Pergamon

Automatica, Vol. 31, No. 12, pp. pp. 1751–1770, 1995

0005-1098/95 $9.50 + 0.00

0005-1098(95)00094-1

Identification and Control — Closed-loop Issues *

PAUL M. J. VAN DEN HOF † and RUUD J. P. SCHRAMA †‡

A survey is given of recent developments in iterative methods of closed-loop identification and control design, where the identification criteria are based on control-relevant cost functions.

Key Words—System identification; robust control; closed-loop identification; experiment-based control design; adaptive control.

Abstract—An overview is given of some current research activities on the design of high-performance controllers for plants with uncertain dynamics, based on approximate identification and model-based control design. In dealing with the interplay between system identification and robust control design, some recently developed iterative schemes are reviewed and special attention is given to aspects of approximate identification under closed-loop experimental conditions.

1. INTRODUCTION

The identification of dynamic models out of experimental data has very often been motivated and supported by the presumed ability to use the resulting models as a basis for model-based control design. As such, control design is considered an important intended-application area for identified models. On the other hand, model-based control design is built upon the assumption that a reliable model of the plant under consideration is available. Without a model, no model-based control design. These statements seem to point to two research areas between which one would expect many interrelations yet to exist. However reality is different.

In the past twenty years identification and control design have shown a development in two separate directions with hardly any relationships.

* Received 20 April 1994; revised 24 February 1995; received in final form 8 June 1995. The original version of this paper was presented at the 10th IFAC Symposium on System Identification, which was held in Copenhagen, Denmark, during 4–6 July 1994. The Published Proceedings of this IFAC meeting may be ordered from: Elsevier Science Limited, The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, U.K. This paper was recommended for publication in revised form by Guest Editors Torsten Söderström and Karl Johan Åström. Corresponding author Dr Paul M. J. Van den Hof. Tel. +31 15 2784509; Fax +31 15 2784717; E-mail vdhof@tudw03.tudelft.nl.

† Mechanical Engineering Systems and Control Group, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands.

‡ Now with Royal Dutch/Shell Group, NAM Business Unit Olie, P.O. Box 33, 3100 AA Schiedam, The Netherlands.

While model-based control design has been developed into robust control, the importance of accurate model descriptions has been amplified. Apart from a nominal plant model, robust control-design methods employ a description of the model uncertainty, i.e. some (hard) upper bound on a specific mismatch between plant and model, in order to be able to evaluate robust stability and/or robust performance of the controlled plant, see e.g. Francis (1987), Maciejowski (1989) and Doyle *et al.* (1992). In the robust control-design paradigm, as a rule, one assumes model and uncertainty to be given a priori. However, one accepts that the (nominal) models that are used in general will not be able to capture all of the dynamics that are present in the plant, as exact modelling is either impossible or too costly.

In system identification, emphasis has long been on aspects of consistency and efficiency, related towards the reconstruction of the "real plant" that underlies the measurement data. However, in real-life situations, models that are identified from data will generally be contaminated with errors due to both aspects of bias (undermodelling) and variance. Even after the introduction of undermodelling issues in identification, as e.g. the asymptotic bias distribution expressions in prediction error methods in Wahlberg and Ljung (1986), it has still not been possible to formulate explicit results for the reliability (uncertainty) of identified approximate models. In this mainstream area of identification, one mainly has to stick to asymptotic confidence intervals that are only valid in the case of consistent modelling see e.g. Ljung (1987). As a result there is a severe problem in explicitly quantifying the accuracy of estimated models.

At the end of the eighties it was pointed out by a number of people that the established techniques for identification and control design were hardly related to each other. This was due to two main points: firstly, it is generally not possible to bound

the uncertainty in identified models; and secondly, it is not clear what kind of (approximate) models are best suited for model-based control design.

Nevertheless, both communities would very likely agree on the relevance and importance of the question: "How can one arrive at appropriate (high performance) controlled plants on the basis of plant models that result from (or at least are validated by) measurement data?".

The challenge to bring identification and control design more closely together and to tackle the problem formulated above, has led to a substantially increased attention for the problem area indicated by "identification for control" (from an identification point of view) or "experiment-based control design" (from a control-design point of view). The core of this problem is briefly indicated next.

a. Identification methods deliver a nominal model of a plant with unknown dynamics. Some methods deliver also an uncertainty region. The nominal model is just an approximation of the plant.
b. Based on this nominal model, a controller has to be designed, assuming a certain level of accuracy (uncertainty) of the nominal model.

The performance achieved by this controller when applied to the plant will be highly dependent on the nominal model and the assumed uncertainty.

From here the research for control-relevant system identification branches into two directions. These directions are depicted in Fig. 1 which relates to the above remarks. The branch on the left illustrates the demand of robust control theory for a quantification of the "model error". The right branch concerns the identification of a nominal model that is suited for high-performance control design.

In this paper we will emphasize the right branch of this problem, but without losing sight of the left branch. However, for a detailed discussion on methods and techniques for estimating model uncertainties, we will refer to the literature.

An interconnection between identification and control design has been investigated before. For example, in Astrom and Wittenmark (1971) probabilistic schemes for simultaneous identification and control design have been proposed, and in Gevers and Ljung (1986) an optimal identification experiment is proposed for control-design model applications. However, similar to the "classical" separation theorem in optimal control, these works consider exact models and aspects of approximation are not taken into account.

In this survey paper we will first elucidate the problem of concern, and we will briefly review the main approaches in the literature. In Section 3, we will present a framework for handling the problem directed towards the matching of criteria that are used in control design and in identification. This leads to a generic form of iterative scheme of repeated identification and control design. Next, in Section 4, we review recent developments in approximate closed-loop identification. Several examples of iterative schemes to solve the problem are presented and evaluated in Section 5, while final remarks conclude the paper.

2. MODELS FOR CONTROL — PRELIMINARIES

2.1. *The high-performance control-design problem*

Let us first have a look at how model-based control design is commonly applied in practice. The basic ingredients are a set of control objectives, some nominal model, and possibly an upper bound on some model-plant mismatch (model error).

Let us denote with P_0 a linear, time-invariant *plant*, represented by its discrete-time transfer function; $\hat{P}$ is a *nominal model* of that plant, and $\mathcal{P}_\Delta(\hat{P}, b)$ refers to an *uncertainty set* induced by the nominal model $\hat{P}$ and an uncertainty structure Δ, while the scalar b is a measure for the "size" of this set. The uncertainty set can for instance represent unstructured weighted additive uncertainty, as

$$\mathcal{P}_A(\hat{P}, b) := \{\tilde{P} \mid |\tilde{P}(e^{i\omega}) - \hat{P}(e^{i\omega})|g(\omega)^{-1} \leq b\} \quad (1)$$

with $g(\omega)$ some real-valued weighting function. We could also think of uncertainties in a multiplicative or structured form, see e.g. Doyle *et al.* (1992).

C will denote a linear time-invariant controller, and (P_0, C) represents the closed-loop system composed of plant P_0 and controller C. We will employ the notion of performance of a controlled system in an abstract way, without having it specified in detail at this moment.

Given some $\hat{P}$ and $\mathcal{P}_\Delta(\hat{P}, b)$, the robust control designer carefully chooses a control criterion and weighting functions, and calculates a controller by some numerical optimization. Next, the designer checks on the new controller by applying it to the nominal model $\hat{P}$ in order to examine e.g. the sensitivity, step response, robustness margins, etc. The designed controller will be accepted if it performs satisfactorily on the nominal model. If so, then the performance achieved for the plant is desired or even required to be similar to the designed nominal performance. Thus one pursues a high plant performance through a high nominal performance.

In this line of thought the design of a high-performance controller involves two prerequisites, again pointing to the two branches in Fig. 1:

(1) the controller must be robust with respect to the mismatch between P_0 and $\hat{P}$; and

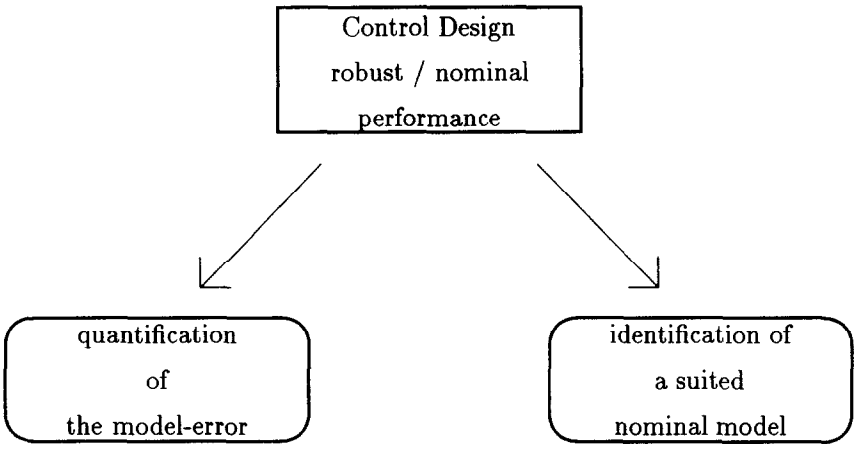

Fig. 1. The two branches of control-relevant system identification.

(2) this mismatch must leave enough room to achieve a high performance.

The quantification of the mismatch between P_0 and $\hat{P}$ can, of course, be done in many different ways. It comes down to the specification of an uncertainty set $\mathcal{P}_\Delta(\hat{P}, b)$ that contains (or is very likely to contain) P_0. Many choices for the uncertainty structure Δ are possible: e.g. additive, multiplicative and coprime factor uncertainty in both unstructured and structured form; it is apparent that the achievable control performance for both $\hat{P}$ and P_0 is dependent on $\hat{P}$, on Δ and on b. The fact that the achievable robust performance is limited for a given uncertainty set $\mathcal{P}_\Delta(\hat{P}, b)$ has been stated frequently in the control theory. However, from an identification point of view, the aim is to *select a nominal model* which does allow the above high-performance control design. Therefore, one can make the following converse observation.

The requirement of a high performance imposes limitations on the allowed structure and size of the uncertainty set $\mathcal{P}_\Delta(\hat{P}, b)$, representing the mismatch between the plant and its nominal model.

For instance, it is well understood that a reasonable fit of the frequency responses around the crossover frequency of the control system is needed for robust performance, see e.g. Stein and Doyle (1991). This is also illustrated by examples in e.g. Schrama (1992a), showing that a seemingly very accurate model in terms of its open-loop transfer function, may very well lead to a destabilizing controller. This supports the earlier statements concerning control-relevant model errors as put forward by Skelton (1989), * who pointed out the need for iterative solutions to the modelling and control design problem. The example of Schrama (1992a) is sketched in the magnitude Bodeplot of Fig. 2, where an eight-order plant P_0, is modelled by two fourth-order models, $\hat{P}_1$ and $\hat{P}_2$. For frequencies smaller than 1.2 rad/s, $\hat{P}_1$ cannot be distinguished from P_0.

When using both models for model-based control design, aiming at a designed bandwidth of 15 rad/s, the controller based on $\hat{P}_1$ will destabilize the model, whereas the controller based on $\hat{P}_2$ will achieve the designed performance. This is inspite of the model errors of $\hat{P}_2$ in the low frequency range. The higher accuracy of $\hat{P}_2$ around the designed bandwidth is the crucial thing here. Larger plant-model deviations are allowed at other frequencies, as long as they do not impair the control design. However the extent of the allowed deviations is unclear without any knowledge of the controller yet to be designed. For more details on this example we refer to Schrama (1992a) and Schrama and Bosgra (1993).

2.2. *Approaches in the literature*

The growing interest for the interaction-area of system identification and robust control, has yielded different lines of research and different problem formulations that have been dealt with. Here we briefly summarize the main lines.

2.2.1. *Quantification of model uncertainty* . Here the reasoning is that in order to obtain iden-

* We acknowledge Michel Gevers for bringing this paper to our attention.

