Learning strategic behavior in social and economic networks

Nicolò Pagan ¹  Florian Dörfler ¹

¹ Automatic Control Laboratory, ETH Zürich, Switzerland

Complex Networks 2018

7th International Conference on Complex Networks and Their Applications

December, 11-13 2018, Cambridge, UK
Motivation

Observations

- Actors decide with whom they want to interact.
Motivation

Observations

▶ Actors decide with whom they want to interact.

▶ Network positions provide benefits to the actors.

Using Social Networks To Advance Your Career

"It's not what you know, it's who you know" is one of those phrases
Motivation

Observations

▶ Actors decide with whom they want to interact.

▶ Network positions provide benefits to the actors.
Motivation

Observations

- Actors decide with whom they want to interact.
- Network positions provide benefits to the actors.

"It's not what you know, it's who you know" is one of those phrases.
Motivation

Observations

- Actors decide with whom they want to interact.
- Network positions provide benefits to the actors.
Social Network positions’ benefits

Social Influence
The more people we are connected to, the more we can influence them.

Social Support
The more our friends’ friends are our friends, the safer we feel. [Heider (1946); Coleman (1990) (Structural Balance theory)]

Brokerage
The more we are on the path between people, the more we can control. [Burt (1992) (Structural Holes theory)]

Degree Centrality
Clustering Coefficient
Betweenness Centrality
Social Network positions’ benefits

**Social Influence**

The more people we are connected to, the more we can influence them.

[Heider (1946); Coleman (1990)] (Structural Balance theory)

**Brokerage**

The more we are on the path between people, the more we can control.

[Burt (1992)] (Structural Holes theory)
Social Network positions’ benefits

Social Influence

The more people we are connected to, the more we can influence them.

Social Support

The more our friends’ friends are our friends, the safer we feel.

[Heider (1946); Coleman (1990)]
(Structural Balance theory)
Social Network positions’ benefits

**Social Influence**

The more people we are connected to, the more we can influence them.

**Social Support**

The more our friends’ friends are our friends, the safer we feel.

[Heider (1946); Coleman (1990)]

(Structural Balance theory)

**Brokerage**

The more we are on the path between people, the more we can control.

[Burt (1992)]

(Structural Holes theory)
Social Network positions’ benefits

**Social Influence**

The more people we are connected to, the more we can influence them.

**Social Support**

The more our friends’ friends are our friends, the safer we feel.

[Heider (1946); Coleman (1990)]
(Structural Balance theory)

**Brokerage**

The more we are on the path between people, the more we can control.

[Burt (1992)]
(Structural Holes theory)

---

**Degree Centrality**

**Clustering Coefficient**

**Betweenness Centrality**
Social Network Formation Model

Ingredients

- Directed weighted network $G$ with $\mathcal{N} = \{1, \ldots, N\}$ agents.

A typical action of agent $i$ is: $a_i = [a_{i1}, \ldots, a_{i,N-1}, a_{i,N}, a_{i,N+1}, \ldots, a_{iN}] \in A = [0, 1]^{N-1}$

Rational agents: every agent $i$ is endowed with a payoff function $V_i$ and is looking for $a^\star_i \in \text{argmax}_{a_i \in A} V_i(a_i, a_{-i})$
Social Network Formation Model

Ingredients

- **Directed weighted** network $G$ with $\mathcal{N} = \{1, \ldots, N\}$ agents.

- $a_{ij} \in [0, 1]$ quantifies the importance of the friendship among $i$ and $j$ from $i$’s point of view.
Social Network Formation Model

Ingredients

- **Directed weighted** network $G$ with $\mathcal{N} = \{1, \ldots, N\}$ agents.
- $a_{ij} \in [0, 1]$ quantifies the importance of the friendship among $i$ and $j$ from $i$’s point of view.
- A typical action of agent $i$ is:

$$a_i = [a_{i1}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{iN}] \in \mathcal{A} = [0, 1]^{N-1},$$

$$\mathcal{A} = [0, 1]^{N-1}.$$
Social Network Formation Model

Ingredients

- Directed weighted network $G$ with $\mathcal{N} = \{1, \ldots, N\}$ agents.

- $a_{ij} \in [0, 1]$ quantifies the importance of the friendship among $i$ and $j$ from $i$'s point of view.

- A typical action of agent $i$ is:
  
  $$a_i = [a_{i1}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{iN}] \in \mathcal{A} = [0, 1]^{N-1},$$

- Rational agents: every agent $i$ is endowed with a payoff function $V_i$ and is looking for
  
  $$a_i^* \in \arg \max_{a_i \in \mathcal{A}} V_i(a_i, a_{-i}).$$
Payoff function

\[ V_i(a_i, a_{-i}) = t_i(a_i, a_{-i}) + u_i(a_i, a_{-i}) - c_i(a_i), \]
Payoff function

\[ V_i(a_i, a_{-i}) = t_i(a_i, a_{-i}) + u_i(a_i, a_{-i}) - c_i(a_i), \]

