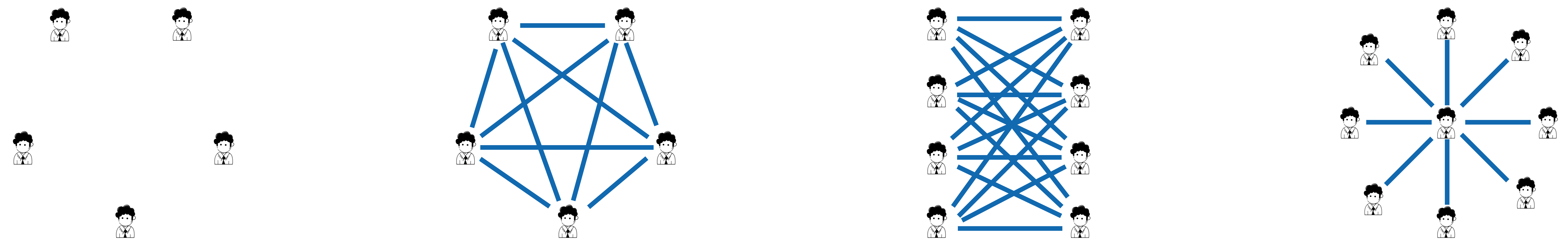
$$\alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0$$

Assumptions.

(i) **Homogeneity**: $\theta_i = \theta$, for all agents i.

(ii) Fully **rational** agents.

Derive **Necessary** and **Sufficient** conditions for Nash equilibrium stability of 4 stylised network motifs.



Empty Complete Complete Balanced Bipartite Star

07

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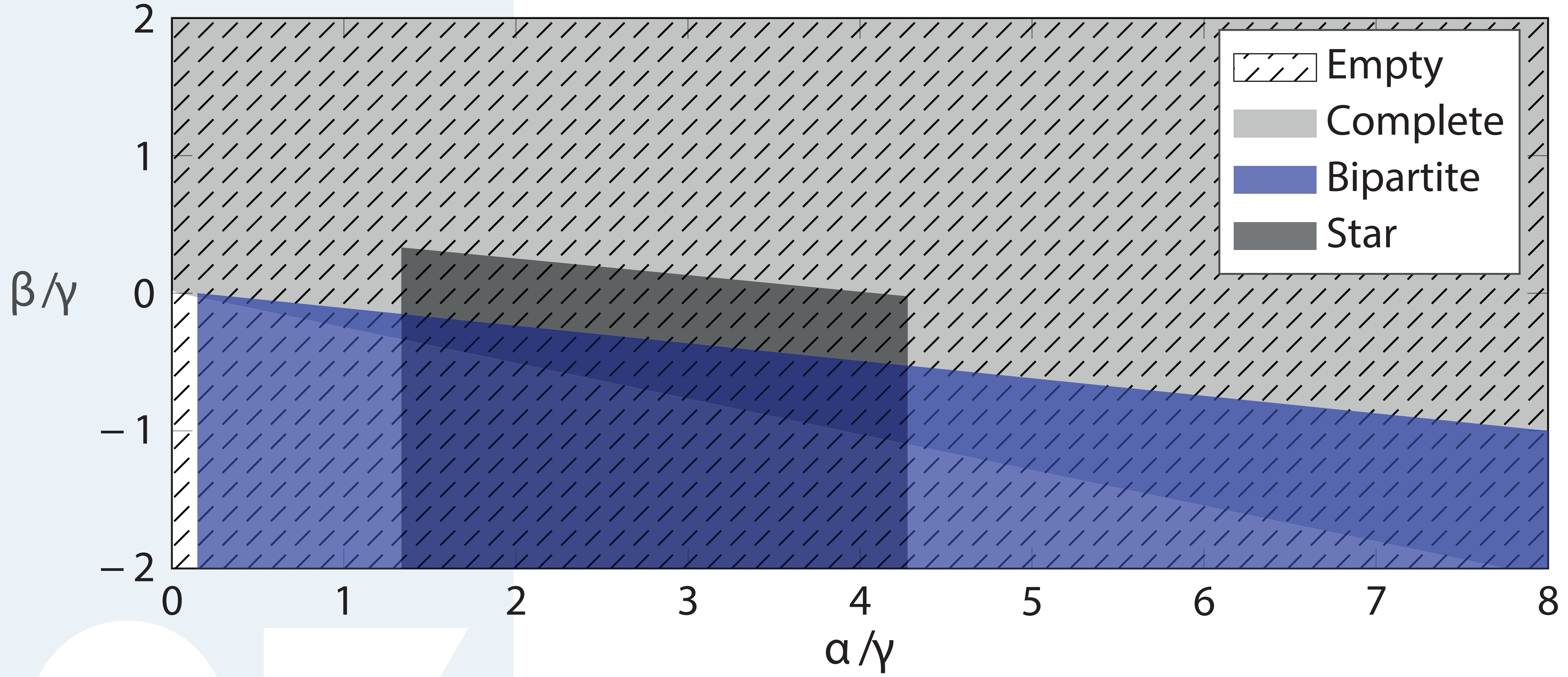

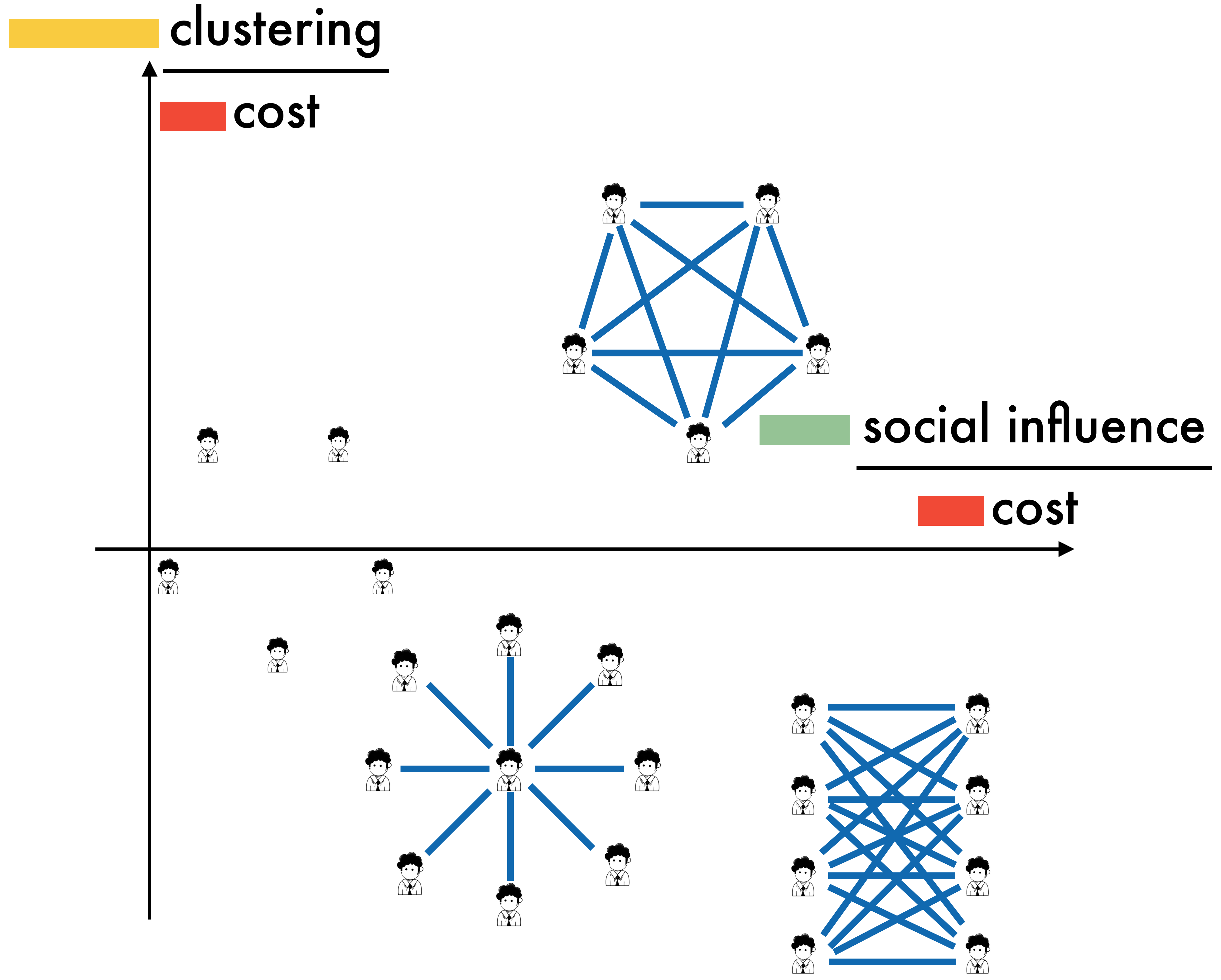

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Using the **Variational Inequality** approach, it is equivalent to

$$\left\langle \nabla V_i\left(a_i, \mathbf{a}_{-i}^\star | \theta\right)\Big|_{a_i^\star}, a_i - a_i^\star \right\rangle \leq 0, \quad \forall a_i \in \mathcal{A}.$$

07

EXAMPLE: COMPLETE NETWORK

$$V_i(a_i, \mathbf{a}_{-i} \,|\, \theta) = \frac{\alpha}{\gamma}\, t_i(a_i, \mathbf{a}_{-i}) + \frac{\beta}{\gamma}\, u_i(a_i, \mathbf{a}_{-i}) - c_i(a_i)$$