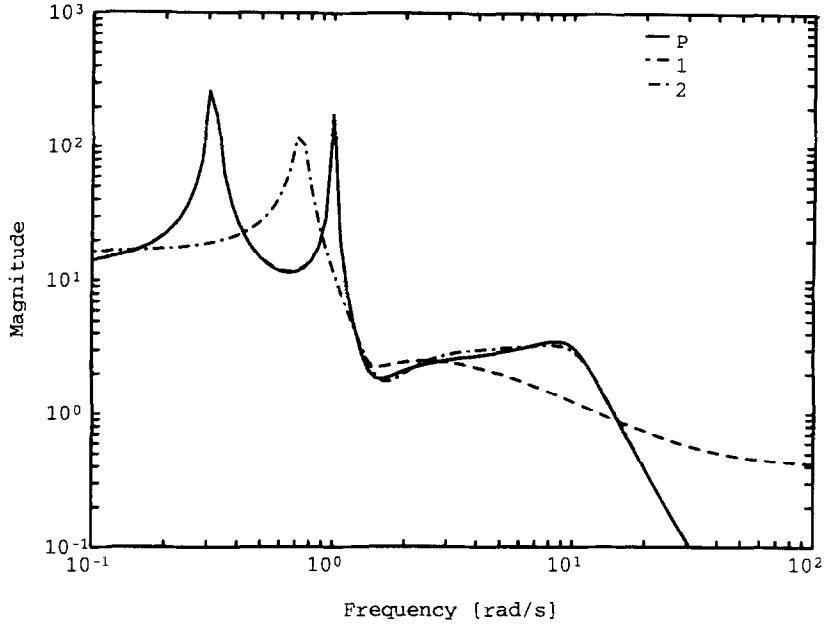

Fig. 2. Log-magnitudes of P_0(—), $\hat{P}_1$(– –), $\hat{P}_2$(– · –).

tified models that are suitable as a basis for robust control design, one has to have available a measure for the model uncertainty, i.e. an upper bound on a mismatch between the plant and the identified model. Starting with assumptions on the class of systems that is feasible, and with assumptions on the class of disturbance signals that is considered to be realistic, one chooses a priori an uncertainty structure Δ. Additionally a model $\hat{P}$ is constructed and a bound b is derived such that the data and the prior assumptions provide evidence for the expression $P_0 \in \mathcal{P}_\Delta(\hat{P}, b)$. Dependent on the type of disturbance signals that are considered, worst-case deterministic or stochastic, this expression becomes "hard" (with probability 1) or "soft" (with probability < 1). The type of priors that are chosen determine the type of results that are obtained. For a discussion on this phenomenon see Ljung *et al.* (1991) and Hjalmarsson (1993).

The worst-case deterministic type of problem has been addressed mainly in terms of frequency response data in Parker and Bitmead (1987), Helmicki *et al.* (1990), Helmicki *et al.* (1991), LaMaire *et al.* (1991), Gu and Khargonekar (1992), Partington (1991) and many others. They use uncertainty sets that allow for an expression like $\|\hat{P} - P_0\|_\infty < b$. One generally does not achieve a minimization of this upper bound over a specified class of models, and the choice of nominal model $\hat{P}$ is just instrumental in arriving at an upper bound of the plant-model mismatch. A more pragmatic approach to the problem directed towards curve-fitting of frequency responses is presented in Hakvoort and Van den Hof (1994).

In the case of time-domain data, a deterministic/worst-case approach with disturbance signals that are norm-bounded, is often referred to as *set-membership identification* or — in a parametric setting — as *parameter-bounding* identification. Accounts are given in Fogel and Huang (1982), Norton (1987) and Milanese and Vicino (1991). As in the previous situation, a (parametric) uncertainty structure Δ is chosen a priori and, based on the available data and prior assumptions, a parametric uncertainty set $\mathcal{P}_\Delta(\hat{P}, b)$ is derived, generally by parametric outer-bounding techniques. This area started off actually long before a connection was made with robust control. Originally it was directed towards the identification of poorly defined systems based on short data sequences. Several norms are used to outer-bound the obtained parametric uncertainty sets (as e.g. in terms of the transfer function magnitude, Wahlberg and Ljung (1992), $\mathcal{H}_\infty$-norms, Kosut *et al.* (1992) and Younce and Rohrs (1992), and ℓ_1-norms, Makila (1991), Jacobson and Nett (1991) and Tse *et al.* (1993)). Direct outer-bounds on the frequency response of the model are considered in Hakvoort (1992) and Hakvoort (1993). Some important characteristics of this line of research are:

- due to the worst-case/deterministic character of the assumed disturbances, the obtained upper bounds on model errors will be very conservative if this worst-case disturbance does

not actually occur;
- the worst-case disturbance signal will typically be highly correlated with ("deliberately playing against") the input signal;
- model uncertainty will generally not vanish as more data become available.

Approaches that consider disturbance signals to be stochastic, but that also account for undermodelling are given in Zhu (1989), Goodwin *et al.* (1992), Bayard (1992) and De Vries and Van den Hof (1995).

Model invalidation is another tool for quantifying model uncertainty. Given a set of priors on the data generating system and the type of disturbances, and a prior uncertainty set $\mathcal{P}_\Delta(\hat{P}, b)$, it is verified whether measured data invalidates these prior assumptions. Accounts of this approach are given in Smith and Doyle (1992) and Poolla *et al.* (1994).

Critical and most interesting discussions on the item hard versus soft bounds (or equivalently worst-case versus stochastic noise) are provided in Hjalmarsson (1993) and Ninness (1993). For a general discussion on the problem of quantifying model uncertainty and worst-case identification we refer to the tutorial papers Ninness and Goodwin (1994) and Makila *et al.* (1994).

Note that in all approaches presented here the control design is not incorporated in the discussion. Control relevance of the identification methods is motivated by the fact that one needs to provide a (hard or soft) bound on a model–plant mismatch. Although the estimation of error bounds on the basis of experimental data has separate intrinsic importance, by itself it is not sufficient for high-performance control design. This is caused by the fact that they are merely upper bounds of the uncertainty that are estimated. As uncertainty can be measured in many shapes and forms, the consequence of over-bounding the plant–model mismatch, and the consequence of chosing a specific uncertainty structure, for the resulting control performance should be taken into account. The key questions here are: which uncertainty structure to use and how to arrive at tight error bounds within this structure?

Whereas the achievable performance is of course limited by plant characteristics like (non)minimum-phase behaviour, and the ability of the plant to be modelled within a linear time-invariant framework, the achievable performance for an LTI plant with a model-based LTI controller, is additionally limited by the mismatch between P_0 and $\hat{P}$, rather than by some upper bound.

2.2.2. *Matching of identification and control-design criteria* . A completely different problem is how to identify models that provide high-performance controllers. This is the motivation for the second area, where most attention has been paid to the identification of nominal models that are suitable for high-performance control design, i.e. models that are accurate especially in those aspects that are essential for consecutive control design. Model–plant mismatches that are considered in the identification criterion have to be matched with the control-design objectives, and the considered uncertainty sets necessarily will become controller dependent. This has led to the construction of iterative schemes of identification, control design and renewal of experiments to obtain controlled plants that exhibit an improving control performance; controllers are tuned experimentally, based on a sequence of identified models. In the sequel of this survey we will specifically pay attention to this approach. Extended references can also be found in the *Workshop Proceedings* by Smith and Dahleh (1994), while the joint design of identification and control is very well advocated in the extended survey paper by Gevers (1993) and in the short survey by Bitmead (1993).

3. INTERPLAY BETWEEN IDENTIFICATION AND CONTROL

3.1. *System set-up*

As a general set-up we will consider the linear time-invariant finite-dimensional feedback interconnection of Fig. 3.

In here u and y are the measurable input and output of the plant; r_1 and r_2 are reference signals, probing signals or disturbances; and v is a disturbance signal. In the sequel of this paper we will mainly deal with the situation that $v = H_0 e$ with H_0 a stable and stably invertible filter, and e_0 a white noise process. The closed-loop dynamics of this feedback interconnection are described by the transfer function

$$T(P_0, C) = \begin{bmatrix} P_0 \\ I \end{bmatrix} [I + CP_0] \begin{bmatrix} C & I \end{bmatrix}, \qquad (2)$$

which maps $col(r_2, r_1)$ into $col(y, u)$. The feedback system is called *stable* if and only if $T(P_0, C) \in \mathcal{R}H_\infty$, where $\mathcal{R}H_\infty$ denotes the usual set of real rational stable transfer functions, see e.g. Francis (1987). All systems, models and controllers in this paper will be considered to be scalar linear, time invariant and finite-dimensional. Whenever appropriate, we will use notation that allows straightforward extension to the multivariable case. For brevity we will often deal with the external signal $r := r_1 + Cr_2$.

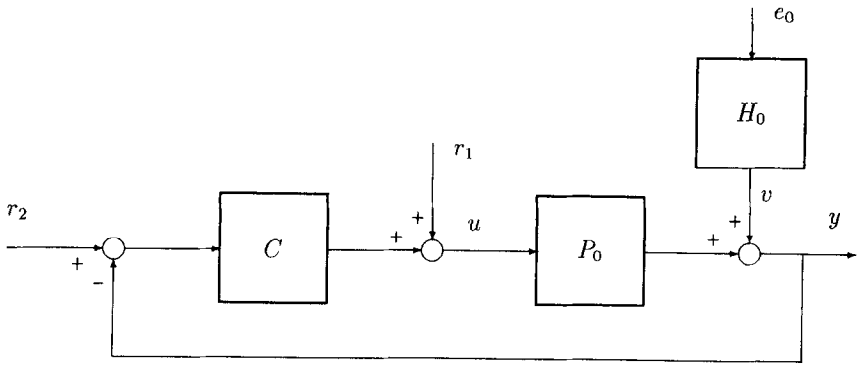

Fig. 3. Feedback configuration.

3.2. *Control-relevant plant–model mismatches*

Given the fact that any model identified from plant data can only be an approximation of the real plant, we consider the question of what is a relevant mismatch between plant and model in view of control design. Let us consider the following two examples.

Example 1. *Robust stability for a given controller.* The controller is based on a nominal model, but it has to stabilize the (unknown) plant. From the small gain theorem we know that for robust stability we can employ the expression

$$(P_0 - \hat{P})C(I + \hat{P}C)^{-1}, \tag{3}$$

where C is known to stabilize the nominal model $\hat{P}$. If the H_∞-norm of (3) is smaller than 1, then C is guaranteed to stabilize P_0 as well.

Example 2. *Robust performance for a given controller.* Here the goal is to find a nominal model $\hat{P}$, such that the performance of the pair $(\hat{P}, C)$ is similar to that of the pair (P_0, C). A typical mismatch that relates to a tracking objective is

$$P_0C(I + P_0C)^{-1} - \hat{P}C(I + \hat{P}C)^{-1}, \tag{4}$$

which can be verified (Schrama, 1992b) to be equivalent to

$$(I + P_0C)^{-1}(P_0 - \hat{P})C(I + \hat{P}C)^{-1}. \tag{5}$$

This mismatch makes sense only if C stabilizes both P_0 and $\hat{P}$.

Note that the two mismatches (3) and (5) differ through a premultiplication by the plant sensitivity $(I + P_0C)^{-1}$. If the error term of (3) is small, then the performances are not (yet) guaranteed to be similar; only a small mismatch (5) reflects whether (P_0, C) and $(\hat{P}, C)$ have similar performances.

We like to stress that these considerations are restricted to the case of a prespecified and fixed controller C. Formally we should have to take into account that that controller C is not fixed but is again based on the identified model $\hat{P}$. This will be addressed further in Section 5.

3.3. *A general measure of performance*

When talking about control performance we have to specify more clearly what we mean. In general terms a performance function of a closed-loop configuration composed of plant P_0 and controller C, is a system property, such as a step response, a sensitivity function, a complementary sensitivity etc. We formalize this control performance function as an element $J(P_0, C)$ in some normed (Banach) space $\mathcal{B}$. The control performance cost is then measured by the norm $\|J(P_0, C)\|_{\mathcal{B}}$, and a corresponding control design method will provide a controller that minimizes this cost. Apart from the arguments P_0 and C in $J(P_0, C)$, the performance function can also be dependent on signal properties (see examples below). Many control-design methods are based on the minimization of a control performance cost, as e.g.:

- *LQ tracking/disturbance rejection.* The criterion

$$\lim_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N-1} E\left\{[y(t) - r(t)]^2 + \lambda u^2(t)\right\} \tag{6}$$

can be reflected by $\|J(P_0, C)\|_2^2$ with

$$J(P_0, C) = [y(t) - r(t) \quad \sqrt{\lambda}u(t)]^T$$

being a two-dimensional quasi-stationary signal. *

* With abuse of notation we use $\|\cdot\|_2$ to indicate the corresponding semi-norm on quasi-stationary (infinite length) signals, defined by $\|x\|_2^2 := \bar{E}[x^T(t)x(t)] = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} E[x^T(t)x(t)]$.

