Core-Selecting Mechanisms in Electricity Markets

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Electricity markets for stability

- Transformation to deregulated competitive markets
- *Stability*: Supply and demand balance at every instance
Electricity markets for stability

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Electricity markets for stability

- Transformation to deregulated competitive markets
- *Stability*: Supply and demand balance at every instance
- Role of electricity markets in ensuring this stability

Intermittent & Uncertain
Control reserves market

- Different supplies depending on **speed and direction (sign)**
- Involves a probabilistic dimensioning criteria
Control reserves market

- Different supplies depending on **speed and direction (sign)**
- Involves a probabilistic dimensioning criteria
Wholesale electricity markets

- Different supplies depending on **bus/node**
- Considers the physics behind the transmission network
Market design criteria

Efficiency: Immunity to strategic manipulations
Market design criteria

*Efficiency:* Immunity to strategic manipulations

How can we **eliminate strategic manipulations** to achieve a stable and an efficient grid?
Outline

Electricity market framework

Characterizing coalition-proofness using the core

Competitive equilibrium using core-selecting mechanisms

Design considerations for core-selecting mechanisms

Numerical results
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Numerical results
Electricity market framework

- Wholesale electricity markets, control reserve markets, and many others; generalization of reverse auctions

Bid profile $\mathcal{B} = \{b_l\}_{l \in L}$

Central Operator (CO)

Bid $b_1$

Bid $b_{|L|}$

Bidder 1

True cost $c_1$

Bidder $|L|$

True cost $c_{|L|}$
Electricity market framework

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Bid profile \( B = \{ b_l \}_{l \in L} \)

Central Operator (CO)

Bidder 1

\( c_1 \)

Bid profile

Payment rule \( p_1 \)

Allocation rule \( x_1^* \)

Bid \( b_1 \)

... 

Bidder \( |L| \)

True cost \( c_{|L|} \)

Payment rule \( p_{|L|} \)

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Bid profile $\mathcal{B} = \{b_l\}_{l \in L}$

Central Operator (CO)

Utility of bidders = Payment − True cost

Utility of CO = − Total payment
Allocation rule as an optimization problem

- Private true cost of bidder $l$

$$c_l : \mathbb{X}_l \rightarrow \mathbb{R}_+ \text{ s.t. } 0 \in \mathbb{X}_l \subset \mathbb{R}_+ \text{ and } c_l(0) = 0$$

- Reported cost of bidder $l$

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- The central operator solves for the economic dispatch
  \[ J(\mathcal{B}) = \min_{x \in \hat{\mathbb{X}}, y \in \mathbb{R}_p} \sum_{l \in \mathcal{L}} b_l(x_l) + d(x, y) \]
  \[ \text{s.t. } g(x, y) \leq 0 \]

- Suppl. cost $d : \mathbb{R}_+^{t|\mathcal{L}|} \times \mathbb{R}_p \rightarrow \mathbb{R}$—e.g., second-stage market

- Additional variables $y \in \mathbb{R}_p$

- Constraints $g : \mathbb{R}_+^{t|\mathcal{L}|} \times \mathbb{R}_p \rightarrow \mathbb{R}_q$—e.g., security constraints
Updating the framework with the allocation rule

Central Operator

\[ J(\mathcal{B}) = \min_{x \in \hat{X}, y} \sum_{l \in L} b_l(x_l) + d(x, y) \text{ s.t. } g(x, y) \leq 0 \]  

\( (CO) \)

The allocation rule \( x^*(\mathcal{B}) \) is the minimizer

\[ \text{Bidder } l \]

\[ b_l, \quad p_l(\mathcal{B}) \in \mathbb{R}, \quad x_l^*(\mathcal{B}) \in \hat{X}_l \]
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Bidder’s utility:  
\[ u_l(\mathcal{B}) = p_l(\mathcal{B}) - c_l(x_l^*(\mathcal{B})) \]
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\[ (CO) \]

▶ Bidder’s utility: \( u_l(\mathcal{B}) = p_l(\mathcal{B}) - c_l(x_l^*(\mathcal{B})) \)

