Volume change reconstruction of Swiss glaciers from length change data

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Received 10 February 2010; revised 17 July 2010; accepted 16 August 2010; published 18 November 2010.

[1] A novel method to reconstruct glacier volume changes from a measured length record is presented and tested for 13 glaciers in the Swiss Alps. The response of a glacier to changes in climate is modeled with a two-parameter dynamical system in the variables "length" and "volume". Driven by a history of equilibrium line altitude (ELA), the model yields variations of glacier length and volume. A dynamically equivalent simple model (DESM) is determined for each glacier by matching modeled and measured length changes. The volume changes predicted with the DESM agree well with measurements for 12 glaciers, whereas agreement is poor for one glacier with topographic breaks in the terminus area. For all glaciers, which are located in different climate regions, the length and volume changes are reproduced with the same ELA history. This agreement shows that the macroscopic glacier response to the climate history is well correlated over a whole mountain range. Modeling the future evolution of the glaciers under a constant present-day climate reveals that fast-reacting glaciers are close to equilibrium, whereas length and volume of the large valley glaciers would be reduced during the next century by an amount similar to the volume lost during the last 150 years.


1. Introduction

[2] Interest in past and future volume changes of mountain glaciers is high, since this class of glaciers is one of the main contributors to sea level rise at present and in the near future [e.g., Arendt et al., 2002; Meier et al., 2007; Bérthier et al., 2010]. Practical interest in the loss rate of glacier volume arises also from questions concerning river hydrology, water resources, and hydroelectric power production [e.g., Burlando and Rosso, 2002; Barnett et al., 2005; Hagg et al., 2007].

[3] Glacier volume changes are only known for a tiny subset of the world’s glaciers and only for the time periods when accurate maps or satellite data are available. Even for a densely populated region like the Alps with a long history of good topographic maps, the earliest accurate glacier maps stem from the 1870s, with just a handful of additional glacier surface elevation data sets [Bauder et al., 2007].

[4] The spatial and temporal coverage of glacier length change records makes them attractive to investigate glacier sensitivity to climate variations [e.g., Oerlemans, 2007; Lüthi and Bauder, 2010] and to infer mass balance variations [e.g., Klok and Oerlemans, 2003; Hoelzle et al., 2003; Oerlemans, 2005; Steiner et al., 2005, 2008]. The link between glacier area and volume changes has been made with kinematic wave theory [Nye, 1963; Hutter, 1983], linear response theory [Klok and Oerlemans, 2003], or by representing a glacier on a macroscopic scale as dynamical system, such as a critically damped harmonic oscillator [Harrison et al., 2003; Lüthi, 2009].

[5] In this contribution we introduce a novel method to infer glacier volume changes from glacier length records. The method is applied to 13 glaciers in the Swiss Alps with long records of glacier volume and length changes. The method relies on finding for each glacier a dynamically equivalent simple model (DESM) that exhibits the same variation of glacier length. A simple, macroscopic description of glacier dynamics is used, which is formulated as a dynamical system in the variables "length" and "volume", henceforth called the LV model [Lüthi, 2009]. The LV model is forced by a history of mass balance, parametrized as the variation of equilibrium line altitude (ELA), and yields variations of glacier length and volume. A DESM is obtained by finding a set of model parameters that lead to a good match of modeled and measured length changes. The volume changes resulting from the DESM agree remarkably well with measurements.

2. Data and Methods

2.1. Length and Volume Change Data

[6] The data set of glacier length changes from the Swiss Glacier Monitoring Network is used in this study
In this study length change records of 13 glaciers, for which volume changes are also available, are used. Figure 1 shows the location of the glaciers; Table 1 lists characteristic quantities. [Glaciological Reports, 1881–2009]. This publicly available data set contains 120 glacier length change records with yearly measurements. In this study length change records of 13 glaciers, for which volume changes are also available, are used. Figure 1 shows the location of the glaciers; Table 1 lists characteristic quantities. [7] The volume change data stem mainly from Bauder et al. [2007], complemented with unpublished data from ongoing work. The glacier volume change data were derived by subtracting two gridded digital elevation models obtained by digital photogrammetric analysis from aerial photographs or from digitized contours on old topographic maps. Further details on data sources and methodology are given by Bauder et al. [2007].

