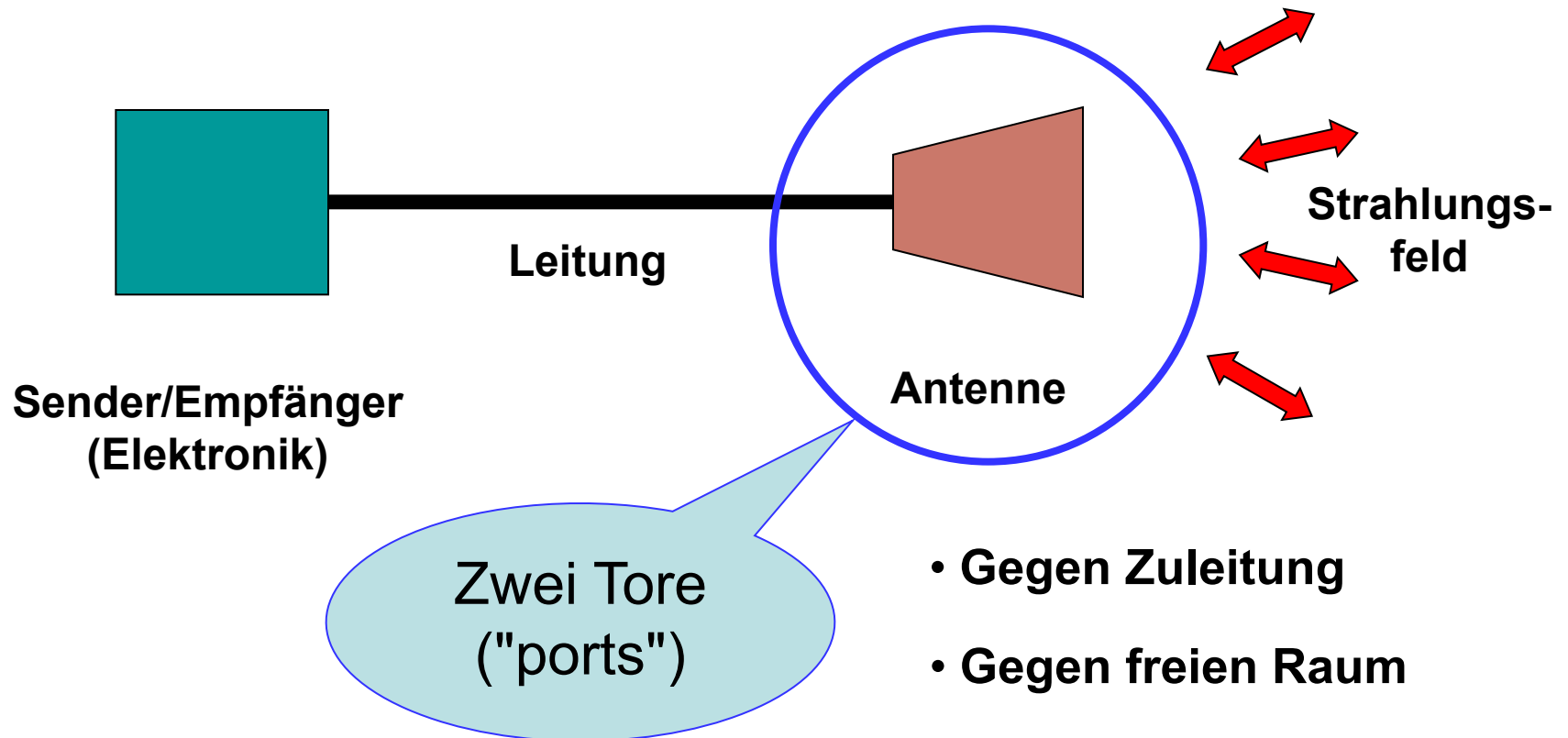


Übersicht

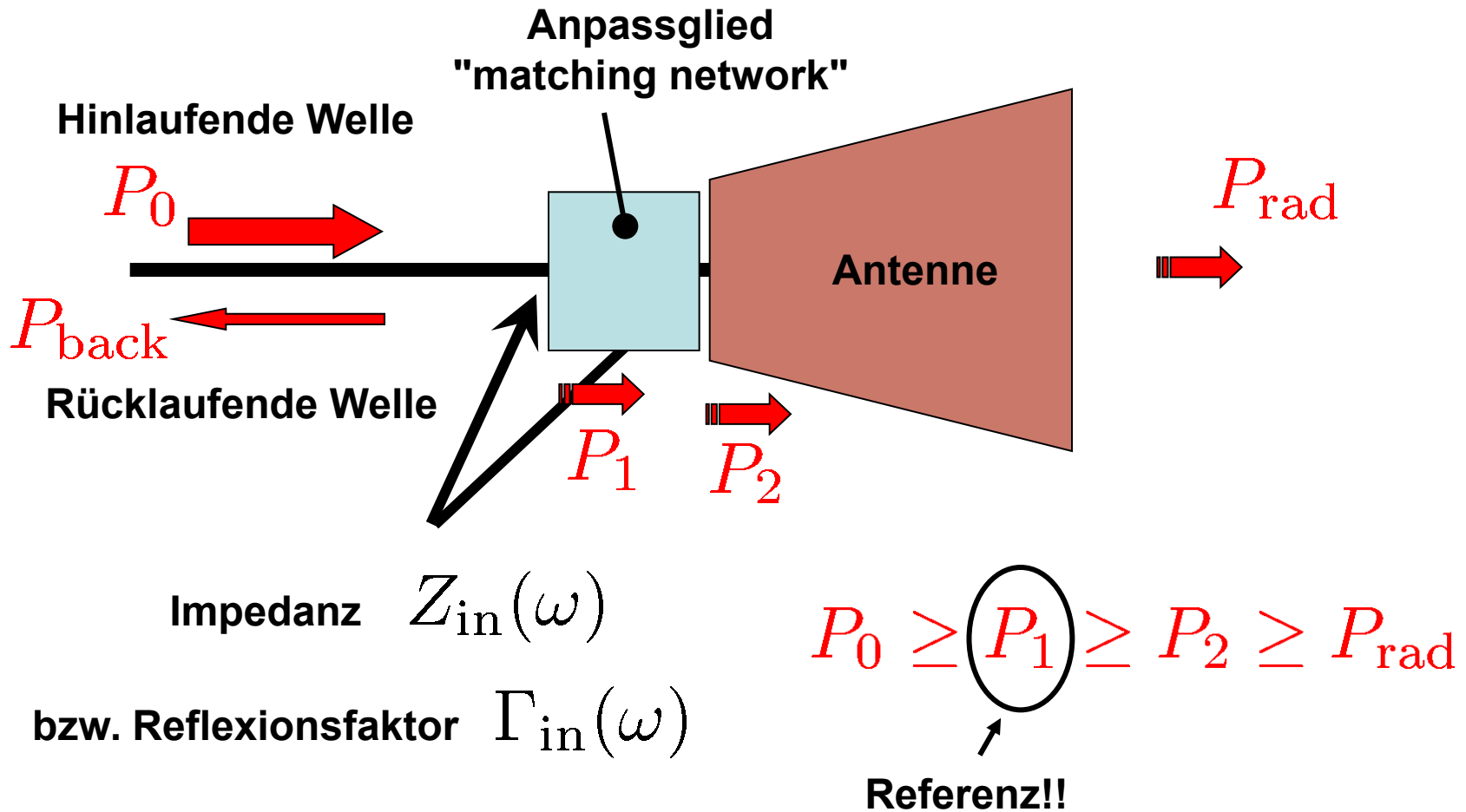
- Antennen aus der Sicht der Speisung
- Antennen aus der Sicht des Fernfeldes
- Richtdiagramm und Strahldichte
- Wirkungsgrad, Richtfaktor, Gewinn, effektive Sendeleistung (ERP)
- Ersatzschaltung einer Antenne, Strahlungswiderstand
- Hertz'scher Dipol

Antennen

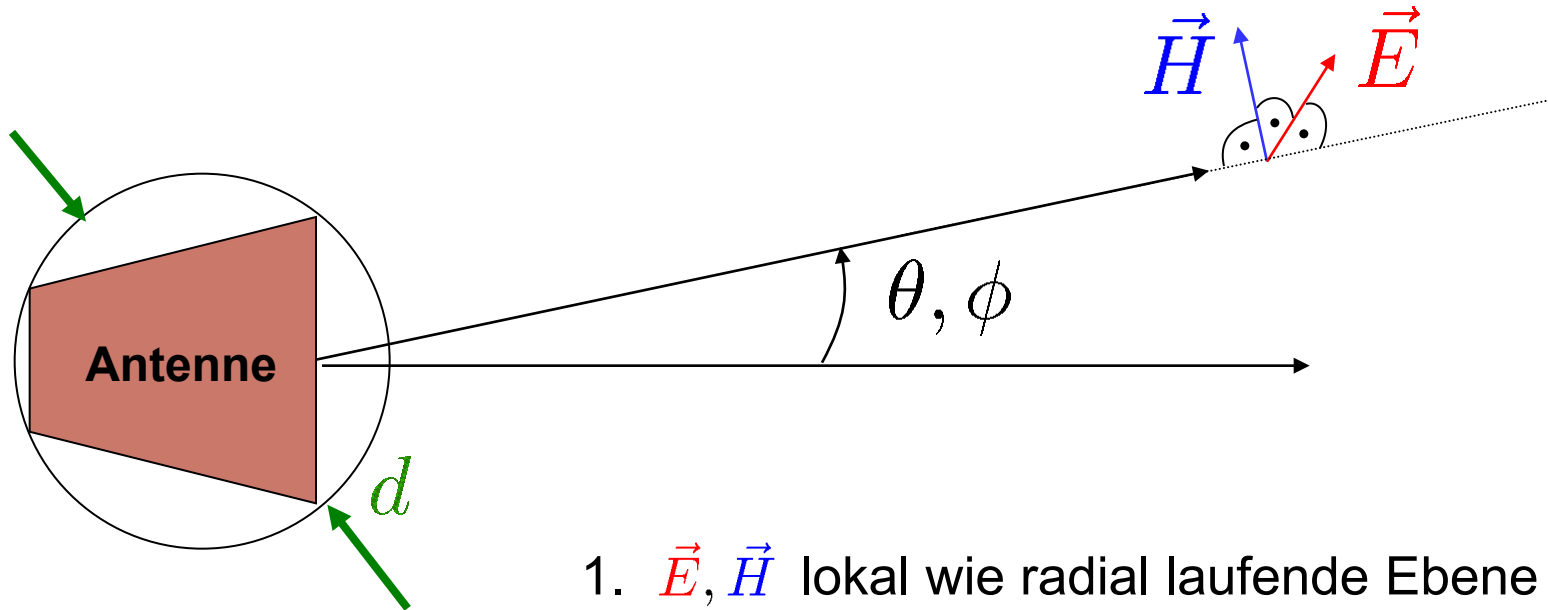
Es geht darum, elektromagnetische Energie in den freien Raum zu bringen bzw. von dort einzufangen.