- *Mixed sensitivity optimization.* The mixed sensitivity design is reflected by the choice

$$J(P_0, C) = \begin{bmatrix} V_1(I + P_0C)^{-1} \\ V_2P_0C(I + P_0C)^{-1} \end{bmatrix} \in \mathcal{R}H_\infty^{2\times 1} \quad (7)$$

with weighting functions V_1, $V_2 \in \mathcal{R}H_\infty$, and the corresponding control performance cost by $\|J(P_0, C)\|_\infty$.

- *H_∞-design based on robustness optimization.* This control-design scheme proposed by McFarlane and Glover (1990) is reflected by the choice for

$$J(P_0, C) = T(P_0, C) \in \mathcal{R}H_\infty^{2\times 2} \quad (8)$$

with $T(P_0, C)$ as defined in (2). The corresponding control performance cost is $\|J(P_0, C)\|_\infty$.

3.4. *A link between identification and control*

Following the starting points as discussed in Section 2, the problem of concern is to identify a model $\hat{P}$ and to design a controller $C_{\hat{P}}$ such that the controlled model and the controlled plant both have a high performance. In other words we are looking for small values of $\|J(P_0, C_{\hat{P}})\|$ and $\|J(\hat{P}, C_{\hat{P}})\|$, where the specific J and norm $\|\cdot\|$ are dictated by the control-design paradigm that is adopted. In this section we will not specify any choices for J and $\|\cdot\|$. When a pair $(\hat{P}, C_{\hat{P}})$ has been derived, it can be evaluated as a candidate solution to the joint problem of identification and control design by using the following triangle inequalities as considered by Schrama (1992a,b):

$$\begin{aligned} &\Big|\, \|J(\hat{P}, C_{\hat{P}})\| - \|J(P_0, C_{\hat{P}}) - J(\hat{P}, C_{\hat{P}})\| \,\Big| \\ &\qquad \le \|J(P_0, C_{\hat{P}})\| \\ &\le \|J(\hat{P}, C_{\hat{P}})\| + \|J(P_0, C_{\hat{P}}) - J(\hat{P}, C_{\hat{P}})\|. \end{aligned} \quad (9)$$

In this triangle inequality we can distinguish:

$\|J(P_0, C_{\hat{P}})\|$ the *achieved performance,*

$\|J(\hat{P}, C_{\hat{P}})\|$ the *designed performance,*

$\|J(P_0, C_{\hat{P}}) - J(\hat{P}, C_{\hat{P}})\|$ the *performance degradation.*

This latter term is due to the fact that $C_{\hat{P}}$ has been designed from $\hat{P}$ rather than from P_0.

Taking as a starting point that we have to obtain a satisfying designed performance (if not we would not be willing to implement the controller on the plant), we can formulate two requirements to achieve a high-performance controlled plant:

i. $\|J(\hat{P}, C_{\hat{P}})\|$ is small, (10)

ii. $\|J(P_0, C_{\hat{P}}) - J(\hat{P}, C_{\hat{P}})\| \ll \|J(\hat{P}, C_{\hat{P}})\|$. (11)

The requirement of (10) pertains to a high *nominal* performance. The strong inequality of (11) embodies the demand of a *robust* performance: if (11) is satisfied, then the difference between the designed performance function $J(\hat{P}, C_{\hat{P}})$ and the achieved performance function $J(P_0, C_{\hat{P}})$ is relatively small. Notice that the latter is not guaranteed by $\|J(P_0, C_{\hat{P}})\| \approx \|J(\hat{P}, C_{\hat{P}})\|$, since these measures are aggregated quantities.

Standard methods for identification and control design can optimize either the model or the controller, each while the other element is fixed. However a *simultaneous* optimization cannot be obtained. This has led to the introduction of several iterative schemes directed towards the use of separate stages of identification and (model-based) control design, see e.g. Zang *et al.* (1991a,b), Hakvoort (1990), Anderson and Kosut (1991), Lee *et al.* (1992) and Schrama (1992a,b).

3.5. *General form of iterative schemes*

The basic principle behind the iterative schemes that have been proposed until now, is the exploration of the triangle inequality (9), in the sense that one aims at minimization of the right part (upper bound of the performance cost), by separate stages of minimization of either of the two terms (10) and (11). Simultaneous optimization of the upper bound (9) over both $\hat{P}$ and $C_{\hat{P}}$ is intractable by common identification and control-design techniques. Instead, separate optimization over $\hat{P}$ (identification) and over C (control design) is performed.

In general terms the model and the controller are obtained according to (indexes refer to step number in the iteration):

$$\hat{P}_{i+1} = \arg\min_{\tilde{P}} \|J(P_0, C_i) - J(\tilde{P}, C_i)\| \quad (12)$$

$$C_{i+1} = \arg\min_{\tilde{C}} \|J(\hat{P}_{i+1}, \tilde{C})\| \quad (13)$$

where $\tilde{P}$, $\tilde{C}$ vary over appropriate model/controller classes, and in the control design one takes account of the constraint:

$$\|J(P_0, C_{i+1}) - J(\hat{P}_{i+1}, C_{i+1})\| << \|J(\hat{P}_{i+1}, C_{i+1})\|. \quad (14)$$

There are a couple of important observations to make here.

- The identification criterion that is reflected in (12), is completely determined by the control performance function $J(P, C)$ and the chosen norm $\|\cdot\|$, thus leading to a really control-oriented identification. The mismatch between plant and model is measured in terms of the control performance costs of plant and model, when controlled by the controller C_i.
- It is a nontrivial problem how to construct identification methods that achieve a criterion

(12). Consider e.g. a weighted sensitivity as control performance function (i.e. (7) with $V_1 = V$ and $V_2 = 0$). Then (12) simplifies to a norm on the weighted mismatch (5), i.e.

$$\|V(I + P_0C)^{-1}(P_0 - \hat{P})C(I + \hat{P}C)^{-1}\|, \quad (15)$$

representing a nontrivial identification problem.

- As the control performance cost refers to a feedback connection of P_0 and C_i, the identification criterion (12) points to the use of measurement data from closed-loop experiments, in order to get information about $J(P_0, C_i)$, (Schrama, 1992a).
- A newly designed controller C_{i+1} will lead to a new performance degradation term (12) which in turn points to performing new identification experiments with this new controller C_{i+1} being implemented on the plant P_0.
- The triangle inequality provides both an upper bound and a lower bound for the achieved performance, see (9). By making the performance degradation term small compared to $\|J(\hat{P}, C_{\hat{P}})\|$ as in (11), the achieved performance is forced to be very close to the designed performance. In this way the control design is forced to provide a robust performance.

Generally design methods will yield their resulting controller through an unconstrained optimization: a criterion is minimized that incorporates some user-chosen weighting functions, reflecting the nominal performance level, as well as an indication on the required robustness. However, there will generally not be a prior guarantee that the achieved robustness is satisfactory, i.e. whether (14) is satisfied. An additional robustness analysis of the designed controller has to certify this. This is also reflected in the (generic) block diagram of an iterative scheme of identification and control, as depicted in Fig. 4. If the robustness test is not passed satisfactorily, different actions may have to be taken as e.g.

- increasing the complexity (order) of the class of controllers considered;
- redesigning the weighting functions applied in the control design;
- identifying a more accurate (higher-order) model.

It has to be noted that in this discussion we have considered the control performance functions to be given a priori. To some extent this neglects the important choice of appropriate weighting functions as e.g. λ in (6) and V_1, V_2 in (7), that in normal practice are being designed (and redesigned) in the control-design stage.

In Section 5 we will take a closer look at different iterative schemes that have been elaborated in the past few years. They all show the basic components as discussed here, and mainly differ in the choice of the control performance cost and in the way the closed-loop identification is treated.

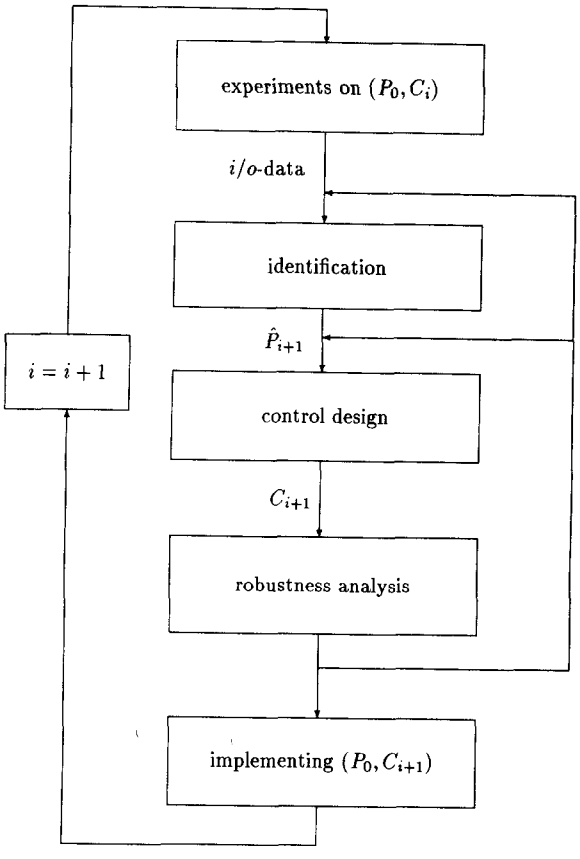

Fig. 4. Iterative scheme of identification and control design.

As we necessarily have to deal with identification of approximate models from closed-loop experiments, we will first review some results concerning approximate closed-loop identification. In that discussion we will limit attention to the prediction error identification framework.

4. APPROXIMATE IDENTIFICATION

4.1. *"Classical" prediction error results*

In a prediction error context, given input and output data of a plant to be modelled we determine the prediction error:

$$\varepsilon(t, \theta) = H(q, \theta)^{-1}[y(t) - P(q, \theta)u(t)] \quad (16)$$

with $H(q, \theta)$ the parametrized (output) noise model and $P(q, \theta)$ the parametrized input/output model,

θ running over some appropriate parameter space Θ, while q is the forward shift operator.

The prediction error estimate (Ljung, 1987) is obtained by minimizing the squared sum of — possibly filtered — prediction errors:

$$\hat{\theta}_N = \arg\min_{\theta\in\Theta} \frac{1}{N}\sum_{t=1}^{N} \varepsilon_F(t,\theta)^2$$

with $\varepsilon_F(t,\theta) = L(q)\varepsilon(t,\theta)$, and $L(q)$ some stable filter.

Under weak regularity conditions this prediction error estimate is known to converge with probability 1 to θ^*, with

$$\theta^* = \arg\min_{\theta} \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{\varepsilon_F}(\omega)\mathrm{d}\omega, \qquad (17)$$

where in the *open-loop case* ($C = 0$): *

$$\Phi_{\varepsilon_F} = \left[|P_0 - P(\theta)|^2\Phi_u + \Phi_v\right]\frac{|L|^2}{|H(\theta)|^2}, \qquad (18)$$

and Φ_u, Φ_v are the spectral densities of input and noise, respectively. A so-called *direct identification* in the *closed-loop case* with controller C (as in Fig. 3) yields a similar expression (17) with (see e.g. Gevers, 1993):

$$\Phi_{\varepsilon_F} = \left\{|S_0[P_0 - P(\theta)]|^2\Phi_r + \frac{|S_0|^2}{|S(\theta)|^2}\Phi_v\right\} \cdot \frac{|L|^2}{|H(\theta)|^2}, \qquad (19)$$

where S_0 is the actual sensitivity $(I + P_0C)^{-1}$ and $S(\theta) = (I + P(\theta)C)^{-1}$ is the sensitivity of the parametrized model.