▶ CO’s utility: \( u_{CO}(\mathcal{B}) = - \sum_{l \in L} p_l(\mathcal{B}) - d(x^*(\mathcal{B}), y^*(\mathcal{B})) \)
Desirable properties for the payment rules

- *Individually rational*: Nonnegative utilities for bidders
- *Efficient*: Sum of all utilities is maximized
- *Incentive-compatible*: Truthfulness is the dominant strategy
Desirable properties for the payment rules

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**Pay-as-bid mechanism**:

\[ p_l(B) = b_l(x^*_l(B)) \]

Not incentive-compatible, not efficient
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- **Locational marginal pricing (LMP) mechanism:**
  \[ p_l(B) = \lambda^*_l(B)x^*_l(B) \]
  If each bidder is a *price-taker*, then it is incentive-compatible
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[Wolfram 1997], [Joskow et al. 2001]
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  - This economic rationale relies on **strong duality**
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The Vickrey-Clarke-Groves (VCG) mechanism
[Vickrey 1961], [Clarke 1971], [Groves 1973]

- Optimal value of (CO) with $x_l = 0$
  \[ J(B_{-l}) \geq J(B) \]

- VCG payment is the externality
  \[ p_l(B) = J(B_{-l}) - \left( J(B) - b_l(x_l^*(B)) \right) \]
  cost of others in the absence of $l$  
  cost of others when $l$ is present
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Theorem 1

Given (CO), the VCG mechanism is

a) Incentive-compatible
b) Efficient
c) Individually rational
The Vickrey-Clarke-Groves (VCG) mechanism

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Theorem 1

*Given (CO), the VCG mechanism is*

  a) *Incentive-compatible*
  b) *Efficient*
  c) *Individually rational*

- Recently been proposed for a broad class of electricity markets

  [Samadi et al. 2012], [Xu and Low 2017], [Sessa et al. 2017]
The lovely but lonely VCG mechanism [Ausubel and Milgrom 2006]

\[ c_3(x_3) = 0.1x_3^2 + 5x_3 \]

\[ c_1(x_1) = 0.1x_1^2 + 12x_2 \]

\[ c_2(x_2) = 0.1x_2^2 + 12x_2 \]

Table: VCG outcomes for the model (CHF) (p: payment, u: utility)

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The lovely but lonely VCG mechanism \cite{Ausubel and Milgrom 2006}

\[ c_1(x_1) = 0.1x_1^2 + 12x_2 \]

\[ c_2(x_2) = 0.1x_2^2 + 12x_2 \]

\[ c_3(x_3) = 0.1x_3^2 + 5x_3 \]

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Truthful bidding is the dominant strategy
The lovely but lonely VCG mechanism [Ausubel and Milgrom 2006]

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The lovely but lonely VCG mechanism  

\[ C_3,2 = 10 \text{ MW} \]
\[ C_3,1 = 10 \text{ MW} \]
\[ C_2,4 = 10 \text{ MW} \]
\[ C_1,4 = 10 \text{ MW} \]
\[ \theta_2 \]
\[ \theta_1 \]
\[ \theta_4 \]
\[ \theta_3 \]
\[ b_1(x_1) = 0 \]
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The lovely but lonely VCG mechanism [Ausubel and Milgrom 2006]

Another important property:

*Coalition-proofness*
- Joint deviation is not profitable for losing bidders
- Bidding with multiple identities is not profitable for any bidder
Which mechanisms attain the \textit{coalition-proofness} property?
Outline

Electricity market framework

Characterizing coalition-proofness using the core

Competitive equilibrium using core-selecting mechanisms

Design considerations for core-selecting mechanisms

Numerical results
Bringing in the core from coalitional game theory

- Bidder’s *revealed* utility: 
  \[ \bar{u}_l(B) = p_l(B) - b_l(x^*_l(B)) \]

- Central operator’s *revealed* utility:
  \[ \bar{u}_{CO}(B) = - \sum_{l \in L} p_l(B) - d(x^*(B), y^*(B)) \]
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- Central operator’s *revealed* utility:

\[
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\]

