Building control

Lecture 4: Feedback

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Building control concepts

Objectives

- Comfort
- Efficiency
- Cost

Controlled (and measured) quantities

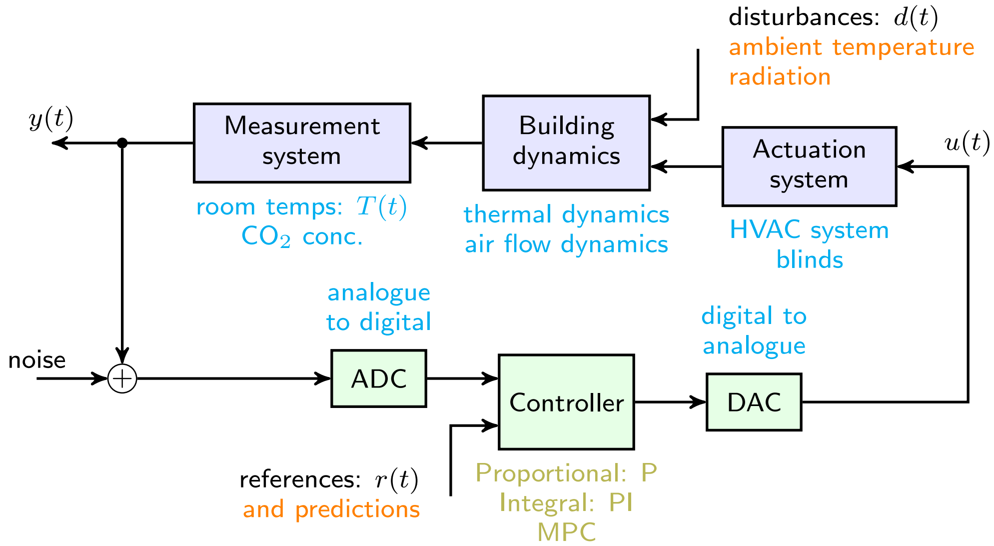
- Room temperatures
- CO_2 levels
- Illumination (and glare)
- Humidity

Actuation

- Ventilation air temperature and flow
- Heating and cooling (HVAC)
- Blinds
- Lighting

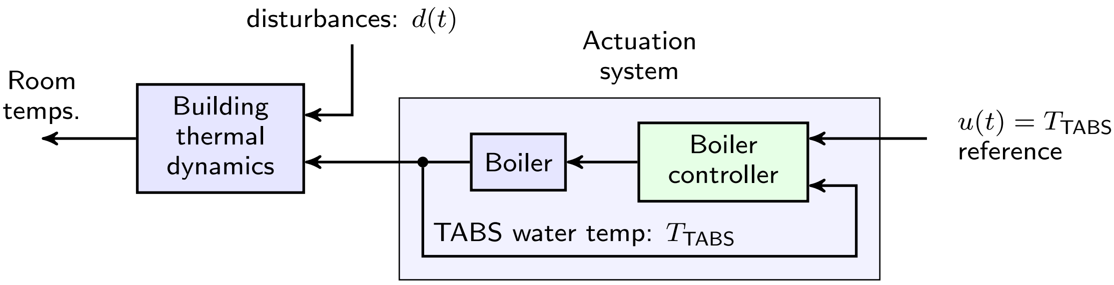
Building control

Feedback control structure



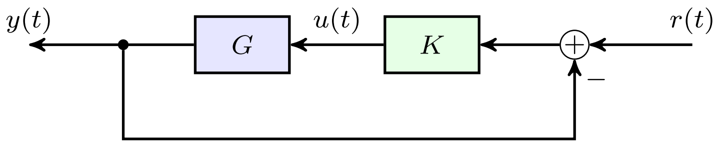
Building control

Control hierarchies



Simple feedback control

Unity gain negative feedback



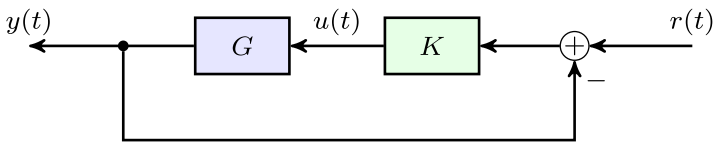
Components:

“Plant”: G

Controller: K

Simple feedback control

Unity gain negative feedback



Components:

“Plant”: G

Controller: K

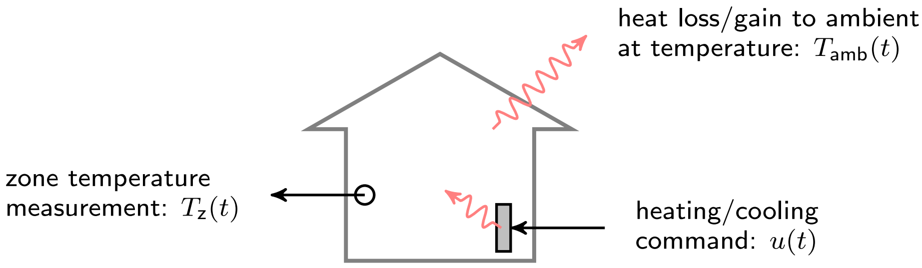
Feedback equations:

$$y(t) = G\,u(t)$$

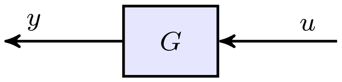
$$u(t) = K(r(t) - y(t))$$

Plant models

A single zone building example



Plant model



Plant models

Differential equation description

m	air mass in zone
c	specific heat capacity for zone volume
k_0	thermal transmittance $\times$ zone area (to ambient)

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State-space representation

$$\begin{aligned}\frac{d}{dt}x(t) &= \frac{-k_0}{mc}\,x(t) + \begin{bmatrix}\frac{k_0}{mc} & \frac{1}{mc}\end{bmatrix}\begin{bmatrix}T_{\mathsf{amb}}(t)\\ u(t)\end{bmatrix}\\ T_{\mathsf{z}}(t) &= x(t)\end{aligned}$$

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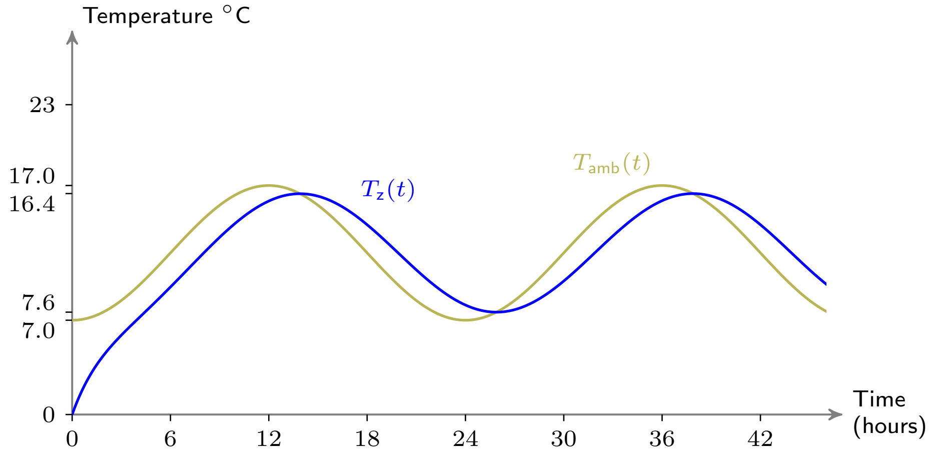
$$T_z(t) = x(t)$$

State: $x(t) = T_z(t)$

$$A = \frac{-k_0}{mc}, \qquad B = \begin{bmatrix}\frac{k_0}{mc} & \frac{1}{mc}\end{bmatrix}, \qquad C = 1, \qquad D = 0.$$

