ETH zürich

# Real-Time Feedback Optimization of Power Systems

Florian Dörfler

Automatic Control Laboratory, ETH Zürich

# Acknowledgements

Adrian Hauswirth

Saverio Bolognani

Gabriela Hug

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Bundesamt für Energie BFE
Swiss Federal Office of Energy SFOE

FNSNF
Fonds national suisse
Schweizerischer Nationalfonds
Fondo nazionale svizzero
Swiss National Science Foundation

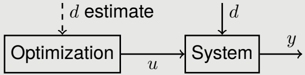
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

HORIZON 2020

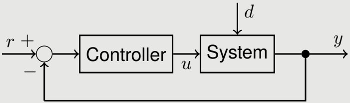
ABB

Rte
Réseau de transport d'électricité

# feedforward optimization vs. feedback control

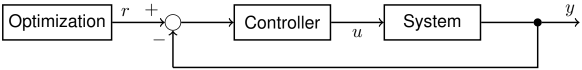



- **complex optimal decision**
- **operational constraints**
- **MIMO (multi-input/output)**
- highly model-based
- computationally intensive



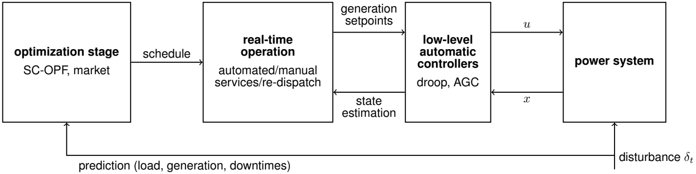

- **robust to model uncertainty**
- **fast response**
- **measurement driven**
- suboptimal operation
- unconstrained operation

→ typically **complementary** methods are combined via **time-scale separation**

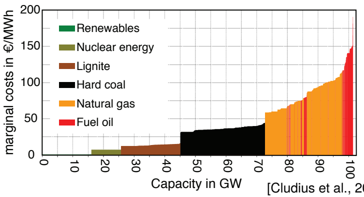



**offline & feedforward** | **real-time & feedback**

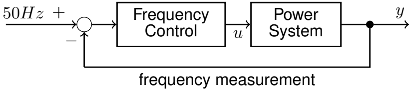
# Example: power systems load / generation balancing




- **optimization stage**
  economic dispatch based on predictions/markets
- **real-time operations**
  unforeseen deviations from schedule (e.g. congestion)
- **low-level automatic control**
  frequency regulation at the individual generators




[Cludius et al., 2014]

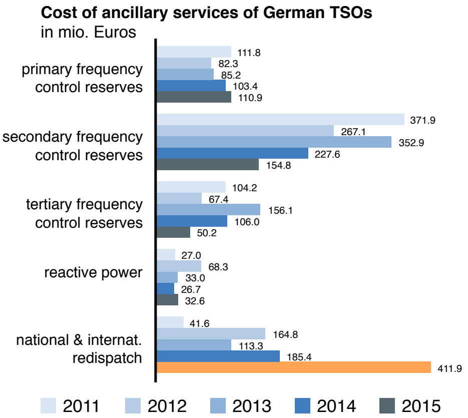


# Price for time-scale separation

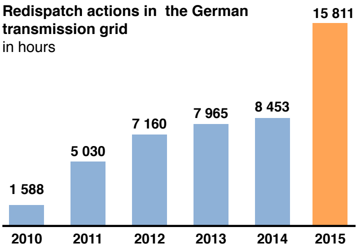
- **re-dispatch** to deal with unforeseen load, congestion, & renewables

⇒ ever more **uncertainty** & **fluctuations** on all time scales

⇒ operation architecture becomes **infeasible & inefficient**




[Bundesnetzagentur, Monitoringbericht 2016]




[Bundesnetzagentur, Monitoringbericht 2016]

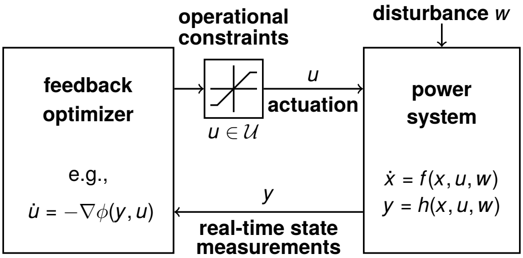
There must be a better way of operation.

# Ancillary services: synopsis and proposal

**Today:** partially automated, provided by separate mechanisms, hitting limits

- real time balancing
- voltage regulation
- loss minimization
- economic re-dispatch
- collapse prevention
- line congestion relief
- reactive power compensation
- frequency control

Central paradigm of future "smart" grids: automation for **real-time operation**




**Proposal:** online optimization algorithms as feedback control

→ robust (feedback)

→ fast response

→ operational constraints

→ steady-state optimal

→ MIMO decision making

# Brief review on related literature

- **historical roots**: optimal routing and queuing in communication networks, e.g., in the internet (TCP/IP) [Kelly et al. 1998/2001, Low, Paganini, and Doyle 2002, Srikant 2012, . . . ]

- lots of recent theory development in **power systems** & other infrastructures

  lots of related work: [Bolognani et al, 2015], [Dall'Anese and Simmonetto, 2016/2017], [Gan and Low, 2016], [Tang and Low, 2017], . . .

  > A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems
  >
  > Daniel K. Molzahn,* *Member, IEEE*, Florian Dörfler,† *Member, IEEE*, Henrik Sandberg,‡ *Member, IEEE*, Steven H. Low,§ *Fellow, IEEE*, Sambuddha Chakrabarti,¶ *Student Member, IEEE*, Ross Baldick,¶ *Fellow, IEEE*, and Javad Lavaei,** *Member, IEEE*

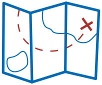
  early adoptions: KKT control [Jokic et al, 2009] and Commelec [Bernstein et al, 2015]

- **MPC version** of "dropping argmin": real-time iteration [Diel et al. 2005], real-time MPC [Zeilinger et al. 2009], . . . and related papers with *anytime* guarantees

- independent literature in **process control** [Bonvin et al. 2009/2010] or **extremum seeking** [Krstic and Wang 2000], . . . and probably much more

- recent **system theory** [Nelson et al. 2017], [Colombino et al. 2018], [Lawrence et al. 2018]

- **algorithms as dynamic control systems** [Lessard et al., 2014], [Wilson et al., 2018]



# OVERVIEW

1. Interconnected dynamics and stability analysis
2. Projected gradient flow on the power flow manifold
3. Numerical experiments

# INTERCONNECTED DYNAMICS

# AND

# STABILITY ANALYSIS

# Stylized problem description

## Optimization Problem

$$\underset{y,u}{\text{minimize}} \quad \phi(y,u)$$

$$\text{subject to} \quad y = (CH + D)u + CRw$$

$$u \in \mathcal{U}$$

$\rightarrow$ gradient control of steady state

$$\dot{u} = \Pi_{\mathcal{U}}\left(-\epsilon\left[CH + D \quad \mathbb{I}\right]^T \nabla\phi\right)(u)$$

