System Identification

Lecture 12: Closed-loop identification

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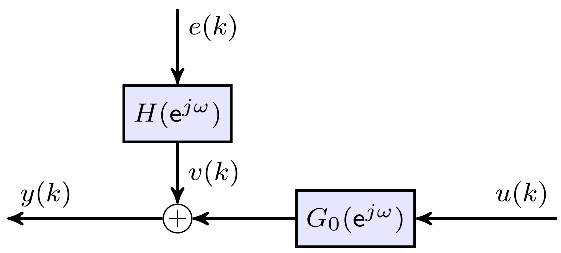
Open-loop identification methods



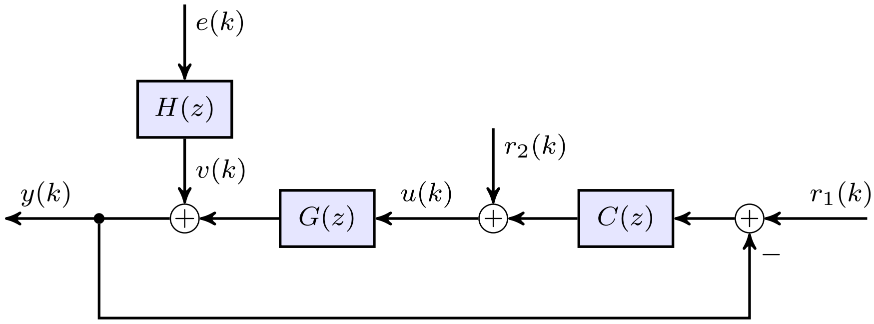
$$Y(\mathrm{e}^{j\omega}) = G(\mathrm{e}^{jw})U(\mathrm{e}^{j\omega}) + \underbrace{V(\mathrm{e}^{j\omega})}_{\text{noise}} + \underbrace{R(\mathrm{e}^{j\omega})}_{\text{transient}}$$

Estimate: $\hat{G}(\mathrm{e}^{j\omega_n}) = \dfrac{\hat{Y}(\mathrm{e}^{j\omega_n})}{\hat{U}(\mathrm{e}^{j\omega_n})}$

Bias: $E\{\hat{G}(\mathrm{e}^{j\omega_n}) - G(\mathrm{e}^{j\omega_n})\} \longrightarrow 0$ as $N \longrightarrow \infty$

Variance: $E\left\{|\hat{G}(\mathrm{e}^{j\omega_n}) - E\{\hat{G}(\mathrm{e}^{j\omega_n})\}|^2\right\} \longrightarrow \dfrac{\phi_v(\mathrm{e}^{j\omega_n})}{\phi_u(\mathrm{e}^{j\omega_n})}$ as $N \longrightarrow \infty$

Closed-loop identification



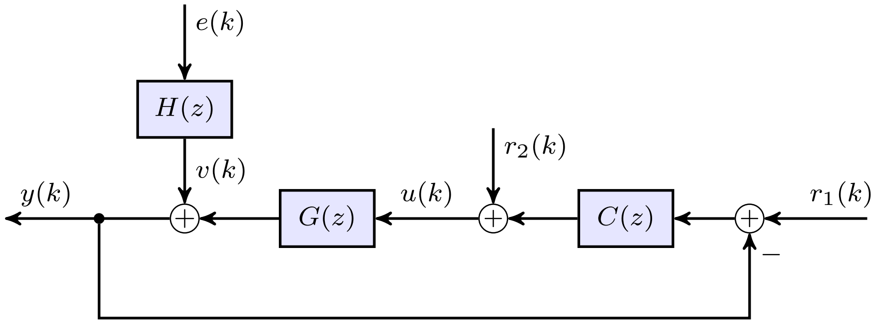
Excitation:

$r_2(k)$ and $r_1(k)$

Measurements

$y(k), u(k), r_2(k)$ and $r_1(k)$

Closed-loop identification



For simplicity define:

$$r(k) = r_2(k) + C(z)r_1(k).$$

This generalises our excitation signal choices.

Closed-loop identification

Motivation for closed-loop identification

- Unstable systems must be operated in closed-loop.
- Operational constraints may require closed-loop.
- Closed-loop controller maintains the system close to the operating point of interest.
- Easier to focus identification on specific operating points.
- Will emphasize plant dynamics close to the cross-over frequency range.
- Closed-loop operation can remove a large-scale zero-frequency response.

