Automatic Control Laboratory
ETH Zurich
Prof. J. Lygeros

D-ITET
Winter 2008
28.01.2008

Signal and System Theory II

This sheet is provided to you for ease of reference only.
***Do not* write your solutions here.**

Exercise 1

1	2	3	Aufgabe
5	10	10	25 Punkte

Consider the continuous time, linear, time invariant system:

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

1. Is the system controllable? Is it observable?
2. Calculate the system output response to a unit step input and an initial condition $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$.
3. Compute the Laplace transform of the state response for the input and initial condition of part 2. Verify that the initial condition is correct using the initial value theorem. Determine what happens to the state as $t \to \infty$ using the final value theorem.

Exercise 2

1	2	3	4	Exercise
5	**5**	**10**	**5**	**25 Points**

Consider the following set of difference equations:

$$\begin{aligned} \theta_{k+1} &= 6\phi_k \\ \phi_{k+1} &= 2\zeta_k \\ \zeta_{k+1} &= a\zeta_k + \tau_k \end{aligned} \tag{1}$$

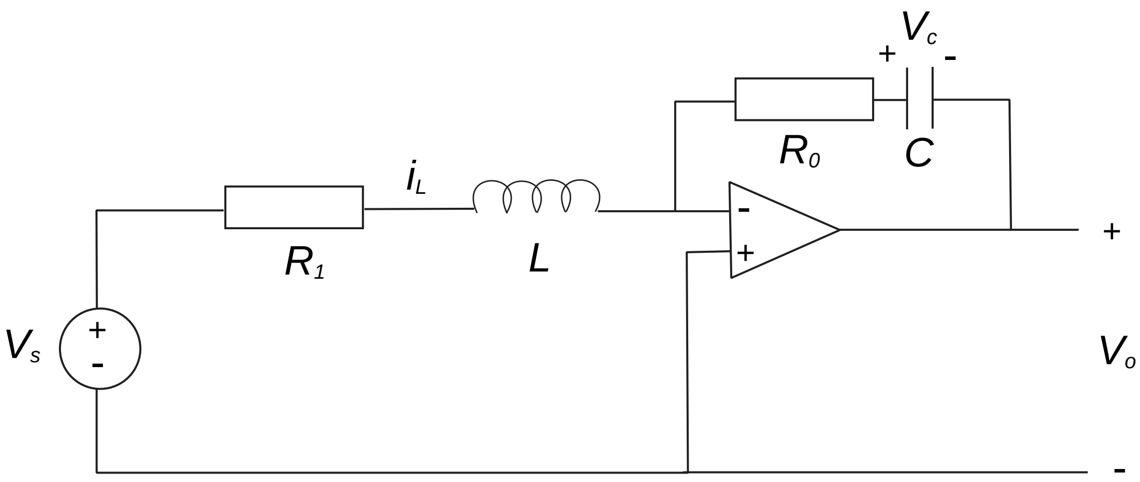
where $a \in \mathbb{R}, \theta_k \in \mathbb{R}, \phi_k \in \mathbb{R}, \zeta_k \in \mathbb{R}, \tau_k \in \mathbb{R}, k \in \mathbb{N}$.

1. Express the system of equations given in (1) in discrete time state space form, using as state, control, and output variables $x_k = \begin{bmatrix} \theta_k \\ \phi_k \\ \zeta_k \end{bmatrix} \in \mathbb{R}^3, u_k = \tau_k \in \mathbb{R}$, and $y_k = \theta_k \in \mathbb{R}$ respectively.
2. Is the resulting discrete time system linear and time invariant? Is it controllable? Is it observable? Justify your answers.
3. Consider now the zero input response of the system ($u_k = 0,\ \forall k = 0, 1, 2, ...$). For what values of a does x_k go to zero after a finite number of steps k, for any initial condition $x_0 \in \mathbb{R}^3$? What is the maximum number of steps before x goes to zero in this case?
4. Your friend from EPFL claims that the system in part 3 (where $x_k \rightarrow 0$ in finite steps) was obtained by sampling a continuous time autonomous, linear time invariant system every T seconds. Do you believe her? Justify your answer.

Exercise 3

1	2	3	4	Exercise
8	6	5	6	25 Points

Consider the following circuit:



1. Write a state space model for this circuit. Use $x_1 = i_L$, $x_2 = V_c$ as the states of the system, $u = V_s$ as input and $y = V_o$ as output. Assume that the operational amplifier is ideal.

2. Notice that the input can affect directly the first state but not the second. Despite of this, a colleague claims that for any $T > 0$ she can steer the system from any initial state $(i_L(0), V_c(0))$ to any final state $(i_L(T), V_c(T))$ by applying a suitable input $V_s(\cdot) : [0, T] \to \mathbb{R}$. Do you agree with her? Justify your answer.

3. Your colleague also claims that for $T > 0$, knowing $V_s(\cdot) : [0, T] \to \mathbb{R}$ and $V_o(\cdot) : [0, T] \to \mathbb{R}$ she can infer the values of $i_L(0)$ and $V_c(0)$. Do you agree with her? Justify your answer.

4. Compute the transfer function of the system. Are there any pole-zero cancellations? Relate this to your answer in parts 2 and 3 of this exercise.

Exercise 4

1	2	3	Aufgabe
8	**7**	**10**	**25 Punkte**

The following differential equation describes the behavior of a simple pendulum under the influence of a constant torque of magnitude μ:

$$\ddot{\theta} + \delta\dot{\theta} + \sin\theta = \mu \tag{2}$$

1. Derive the state-space representation of the above system. Is the system linear or nonlinear? Is it time varying? Justify your answers.
2. Determine all the equilibrium points of the system for $\mu = \frac{\sqrt{2}}{2}$.
3. Using linearization, determine (whenever possible) the stability of the equilibria that you found in question 2 for the cases $\delta > 0$ and $\delta = 0$.