Beschreibung des Antenneneingangs



Beschreibung des Fernfeldes



nur für
 $r \gg d$

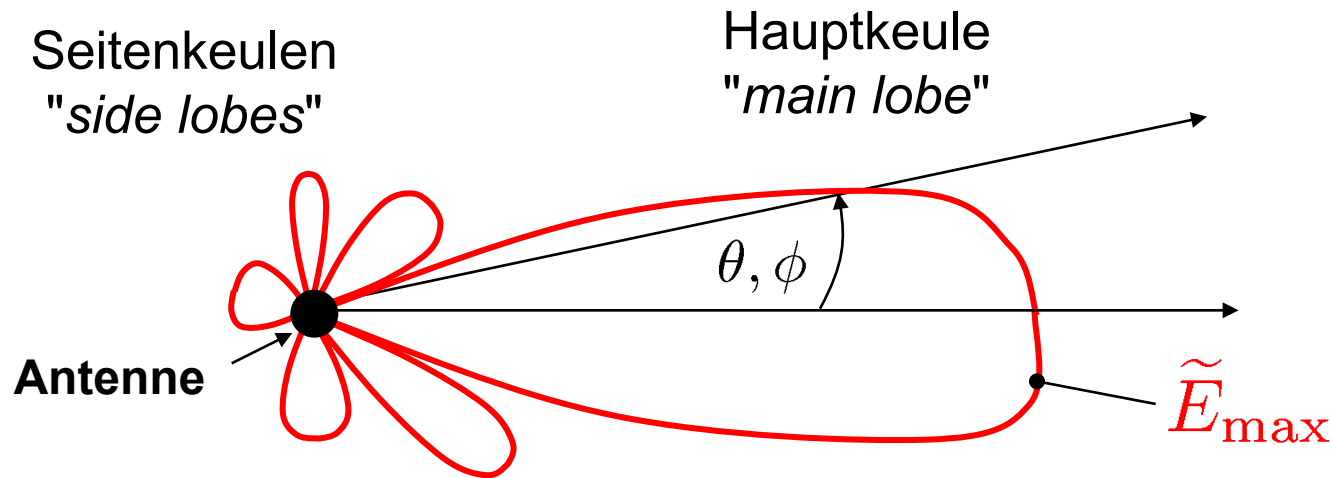
1. $\vec{E}, \vec{H}$ lokal wie radial laufende Ebene Welle

2.
$$\vec{E}(r, \theta, \phi) = \frac{1}{r} \tilde{E}(\theta, \phi) \vec{e}_{\text{pol}}(\theta, \phi)$$

Stärke
der Strahlung

Polarisation

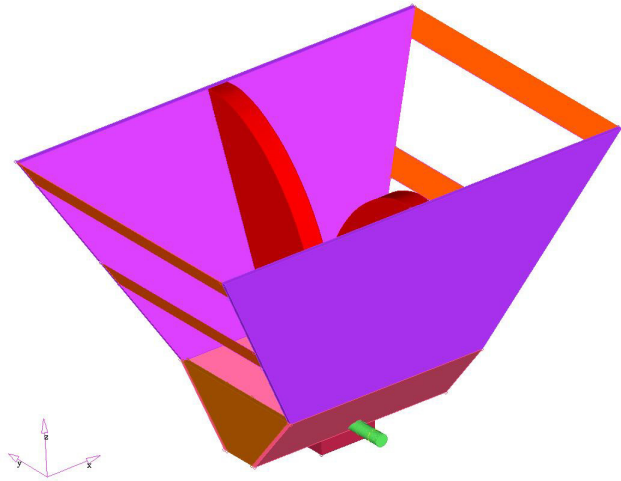
Beschreibung des Fernfeldes



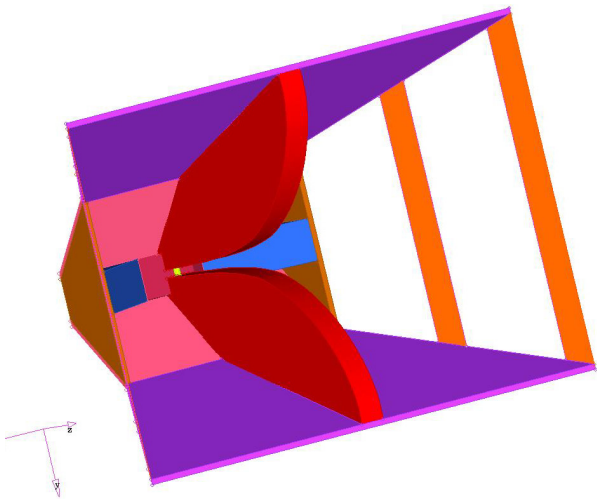
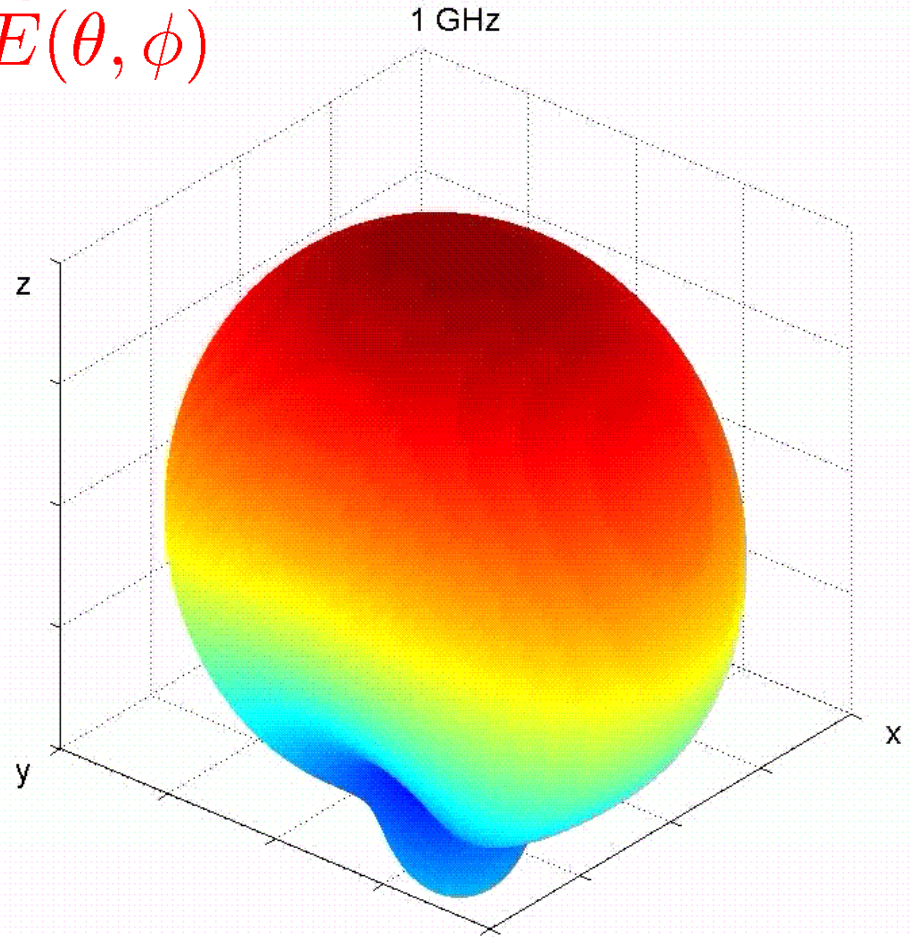
$$\vec{E}(r, \theta, \phi) = \frac{1}{r} \tilde{E}(\theta, \phi) \vec{e}_{\text{pol}}(\theta, \phi)$$

Strahlungsdiagramm
 Richtdiagramm
 Antennendiagramm
 Richtcharakteristik
 Strahlungscharakteristik
 Empfangscharakteristik
 "radiation pattern"

Beispiel ("double rigged horn antenna")



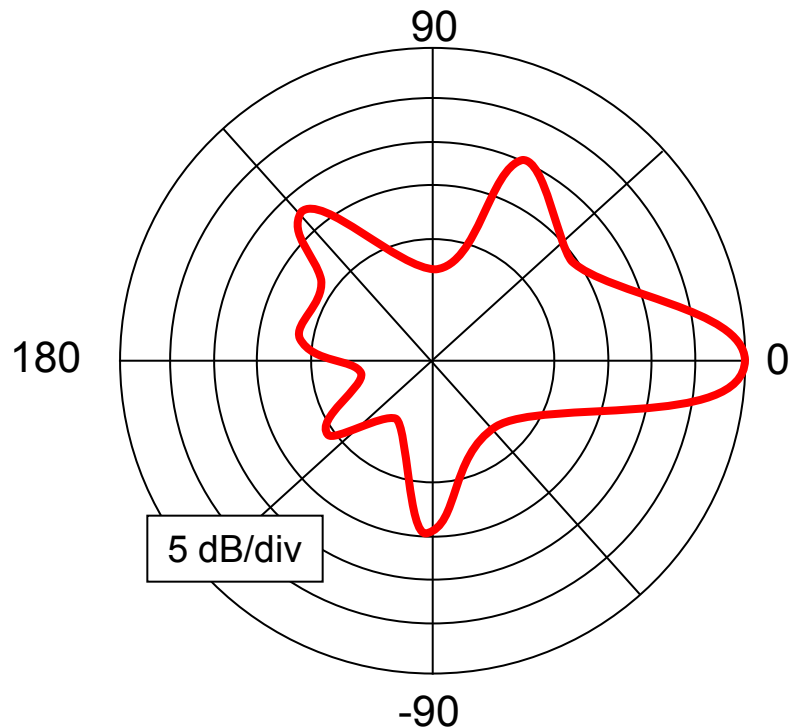
$$\tilde{E}(\theta, \phi)$$



Felder & Komponenten II

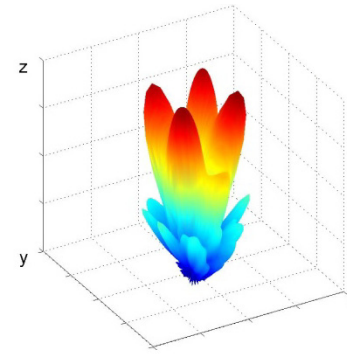
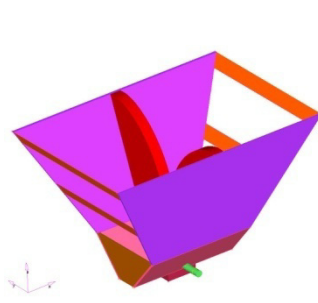
Weiteres zum Richtdiagramm

- Enthält (meist) **keine** Information zur Polarisation
- 3D-Darstellung gibt nur qualitativen Eindruck
- Darstellung in einzelnen Ebenen: $\tilde{E}(\theta) \Big|_{\phi=\text{const}}$

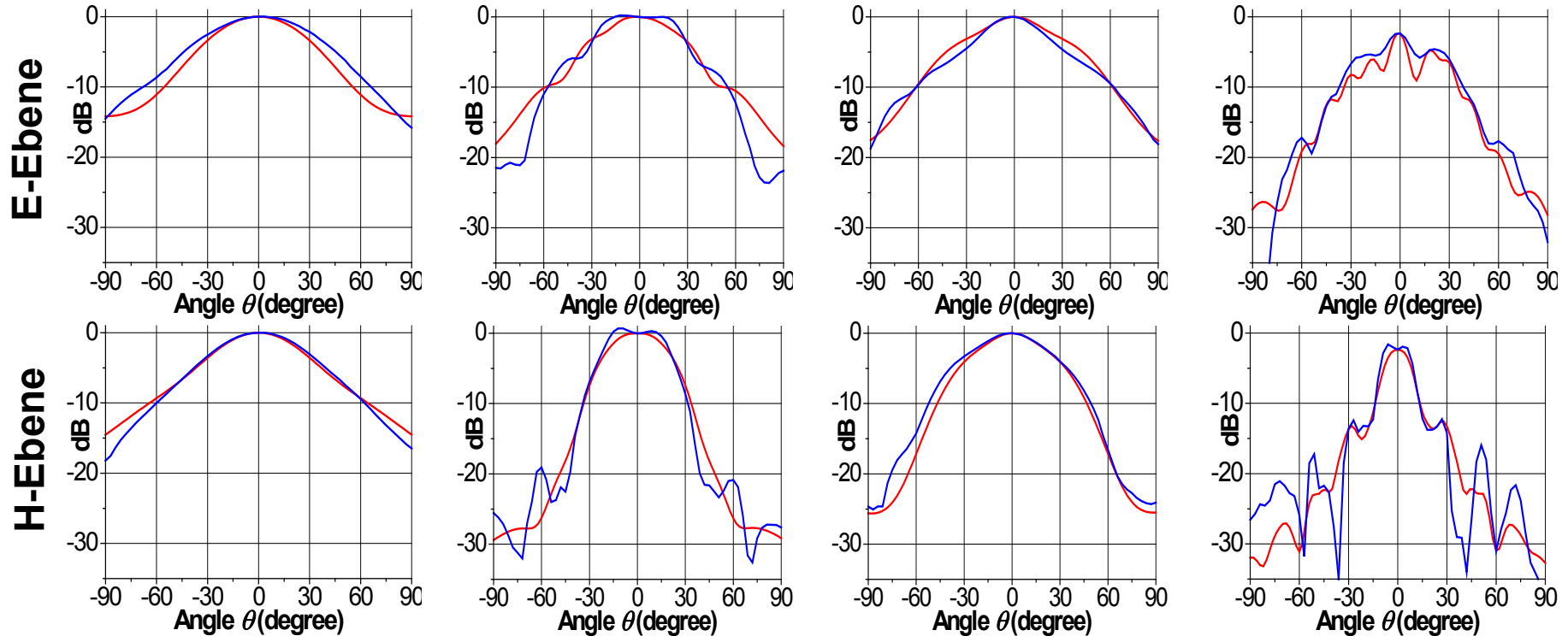


auch kartesische Darstellung!

