Automatic Control Laboratory
ETH Zurich
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D-ITET
Repeat Examination Winter 2012/13
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Signal and System Theory II

This sheet is provided to you for ease of reference only.
***Do not* write your solutions here.**

Exercise 1

1	2	3	4	Exercise
7	6	5	7	25 Points

Consider the linear time-invariant single input, single output systems

$$\begin{aligned} \dot{x}_1(t) &= Ax_1(t) + b_1 u_1(t), \\ y_1(t) &= C_1 x_1(t), \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x}_2(t) &= Ax_2(t) + b_2 u_2(t), \\ y_2(t) &= C_2 x_2(t), \end{aligned} \tag{2}$$

where $x_1(t), x_2(t) \in \mathbb{R}^2$, $u_1(t), u_2(t) \in \mathbb{R}$, $y_1(t), y_2(t) \in \mathbb{R}$, $A \in \mathbb{R}^{2\times 2}$, $b_1, b_2 \in \mathbb{R}^{2\times 1}$ and $C_1, C_2 \in \mathbb{R}^{1\times 2}$. Consider also the two input, single output system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + [b_1 \ b_2]u(t), \\ y(t) &= (C_1 + C_2)x(t), \end{aligned} \tag{3}$$

where $x(t) \in \mathbb{R}^2$, $u(t) \in \mathbb{R}^2$ and $y(t) \in \mathbb{R}$.

1. Show that if either (1) or (2) is controllable, then (3) is also controllable.
2. If both (1) and (2) are not controllable, determine necessary and sufficient conditions on b_1, b_2 which ensure that (3) is controllable.
3. Assume that both (1) and (2) are observable. Show that (3) is not necessarily observable. It suffices to provide an example.
4. Consider the closed loop system formed by setting

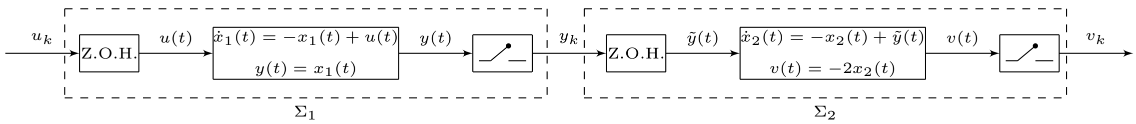
   $$u_1(t) = Kx_1(t) + r(t),$$

   in system (1). Here $K \in \mathbb{R}^{1\times 2}$ and $r(t) \in \mathbb{R}$ is an auxiliary input. Show that the closed loop system is controllable from $r(t)$ if and only if (1) is controllable from $u_1(t)$.

Exercise 2

1	2	Exercise
10	15	25 Points

1. Two continuous time systems are first discretized using a zero order hold and sample operation and then interconnected, as shown in the figure below. Assume the sampling time is T and all the quantities are scalar, i.e., $x_1(t), x_2(t), u(t), \ldots \in \mathbb{R}$.

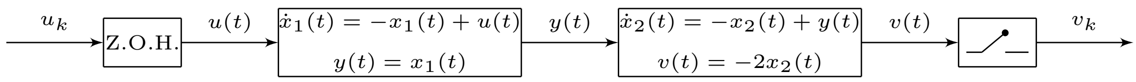


   (a) Find the state space description for the discrete time system Σ_1.

   (b) Find the state space description for the discrete time system Σ_2.

   (c) Find the state space description for the combined system from u_k to v_k.

2. Two continuous time systems are interconnected and then discretized using a zero order hold and sample operation as shown in the figure below. The sampling time is again T and all quantities are scalar.



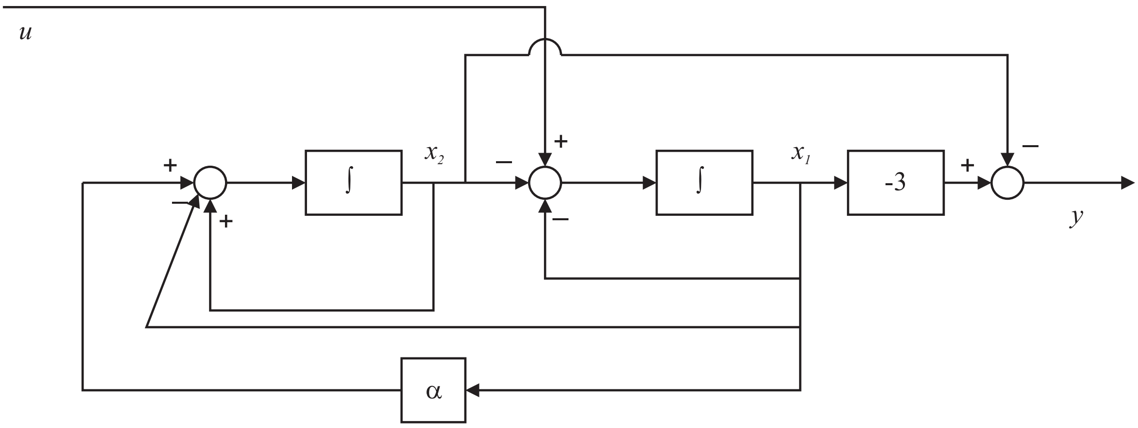
   (a) Find the state space description for the two continuous time systems connected in series from $u(t)$ to $v(t)$

   (b) Find the state space description for the discrete time system from u_k to v_k. Hint: For $R_1, R_2 \in \mathbb{R}^{n\times n}$ if $R_1R_2 = R_2R_1 \Rightarrow e^{R_1+R_2} = e^{R_1}e^{R_2}$

Exercise 3

1	2	3	4	5	Exercise
5	5	7	4	4	25 Points

Consider the following system:



1. Derive the state-space representation in standard form.
2. For which values of α is the system controllable? For the values of α for which controllability is lost compute the set of states that can be reached from the origin.
3. Set $\alpha = -2$. Compute the transfer function of the system. Are there any pole-zero cancellations? What can you conclude?
4. Set $\alpha = 2$. Determine the set of states for which $y(t) = 1$. Determine all equilibrium states of the system when $u(t) = 0$ for all $t \geq 0$.
5. Set again $\alpha = 2$ and assume $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$. Using your answer to part 4, determine whether it is possible to find an input $u(t)$ such that $u(t) = 0$ for all $t \geq 1$ and $y(t) = 1$ for all $t \geq 1$.

Exercise 4

1	2	3	4	Exercise
3	**6**	**13**	**3**	**25 Points**

Consider the differential equation

$$\ddot{y}(t) + 2\dot{y}^3(t) + y^2(t) - 2y(t) + 1 = c^2,$$

where $y(t) \in \mathbb{R}$ is the system output and $c \in \mathbb{R}$ is a constant.

1. Is the system linear? Is it autonomous? Is it time invariant?

2. Using $x_1(t) = \dot{y}(t)$ and $x_2(t) = y(t)$ as states, write the system in state space form

$$\begin{aligned}\dot{x}(t) &= f(x(t), u(t))\\ y(t) &= g(x(t), u(t)).\end{aligned}$$

   What is the dimension of the system?

3. Find all equilibria of the system. Linearize the system around the equilibria and analyze their stability for the case $c = 2$.

4. For which values of c is your stability analysis in part 3 inconclusive for each equilibrium?