2.2. LV model

[8] We use a simplified representation of glacier dynamics as a two-variable dynamical system in the variables “length” $L$ and “volume” $V$ [Lüthi, 2009]. The dynamical system is formulated for unit width and reproduces the essential influence of mass balance and ice dynamics on glacier geometry on a macroscopic scale. Figure 2 illustrates the building blocks of the dynamical system: two reservoirs of volumes $V_A$ and $V_B$ which are linked by a flux element located at horizontal coordinate $G$. The ice flux through a vertical section at the equilibrium line is determined by ice thickness and surface slope according to the shallow ice approximation [Hutter, 1983]. The resulting dynamical system is [Lüthi, 2009, equation (40)]

$$\frac{dV}{dt} = \gamma V + \gamma ZL - \frac{\gamma m_b}{2} L^2$$

(1a)

$$\frac{dL}{dt} = \frac{1}{\tau_o} \left[ \left( \frac{V}{a} \right)^{1/2} - L \right],$$

(1b)

where $\gamma = \partial b/\partial z$ is the vertical gradient of local mass balance rate $b$ (in units of meter ice equivalent per year) and $m_b = \tan \beta$ is bed slope. Equation (1b) is a relaxation equation for the current glacier length $L$ with time constant $\tau_o$ (in years; $\tau_o$ should not be confused with the volume time scale $\tau_v$). The steady state length for the current volume $V$ is determined by a volume-length scaling relation of the form $V = a L^\mu$ with parameters $\mu = 7/5$ and $a$, which depends on $\gamma$ and $m_b$ [Lüthi, 2009, equation (21)], and which is continually updated for the current glacier length [Lüthi, 2009, equations (37) and (38)]. The dynamical system (1) is driven by a forcing in the term $Z(t) = Z_0 - z_{ELA}(t)$, where $Z_0$ is the highest point of the bed and $z_{ELA}(t)$ is the timedependent elevation of the equilibrium line (Figure 2).

[9] Each model run is initialized with a set of parameters $(\gamma, Z_0, \beta)$ for which a steady state is calculated and subsequently driven with an ELA history ($Z_0$ designates the value of $Z$ in the year 1600 when the ELA history begins). The dynamical system (1) is solved numerically with the PyDSTool toolkit [Clewley et al., 2004], which provides accurate and efficient ODE solvers.

2.3. Equilibrium Line History

[10] To drive the LV model and hence calculate glacier response, an ELA history is prescribed as forcing function in the $Z$ term of equation (1). In the LV model, local mass balance rate is given as linear function of elevation $b(z) = \gamma(z - z_{ELA})$, with a constant vertical gradient of local mass

Table 1. Characteristic Quantities of the 13 Glaciers in the Swiss Alps Which Were Used in This Study*

<table>
<thead>
<tr>
<th>Name</th>
<th>$L$ (km)</th>
<th>$A$ (km$^2$)</th>
<th>LC</th>
<th>VC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grosser Aletsch</td>
<td>20.5</td>
<td>82.2</td>
<td>118</td>
<td>6</td>
</tr>
<tr>
<td>Allalin</td>
<td>6.0</td>
<td>9.9</td>
<td>114</td>
<td>9</td>
</tr>
<tr>
<td>Basodino</td>
<td>1.8</td>
<td>2.8</td>
<td>84</td>
<td>6</td>
</tr>
<tr>
<td>Corbassière</td>
<td>9.0</td>
<td>16.0</td>
<td>71</td>
<td>4</td>
</tr>
<tr>
<td>Gomer</td>
<td>13.0</td>
<td>55.2</td>
<td>112</td>
<td>4</td>
</tr>
<tr>
<td>Gries</td>
<td>5.0</td>
<td>5.0</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>Morteratsch</td>
<td>7.5</td>
<td>16.6</td>
<td>122</td>
<td>4</td>
</tr>
<tr>
<td>Rhone</td>
<td>9.0</td>
<td>15.9</td>
<td>128</td>
<td>7</td>
</tr>
<tr>
<td>Schwarzberg</td>
<td>4.0</td>
<td>5.3</td>
<td>77</td>
<td>8</td>
</tr>
<tr>
<td>Silvretta</td>
<td>3.0</td>
<td>2.8</td>
<td>49</td>
<td>8</td>
</tr>
<tr>
<td>Trift</td>
<td>7.0</td>
<td>20.5</td>
<td>17</td>
<td>7</td>
</tr>
<tr>
<td>Unteraar</td>
<td>11.5</td>
<td>22.7</td>
<td>114</td>
<td>7</td>
</tr>
<tr>
<td>Zinal</td>
<td>6.8</td>
<td>13.4</td>
<td>114</td>
<td>4</td>
</tr>
</tbody>
</table>

