Automatic Control Laboratory
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D-ITET
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Signal and System Theory II

This sheet is provided to you for ease of reference only.
***Do not* write your solutions here.**

1 Exercise 1

1	2	3	4	Exercise
5	5	8	7	25 Points

Consider the continuous time, linear, time invariant system:

$$\dot{x} = Ax$$

where

$$A = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

1. Determine **all** equilibria of the system. Draw their locations in the x_1 - x_2 plane.
2. Is the equilibrium $\hat{x} = 0$ stable? Is it asymptotically stable?
3. Compute the eigenvalues and eigenvectors of the matrix A. Hence determine the state transition matrix e^{At}.
4. Draw the phase plane plot of the system ($x_2(t)$ versus $x_1(t)$). Relate the plot to your answer in part (2).

2 Exercise 2

1	2	3	4	Exercise
5	5	5	10	25 Points

Consider the discrete time, linear time invariant system:

$$x_{k+1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x_k = Ax_k \quad (1)$$

$$y_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_k = Cx_k \quad (2)$$

1. Is the system controllable? Is it observable?
2. Show that the initial state x_0 of the system can be computed using only 3 output measurements y_0, y_1, y_2.
3. A discrete time observer constructs an estimate $\hat{x}_k$ of the state x_k starting from an arbitrary initial guess $\hat{x}_0$ and iterating

$$\hat{x}_{k+1} = A\hat{x}_k + L(y_k - C\hat{x}_k) \quad (3)$$

   Derive the dynamics of the estimation error $e_k = x_k - \hat{x}_k$.
4. You would like to determine a gain matrix $L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$ such that the dynamics of the error $e_k = x_k - \hat{x}_k$ are stable. You may either place all three of the eigenvalues of e_k at $\lambda = -2$ or at $\lambda = \frac{1}{2}$. Which would you choose? Design L to achieve the eigenvalues of your choice. Comment on the advantages and disadvantages of this estimation method over the one in part (2).

3 Exercise 3

1	2	3	4	Exercise
5	8	7	5	25 Points

Consider the linear, time invariant system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

1. Is the system controllable? Is it observable? Is it stable under the input $u(t) = 0$ for all t?

2. Assume that state feedback $u = Kx + v$ for some $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$ is applied to the system. Derive the dynamics of the resulting "closed loop" system and compute its transfer function from v to y.

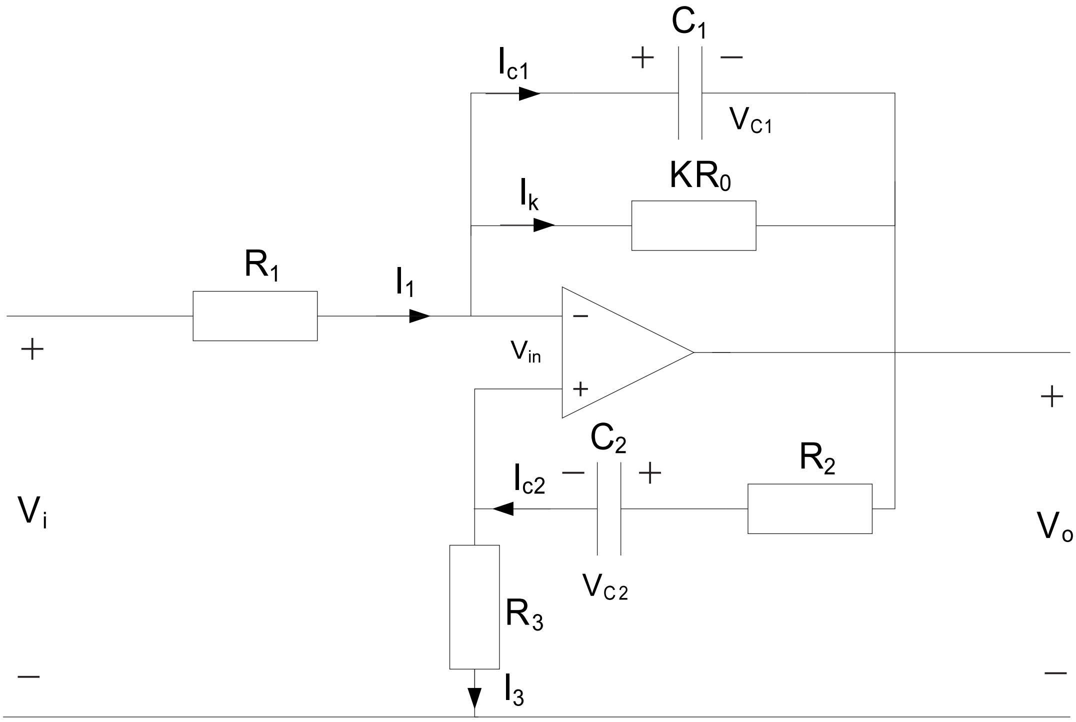
3. Compute values for k_1 and k_2 such that the poles of the closed loop transfer function in part (2) are both at $s = -1$. Is the closed loop system stable with $v(t) = 0$ for all t?

4. Notice that only the first state is measured directly through y, but the values of both states are needed for the state feedback in parts (2)-(3). What would you do if you were asked to implement the state feedback in parts (2)-(3) using only the measured y?

4 Exercise 4

1	2	3	Exercise
10	8	7	25 Points

You are given the following electrical circuit. Assume that the operational amplifier is ideal, that $K > 0$ and that $C_1 = C_2 = 1F$ and $R_1 = R_2 = R_3 = R_0 = 1\Omega$.



1. Derive the state space equations of the system using as states $x_1 = V_{C1}$ and $x_2 = V_{C2}$, as input $u = V_i$ and as output $y = V_o$.
2. Compute the transfer function of the system.
3. Analyze the stability of the system and discuss qualitatively its response as K changes.