$$\theta = \{\alpha, \beta, \gamma\}$$

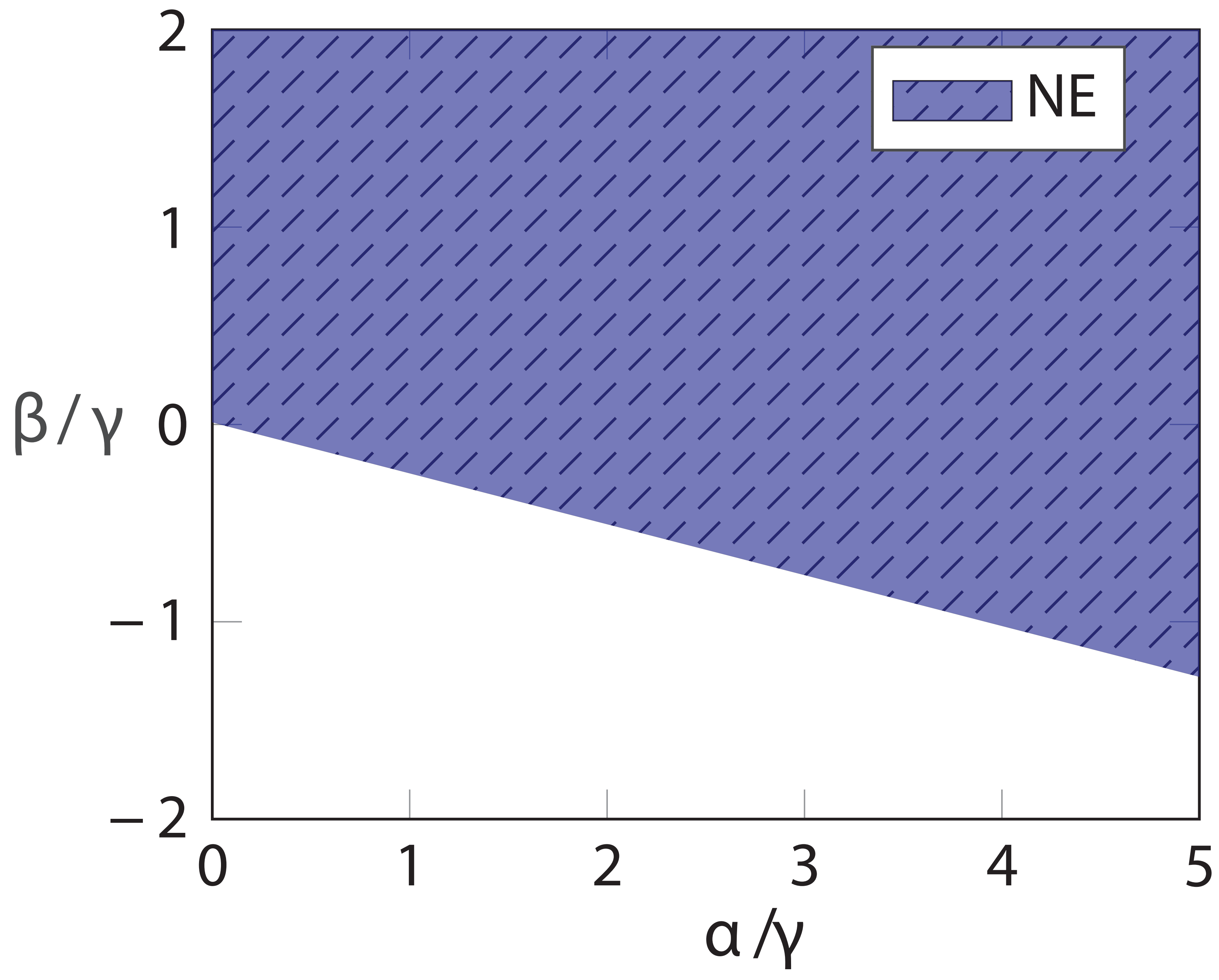
$$\alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0$$

Theorem.

Let $\mathscr{G}^{CN}$ be a complete network of N homogeneous, rational agents. Define:

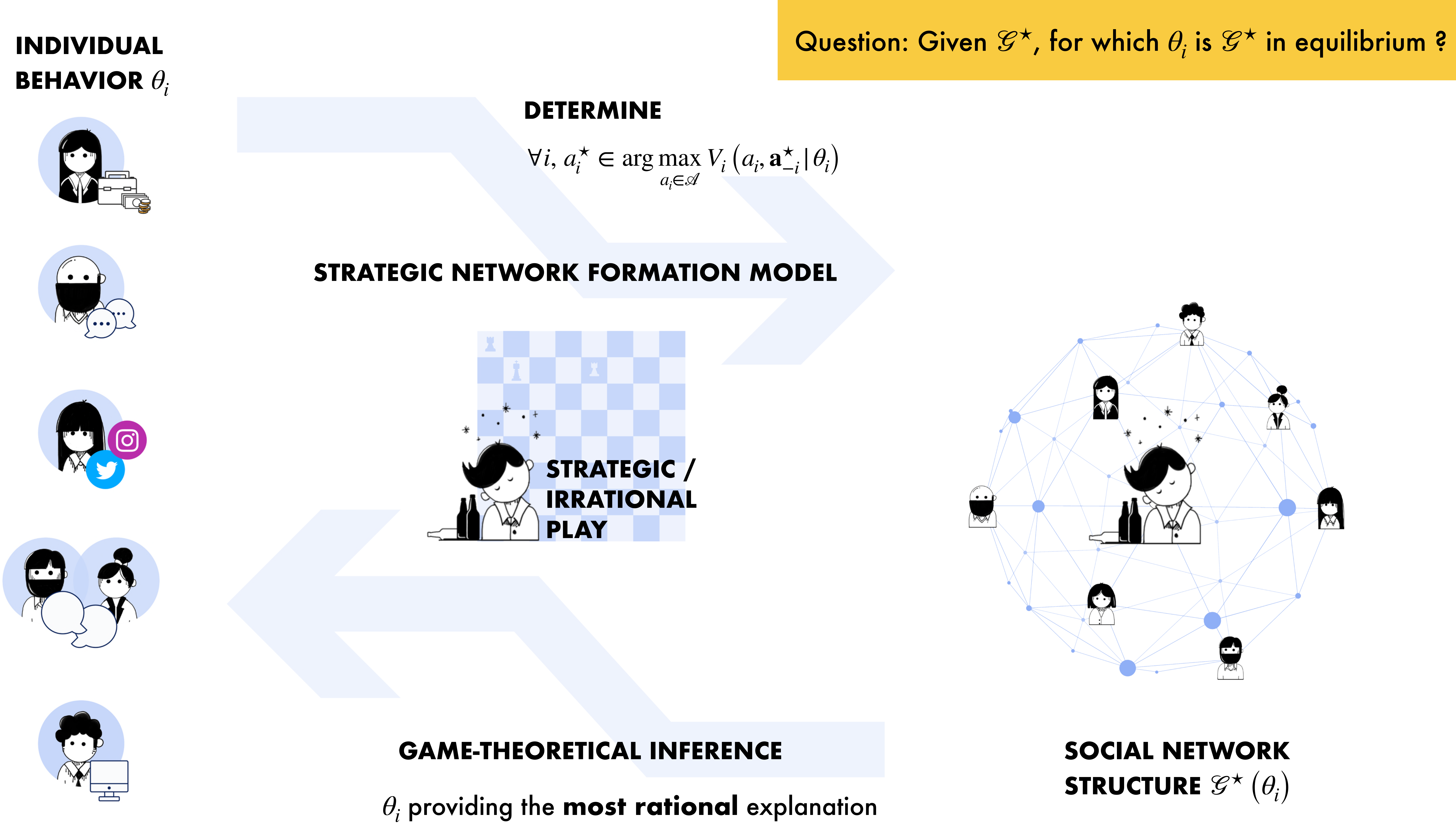
$$\bar{\gamma}_{NE} := \begin{cases} \alpha\delta\left(1 + \delta(2N-3)\right) + \beta\,(N-2), & \text{if } \beta > 0 \\ \alpha\delta\left(1 + \delta(2N-3)\right) + 2\beta\,(N-2), & \text{if } \beta \leq 0, \end{cases}$$

then $\mathscr{G}^{CN}$ is a Nash equilibrium if and only if $\gamma \leq \bar{\gamma}_{NE}$.



08


Question: Given $\mathscr{G}^\star$, for which θ_i is $\mathscr{G}^\star$ in equilibrium ?
INDIVIDUAL BEHAVIOR θ_i
DETERMINE
$\forall i,\ a_i^\star \in \arg\max_{a_i \in \mathscr{A}} V_i\left(a_i, \mathbf{a}_{-i}^\star \,|\, \theta_i\right)$
STRATEGIC NETWORK FORMATION MODEL
STRATEGIC PLAY
GAME-THEORETICAL INFERENCE
$\forall i,$ find θ_i s.t. $V_i\left(a_i, \theta_i \,|\, \mathbf{a}_{-i}^\star\right) \leq V_i\left(\theta_i \,|\, a_i^\star, \mathbf{a}_{-i}^\star\right), \forall a_i \in \mathscr{A}$
SOCIAL NETWORK STRUCTURE $\mathscr{G}^\star\left(\theta_i\right)$



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DETERMINE
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STRATEGIC NETWORK FORMATION MODEL
STRATEGIC / IRRATIONAL PLAY
GAME-THEORETICAL INFERENCE
θ_i providing the **most rational** explanation
SOCIAL NETWORK STRUCTURE $\mathscr{G}^\star\left(\theta_i\right)$

INVERSE OPTIMIZATION PROBLEM

Error function.

Deviation from Nash equilibrium:

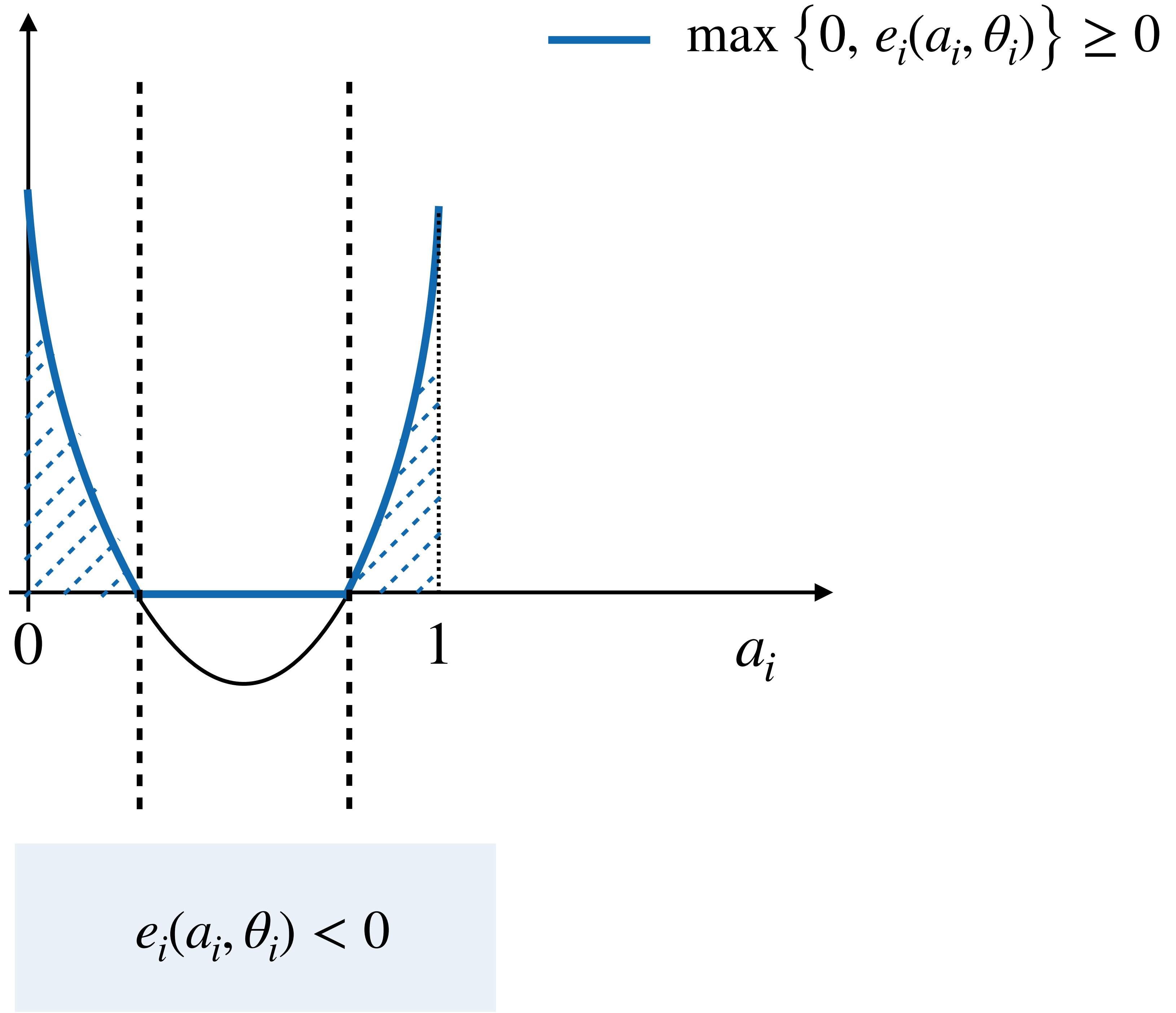
$$e_i(a_i, \theta_i) := V_i\left(a_i, \mathbf{a}^\star_{-i} \,|\, \theta_i\right) - V_i\left(a_i^\star, \mathbf{a}^\star_{-i} \,|\, \theta_i\right)$$

Positive error corresponds to a violation of the Nash equilibrium condition:

$$\max\left\{0,\, e_i(a_i, \theta_i)\right\} \geq 0$$

Distance function.

$$d_i(\theta_i) := \int_{\mathscr{A}} \left(\max\left\{0,\, e_i(a_i, \theta_i)\right\}\right)^2 da_i$$



No violations: can be neglected

09

INVERSE OPTIMIZATION PROBLEM

Problem [Minimum NE-Distance Problem].