- Objective value under the profile \( B_S = \{b_l\}_{l \in S}, S \subseteq L \)

\[
J(B_S) = \min_{x \in \hat{X}, y} \sum_{l \in S} b_l(x_l) + d(x, y) \\
\text{s.t. } g(x, y) \leq 0, \ x_{-S} = 0
\]
Bringing in the core from coalitional game theory

- Bidder’s revealed utility: $\bar{u}_l(B) = p_l(B) - b_l(x_l^*(B))$

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$$J(B_S) = \min_{x \in \hat{x}, y \in S} \sum_{l \in S} b_l(x_l) + d(x, y)$$

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Bringing in the core from coalitional game theory

- Bidder’s revealed utility: \( \tilde{u}_l(B) = p_l(B) - b_l(x^*_l(B)) \)
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- The core: set of revealed utilities that cannot be improved upon by forming coalitions

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J(B_S) = \min_{x \in \hat{x}, y \in S} \sum_{l \in S} b_l(x_l) + d(x, y) \\
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- The core: set of revealed utilities that cannot be improved upon by forming coalitions

\[
Core(B) = \left\{ \tilde{u} \in \mathbb{R} \times \mathbb{R}_{+}^{L} \mid \tilde{u}_{CO} + \sum_{l \in L} \tilde{u}_l = -J(B), \right. \\
\left. \tilde{u}_{CO} + \sum_{l \in S} \tilde{u}_l \geq -J(B_S), \forall S \subset L \right\}
\]

\[
J(B_S) = \min_{x \in \mathbb{R}^L, y \in \mathbb{R}^S} \sum b_l(x_l) + d(x, y) \\
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  \]

  \[
  J(B_S) = \min_{x \in X, y \in S} \sum_{l \in S} b_l(x_l) + d(x, y)
  \]

  subject to: \( g(x, y) \leq 0, \quad x_{-S} = 0 \)
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  \[
  Core(B) = \left\{ \bar{u} \in \mathbb{R} \times \mathbb{R}_+^{\left| L \right|} \mid \bar{u}_{CO} + \sum_{l \in L} \bar{u}_l = -J(B), \underbrace{\bar{u}_{CO} + \sum_{l \in S} \bar{u}_l \geq -J(B_S), \forall S \subset L}_{\text{efficient}, \text{individ. rational}} \right\}
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Core(\mathcal{B}) = \left\{ \tilde{u} \in \mathbb{R} \times \mathbb{R}^{\left|L\right|} \mid \tilde{u}_{CO} + \sum_{l \in L} \tilde{u}_l = -J(\mathcal{B}), \begin{array}{c}
\text{individ. rational} \\
\text{efficient}
\end{array} \right. \}
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\text{ s.t. } g(x, y) \leq 0, \ x_{-S} = 0
\]
Characterization of coalition-proof mechanisms

- Core-selecting payment rule

\[ p_l(B) = b_l(x_l^*(B)) + \bar{u}_l(B), \forall l, \text{ where } \bar{u} \in Core(B) \]

- Equivalently, revealed utilities lie in the core

\[ \bar{u}_{PAB} \in Core(B) \]
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**Theorem 2**

Core-selecting mechanisms \( \iff \) Coalition-proof mechanisms
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**Theorem 2**

Core-selecting mechanisms \iff Coalition-proof mechanisms

- Pay-as-bid is core-selecting since

\[ \bar{u}^{\text{PAB}}_l(B) = 0, \quad \forall l \in L, \quad \bar{u}^{\text{PAB}}_C(B) = -J(B) \]
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\[ \bar{u}_l^{PAB}(B) = 0, \forall l \in L, \quad \bar{u}_C^{PAB}(B) = -J(B) \implies \bar{u}_P^{PAB} \in Core(B) \]
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*Core-selecting mechanisms \iff Coalition-proof mechanisms*

- Pay-as-bid is core-selecting since

\[ \bar{u}^{\text{PAB}}_l(B) = 0, \forall l \in L, \quad \bar{u}^{\text{PAB}}_\text{CO}(B) = -J(B) \implies \bar{u}^{\text{PAB}} \in \text{Core}(B) \]

