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Signal and System Theory II

This sheet is provided to you for ease of reference only.
***Do not* write your solutions here.**

Exercise 1

1	2	3	Exercise
9	7	9	25 Points

Consider the discrete time linear system

$$\begin{cases} x_{k+1} &= Ax_k + Bu_k \\ y_k &= Cx_k + Du_k \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$, $y_k \in \mathbb{R}^p$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and $D \in \mathbb{R}^{p\times m}$.

1. A discrete time observer constructs an estimate $\hat{x}_k$ of the state x_k of the form

$$\begin{cases} \hat{x}_{k+1} &= A\hat{x}_k + Bu_k + L(y_k - \hat{y}_k) \\ \hat{y}_k &= C\hat{x}_k + Du_k \end{cases}$$

   starting with $\hat{x}_0 = 0$. Derive the dynamics of the error $e_k = x_k - \hat{x}_k$. Under what conditions will $\hat{x}_k$ converge to x_k as k tends to infinity?

2. Consider now the system (1) with

$$A = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -12 \\ 0 & 1 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad D = 0 \tag{2}$$

   Is the system observable? Is it controllable? Is the system with $u_k = 0$ for all $k = 0, 1, \dots$ stable?

3. Design an observer of the form given in part 1 for the system in part 2. What dimensions should the matrix L have? Select the entries of the matrix L so that the error dynamics have all eigenvalues equal to 0.5.

Exercise 2

1	2	3	4	5	Exercise
3	5	6	7	4	25 Points

Consider the dynamical system

$$\ddot{y}(t) + \dot{y}(t) + y^3(t) - y(t) = \gamma \cos(\omega t) \tag{3}$$

The parameter γ is constant.

1. Is the system autonomous? Is it linear? What is the dimension of the system? Justify your answers.
2. Write the system in state space form using $x_1 = y$ and $x_2 = \dot{y}$ as states.
3. Determine all equilibria of the system when $\gamma = 0$.
4. Using linearization determine the stability of all equilibria computed in part 3.
5. Based on the eigenvalues of the linearization computed in part 4 try to guess what the phase plane plot ($x_1(t) - x_2(t)$ parameterized by t) of the system may look like. Take $\gamma = 0$.

Exercise 3

1	2	3	4	Exercise
5	7	6	7	25 Points

Consider the following continuous-time linear, time invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (4)$$
$$y(t) = Cx(t), \quad (5)$$

with

$$A = \begin{bmatrix} -2.5 & 0.5 \\ 0.5 & -2.5 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Let $T \in \mathbb{R}^{2\times 2}$ be an invertible matrix. Define $\hat{x}(t) = Tx(t)$ and consider also the system

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t) \quad (6)$$
$$y(t) = \hat{C}\hat{x}(t). \quad (7)$$

1. Compute $\hat{A}, \hat{B}, \hat{C}$ as a function of A, B, C and T.
2. Find a matrix T such that $\hat{A}$ is a diagonal matrix $\hat{A} = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$. Justify why this calculation is possible. What will be the values of d_1 and d_2?
3. Using your answer in part 2, determine whether the system (6)-(7) is controllable and observable, without calculating the controllability and observability matrices. What can you conclude about the controllability and observability of the original system?
4. How should the initial condition $x(0)$ of the system (4)-(5) be chosen, so that the zero input response is $x(t) = x(0)e^{d_1 t}$? (Hint: Base your answer on part 2).

Exercise 4

1	2	3	4	Exercise
7	6	5	7	25 Points

Consider the following system:

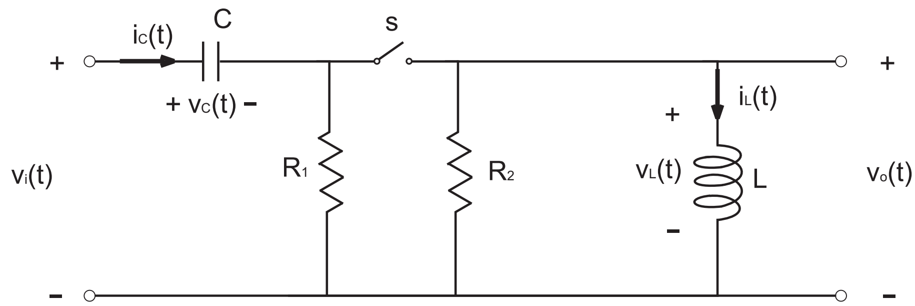

Figure 1: Electrical circuit with switch.

1. Using $u = V_i$ as an input and $y = V_0$ as an output derive the state space equations of the system when the switch s is open. Repeat for the case where s is closed.

2. Is the system observable with the switch open? Is it observable with the switch closed? Justify your answers both mathematically and intuitively.

3. Is the system controllable with the switch open? Is it controllable with the switch closed? An intuitive explanation is sufficient.

4. Assume that $R_1 = R_2 = \frac{2}{3}\Omega$, $C = 1F$ and $L = \frac{1}{2}H$, $V_i(t) = 0V$ for all $t \geq 0$, $V_c(0) = 1V$ and $i_L(0) = 1A$. Compute the system response starting at $t = 0$ assuming that the switch remains open until $t = 1$ and then closes and remains closed for all $t > 1$.