## LTI Dynamics

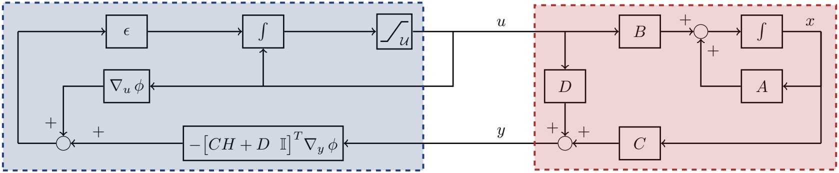
$$\dot{x} = Ax + Bu + Qw$$

$$y = Cx + Du$$

with $A$ Hurwitz & steady-state maps

$$x = \underbrace{-A^{-1}B}_{H}\, u \underbrace{-A^{-1}Q}_{R}\, w$$

$$y = (CH + D)u + CRw$$



# Stability, feasibility, & asymptotic optimality of closed loop

**Theorem:** Assume that

- LTI system asymptotically **stable**: $\exists \gamma > 0\,,\, \exists P \succ 0 :\ PA + A^T P \preceq -\gamma P$
- **regularity** of cost function $\phi$: compact level sets and $\ell$-Lipschitz gradient
- sufficient **time-scale separation** (small gain): $0 \leq \epsilon \leq \epsilon^\star \triangleq \frac{\gamma}{2\ell \|H\|}$

Then the closed-loop system is **stable** and **globally converges** to the critical points of the **optimization problem** while remaining **feasible** at all times.

**Proof:** **LaSalle/Lyapunov** analysis inspired from singular perturbation theory

$$\Psi_\delta(u,e) \;=\; \delta \cdot \underbrace{e^T P e}_{\text{LTI Lyapunov function}} \;+\; (1-\delta) \cdot \underbrace{\phi(e,u)}_{\text{objective function}}$$

with steady-state **error coordinate** $e = x - Hu - Rw$ & **coefficient** $\delta \in (0,1)$

$\rightarrow$ derivative $\dot{\Psi}_\delta(u,e)$ is non-increasing if $\epsilon \leq \epsilon^\star$ and for optimal choice of $\delta$

# Example: optimal constrained frequency control

## Dynamic model:

- linearized swing dynamics
- 1st-order turbine-governor
- primary frequency control
- DC power flow approximation

$$\left.\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= -M^{-1}\left(D\omega + \mathbf{B}\theta - p + p^L(t)\right) \\ \dot{p} &= -K\left(R^{-1}\omega + p - p^C\right) \end{aligned}\right\}$$

$\dot{x} = Ax + Bu + Qw$ where

$$x = \begin{bmatrix} \theta \\ \omega \\ p \end{bmatrix}, \; u = p^C, \; w = p^L(t)$$

## Measurements:

$$y = \begin{bmatrix} \mathbb{0} & 1 & 0 & \dots & 0 & \mathbb{0} \\ \mathbf{B}^\ell & & & \mathbb{0} & & \mathbb{0} \\ \mathbb{0} & & & \mathbb{0} & & I \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ p \end{bmatrix} = \begin{bmatrix} \text{frequency at node 1} \\ \text{selected line flows} \\ \text{active power injections} \end{bmatrix}$$

# Example: optimal constrained frequency control

- **optimization problem**

$$\begin{aligned} \underset{y,u}{\text{minimize}} \quad & \phi(y) \\ \text{subject to} \quad & y = CHu + CRw \\ & u \in \mathcal{U} \end{aligned}$$

where $y = CHu + CRw$ is the steady-state input-output map

- **economic cost** and **operational limits** are encoded in

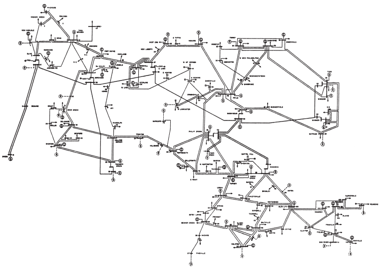
$$\phi(y) = \underbrace{\text{cost}(y)}_{\text{DC OPF}} + \underbrace{\tfrac{1}{2}\|\max\{0, \underline{y} - y\}\|_{\Xi}^2 + \tfrac{1}{2}\|\max\{0, y - \overline{y}\}\|_{\Xi}^2}_{\text{operational limits (line flows, frequency, \dots)}}$$

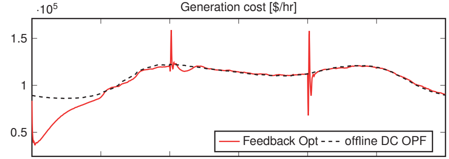
- $\mathcal{U}$ describes the **saturation constraints** on the actuation

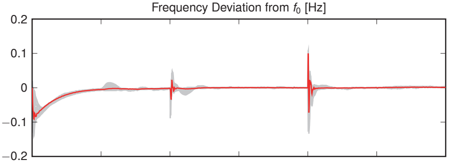
$\rightarrow$ **control** $\dot{u} = \Pi_{\mathcal{U}}(\ldots \nabla\phi) \equiv$ optimal Automatic Generation Control (AGC)

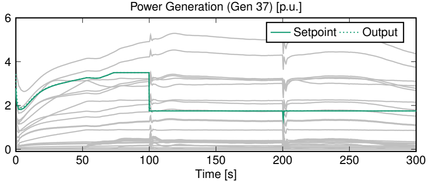
# Response to contingencies

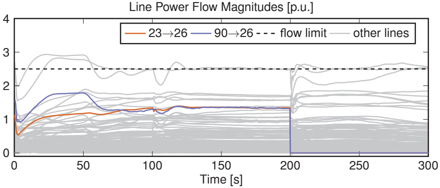
Generator outage & double line tripping in IEEE 118-bus test system





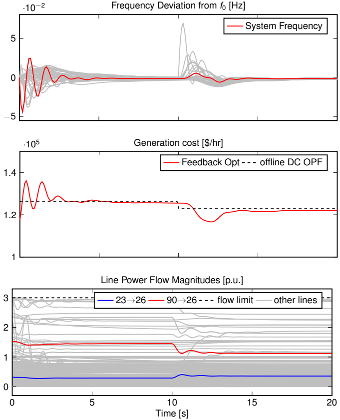



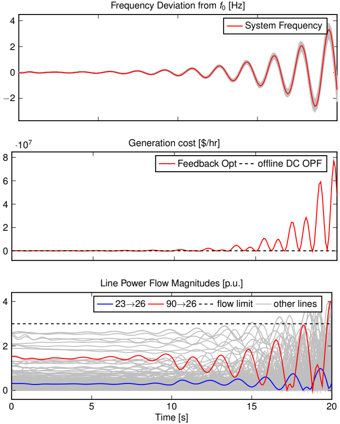
# How conservative is $\epsilon \leq \epsilon^\star$ ?