- Core-selecting payments are upper bounded by the VCG payments

\[ \bar{u}^{\text{VCG}}_l(B) = J(B_{-l}) - J(B) = \max \{ \bar{u}_l | \bar{u} \in \text{Core}(B) \} \]
Characterization of coalition-proof mechanisms

- Core-selecting payment rule
  \[ p_l(B) = b_l(x_l^*(B)) + \bar{u}_l(B), \quad \forall l, \text{ where } \bar{u} \in Core(B) \]

- Equivalently, revealed utilities lie in the core

Theorem 2

Core-selecting mechanisms \iff Coalition-proof mechanisms

- Pay-as-bid is core-selecting since
  \[ \bar{u}_{l}^{PAB}(B) = 0, \forall l \in L, \quad \bar{u}_{CO}^{PAB}(B) = -J(B) \implies \bar{u}^{PAB} \in Core(B) \]

- Core-selecting payments are upper bounded by the VCG payments
  \[ \bar{u}_{l}^{VCG}(B) = J(B_{-l}) - J(B) = \max \{ \bar{u}_l | \bar{u} \in Core(B) \} \]
For which markets can we ensure that the VCG mechanism is core-selecting?
Polymatroid + Convex bids $\implies$ Core-selecting VCG

- Let $[t] = \{1, \ldots, t\}$ be the set of types of supplies
- *Contra-Polymatroid*: a polytope $\mathcal{P}$ defined by a nondecreasing supermodular function $f : 2^{[t]} \to \mathbb{R}_+$

$$\mathcal{P} = \left\{ x \mid \bar{X}_i \geq x_i \geq 0, \forall i \in L, \text{ and } \sum_{\tau \in T} \sum_{i \in L} x_{i,\tau} \geq f(T), \forall T \subseteq [t] \right\}$$
Polymatroid + Convex bids $\implies$ Core-selecting VCG

- Let $[t] = \{1, \ldots, t\}$ be the set of types of supplies
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production capacity constraints
Polymatroid + Convex bids $\implies$ Core-selecting VCG

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\[
\mathcal{P} = \left\{ x \mid \begin{array}{c}
\tilde{X}_i \geq x_i \geq 0, \forall i \in L, \\
\text{production capacity constraints}
\end{array}
\right\}
\]

\[
\text{and } \sum_{\tau \in T} \sum_{i \in L} x_{i,\tau} \geq f(T), \forall T \subseteq [t]
\]

\[
\text{procurement constraints for different subsets of types}
\]
Polymatroid + Convex bids $\implies$ Core-selecting VCG

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- **production capacity constraints**
- **procurement constraints for different subsets of types**

**Theorem 3**

*Bids are convex (or marginally nondecreasing) and the constraint set is a contra-polymatroid*

$\implies$ *The VCG mechanism is core-selecting*
Optimal power flow problems, and control reserves markets do not satisfy this restrictive condition...

The VCG mechanism is in general not core-selecting!
Optimal power flow problems, and control reserves markets do not satisfy this **restrictive** condition...

The VCG mechanism is in general not core-selecting!

- The maximal point is not in the core
Is the LMP mechanism core-selecting?
Outline

Electricity market framework

Characterizing coalition-proofness using the core

Competitive equilibrium using core-selecting mechanisms

Design considerations for core-selecting mechanisms

Numerical results
Bringing in the competitive equilibrium

▶ An allocation $x^* \in \mathbb{R}^{t|L|}$ and a set of price functions $\{\psi_l\}_{l \in L}$, where $\psi_l : \mathbb{R}_+^t \to \mathbb{R}$ and $\psi_l(0) = 0$, constitute a competitive equilibrium (CE) if and only if

(i) $x_l^* \in \arg \max_{x_l \in X_l} \psi_l(x_l) - c_l(x_l), \ \forall l \in L,$

(ii) $x^* \in \arg \min_{x \in \mathbb{R}_+^{t|L|}} \left\{ \min_{y : h(x,y)=0, g(x,y) \leq 0} \sum_{l \in L} \psi_l(x_l) + d(x,y) \right\}.$