Kartesische Darstellung



— Messung
— FVTD



2 GHz

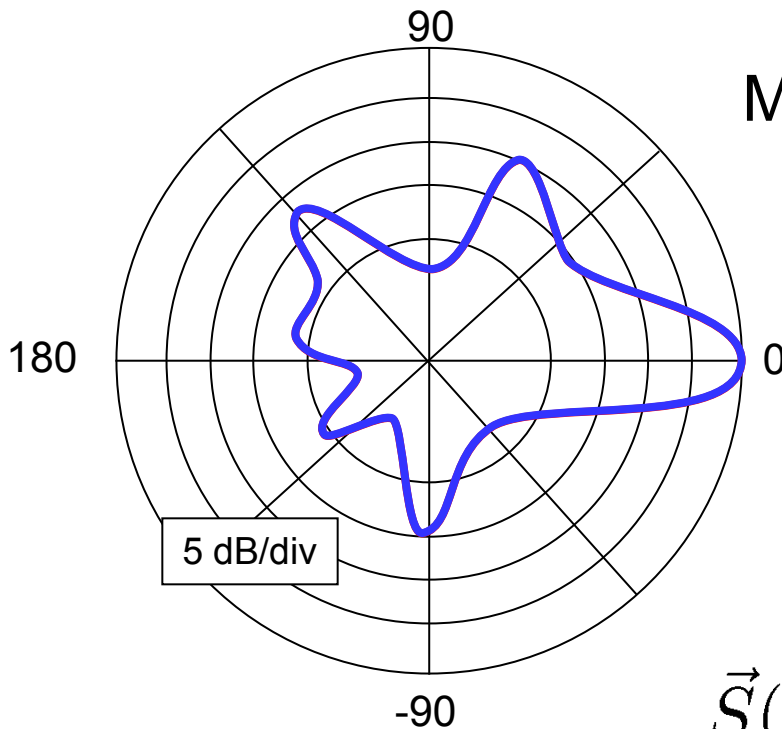
4 GHz

8 GHz

16 GHz

Weiteres zum Richtdiagramm

Normalerweise bezogen auf $\tilde{E}(\theta, \phi) [\text{V}]$



Manchmal auch auf **Strahldichte**

$$S(\theta, \phi) := \frac{\tilde{E}^2(\theta, \phi)}{Z_w} [\text{W}]$$

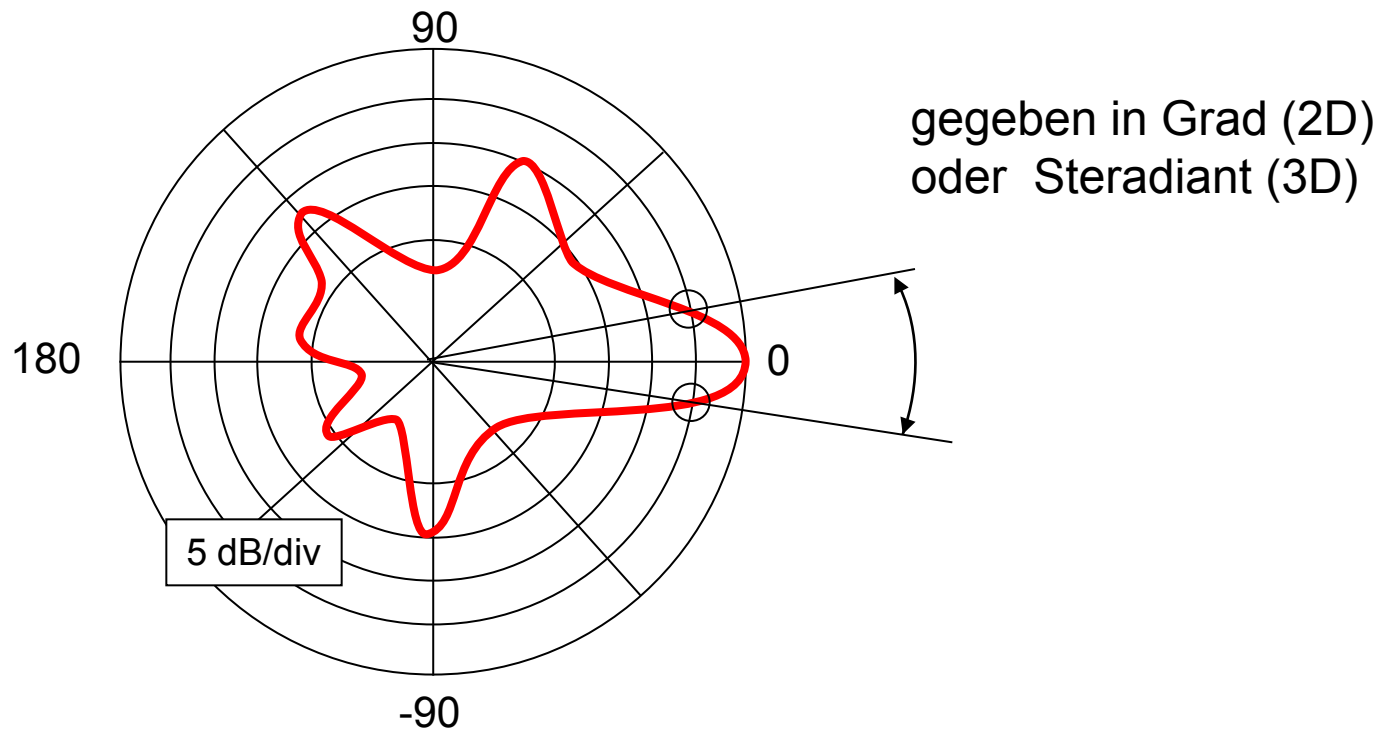
Achtung:
Nicht Poynting-Vektor!

$$\vec{S}(r, \theta, \phi) \hat{=} \frac{S(\theta, \phi)}{r^2} \vec{e}_r \left[\frac{\text{W}}{\text{m}^2} \right]$$

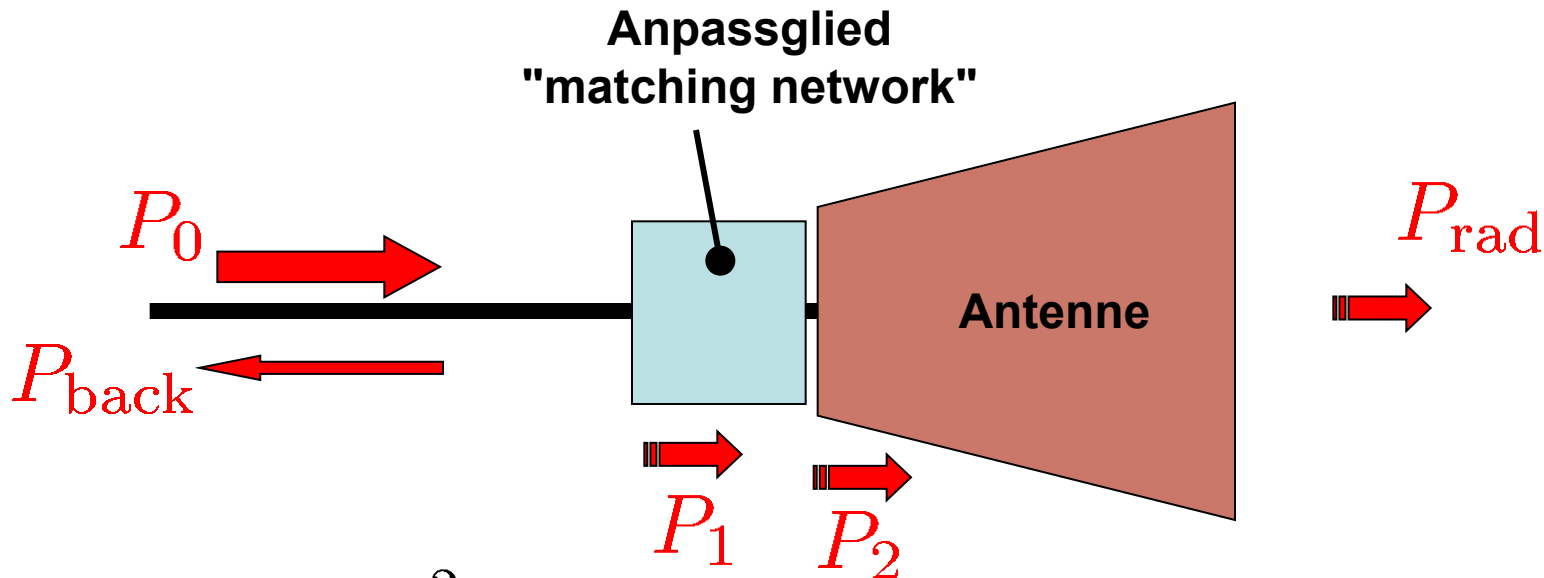
Felder & Komponenten II

Weiteres zum Richtdiagramm

- (x-dB-)Keulenbreite ("*beam width*")



Strahlungsleistung und Wirkungsgrad

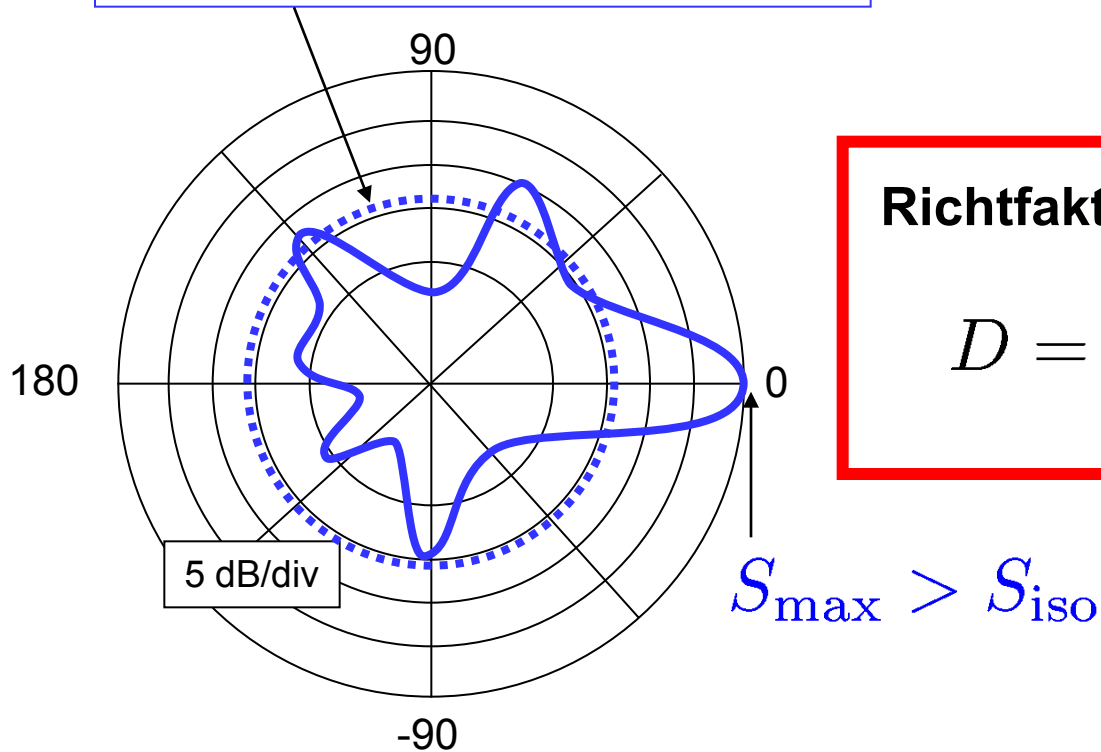


$$P_{\text{rad}} = \int_0^{\pi} \int_0^{2\pi} S(\theta, \phi) d\phi \sin \theta d\theta \quad [\text{W}]$$