What we are aiming at is to find — based on signal measurements — a model $P(\theta^*)$ that is obtained as the minimizing argument of an identification (approximation) criterion, that can be flexibly tuned, e.g. by appropriate choices of L and Φ_r, to our needs in view of the control performance function. In order to achieve this, the criterion should not be dependent on the unknown noise spectrum Φ_v.

Note that in the open-loop case, with a fixed noise model $H(q,\theta) = 1$, as in the case of an output-error model structure, it can be verified that

$$\theta^* = \arg\min_{\theta} \|[P_0 - P(\theta)]H_uL\|_2, \qquad (20)$$

with H_u a (stable) spectral factor of Φ_u.

In the closed-loop case it follows from (19) that there does not exist a simple choice of noise model such that the Φ_v-dependent term in (19) will become independent of θ. As a result, the identification criterion (and thus the obtained model $\hat{P}$) will be essentially dependent on Φ_v, which is unknown. This is typical for "classical" closed-loop prediction error methods, see e.g. Soderstrom and Stoica (1989).

If open-loop techniques like (20) have to be used to obtain a control-relevant mismatch like e.g. (15), one can observe that input spectrum Φ_u and/or prefilter L have to be chosen dependent on the — unknown — plant P_0 and on the parametrized model $P(\theta)$.

The closed-loop expression (19) nevertheless shows some clear resemblance with the desired approximation criterion (15) if we consider the first term in the expression (19). The weighting factor S_0 which is present in both expressions apparently is a weighting function that is obtained by performing the identification under closed-loop conditions. This is quite understandable if one realizes that it is exactly this sensitivity S_0 that determines the relation between the external signal r and the input signal u.

In an attempt to construct closed-loop approximate identification methods that have an explicitly tunable bias expression one can consider the following alternative. For simplicity we will assume the signal r to be available from measurements. If we know the controller C, one could consider a parametrized model $P(\theta)$, $\theta \in \Theta$, and identify θ through

$$\varepsilon(t,\theta) = y(t) - \frac{P(\theta)}{1 + P(\theta)C}r(t) \qquad (21)$$

by least-squares minimization of the prediction error $\varepsilon(t)$.

This alternative leads to a complicatedly parametrized model set, and as a result it is not attractive, although it provides us with an explicitly tunable approximation criterion, given by (17) with

$$\Phi_{\varepsilon_F} = |P_0(I + P_0C)^{-1} - P(\theta)(I + P(\theta)C)^{-1}|^2 \cdot |L|^2\Phi_r.$$

Note that for validity of the asymptotic prediction error analysis leading to the above expression, the parameter set Θ has to be a connected subset of $\mathcal{R}^d$ being restricted to contain only models that generate stable predictors $P(\theta)[1 + CP(\theta)]^{-1}$.

In the next subsections we will present some recent developments in the area of closed-loop approximate identification, aiming at providing solutions to the problem sketched above.

4.2. *Two-stage method*

As an alternative method that can provide an explicitly tunable approximation criterion, a two-

* For brevity the arguments $e^{i\omega}$ are suppressed in the frequency domain expressions.

stage identification method is proposed in Van den Hof and Schrama (1993).

The idea is that from closed-loop data one can first identify the plant sensitivity S_0 as a black box transfer function $\hat{S}$, using measured data $\{r, u\}$. Since $u(t) = S_0(q)r(t) - C(q)S_0(q)v(t)$ and v and r are uncorrelated, this is an open-loop type of identification problem. In the second step of the procedure one identifies P_0 from:

$$y(t) = P(\theta)\hat{u}_r(t) + \varepsilon(t) \tag{22}$$

$$\text{with} \quad \hat{u}_r(t) := \hat{S}r(t) \tag{23}$$

applying e.g. an output-error model structure in (22). The signal $\hat{u}_r(t)$ is simply constructed by the measured signal r and the estimate $\hat{S}$ being the result of the first step.

It can be shown that the approximation criterion in the second step of this procedure is determined by the spectrum

$$\Phi_{\varepsilon_F} = \left|[P_0 - P(\theta)]S_0 + P(\theta)[S_0 - S(\beta^*)]\right|^2 \cdot \Phi_r|L|^2. \tag{24}$$

In this expression β^* is the (asymptotically) estimated parameter in the first step and L is a prefilter, filtering the prediction error in the second step. Note that when the first step is executed sufficiently accurately, i.e. $S(\beta^*) \to S_0$, then the expression above tends to a simple weighted additive mismatch $P_0 - P$, where the weighting incorporates the actual plant sensitivity S_0. This has a clear resemblance with the robust control performance criterion (15).

4.3. *Dual Youla parametrization*

The basic idea behind this method is introduced by Hansen and Franklin (1988) in view of closed-loop experiment design. It was further elaborated and modified in Hansen *et al.* (1989), and also employed for approximate identification in Schrama (1991), Schrama (1992b) and Anderson and Kosut (1991). It utilizes the (dual) Youla parametrization of all plants that are stabilized by a given (known) controller. In order to describe this method, we need the following concepts.

Definition 1. (Vidyasagar (1985)). A linear, time-invariant, finite-dimensional plant P has a *right coprime factorization (rcf)* over $\mathcal{R}H_\infty$ if there exist $N, D, X, Y \in \mathcal{R}H_\infty$ such that $P = ND^{-1}$ and $XN + YD = I$. A *rcf* (N, D) is *normalized* if $N^*N + D^*D = I$.

Through coprime factorizations a (possibly unstable) plant is represented by a quotient of two stable transfer functions. Coprimeness refers to the property that the factorization does not exhibit canceling terms that contain unstable zeros.

We can employ the following result from stability analysis.

Proposition 2. (Desoer *et al.* (1980)). Let C be a controller with *rcf* (N_c, D_c), and let P_x with *rcf* (N_x, D_x) be any system that is stabilized by C. Then the plant P_0 is stabilized by C if and only if there exists an $R \in \mathcal{R}H_\infty$ such that

$$P_0 = (N_x + D_cR)(D_x - N_cR)^{-1}.$$

The above proposition shows a parametrization of the class of all plants that are stabilized by the given C. The parametrization is depicted in a block diagram in Fig. 5. Note that $N_xD_x^{-1}$ is just any (nominal, auxiliary) system that is stabilized by C. In the case of a stable controller C, a valid choice is given by $N_x = 0$, $D_x = I$.

Given the feedback configuration as presented in Fig. 3, it can be shown (Schrama, 1992b) that the unique value of R that corresponds to the real plant (P_0) in this dual Youla parametrization is determined by

$$R = D_c^{-1}[I + P_0C]^{-1}(P_0 - P_x)D_x, \tag{25}$$

and that we can rewrite the noise contribution $v(t) = H_0(q)e_0(t)$ on the output, as in Fig. 3, into the form as indicated in Fig. 5 with

$$S = D_c^{-1}[I + P_0C]^{-1}H_0. \tag{26}$$

Now defining the signals z, x as indicated in Fig. 5 and writing the node equations $x = D_x^{-1}(u + N_cz)$ and $y = N_xx + D_cz$, it follows that:

$$z = (D_c + P_xN_c)^{-1}(y - P_xu), \tag{27}$$

$$x = (D_x + CN_x)^{-1}(u + Cy) \tag{28}$$

and

$$z = Rx + Se_0. \tag{29}$$

Moreover one can make the following observations:

- the signals z and x can be reconstructed from data through known filters, provided the controller C is known;
- signal x is uncorrelated with e_0, as $u + Cy = r_1 + Cr_2$, and the external signals r_1, r_2 are assumed to be uncorrelated with e_0.

This shows that we can identify a model $\hat{P}$ of P_0 through identification of R from reconstructed measurements $\{z, x\}$ according to (29). Since x and e_0 are uncorrelated, the identification of R forms an open-loop identification problem. This implies that an approximate model of R can be obtained, where the asymptotic identification criterion is not dependent on the noise contribution on the data.

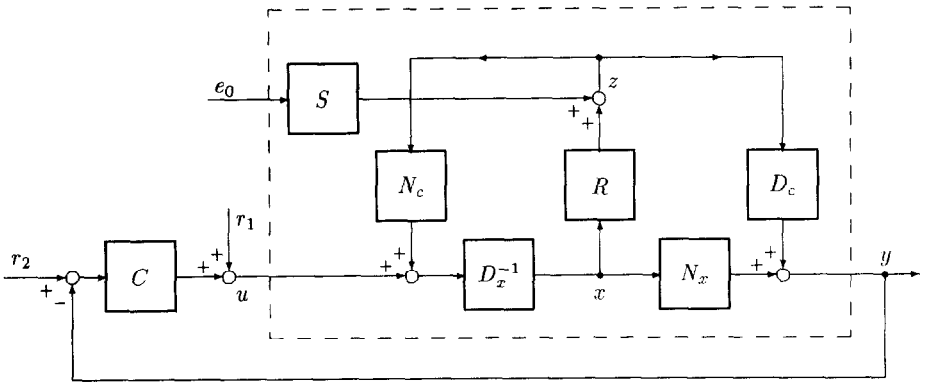

Fig. 5. Dual Youla representation of the data generating system.

Using e.g. an output-error model structure and a prefilter L, the filtered prediction error becomes

$$\varepsilon_F(t, \theta) = L(q)[z(t) - R(q, \theta)x(t)].$$

A corresponding least-squares identification criterion leads to the asymptotic estimate

$$\theta^* = \arg\min_{\theta\in\Theta} \frac{1}{2\pi} \int_{-\pi}^{\pi} |L|^2 |R - R(\theta)|^2 \Phi_x d\omega, \quad (30)$$

and the resulting model $P(\theta^*)$ is then calculated according to Proposition 2, by

$$P(\theta^*) = [N_x + D_c R(\theta^*)][D_x - N_c R(\theta^*)]^{-1}. \quad (31)$$

Careful evaluation of the expressions (30),(31) and (25) shows that in the scalar case we can rewrite the integrand of (30) as

$$\left| \frac{L}{D_c(I + P_0 C)} [P_0 - P(\theta)] \frac{1}{(I + P(\theta)C)} \right|^2 \Phi_r. \quad (32)$$

As a result, the identification criterion can be explicitly tuned to the performance degradation (15) provided a 2-norm is used in the performance degradation, and the identification design variables are chosen to satisfy $LH_r = N_c V$. In here, H_r is a stable spectral factor of Φ_r.

The following remarks can be made with respect to this identification method.

- The identification method fruitfully uses knowledge of the controller that is implemented when experiments are performed. Knowledge of this controller is instrumental in recasting the closed-loop identification problem into an open-loop one.
- Due to this configuration, an estimated factor $R(\theta^*)$ that is stable, will automatically yield a model $P(\theta^*)$ that is guaranteed to be stabilized by C.
- Having identified the parameter θ^* and the corresponding transfer function $R(\theta^*)$ with a fixed McMillan degree n_r, the McMillan degree of $P(\theta^*)$ will generally be much larger. This is due to the required reparametrization as presented in (31). This implies that in the identification as discussed above, the complexity (McMillan degree) of the resulting model P is not simply tunable. Constructing an appropriate parameter space Θ in such a way that the corresponding set of models $\{P(\theta), \theta \in \Theta\}$ has a fixed McMillan degree, is definitely a nontrivial parametrization problem, that has not been solved yet.
- The asymptotic identification criterion, as reflected by (32), is not dependent on the chosen auxiliary model P_x or its factorization.

4.4. *Coprime factor identification*

Closely related to the previous method is an identification method that is directed towards identification of coprime factors of P_0. It was introduced in Schrama (1991), and further elaborated in Schrama (1992b) and Van den Hof *et al.* (1995). Similarly as above it employs the fact that the signal $r := u + Cy$ can be constructed from data and knowledge of the controller, and that this signal is uncorrelated to the noise signal e_0.

Consider a stable filter F that generates an auxiliary signal according to $x = Fr$, as depicted in Fig. 6a. Then we can write

$$y = N_0 x + S_0 H_0 e_0, \quad (33)$$

$$u = D_0 x - C S_0 H_0 e_0, \quad (34)$$

with $N_0 = P_0 S_0 F^{-1}$, $D_0 = S_0 F^{-1}$ and $S_0 = (I + CP_0)^{-1}$. Since x and e_0 are uncorrelated we can

identify N_0 and D_0 in an open-loop way, thus utilizing the possibility of an explicitly tunable identification criterion. The plant model $\hat{P}$ is then constructed as $\hat{P} = N(\hat{\theta})D(\hat{\theta})^{-1}$.