▶ Consistency conditions: Allocations are optimal at the prices
▶ Extends the traditional definition with linear prices
Bringing in the competitive equilibrium

- An allocation \( x^* \in \mathbb{R}^{t|L|}_+ \) and a set of price functions \( \{\psi_l\}_{l\in L} \), where \( \psi_l : \mathbb{R}^t_+ \to \mathbb{R} \) and \( \psi_l(0) = 0 \), constitute a competitive equilibrium (CE) if and only if

\[
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(ii) \quad & x^* \in \arg\min_{x \in \mathbb{R}^{t|L|}_+} \left\{ \min_{y: h(x,y) = 0} \sum_{l \in L} \psi_l(x_l) + d(x,y) \right\}.
\end{align*}
\]

- Consistency conditions: Allocations are optimal at the prices
- Extends the traditional definition with linear prices

Lemma 1

*Competitive equilibrium exists only for \( x^*(C) \), that is, the optimal allocation of the market under true costs*
Competitive equilibrium using mechanism design

\[
x_l^*(C) \in \arg \max_{x_l \in X_l} \psi_l(x_l) - c_l(x_l), \forall l \in L
\]

\[
x^*(C) \in \arg \min_{x \in \mathbb{R}^{t|L|}} \left\{ \min_{y: h(x,y)=0} \sum_{l \in L} \psi_l(x_l) + d(x, y)\right\}
\]
Competitive equilibrium using mechanism design

- A mechanism ensures the existence of a CE, if for any \( C \), \( \exists \{ \psi_l \}_{l \in L} \) s.t. \( \psi_l(x^*_l(C)) = p_l(C) \), and \( (\{ \psi_l \}_{l \in L}, x^*(C)) \) constitutes a CE

\[
x^*_l(C) \in \arg \max_{x_l \in X_l} \psi_l(x_l) - c_l(x_l), \quad \forall l \in L
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- Under strong duality, the LMP mechanism satisfies this with

\[
\psi_l(x) = \lambda^*(C)x, \quad \forall x \in \mathbb{R}^t_+, \quad \forall l \in L,
\]

where \( \lambda^*(C) \in \mathbb{R}^t \) concatenates the Lagrange multipliers

- Remark: Sets a linear price for each supply type
Competitive equilibrium using mechanism design

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  where $\lambda^*(C) \in \mathbb{R}^t$ concatenates the Lagrange multipliers.

- **Remark**: Sets a linear price for each supply type.

- First condition implies *price-taker incentive-compatibility*

\[
\begin{align*}
x_l^*(C) &\in \arg \max_{x_l \in \mathcal{X}_l} \lambda^*(C)x_l - c_l(x_l), \quad \forall l \in L \\
x^*(C) &\in \arg \min_{x \in \mathbb{R}_+^{t|L|}} \left\{ \min_{y: h(x,y) = 0, g(x,y) \leq 0} \sum_{l \in L} \lambda^*(C)x_l + d(x,y) \right\}
\end{align*}
\]
Equivalence of the core and the competitive equilibrium

Theorem 4

A mechanism is core-selecting if and only if it ensures the existence of a CE

- The proof involves price functions of the form

\[ \psi_l(x) = \begin{cases} 
0 & x = 0 \\
cl(x) + \bar{u}_l & x \in X_l \setminus \{0\} \\
\infty & \text{otherwise}
\end{cases} \]

- Extends the line of works from [Shapley and Shubik 1971], [Bikhchandani and Ostroy 2002]
Equivalence of the core and the competitive equilibrium

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- Two implications:
  - LMP is core-selecting
  - VCG payments upper bound LMP payments
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Two implications:

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Core-selecting mechanisms exist even under nonconvex bids and nonconvex constraint sets
Outline

Electricity market framework

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Design considerations for core-selecting mechanisms

Numerical results
Core-selecting is in general **not incentive-compatible** and there are **many points** to choose from the core...

Can core-selecting mechanisms **approximate incentive-compatibility** without the price-taking assumption?
Approximating incentive-compatibility using core-selecting

- We quantify the violation of incentive-compatibility under any core-selecting mechanism.