Given a network $\mathscr{G}^\star$ of N agents, for all agents i find the vectors of preferences $\theta_i^\star$ such that

$$\theta_i^\star \in \arg\min_{\theta_i \in \Theta} \; d_i(\theta_i)$$

Theorem [Smoothness & convexity of distance function].

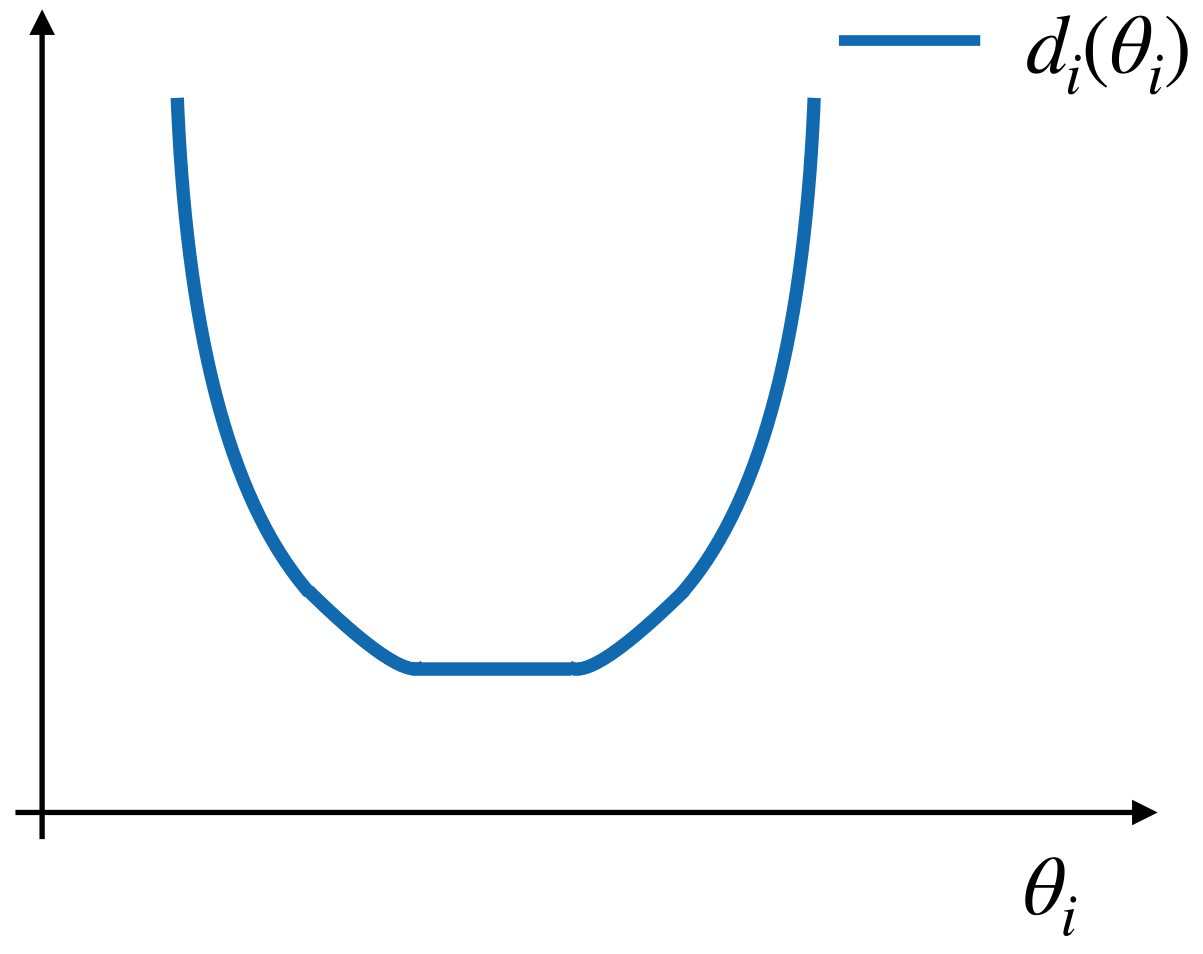
Let

$$d_i(\theta_i) = \int_{\mathscr{A}} \left(\max\left\{0,\, e_i(a_i, \theta_i)\right\} \right)^2 da_i$$

Then $d_i(\theta_i)$ is continuously differentiable, and its gradient reads as

$$\nabla_\theta d_i(\theta) = \int_{\mathscr{A}} 2\, \nabla_{\theta_i}\left(e_i(a_i, \theta_i)\right) \max\left\{0,\, e_i(a_i, \theta_i)\right\} da_i \,.$$

Moreover, $d_i(\theta_i)$ is convex.



INVERSE OPTIMIZATION PROBLEM - SOLUTION

First-order optimality condition

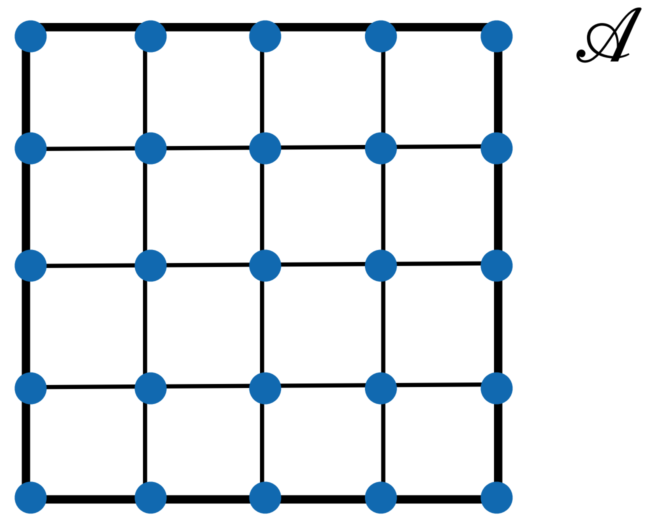
$$0 = \nabla_{\theta_i}\big(d_i(\theta_i)\big) = 2\int_{\mathcal{A}} \nabla_{\theta_i}\big(e_i(a_i,\theta_i)\big)\max\left\{0,\, e_i(a_i,\theta_i)\right\} da_i\,.$$

max operator within $(N-1)$ - dimensional integral

INVERSE OPTIMIZATION PROBLEM - SOLUTION

Search for an approximate solution: Consider a finite set of possible actions (samples)

$$\left\{a_i^j\right\}_{j=1}^{n_i} \subset \mathscr{A}$$



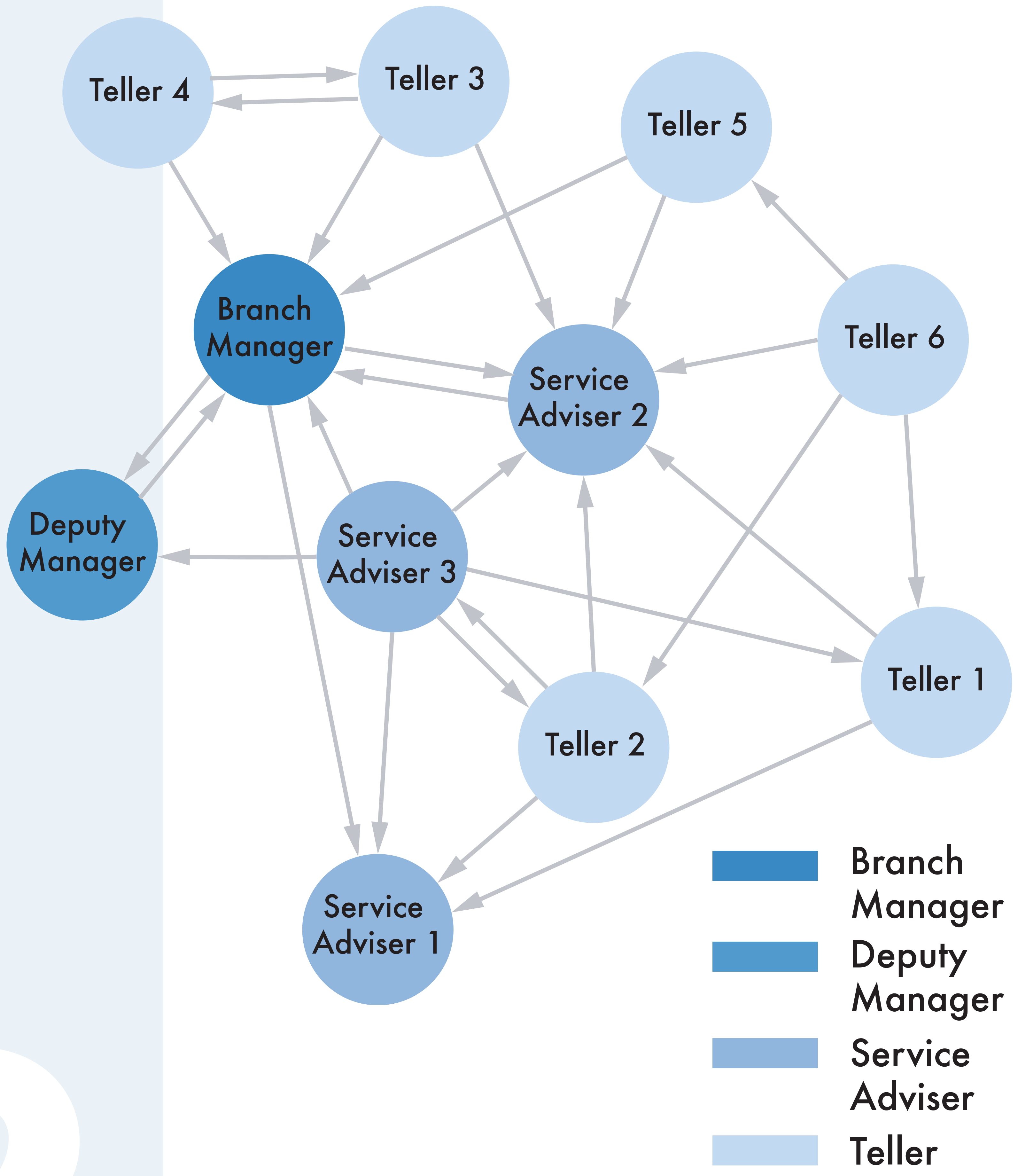
and let $e_i(a_i^j, \theta_i)$ be the corresponding error.