*Length (2004), area (1999), and length (LC) and volume (VC) change records.
The history of ELAs was based on gridded values for temperature and precipitation from a reconstruction of European climate since 1600 [Casty et al., 2005]. A bilinear relation between ELA and changes of temperature $\Delta T$ and scaled precipitation variation $\Delta P/P_{\text{ref}}$ was assumed of the form

$$z_{\text{ELA}}(t) = a + b \Delta T(t) + c \left( 1 - \frac{\Delta P(t)}{P_{\text{ref}}} \right),$$

which is equivalent to a standard climate-ELA relation [Ohmura et al., 1992, equation (1)] if the derivatives $\partial T/\partial z$ and $\partial P/\partial T$ are constant. The values of the constants were determined by fitting the parametrized ELA history to reconstructed ELA variations for several Swiss glaciers [Huss et al., 2008, also unpublished data, 2009]. The best agreement between the climate parameters and ELA reconstructions was found for summer (JJA) temperature and yearly precipitation, with the constants $a = 2738$ m, $b = 101$ m K$^{-1}$, $c = 200$ m, and $P_{\text{ref}} = 2000$ mm. A constant value of $\gamma = 0.008$ a$^{-1}$ gave the best agreement for all glaciers and was assumed throughout this study.

Figure 3 shows that the modeled length change depends heavily on the climate history assumed between 1650 and 1850 (the “Little Ice Age” (LIA)), especially for glaciers with a long response time such as Grosser Aletschgletscher. As has been discussed elsewhere [Lüthi and Bauder, 2010], the ELA reconstruction based on temperature and precipitation alone is not suitable to produce any of the big and rapid glacier advances observed during the LIA. To achieve a match to measured length changes before 1910, the ELA had to be lowered by 100–200 m for certain periods in the time span 1650–1850 (Figure 3). No match for length changes

![Figure 3](image_url)

Figure 3. (a) Length changes for LV models, driven by different ELA histories, that fit best the measured length changes of Grosser Aletschgletscher (symbols). Dotted line corresponds to a climate reconstruction alone; dashed and solid lines indicate altered ELA histories. (b) The three climate scenarios, plotted as 11 year smoothed ELA variations, which were used to drive the LV model.

![Figure 4](image_url)

Figure 4. DESM length changes (lines) for all glaciers are compared to measured length changes (symbols). Glacier names are given next to curves.
before 1860 was attempted so that the assumed ELA history before 1860 is somewhat arbitrary.

2.4. Optimization Procedure

[12] The method employed to infer glacier volume change from length variations relies on finding a DESM that reproduces the length measurements. To this aim, the LV model was initialized to a steady state for a pair of model parameters $b$ and $Z_0$ and subsequently forced with the ELA history since 1600, shown in Figure 3b (solid line). For each glacier, a DESM (i.e., a pair of parameters $b$ and $Z_0$) was found by minimizing the root mean square of the difference between measured and modeled absolute length changes with an optimization procedure (the r-algorithm [Shor, 1985]).

[13] An alternative procedure, which does not require any assumptions about the climate history before the first measurement, is to determine initial length and volume ($L_0$, $V_0$) within the same optimization procedure used to determine the parameters $b$ and $Z_0$ of the DESM. For glaciers with a long length change record, the difference in model parameters is less than 1%, and calculated length changes are almost identical. For glaciers with a short length change history (compared to their response time), the alternative procedure yields DESM model parameters which lead to unrealistic length changes for the time span between 1850 and the first measurements. In this study we used the first approach, starting from a steady state in 1600 and using the ELA history of Figure 3b (solid line). With this approach, all model glaciers are in a consistent transient state in 1850, which results from the same climate history.