Open-loop ambient response

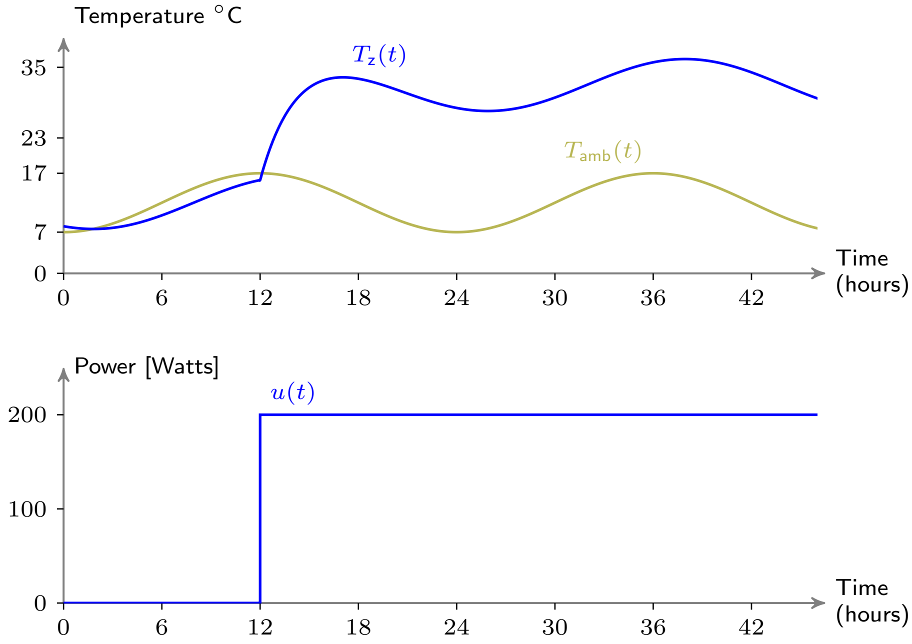


Model parameters

Zone volume	V_z	60 m^3
Air density	ρ	1.225 kg/m^3
Air mass	m	$1000\rho V_z$ g

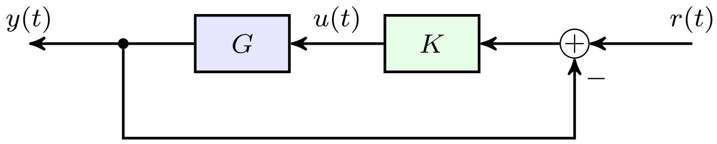
Specific heat capacity	c	1.01 J/g/K
Total thermal transmittance	k_0	10 W/K

Open-loop heat input response



Closed-loop proportional (P) control

Feedback loop



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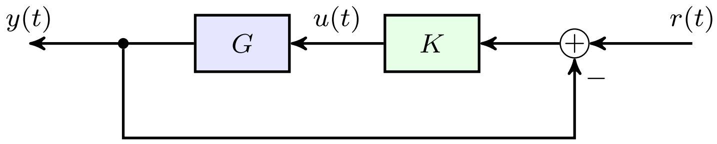


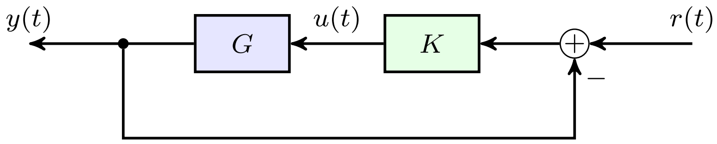
Proportional control law

$$u(t) = K_{\mathsf{P}}\,(r(t) - y(t))$$

$r(t)$ is the zone temperature reference.

Closed-loop proportional (P) control

Feedback loop



Proportional control law

$$u(t) = K_{\mathsf{P}}\,(r(t) - y(t))$$

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Closed-loop system equations

$$\begin{aligned} \frac{d}{dt}x(t) &= -\left(\frac{k_0}{mc} + \frac{K_{\mathsf{P}}}{mc}\right)x(t) + \frac{k_0}{mc}T_{\mathsf{amb}}(t) + \frac{K_{\mathsf{P}}}{mc}r(t) \\ T_{\mathsf{z}}(t) &= x(t) \end{aligned}$$

Closed-loop proportional (P) control

Open-loop (uncontrolled) equilibrium

$$T_{\mathsf{z}} = T_{\mathsf{amb}} + \frac{u}{k_0}.$$

Closed-loop proportional (P) control

Open-loop (uncontrolled) equilibrium

$$T_{\mathsf{z}} = T_{\mathsf{amb}} + \frac{u}{k_0}.$$

Closed-loop (controlled) equilibrium

$$T_{\mathsf{z}} = \frac{k_0}{k_0 + K_{\mathsf{P}}} T_{\mathsf{amb}} + \frac{K_{\mathsf{P}}}{k_0 + K_{\mathsf{P}}} r, \qquad K_{\mathsf{P}} > 0.$$

