Lasserre hierarchy for stability region approximation

Tutorial on occupation measures and Positivstellensätze

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Visiting ETHZ until April 10th

Outline

1. **Transient stability analysis**
2. **Stability regions**
3. **Occupation measures**
4. **Lasserre hierarchy**

Outline

Rotor angle transient stability

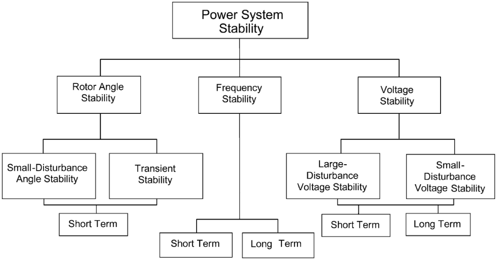

Figure: Kundur et al. Definition and Classification of Power System Stability.

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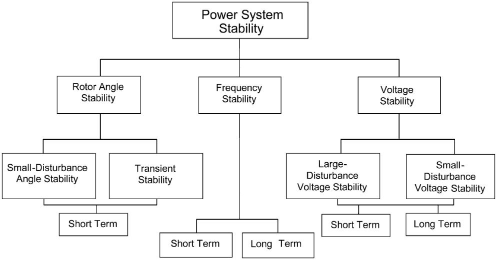

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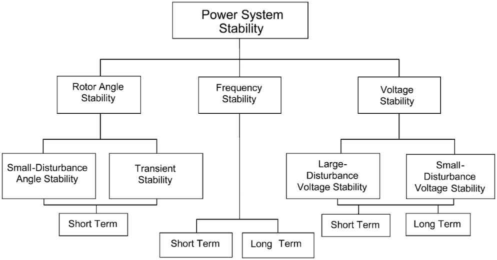

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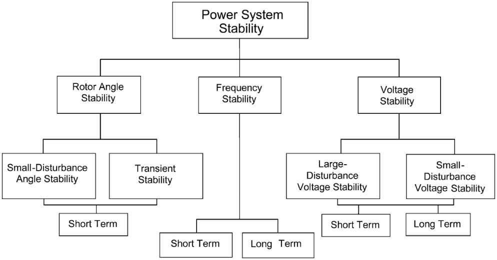

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- **Fast dynamics**: duration $\simeq$ 100 ms $\rightarrow$ short term stability is OK
- **Loss of synchronism** $\Leftrightarrow$ angle explodes $\rightarrow$ state constraints
- **Large perturbation**: nonlinear dynamics $\rightarrow$ nonlinear tools needed

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- Maximal positively invariant set [A. Oustry et al, CDC 2019]

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Context: polynomial differential systems

Autonomous ODE with polynomial dynamics and polynomial constraints:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) \tag{1a}$$

$$g_i(\mathbf{x}(t)) \geq 0 \qquad i = 1, \ldots, m \tag{1b}$$

$t \geq 0$, $\mathbf{x} \in \mathbb{R}^n, \mathbf{f} \in \mathbb{R}[\mathbf{x}]^n$, $\mathbf{g} := (g_1, \ldots, g_m) \in \mathbb{R}[\mathbf{x}]^m$.

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(1b) summed up as $\mathbf{x}(t) \in \mathbf{X}$ where
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N.B.: sin, cos, $\sqrt{\cdot}$ in $\mathbf{f}$ tackled through change of variables
ex: $\{\sin(\theta), \cos(\theta)\}_{\theta \in \mathbb{R}} \leftrightarrow \{(s, c) \in \mathbb{R}^2 : s^2 + c^2 = 1\}$

Infinite time region of attraction

$\mathbf{0} = \bar{\mathbf{x}} \in \mathbf{f}^{-1}(\{\mathbf{0}\}) \cap \mathbf{X}$ locally asymptotically stable equilibrium point

$$\mathbf{A}_\infty^{\bar{\mathbf{x}}} := \left\{ \mathbf{x}_0 \in \mathbf{X} : \mathbf{X} \ni \mathbf{x}(t|\mathbf{x}_0) \underset{t\to\infty}{\longrightarrow} \bar{\mathbf{x}} \right\}$$

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v Lyapunov function :

- v positive definite on $\mathbf{D} \subset \mathbf{X}$ ($\mathbf{0} \in \mathbf{D}$)
- $\dot{v} := \nabla v \cdot \mathbf{f}$ negative definite on $\mathbf{D}$

$\Rightarrow$ any $\{\mathbf{x} \in \mathbb{R}^n : v(\mathbf{x}) \leq c\} \subset \mathbf{D}$ is a positively invariant subset of $\mathbf{A}_{\infty}^{\bar{\mathbf{x}}}$.

