Do Self-driving Cars Swallow Public Transport?

A Game-theoretical Perspective on Transportation Systems

Nicolas Lanzetti[1,2] Gioele Zardini[1,2] Maximilian Schiffer[1,3] Michael Ostrovsky[4] Marco Pavone[1]

[1]Autonomous Systems Lab (ASL), Stanford University

[2]Automatic Control Laboratory (IfA) & Institute for Dynamic Systems and Control (IDSC), ETH Zürich

[3]TUM School of Management, Technische Universität München

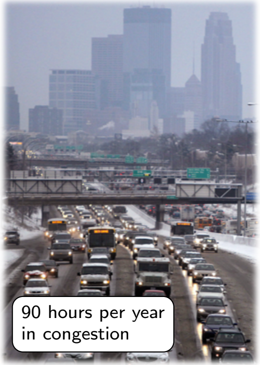
[4]Stanford Graduate School of Business, Stanford University

INFORMS Annual Meeting

22nd October, 2019

Motivation

Challenges

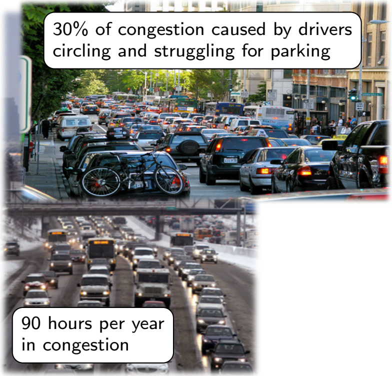


90 hours per year in congestion

Data from: INRIX, International Parking Institute, Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

Motivation

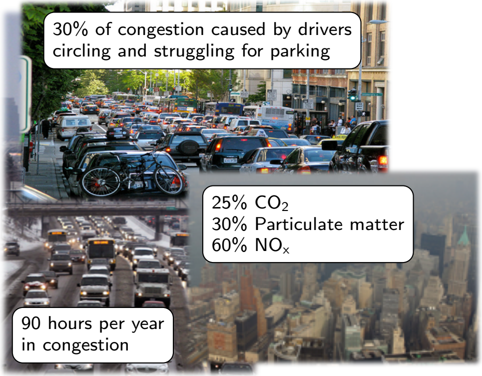
Challenges



Data from: INRIX, International Parking Institute,
Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

Motivation

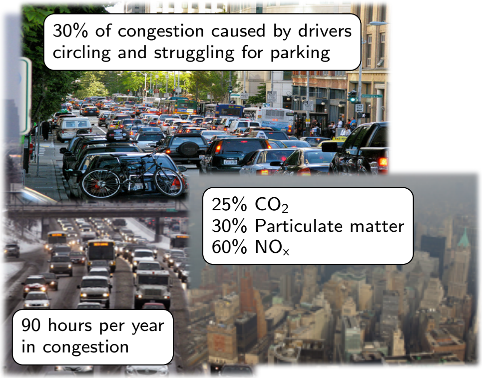
Challenges



Data from: INRIX, International Parking Institute,
Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

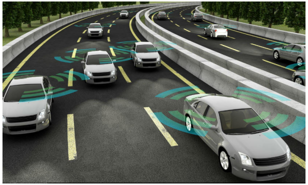
Motivation

Challenges



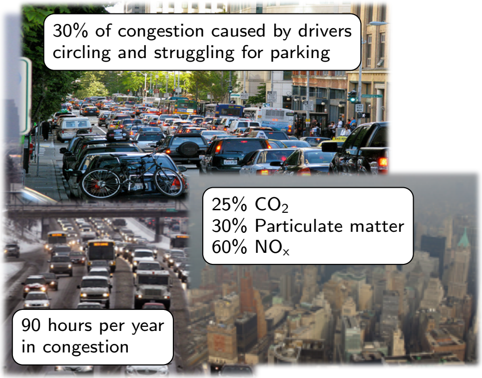
Data from: INRIX, International Parking Institute, Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

Autonomous Mobility-on-Demand Systems



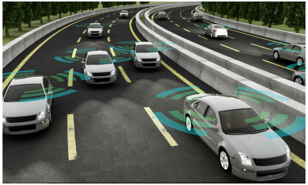
Motivation

Challenges



Data from: INRIX, International Parking Institute, Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