Closed-loop identification

Methods

- Direct (open-loop) methods:

$$\hat{G} = \hat{Y}_N / \hat{U}_N \qquad \text{or} \qquad \hat{G} = \hat{\phi}_{yu} / \hat{\phi}_u .$$

- Indirect methods:

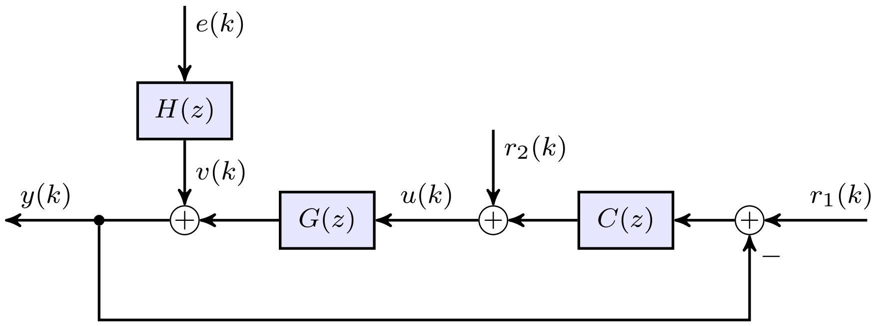
$$y = \frac{G(z)}{(1 + G(z)C(z))}\, r.$$

- Input-output methods:

$$y = \frac{G(z)}{(1 + G(z)C(z))}\, r, \quad \text{and} \quad u = \frac{1}{(1 + G(z)C(z))}\, r.$$

- Dual-Youla methods.

Direct methods: closed-loop identification with open loop methods



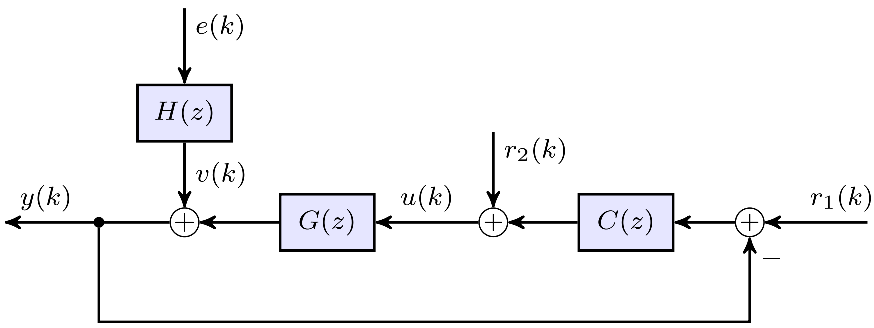
Open-loop spectral estimation methods:

$$\hat{\phi}_{yu}(\mathrm{e}^{j\omega_n}) = \frac{1}{N} Y_N(\mathrm{e}^{j\omega_n}) \overline{U_N}(\mathrm{e}^{j\omega_n})$$

$$\hat{\phi}_{u}(\mathrm{e}^{j\omega_n}) = \frac{1}{N} \left| U_N(\mathrm{e}^{j\omega_n}) \right|^2$$

$$\hat{G}(\mathrm{e}^{j\omega_n}) = \frac{\hat{\phi}_{yu}(\mathrm{e}^{j\omega_n})}{\hat{\phi}_{u}(\mathrm{e}^{j\omega_n})}$$

Direct closed-loop identification methods



Closed-loop transfer functions:

$$y = SGr_2 + SGCr_1 + Sv \quad = \quad SGr + Sv$$

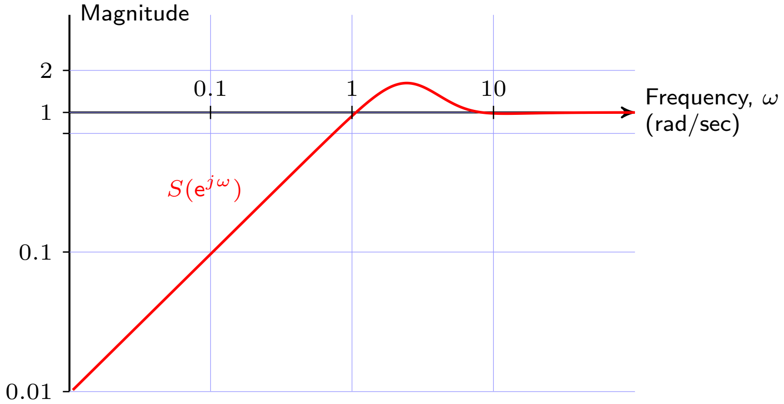
$$u = Sr_2 + SCr_1 - SCv \quad = \quad Sr - SCv.$$

where, $S(\mathrm{e}^{j\omega}) = \dfrac{1}{1 + C(\mathrm{e}^{j\omega}) G(\mathrm{e}^{j\omega})}$ (assumed stable)

Sensitivity function weighting

Closed-loop transfer functions:

$$
\begin{aligned}
y &= SGr + Sv \\
u &= Sr - SCv
\end{aligned}
\qquad S = \frac{1}{1+CG}
$$



Direct closed-loop identification methods

Spectral analysis

Assume that $\phi_{rv} = 0$.

