Automatic Control Laboratory
ETH Zurich
Prof. J. Lygeros

D-ITET
Winter 2011
12.02.2011

Signal and System Theory II

This sheet is provided to you for ease of reference only.
***Do not* write your solutions here.**

Exercise 1

1	2	3	Exercise
10	9	8	27 Points

Consider the following electrical network.

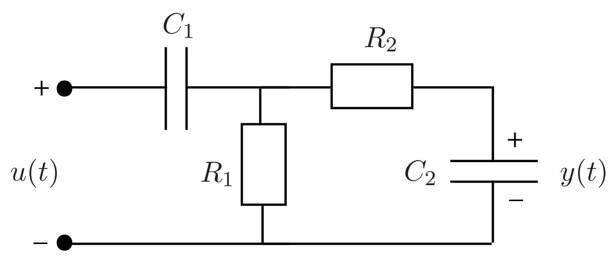

Figure 1: Electrical circuit

1. Define an appropriate state vector $x(t)$ and derive a state-space model of the system

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t),$$

   with input $u(t)$ and output $y(t)$ as shown in Figure 1. What is the dimension of the system?

2. Assume $C_1 = C_2 = 1F$ and $R_1 = R_2 = 1\Omega$. Calculate the transfer function $G(s)$.

3. Use your answer in part 2 to calculate the steady state output response of the system to the input signal $u(t) = sin(t)$.

Exercise 2

1	2	3	Exercise
7	9	10	26 Points

Consider the following discrete time linear system

$$z(k+2) - 0.7z(k+1) + 0.1z(k) = 0.5u(k)$$
$$y(k) = z(k+1).$$

1. Let $x(k) = \begin{bmatrix} z(k+1) \\ z(k) \end{bmatrix} \in \mathbb{R}^2$, and write the system in state space form

$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k).$$

   What is the dimension of the system?

2. Is the system controllable? Is it observable? If we set $u(k) = 0$ for all k, is the system asymptotically stable? Justify your answers.

3. Consider now an observer that generates an estimate $\hat{x}(k)$ of the state $x(k)$, by starting with an arbitrary $\hat{x}(0)$ and evolving according to

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - C\hat{x}(k)).$$

   Derive a state space model for the evolution of the observation error $e(k) = \hat{x}(k) - x(k)$. Determine the entries of the matrix $L \in \mathbb{R}^{2\times1}$ so that the eigenvalues of the observation error dynamics are both equal to 0.1.

Exercise 3

1	2	3	4	Exercise
5	7	9	6	27 Points

Consider the system

$$\dot{y}(t) = \frac{z(t)}{z(t)^2+1}y(t)$$
$$\dot{z}(t) = -\frac{z(t)}{z(t)^2+1}y(t),$$

where $y(t), z(t) \in \mathbb{R}$.

1. Write the system in state space form. What is the dimension of the system? Is the system autonomous? Is it linear?
2. Compute all equilibria of the system.
3. Let $(\hat{y}, \hat{z})$ denote an equilibrium of the system. Linearize the system about this equilibrium. Can you determine the stability of $(\hat{y}, \hat{z})$ from the linearization when $\hat{y} > 0$? What about $\hat{y} < 0$? Repeat for $\hat{z} > 0$ and $\hat{z} < 0$.
4. Draw all equilibria of the system on the (y, z) plane. By differentiating the function $V(y, z) = y + z$ show that all lines of the form $y + z = c$ for constant $c \in \mathbb{R}$ are invariant sets. Draw the line for $c = 1$ on your plot and use your results from part 3 to argue about how different points on $y + z = 1$ are going to move under the dynamics of the system.

Exercise 4

1	2	3	Exercise
6	4	10	20 Points

Consider a second order, continuous time, linear, time invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t).$$

Assume that the state transition matrix is given by:

$$e^{At} = \frac{1}{3}\begin{bmatrix} 2e^{-t} + e^{2t} & -2e^{-t} + 2e^{2t} \\ -e^{-t} + e^{2t} & e^{-t} + 2e^{2t} \end{bmatrix}.$$

1. Determine the eigenvalues of the system directly from e^{At}.
   If we set $u(t) = 0$ for all t, is the sytem stable? Is the system asymptotically stable?
2. Using your answer in part 1, can you determine whether the eigenvectors of A are linearly independent without computing them?
3. Compute the system matrix A.