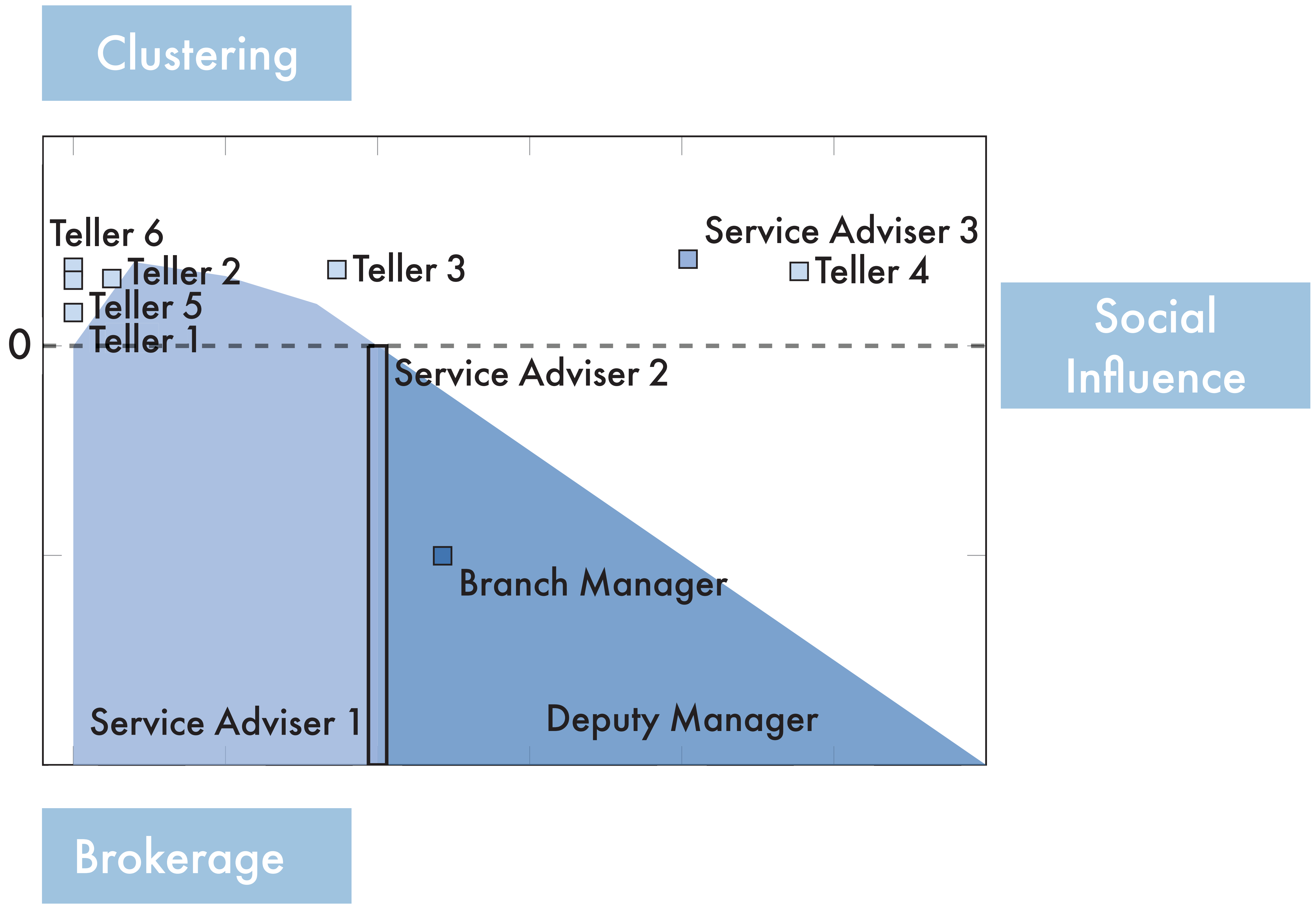
Approximate the distance function as $\tilde{d}_i(\theta_i) := \sum_{j=1}^{n_i} \left(\max\left\{0,\, e_i(a_i^j, \theta_i)\right\}\right)^2 \approx \int_{\mathscr{A}} \left(\max\left\{0,\, e_i(a_i, \theta_i)\right\}\right)^2 dx$

The solution $\hat{\theta}_i \in \arg\min_{\theta_i \in \Theta} \tilde{d}_i(\theta_i)$ is similar to the solution of a Generalized Least Square Regression Problem.

Note: The estimate needs to be unbiased due to the positiveness of the error terms.