3. Results

[14] Figure 4 shows that for all of the 13 investigated glaciers, the DESM closely reproduces the measured length changes. For most of the 91 glaciers from the Swiss Glacier Monitoring Service a similarly good agreement was obtained [Lüthi and Bauder, 2010], with the exception of strongly debris-covered glacier tongues and for glacier tongues moving through strong breaks in bed slope during their advance or retreat.

[15] The DESM also calculates volume changes which are compared to measurements in Figure 5. Since the LV model is formulated for a unit width (i.e., without consideration of lateral extent), the modeled volumes $V_{LV}$ of a longitudinal section have to be scaled by a factor $W$, which is equivalent to a glacier width averaged vertically and along flow. The values of $W$ are found by linear scaling of the modeled volume changes $\Delta V_{LV}$ to measured volume changes, resulting in scaled model volumes $\Delta V_{s,2}$ or including an offset, as $\Delta V_{s,2} = W_2 \Delta V_{LV} + V_0$.

[16] The agreement between the modeled and measured volume changes shown in Figure 5 is generally good, with a slightly better agreement for $\Delta V_{s,2}$. We emphasize that the modeled volume changes are a result of the LV model run which matched best the length changes. No tuning of the modeled volumes was performed except for the linear scaling described above.
Table 2. Comparison of Characteristic Glacier Parameters and Model Results

<table>
<thead>
<tr>
<th>Name</th>
<th>$\beta_{\text{tot}}$ (deg)</th>
<th>$\beta_{\text{low}}$ (deg)</th>
<th>$\beta_{L,V}$ (deg)</th>
<th>$L_{\text{tot}}$ (km)</th>
<th>$L_{L,V}$ (km)</th>
<th>$V_F$ (km$^3$)</th>
<th>$V_{L,V}$ (km$^3$)</th>
<th>$H_m$ (m)</th>
<th>$\tau_s$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grosser Aletsch</td>
<td>5.0</td>
<td>3.2</td>
<td>3.0</td>
<td>20.5</td>
<td>12.5</td>
<td>15.4</td>
<td>14.3</td>
<td>255</td>
<td>146</td>
</tr>
<tr>
<td>Allalin</td>
<td>8.5</td>
<td>4.8</td>
<td>9.0</td>
<td>6.0</td>
<td>22.8</td>
<td>0.9</td>
<td>6.2</td>
<td>206</td>
<td>19</td>
</tr>
<tr>
<td>Basodino</td>
<td>15.0</td>
<td>15.7</td>
<td>16.0</td>
<td>1.8</td>
<td>2.3</td>
<td>0.036</td>
<td>0.2</td>
<td>68</td>
<td>30</td>
</tr>
<tr>
<td>Corbassière</td>
<td>8.4</td>
<td>7.2</td>
<td>15.0</td>
<td>9.0</td>
<td>2.1</td>
<td>1.5</td>
<td>0.7</td>
<td>68</td>
<td>35</td>
</tr>
<tr>
<td>Gorner</td>
<td>8.0</td>
<td>1.2</td>
<td>2.8</td>
<td>13.0</td>
<td>14.7</td>
<td>6.1</td>
<td>8.1</td>
<td>281</td>
<td>151</td>
</tr>
<tr>
<td>Gries</td>
<td>7.4</td>
<td>3.9</td>
<td>6.8</td>
<td>5.0</td>
<td>7.1</td>
<td>0.8</td>
<td>1.5</td>
<td>149</td>
<td>57</td>
</tr>
<tr>
<td>Morteratsch</td>
<td>11.4</td>
<td>4.5</td>
<td>4.0</td>
<td>7.5</td>
<td>7.5</td>
<td>1.1</td>
<td>1.6</td>
<td>185</td>
<td>129</td>
</tr>
<tr>
<td>Rhone</td>
<td>6.6</td>
<td>5.0</td>
<td>10.0</td>
<td>9.0</td>
<td>4.2</td>
<td>2.2</td>
<td>1.3</td>
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<td>Schwarzberg</td>
<td>9.0</td>
<td>6.6</td>
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<td>4.0</td>
<td>2.6</td>
<td>0.4</td>
<td>0.6</td>
<td>68</td>
<td>24</td>
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<tr>
<td>Silvretta</td>
<td>8.4</td>
<td>7.0</td>
<td>15.2</td>
<td>3.0</td>
<td>1.1</td>
<td>0.2</td>
<td>0.1</td>
<td>55</td>
<td>48</td>
</tr>
<tr>
<td>Trift</td>
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<td>9.5</td>
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<td>7.0</td>
<td>34.6</td>
<td>1.1</td>
<td>8.9</td>
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<td>Unterar</td>
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<td>2.6</td>
<td>4.8</td>
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<td>3.1</td>
<td>162</td>
<td>110</td>
</tr>
<tr>
<td>Zinal</td>
<td>10.1</td>
<td>6.6</td>
<td>7.8</td>
<td>6.8</td>
<td>2.4</td>
<td>1.0</td>
<td>0.6</td>
<td>93</td>
<td>91</td>
</tr>
</tbody>
</table>