Simulation on IEEE 118-bus test case

still **stable for** $\epsilon = 2\,\epsilon^\star$




**unstable for** $\epsilon = 10\,\epsilon^\star$




**Note:** conservativeness ranges from 1.2 to 1000, depending on penalty scalings.

# Highlights and comparison of our contributions

## Weak assumptions on plant

- internal stability

→ no observability/controllability needed

- reduced model dependency

→ need only steady-state map $H$

## Weak assumptions on cost

- Lipschitz gradient + properness

→ no (strict/strong) convexity required

- **convexity** $\Rightarrow$ global convergence

**take-home msg:** online optimization algorithms can be applied to dynamics

## Parsimonious but powerful setup

- potentially conservative bound, but

→ **minimal assumptions** on optimization problem & plant

→ constraints assured by **general plant dynamics** (no primal/dual)

[Jokic et al. 2009], [Zhao et al. 2013]

→ directly **useful for design** (no LMIs)

[Nelson et al. 2017], [Colombino et al. 2018]

- proof **can be extended** to other algorithms & nonlinear dynamics

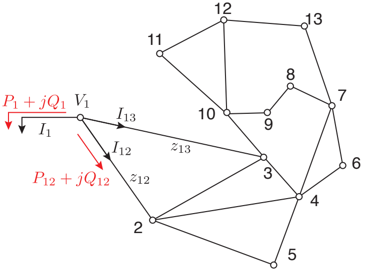
→ Menta, Hauswirth, Bolognani, Hug & Dörfler (2018) **"Stability of Dynamic Feedback Optimization with Applications to Power Systems"**

# PROJECTED GRADIENT FLOW

# ON THE POWER FLOW MANIFOLD

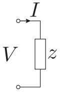
# Steady-state AC power flow, constraints, and objectives

- **quasi-stationary** dynamics



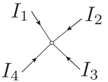

| | | | |
|---|---|---|---|
| $V_k$ | nodal voltage | $z_{kl}$ | line impedance |
| $I_k$ | current injection | $I_{kl}$ | line current |
| $P_k, Q_k$ | power injections | $P_{kl}, Q_{kl}$ | power flow |

Ohm's Law




$V = zI$

Current Law




$0 = I_1 + \ldots + I_k$

AC power $S = P + jQ = VI^*$

**AC power flow equations**

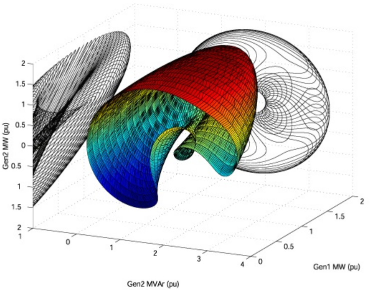
$$S_k = \sum_{l \in N(k)} \frac{1}{z_{kl}^*} V_k (V_k^* - V_l^*) \quad \forall k \in \mathcal{N}$$

(all variables and parameters are $\mathbb{C}$-valued)

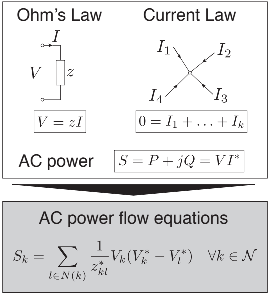
- **objective:** economic dispatch, minimize losses, distance to collapse, etc.
- **operational constraints:** generation capacity, voltage bands, congestion, etc.
- **control:** state measurements and actuation via generation set-points

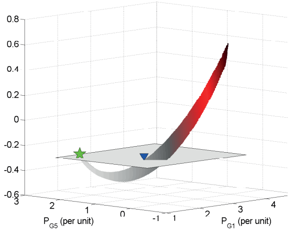
# What makes power flow optimization interesting?

## graphical illustration of AC power flow




[Hiskens, 2001]

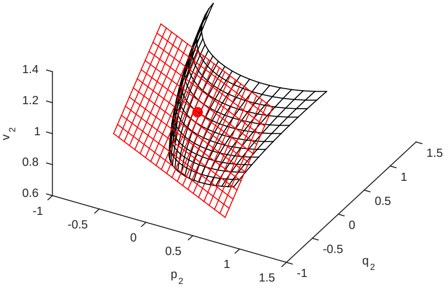



[Molzahn, 2016]

- imagine **constraints slicing** this set
  $\Rightarrow$ nonlinear, non-convex, disconnected
- additionally the parameters are $\pm 20\%$ **uncertain** ... this is only steady state!

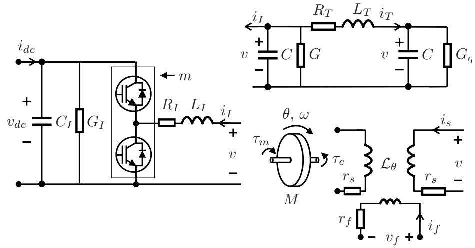
# Key insights about our physical equality constraint




- AC power flow is complex but it defines a **smooth manifold**

→ local tangent plane approximations, local invertibility, & generic LICQ

→ Bolognani & Dörfler (2015)
**"Fast power system analysis via implicit linearization of the power flow manifold"**



- AC power flow is **attractive steady state** for ambient physical dynamics

→ physics enforce feasibility even for non-exact (e.g., discrete) updates

→ Gross, Arghir, & Dörfler (2018)
**"On the steady-state behavior of a nonlinear power system model"**

# Control specifications as Optimal Power Flow (OPF)

## Real-time optimal power flow

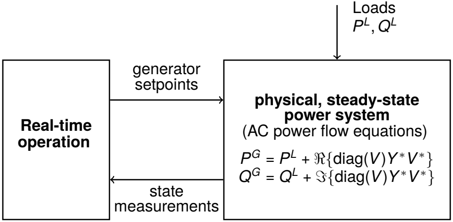
- minimize objective
- satisfy AC power flow laws
- respect generation capacity
- no over-/under-voltage
- no congestion

$$\begin{aligned} \text{minimize} \quad & \phi(P, Q, V) \\ \text{subject to} \quad & P^G + jQ^G = P^L + jQ^L + \text{diag}(V) Y^* V^* \\ & \underline{P}_k \leq P_k^G \leq \overline{P}_k, \ \underline{Q}_k \leq Q_k^G \leq \overline{Q}_k \\ & \underline{V}_k \leq V_k \leq \overline{V}_k \\ & |P_{kl} + jQ_{kl}| \leq \overline{S}_{kl} \end{aligned}$$

where $\phi(P, Q, V)$ can be cost of generation, distance to voltage collapse, etc.

