Exercise 7 - Recap

1 The 3D Schrödinger Equation

The 3D Schrödinger equation is given by

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{-\hbar^2}{2m}\nabla^2\Psi + V\Psi.$$

Similarly as in the 1D, we assume the potential is time-independent. Then, we can use separation of variable and obtain the TISE

$$\hat{H}\psi = E\psi,$$

where $\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$.

2 Hydrogen Atom

For the hydrogen atom the potential is given by Coulomb attractation. That is,

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}.$$

By rewriting the TISE in spherical coordinates we obtain

$$\psi_{n,l,m_l} = R_{n,l}(r)Y_l^{m_l}(\theta,\varphi),$$

where $R_{n,l}(r)$ and $Y_l^{m_l}(\theta,\varphi)$ can be found in tables. The energy levels are

$$E_n = -\left(\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right) \approx -\frac{13.6\,\mathrm{eV}}{n^2}.$$

We can also define the Bohr radius, denoted by a and given by

$$a = \frac{4\pi\varepsilon_0\hbar^2}{me^2} \approx 0.0529\,\mathrm{nm}.$$

Remark. Since E_n's are discrete only the phototon energies $E_2 - E_1, E_3 - E_1, \ldots$ can be absorbed by the hydrogen atom.