Automatic Control Laboratory
ETH Zurich
Prof. J. Lygeros

D-ITET
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Signal and System Theory II

This sheet is provided to you for ease of reference only.
***Do not* write your solutions here.**

Exercise 1

1	2	3	4	5	Exercise
2	4	4	5	10	25 Points

Consider the system

$$\begin{aligned} \dot{x}_1(t) &= 2\beta x_1(t) + \beta^2 x_2(t) + u(t) \\ \dot{x}_2(t) &= x_1(t) + 2\beta x_2(t) \\ y(t) &= x_1(t) + x_2(t), \end{aligned} \tag{1}$$

where $u(t)$ is the system input, $y(t)$ the system output, and $\beta \in \mathbb{R}$ is a constant parameter.

1. Is the system linear? Is it time invariant?
2. For which values of β is the system observable?
3. For which values of β is the system controllable?
4. For which values of β is the system asymptotically stable under the input $u(t) = 0$?
5. Consider now $\beta = 2$. Assume that a state feedback $u(t) = Kx(t) + v(t)$, where $K = [k_1 \;\; k_2]$, is applied to the system (1).
   (a) Derive the dynamics of the resulting closed loop system.
   (b) Find k_1, k_2 such that the closed loop system has poles at -1 and -2.

Exercise 2

1	2	3	4	Exercise
3	5	10	7	25 Points

Consider the differential equation

$$\ddot{y}(t) - \dot{y}(t)^2 y(t) + y(t)^2 - 1 = 0.$$

where $y(t) \in \mathbb{R}$ is the system output.

1. Is the system linear? Is the system autonomous? Is it time invariant?
2. Using the states $x_1(t) = \dot{y}(t)$, $x_2(t) = y(t)$, write the system in state space form
$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= g(x(t), u(t)). \end{aligned}$$
3. Find all equilibria of the system and, wherever possible, determine their stability using linearization.
4. In Figure 1 you are given a phase-plane plot of the system in the vicinity of one equilibrium from part 3. Which equilibrium could the phase-plane plot correspond to? Justify your answer.

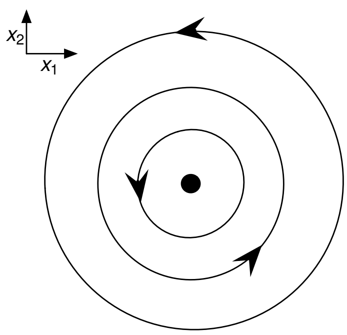

Figure 1: Phase-plane plot around one equilibrium.

Exercise 3

1	2	3	Exercise
10	8	7	25 Points

Consider the following circuit:

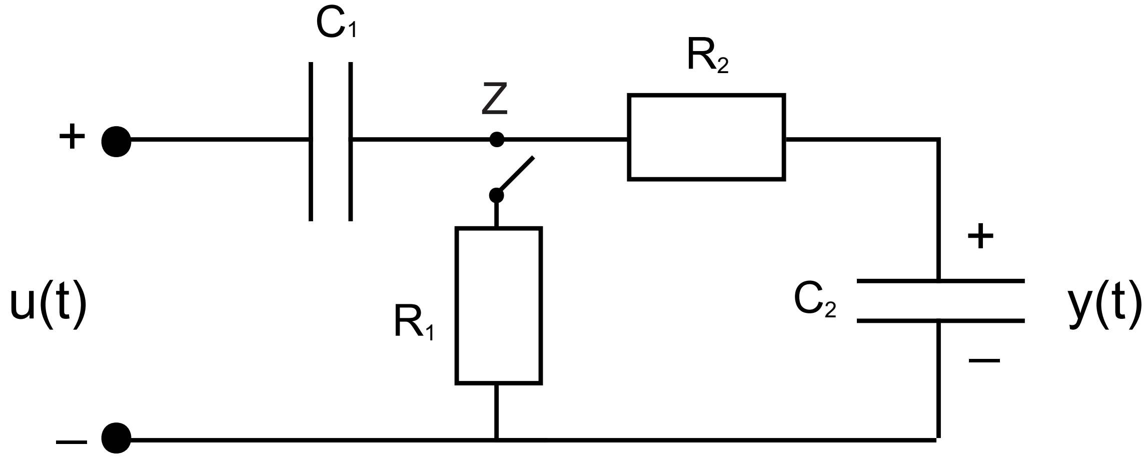

Figure 2: Electrical circuit

1. Let $Z(t) = 1$ represent the case where the switch Z is closed and R_1 is connected and $Z(t) = 0$ the case where the switch Z is open and R_1 is not connected. Define an appropriate state vector $x(t)$ and derive a state-space model of the system of the form

$$\dot{x}(t) = A(Z(t))x(t) + B(Z(t))u(t)$$
$$y(t) = Cx(t) + Du(t),$$

with input $u(t)$ and output $y(t)$ as shown in Figure 2.

2. Assume $C_1 = C_2 = 1F$ and $R_1 = R_2 = 1\Omega$ and consider the case where the switch is open for all times. Calculate the transfer function $G(s)$. Are there any pole-zero cancellations? What can you conclude?

3. Consider the case where the switch is open for all times. Derive the zero-input response of the system for the initial state $x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

   Hint: Use your calculations from part 2.

Exercise 4

1	2	3	Exercise
9	8	8	25 Points

1. Consider the continuous time system

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t) = \begin{bmatrix} -2 & -6 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t). \qquad (2)$$

   (a) Compute the eigenvalues and the eigenvectors of $\bar{A}$.

   (b) Suppose $u(t) = 0$ for all t. Is the system asymptotically stable?

   (c) Compute the matrix exponential $e^{\bar{A}t}$, as a function of $t > 0$.

2. Suppose that you want to discretize the system (2) using a zero order hold, that is, $x_k = x(kT)$ and $u(t) = u_k$ for all $t \in [kT, (k+1)T)$.

   (a) Compute the matrices A and B for the corresponding discrete time system,

$$x_{k+1} = Ax_k + Bu_k, \qquad (3)$$

   for an arbitrary $T > 0$.

   (b) Are there values of T such that the discrete time system (3) is not asymptotically stable? Justify your answer.

3. A lazy friend of yours discretizes system (2) using

$$x_{k+1} = \hat{A}x_k + \hat{B}u_k = (I + \bar{A}T)x_k + \begin{bmatrix} T - T^2 \\ 0 \end{bmatrix} u_k. \qquad (4)$$

   Find the values of T such that the discrete time system (4) is not asymptotically stable.