Lemma 2

The maximum gain of bidder $l$ by a unilateral deviation from its true cost is tightly upperbounded by

$$\bar{u}_l^{\text{VCG}}(C_l, B_{-l}) - \bar{u}_l(C_l, B_{-l})$$
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**Lemma 2**

*The maximum gain of bidder \( l \) by a unilateral deviation from its true cost is tightly upperbounded by*

\[
\bar{u}^\text{VCG}_l(C_l, B_{-l}) - \bar{u}_l(C_l, B_{-l})
\]

**Idea:** The closer you get to the VCG payments, the better you approximate incentive-compatibility.
Maximum payment core-selecting mechanism

- Maximum payment core-selecting (MPCS) mechanism:

\[ \bar{u}^{\text{MPCS}}(B) = \arg \max_{u \in \text{Core}(B)} \sum_{l \in L} u_l - \epsilon \left\| u_l - \bar{u}_l^{\text{VCG}}(B) \right\|^2 \]

**Theorem 5**

The MPCS mechanism minimizes the sum of maximum gains from unilateral deviations
Maximum payment core-selecting mechanism

- Maximum payment core-selecting (MPCS) mechanism:

\[
\tilde{u}^{\text{MPCS}}(\mathcal{B}) = \arg\max_{u \in \text{Core}(\mathcal{B})} \sum_{l \in L} u_l - \epsilon \left\| u_l - \tilde{u}^{\text{VCG}}_l(\mathcal{B}) \right\|_2^2
\]

**Theorem 5**

The MPCS mechanism minimizes the sum of maximum gains from unilateral deviations

- Problem size is **exponential in the number of bidders**!
  - Characterizing the core requires solutions to the market under \(2^{|L|}\) subsets of bidders
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**Theorem 5**

The MPCS mechanism minimizes the sum of maximum gains from unilateral deviations

- Problem size is **exponential in the number of bidders!**
  - Characterizing the core requires solutions to the market under \(2^{|L|}\) subsets of bidders
  - Can be tackled via **iterative constraint generation**
    - [Dantzig et al. 1954], [Hallefjord et al. 1995]
Comparison of revealed utilities under different mechanisms

\[ \bar{u}_1, \bar{u}_2 \]

Core

\[ (0, 0) \]

\[ \bar{u}_1^{\text{LMP}}, \bar{u}_1^{\text{MPCS}}, \bar{u}_1^{\text{VCG}} \]

\[ \bar{u}_2^{\text{LMP}}, \bar{u}_2^{\text{MPCS}}, \bar{u}_2^{\text{VCG}} \]

\[ (\bar{u}_1^{\text{VCG}}, \bar{u}_2^{\text{VCG}}) \]
Comparison of revealed utilities under different mechanisms

\[ \begin{align*}
\bar{u}_1 &= \bar{u}_{VCG}^1, \\
\bar{u}_2 &= \bar{u}_{VCG}^2
\end{align*} \]

The MPCS mechanism:

+ Does not rely on price-taker assumption
+ Equivalent to the VCG if VCG is core-selecting
- Payments are nonlinear and bidder-dependent

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+ Does not rely on price-taker assumption
+ Equivalent to the VCG if VCG is core-selecting
- Payments are nonlinear and bidder-dependent
We extend our model to exchanges (and two-sided markets)

Can we quantify the budget-balance of the MPCS mechanism?
Budget-balance in exchanges

- Exchange relaxes the domains of the functions to $\mathbb{R}^t$

  $$c_l : \mathbb{X}_l \rightarrow \mathbb{R} \text{ s.t. } 0 \in \mathbb{X}_1 \subset \mathbb{R}^t \text{ and } c_l(0) = 0$$

  $$b_l : \hat{\mathbb{X}}_l \rightarrow \mathbb{R} \text{ s.t. } 0 \in \hat{\mathbb{X}}_1 \subset \mathbb{R}^t \text{ and } b_l(0) = 0$$

- All the results hold in exchanges (e.g., coalition-proofness, CE)
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- Another important property:
  - Budget-balance: $u_{CO} \geq 0$ (Central operator’s utility)
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- The LMP mechanism is budget-balanced