Apparently there seems to be a lot of freedom in the choice of F. However this is limited if we restrict N_0 and D_0 to be stable and the signal x to be bounded, as is shown in Van den Hof *et al.* (1995):

Proposition 3. The filter F yields stable mappings $(y, u) \rightarrow x$ and $x \rightarrow (y, u)$ if and only if there exists an auxiliary system P_x with $rcf(N_x, D_x)$, stabilized by C, such that $F = (D_x + CN_x)^{-1}$. For all such F the induced factorization $P_0 = N_0 D_0^{-1}$ is right coprime. □

This Proposition shows the clear resemblance of this scheme with the previous dual Youla parametrization. The choice of F as given in the Proposition shows that the resulting signal x matches the same signal in the previous section, cf. (28).

As in the previous method an intermediate signal x is reconstructed from data (Fig. 6a). Next the two transfer functions between x and (y, u) are identified according to the scheme of Fig. 6b.

If we use a corresponding model structure

$$\varepsilon(t, \theta) = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} y - N(\theta)x \\ u - D(\theta)x \end{bmatrix} \tag{35}$$

then a least-squares identification criterion will yield the asymptotic estimate determined by

$$\theta^* = \arg\min_{\theta} \frac{1}{2\pi} \int_{-\pi}^{\pi} \begin{bmatrix} N_0 - N(\theta) \\ D_0 - D(\theta) \end{bmatrix}^* \begin{bmatrix} |L_1|^2 & 0 \\ 0 & |L_2|^2 \end{bmatrix} \begin{bmatrix} N_0 - N(\theta) \\ D_0 - D(\theta) \end{bmatrix} \Phi_x d\omega. \tag{36}$$

According to (33) and (34) and the specific choice of F, the coprime plant factors that can be identified from closed-loop data satisfy

$$\begin{pmatrix} N_0 \\ D_0 \end{pmatrix} = \begin{pmatrix} P_0(I + CP_0)^{-1}(I + CP_x)D_x \\ (I + CP_0)^{-1}(I + CP_x)D_x \end{pmatrix}. \tag{37}$$

Now the question is how we can reformulate the identification criterion (36) into an expression in terms of P_0 and $P(\theta) = N(\theta)D(\theta)^{-1}$. If we would be able to construct a parametrization for $N(\theta), D(\theta)$ that satisfies

$$\begin{bmatrix} N(\theta) \\ D(\theta) \end{bmatrix} = \begin{bmatrix} P(\theta)(I + CP(\theta))^{-1} \\ (I + CP(\theta))^{-1} \end{bmatrix} (I + CP_x)D_x, \tag{38}$$

then the integrand expression in (36) can be written as

$$\left\{ |P_0(I + CP_0)^{-1} - P(\theta)(I + CP(\theta))^{-1}|^2 |L_1|^2 + |(I + CP_0)^{-1} - (I + CP(\theta))^{-1}|^2 |L_2|^2 \right\} \Phi_r. \tag{39}$$

The latter expression induces a very flexible approximation criterion in which clearly control-relevant mismatches can be recognized.

Note that the restriction to the parametrization (38) is nontrivial. A second remark is that the dynamics that are present in the coprime factors N_0, D_0 strongly depend on the choice of the auxiliary system P_x. This also holds for the question of whether the factors can be accurately modelled by restricted complexity models. If both factors exhibit (very) high-order dynamics, then approximate identification of these factors may lead to an inaccurate plant model. This implies that somehow one has to get rid of the common dynamics in both factors, and thus also simplifying the parametrization restriction (38). In Van den Hof *et al.* (1995) an algorithm is presented in which the freedom in choosing P_x is employed to arrive at coprime factors that are — nearly — normalized. This means that the factors (N_0, D_0) have minimal McMillan degree. It is achieved through a similar strategy as in the two-stage method, described earlier. First an accurate — high-order — estimate is made of N_0 and D_0; the resulting coprime factors are normalized in a normalization procedure, and these latter factors are subsequently used as an auxiliary model. The factors N_0 and D_0 that result from this choice of P_x and D_x will be almost normalized (in the sense that $N^*N + D^*D \sim I$), and can be accurately identified by using a model structure for $N(\theta)$, $D(\theta)$, given by

$$N(\theta) = b(q^{-1}, \theta)f(q^{-1}, \theta)^{-1}, \tag{40}$$

$$D(\theta) = a(q^{-1}, \theta)f(q^{-1}, \theta)^{-1}, \tag{41}$$

with a, b and f polynomials of specified degree. This parametrization now approximately satisfies the restrictions of (38). Moreover it guarantees that the McMillan degree of the resulting plant model is equal to the McMillan degree of the estimated coprime factors.

As a final note we remark that due to this coprime factor framework the identification methods discussed in the previous Sections 4.3 and 4.4 do not meet any problems in handling unstable plants and/or unstable controllers.

5. ITERATIVE SCHEMES UNDER CONSTRUCTION

5.1. *Introduction*

While in the previous section we discussed the developments in approximate closed-loop identification, we will now return to the interaction with the control design, by addressing several iterative

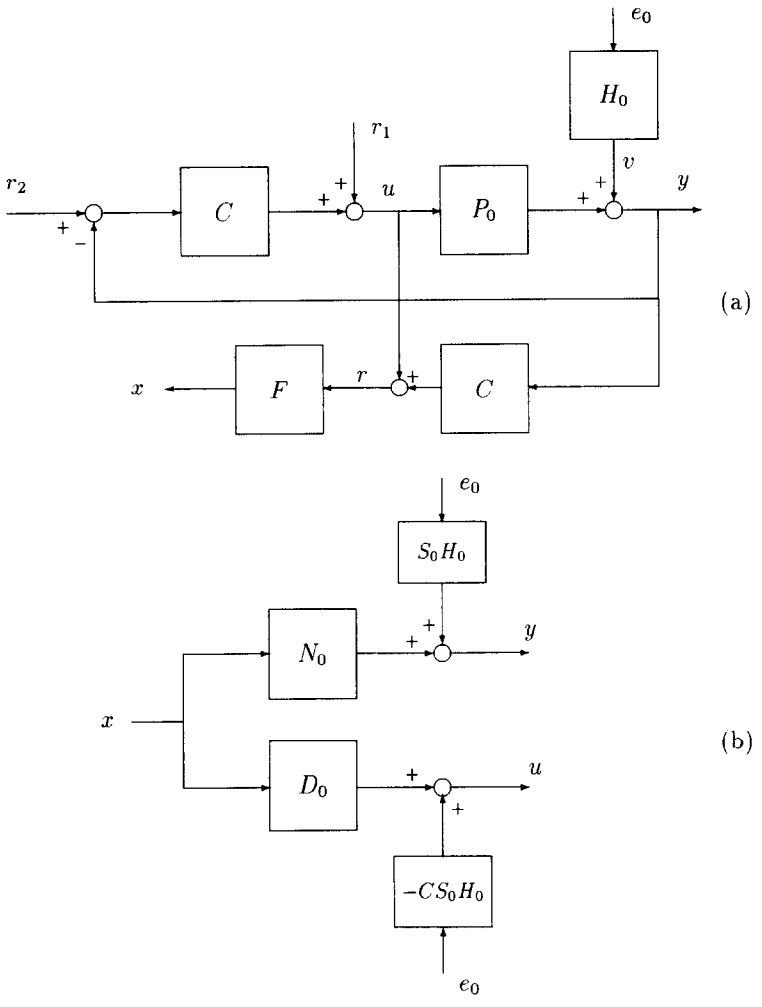

Fig. 6. Coprime factor identification from closed-loop data: (a) construction of auxiliary signal x; and (b) open-loop identification of coprime factors.

methods. We will discuss three different approaches, each based on a specific control-design strategy: LQ-control, Internal Model Control (IMC) and a $\mathcal{H}_\infty$-design based on robustness optimization. Besides this design strategy, the main characteristics lie in the treatment of the approximate identification and in verifying robustness issues, i.e. in tuning the control design in order to prevent large performance degradations (cf. (14)). All schemes follow the basic iterative strategy as presented in Section 3. As the method based on $\mathcal{H}_\infty$-design allows a stability robustness test utilizing experimental data, we will specifically pay attention to this.

5.2. *LQ-control*

Zang *et al.* (1991a,b) developed an iterative identification and control-design strategy, taking an infinite horizon LQ-control objective as a starting point. The seeds for this approach were already planted in Bitmead *et al.* (1990), while it is further refined in Zang *et al.* (1992) and Partanen and Bitmead (1993). The global control performance cost is denoted by

$$\|J_{LQ}(P_0, C)\|_2^2 = \bar{E}[(y(t)-r_2(t))^2 + \lambda u(t)^2], \quad (42)$$

where

$$J_{LQ}(P_0, C) = \begin{bmatrix} y(t) - r_2(t) \\ \sqrt{\lambda} u(t) \end{bmatrix}. \quad (43)$$

The system configuration is taken as sketched in Fig. 3, with $r_1 \equiv 0$.* The triangle inequality now becomes

$$\|J_{LQ}(P_0, C)\|_2 \le \|J_{LQ}(\hat{P}, C)\|_2 + \|J_{LQ}(P_0, C) - J_{LQ}(\hat{P}, C)\|_2, \quad (44)$$

* For simplicity of notation we have neglected the fact that the optimal LQ controller will generally be a two degree of freedom controller.

where $J_{LQ}(\hat{P}, C) = \begin{bmatrix} y_c(t) - r_2(t) \\ \sqrt{\lambda} u_c(t) \end{bmatrix}$, and $y_c(t)$, $u_c(t)$ the corresponding output/input signal in the design loop, i.e.

$$y_c = \hat{P}\hat{S}Cr_2 + \hat{S}\hat{H}e_1, \tag{45}$$

$$u_c = \hat{S}Cr_2 - C\hat{S}\hat{H}e_1. \tag{46}$$

In this expression $\hat{S}$ and $\hat{H}$ denote the sensitivity function of the i/o model $\hat{P}$ and the estimated noise model, respectively. The signal e_1 is the assumed noise signal in the design loop.

Through careful evaluation of the performance degradation term, $J_{LQ}(P_0, C) - J_{LQ}(\hat{P}, C)$, it follows that the (squared) norm of this term becomes

$$\bar{E}[(y(t) - y_c(t))^2 + \lambda(u(t) - u_c(t))^2]. \tag{47}$$

This is the related identification criterion that is induced by (42). In order to come to an identification set-up in which this criterion is minimized, (47) can be rewritten as

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \left\{ |S_0 H_0|^2 (1 + \lambda|C|^2)\Phi_{e_0} + |CS_0[P_0 - P(\theta)]S(\theta)|^2 (1 + \lambda|C|^2)\Phi_{r_2} \right\} d\omega. \tag{48}$$

In this latter derivation, Zang *et al.* (1991b), have taken $e_1 \equiv 0$. As an identification set-up, it is proposed to apply a direct-type closed-loop prediction error criterion conformable to (19) with $r := C(q)r_2$, with a fixed noise model $H(q, \theta) \equiv 1$ and a prefilter L satisfying

$$|L(\theta)|^2 = (1 + \lambda|C|^2)|S(\theta)|^2 = \frac{1 + \lambda|C|^2}{|1 + CP(\theta)|^2}. \tag{49}$$

Such a choice of L makes the closed-loop identification criterion (17) and (19) equivalent to (48). (Note that we may neglect the θ-independent terms in the integrands.) A remaining problem is the fact that this optimal prefilter is again a function of the unknown parameter θ, which implies that the required model structure is not a regular output-error model structure. An "approximate" solution is provided by using the model estimate from the previous step in the iteration, i.e. $L(\theta) = L(\hat{\theta}_{i-1})$. In this way the prefilter becomes fixed, but the two criteria (17) and (19), and (48) no longer match exactly.

Remark 4. A similar approach to the identification problem is taken in Hakvoort (1990) and Hakvoort *et al.* (1994). There, the design loop is chosen to be contaminated by the same noise e_0 as in the achieved loop. For that case the same choice of optimal prefilter $L(\theta)$ (49) is obtained, and conditions are given for which a choice $L(\theta) = L(\hat{\theta}_{i-1})$ provides equivalent criteria (17),(19) and (48).