*Mean bed slopes $\beta_{\text{tot}}$ for the whole glacier and $\beta_{\text{low}}$ for the terminus area were read from a 2004 topographic map; $\beta_{L,V}$ is the DESM bed slope. Approximate total glacier length in 2004 $L_{\text{tot}}$ was read from a map; $L_{L,V}$ is the corresponding DESM length. Glacier volumes $V_F$ for 1999 were taken from Farinotti et al. [2009b]; $V_{L,V}$ is the linearly scaled DESM volume. The DESM also predicts mean ice thickness $H_m$ and the volume time scale $\tau_s$. 

[17] Estimated volumes are inaccurate for Grosser Aletschgletscher in 1926, Gornergletscher in 1931 (Figure 5f), and Unteraargletscher in 1927 (Figure 5d). The only two volumes that could not be closely reproduced are those of Allalingletscher in 1879 and 1932 (Figure 5c) and are likely attributable to a topographic effect. During the LIA, Allalingletscher reached into a flat valley floor where it dammed several times the river Vispa to form Lake Mattmark. During the retreat over the steep slope, two big ice avalanches in 1965 and 1999 removed parts of the terminus [Röthlisberger and Kasser, 1978; Funk-Salami, 2001], which now has receded into a flat area. The behavior of this glacier seems to be too complicated to be reproduced with such a simple model.

4. Discussion

4.1. Dynamically Equivalent Simple Model

[18] For each of the 13 analyzed glaciers, a DESM was found that closely reproduces the measured absolute length changes under the same ELA history. These DESMs also yield volume variations that reproduce measured volume changes. The existence of a DESM for all 13 glaciers implies that the details of glacier geometry and flow have little influence on the overall behavior of the macroscopic quantities “area” (represented as “length”) and “volume”. This statement might be invalid for glaciers with strong variations in terminus geometry, as discussed above for the example of Allalingletscher. Close to a steady state the LV model is equivalent to a driven, linearly damped harmonic oscillator [Harrison et al., 2003] which is slightly over-damped [Lüthi, 2009]. Therefore, valley glaciers essentially behave like this well-studied dynamical system.

4.2. Geometric Parameters

[19] The DESMs share characteristic quantities with the glaciers they represent: bed slope, vertical extent, and length. Ideally, glacier length and mean slope would agree, and the DESM parameters could be read from a map. It is instructive to compare the inferred quantities for the individual glaciers: Table 2 shows bed inclinations for the whole glacier $\beta_{\text{tot}}$, the lower part of the glacier $\beta_{\text{low}}$, and the inclination of the best-fitting LV model $\beta_{L,V}$. For several glaciers, the bed slope of the lower part $\beta_{\text{low}}$ (roughly from the point at which the glacier flows through a valley of approximately constant width) is picked by optimizing on the length changes (Grosser Aletsch, Basodino, Gorner, Morteratsch, Trift). All other glaciers have a more complex topography, which precludes inference of the DESM slope.