Antennenwirkungsgrad $\eta = \frac{P_{\text{rad}}}{P_{1,2}}$

Richtfaktor D ("directivity")

Richtdiagramm eines isotropen Strahlers gleicher Leistung:



$$\cancel{S_{\text{iso}}(\theta, \phi)} = \frac{P_{\text{rad}}}{4\pi}$$

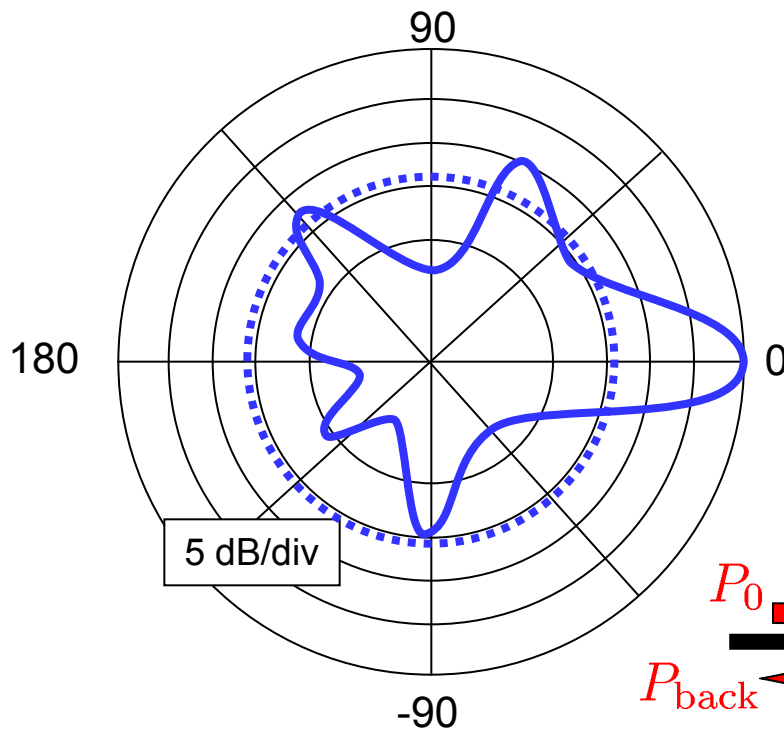
Richtfaktor:

$$D = \frac{S_{\max}}{S_{\text{iso}}} = 4\pi \frac{S_{\max}}{P_{\text{rad}}}$$

Antennen-Gewinn G ("gain")

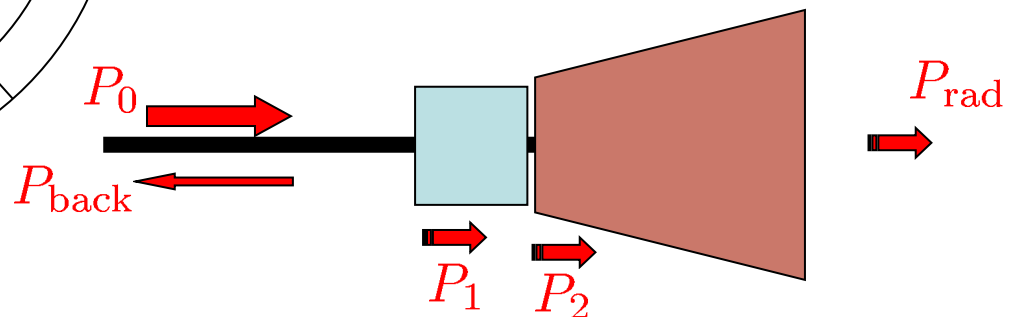
$$D = \frac{S_{\max}}{S_{\text{iso}}} = 4\pi \frac{S_{\max}}{P_{\text{rad}}}$$

$$\eta = \frac{P_{\text{rad}}}{P_{1,2}}$$



Antennen-Gewinn:

$$G = 4\pi \frac{S_{\max}}{P_{1,2}} = \eta \cdot D$$



Felder & Komponenten II

Antennen-Gewinn: Beispiel 1

Horn mit Spiegel: gemäss Hersteller **G = 42 dB**

$$f = 1 \dots 18 \text{ GHz}$$

gemessen: **G = 32 dB**

Nur 10% der erwarteten Leistung
im Bereich der Hauptkeule!



Antennen-Gewinn: Beispiel 2

$$f = 380 \dots 400 \text{ MHz}$$

Undercover-Antenne: Bestehendes System **$G = -26 \text{ dB}$**
(Draht-Dipol, fast auf der Haut)



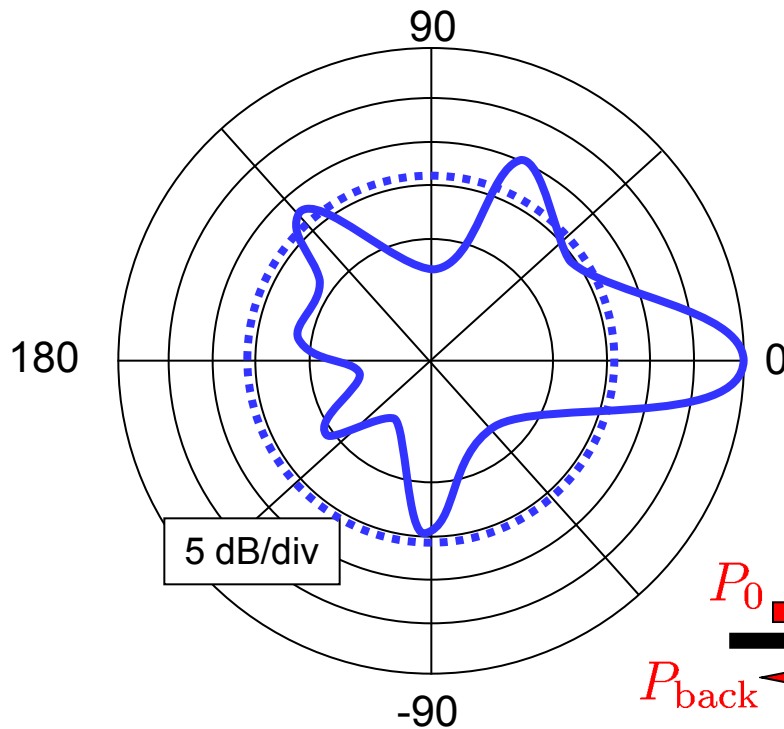
Prototyp: **$G = -12 \text{ dB}$**

Ideale Antenne hätte
 $G = 0 \text{ dB}$ wegen
Rundstrahlcharakteristik!

Effektive Sendeleistung ("effective radiated power", ERP)

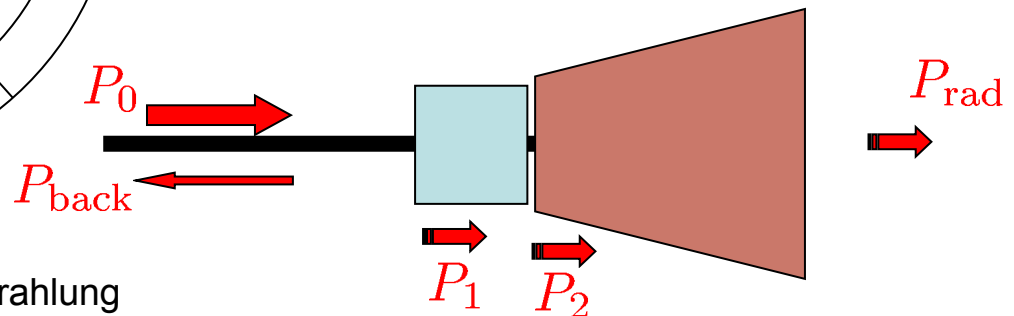
Welche Leistung wäre nötig, wenn die tatsächliche Antenne ein **Kugelstrahler** wäre, der die gleiche A **Juristenfutter bei NISV*** in der Hauptstrahlrichtung?