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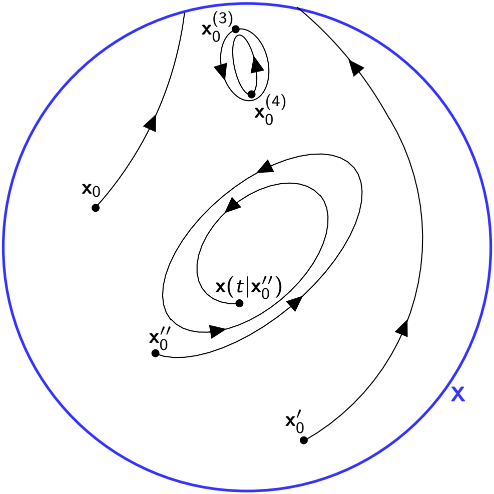
Performances of SOS-based Lyapunov methods

- Finding polynomial Lyapunov function $\Leftrightarrow$ LMI feasibility (convex)
- Optimizing sublevel set $\Leftrightarrow$ solving BMIs (no proof of convergence)

Infinite time ROA: illustration



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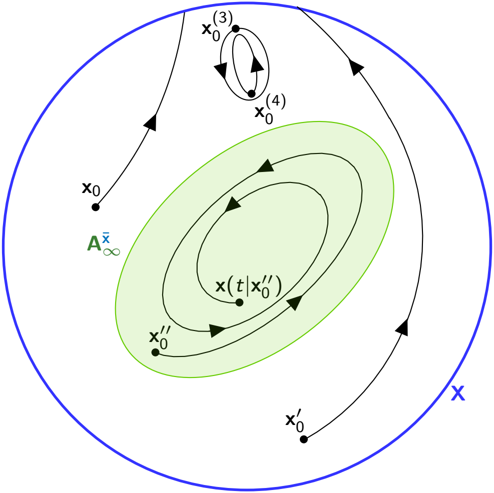

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Target set $\mathbf{\Omega} := \{\mathbf{x} \in \mathbb{R}^n : h_i(\mathbf{x}) \geq 0 \; \forall i = 1, \ldots, m'\} \subset \mathbf{X}$, $\mathbf{h} \in \mathbb{R}[\mathbf{x}]^{m'}$
Time horizon $T > 0$

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Performances of Lasserre hierarchy

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- Parameter dependent: $T, \boldsymbol{\Omega}$

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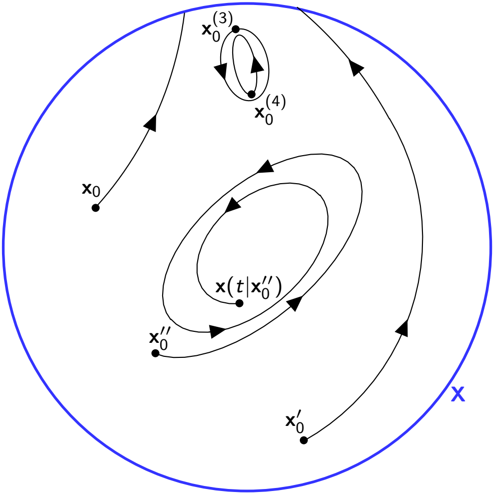

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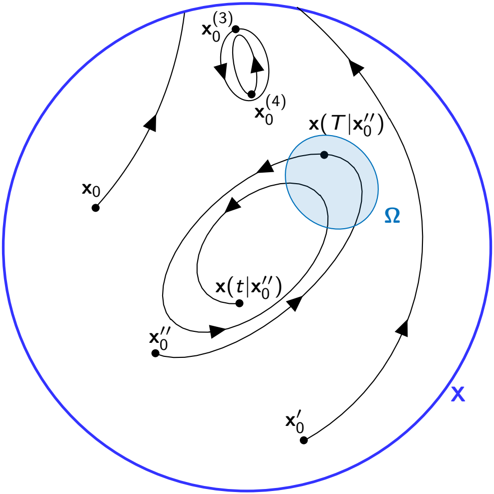

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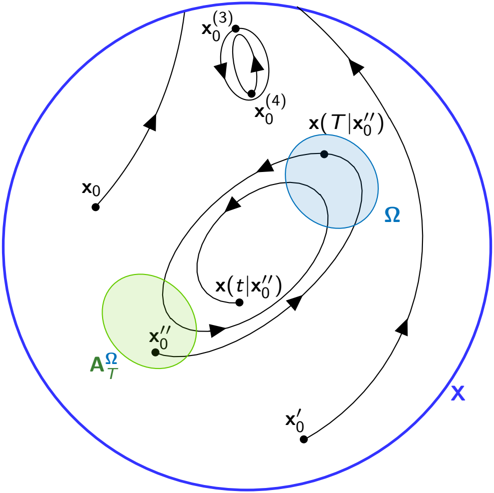