The LQ controller is obtained by minimizing the performance cost

$$\|J_{LQ}(\hat{P}, C)\|_2^2 = \bar{E}[(y_c(t) - r_2(t))^2 + \lambda u_c(t)^2]. \tag{50}$$

This design strategy does not account for any mismatch between the nominal model and the actual plant. This implies that during the control design we have to verify whether (14) is satisfied. Only under this condition can one guarantee that the achieved performance will be similar to the designed one. Zang *et al.* (1991b) propose a local design criterion that accounts for this mismatch. This local design criterion should prevent a decreasing designed (nominal) performance cost which would dramatically increase the performance degradation cost. It takes the form of

$$\|J_{LQ,local}(\hat{P}, C)\|_2^2 = \bar{E}\{[F_1(q)(y_c(t) - r_2(t))]^2 + \lambda[F_2(q)u_c(t)]^2\}, \tag{51}$$

with $F_1(e^{i\omega}) = \left(\frac{\Phi_{y-r_2}}{\Phi_{y_c-r_2}}\right)^{1/2}$, $F_2(e^{i\omega}) = \left(\frac{\Phi_u}{\Phi_{u_c}}\right)^{1/2}$.

The spectra in this expression are estimated on the basis of measurement data from the actual system and of simulation data from the designed loop.

Recent contributions to this iterative scheme are presented in Zang *et al.* (1995).

5.3. *Internal Model Control (IMC)*

In Lee *et al.* (1992), Lee *et al.* (1993a) and Lee *et al.* (1993b) an iterative scheme of identification and control design is proposed based upon the internal model control (IMC) design paradigm. The resulting iterative scheme is often indicated as the "windsurfer approach" * (Anderson and Kosut, 1991), referring to the presumed iterative way that people take when learning windsurfing: starting with a control system with moderate bandwidth, while experimentally refining the model and increasing the bandwidth of the controlled "plant".

The control performance is based on

$$J_{IMC}(P_0, C) = \frac{P_0 C}{1 + P_0 C} - T_d, \tag{52}$$

with $J_{IMC} \in \mathcal{R}H_\infty$, T_d is some prechosen desired complementary sensitivity, and the performance cost is taken to be the $\mathcal{H}_\infty$-norm of J_{IMC}. T_d is chosen to be of the form $(\frac{\lambda}{s+\lambda})^{n+1}$, with $\lambda \in \mathcal{R}$ the closed-loop bandwidth, and n a prechosen integer.

The choice of control performance cost induces the performance degradation

$$\|J_{IMC}(P_0, C) - J_{IMC}(\hat{P}, C)\|_\infty$$

* Due to local climate conditions this term is not so often used by the north-west European iterators.

$$= \| \frac{P_0 C}{1 + P_0 C} - \frac{\hat{P} C}{1 + \hat{P} C} \|_\infty. \quad (53)$$

The mismatch between complementary sensitivity functions has already been shown to be a control-relevant plant-model mismatch, see e.g. (4) and (5). In Lee *et al.* (1992) the closed-loop identification scheme based on the dual Youla parametrization is employed to identify a stable transfer function R, as described in Section 4.3 and equation (29). By choosing a least-squares prediction error criterion with a prefilter $L = N_c$, the resulting parameter estimate will converge to (assuming $\Phi_r \equiv 1$)

$$\theta^* = \arg\min_\theta \int_{-\pi}^{\pi} | \frac{P_0 C}{1 + P_0 C} - \frac{P(\theta) C}{1 + P(\theta) C} |^2 d\omega, \quad (54)$$

where the plant model $P(\theta)$ is parametrized according to (31). In this setting the $\mathcal{H}_\infty$-norm of the performance degradation is replaced by the 2-norm of the least-squares prediction error criterion. Due to the parametrization (31) the estimated plant model will generally be of high order; a subsequent model-reduction procedure based on frequency-weighted balanced reduction, conformable to (53) is applied to keep control over the model order of $\hat{P}$ and the resulting order of the controller.

The control design uses IMC-design techniques for a nominal design that achieves the predesigned closed-loop bandwidth. During the iterations of identification and control design, the nominal closed-loop bandwidth λ is gradually increased as the identified model becomes more accurate around and beyond the designed closed-loop bandwidth. When increasing λ for a specific model $\hat{P}_i$, the performance degradation is monitored through step response experiments on the achieved and designed loop. When the responses are different, this indicates either the limit of validity of the current model $\hat{P}_i$, and thus the need for a new model $\hat{P}_{i+1}$, or the limits of performance that can be achieved. In Lee *et al.* (1993b), considerations of variance of the model estimates are also gathered into the iterative scheme. While the original method has been worked out for stable plants and stable controllers only, a further extension to the handling of unstable plants is presented in Lee (1994).

5.4. *H_∞-design based on robustness optimization*

In the work of Schrama (1992a,b), Schrama and Van den Hof (1992) and Schrama and Bosgra (1993), an iterative scheme of identification and control design is elaborated, utilizing the robust control-design method of Bongers and Bosgra-(1990) and McFarlane and Glover (1990). The control-design criterion is:

$$C = \arg\min_{\tilde{C}} \| V T(\hat{P}, \tilde{C}) W \|_\infty. \quad (55)$$

$T(\hat{P}, C)$ is the 2×2 transfer matrix as defined in (2) that embodies all feedback properties of the closed-loop system, including disturbance and noise attenuation, sensitivity and robustness margins, and with V and W appropriate stable weighting functions. Through specific choices of V and W this method reduces to specific methods like weighted-sensitivity minimization or mixed-sensitivity minimization (see also Section 3.3).

For $V = W = I$, the following robustness result is known from Glover and McFarlane (1989) and Bongers and Bosgra (1990).

Proposition 5. Let $T(\hat{P}, C)$ be stable, and let $(\hat{N}_n, \hat{D}_n)$ be a normalized *rcf* of $\hat{P}$. Define the uncertainty set $\mathcal{P}_\Delta(\hat{P}, \gamma) := \{P_\Delta = (\hat{N}_n + \Delta N)(\hat{D}_n + \Delta D)^{-1}, \left\| \begin{matrix} \Delta N \\ \Delta D \end{matrix} \right\|_\infty < \gamma\}$. Then all plants in $\mathcal{P}_\Delta(\hat{P}, \gamma)$ are stabilized by C if and only if $\| T(\hat{P}, C) \|_\infty \leq \gamma^{-1}$. □

As the control design amounts to minimizing $\| T(\hat{P}, C) \|_\infty$ this corresponds to a maximization of the stability robustness margin with respect to (stable) perturbations of normalized *rcf*'s of the plant model. Additionally the resulting controller pursues some traditional control objectives like a small sensitivity at the lower frequencies and a small complementary sensitivity at the higher frequencies. The bandwidth of the designed loop will be close to the crossover frequency of the nominal model $\hat{P}$. This controller is also known to optimize the stability robustness margin in the sense of the gap metric, see Bongers and Bosgra (1990) and Georgiou and Smith (1990).

In the given situation the control performance function is defined by

$$J_{RO}(P_0, C) = T(P_0, C), \quad (56)$$

while its cost is measured as the corresponding $\mathcal{H}_\infty$-norm. The induced performance degradation becomes

$$\| T(P_0, C) - T(\hat{P}, C) \|_\infty. \quad (57)$$

It can simply be verified that this performance degradation is an extended version of the degradation (53) in the case of the IMC design.

5.4.1. *Identification.* The identification method has to provide a model $\hat{P}$ that (asymptotically) minimizes the performance degradation (57). In the proposed iterative scheme an identification criterion is chosen that equals $\| T(P_0, C) - T(\hat{P}, C) \|_2$, where minimization of the 2-norm is used to obtain a reduction of the ∞-norm of the corresponding mismatch. This replacement is justified by the

fact that an accurate L_2-approximation implies an accurate L_∞-approximation, provided that the error term is sufficiently smooth and small, see e.g. Caines and Baykal-Gürsoy (1989). Identification of $\hat{P} = \arg\min_{\tilde{P}} \|T(P_0, C) - T(\tilde{P}, C)\|_2$ is obtained by applying the coprime factor identification as described in Section 4.4. Under the parametrization restriction (38) it can be verified that

$$T(P_0, C) - T(P(\theta), C) = \begin{bmatrix} N_0 - N(\theta) \\ D_0 - D(\theta) \end{bmatrix} D_x^{-1}[I + CP_x]^{-1} \begin{bmatrix} C & I \end{bmatrix}. \tag{58}$$

Minimization of the 2-norm of this expression can be achieved through least-squares identification as in (36) with $L_1 = L_2 = L$, satisfying $|L|^2\Phi_r = 1 + |C|^2$. In the first versions of this iterative scheme, the identification was actually performed based on frequency response data of the plant. Later extensions in Van den Hof *et al.* (1995) also show the use of time-domain data.

5.4.2. *Control performance enhancement* . The controller C of (55) with $V = W = I$ depends solely on the nominal model $\hat{P}$. As a result the achieved performance $\|T(P_0, C)\|_\infty$ may be substantially different from the designed performance $\|T(\hat{P}, C)\|_\infty$. In order to preclude large performance degradations the nominal control design is furnished with weighting functions. Like in McFarlane and Glover (1990) this iterative scheme uses just a simple scalar weight, $W^{-1} = V = diag(\alpha, 1)$, leading to the control design

$$C = \arg\min_{\tilde{C}} \|T(\alpha\hat{P}, \frac{\tilde{C}}{\alpha})\|_\infty. \tag{59}$$

The resulting controller yields optimal robustness for $\mathcal{H}_\infty$-bounded perturbations of normalized *rcf*'s of the weighted model $\alpha\hat{P}$. Additionally, the bandwidth of the feedback system will be around the crossover frequency of $\alpha\hat{P}$. When the frequency response of the nominal model $\hat{P}$ rolls off in the range where $|\alpha\hat{P}(e^{i\omega})| \sim 1$, one can push out the designed bandwidth by increasing the weight α, thus allowing more control action. In other words, a large α corresponds to a high nominal performance.

Note that the introduction of this weighting does not hinder the identification part, as it simply acts as a scalar weight on $\hat{P}$, P_0 and C.

At each control-design step the weight α is increased just gradually in order to enhance the nominal performance, while the performance degradation has to remain acceptably small. This implies that at iteration step i, having available $\hat{P}_{i+1}$, α_i, C_i, a choice is made for $\alpha_{i+1} > \alpha_i$ requiring that

$$\|T(\alpha_{i+1}P_0, \frac{C_{i+1}}{\alpha_{i+1}}) - T(\alpha_{i+1}\hat{P}_{i+1}, \frac{C_{i+1}}{\alpha_{i+1}})\|_\infty << \|T(\alpha_{i+1}\hat{P}_{i+1}, \frac{C_{i+1}}{\alpha_{i+1}})\|_\infty, \tag{60}$$

while the sum of left- and right-hand expression has to remain sufficiently small.

The right-hand side of (60) is actually minimized in the control design and thus can be calculated simply. The left-hand side is not directly available, as experiments with the new controller C_{i+1} are not yet available. From the identification in step i we have obtained knowledge of

$$\|T(\alpha_iP_0, \frac{C_i}{\alpha_i}) - T(\alpha_i\hat{P}_{i+1}, \frac{C_i}{\alpha_i})\|_2, \tag{61}$$

which is clearly different from the left-hand side of (60). In the proposed iterative scheme a high-order frequency response estimate of P_0 is employed in order to estimate this left-hand side of (60) for each new candidate α_{i+1}.

5.4.3. *Robust stability analysis* . Having selected α_{i+1} and calculated C_{i+1}, there is no formal guarantee that the plant P_0 is stabilized by C_{i+1}. This is due to the fact that the design (55) is an unconstrained optimization; although the robustness is optimized, there is no prior guarantee about the extent of the robustness margin. Moreover, (60) can be verified only by replacing P_0 by some estimate of P_0.

In order to test the robust stability before actually implementing the newly-designed controller, we can exploit a robust stability result that utilizes an uncertainty structure on (coprime factors of) $\hat{P}_{i+1}$ that is controller dependent.