[20] Bed slopes of several DESM are much steeper than for the real glaciers. The effect of a steeper slope in the LV model is a more dynamic behavior, i.e., a shorter response time. This might hint to the importance of basal motion which is completely ignored in the LV model. On a steeper slope, more ice is transported from the accumulation area to the ablation area (for constant ice thickness) which mimics the effect of basal motion.

[21] Modeled and measured glacier lengths do not agree for obvious reasons. Most glaciers have a wide accumulation area and a narrow ablation area, with different mass balance gradients, or even constant accumulation rates in large parts of the accumulation area. A modeled length that far exceeds the real length might indicate that a glacier has a wide accumulation area compared to the width of its ablation area.

[22] A detailed correspondence between geometrical parameters of glaciers and LV models could not be found. Unpublished model experiments with a transient full system flow line model indicate that a DESM for a simple glacier on kinked bed can only be found by matching the dynamical response under a given ELA history [Singer, 2009]. Neither mean slope, nor length-weighted slopes of glacier parts are sufficient to infer DESM parameters.

4.3. Climate

[23] Even though climatic conditions differ considerably between different parts of the Swiss Alps, glacier mass balance variations seem to be closely linked. In this study we have found that the macroscopic response of glaciers in different climates can be explained by the same ELA variation. Glacier length change depends on the relative magnitudes of ice flux toward the glacier terminus and the local mass balance rate near the glacier tongue. While the former varies only slowly through geometry adjustment, the latter varies rapidly and is mainly driven by summer ablation. Previous studies [e.g., Vincent et al., 2004] have shown that...
the summer climate, which affects glacier mass balance, is similar over the entire Alps.

The measured length changes could only be reproduced with an ELA history which was altered from the history calculated from temperature and precipitation records. The ELA had to be lowered by 150–200 m during some periods of the LIA, which would correspond to an additional temperature change of −2 K or a precipitation change of 2000 mm a−1, both of which are very large and seem unlikely. Whether such an ELA lowering was caused by increased precipitation within the inner Alpine region or was due to a different energy (i.e., radiation) budget, is still a matter of debate [Nesje and Dahl, 2003; Vincent et al., 2005].

4.4. Volume Change

The volume changes are well reproduced by the DESM except for the large valley glaciers in the 1920s–1930s and for Allalingletscher. It is tempting to infer absolute glacier volumes from the scaled LV model volumes. The LV model volumes $V_{s,2}$ for the year 1999 are given in Table 1. For Grosser Aletschgletscher, the agreement with the volume for 1999 calculated by Farinotti et al. [2009] is striking and is within 30% for Gorner, Morteratsch, Unteraar, and Zinal. For the other glaciers, there is no agreement, and volumes differ by a factor of up to 8 in the case of Basodino. The agreement seems to be restricted to valley glaciers with simple topography.

4.5. Time Scales and Steady States

In a varying climate, glaciers are rarely in a steady state. How fast and to what extent a glacier reacts to changes in climate is largely determined by a single parameter, the volume time scale $\tau_v$, [Johannesson et al., 1989; Harrison et al., 2001]. The volume time scale depends on a combination of geometric parameters and is inversely proportional to the mass balance gradient $\gamma$ [Harrison et al., 2003; Lüthi, 2009]

$$\tau_v = -\left(\gamma + \frac{b_L}{H_e}\right)^{-1} = \frac{H_e}{\gamma (m_b L - Z - H_e)}.$$  

The volume time scale depends on the balance rate at the terminus $b_L = \gamma (Z_0 - m_b L - z_{ELA}) = \gamma (Z - m_b L)$ and on the effective ice thickness $H_e = (dL/dV)^{-1}$, which is the slope of the volume-length relationship of steady state glaciers (shown with dotted lines in Figure 6). The volume time scales calculated from the DESM parameters are given in Table 2.

How much the glacier lengths and volumes differ from the closest steady state values can be assessed by comparing the instantaneous length and volume to the locus of all steady states, calculated by varying $Z$ at constant $\gamma$ and $\beta$. Figure 6 shows the trajectories of the individual glaciers in phase space diagrams (solid lines), together with their steady states (dotted lines). As would be expected, the glacier volumes exceeded their steady state values during the LIA maximum about 1850 (the hook-like features at the right ends of the lines in Figure 6). Since 1900, all glaciers were mostly below their steady state volumes and were therefore in a retreat configuration.