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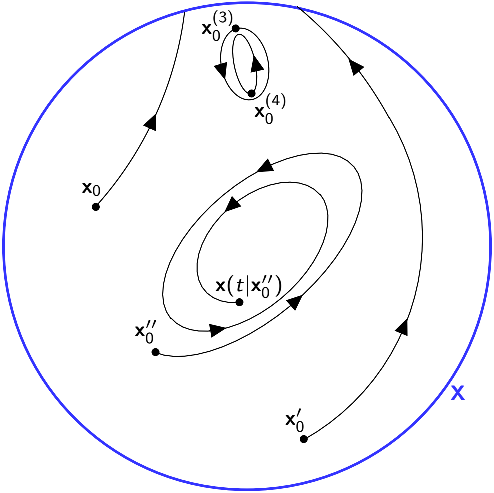

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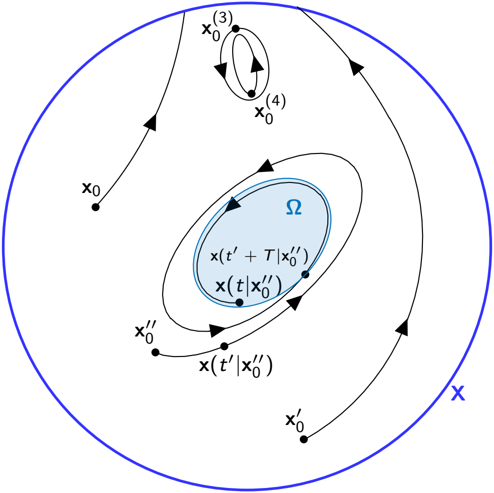

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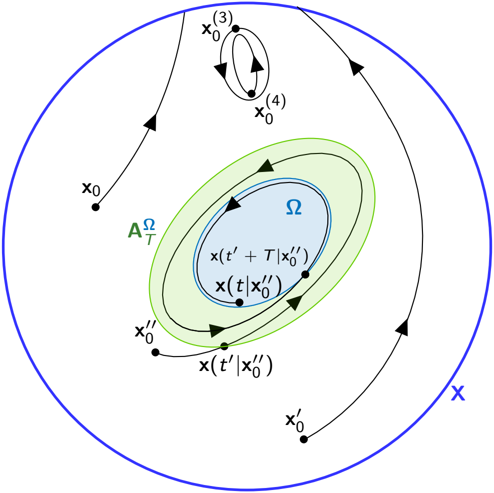

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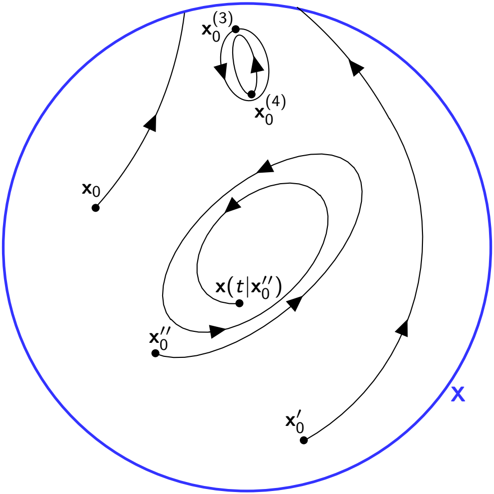
- Needs technical assumptions such as $\int_{\mathbf{X}_\infty^c} \tau(\mathbf{x}_0)\, \mathrm{d}\mathbf{x}_0 < \infty$

MPI set: illustration



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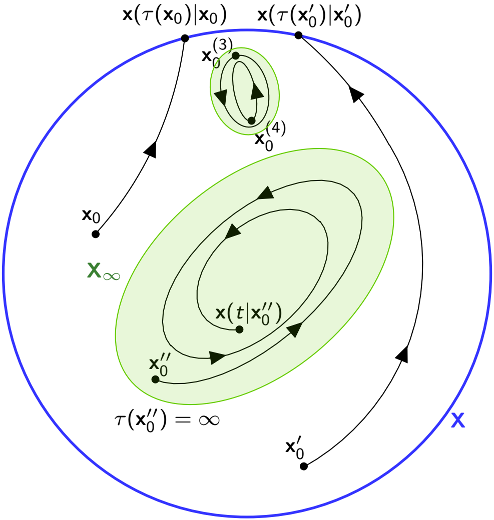

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- Initial measure $\alpha = \mathbb{P}_0$, terminal measure $\omega = \mathbb{P}_T$

Interpretation of occupation measures

$$\mu([a,b] \times \mathbf{Y}) = \mathbb{E}\left[\text{time spent by } \mathbf{x}(t|\mathbf{x}_0) \text{ in } \mathbf{Y} \text{ between } a \;\&\; b\right]$$