- **Social influence** on friends,

\[ t_i(a_i, a_{-i}) = \sum_{j \neq i} a_{ji} \]
Payoff function

\[ V_i(a_i, a_{-i}) = t_i(a_i, a_{-i}) + u_i(a_i, a_{-i}) - c_i(a_i), \]

**Social influence** on friends, on friends of friends, with \( \delta_i \in [0, 1] \):

\[ t_i(a_i, a_{-i}) = \sum_{j \neq i} a_{ji} + \delta_i \sum_{k \neq j \neq i} a_{kja_{ji}} \]

paths of length 2
Payoff function

\[ V_i(a_i, a_{-i}) = t_i(a_i, a_{-i}) + u_i(a_i, a_{-i}) - c_i(a_i), \]

- **Social influence** on friends, on friends of friends, ..., with \( \delta_i \in [0, 1] \):

\[
t_i(a_i, a_{-i}) = \sum_{j \neq i} a_{ji} + \delta_i \sum_{k \neq j \neq i} a_{kj}a_{ji} + \underbrace{\delta_i^2 \sum_{l \neq k \neq j \neq i} a_{lk}a_{kj}a_{ji}}_{\text{paths of length 3}}.
\]

[Jackson and Wolinsky (1996)]
Payoff function

\[ V_i(a_i, a_{\sim i}) = t_i(a_i, a_{\sim i}) + u_i(a_i, a_{\sim i}) - c_i(a_i), \]

- **Social influence** on friends, on friends of friends, ... with \( \delta_i \in [0, 1] \):

\[
t_i(a_i, a_{\sim i}) = \sum_{j \neq i} a_{ji} + \delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji} + \]
\[
\underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji},}_{\text{paths of length 3}}
\]

[Jackson and Wolinsky (1996)]

- **Clustering coefficient**: number of closed triads:

\[
u_i(a_i, a_{\sim i}) = \sum_{j \neq i} a_{ij} \left( \sum_{k \neq i, j} a_{ik} a_{kj} \right),
\]

[Burger and Buskens (2009)]
Payoff function

\[ V_i(a_i, a_{-i}) = t_i(a_i, a_{-i}) + u_i(a_i, a_{-i}) - c_i(a_i), \]

- **Social influence** on friends, on friends of friends, \ldots with \( \delta_i \in [0, 1] \):
  
  \[
  t_i(a_i, a_{-i}) = \sum_{j \neq i} a_{ji} + \delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji} + \underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji}}_{\text{paths of length 3}}
  \]

  [Jackson and Wolinsky (1996)]

- **Clustering coefficient**: number of closed triads:
  
  \[
  u_i(a_i, a_{-i}) = \sum_{j \neq i} a_{ij} \left( \sum_{k \neq i, j} a_{ik} a_{kj} \right),
  \]

  [Burger and Buskens (2009)]

- **Cost of maintaining ties**: \( c_i(a_i) = \sum_{j \neq i} a_{ij} \).
Payoff function

\[ V_i(a_i, a_{-i} | P_i) = \alpha_i t_i(a_i, a_{-i}) + \beta_i u_i(a_i, a_{-i}) - \gamma_i c_i(a_i), \]
\[ \alpha_i \geq 0, \beta_i \in \mathbb{R}, \gamma_i > 0 \]
\[ P_i = \{ \alpha_i, \beta_i, \gamma_i, \delta_i \} \]

- **Social influence** on friends, on friends of friends, … with \( \delta_i \in [0, 1] \):

\[
t_i(a_i, a_{-i}) = \sum_{j \neq i} a_{ji} + \delta_i \sum_{k \neq j \neq i} a_{kja_{ji}} + \left( \sum_{l \neq k \neq j \neq i} a_{lk}a_{kj}a_{ji} \right),
\]

paths of length 2

paths of length 3

[Jackson and Wolinsky (1996)]

- **Clustering coefficient**: number of closed triads:

\[
u_i(a_i, a_{-i}) = \sum_{j \neq i} a_{ij} \left( \sum_{k \neq i, j} a_{ik}a_{kj} \right),
\]

[Burger and Buskens (2009)]

- **Cost** of maintaining ties:

\[ c_i(a_i) = \sum_{j \neq i} a_{ij}. \]
Nash Equilibrium

Definition (Nash equilibrium).
The network $G^*$ is a NE if for all agents $i$,

$$V_i(a_i, a_{-i}^*|P_i) \leq V_i(a_i^*, a_{-i}^*|P_i), \forall a_i \in \mathcal{A}.$$
Nash Equilibrium

**Definition (Nash equilibrium).**
The network $G^*$ is a NE if for all agents $i$,

$$V_i(a_i, a_{-i}^*|P_i) \leq V_i(a_i^*, a_{-i}^*|P_i), \forall a_i \in \mathcal{A}.$$
Nash Equilibrium

Definition (Nash equilibrium).
The network $G^*$ is a NE if for all agents $i$,

$$V_i(a_i, a_{-i}^*|P_i) \leq V_i(a_i^*, a_{-i}^*|P_i), \forall a_i \in A.$$
Nash Equilibrium

Definition (Nash equilibrium).
The network $G^*$ is a NE if for all agents $i$,\[ V_i(a_i, a_{-i}^*|P_i) \leq V_i({a_i}^*, a_{-i}^*|P_i), \forall a_i \in \mathcal{A}. \]
Nash Equilibrium

Definition (Nash equilibrium).
The network $G^*$ is a NE if for all agents $i$,

$$V_i(a_i, a_{-i}^*|P_i) \leq V_i(a_i^*, a_{-i}^*|P_i), \forall a_i \in \mathcal{A}.$$
Homogeneous agents

Assumption

Individual preferences $P_i = P$, for all agents $i$. 
Homogeneous agents

Assumption

Individual preferences $P_i = P$, for all agents $i$.