Closed-loop proportional (P) control

Open-loop (uncontrolled) equilibrium

$$T_{\mathsf{z}} \; = T_{\mathsf{amb}} + \frac{u}{k_0}.$$

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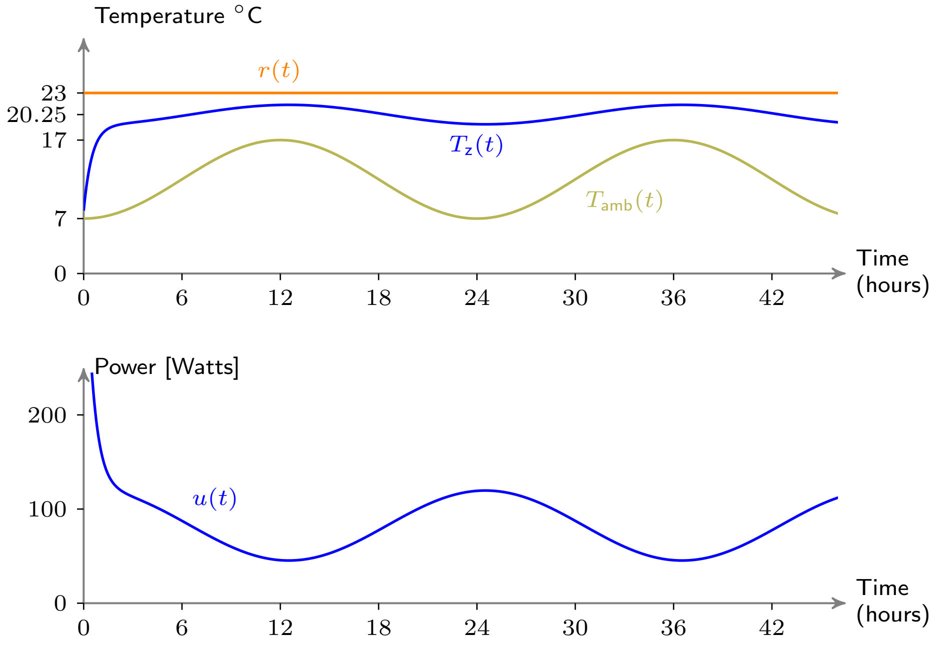
$$T_{\mathsf{z}} \; = \frac{k_0}{k_0 + K_{\mathsf{P}}} T_{\mathsf{amb}} \; + \; \frac{K_{\mathsf{P}}}{k_0 + K_{\mathsf{P}}} r, \qquad K_{\mathsf{P}} > 0.$$

Equilibrium error

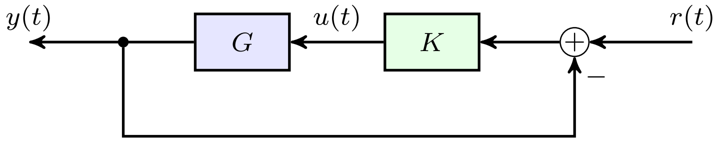
$$r - T_{\mathsf{z}} \; = \; \frac{-k_0}{k_0 + K_{\mathsf{P}}} T_{\mathsf{amb}} \; + \; \frac{k_0}{k_0 + K_{\mathsf{P}}} r, \qquad K_{\mathsf{P}} > 0.$$

Closed-loop proportional control



Closed-loop proportional plus integral (PI) control

Feedback loop



Proportional plus integral control law

Define $e(t) = r(t) - y(t)$

The PI controller is,

$$u(t) \;=\; \underbrace{K_{\mathsf{P}}\, e(t) + K_{\mathsf{I}} \int_0^t e(t)\, dt}_{K\, e(t)}\,.$$

Closed-loop proportional plus integral (PI) control

Dynamic controller

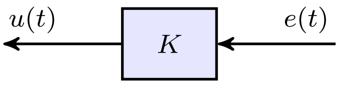
Define a controller state, $\quad x_{\mathsf{K}}(t) = \int_0^t e(t)\, dt.$