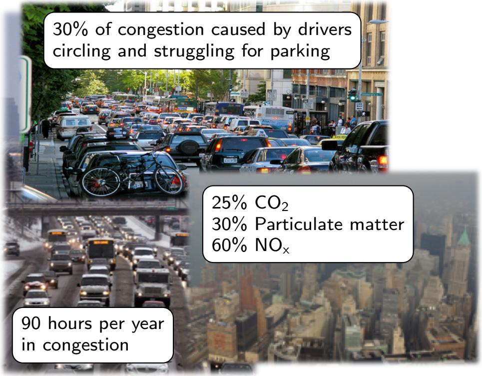
Autonomous Mobility-on-Demand Systems



- Ride-hailing fleet of (electric) self-driving cars.

Motivation

Challenges



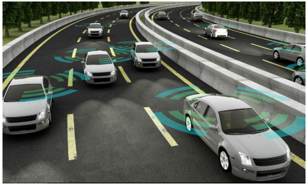
30% of congestion caused by drivers circling and struggling for parking

25% CO_2
30% Particulate matter
60% NO_x

90 hours per year in congestion

Data from: INRIX, International Parking Institute, Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

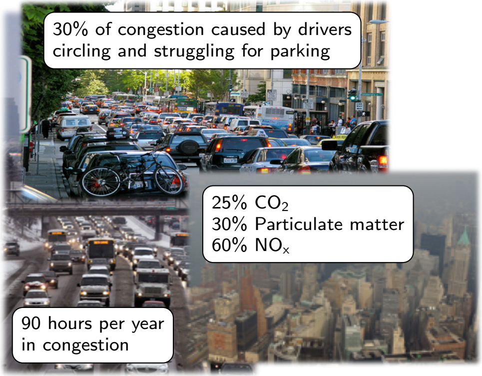
Autonomous Mobility-on-Demand Systems



- Ride-hailing fleet of (electric) self-driving cars.
- Controlled by a central operator that

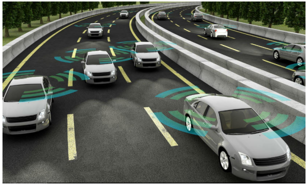
Motivation

Challenges



Data from: INRIX, International Parking Institute, Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

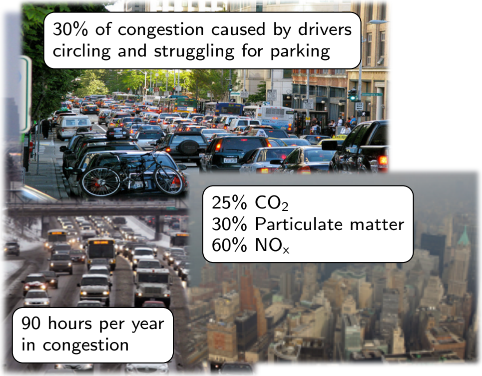
Autonomous Mobility-on-Demand Systems



- Ride-hailing fleet of (electric) self-driving cars.
- Controlled by a central operator that
  - assigns customer requests to vehicles;

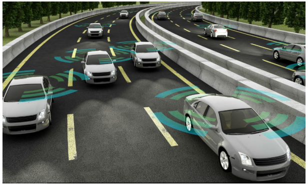
Motivation

Challenges



Data from: INRIX, International Parking Institute,
Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

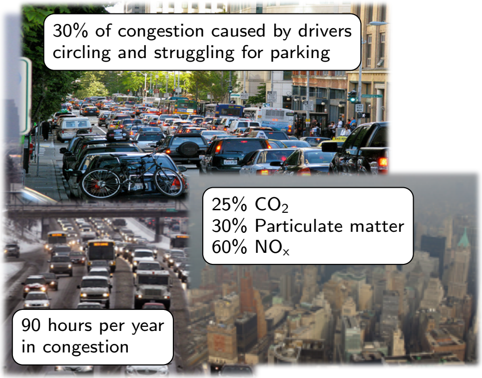
Autonomous Mobility-on-Demand Systems



- Ride-hailing fleet of (electric) self-driving cars.
- Controlled by a central operator that
  - assigns customer requests to vehicles;
  - decides on vehicle routes;