- The VCG mechanism is not always budget-balanced

[Myerson and Satterthwhite 1983], [Krishna and Perry 1998]
Budget-balance in exchanges

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- The VCG mechanism is *not* always budget-balanced

  [Myerson and Satterthwhite 1983], [Krishna and Perry 1998]

**Theorem 6**

*Any core-selecting mechanism is budget-balanced*
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Numerical results
AC-OPF problem with a duality gap

- A simulation based on the 5-bus model in [Bukhsh et al. 2013]

![Diagram of a 5-bus model with bus connections and admittances.]
AC-OPF problem with a duality gap

- A simulation based on the 5-bus model in [Bukhsh et al. 2013]
- SDP relaxation is not tight $\Rightarrow$ Nonzero duality gap [Lavaei and Low 2012]
- Can be solved via second level of moment hierarchy
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<table>
<thead>
<tr>
<th>Table: Generator data and market outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gen.</td>
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**Table:** Generator data and market outcome

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<td>$0.1x_1^2 + 4x_1$</td>
<td>246.0</td>
<td>$7038.0$</td>
<td>$12772.3$</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>$0.1x_2^2 + 1x_2$</td>
<td>98.2</td>
<td>$1061.5$</td>
<td>$2435.6$</td>
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</table>

- There are no linear prices that would constitute a CE

**Reason:** Since the bids are strictly convex, CE assigns bidder 1 a linear price equal to its marginal cost at $x_1^* = 246$MW, that is, $\$53.2$/MW. This yields the payment $\$13087.2$. 

\[\]
AC-OPF problem with a duality gap

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**This payment is greater than its VCG payment, cannot be in the core!**
Swiss reserve procurement auctions

- Two-stage stochastic weekly market for secondary and tertiary reserves [Abbaspourtoobati and Zima 2016]
- Mutually exclusive bids are submitted

\[
J(B) = \min_{x \in \hat{X}, y} \sum_{l \in L} b_l(x_l) + d(y)
\]

s.t. \( g(x, y) \leq 0 \)

- Power to be purchased in the weekly market \( x \in \hat{X} \subset \mathbb{R}_+^{t|L|} \)
- Power to be purchased in the daily market \( y \in \mathbb{R}^p_+ \)
- Expected daily market cost \( d : \mathbb{R}^p_+ \to \mathbb{R} \)
- Reserves ensure a deficit probability of less than 0.2%
Swiss reserve procurement auctions

- Based on 2014 data—67 bidders

**Table:** Total payments of the two-stage auction

<table>
<thead>
<tr>
<th>Total Pay-as-bid payment</th>
<th>2.293 million CHF</th>
</tr>
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<tr>
<td>Total MPCS payment</td>
<td>2.437 million CHF</td>
</tr>
<tr>
<td>Total VCG payment</td>
<td>2.529 million CHF</td>
</tr>
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</table>

- Computation times for different mechanisms
  - VCG: 580.6 seconds
  - MPCS: 659.2 seconds
Two-sided markets with DC-OPF constraints

\[ c_3(x_3) = x_3^2 + x_3, \quad x_3 \geq 0 \]

\[ c_1(x_1) = 5x_1^2 + 4x_1, \quad x_1 \geq 0 \]

\[ c_2(x_2) = 4x_2^2 + 5x_2, \quad x_2 \geq 0 \]

\[ c_4(x_4) = x_4^2 + 20x_4, \quad -8 \leq x_4 \leq 0 \]
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Table: Budget-balance comparison

<table>
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<td>( u_{CO} )</td>
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Conclusion

▶ Summary
  ▶ Derived conditions for core-selecting VCG mechanism
  ▶ Showed the equivalence of the core and the competitive equilibrium
  ▶ Designed core-selecting mechanisms with desirable properties
  ▶ Verified with OPF test systems and Swiss reserve market

▶ Outlook
  ▶ Ways to reallocate the budget surplus
  ▶ Learning in a repeated setting
Thank you for your attention

The results from this talk appear in

- ArXiv:1811.09646 (under review)
- ArXiv:1711.06774 (under review)

You may contact me: okaraca@ethz.ch