Challenging **specifications on the closed-loop trajectories**:

1. stay on the manifold at all times
2. satisfy constraints at all times
3. converge to the OPF solution



# Real-time optimization on the power flow manifold

## Real-time optimal power flow

- minimize objective
- satisfy AC power flow laws
- respect generation capacity
- no over-/under-voltage
- no congestion

$$\begin{aligned} \text{minimize} \quad & \phi(P,Q,V) \\ \text{subject to} \quad & P^G + jQ^G = P^L + jQ^L + \text{diag}(V) Y^* V^* \\ & \underline{P}_k \leq P_k^G \leq \overline{P}_k, \ \ \underline{Q}_k \leq Q_k^G \leq \overline{Q}_k \\ & \underline{V}_k \leq V_k \leq \overline{V}_k \\ & |P_{kl} + jQ_{kl}| \leq \overline{S}_{kl} \end{aligned}$$

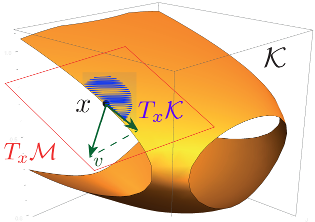
## Prototype of real-time OPF

$$\begin{aligned} \text{minimize} \quad & \phi(x) \\ \text{subject to} \quad & x \in \mathcal{K} = \mathcal{M} \cap \mathcal{X} \end{aligned}$$

$\phi : \mathbb{R}^n \to \mathbb{R}$ objective function

$\mathcal{M} \subset \mathbb{R}^n$ AC power flow manifold

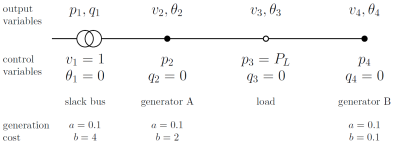
$\mathcal{X} \subset \mathbb{R}^n$ operational constraints

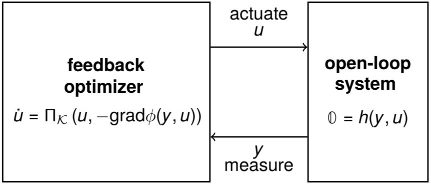



Projection of trajectory $v$ in **feasible cone**

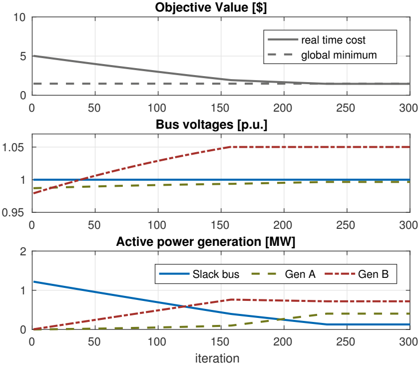
$$\Pi_{\mathcal{K}}(x,v) \in \arg \min_{w \in T_x \mathcal{K}} ||v - w||$$

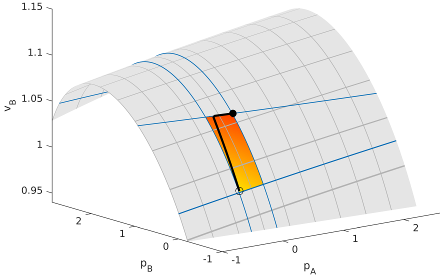
# Simple illustrative case study



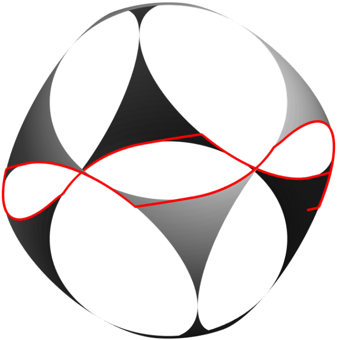




$\rightarrow$ closed loop is projected grad descent



# Projected gradient descent on manifolds



$$\mathcal{K} = \left\{x : \|x\|_2^2 = 1\,,\ \|x\|_1 \leq \sqrt{2}\right\}$$

**Theorem (simplified)**

Let $x : [0, \infty) \to \mathcal{K}$ be a Carathéodory solution of the initial value problem

$$\dot{x} = \Pi_{\mathcal{K}}\left(x, -\text{grad}\phi(x)\right)\ , \quad x(0) = x_0\ .$$

If $\phi$ has compact level sets on $\mathcal{K}$, then $x(t)$ will converge to a critical point $x^\star$ of $\phi$ on $\mathcal{K}$.

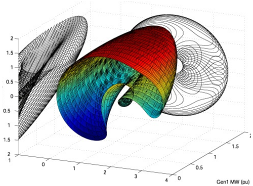
$\longrightarrow$ Hauswirth, Bolognani, Hug, & Dörfler (2016)
**"Projected gradient descent on Riemanniann manifolds with applications to online power system optimization"**

**Hidden assumption**: existence of a Carathéodory solution $x(t) \in \mathcal{K}$

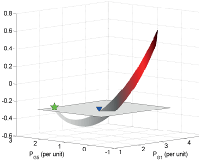
$\rightarrow$ when does it exist, is forward complete, unique, and sufficiently regular ?

(in absence of convexity, Euclidean space, and other regularity properties)

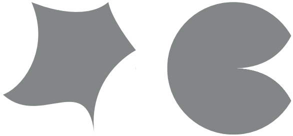
# Analysis via projected systems hit mathematical bedrock




nonlinear power flow manifold




disconnected regions



cusps & corners (convex and/or inward)

| | constraint set | gradient field | metric | manifold |
|---|---|---|---|---|
| existence (Krasovski) | loc. compact | loc. bounded | - | $C^1$ |
| Krasovski $=$ Carathéodory | Clarke regular | $C^0$ | $C^0$ | $C^1$ |
| uniqueness of solutions | prox regular | $C^{0,1}$ | $C^{0,1}$ | $C^{1,1}$ |

$\rightarrow$ also forward-Lipschitz continuity of time-varying constraints

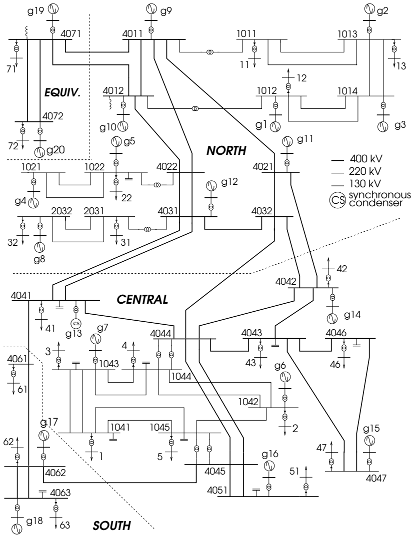
$\rightarrow$ Hauswirth, Bolognani, Hug, & Dörfler (2018)
**"Projected Dynamical Systems on Irregular Non-Euclidean Domains for Nonlinear Optimization"**

$\rightarrow$ Hauswirth, Subotic, Bolognani, Hug, & Dörfler (2018)
**"Time-varying Projected Dynamical Systems with Applications to Feedback Optimization of Power Systems"**