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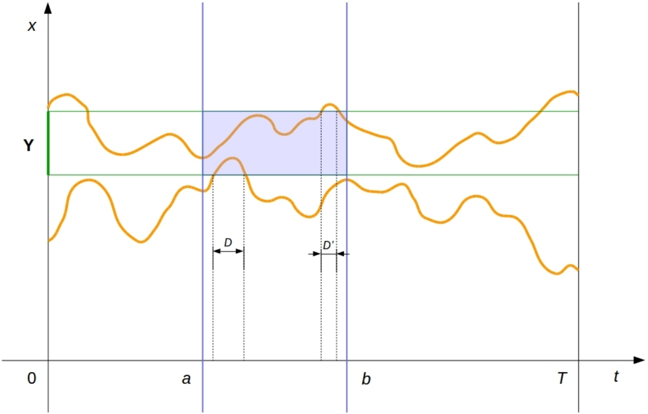

Figure: Example of occupation measure: $\mu([a,b]\times\mathbf{Y}) = \frac{D}{2} + \frac{b-a-D'}{2}$

Transport of occupation measures

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Fluid mechanics interpretation:

- Initial state α of a fluid transported to terminal state ω by $\mathbf{f}$'s flow.
- μ **represents** the trajectories of fluid particles under dynamics $\mathbf{f}$.

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$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \ \& \ \mathbf{x} \in \mathbf{X} \text{ on } [0, T] \quad \Leftrightarrow \quad \frac{\partial \mu}{\partial t} + \operatorname{div}(\mathbf{f}\ \mu) = \delta_0 \alpha - \delta_T \omega \qquad (2)$$

Fluid mechanics interpretation:

- Initial state α of a fluid transported to terminal state ω by **f**'s flow.
- μ **represents** the trajectories of fluid particles under dynamics **f**.

Consequence: spt $\omega \subset \mathbf{\Omega} \overset{\text{def}}{\Longrightarrow}$ spt $\alpha \subset \mathbf{A}_T^{\mathbf{\Omega}}$

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Idea: maximize spt α while spt $\omega \subset \boldsymbol{\Omega}$ & (2) holds.

Linear programming on occupation measures

$$\begin{array}{ll} \max & \alpha(\mathbf{X}) \overset{\text{def}}{=} \int_{\mathbf{X}} 1 \; d\alpha \\ & \partial_t \mu + \operatorname{div}(\mathbf{f} \; \mu) = \delta_0 \alpha - \delta_T \omega \\ \text{s.t.} & \operatorname{spt} \; \omega \subset \mathbf{\Omega} \\ & d\alpha(\mathbf{x}) \leq 1 \; \mathrm{d}\mathbf{x} \end{array}$$

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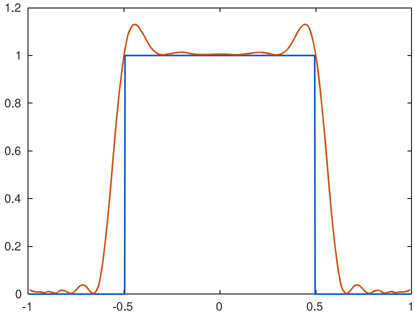

Figure: Possible plot of w VS $\mathbb{1}_{\mathbf{A}_T^{\mathbf{\Omega}}}$.

Outline

How do we represent positive continuous functions ?

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Any (nonnegative) continuous function on the **compact $\mathbf{X}$** can be approximated uniformly by (nonnegative) polynomials:

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Consequence on the dual

Look for $v \in \mathcal{P}([0, T] \times \mathbf{X})$ and $w \in \mathcal{P}(\mathbf{X})_+$ satisfying all constraints

Some useful definitions

Sums of squares

- $\sigma \in \Sigma(\mathbb{R}^n)$ *iff* $\sigma = p_1^2 + \cdots + p_k^2$ for some $p_i \in \mathbb{R}[\mathbf{x}]$

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- $q(x) = 1 + x^2 + (1 + x^4)\, x\, (1 - x) \in \Sigma([0, 1])$

Writing the SOS problem

Putinar's Positivstellensatz

Notation: $\mathcal{P}(\mathbf{X})_{++} := \{p \in \mathbb{R}[\mathbf{x}] : \forall \mathbf{x} \in \mathbf{X}, p(\mathbf{x}) > 0\}$

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N.B: testing if a polynomial is in a $\Sigma_d(\cdot)$ is an LMI (convex optimization).

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- Conclusion: $\{\mathbf{x} \in \mathbb{R}^n : w_d(\mathbf{x}) \geq 1\}$ is a **converging outer approximation** of $\mathbf{A}_T^{\boldsymbol{\Omega}}$ (in the sense of the volume).

Extensions and project at ETH

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Project at ETH:

- Apply existing results to compare performances of droop control & virtual oscillator control for power converters
- Mix existing results with multiple time scale [I. Subotić et al, preprint 2019] framework to increase scalability?

Questions

Thank you for your attention!

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