Then,

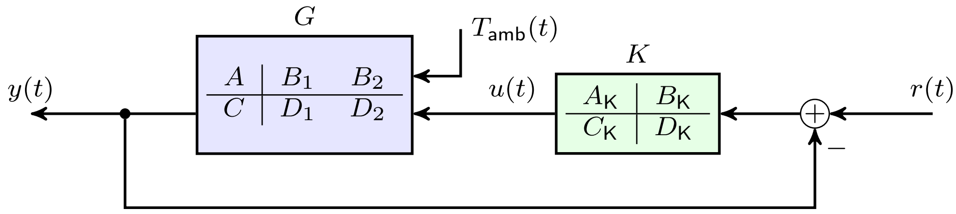
$$\frac{d}{dt} x_{\mathsf{K}}(t) = e(t).$$

Controller state-space representation

$$\begin{aligned} \frac{d}{dt} x_{\mathsf{K}}(t) \;&=\; 0\, x_{\mathsf{K}}(t) \;+\; 1\, e(t) \\ u(t) \;&=\; K_{\mathsf{I}}\, x_{\mathsf{K}}(t) \;+\; K_{\mathsf{P}}\, e(t). \end{aligned}$$



Closed-loop state-space interconnection



State-space representation

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ x_{\mathsf{K}}(t) \end{bmatrix} = \begin{bmatrix} A - B_2 D_{\mathsf{K}} C & B_2 C_{\mathsf{K}} \\ -B_{\mathsf{K}} C & A_{\mathsf{K}} \end{bmatrix} \begin{bmatrix} x(t) \\ x_{\mathsf{K}}(t) \end{bmatrix} + \begin{bmatrix} B_1 & B_2 D_{\mathsf{K}} \\ 0 & B_{\mathsf{K}} \end{bmatrix} \begin{bmatrix} T_{\mathsf{amb}}(t) \\ r(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_{\mathsf{K}}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} T_{\mathsf{amb}}(t) \\ r(t) \end{bmatrix}$$

Assumes $D_1 = 0$ and $D_2 = 0$ for simplicity.

PI control properties

Stability

Check the eigenvalues of the closed-loop "A" matrix:

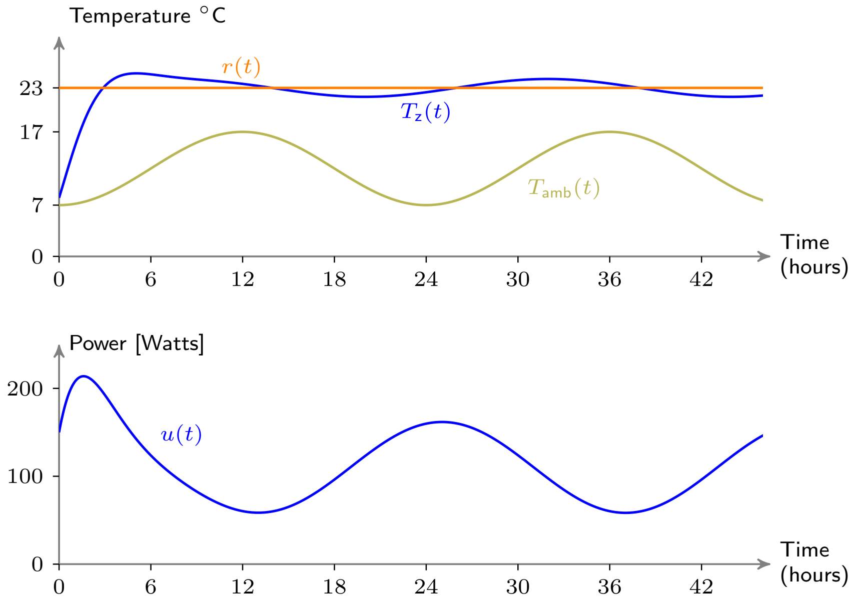
$$\begin{bmatrix} A - B_2 D_{\mathsf{K}} C & B_2 C_{\mathsf{K}} \\ -B_{\mathsf{K}} C & A_{\mathsf{K}} \end{bmatrix} = \begin{bmatrix} -\dfrac{k_0 + K_{\mathsf{P}}}{mc} & \dfrac{K_{\mathsf{I}}}{mc} \\ -1 & 0 \end{bmatrix}$$

Steady-state error

$$\frac{dx_{\mathsf{K}}(t)}{dt} = 0 \quad \Longrightarrow \quad 0 = -1\,x + 0\,x_{\mathsf{K}} + 1\,r.$$