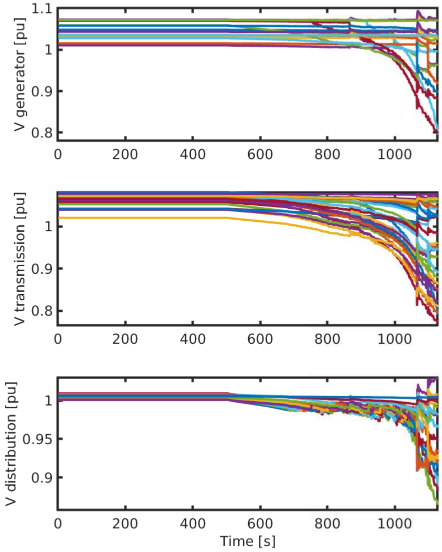
# NUMERICAL EXPERIMENTS

# Voltage stability in the Nordic system

- historically known for voltage collapse (Southern Sweden ’83)
- high-fidelity model of Nordic system (RAMSES + python + MATLAB)
- heavily loaded system
- large transfers between north and central areas
- all loads equipped with LTCs
- generators equipped with Automatic Voltage Regulators and Over Excitation Limiters
- frequency regulation through speed governor control



# Voltage collapse

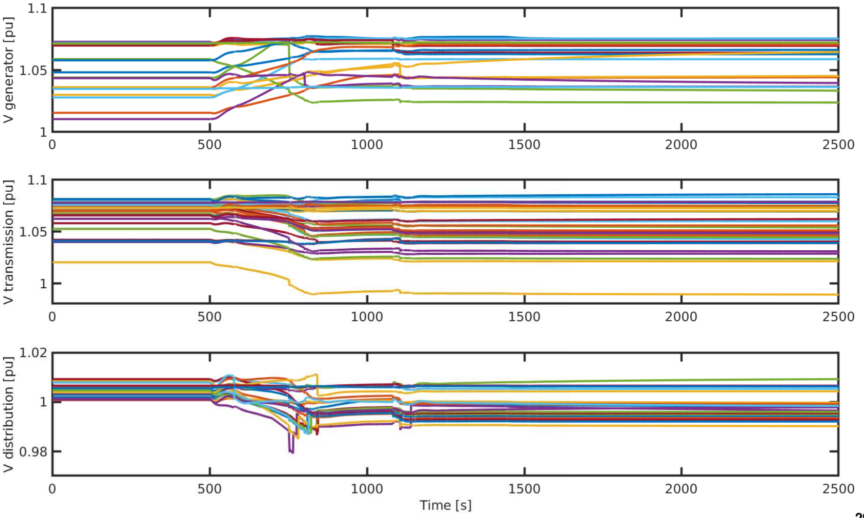



- 250 MW load ramp from $t = 500\,\text{s}$ to $t = 800\,\text{s}$
- extra demand is balanced by primary frequency control
- cascade of activation of over-excitation limiters
- LTCs increase power demand of distribution buses
- . . . voltage collapse
- very hard (nearly impossible) to mitigate via conventional controls

**Assume we can control AVR set-points in real time . . .**

# Voltage collapse averted !

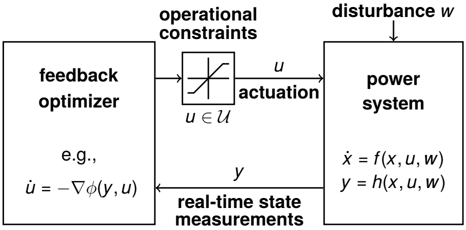
**objective** $\phi(P,Q,V) = -\log\det\big(\text{load flow Jacobian}\big) = \text{distance to collapse}$



# The tracking problem

- power system affected by **exogeneous time-varying inputs** $w$

$\rightarrow$ disturbances may lead to **infeasible** states $\rightarrow$ ill-defined dynamics



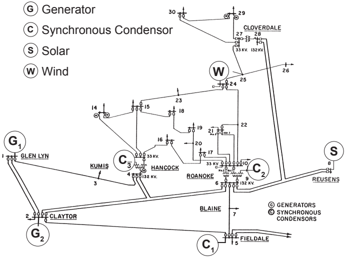

- $\mathcal{U}$ accounts for **hard constraints** on controllable variables $u$ (e.g., generation limits)

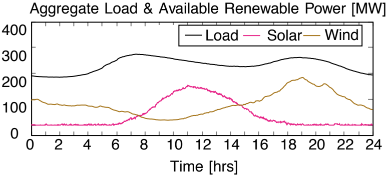
$\rightarrow$ gradient projection becomes **input saturation** (saturated proportional feedback control)

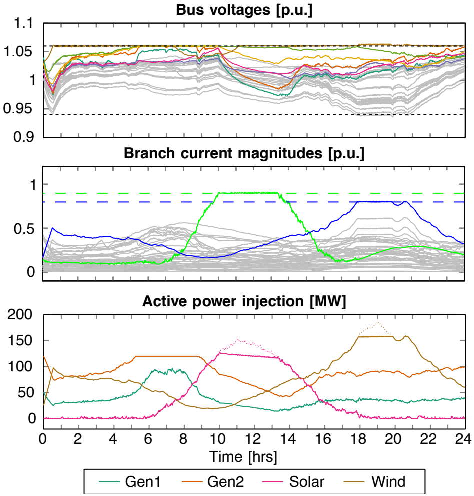
- **soft constraints** via penalty in $\phi$ for non-controllable variables (e.g., voltage limits)

$\rightarrow$ gradient of penalty functions becomes a **proportional control** (e.g., droop)

# Transient tracking performance under disturbances

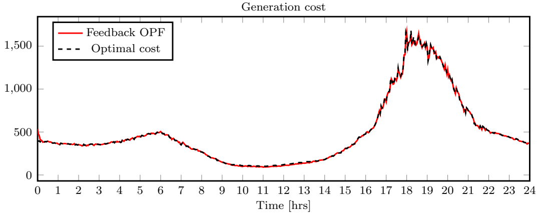


# The tracking problem: optimality and robustness

- practically **exact tracking** of ground-truth OPF (knowing exact disturbance & without computation delay)
- transient trajectory **feasibility**
- **robustness** to model mismatch (asymptotic optimality under wrong model)




| | offline optimization | | | feedback optimization | | |
|---|---|---|---|---|---|---|
| model uncertainty | feasible ? | $\phi - \phi^*$ | $\|v - v^*\|$ | feasible ? | $\phi - \phi^*$ | $\|v - v^*\|$ |
| loads $\pm 40\%$ | no | 94.6 | 0.03 | yes | 0.0 | 0.0 |
| line params $\pm 20\%$ | yes | 0.19 | 0.01 | yes | 0.01 | 0.003 |
| 2 line failures | no | -0.12 | 0.06 | yes | 0.19 | 0.007 |

**conclusion:** simple algorithm performs extremely well & robust

# SUMMARY AND CONCLUSIONS

# Summary and conclusions

**Summary:**

- necessity of **real-time power system operation**
- our starting point: **online optimization as feedback control**
- **technical approach:** singular perturbation & manifold optimization
- **unified framework** accommodating various constraints & objectives

**Ongoing work and open problems:**

- **quantitative certificates** for robustness, tracking performance, etc.
- **implementation issues:** discretization, distributed, state estimation, communication, etc. $\rightarrow$ microgrid experiments and RTE collaboration
- **extensions:** stochastic disturbances, transient optimality à la MPC, model-free à la extremum seeking, Nash-seeking in antagonistic context, etc.