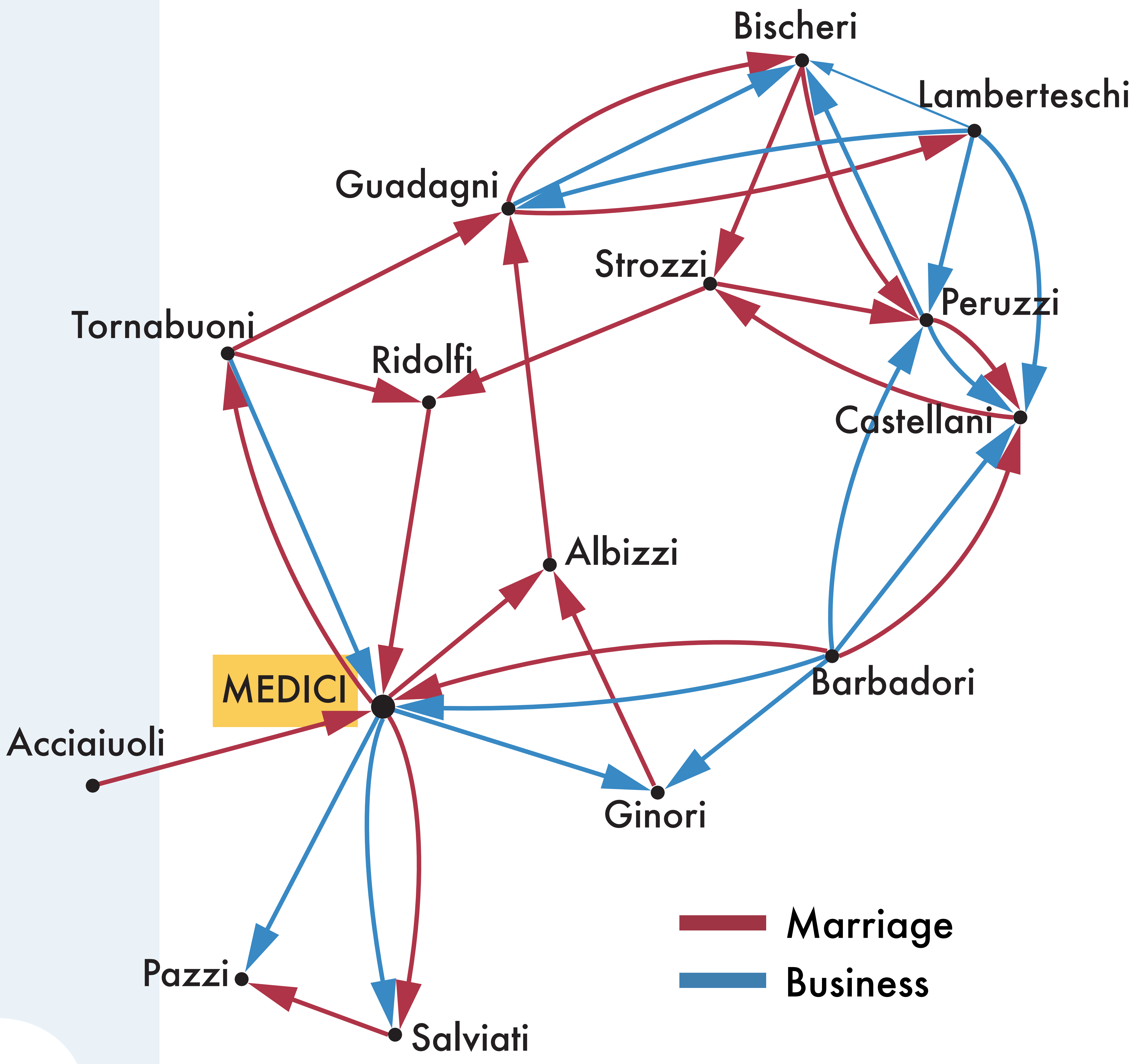
AUSTRALIAN BANK

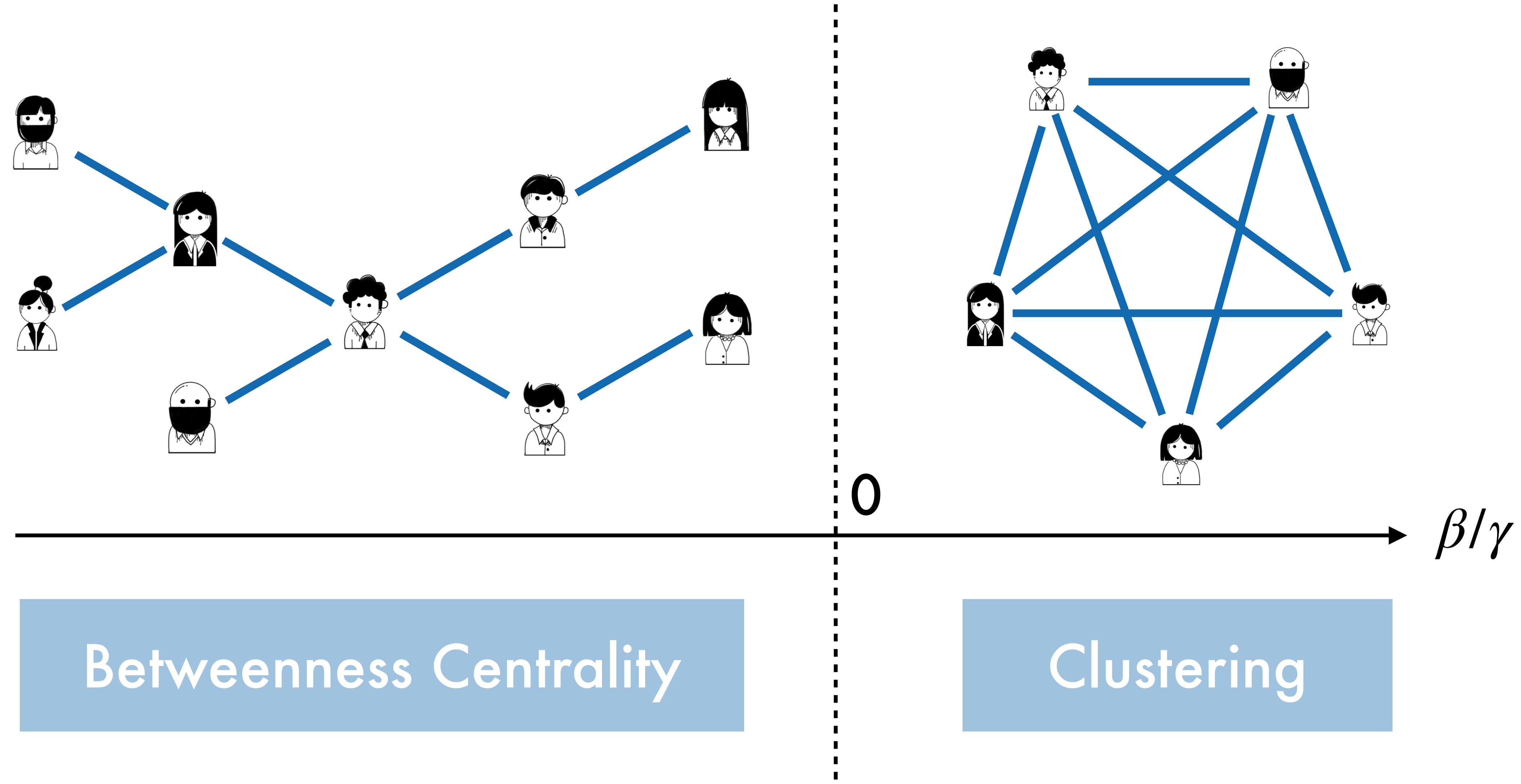


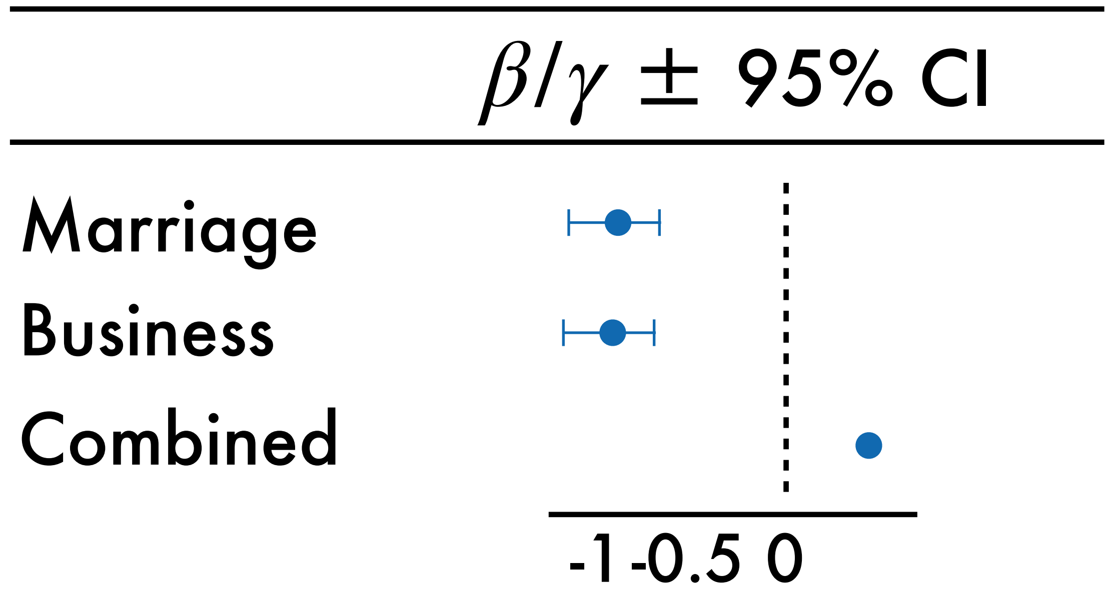


[Pattison, P., Wasserman, S., Robins, G. & Kanfer, A. M. Statistical evaluation of algebraic constraints for social networks. J. Math. Psychology 44, 536–568 (2000)]

RENAISSANCE FLORENCE NETWORK



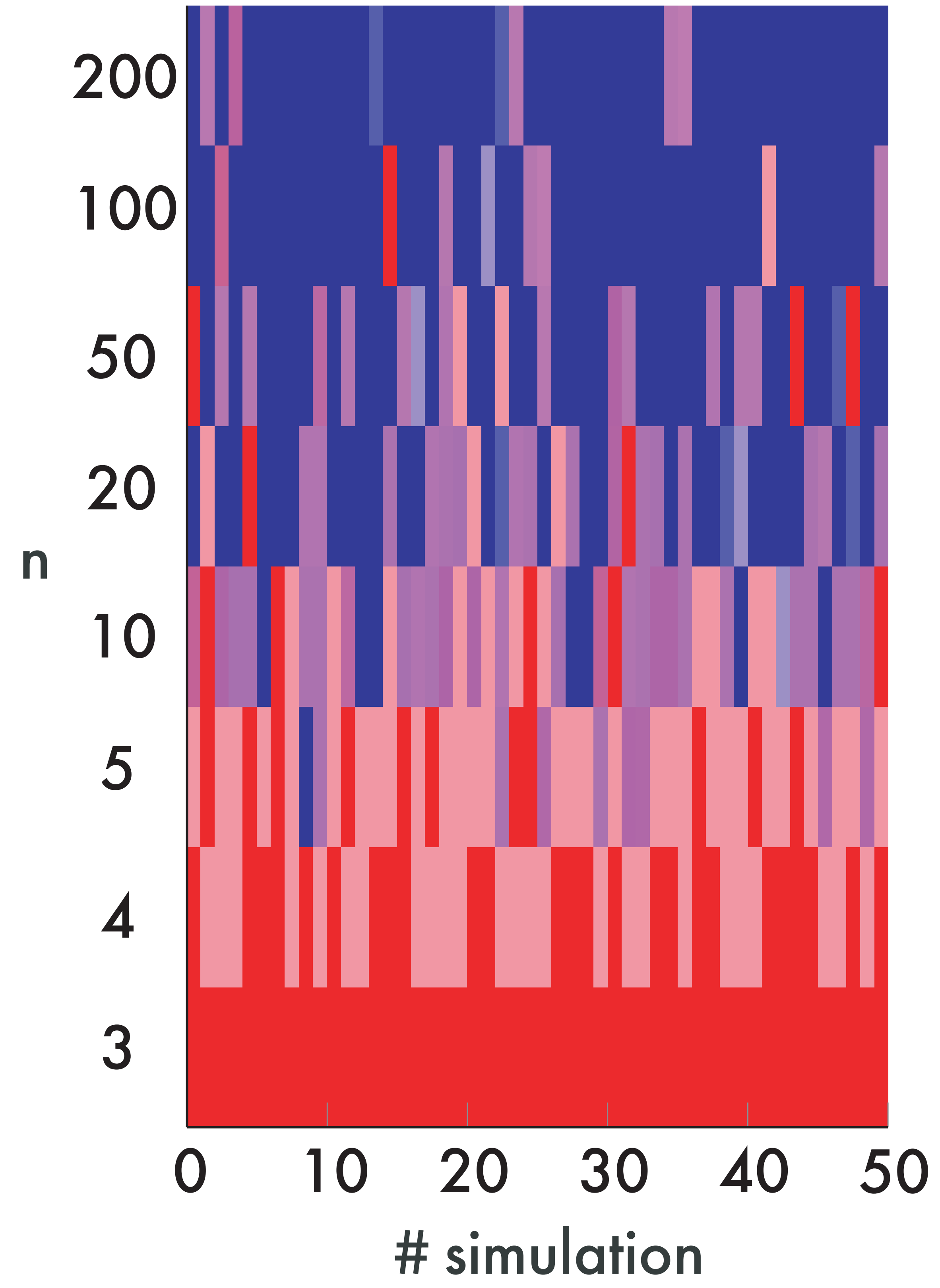


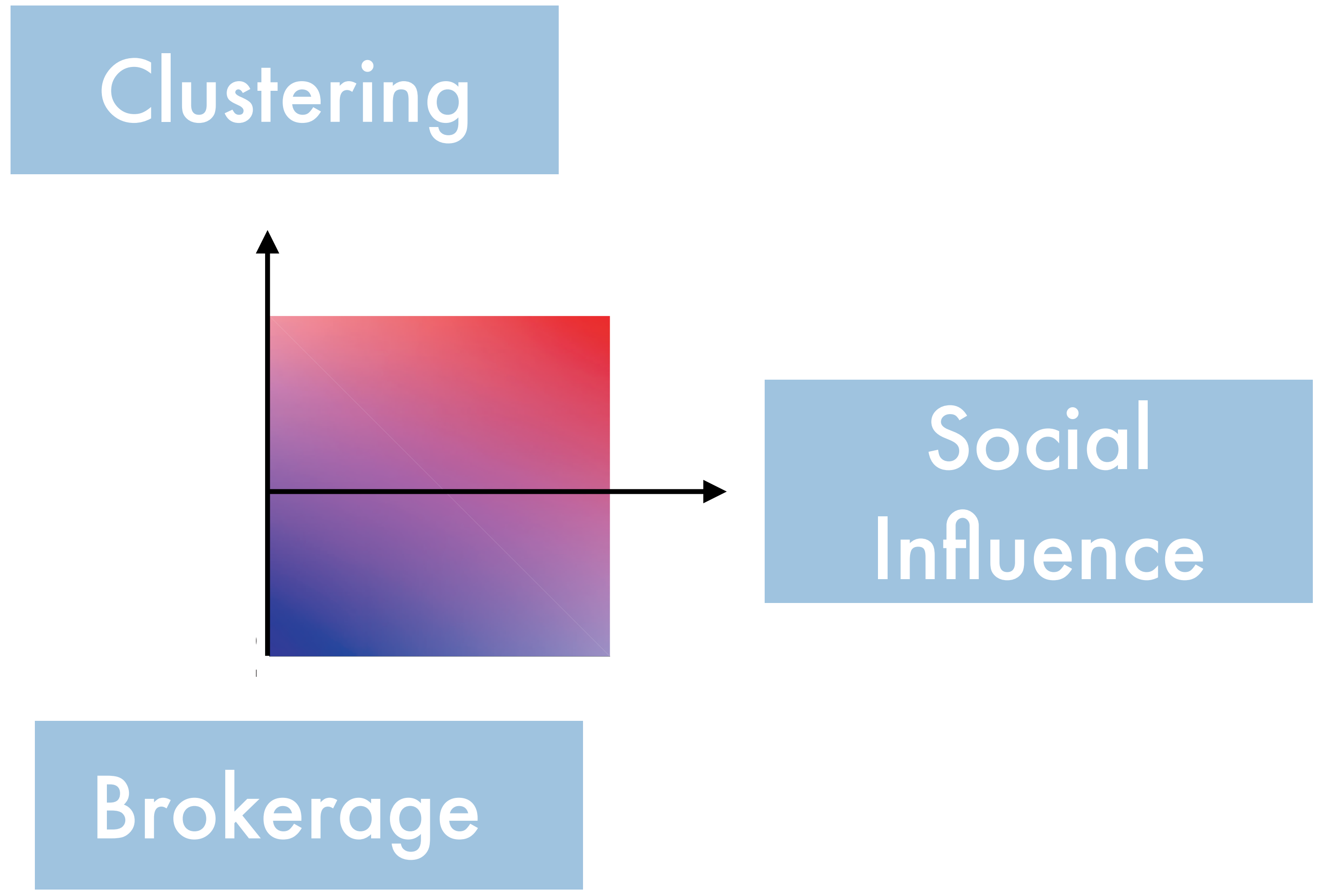


[Padgett, J. F., & Ansell, C. K. (1993). Robust Action and the Rise of the Medici, 1400-1434. *American Journal of Sociology*, 98(6), 1259-1319]