Proposition 6. (Schrama (1992b).) Let controller C_i stabilize both the model $\hat{P}_{i+1}$ and the plant P_0. Let $\alpha_{i+1}\hat{P}_{i+1}$ have a normalized *rcf* $(\hat{N}_n, \hat{D}_n)$ and let $\alpha_{i+1}^{-1}C_i$ have a normalized *rcf* (N_c, D_c). Let $\alpha_{i+1}P_0$ have an *rcf* (N_0, D_0) that satisfies $N_0 = \hat{N} + \Delta N$ and $D_0 = \hat{D} + \Delta D$, with $\|[\Delta N\ \Delta D]\|_\infty < \gamma$. Then P_0 is stabilized by C_{i+1} if

$$\|[\alpha_{i+1}\hat{D}_n + C_{i+1}\hat{N}_n]^{-1}[C_{i+1} - C_i]D_c\|_\infty < \gamma^{-1}. \tag{62}$$

This result is based on a Youla parametrization for both the controller C_{i+1} and the plant P_0. The resulting stability test is non-conservative in the case $C_{i+1} = C_i$, as in that case it matches with the Youla parametrization. Note that the left-hand side of (62) is completely known. This stability test calls for identification methods that provide a $\mathcal{H}_\infty$-error bound on the normalized *rcf* of the weighted model $\alpha_{i+1}\hat{P}_{i+1}$. Once an upper bound γ can be found from data, (62) can be verified. In Schrama (1992b) high-order frequency domain estimates of P_0 are taken to construct the required error bounds and to verify robust stability.

5.5. *Other approaches*

Apart from the three methods presented in this section so far, there are a couple of alternatives that all incorporate iterations between model identification and control design.

In Liu and Skelton (1990) an iterative method is presented that relies on closed-loop impulse response experiments for identifying a model using the q-Markov cover theory; the model matches the first sequence of Markov parameters and the first elements of the output covariance function of the closed-loop plant. An open-loop model is reconstructed by employing knowledge of the controller and through application of a model reduction technique. This identification is iterated with a control-design scheme that minimizes the control energy of the closed loop subjected to inequality constraints on the output variance.

In Astrom (1993) considerations are given for the matching of criteria in identification and control. For several control-design strategies the relevance of least-squares identification in closed loop using appropriate data filters is discussed. This leads to results that are in line with the ones obtained for LQ-control as discussed in Section 5.2.

In Graebe *et al.* (1993) and Graebe and Goodwin (1993) an iterative scheme is proposed that is based on closed-loop identification of multiplicative model increments, a stochastic embedding approach for quantifying the model uncertainty and an IMC control-design method. With each iteration the model complexity is increased so as to capture more of the relevant plant dynamics in the model. Due to the model uncertainty quantification a bandwidth of robust performance can be predicted at each iteration.

In a number of approaches one does not explicitly use closed-loop experiments. Bayard *et al.* (1992) matches the identification and control design for a mixed sensitivity type of control design, where the actual identification is replaced by a control-relevant weighted curve fit on the frequency response of the (noiseless) plant. In Rivera *et al.* (1992), considering several control objectives, an account is given on the construction of control-relevant prefilters to be applied in open-loop identification in order to arrive at control-relevant models. Shook *et al.* (1992) propose data prefilters in order to provide an identification criterion that matches a generalized predictive control criterion; this method is worked out for a noise-free situation.

5.6. *Evaluation*

Despite their differences, the iterative schemes presented in the previous subsections find their roots in a similar philosophy to the problem, for which a generic formulation has been given in Section 3. Basic features are:

- An appropriate nominal model is indispensable for achieving a high performance controlled plant.
- Approximate identification in closed loop serves to match the achieved and designed performance as close as possible.
- Generally the iterative algorithms start off with a moderate nominal performance requirement (loose controller) and pursue a nominal performance improvement by gradually increasing the nominal performance requirement (cf. increasing designed bandwidth in the IMC-design and increasing α in the robustness optimization of Section 5.4).
- Decision points occur whenever one is no longer able to achieve a performance degradation cost that is (far) lower than the designed performance cost. In that situation one either has to increase the complexity of the model (to reduce the performance degradation) or to increase the complexity of the controller (to reduce the nominal performance cost), or to realize that one may have reached the limits of achievable performance.
- Convergence of the iterative schemes is obtained through monitoring the iterations, rather than by formal theoretical justification.

The approach that is present in these iterative schemes can be viewed as a means of letting experiments reveal what control performance can be achieved for the plant under consideration. In this way one can "learn" about the achievable plant performance as the iterations go by, by successively retuning the controller based on information obtained from experiments. Referring back to the problem as stated in the introduction of this paper, the surveyed area offers a clear approach to the goal of arriving at appropriate controlled plants on the basis of experiment-based plant models. However, we are not yet in a situation in which full answers are provided, and many questions remain to be solved. We list only a few of the open problems.

- Attention is focused to bias aspects of the identified models, rather than to variance aspects. A recent contribution that supports the approach presented here and that focuses on variance aspects in the considered identification setting, is given in Hjalmarsson *et al.* (1994a).
- Attention is restricted to classical prediction error methods leading to the use of $\mathcal{H}_2$-norms in model approximation, while actually from a point of view of control performance cost an $\mathcal{H}_\infty$ criterion would be most suitable; see e.g. the methods in Sections 5.3 and 5.4. This

mismatch of norms requires attention in future research, and may lead to the explicit use of $\mathcal{H}_\infty$-norms in identification procedures.

- The possible convergence of these schemes has to be further analysed, and conditions have to be formulated for avoiding divergent behaviour.
- The prediction of performance degradation every time a new controller is designed calls for the explicit use of uncertainty models. This point refers to the problem of satisfying (14) after a controller redesign. In this uncertainty modelling, uncertainty structures have to be used that are specifically suited for the considered control performance cost, in the sense that non-conservative statements can be made with respect to the left-hand side of (14).

One can argue whether one needs closed-loop experiments and closed-loop identification in order to obtain identified models that serve our goal. As apparent in the analysis of this paper, performing the identification in closed loop provides a weighting of the identification criterion that can exactly support the intended control application of the model. The controller shapes the input of the plant to a form that stresses those components that are control relevant. The experimental situation under which models are obtained is closely matched to the experimental situation in which the model and its induced controller, have to perform particularly well. A similar situation can be approximated by open-loop identification employing weighting functions (prefilters) that are plant and controller dependent. Consequently, that similarity can only be approximative.

The presented iterative ways of increasing the plant performance are specifically directed towards situations in which the achievable plant performance is not known a priori. In the case in which one knows that a specific (moderate) plant performance can be obtained and which control-design weights have to be used, it may not be too hard to find appropriate prefilters in open-loop identification, that will provide a model that supports the required control design.

When arriving at a model that is suited for high-performance control, one is not automatically assured of an overall accurate open-loop model. It has been shown in a number of situations that one can easily arrive at high-performance control systems with only a moderate, or even a bad, open-loop performance of the nominal model, see e.g. Zang *et al.* (1991a) and Schrama (1992a) as also illustrated in Section 2. If one additionally wishes a model that satisfies the experimenter's prior knowledge or that is an accurate open-loop description of the plant, this will have to be considered as an additional model requirement, for which a price has to be paid, generally in terms of a higher model order and/or more experiments.

We also would like to point to the "inner" iterative loop that is present in the block diagram in Fig. 4. Especially in situations where experiments are expensive and time consuming, it can be advantageous to fully exploit the possibilities of this "inner" loop, i.e. repeating identification and control design without renewing the experiments.

We have focused on the problem of arriving at well-controlled plants by separate though related stages of model identification and model-based control design. An interesting account of a direct controller tuning based on experimental data is given in Hjalmarsson *et al.* (1994b).

Successful real-life applications of high-performance control design will have to be the ultimate justification for the methods presented in this survey. While most of the techniques presented here are supported by simulation examples, it is gratifying that also a couple of real-life applications have already been reported. The iterative scheme of Zang and co-workers with LQ control has been applied to a sugar crane crushing mill, see Partanen and Bitmead (1995), while the scheme of Schrama and co-workers with the $\mathcal{H}_\infty$-design has been applied to a servomechanical system in Schrama and Bosgra (1993). One single step in this latter scheme has also been applied to the pick-up mechanism in a Compact Disc player, see De Callafon *et al.* (1994).

6. FINAL REMARKS

In this paper we have reviewed some aspects of the research related to the problem of designing a high-performance controller for a plant with unknown dynamics through separate identification and control design. The underlying ideas in this area are quite close to adaptive control. Whereas in adaptive control the procedures of identification and control design are completely intertwined, the approaches discussed here attempt to provide a separate analysis of both steps, and to accomplish a joint performance criterion of both parts.

Iterative procedures in which the performance level of the controlled plant is gradually increased are actually quite appealing. In engineering practice a product is commonly accomplished by a number of successive improvements rather than by a one-step design that starts from scratch. Why should control engineering be different? The implication of this is that one should not focus on an ideal but possibly unachievable desired control performance. Instead one better aims at an improvement of the performance that has already been achieved.

Acknowledgements— The authors gratefully acknowledge their colleagues at Delft University of Technology for fruitful discussions and their contributions to this work.

REFERENCES

Anderson, B. D. O. and R. L. Kosut (1991). Adaptive robust control: on-line learning. In *Proc. 30th IEEE Conf. Decision and Control*, Brighton, U.K., pp. 297–298.

Astrom, K. J. (1993). Matching criteria for identification and control. In *Proc. 2nd European Control Conf.*, Groningen, The Netherlands, pp. 248–251.

Astrom, K. J. and B. Wittenmark (1971). Problems of identification and control. *J. Math. Anal. & Applic.*, **34**, 90–113.

Bayard, D. S. (1992). Statistical plant set estimation using Schroeder-phased multisinusoidal input design. In *Proc. Am. Control Conf.*, Chicago, IL, pp. 2988–2995.

Bayard, D. S., Y. Yam and E. Mettler (1992). A criterion for joint optimization of identification and robust control. *IEEE Trans. Autom. Control*, **AC-37**, 986–991.

Bitmead, R. R., M. Gevers and V. Wertz (1990). *Adaptive Optimal Control, The Thinking Man's GPC*. Prentice-Hall, Englewood Cliffs, NJ.

Bitmead, R. R. (1993). Iterative control-design approaches. In *Preprint 12th IFAC World Congress*, Sydney, Australia, **9**, pp. 381–384.

Bongers, P. M. M. and O. H. Bosgra (1990). Low order robust $\mathcal{H}_\infty$ controller synthesis. *Proc. 29th IEEE Conf. Decision and Control*, Honolulu, HI, pp. 194–199.

Caines, P. E. and M. Baykal-Gürsoy (1989). On the L_∞ consistency of L_2 estimators. *Syst. Control Letters*, **12**, 71–76.

De Callafon, R. A., P. M. J. Van den Hof and D. K. de Vries (1994). Identification and control of a compact disc mechanism using fractional representations. In *Preprint 10th IFAC Symp. on System Identification*, 4–6 July 1994, Copenhagen, Denmark, Vol. 2, pp. 121–126.

Desoer, C. A., R. W. Liu, J. Murray and R. Saeks (1980). Feedback system design: the fractional representation approach to analysis and synthesis. *IEEE Trans. Autom. Control*, **AC-25**, 399–412.

De Vries, D. K. and P. M. J. Van den Hof (1995). Quantification of uncertainty in transfer function estimation: a mixed deterministic–probabilistic approach. *Automatica*, **31**, 543–557.

Doyle, J. C., B. A. Francis and A. R. Tannenbaum (1992). *Feedback Control Theory*. MacMillan, New York.

Fogel, E. and Y. F. Huang (1982). On the value of information in system identification — bounded noise case. *Automatica*, **18**, 229–238.

Francis, B. A. (1987). *A Course in $\mathcal{H}_\infty$ Control Theory*. Springer, Berlin.

Georgiou, T. T. and M. C. Smith (1990). Optimal robustness in the gap metric. *IEEE Trans. Autom. Control*, **AC-35**, 673–686.

Gevers, M. and L. Ljung (1986). Optimal experiment design with respect to the intended model application. *Automatica*, **22**, 543–554.

Gevers, M. (1993). Towards a joint design of identification and control? In H. L. Trentelman and J. C. Willems (Eds), *Essays on Control: Perspectives in the Theory and its Applications*. Birkhäuser, Boston, pp. 111–151.