# Thanks !

**Florian Dörfler**

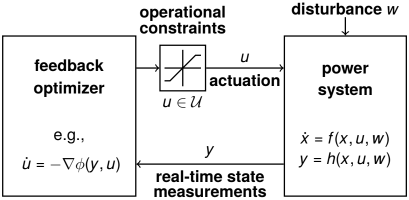
http://control.ee.ethz.ch/~floriand

dorfler@ethz.ch

# BACK-UP SLIDES . . . SINCE YOU ASKED FOR IT

# LITERATURE COMPARISON

# Emerging research area: online optimization in closed loop




## Optimization perspective

Algorithms as dynamical systems

[Lessard et al., 2014], [Wilson et al., 2018]

→ **implemented via the physics**

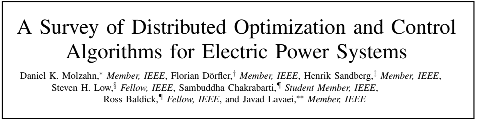
## Control perspective

Existing feedback systems interpreted as solving opt. problem

→ **general objective + constraints**

Lots of recent development: theory and power system applications

[Bolognani et. al, 2015], [Cady et al., 2015], [Dall'Anese and Simmonetto, 2016/2017], [Gan and Low, 2016], [Tang and Low, 2017], ...

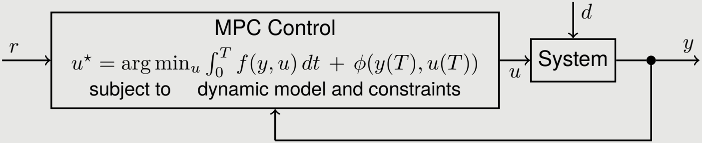


A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems

Daniel K. Molzahn,* *Member, IEEE*, Florian Dörfler,† *Member, IEEE*, Henrik Sandberg,‡ *Member, IEEE*, Steven H. Low,§ *Fellow, IEEE*, Sambuddha Chakrabarti,¶ *Student Member, IEEE*, Ross Baldick,¶ *Fellow, IEEE*, and Javad Lavaei,** *Member, IEEE*

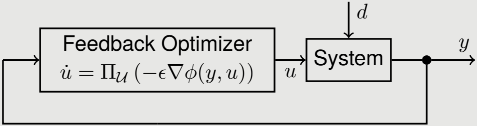
# Model Predictive Control vs. feedback optimization

## MPC




- highly model-based
- computationally intensive
- **optimal trajectories**
- **stabilization**

## Feedback optimization ← drop arg min, stage cost, & dynamic model

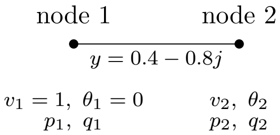


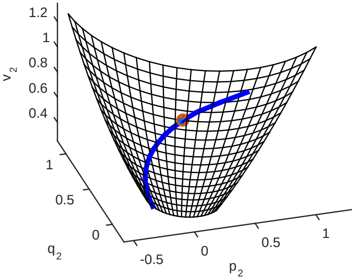

- **model-free (robust) design**
- **fast response**
- suboptimal trajectories
- requires stable system

# TECHNICAL INGREDIENT I:

# THE POWER FLOW MANIFOLD

# Geometric perspective: the power flow manifold




- **variables:** all of $x = (|V|, \theta, P, Q)$
- **power flow manifold:**

$\mathcal{M} = \{x : h(x) = \mathbb{0}\}$

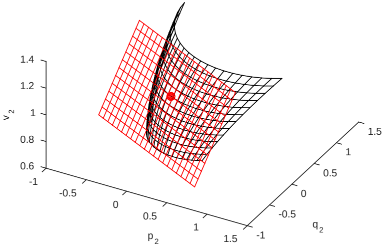
$\rightarrow$ submanifold in $\mathbb{R}^{2n}$ or $\mathbb{R}^{6n}$ (3-phase)

- **tangent space** $\frac{\partial h(x)}{\partial x}\Big|_{x^*}^{\top} (x - x^*) = \mathbb{0}$

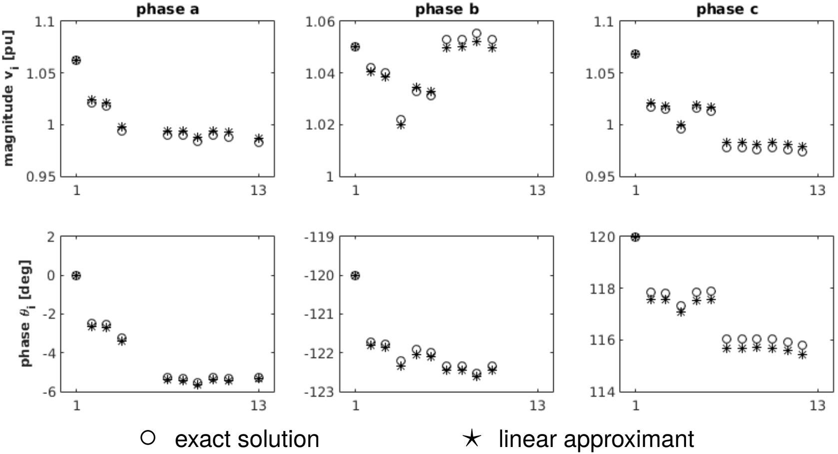
$\rightarrow$ best linear approximant at $x^*$

- **accuracy** depends on curvature $\frac{\partial^2 h(x)}{\partial x^2}$

$\rightarrow$ constant in rectangular coordinates



# Accuracy illustrated with unbalanced three-phase IEEE13




**dirty secret:** power flow manifold is very flat (linear) near usual operating points

→ **Matlab/Octave code** @ `https://github.com/saveriob/1ACPF`

# Coordinate-dependent linearizations reveal old friends

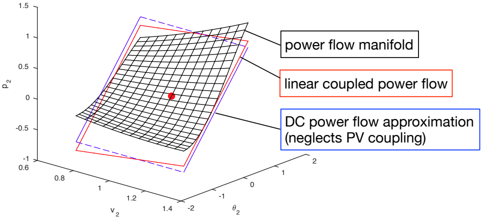
- **flat-voltage/0-injection point:** $x^* = (|V|^*, \theta^*, P^*, Q^*) = (\mathbb{1}, \mathbb{0}, \mathbb{0}, \mathbb{0})$

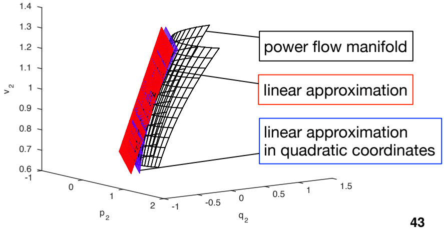
$\rightarrow$ tangent space parameterization $\begin{bmatrix} \Re(Y) & -\Im(Y) \\ -\Im(Y) & \Re(Y) \end{bmatrix} \begin{bmatrix} |V| \\ \theta \end{bmatrix} = \begin{bmatrix} P \\ Q \end{bmatrix}$

is **linear coupled power flow** and $\Re(Y) \approx \mathbb{0}$ gives **DC power flow** approximation