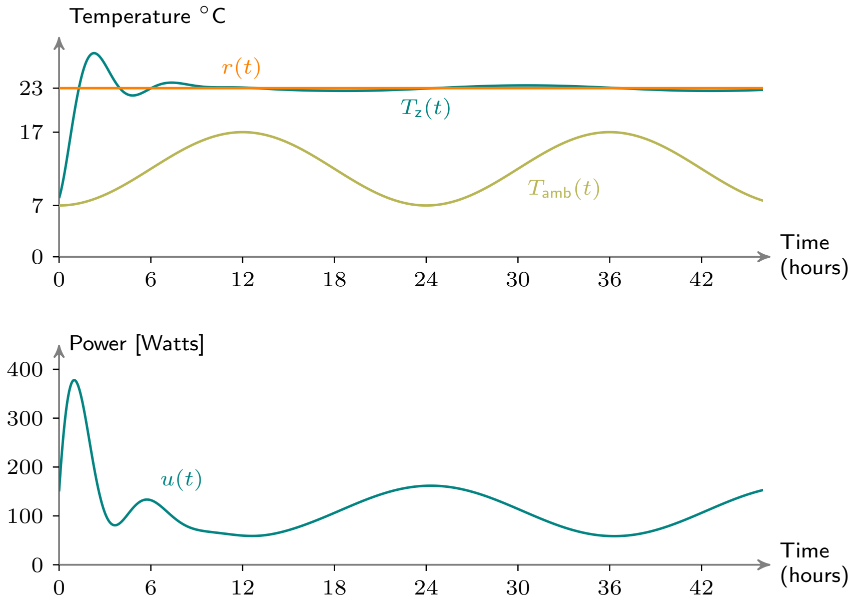
And so $\lim_{t \longrightarrow \infty} e(t) = r(t) - y(t) = 0.$

Closed-loop integral control



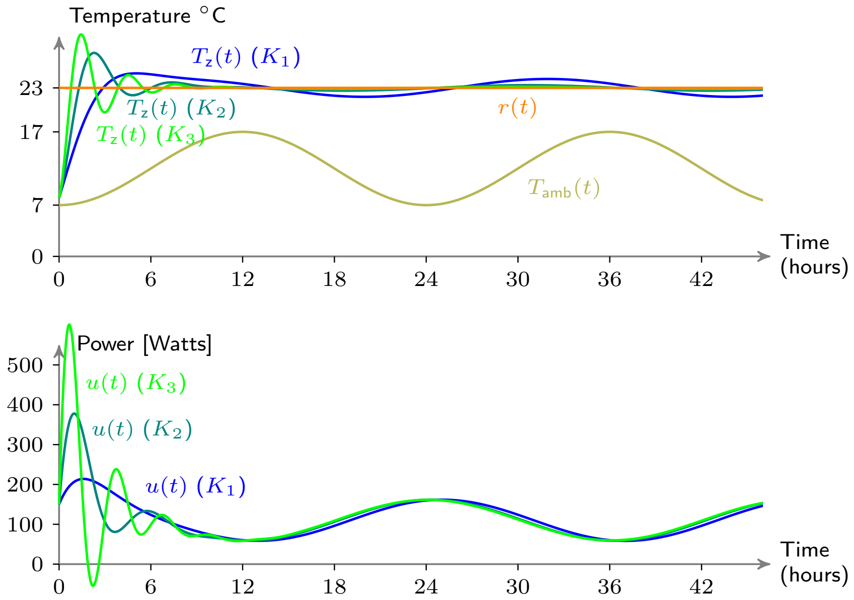
PI control gains: $K_P = 10$, $K_I = 0.0025$

Closed-loop integral control



PI control gains: $K_P = 10$, $K_I = 0.01$

Closed-loop integral control (comparions)



PI control gains: $K_P = 10$, $K_I = 0.0025$ (K_1)
$K_I = 0.01$ (K_2)
$K_I = 0.025$ (K_3)

Feedback limitations

Poor performance for some signals

There are always exogeneous input signal (here $r(t)$ or $T_{amb}(t)$) for which the closed-loop system will perform worse than having no control at all.

These signals are usually faster than the typical response of the closed-loop system.

Closed-loop integral control: disturbance response