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Signal and System Theory II

This sheet is provided to you for ease of reference only.
***Do not* write your solutions here.**

1 Exercise 1

1	2	3	4	Exercise
5	5	8	7	25 Points

Consider the continuous time, linear, time invariant system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned}$$

where

$$A = \begin{bmatrix} 1 & \alpha \\ 0 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

and $\alpha \in \mathbb{R}$.

1. Determine for which α the system is controllable.
2. Determine for which α the system is observable.
3. Let $\alpha = 5$ and set $u(t) = Kx(t)$ where $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \in \mathbb{R}^{1\times 2}$. Determine the values of k_1, k_2 for which the poles of the resulting system are equal to -1 and -7.
4. Is the resulting system in part 3 stable? Justify your answer.

2 Exercise 2

1	2	3	4	Exercise
5	5	5	10	25 Points

Consider the following discrete time system:

$$m\phi_{k+2} + d\phi_{k+1} - (\nu + 1)\phi_k = b\mu_k$$

where m, d, and b are real, positive, constant parameters and $\nu \in \mathbb{R}$ is a design parameter.

1. Write the system in state space form using $x_k = \begin{bmatrix} \phi_k & \phi_{k+1} \end{bmatrix}^T$ as the state vector and $u_k = \mu_k$ as the input.
2. What is the dimension of the system? Is the system autonomous? Is it linear?
3. Consider $m = 2$, $d = 1$, and $b = 3$. Is the system stable for $\nu = -1$? Is the system stable for $\nu = 0$? Is the system stable for $\nu = 2$?
4. Consider $m = 2$, $d = 1$, $b = 3$, and $\nu = 2$. Show that setting $u_k = \begin{bmatrix} -1 & \frac{1}{3} \end{bmatrix} x_k$ results in $x_k = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ for all $k \geq 2$, for any $x_0 \in \mathbb{R}^2$.

3 Exercise 3

1	2	3	Exercise
9	8	8	25 Points

Consider the electric circuit of Figure 1:

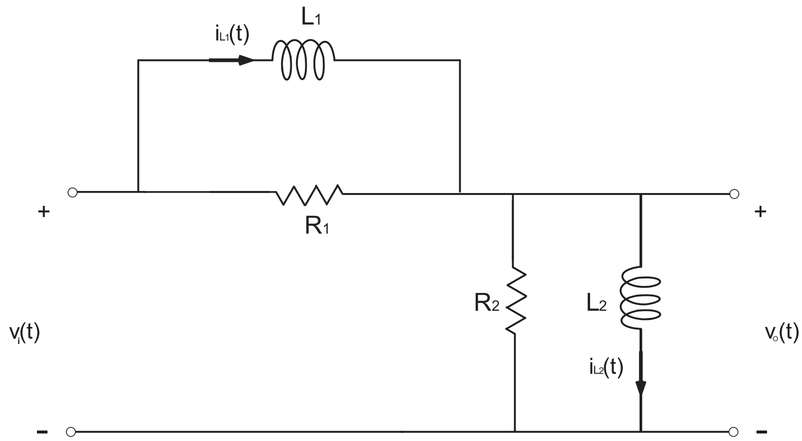

Figure 1: Electric circuit

1. Provide the state space representation of the system, with $x = [i_{L_1}\ i_{L_2}]^T$ as states, V_i as input and V_0 as output.
2. Let $R_1 = R_2 = 1\Omega$ and $L_1 = L_2 = 1H$. Is the system controllable? Is it observable?
3. Compute the zero input response of the system ($V_i = 0$), if $x(0) = [1\ -1]^T$.

4 Exercise 4

1	2	3	Exercise
8	9	8	25 Points

Consider the following linear, time invariant systems:

$$\begin{aligned} &\text{System (1)} && \text{System (2)} \\ \dot{x}(t) &= A_1 x(t) + Bu(t) & \dot{x}(t) &= A_2 x(t) + Bu(t) \\ &\text{System (3)} && \text{System (4)} \\ \dot{x}(t) &= (A_1 + A_2)x(t) + Bu(t) & \dot{x}(t) &= A_1 A_2 x(t) + Bu(t) \end{aligned}$$

Assume that in all cases $x(t) \in \mathbb{R}^2, u(t) \in \mathbb{R}, A_1, A_2 \in \mathbb{R}^{2\times 2}$ and $B \in \mathbb{R}^{2\times 1}$.

1. Assume that systems (1) and (2) are both controllable. Show that system (3) is not necessarily controllable. (It suffices to provide an example)
2. Assume that system (1) is controllable and system (2) is uncontrollable. Show that system (3) is controllable.
3. Assume that systems (1) and (2) are both controllable. Show that system (4) is not necessarily controllable. (It suffices to provide an example)