- nonlinear change to **quadratic coordinates** $|V| \rightarrow |V|^2$

$\rightarrow$ linearization $\equiv$ (non-radial) **LinDistFlow** [M. Baran and F.F. Wu, '88] $\rightarrow$ more exact in $|V|$



# TECHNICAL INGREDIENT II:

# MANIFOLD OPTIMIZATION

# Unconstrained manifold optimization: the smooth case

- **geometric objects:**

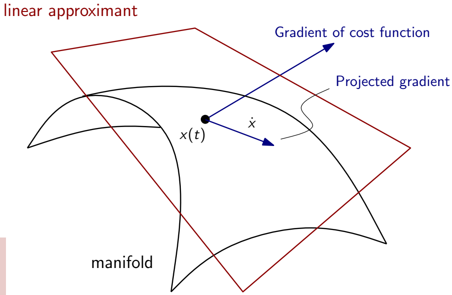
| | | | |
|---|---|---|---|
| manifold | $\mathcal{M} = \{x : \; h(x) = \mathbb{0}\}$ | objective | $\phi : \mathcal{M} \to \mathbb{R}$ |
| tangent space | $T_x\mathcal{M} = \ker \frac{\partial h(x)}{\partial x}^\top$ | Riemann metric (degree of freedom) | $g : T_x\mathcal{M} \times T_x\mathcal{M} \to \mathbb{R}$ |

- **target state:** local minimizer on the manifold $x^\star \in \arg\min_{x \in \mathcal{M}} \phi(x)$

- **always feasible** $\leftrightarrow$ trajectory/sequence $x(t)$ remains on manifold $\mathcal{M}$

- continuous-time **gradient descent** on $\mathcal{M}$:

1. grad $\phi(x)$: **gradient** of cost function in ambient space
2. $\Pi_{\mathcal{M}}\,(x, -\text{grad}\phi(x))$: **projection** of gradient on tangent space $T_x\mathcal{M}$
3. **flow** on manifold: $\dot{x} = \Pi_{\mathcal{M}}\,(x, -\text{grad}\phi(x))$



# Constrained manifold optimization: the wild west

dealing with **operational constraints** $g(x) \leq 0$

1. **penalty** in cost function $\phi$

$\rightarrow$ barrier: not practical for online implementation

$\rightarrow$ soft penalty: practical but no real-time feasibility

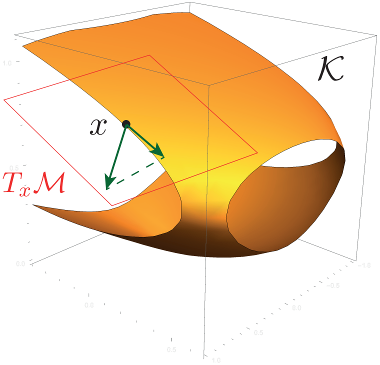
2. **dualization** and gradient flow on Lagrangian

$\rightarrow$ poor performance & no real-time feasibility

$\rightarrow$ theory: close to none available on manifolds

$\longrightarrow$ Hauswirth, Bolognani, Hug, & Dörfler (2018)

**"Generic Existence of Unique Lagrange Multipliers in AC Optimal Power Flow"**




3. **projection** of gradient flow trajectory $x(t)$ on feasible set $\mathcal{K} = \mathcal{M} \cap \{g(x) \leq 0\}$

$$\dot{x} \;=\; \Pi_{\mathcal{K}}\left(x, -\text{grad}\phi(x)\right) \;\in\; \arg\min_{v \in T_x\mathcal{K}} \|-\text{grad}\,\phi(x) - v\|_g$$

where $T_x\mathcal{K} \subset T_x\mathcal{M}$ is inward tangent cone

# Implementation issue: how to **induce** the gradient flow?

## Open-loop system

$\dot{x}_1 = u$ controlled generation

$\mathbb{0} = h(x_1, x_2, w)$ AC power flow manifold relating $x_1$ & other variables

## Desired closed-loop system

$\dot{x}_1 = f_1(x_1, x_2)$ desired projected

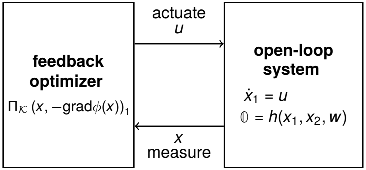
$\dot{x}_2 = f_2(x_1, x_2)$ gradient descent

where $f(x) = \Pi_{\mathcal{K}}(x, -\text{grad}\phi(x))$

Solution: physics are **non-singular** $\rightarrow \mathbb{0} = h(x_1, x_2, w)$ can be solved for $x_2$

### Feedback equivalence

The trajectories of the desired closed loop **coincide** with those of the open loop under the feedback $u = f_1(x_1, x_2)$.




$\rightarrow$ closed-loop trajectory remains feasible at all times and converges to optimality

$\rightarrow$ no need to numerically solve the optimization problem or power flow equation

# Implementation issue: discrete-time manifold optimization

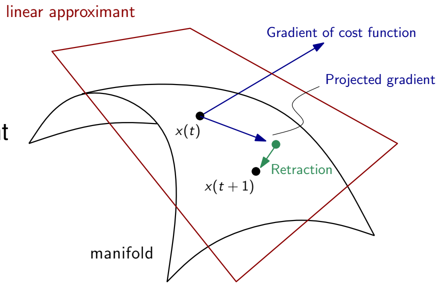
- **always feasible** $\leftrightarrow$ trajectory/sequence $x(t)$ remains on manifold $\mathcal{M}$

- **discrete-time** gradient descent on $\mathcal{M}$:

1. grad $\phi(x)$: **gradient** of cost function
2. $\Pi_{\mathcal{M}}\left(x, -\text{grad}\phi(x)\right)$: **projection** of gradient
3. **Euler integration** of gradient flow:

$$\tilde{x}(t+1) = x(t) - \varepsilon\,\Pi_{\mathcal{M}}\left(x, -\text{grad}\phi(x)\right)$$

4. **retraction step**: $x(t+1) = \mathcal{R}_{x(t)}\left(\tilde{x}(t+1)\right)$




**Discrete-time control implementation:**

$\rightarrow$ manifold is attractive steady state for ambient dynamics

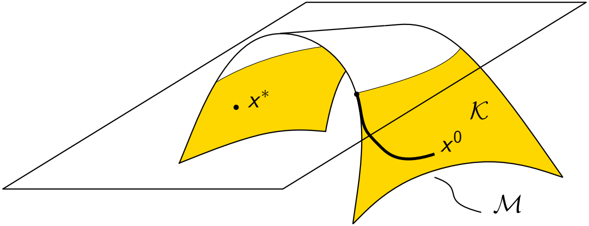
$\rightarrow$ retraction is taken care of by the physics: “nature enforces feasibility”

$\rightarrow$ can be made rigorous using singular perturbation theory (Tikhonov)

# FURTHER NUMERICAL STUDIES

# Trajectory feasibility

The feasible region $\mathcal{K} = \mathcal{M} \cap \mathcal{X}$ often has **disconnected components**.




