1 Playing with tensors

a) Write \( a_{ij}b_{jk} = c_{ik} \) in matrix notation.

b) Show that \( \delta_{ij}\delta_{ij} = 3 \).

c) Show that \( \delta_{ik}\delta_{jm}\delta_{ij} = \delta_{km} \).

d) Write down all intermediate steps to prove why we can write (p. 43)

\[
e_{ij} = \frac{1}{2} \left[ \delta_{ij} - \delta_{pq} \left( - \frac{\partial u_p}{\partial x_i} + \delta_{pi} \right) \left( - \frac{\partial u_q}{\partial x_j} + \delta_{qj} \right) \right]
\]

\[
= \frac{1}{2} \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{\partial u_p}{\partial x_i} \frac{\partial u_p}{\partial x_j} \right].
\]

2 Basal shear stress

Consider a glacier of \( H = 70 \text{ m} \) thickness at a surface slope of \( \alpha = -10^\circ \). Assume a stress state at the bedrock of the form (in the K-system):

\[
\sigma = \begin{pmatrix}
\delta & 0 & \gamma \\
0 & \delta & 0 \\
\gamma & 0 & \delta
\end{pmatrix}.
\]

with \( \delta = -\rho g H \cos(-\alpha) \) and \( \gamma = \rho g H \sin(-\alpha) \), see Section 2.2 of the script. Suppose there is a “bump” on the bedrock with faces that are inclined with respect to the horizontal by \( \beta_u = 30^\circ \) (upstream face) and \( \beta_d = -50^\circ \) (downstream face).

Compute the normal and tangential stresses acting on the upstream and downstream faces. (Hint: Compute first the unit normals to the two faces in the K-system of coordinates)

Note: The basal shear stress describes how much force is being transmitted from the glacier to the bedrock – and vice versa.
3 Strain rates

Strain rates can be determined by measuring the movement of stakes drilled into the ice. Assume that you have installed 3 stakes $A$, $B$ and $C$ on Rhone glacier in 2013: $B$ is located on the same flow line than $A$ but 100 m downstream, while $C$ is located in 100 m distance from $A$ across the glacier. When you come back after a year, Stakes $A$ and $B$ have moved by 30 m and 33 m, respectively, along the flow line. In addition, Stake $C$ has moved by 29 m along flow line, and with 2 m across the glacier and away from $A$.

(a) Represent the movement of Stakes A, B and C between 2013 and 2014 on a drawing.

(b) Calculate the components $\dot{\varepsilon}_{xx}$, $\dot{\varepsilon}_{yy}$, and $\dot{\varepsilon}_{xy}$ of the strain rate tensor.

(c) What can you say about the vertical strain rate?
   *(hint: use relation $\dot{\varepsilon}_{ii} = 0$ due to the incompressibility of ice)*

(d) Where are the stakes most probably located, in the accumulation area or in the ablation area?

(e) What are the principal strain rates in the horizontal plan (2D).
   *(hint: principal strain rates are the eigenvalues of the strain rate tensor, similarly to principal stresses)*