$$G = 4\pi \frac{S_{\max}}{P_{1,2}}$$



$$\frac{P_{\text{eff}}}{4\pi} = S_{\max} = \frac{G P_{1,2}}{4\pi}$$

$$P_{\text{eff}} = G \cdot P_{1,2}$$

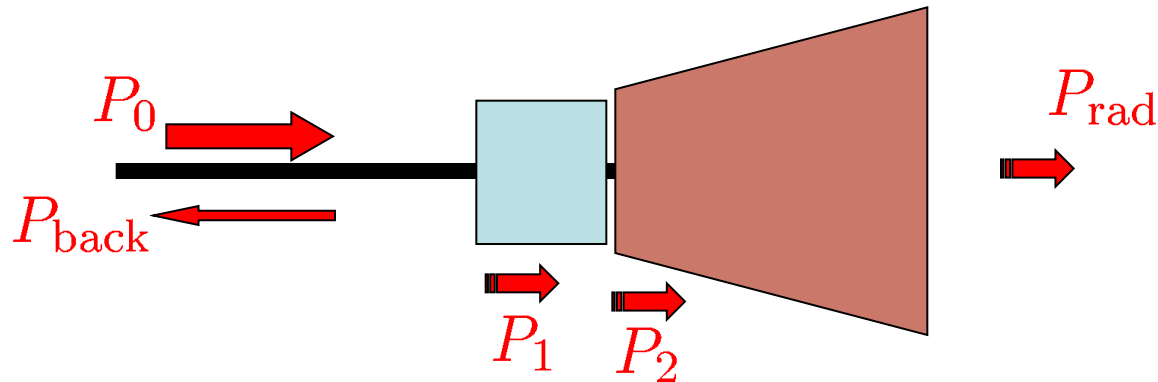


*Verordnung über die Nicht ionisierende Strahlung

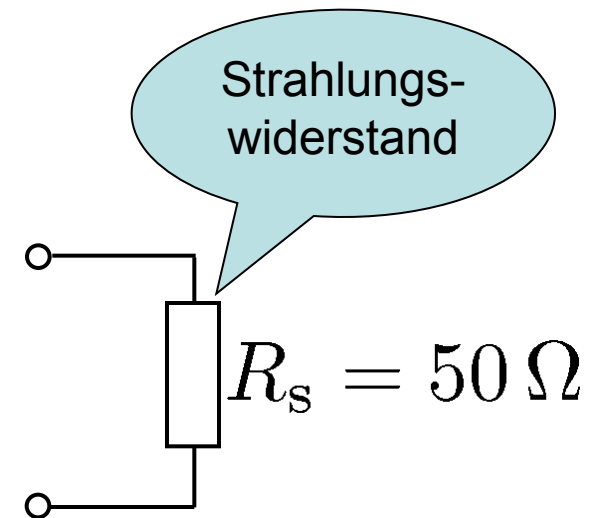
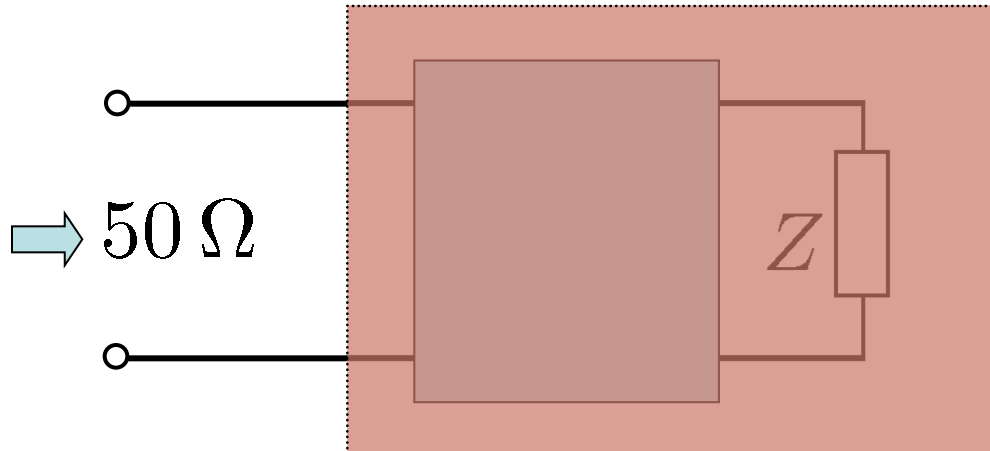
$G = 20\text{dB} \rightarrow 10\text{ W reale Leistung ergeben } 1\text{ kW ERP}$

Felder & Komponenten II

Ersatzschaltung einer Antenne



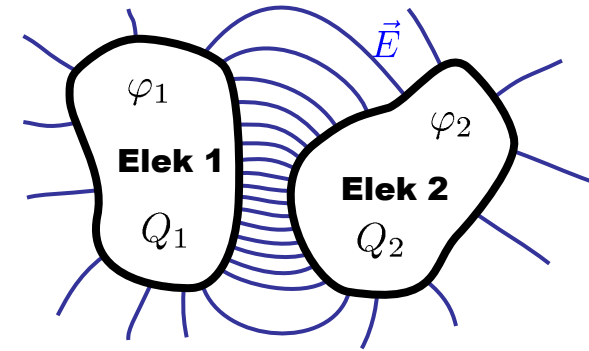
1. Ideale Antenne: $P_0 = P_1 = P_2 = P_{\text{rad}}$



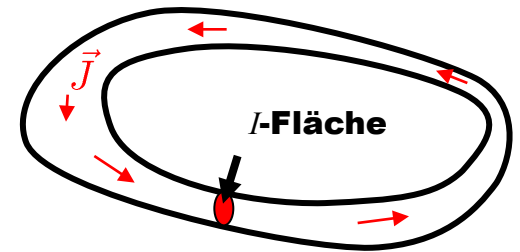
Felder & Komponenten II

Feld $\longleftrightarrow$ Netzwerk

$$C = \frac{1}{U^2} \iiint_{V_\infty} \vec{D} \cdot \vec{E} dV = \frac{1}{U^2} \iiint_{V_\infty} \epsilon |\vec{E}|^2 dV$$



$$L = \frac{1}{I^2} \iiint_{V_\infty} \vec{B} \cdot \vec{H} dV = \frac{1}{I^2} \iiint_{V_\infty} \mu |\vec{H}|^2 dV$$



$$R = \frac{1}{I^2} \iiint_{V_J} \vec{J} \cdot \vec{E} dV = \frac{1}{I^2} \iiint_{V_J} \sigma |\vec{E}|^2 dV$$

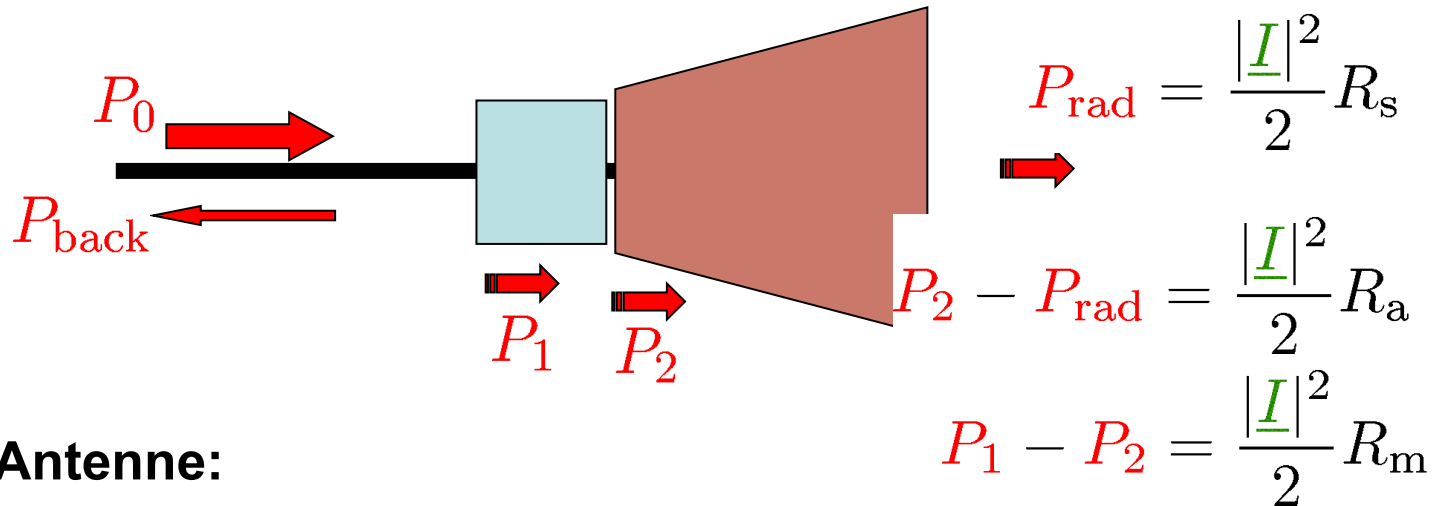
Poynting-Theorem (Phasor)

$$-\oint_{\partial V} \vec{S} \cdot d\vec{F} = \iiint_V \left(\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} + \varepsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} \cdot \vec{E} \right) dV$$

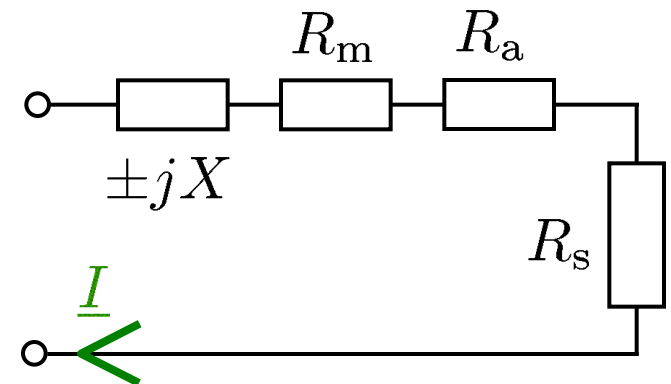
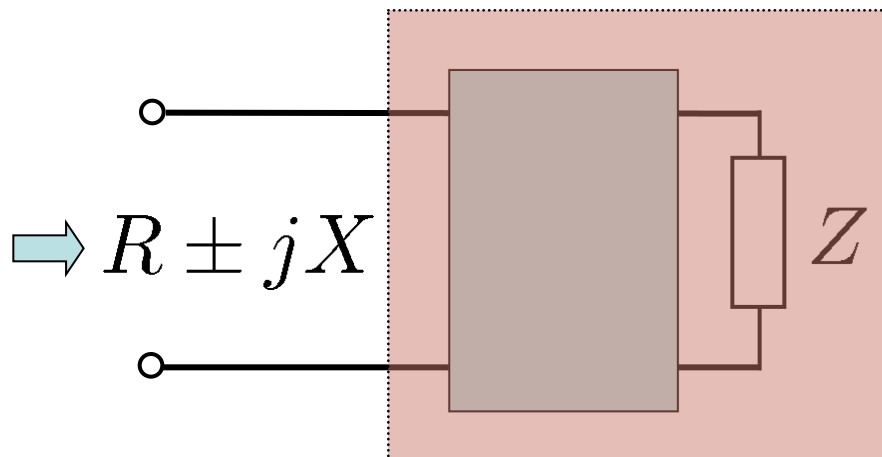
$$-\oint_{\partial V} \underline{\vec{S}} \cdot d\vec{F} = \frac{1}{2} \iiint_V \left\{ j\omega \left(\mu \underline{\vec{H}} \cdot \underline{\vec{H}} + \varepsilon \underline{\vec{E}} \cdot \underline{\vec{E}} \right) + \sigma \underline{\vec{E}} \cdot \underline{\vec{E}} \right\} dV$$

$$-\oint_{\partial V} \underline{\vec{S}} \cdot d\vec{F} = \frac{1}{2} \iiint_V \left\{ j\omega \left(\mu \underline{\vec{H}} \cdot \underline{\vec{H}}^* - \varepsilon^* \underline{\vec{E}} \cdot \underline{\vec{E}}^* \right) + \sigma^* \underline{\vec{E}} \cdot \underline{\vec{E}}^* \right\} dV$$

