A Probability Theory

Let Y be a discrete random variable (DV) with sample space $\mathcal{Y} = \mathbb{N}$ and probability function $P(Y = y)$. Let X be a continuous random variable (CV) with sample space $\mathcal{X} = \mathbb{R}$ and probability density function $\rho(x)$.

A.1 Normalization

$$\text{DV:} \ \sum_{y=0}^{+\infty} P(Y = y) = 1$$
$$\text{CV:} \ \int_{-\infty}^{+\infty} \rho(x)\,\mathrm{d}x = 1$$

A.2 Expected Value

$$\begin{aligned}
\text{DV:} \qquad \langle Y \rangle &= \sum_{y=0}^{+\infty} y P(Y = y) \\
\langle f(Y) \rangle &= \sum_{y=0}^{+\infty} f(y) P(Y = y) \\
\text{CV:} \qquad \langle X \rangle &= \int_{-\infty}^{+\infty} x \rho(x)\,\mathrm{d}x \\
\langle f(X) \rangle &= \int_{-\infty}^{+\infty} f(x) \rho(x)\,\mathrm{d}x
\end{aligned}$$

Linearity of the expected value:

$$\langle aY + b \rangle = a \cdot \langle Y \rangle + b \qquad a, b \in \mathbb{R}$$
$$\langle aX + b \rangle = a \cdot \langle X \rangle + b \qquad a, b \in \mathbb{R}$$

A.3 Variance

$$\begin{aligned}
\mathrm{Var}(Y) = \sigma_Y^2 &= \langle (Y - \langle Y \rangle)^2 \rangle \\
&= \langle Y^2 - 2\langle Y \rangle Y + \langle Y \rangle^2 \rangle \\
&= \langle Y^2 \rangle - 2\langle Y \rangle \langle Y \rangle + \langle Y \rangle^2 \\
&= \langle Y^2 \rangle - \langle Y \rangle^2 \\
\mathrm{Var}(X) = \sigma_X^2 &= \langle (X - \langle X \rangle)^2 \rangle \\
&= \langle X^2 - 2\langle X \rangle X + \langle X \rangle^2 \rangle \\
&= \langle X^2 \rangle - 2\langle X \rangle \langle X \rangle + \langle X \rangle^2 \\
&= \langle X^2 \rangle - \langle X \rangle^2
\end{aligned}$$

$$\text{DV:} \ \sigma_Y^2 = \sum_{y=0}^{+\infty} (y - \langle Y \rangle)^2 P(Y = y)$$
$$\text{CV:} \ \sigma_X^2 = \int_{-\infty}^{+\infty} (x - \langle X \rangle)^2 \rho(x)\,\mathrm{d}x$$