Then,

$$\phi_{yu} = |S|^2 G\phi_r - |S|^2 \bar{C}\phi_v$$

$$\phi_u = |S|^2 \phi_r + |S|^2 |C|^2 \phi_v$$

$$\hat{G} = \frac{\hat{\phi}_{yu}}{\hat{\phi}_u} \approx \frac{|S|^2 G\hat{\phi}_r - |S|^2 \bar{C}\hat{\phi}_v}{|S|^2\hat{\phi}_r + |S|^2|C|^2\hat{\phi}_v} \approx \frac{G\hat{\phi}_r - \bar{C}\hat{\phi}_v}{\hat{\phi}_r + |C|^2 \hat{\phi}_v}$$

Indirect closed-loop identification methods

Closed-loop transfer functions:

$$\begin{aligned} y &= SGr + Sv \qquad \text{define: } T_{yr} = SG, \\ &= T_{yr} r + Sv. \end{aligned}$$

Identification problem (open-loop)

If $\phi_{vr} = 0$,

$$\hat{T}_{yr}(\mathrm{e}^{j\omega_n}) = \frac{Y_N(\mathrm{e}^{j\omega_n})}{R_N(\mathrm{e}^{j\omega_n})} \qquad \text{(asymptotically unbiased).}$$

Solving for $\hat{G}(\mathrm{e}^{j\omega_n})$

$$T_{yr}(z) = \frac{G(z)}{1 + C(z)G(z)} \quad \Longrightarrow \quad G(z) = \frac{T_{yr}}{1 - T_{yr}(z)C(z)}$$

$$\rightsquigarrow \quad \hat{G}(\mathrm{e}^{j\omega_n}) = \frac{\hat{T}_{yr}(\mathrm{e}^{j\omega_n})}{1 - \hat{T}_{yr}(\mathrm{e}^{j\omega_n})C(\mathrm{e}^{j\omega_n})}$$

Joint input-output methods

Closed-loop transfer functions:

$$\begin{aligned} y &= SGr + Sv \quad \text{define:} \quad & T_{yr} &= SG, \\ u &= Sr - SCv & T_{ur} &= S \end{aligned}$$

Identification problems

If $\phi_{vr} = 0$,

$$\hat{T}_{yr}(\mathrm{e}^{j\omega_n}) = \frac{Y_N(\mathrm{e}^{j\omega_n})}{R_N(\mathrm{e}^{j\omega_n})} \qquad \text{(asymptotically unbiased).}$$

Similarly, if $\phi_{vr} = 0$,

$$\hat{T}_{ur}(\mathrm{e}^{j\omega_n}) = \frac{U_N(\mathrm{e}^{j\omega_n})}{R_N(\mathrm{e}^{j\omega_n})} \qquad \text{(asymptotically unbiased).}$$

Joint input-output methods

Closed-loop identification approach

$$\frac{T_{yr}}{T_{ur}} = \frac{SG}{S} = G \qquad \rightsquigarrow \qquad \hat{G}(\mathrm{e}^{j\omega_n}) = \frac{\hat{T}_{yr}(\mathrm{e}^{j\omega_n})}{\hat{T}_{ur}(\mathrm{e}^{j\omega_n})}$$

Key points:

- The estimates, $\hat{T}_{yr}$ and $\hat{T}_{ur}$ may be unbiased. Their ratio is not.
- The estimated spectra are weighted by $S(\mathrm{e}^{j\omega})$ or $S(\mathrm{e}^{j\omega})C(\mathrm{e}^{j\omega})$.
- The noise enters in a complicated manner.