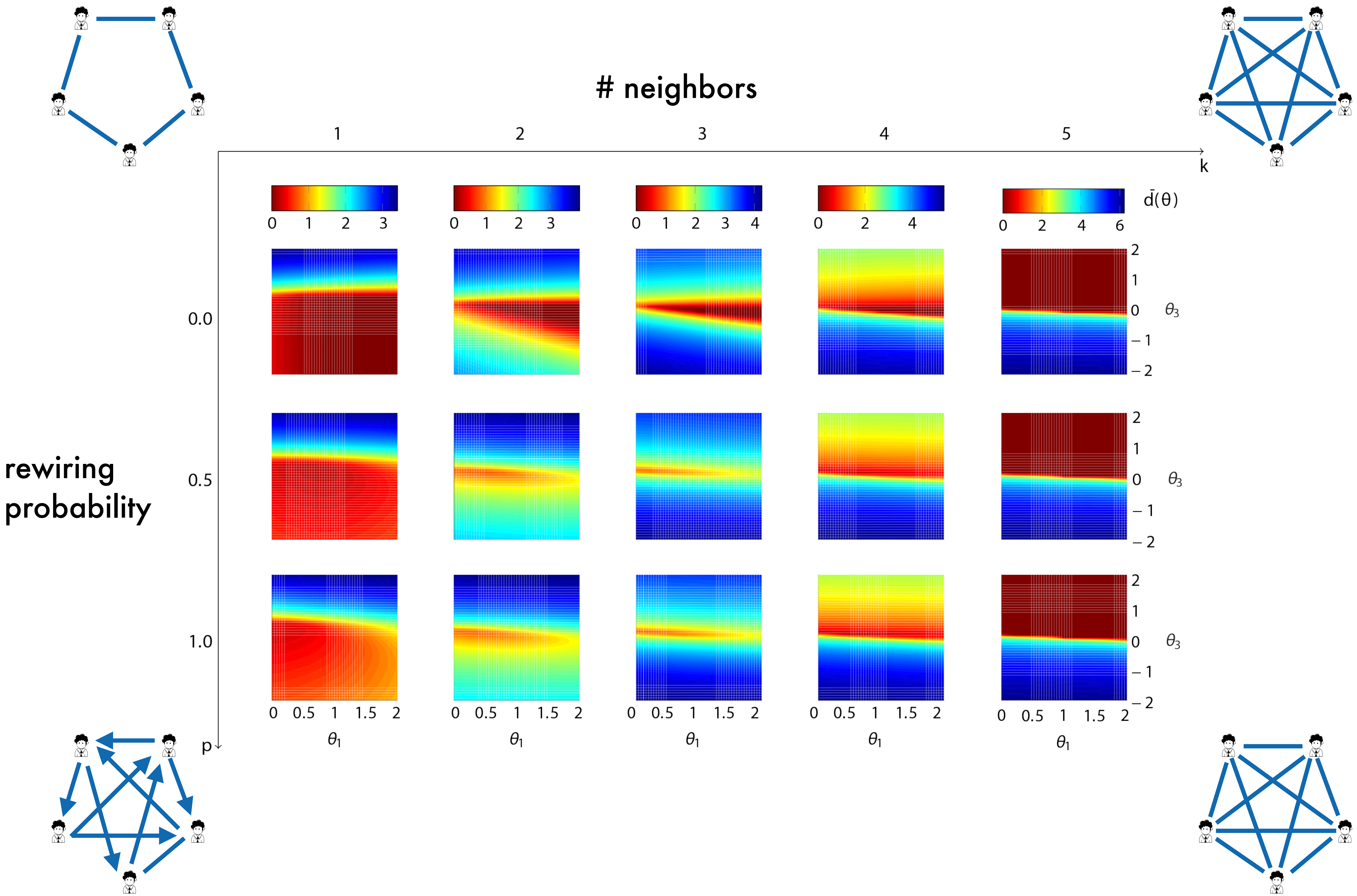
PREFERENTIAL ATTACHMENT MODEL

Nodes are introduced sequentially.
Each newborn **receives 2 incoming ties** from existing nodes (randomly selected, proportionally to the outdegree), and **creates 2 outgoing ties** to existing nodes (randomly selected, proportionally to the indegree).





SMALL-WORLD NETWORKS



SUMMARY & OPEN DIRECTIONS

Starting from the strategic network formation literature, we proposed a new model:

- sociologically well-founded,
- mathematically tractable, and
- statistically robust,

capable of reverse-engineering human behavior from easily accessible data on the network structure.

We provided evidence that our results are consistent with empirical, historical, and sociological observations.

Our method offers socio-strategic interpretations of random network models.

The model can be adapted to further specifications of the payoff function.

Incorporating prior knowledge on the action space of the agents can reduce the computational burden.

Actors' attributes have not yet been considered.

[N. Pagan & F. Dörfler, “*Game theoretical inference of human behavior in social networks*”, Nature Communications, 2019]

NICOLÒ PAGAN
FLORIAN DÖRFLER

AUTOMATIC
CONTROL
LABORATORY
IfA

ETH zürich

BACK UP SLIDES

NASH AND PAIRWISE NASH EQUILIBRIA

Definition.

The network $\mathscr{G}^\star$ is a **Nash Equilibrium** if

- for all agents i:
  $V_i\left(a_i, \mathbf{a_{-i}}^\star \mid \theta_i\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star \mid \theta_i\right), \forall a_i \in \mathscr{A}$.

Definition.

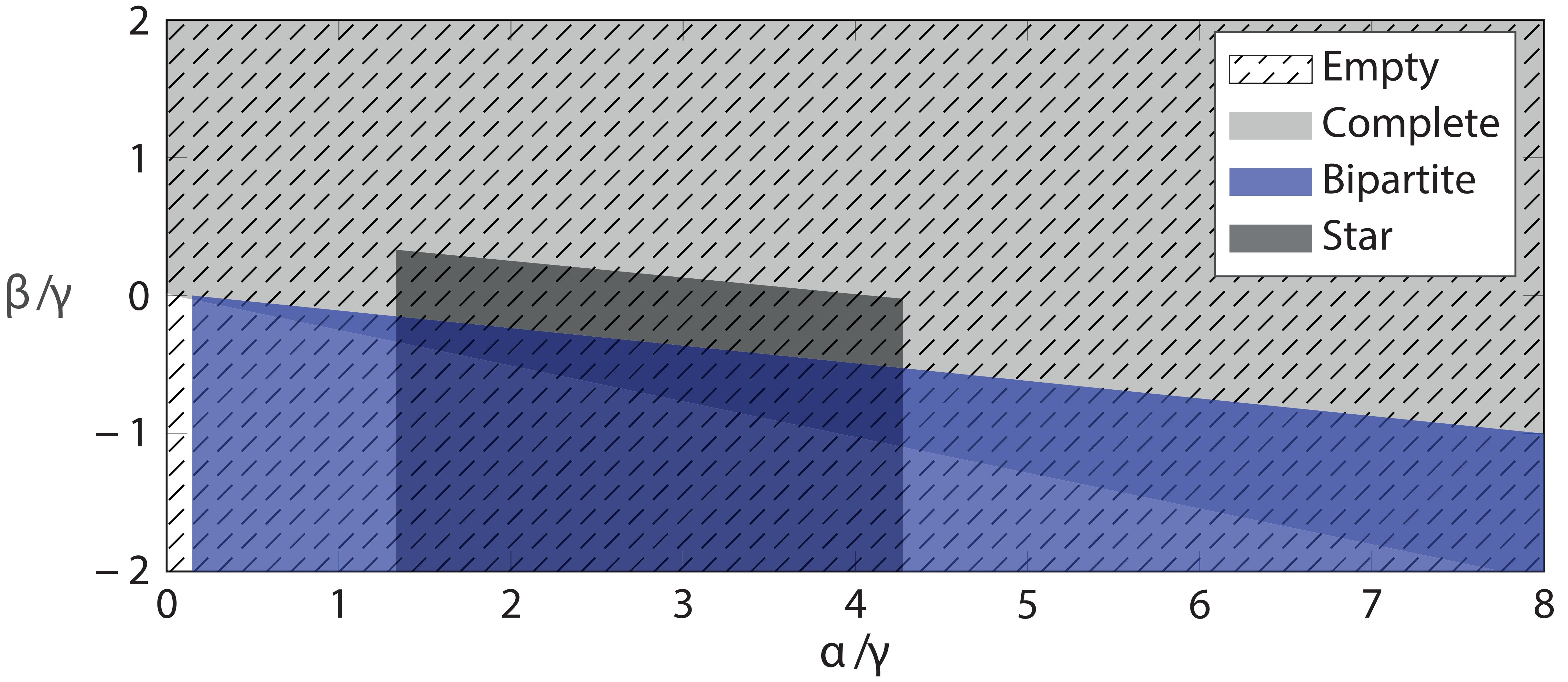
The network $\mathscr{G}^\star$ is a **Pairwise-Nash Equilibrium** if