Ersatzschaltung einer Antenne

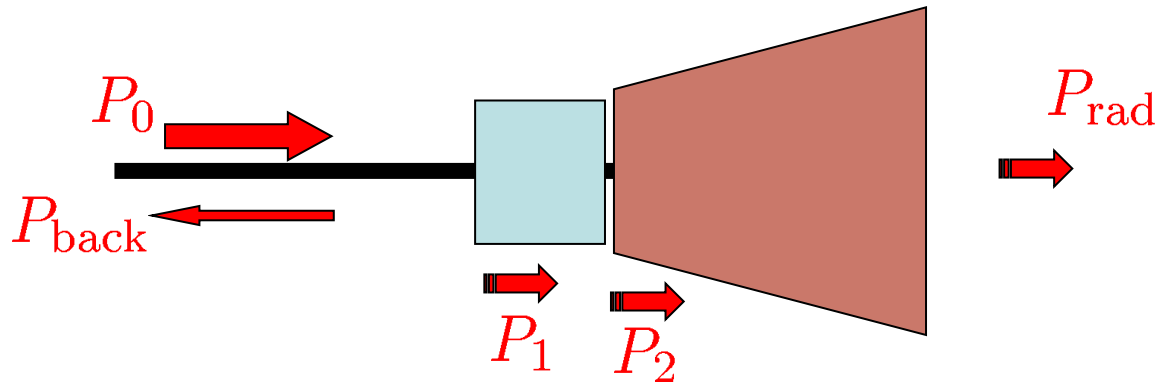


2. Reale Antenne:

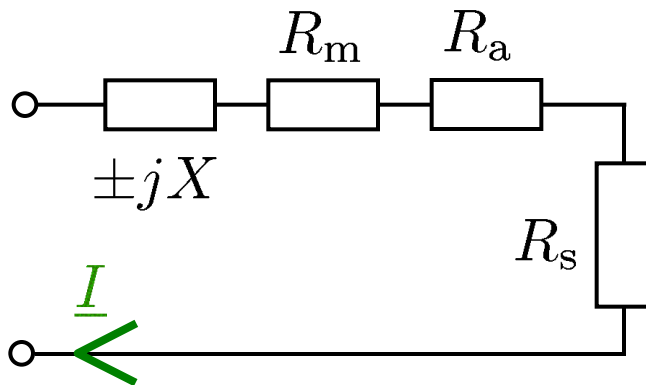


Felder & Komponenten II

Ersatzschaltung einer Antenne

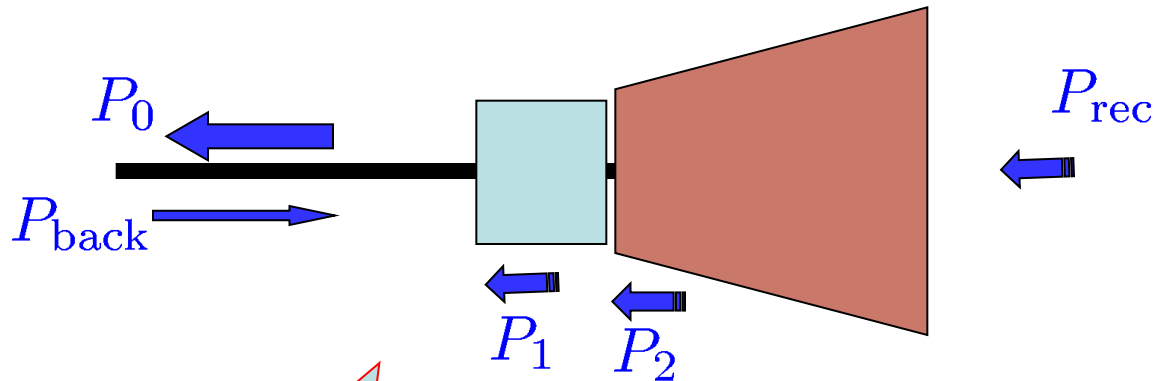


2. Reale Antenne:



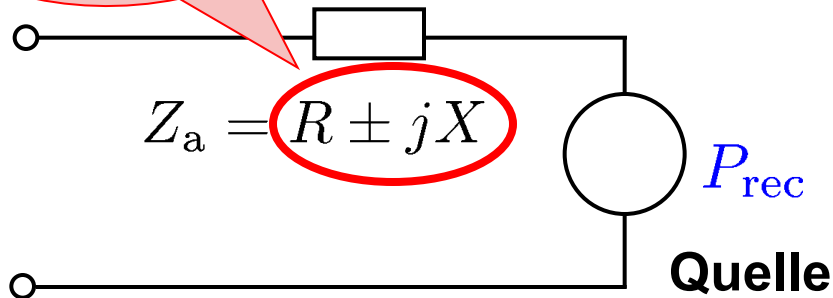
kompliziertere Ersatzschaltungen
sind möglich!
(z.B. um grösseren Frequenzbereich
abzudecken)

Antenne beim Empfangen



Wie beim Senden!

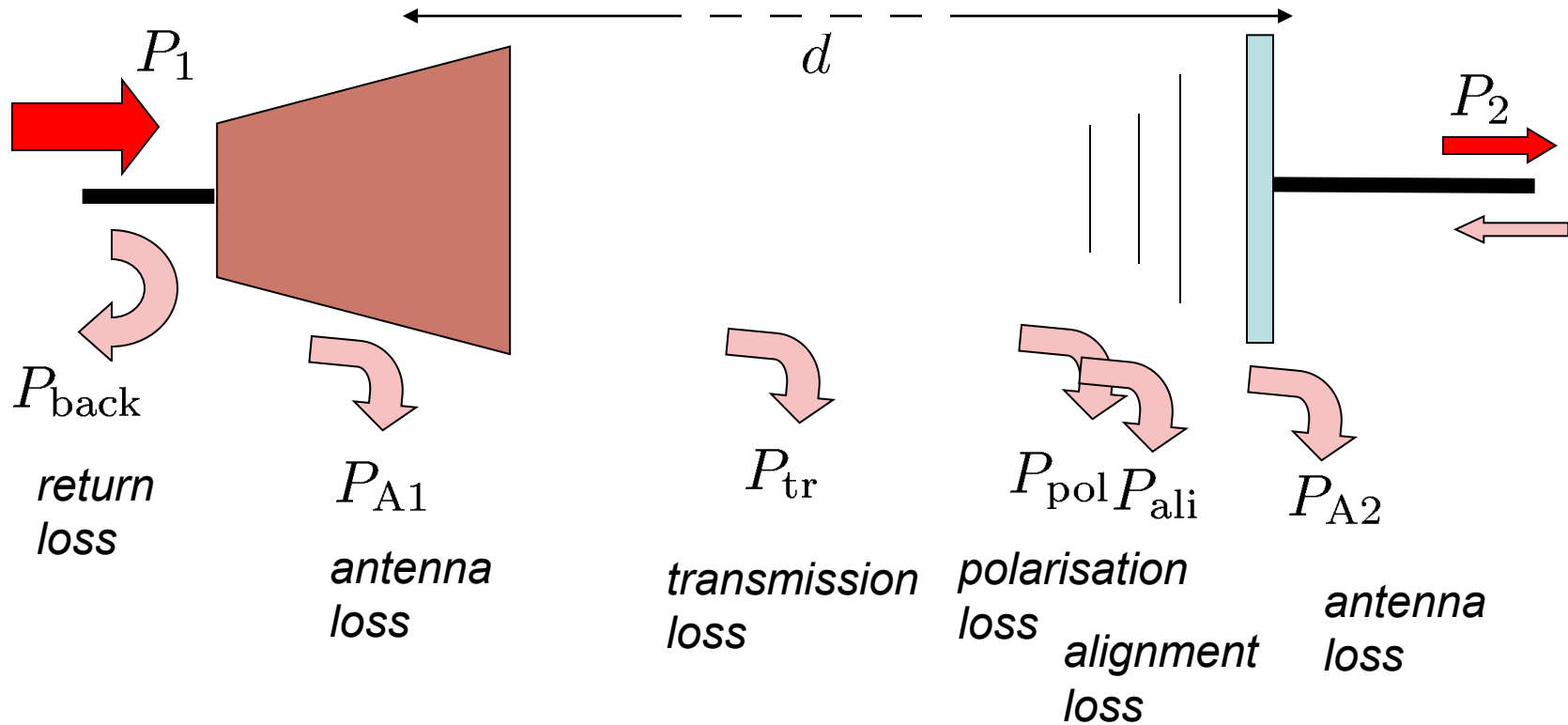
Reziprozität



**Empfangscharakteristik
=
Sendecharakteristik**

Gewinn G
Richtfaktor D

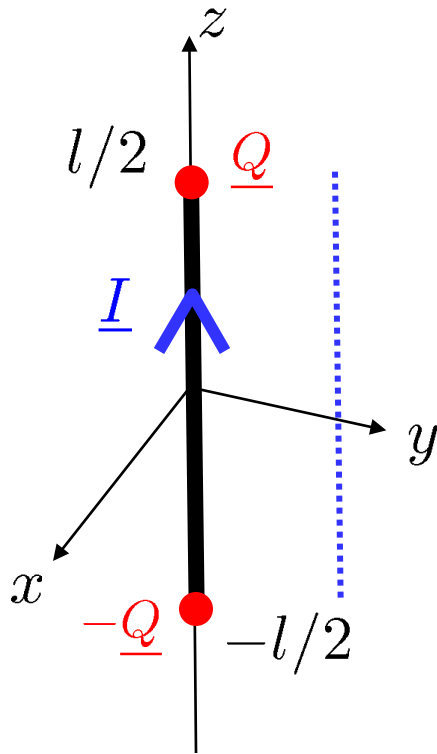
Drahtlose Übertragungsstrecke



Leistungsanpassung wo immer möglich!!

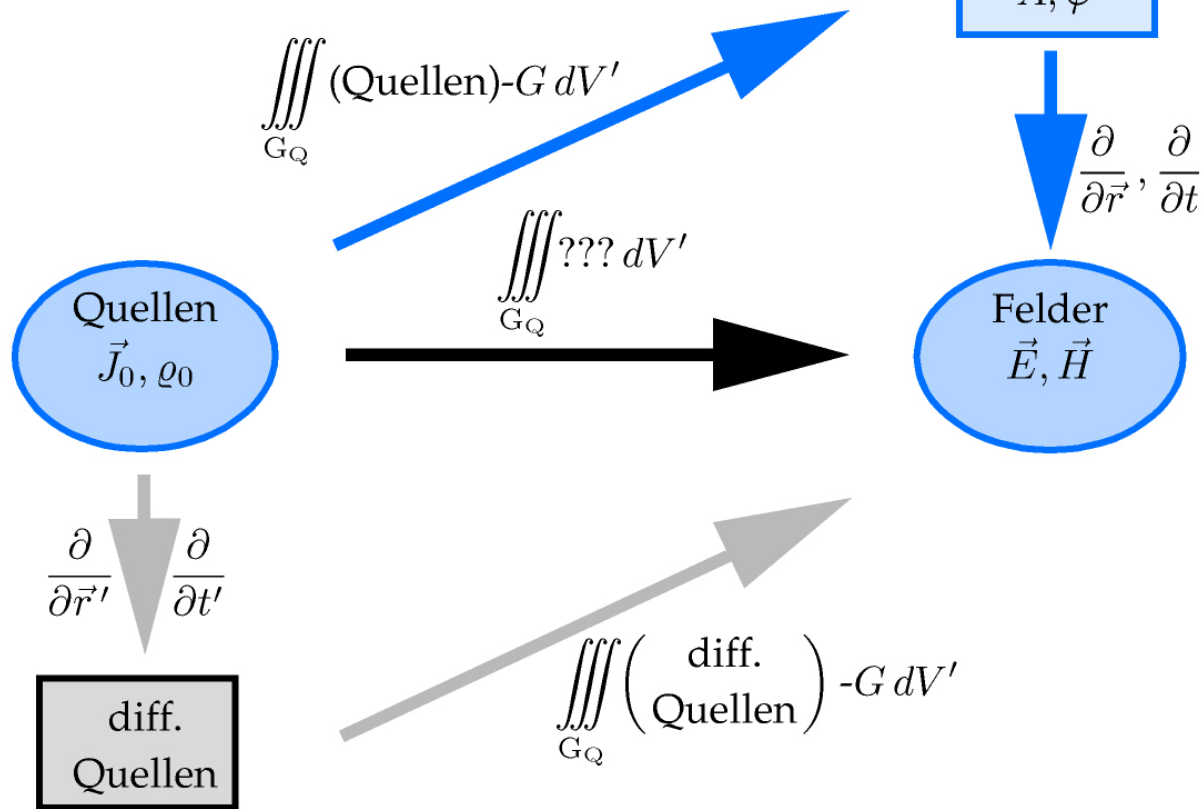
Hertz'scher Dipol

Siehe auch im Buch, Abschnitt 12.1.4



**Berechnung des Feldes
explizit und analytisch
möglich!**

$$\underline{\vec{A}}(\vec{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\underline{\vec{J}} \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV'$$



$$\underline{\vec{E}} = -\text{grad } \underline{\varphi} - j\omega \underline{\vec{A}}$$

$$\underline{\vec{H}} = \frac{1}{\mu} \text{rot } \underline{\vec{A}}$$

(Kapitel 6, Fuk I)

Bild aus dem Buch!