Ratio distributions

Mean of a ratio of stochastic variables

Ratio distributions

Probability density function of a ratio of stochastic variables

Suppose we have two normal distributions, $v \in \mathcal{N}(0,1)$ and $w \in \mathcal{N}(0,1)$.

$$f_v(v) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}v^2}, \qquad f_w(w) = \frac{1}{\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}w^2}.$$

The ratio, $z = v/w$, is a stochastic variable with PDF:

$$f_z(z) = \frac{1}{\pi} \frac{1}{1+z^2} \qquad \text{Cauchy distribution.}$$

The mean and variance of a Cauchy distribution are undefined.

Youla parametrisations

Coprime factorisations

$$G_0(s) = \frac{N_0(s)}{D_0(s)},$$

with $N_0(s), D_0(s)$ stable and coprime (no common zeros).

Coprime factorisations are not unique.

For example: $$G(s) = \frac{(s-2)}{(s+5)(s-5)}, \quad \text{(unstable, non-minimum phase)}$$

$$N_0(s) = \frac{(s-2)}{(s+5)(s+\alpha)},$$

$$D_0(s) = \frac{(s-5)}{(s+\alpha)}, \quad \alpha > 0.$$

Youla parametrisations

Bezout identity

The transfer functions $N_0(s)$ and $D_0(s)$ are coprime

iff there exists $U(s)$ and $V(s)$ such that,

$$U(s)N_0(s) + V(s)D_0(s) = I.$$

Normalised coprime factorisations

A coprime factorisation is "normalised" if,

$$D_0^*(s)D_0(s) + N_0^*(s)N_0(s) = I.$$

MATLAB: `sncfbal`

Youla parametrisations

All stabilising controllers

If we have a controller, C_0, which stabilises G_0, with

$$C_0 = \frac{X_0}{Y_0}, \qquad (X_0,\ Y_0 \text{ a coprime factorisation})$$

then, all controllers, C, stabilising $G_0 = N_0/D_0$, have the form,

$$C_Q = \frac{X_0 + QD_0}{Y_0 - QN_0}, \qquad \text{with } Q \text{ stable.}$$

Dual-Youla methods

Control design

Given a particular plant, $G(s)$,

select $C(s)$ from the set of all controllers stabilising $G(s)$.

Closed-loop identification

Given a particular controller, $C(s)$,

select $G(s)$ from the set of all plants stabilised by $C(s)$.

Both problems can be formulated as a search over stable $Q(s)$.

Dual-Youla methods

Usual identification formulation:

$$y = Gu + He, \quad e \in \mathcal{N}(0,1).$$

Find G (and possibly H) from the data. We assume that H is stable and stably invertible.

Dual-Youla formulation:

$$Dy = Nu + Fe, \quad e \in \mathcal{N}(0,1) \quad D,\ N,\ \text{and } F \text{ stable.}$$

Find D, N (and possibly F) from the data.

Plant is stabilised by $C_0 = X_0/Y_0$.

Dual-Youla methods

Parameterisation with R and F:

$$G_R = \frac{N}{D} = \frac{N_0 + RY_0}{D_0 - RX_0}, \qquad R \text{ is stable.}$$

$$H_{R,F} = \frac{F}{D} = \frac{F}{D_0 - RX_0}, \qquad F \text{ is stable and stably invertible.}$$

Equivalent open-loop ID experiment:

$$(D_0 - RX_0)y = (N_0 + RY_0)u + Fe$$

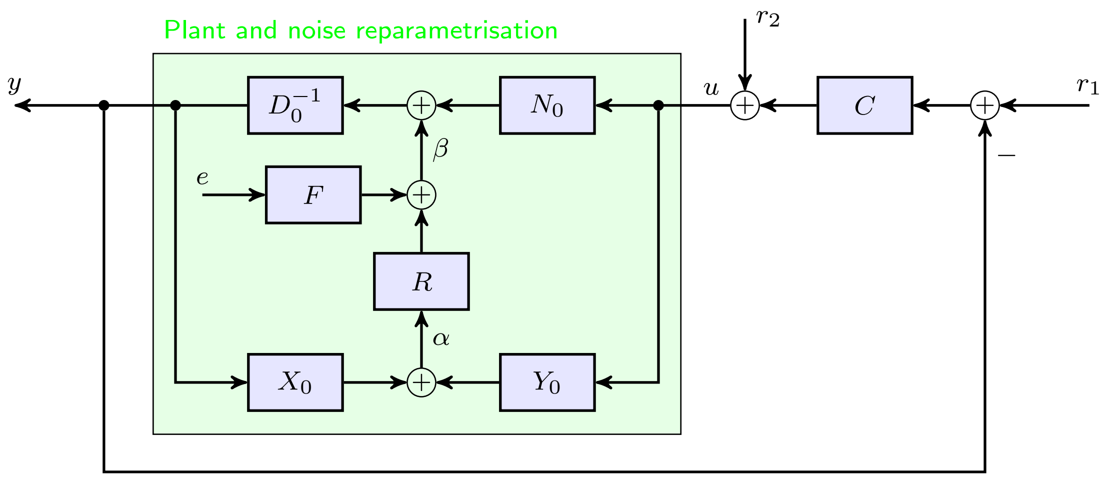
and rearranging gives,

$$\underbrace{D_0 y - N_0 u}_{=:\,\beta} = R\,\underbrace{(X_0 y + Y_0 u)}_{=:\,\alpha} + Fe$$

"Open-loop" system: $\beta = R\alpha + Fe$ with R and F stable.

Dual-Youla methods

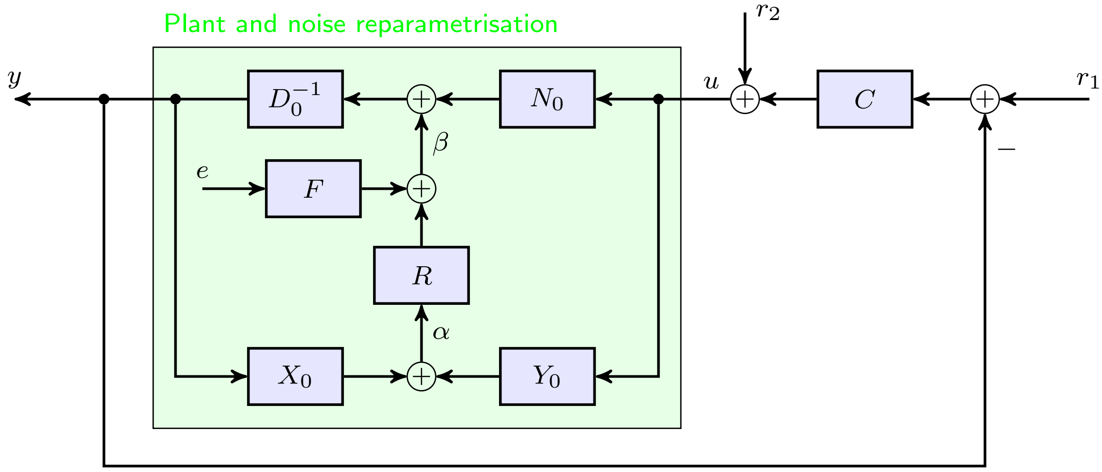
Reparametrised plant and noise structure



Dual-Youla methods

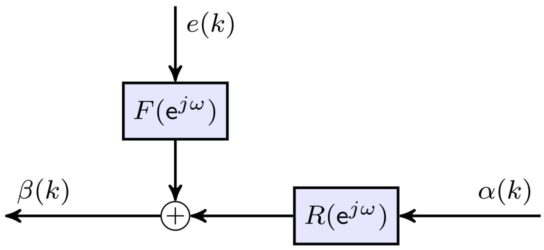
Key feature

The transfer function from β to α is zero.



Dual-Youla methods

Equivalent open-loop ID problem



$$\beta = D_0 y - N_0 u \qquad \text{(filtered input and output signals)}$$

$$\alpha = X_0 y + Y_0 u$$

$$= X_0 y + Y_0 \left(r_2 + \frac{X_0}{Y_0}(r_1 - y) \right) \quad = X_0 y + Y_0 \left(r - \frac{X_0}{Y_0} y \right)$$

$$= Y_0 r \qquad \text{(filtered excitation signal)}$$

Dual-Youla methods

Procedure

Given a stabilizing controller, C_0,

1. Factorise: $C_0 = X_0/Y_0$.
2. Choose excitation, r. (Note $\alpha = Y_0 r$ filtering).
3. Run closed-loop experiments with C_0, measuring y and u.
4. Choose an initial model, $G_0 = N_0/D_0$ (must be stabilised by C_0).
5. Filter measurements, $\beta = D_0 y - N_0 u$ (time or frequency domain).
6. Filter excitation, $\alpha = Y_0 r$.
7. Estimate $\hat{R}$ (and $\hat{F}$) from $\beta = R\alpha + Fe$.
8. Calculate plant estimate, $\hat{G} = (N_0 + \hat{R}Y_0)/(D_0 - \hat{R}X_0)$.

Bibliography

Indirect methods

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