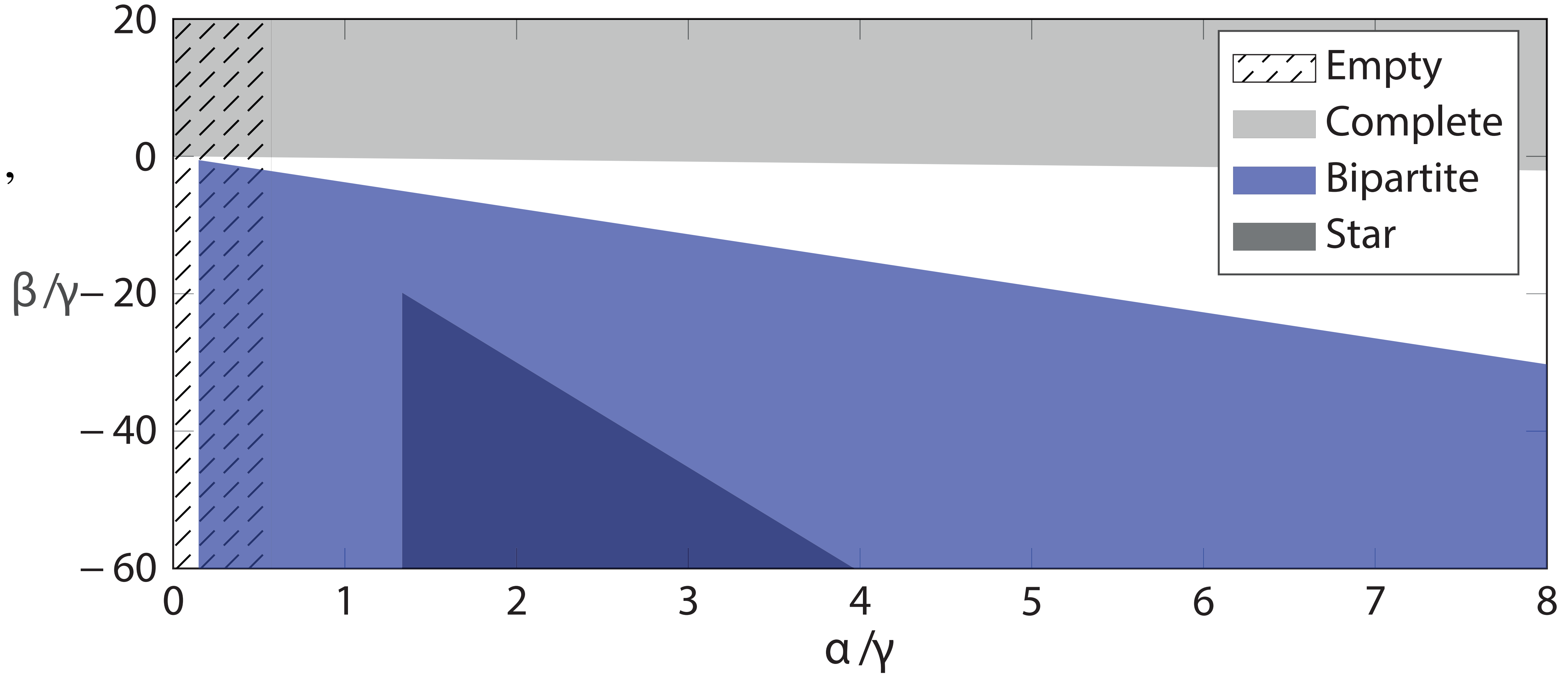
- for all pairs of distinct agents (i, j):
  $V_i\left(a_{ij}, \mathbf{a}^\star_{i-(i,j)}, \mathbf{a}^\star_{-i}\right) \leq V_i\left(a^\star_{ij}, \mathbf{a}^\star_{i-(i,j)}, \mathbf{a}^\star_{-i}\right), \forall a_{ij} \in [0,1],$
- for all pairs of distinct agents (i, j):

$$V_i\left(a_{ij}, a_{ji}, \mathbf{a}^\star_{-(i,j)}\right) > V_i\left(a^\star_{ij}, a^\star_{ji}, \mathbf{a}^\star_{-(i,j)}\right)$$

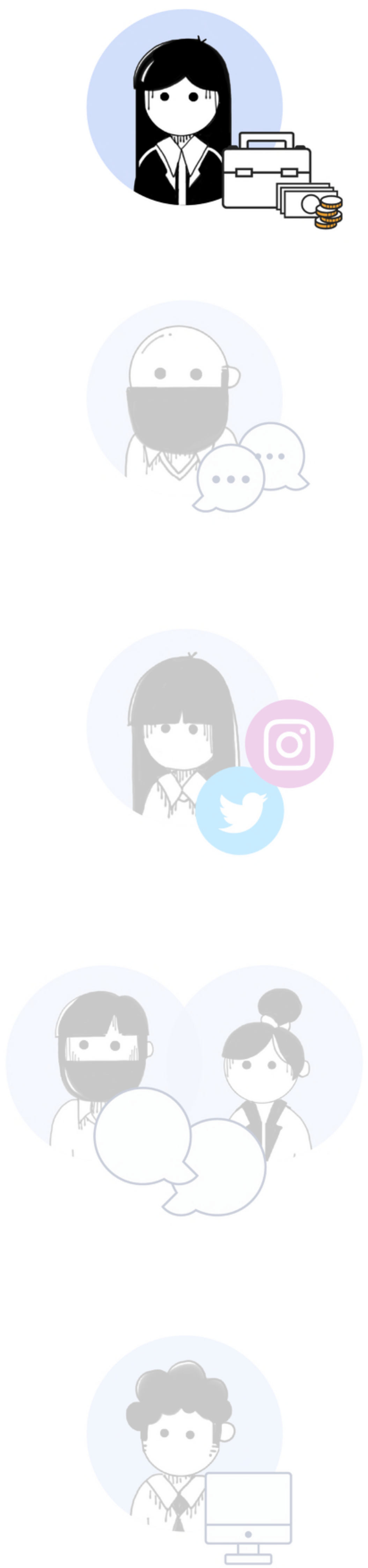
$$\Downarrow$$

$$V_j\left(a_{ij}, a_{ji}, \mathbf{a}^\star_{-(i,j)}\right) < V_j\left(a^\star_{ij}, a^\star_{ji}, \mathbf{a}^\star_{-(i,j)}\right).$$