- **feedforward (OPF)**
  - optimizer $x^\star = \arg\min_{x \in \mathcal{K}} \phi(x)$ can be in different **disconnected component**

  $\rightarrow$ no feasible trajectory exists: $x_0 \rightarrow x^\star$ must **violate constraints**

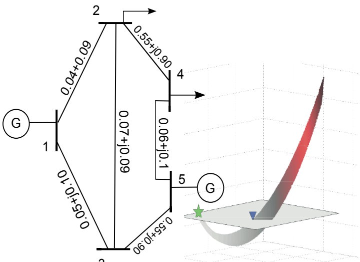
- **feedback (gradient descent)**

  $\rightarrow$ continuous closed-loop trajectory $x(t)$ guaranteed to be **feasible**

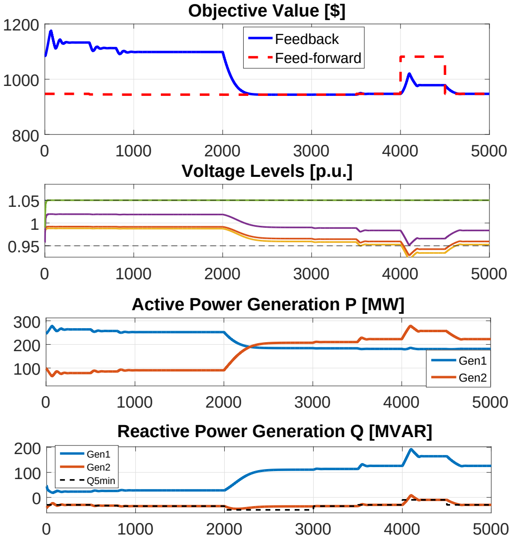
  $\rightarrow$ convergence of $x(t)$ to a **local minimum** is guaranteed

# Illustration of continuous trajectories & reachability

5-bus system known to have two disconnected feasible regions:



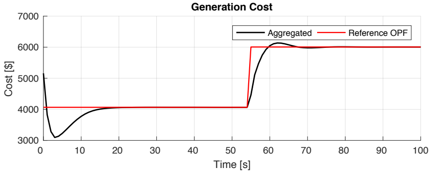

[Molzahn, 2016]

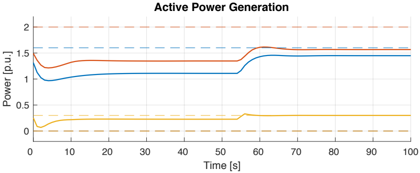


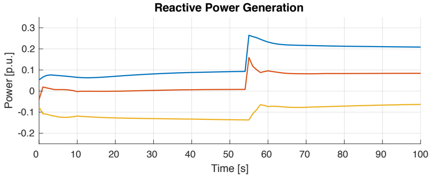

- [0s,2000s]: separate feasible regions
- [2000s,3000s]: loosen limits on reactive power $\underline{Q}_2$ $\rightarrow$ regions merge
- [4000s,5000s]: tighten limits on $\underline{Q}_2$ $\rightarrow$ vanishing feasible region

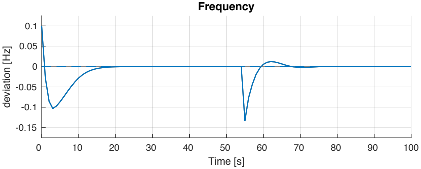
# Feedback optimization with frequency

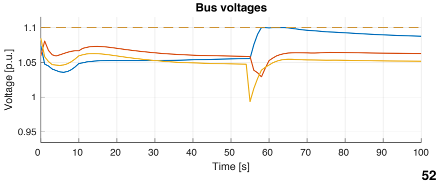
- **frequency** $\omega$ as global variable
- **primary control**: $P = P_G - K\omega$
- **secondary frequency control** incorporated via dual multiplier
- 20% step increase in load



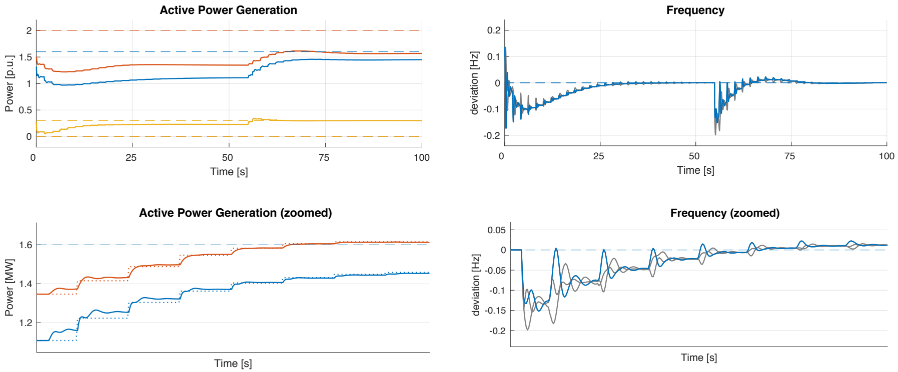



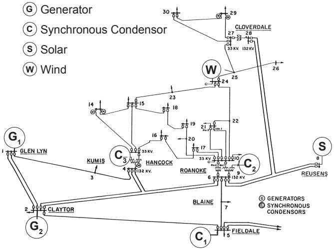
# Same feedback optimization with grid dynamics

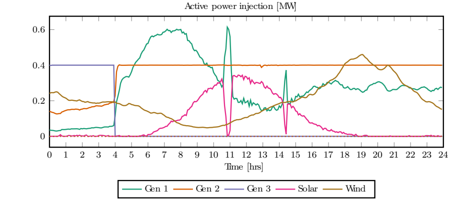


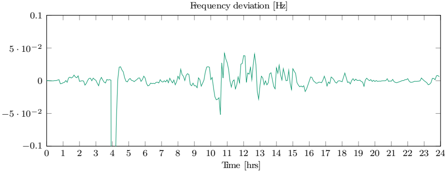

- **dynamic grid model:** swing equation & simple turbine governor
- **work in progress** based on singular perturbation methods

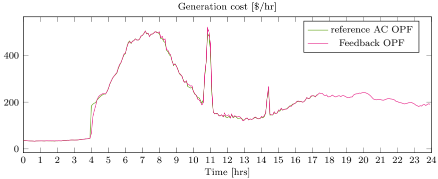
  ⇒ dynamic and quasi-stationary dynamics are “close” and converge to the same optimal solutions under “sufficient” time-scale separation

# Feedback optimization in dynamic IEEE 30-bus system




- **events:**

→ generator outage at 4:00

→ PV generation drops at 11:00 and 14:15

⇒ feedback optimization can provide **all ancillary services** + optimal + constraints + robust + scalable + …