Quellpunkt $\vec{r}', t'$

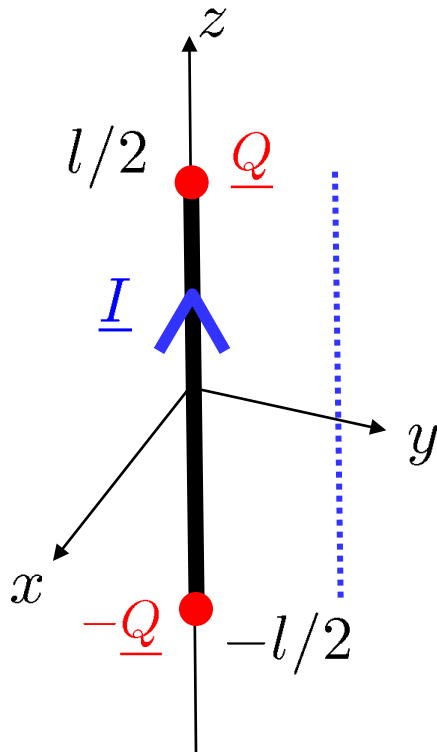
$$R := |\vec{r} - \vec{r}'|$$

$$t' := t - \frac{R}{v}$$

Aufpunkt $\vec{r}, t$

Felder & Komponenten II

Hertz'scher Dipol



1. Berechnung des Vektorpotentials $\underline{\vec{A}}(\vec{r})$ für alle "fernen" Punkte $\vec{r}$
2. Berechnung der Felder aus $\underline{\vec{A}}(\vec{r})$
3. Diskussion des Feldverlaufs
4. Berechnung der Antennenparameter (soweit sinnvoll)

Spezielle (partikuläre) Lösungen im stationären Zustand

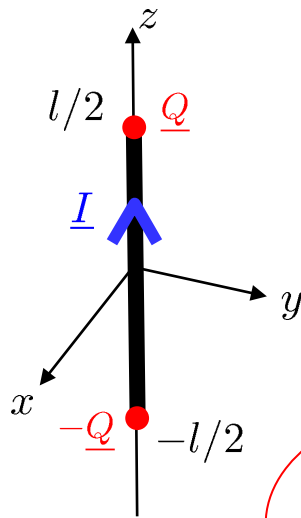
$$\underline{\vec{E}}(\vec{r}) = \frac{-1}{4\pi\epsilon} \iiint_{V'} \frac{\left(j\omega\mu\epsilon \underline{\vec{J}}(\vec{r}') + \text{grad}' \underline{\varrho}(\vec{r}') \right) \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r} - \vec{r}'|} dV'$$

$$\underline{\vec{H}}(\vec{r}) = \frac{1}{4\pi} \iiint_{V'} \frac{\text{rot}' \underline{\vec{J}}(\vec{r}') \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r} - \vec{r}'|} dV'$$

$$\underline{\varphi}(\vec{r}) = \frac{1}{4\pi\epsilon} \iiint_{V'} \frac{\underline{\varrho}(\vec{r}') \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r} - \vec{r}'|} dV'$$

$$\underline{\vec{A}}(\vec{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\underline{\vec{J}}(\vec{r}') \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r} - \vec{r}'|} dV'$$

Hertz'scher Dipol: Vektorpotential $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\underline{\vec{J}} \cdot e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dV' \quad (7.34)$$

$$= \frac{\mu \underline{I}}{4\pi} \vec{e}_z \int_{-l/2}^{l/2} \frac{e^{-jk\sqrt{x^2+y^2+(z-z')^2}}}{\sqrt{x^2+y^2+(z-z')^2}} dz'$$

$$dV' = dF \cdot dz'$$

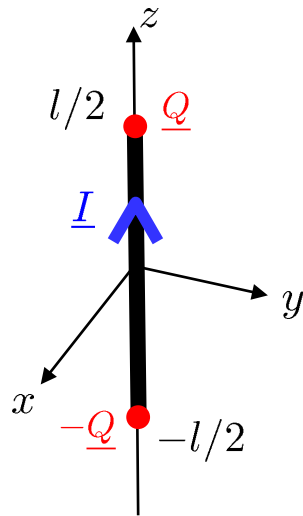
$$\iint \underline{\vec{J}} dF = \underline{I} \cdot \vec{e}_z$$

$$= \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jk\sqrt{x^2+y^2+z^2}}}{\sqrt{x^2+y^2+z^2}} = \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jkr}}{r}$$

$$x^2 + y^2 + z^2 \gg l^2$$

$$kl \ll 1$$

Hertz'scher Dipol: Felder aus $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu \underline{I} l}{4\pi} \underline{\vec{e}}_z \frac{e^{-jkr}}{r}$$

$$\underline{\vec{e}}_z = \underline{\vec{e}}_r \cos \theta - \underline{\vec{e}}_\theta \sin \theta$$

$$\underline{\varphi} = \frac{\text{div } \underline{\vec{A}}}{-j\omega\mu\epsilon}$$

Lorentz-Eichung (6.103)

$$\underline{\vec{E}} = -\text{grad } \underline{\varphi} - j\omega \underline{\vec{A}} = \frac{\text{grad div } \underline{\vec{A}}}{j\omega\mu\epsilon} - j\omega \underline{\vec{A}}$$

$$\underline{\vec{H}} = \frac{1}{\mu} \text{rot } \underline{\vec{A}}$$

$$\underline{A}_r(r, \theta) = \underline{A}_z(r) \cos \theta = \frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \cos \theta$$

$$\underline{A}_\theta(r, \theta) = -\underline{A}_z(r) \sin \theta = -\frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \sin \theta$$

Felder & Komponenten II

Koordinatenformen

$$\vec{v} = v_x \vec{e}_x + v_y \vec{e}_y + v_z \vec{e}_z \quad \text{z.B.} \quad v_y = v_y(x, y, z)$$

$$\text{rot } \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \vec{e}_x + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \vec{e}_y + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \vec{e}_z$$

$$\text{div } \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

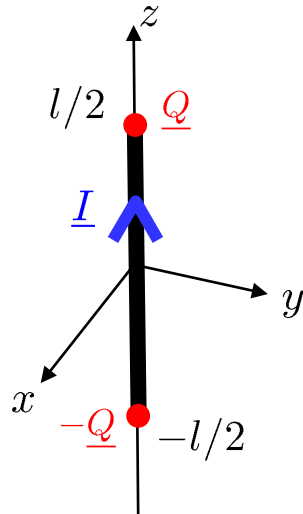
Vgl. S. 154 im Buch

$$\vec{v} = v_\rho \vec{e}_\rho + v_\phi \vec{e}_\phi + v_z \vec{e}_z$$

$$\text{rot } \vec{v} = \left(\frac{1}{\rho} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right) \vec{e}_\rho + \left(\frac{\partial v_\rho}{\partial z} - \frac{\partial v_z}{\partial \rho} \right) \vec{e}_\phi + \frac{1}{\rho} \left(\frac{\partial(\rho v_\phi)}{\partial \rho} - \frac{\partial v_\rho}{\partial \phi} \right) \vec{e}_z$$

$$\text{div } \vec{v} = \frac{1}{\rho} \frac{\partial(\rho v_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Hertz'scher Dipol: Felder aus $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jkr}}{r}$$

$$\underline{\vec{E}} = \frac{\text{grad div } \underline{\vec{A}}}{j\omega\mu\epsilon} - j\omega \underline{\vec{A}}$$

$$\underline{\vec{H}} = \frac{1}{\mu} \text{rot } \underline{\vec{A}}$$

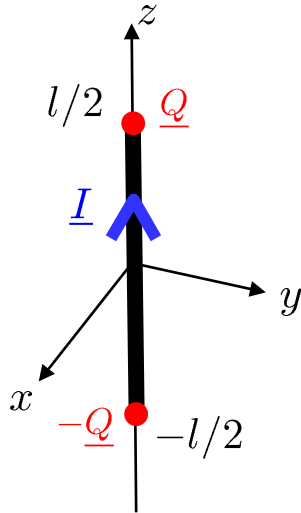
$$\underline{A}_r(r, \theta) = \frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \cos \theta$$

$$\underline{A}_\theta(r, \theta) = -\frac{\mu \underline{I} l}{4\pi} \frac{e^{-jkr}}{r} \sin \theta$$

$$\text{rot } \underline{\vec{A}} = \frac{\mu \underline{I} l}{4\pi} \left(jk + \frac{1}{r} \right) \vec{e}_\phi \frac{e^{-jkr}}{r} \sin \theta$$

$$\text{div } \underline{\vec{A}} = -\frac{\mu \underline{I} l}{4\pi} \left(jk + \frac{1}{r} \right) \frac{e^{-jkr}}{r} \cos \theta$$

Hertz'scher Dipol: Felder aus $\underline{\vec{A}}(\vec{r})$



$$\underline{\vec{A}}(\vec{r}) = \frac{\mu \underline{I} l}{4\pi} \vec{e}_z \frac{e^{-jkr}}{r}$$

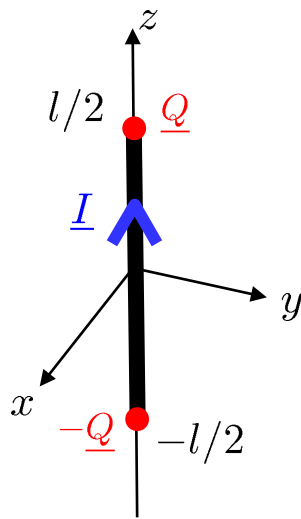
$$\underline{E}_r = \frac{\underline{I} l}{2\pi j \omega \epsilon} \frac{e^{-jkr}}{r^3} (1 + jkr) \cos \theta$$

$$\underline{E}_\theta = \frac{\underline{I} l}{4\pi j \omega \epsilon} \frac{e^{-jkr}}{r^3} (1 + jkr - k^2 r^2) \sin \theta$$

$$\underline{H}_\phi = \frac{\underline{I} l}{4\pi} \frac{e^{-jkr}}{r^2} (1 + jkr) \sin \theta$$

weitere Komponenten verschwinden!

Hertz'scher Dipol: Feldverlauf



$$\text{Dipolmoment } \underline{p} := -\frac{\underline{I}l}{4\pi j\omega\epsilon} = -\frac{\underline{I}l Z_w}{4\pi jk}$$

$$\underline{E}_r = -2\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr) \cos\theta \sin\theta$$

$$\underline{E}_\theta = -\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr - k^2 r^2) \sin\theta \sin\theta$$

$$\underline{H}_\phi = -\underline{p} \frac{jk}{Z_w} \frac{e^{-jkr}}{r^2} (1 + jkr) \sin\theta$$

Hertz'scher Dipol: Nahfeld

$$kr \ll 1$$

$$\underline{E}_r = -2\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr) \cos \theta = -2 \frac{\underline{p}}{r^3} \cos \theta$$

$$\underline{E}_\theta = -\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr - k^2 r^2) \sin \theta = -\frac{\underline{p}}{r^3} \sin \theta$$

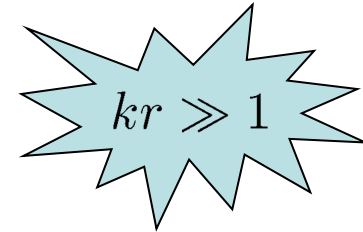
Elektrisches
Feld
dominiert!

$$\underline{H}_\phi = -\underline{p} \frac{jk}{Z_w} \frac{e^{-jkr}}{r^2} (1 + jkr) \sin \theta = -\underline{p} \frac{jk}{Z_w} \frac{1}{r^2} \sin \theta$$

90° phasenverschoben

"keine" radiale Leistung im Mittel?! $\Re \underline{\underline{S}} = \frac{1}{2} \Re(\underline{\vec{E}} \times \underline{\vec{H}}^*)$

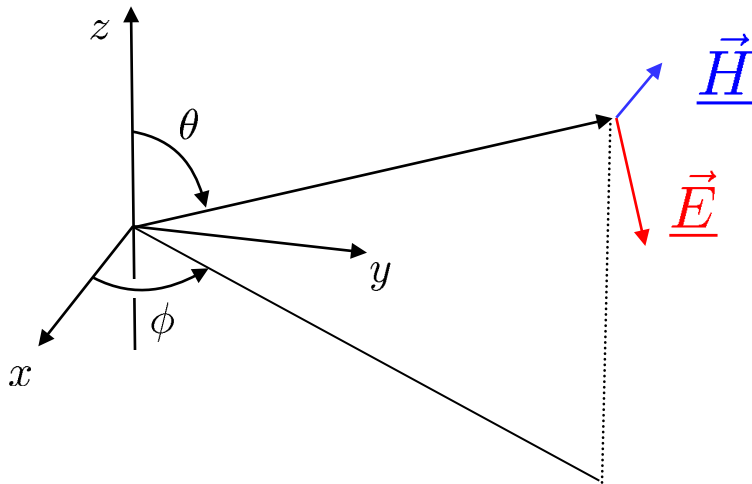
Hertz'scher Dipol: Fernfeld



$$\underline{E}_r = -2\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr) \cos \theta = \underline{0}$$

$$\underline{E}_\theta = -\underline{p} \frac{e^{-jkr}}{r^3} (1 + jkr - k^2 r^2) \sin \theta = \underline{p} k^2 \frac{e^{-jkr}}{r} \sin \theta$$

$$\underline{H}_\phi = -\underline{p} \frac{jk}{Z_w} \frac{e^{-jkr}}{r^2} (1 + jkr) \sin \theta = \underline{p} \frac{k^2}{Z_w} \frac{e^{-jkr}}{r} \sin \theta = \frac{1}{Z_w} \underline{E}_\theta$$



Ebene Welle in
radialer Richtung!