Figure 6 illustrates that glaciers with short volume time scales $\tau_v$ (Trift, Allalin, Schwarzberg, and Gries) stayed relatively close to their respective steady states throughout the 20th century and were probably balanced.
around 1960, whereas glaciers with volume time scales exceeding 100 years (Grosser Aletsch, Gorner, Morteratsch, and Unteraar) are far out of equilibrium.

[29] The “dynamic health” of the glaciers can be analyzed by modeling the future length and volume evolution for the present climate, with the ELA held constant at its average value between 1988 and 2008. The model predicts that most glaciers will undergo considerable geometry changes until they are in equilibrium with climate. Figure 7 shows that fast-reacting glaciers will be reduced little in size, whereas the slowly reacting glaciers will lose the same amount of volume as they did during the last 150 years until they reach a new steady state after centuries.

5. Conclusions

[30] A novel method has been presented to infer glacier volume changes from length changes with help of a two-parameter dynamical system of macroscopic glacier dynamics. For 12 glaciers, a dynamically equivalent simple model (DESM) was found which reproduces their length and volume changes. The method failed for one glacier with a terminus area of strongly varying topography.

[31] The proposed method can be used to interpolate measured glacier length and glacier volume variations in a physically meaningful way. The only requirement for this process is an ELA history to drive the model and length change data for the duration of roughly one volume time scale. Both requirements might be difficult to determine for mountain areas with sparser data than is available in the Alps. To obtain absolute volume changes, the DESM volume changes have to be scaled by at least one measurement, which requires the difference of two digital elevation models.

[32] Given the good agreement between modeled and measured volume changes, the presented method can be used to assess the dynamic health of glaciers through comparison of their present state with the steady state they would reach under a constant climate. The method can also be used to predict length and volume changes under scenarios of future climate. Since the parameters influencing the area adjustment time scale are computed from the time-varying glacier length, the LV model will correctly reproduce the increasing area and volume response times of shorter glaciers on a bed of constant slope [Lüthi, 2009]. The predictions of length and volume will stay accurate as long as the glacier terminus does not move through a strong topographic break or the character of the glacier terminus changes completely (e.g., from a valley glacier to a wide slope or a cirque, or the formation of a proglacial lake). If the model glacier approaches a critical minimum size [Lüthi, 2009, section 6.30], the assumptions of the LV model are not fulfilled anymore and a different model should be used.

[33] Length and volume change records for many glaciers in different climatic areas of the Alps can be reproduced with a single ELA history and a single vertical gradient of mass balance rate. This result is of significance since it implies that the climatic forcing, which causes glacier changes, can be similar over a whole mountain range. This opens up the possibility of inference of glacier length and volume changes over a whole mountain range from only a few map data points. Whether such an approach also works for mountain ranges with large spatial variations in precipitation regime (e.g., coastal ranges) has to be investigated.

**Figure 7.** Phase space diagrams of modeled past (solid) and future (gray) length and volume changes for constant climate. Meaning of lines and symbols same as in Figure 6.
[34] Glacier length change data are available for many glaciers in the Alps and elsewhere. Complementary length change data can be obtained from different sources such as old maps, photographs, paintings, and geomorphological evidence [e.g., Nussbaumer et al., 2005]. If volume change data are not available, an area–volume scaling relation [Bahr et al., 1997; Lüthi, 2009] for climate phases close to equilibrium, possibly adapted to known glacier volumes, could be used. Estimates of ice volume changes of a whole mountain range could then be inferred.

35 Acknowledgments. We acknowledge the inspiring discussions with W. Harrison and M. Huss. Results of ELA changes and mass balance parametrization were made available by M. Huss, and D. Farinotti provided ice volume estimates. B. Nedela carefully digitized old topographic maps, and the DEMs were produced by H. Bösö. This effort received support from swisstopo, ETH research grant TH–17 06–1, and BAFU project CCHydro. Valuable comments by W. Harrison and two anonymous referees helped to clarify the presentation. This work was funded by Swiss National Science Foundation project FUGE (grant 406140_125997/1) and EU–FP7 project ACQWA (grant 212250).

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