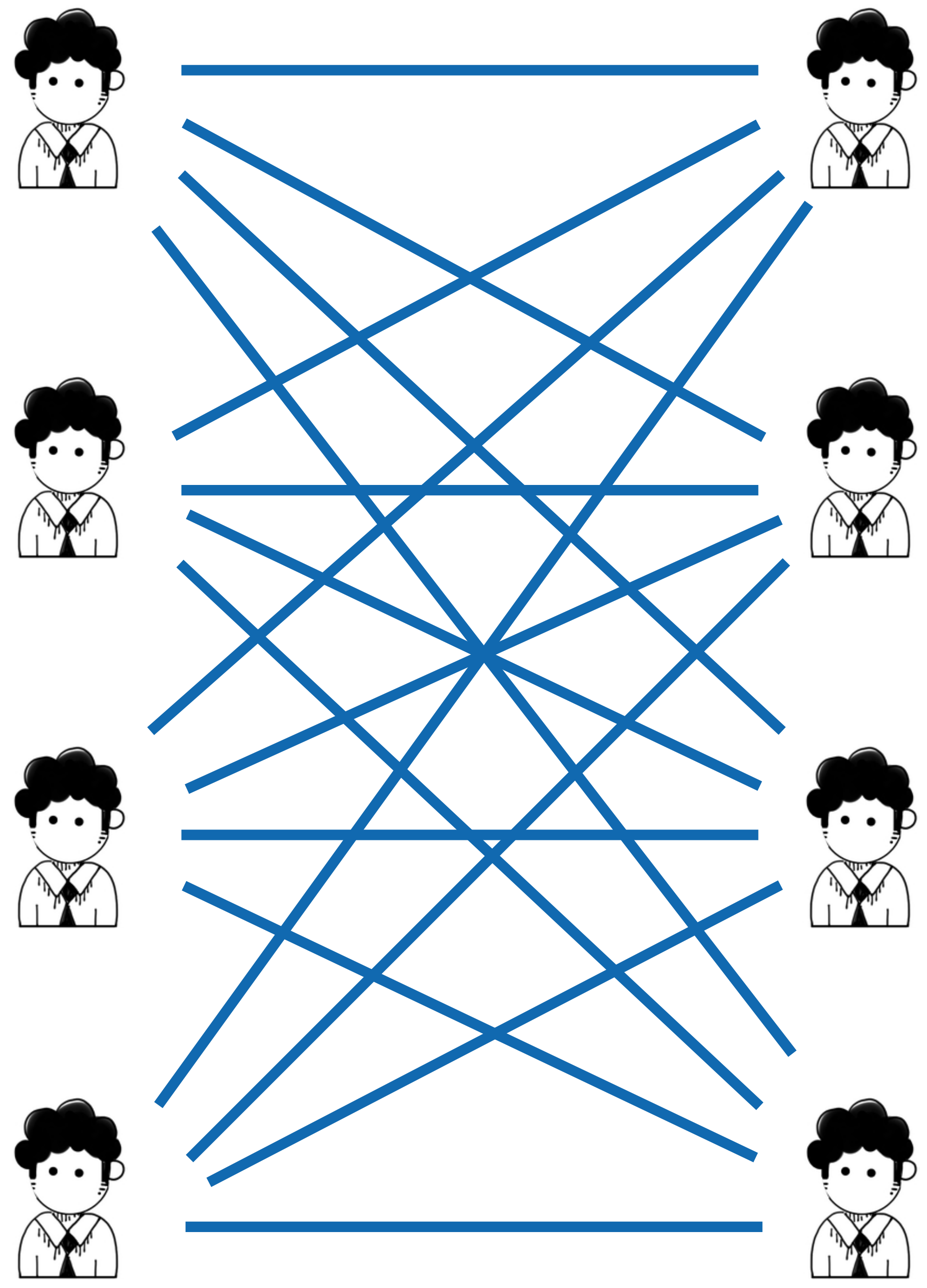




STRATEGIC NETWORK FORMATION MODEL

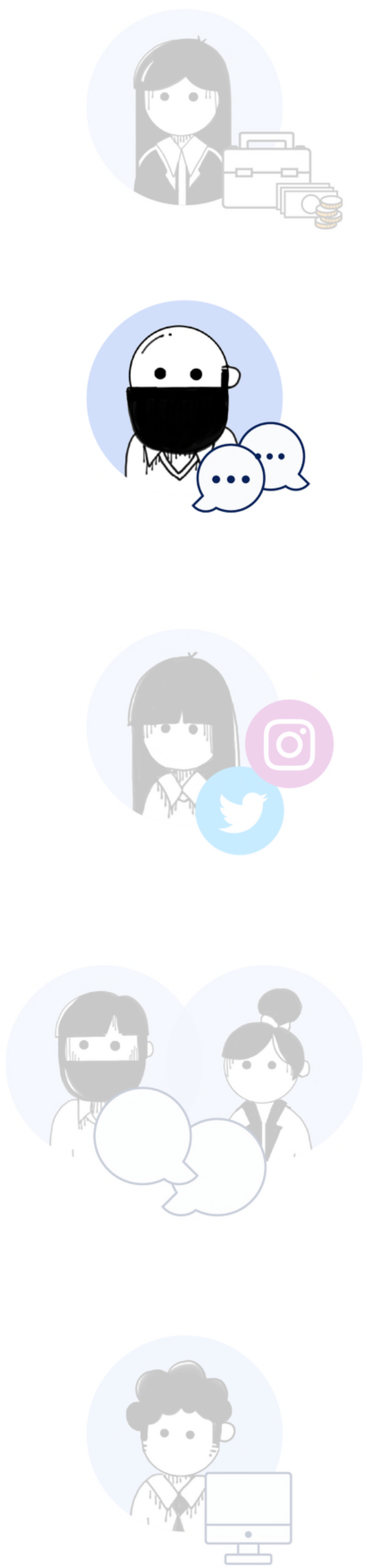


Assumption: Homogeneous agents

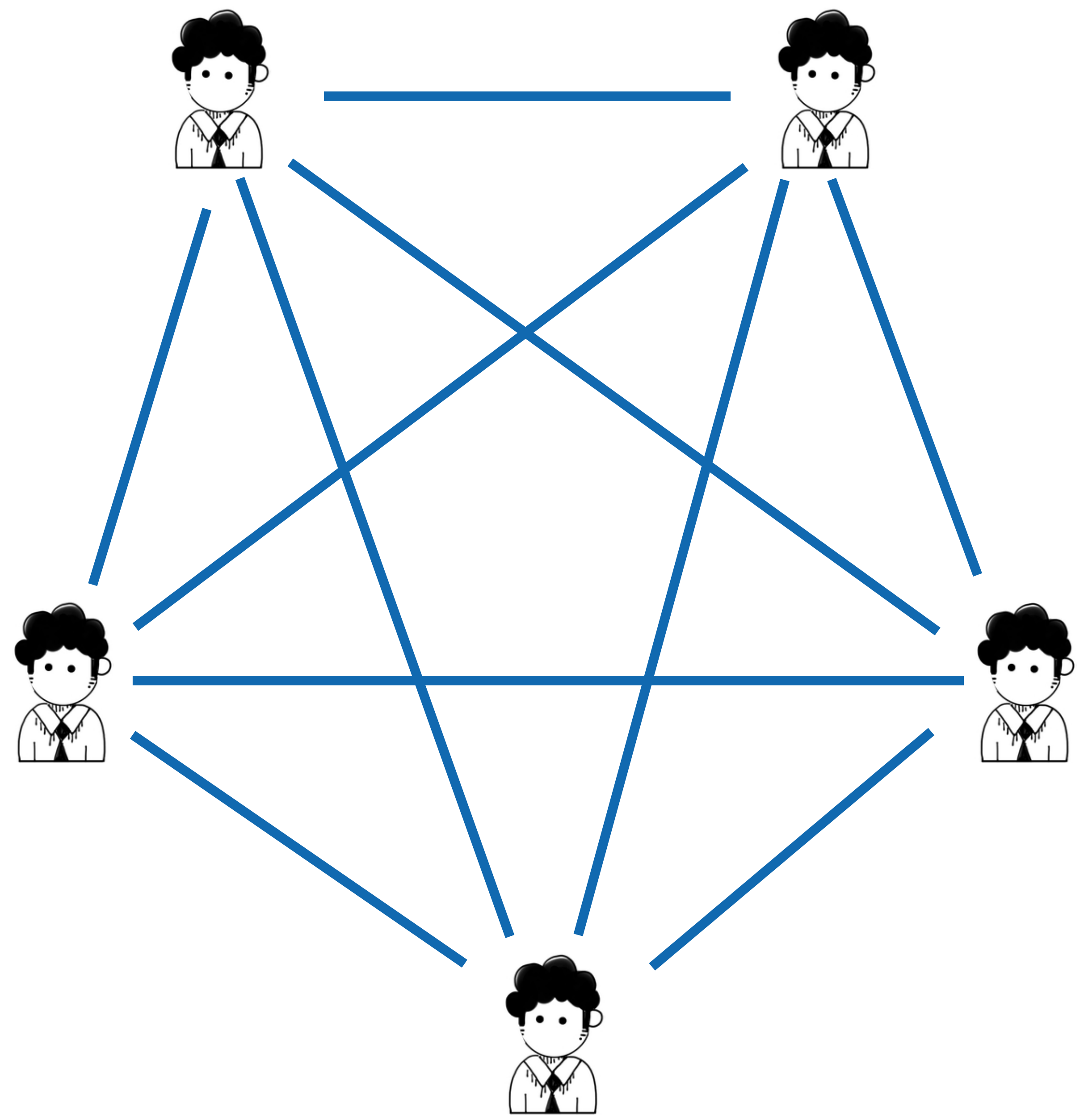


[Buechel, B. & Buskens, V. The dynamics of closeness and betweenness. J.Math. Sociol. 37, 159–191 (2013)]

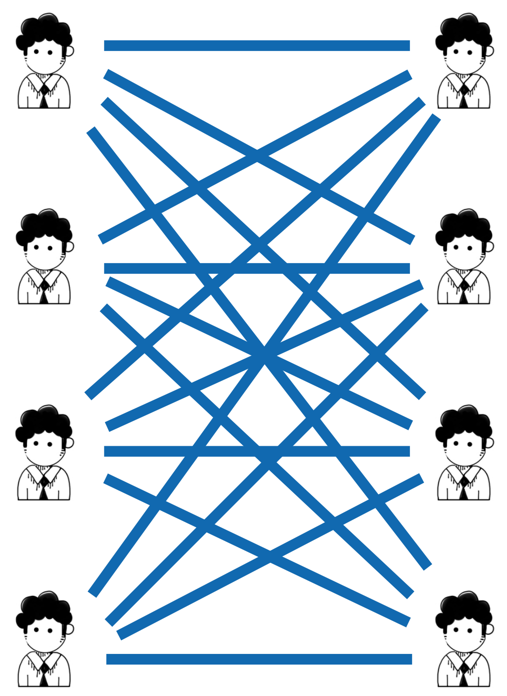
STRATEGIC NETWORK FORMATION MODEL



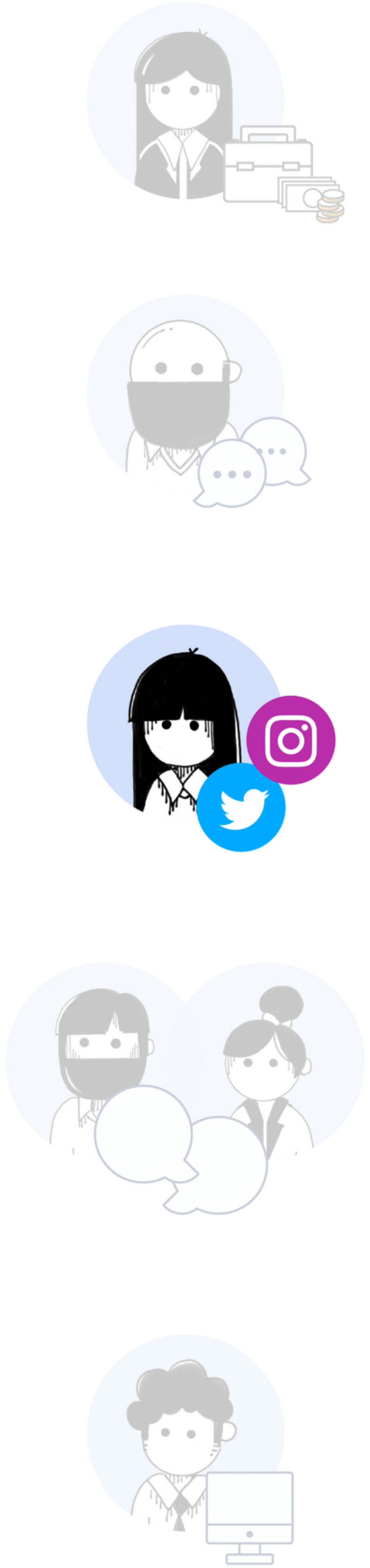
Assumption: Homogeneous agents



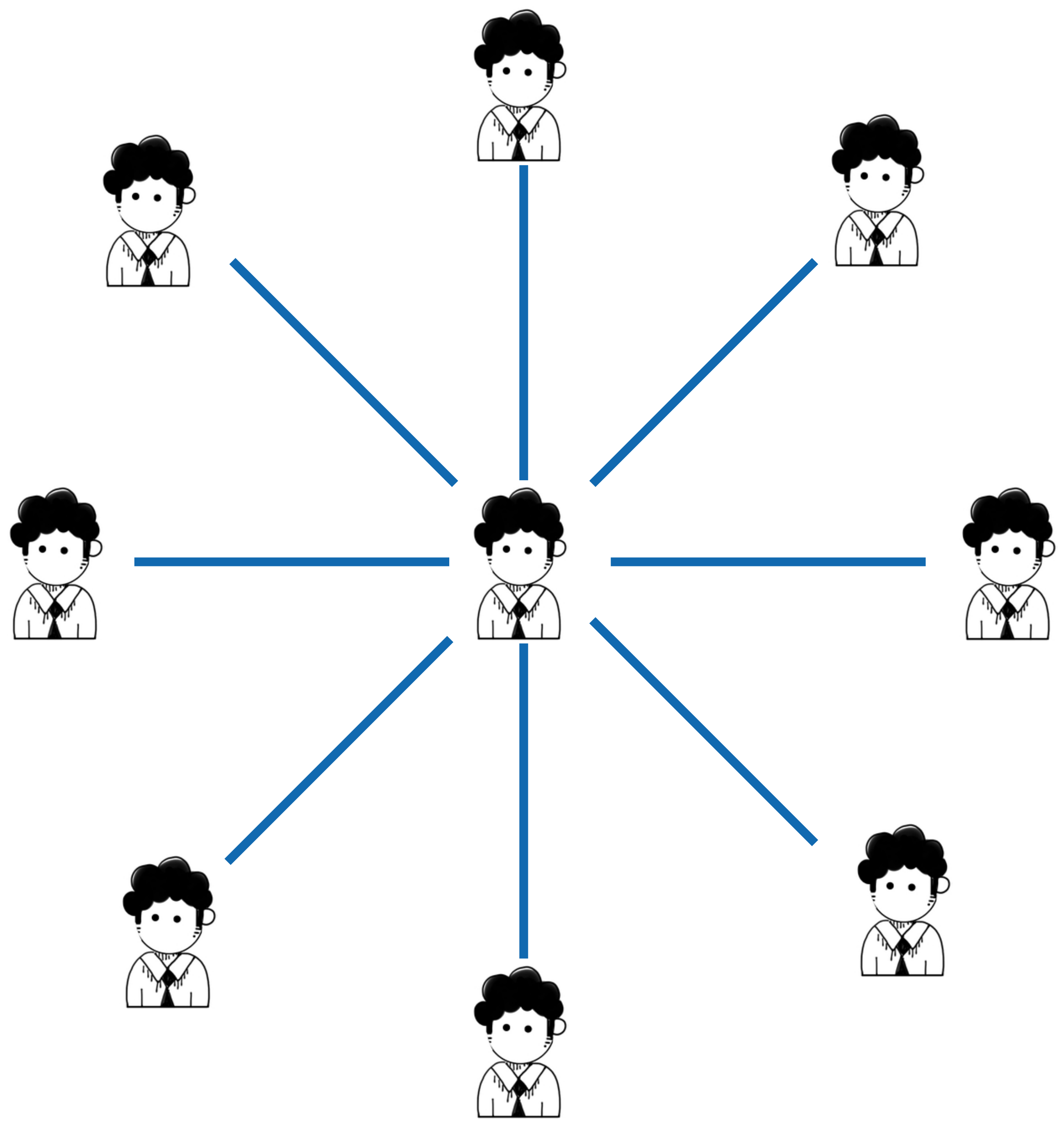
[Burger, M. J. & Buskens, V. Social context and network formation: an experimental study. Social Networks 31, 63–75 (2009).]



STRATEGIC NETWORK FORMATION MODEL



Assumption: Homogeneous agents



[Buechel, B. In Networks, Topology and Dynamics. Springer Lecture Notes in Economic and Mathematical Systems Vol. 613, 95–109 (Springer, 2008)]