Hertz'scher Dipol: Strahldichte

$$\underline{E}_\theta = \underline{p} k^2 \frac{e^{-jkr}}{r} \sin \theta$$

$$\underline{H}_\phi = \underline{p} \frac{k^2}{Z_w} \frac{e^{-jkr}}{r} \sin \theta = \frac{1}{Z_w} \underline{E}_\theta$$

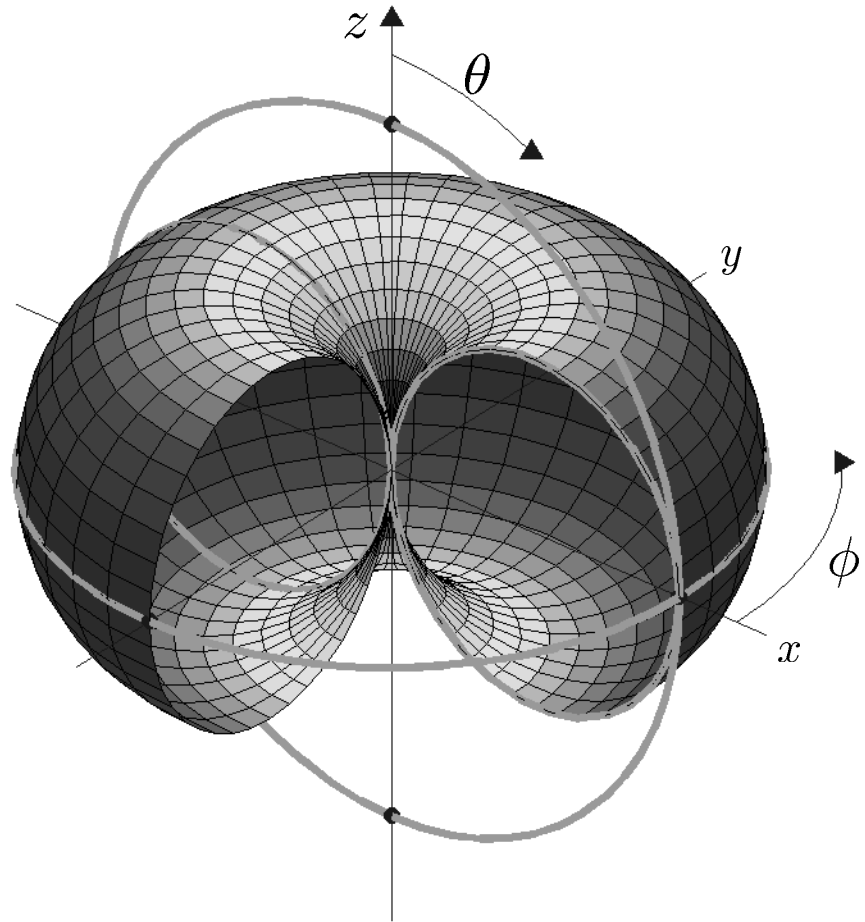
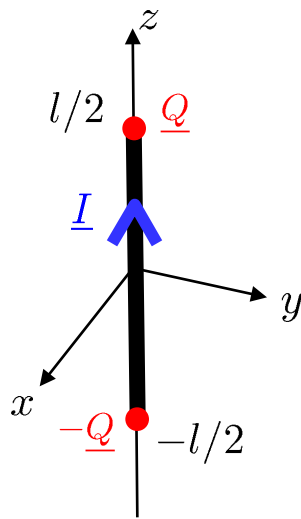
$$\underline{p} := -\frac{\underline{I}l}{4\pi j\omega\varepsilon} = -\frac{\underline{I}lZ_w}{4\pi jk}$$

$$S = \frac{|\underline{I}|^2 Z_w}{8} \left(\frac{l}{\lambda} \right)^2 \sin^2 \theta$$



$$S = r^2 \frac{1}{2} \Re \left\{ \underline{\vec{E}} \times \underline{\vec{H}}^* \right\} \cdot \vec{e}_r = \frac{r^2}{2} \Re \left\{ \underline{E}_\theta \underline{H}_\phi^* \right\} = \frac{|\underline{p}|^2 k^4}{2Z_w} \sin^2 \theta$$

Hertz'scher Dipol: Strahlungsdiagramm



$$S = \frac{|\underline{I}|^2 Z_w}{8} \left(\frac{l}{\lambda} \right)^2 \sin^2 \theta$$

Hertz'scher Dipol: Abgestrahlte Leistung

$$\underline{E}_\theta = \underline{p} k^2 \frac{e^{-jkr}}{r} \sin \theta$$

$$\underline{H}_\phi = \frac{1}{Z_w} \underline{E}_\theta$$

$$S = \frac{|\underline{I}|^2 Z_w}{8} \left(\frac{l}{\lambda} \right)^2 \sin^2 \theta$$

$$S_{av} = \frac{P_{rad}}{4\pi} = \frac{|\underline{I}|^2 Z_w}{12} \left(\frac{l}{\lambda} \right)^2$$

Abgestrahlte Leistung

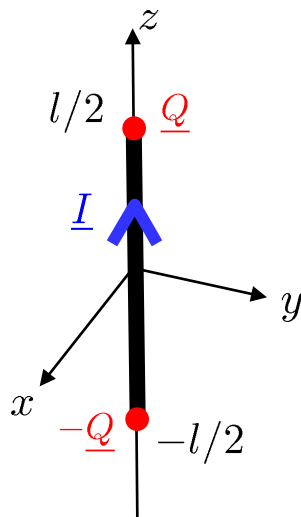
$$P_{rad} = \int_0^\pi \int_0^{2\pi} S(\theta) d\phi \sin \theta d\theta = \frac{|\underline{I}|^2 Z_w \pi}{3} \left(\frac{l}{\lambda} \right)^2$$

$$\int_0^\pi \sin^3 \theta d\theta = \frac{4}{3}$$

$$D = \frac{S_{max}}{S_{av}} = \frac{3}{2}$$

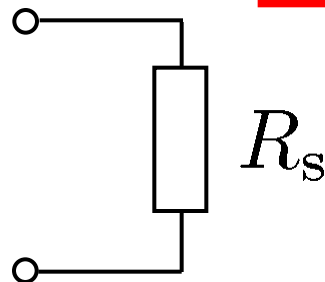
Richtfaktor ("*directivity*")

Hertz'scher Dipol: Strahlungswiderstand



$$P_{\text{rad}} = \frac{|\underline{I}|^2 Z_w \pi}{3} \left(\frac{l}{\lambda} \right)^2 = \frac{|\underline{I}|^2}{2} R_s$$

$$R_s = Z_w \frac{2\pi}{3} \left(\frac{l}{\lambda} \right)^2$$

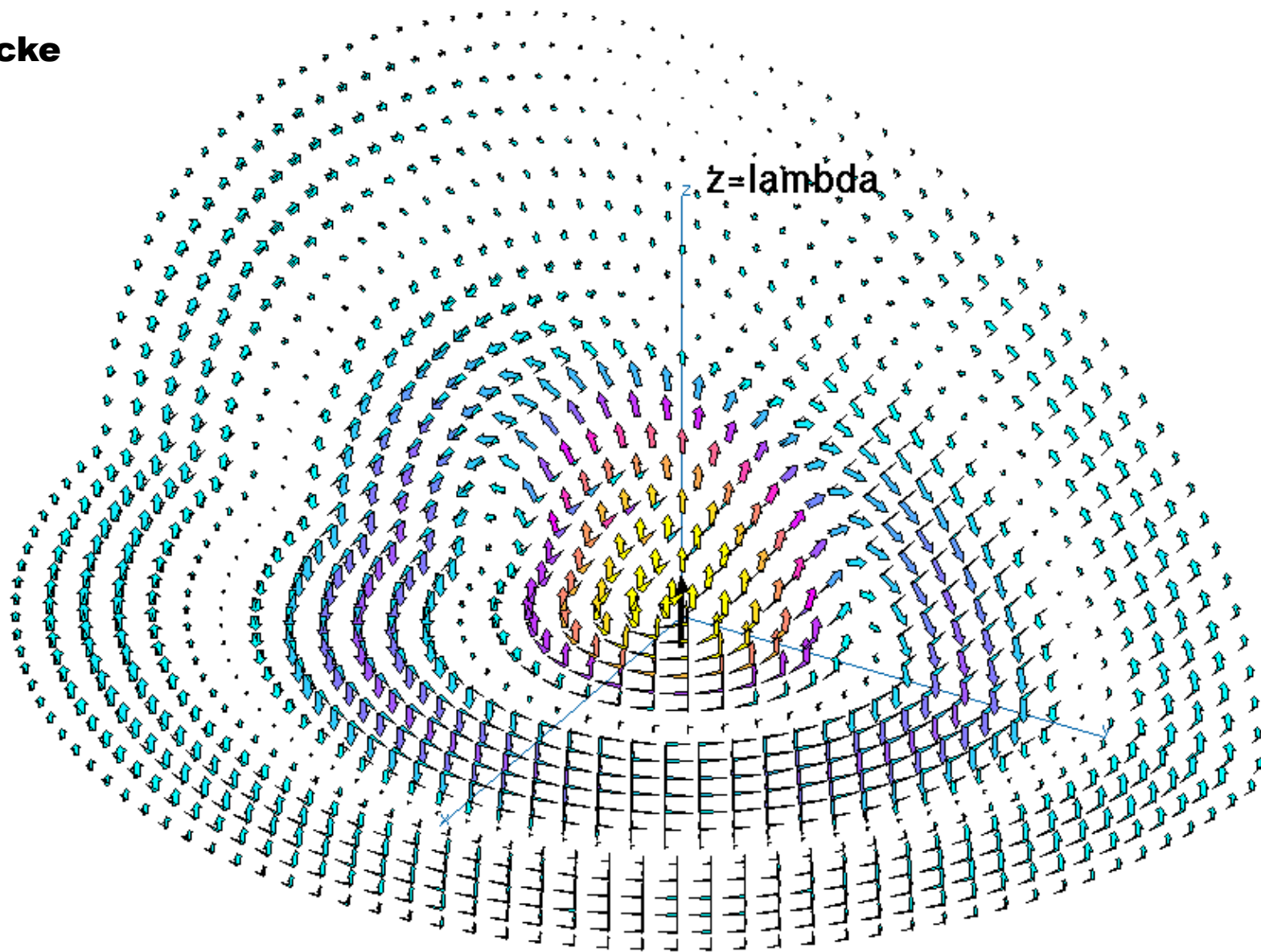


Sehr viel kleiner als
die Wellenimpedanz
des Raumes!

Hertz'scher Dipol

E-Feld: Pfeile

H-Feld: Dreiecke



Felder & Komponenten II