Result

Analytic characterization of individual behavior, based on some specific network motifs.
Homogeneous agents

Assumption

Individual preferences $P_i = P$, for all agents $i$.

Result

Analytic characterization of individual behavior, based on some specific network motifs.

Heterogeneous Agents

Let the unknown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$. Define:

$$\Phi_i(a_i, P_i) = V_i(a_i, a^*_{-i}|P_i) - V_i(a^*_i, a^*_{-i}|P_i).$$
Heterogeneous Agents

Let the unknown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$. Define:

$$
\Phi_i(a_i, P_i) = V_i(a_i, a^*_i|P_i) - V_i(a^*_i, a^*_i|P_i).
$$

Note: $\Phi_i(a_i, P_i) > 0$ if $P_i$ violates the Nash equilibrium condition.
Heterogeneous Agents

Let the unknown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$. Define:

$$\Phi_i(a_i, P_i) = V_i(a_i, a_i^*|P_i) - V_i(a_i^*, a_i^*|P_i).$$

Note: $\Phi_i(a_i, P_i) > 0$ if $P_i$ violates the Nash equilibrium condition.

Integrate over the all action space, taking into account only the violations.

$$\Psi_i(P_i) = \int_{\mathcal{A}} \max \{0, \Phi_i(a_i, P_i)\} \, da_i.$$
Heterogeneous Agents

Let the unknown heterogeneous individual preferences set \( P_i = \{ \alpha_i, \beta_i, \gamma_i, \delta_i \} \).

Define:

\[
\Phi_i(a_i, P_i) = V_i(a_i, a^*_i | P_i) - V_i(a^*_i, a^*_{-i} | P_i).
\]

Note: \( \Phi_i(a_i, P_i) > 0 \) if \( P_i \) violates the Nash equilibrium condition.

Integrate over the all action space, taking into account only the violations.

\[
\Psi_i(P_i) = \int_{\mathcal{A}} \max \{ 0, \Phi_i(a_i, P_i) \} \, da_i.
\]

Minimize the violations

\[
\hat{P}_i \in \arg \min_{P_i \in \mathcal{P}} \{ \Psi_i(P_i) \}.
\]
Heterogeneous Agents

Let the unknown heterogeneous individual preferences set \( P_i = \{ \alpha_i, \beta_i, \gamma_i, \delta_i \} \).

Define:

\[
\Phi_i(a_i, P_i) = V_i (a_i, a^*_i | P_i) - V_i (a^*_i, a^*_i | P_i). 
\]

Note: \( \Phi_i(a_i, P_i) > 0 \) if \( P_i \) violates the Nash equilibrium condition.

Integrate over the all action space, taking into account only the violations.

\[
\Psi_i(P_i) = \int_{\mathcal{A}} \max \{0, \Phi_i(a_i, P_i)\} \, da_i.
\]

Minimize the violations

\[
\hat{P}_i \in \arg\min_{P_i \in \mathcal{P}} \{ \Psi_i(P_i) \} \iff \hat{P}_i \in \arg\max_{P_i \in \mathcal{P}} \{ L_i(P_i) \},
\]

where \( L_i(P_i) = -\Psi_i(P_i) \) is the likelihood function.
Behavioral analysis on real-world networks

Figure: Australian bank data set, Pattison et al. (2000).
Behavioral analysis on real-world networks

Low hierarchical positions occupy the periphery of the network.
Behavioral analysis on real-world networks

- Low hierarchical positions occupy the periphery of the network.
- Presence of star-like motifs embedded in the network (e.g. Branch Manager).
Behavioral analysis on real-world networks

- Competitive behavior and reciprocity of high-ranking positions.
- Low-ranking positions inclined towards social support.

Figure: Maximum likelihood estimate of strategic behavior $\hat{P}_i$.

$$V_i(a_i, a_{-i}|P_i) = \alpha_i t_i(a_i, a_{-i}) + \beta_i u_i(a_i, a_{-i}) - \gamma_i c_i(a_i), \quad \alpha_i \geq 0, \; \beta_i \in \mathbb{R}, \; \gamma_i > 0.$$
Summary

\[ V_i(a_i, a_{-i} | P_i) = \alpha_i t_i(a_i, a_{-i}) + \beta_i u_i(a_i, a_{-i}) - \gamma_i c_i(a_i), \quad \alpha_i \geq 0, \beta_i \in \mathbb{R}, \gamma_i > 0. \]