Goodwin, G. C., M. Gevers and B. M. Ninness (1992). Quantifying the error in estimated transfer functions with application to model order selection. *IEEE Trans. Autom. Control*, **AC-37**, 913–928.

Glover, K. and D. McFarlane (1989). Robust stabilization of normalized coprime factor plant descriptions with $\mathcal{H}_\infty$-bounded uncertainty. *IEEE Trans. Autom. Control*, **AC-34**, 821–830.

Graebe, S. F., M. R. West and G.C. Goodwin (1993). A laboratory scale application of incremental estimation and stochastic embedding. In *Proc. 1993 Am. Control Conf.*, San Francisco, CA, pp. 3028–3032.

Graebe, S. F. and G. C. Goodwin (1993). An incremental estimation technique for predicting a bandwidth of robust performance. In *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, pp. 2260–2265.

Gu, G. and P. P. Khargonekar (1992). A class of algorithms for identification in $\mathcal{H}_\infty$. *Automatica*, **28**, 299–312.

Hakvoort, R. G. (1990). Optimal experiment design for prediction error identification in view of feedback design. In *Selected Topics in Identification, Modelling and Control*, 2. Delft University Press, pp. 71–78.

Hakvoort, R. G. (1992). Worst-case system identification in ℓ_1: error bounds, optimal models and model reduction. In *Proc. 31st IEEE Conf. Decision and Control*, Tucson, AZ, pp. 499–504.

Hakvoort, R. G. (1993). Worst-case system identification in $\mathcal{H}_\infty$: error bounds and optimal models. In *Preprint 12th IFAC World Congress*, Sydney, Vol. 8, pp. 161–164.

Hakvoort, R. G. and P. M. J. Van den Hof (1994). Frequency domain curve fitting with maximum amplitude criterion and guaranteed stability. *Int. J. Control*, **60**, 809–825.

Hakvoort, R. G., R. J. P. Schrama and P. M. J. Van den Hof (1994). Approximate identification with closed-loop performance criterion and application to LQG feedback design. *Automatica*, **30**, 679–690.

Hansen, F. R. and G. F. Franklin (1988). On a fractional representation approach to closed-loop experiment design. In *Proc. Am. Control Conf.*, Atlanta, GA, pp. 1319–1320.

Hansen, F., G. Franklin and R. Kosut (1989). Closed-loop identification via the fractional representation: experiment design. In *Proc. Am. Control Conf.*, Pittsburgh, PA, pp. 1422–1427.

Helmicki, A. J., C. A. Jacobson and C. N. Nett (1990). Identification in $\mathcal{H}_\infty$: a robustly convergent nonlinear algorithm. In *Proc. Am. Control Conf.*, San Diego, CA, pp. 386–391.

Helmicki, A. J, C. A. Jacobson and C. N. Nett (1991). Control-oriented system identification: a worst-case/deterministic approach in $\mathcal{H}_\infty$. *IEEE Trans. Autom. Control*, **AC-36**, 1163–1176.

Hjalmarsson, H. (1993). *Aspects of Incomplete Modelling in System Identification*. PhD thesis, Linköping Studies in Science and Technology, No. 298, Linköping, Sweden.

Hjalmarsson, H., M. Gevers, F. De Bruyne and J. Leblond (1994a). Identification for control: closing the loop gives more accurate controllers. In *Proc. 33rd IEEE Conf. Decision and Control*, Lake Buena Vista, FL, 14–16 December, pp. 4150–4155.

Hjalmarsson, H., S. Gunnarsson and M. Gevers (1994b). A convergent iterative restricted complexity control-design scheme. In *Proc. 33rd IEEE Conf. Decision and Control*, Lake Buena Vista, FL, 14–16 December , pp. 1735–1740.

Jacobson, C. A. and C. N. Nett (1991). Worst-case identification in ℓ_1: optimal algorithms and error bounds. In *Proc. Am. Control Conf.*, Boston, MA, pp. 3152–5157.

Kosut, R. L., M. K. Lau and S. P. Boyd (1992). Set-membership identification of systems with parametric and nonparametric uncertainty. *IEEE Trans. Autom. Control*, **AC-37**, 929–941.

LaMaire, R. O., L. Valavani, M. Athans and G. Stein (1991). A frequency domain estimator for use in adaptive control systems. *Automatica*, **27**, 23–38.

Lee, W. S., B. D. O. Anderson, R. L. Kosut and I. M. Y. Mareels (1992). On adaptive robust control and control-relevant system identification. In *Proc. Am. Control Conf.*, Chicago, IL, pp. 2834–2841.

Lee, W. S., B. D. O. Anderson, R. L. Kosut and I. M. Y. Mareels (1993a). A new approach to adaptive robust control. *Int. J. Adaptive Control and Signal Proc.*, **7**, 183–211.

Lee, W. S., B. D. O. Anderson, R. L. Kosut and I. M. Y. Mareels (1993b). On robust performance improvement through the windsurfer approach to adaptive robust control. In *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, pp. 2821–2827.

Lee, W. S. (1994). *Iterative Identification and Control Design for Robust Performance*. PhD thesis, Australian National University, Canberra, September 1994.

Liu, K. and R. E. Skelton (1990). Closed-loop identification and iterative controller design. In *Proc. 29th IEEE Conf. Decision and Control*, Honolulu, HI, pp. 482–487.

Ljung, L. (1987). *System Identification: Theory for the User*. Prentice-Hall, Englewood Cliffs, NJ.

Ljung, L., B. Wahlberg and H. Hjalmarsson (1991). Model quality: the role of prior knowledge and data information. In *Proc. 30th IEEE Conf. Decision and Control*, Brighton, U.K., pp. 273–278.

Maciejowski, J. M. (1989). *Multivariable Feedback Design*. Addison-Wesley, New York.

Makila, P. M. (1991). Robust identification and Galois sequences. *Int. J. Control*, **54**, 1189–1200.

Makila, P. M., J. R. Partington and T. K. Gustafsson (1994). Robust identification. In *Preprint 10th IFAC Symp. on System Identification*, 4–6 July 1994, Copenhagen, Denmark, Vol. 1, pp. 45–63.

McFarlane, D. and K. Glover (1990). *Robust Controller Design Using Normalized Coprime Factor Plant Descriptions*. Springer, Berlin.

Milanese, M. and A. Vicino (1991). Optimal estimation theory for dynamic systems with set membership uncertainty: an overview. *Automatica*, **27**, 997–1009.

Ninness, B. M. (1993). *Stochastic and Deterministic Modelling*. PhD thesis, The University of Newcastle, NSW, Australia.

Ninness, B. M. and G. C. Goodwin (1994). Estimation of model quality. In *Preprint 10th IFAC Symp. on System Identification*, 4–6 July 1994, Copenhagen, Denmark, Vol. 1, pp. 25–44.

Norton, J. P. (1987). Identification and application of bounded-parameter models. *Automatica*, **23**, 497–507.

Parker, P. J. and R. R. Bitmead (1987). Adaptive frequency response identification. In *Proc. 26th Conf. Decision and Control*, Los Angeles, CA, pp. 348–353.

Partanen, A. G. and R. R. Bitmead (1993). Two stage iterative identification/controller design and direct experimental controller refinement. In *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, pp. 2833–2838.

Partanen, A. G. and R. R. Bitmead (1995). The application of an iterative identification and controller design to a sugar cane crushing mill. *Automatica*, **31**, 1547–1563.

Partington, J. R. (1991). Robust identification in $\mathcal{H}_\infty$. *J. Math. Analysis & Applic.*, **166**, 428–441.

Poolla, K., P. Khargonekar, A. Tikku, J. Krause and K. Nagpal (1994). A time-domain approach to model invalidation. *IEEE Trans. Autom. Control*, **AC-39**, 951–959.

Rivera, D. E., J. F. Pollard and C. E. Garcia (1992). Control-relevant prefiltering: a systematic design approach and case study. *IEEE Trans. Autom. Control*, **AC-37**, 964–974.

Schrama, R. J. P. (1991). An open-loop solution to the approximate closed-loop identification problem. In *Proc. 9th IFAC/IFORS Symp. Identification and System Parameter Estimation*, Budapest, Hungary, pp. 1602–1607.

Schrama, R. J. P. and P. M. J. Van den Hof (1992). An iterative scheme for identification and control design based on coprime factorizations. In *Proc. Am. Control Conf.*, Chicago, IL, pp. 2842–2846.

Schrama, R. J. P. (1992a). Accurate models for control design: the necessity of an iterative scheme. *IEEE Trans. Autom. Control*, **AC-37**, 991–994.

Schrama, R. J. P. (1992b). *Approximate Identification and Control Design*. PhD thesis, Delft University of Technology, The Netherlands.

Schrama, R. J. P. and O. H. Bosgra (1993). Adaptive performance enhancement by iterative identification and control design. *Int. J. Adaptive Control & Signal Proc.*, **7**, 475–487.

Shook, D. S., C. Mohtadi and S. L. Shah (1992). A control-relevant identification strategy for GPC. *IEEE Trans. Autom. Control*, **AC-37**, 975–980.

Skelton, R. E. (1989). Model error concepts in control design. *Int. J. Control*, **49**, 1725–1753.

Smith, R. S. and J. C. Doyle (1992). Model invalidation: a connection between robust control and identification. *IEEE Trans. Autom. Control*, **AC-37**, 942–952.

Smith, R. and M. Dahleh (Eds) (1994). *The Modelling of Uncertainty in Control Systems*. Lecture Notes in Control and Information Science, **192**, Springer, London.

Soderstrom, T. and P. Stoica (1989). *System Identification*. Prentice-Hall, Hemel Hempstead, U.K.

Stein, G. and J. C. Doyle (1991). Beyond singular values and loop shapes. *J. Guidance, Control & Dynamics*, **14**, 5–16.

Tse, D. N. C., M. A. Dahleh and J. N. Tsitsiklis (1993). Optimal asymptotic identification under bounded disturbances. *IEEE Trans. Autom. Control*, **AC-38**, 1176–1190.

Van den Hof, P. M. J. and R. J. P. Schrama (1993). An indirect method for transfer function estimation from closed-loop data. *Automatica*, **29**, 1523–1527.

Van den Hof, P. M. J., R. J. P. Schrama, R. A. de Callafon and O. H. Bosgra (1995). Identification of normalized coprime plant factors from closed-loop experimental data. *European J. Control*, **1**, (to appear). A preliminary version appeared in *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, pp. 62–74.

Vidyasagar, M. (1985). *Control System Synthesis: A Factorization Approach*. MIT Press, Cambridge, MA.

Wahlberg B. and L. Ljung (1986). Design variables for bias distribution in transfer function estimation. *IEEE Trans. Autom. Contr.*, **AC-31**, 134–144.

Wahlberg, B. and L. Ljung (1992). Hard frequency domain model error bounds from least-squares like identification techniques. *IEEE Trans. Autom. Control*, **AC-37**, 900–912.

Younce, R. C. and C. E. Rohrs (1992). Identification with nonparametric uncertainty. *IEEE Trans. Autom. Control*, **AC-37**, 715–728.

Zang, Z., R. R. Bitmead and M. Gevers (1991a). Iterative Model Refinement and Control Robustness Enhancement. Report 91.137, CESAME, Catholic University Louvain-la-Neuve, Belgium.

Zang, Z., R. R. Bitmead and M. Gevers (1991b). $\mathcal{H}_2$ iterative model refinement and control robustness enhancement. In *Proc. 30th IEEE Conf. Decision and Control*, 11–13 December 1991, Brighton, U.K., pp. 279–284.

Zang, Z., R. R. Bitmead and M. Gevers (1992). Disturbance rejection: on-line refinement of controllers by closed-loop modelling. In *Proc. 1992 Am. Control Conf.*, Chicago, IL, pp. 2829–2833.

Zang, Z., R. R. Bitmead and M. Gevers (1995). Iterative weighted least-squares identification and weighted LQG-control. *Automatica*, **31**, 1577–1594.

Zhu, Y. C. (1989). Estimation of transfer functions: asymptotic theory and a bound of model uncertainty. *Int. J. Control*, **49**, 2241–2258.