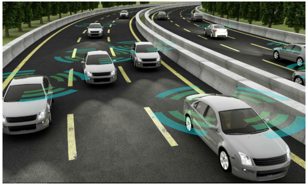
Motivation

Challenges



Data from: INRIX, International Parking Institute, Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

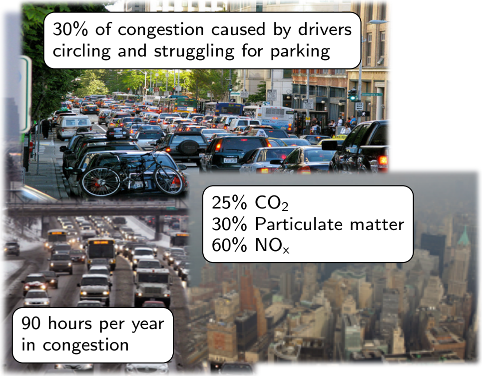
Autonomous Mobility-on-Demand Systems



- Ride-hailing fleet of (electric) self-driving cars.
- Controlled by a central operator that
  - assigns customer requests to vehicles;
  - decides on vehicle routes;
  - rebalances the fleet.

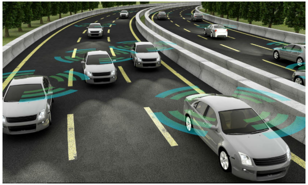
Motivation

Challenges



Data from: INRIX, International Parking Institute, Statistical Pocketbook 2018, Aptiv, World Economic Forum, BCG.

Autonomous Mobility-on-Demand Systems



- Ride-hailing fleet of (electric) self-driving cars.
- Controlled by a central operator that
  - assigns customer requests to vehicles;
  - decides on vehicle routes;
  - rebalances the fleet.

-30% travel time | **-44% parking places** | **-66% emissions**

Data from: Aptiv, World Economic Forum, BCG.

Motivation

Optimists

- fewer, better utilized vehicles;
- improved pooling, fair matching;
- less congestion, balanced routing.

10,603 views | Feb 8, 2016, 01:08pm

The Virtuous Cycle Between Driverless Cars, Electric Vehicle[s] Car-Sharing Services

Motivation

Optimists

- fewer, better utilized vehicles;
- improved pooling, fair matching;
- less congestion, balanced routing.

10,603 views | Feb 8, 2016, 01:08pm

The Virtuous Cycle Between Driverless Cars, Electric Vehicle Car-Sharing Services

Pessimists

- increased traffic;
- worsened modal split;
- cannibalization of public transport.

July 20, 2018

Pave Over the Subway? Cities Face Tough Bets on Driverless Cars

Motivation

Optimists

- fewer, better utilized vehicles;
- improved pooling, fair matching;
- less congestion, balanced routing.

10,603 views | Feb 8, 2016, 01:08pm

The Virtuous Cycle Between Driverless Cars, Electric Vehicle Car-Sharing Services

Pessimists

- increased traffic;
- worsened modal split;
- cannibalization of public transport.

July 20, 2018

Pave Over the Subway? Cities Face Tough Bets on Driverless Cars

Can AMoD systems cannibalize public transport?

Aims & Scope

Contribution

We present the first algorithmic framework that

- captures the dynamics between multiple mobility service providers and customers;

Aims & Scope

Contribution

We present the first algorithmic framework that

- captures the dynamics between multiple mobility service providers and customers;
- considers constraints of a complex real-world transportation network; and

Aims & Scope

Contribution

We present the first algorithmic framework that

- captures the dynamics between multiple mobility service providers and customers;
- considers constraints of a complex real-world transportation network; and
- allows for multimodal customer decisions.

Problem Setting – Who is playing?

Stakeholder	Role	Goal

Problem Setting – Who is playing?

	Stakeholder	**Role**	**Goal**
	Mobility Service Providers	Offer mobility services	Profit

Problem Setting – Who is playing?

	Stakeholder	Role	Goal
	Mobility Service Providers	Offer mobility services	Profit
	Municipalities	Offer mobility services	Social welfare

Problem Setting – Who is playing?

	Stakeholder	**Role**	**Goal**
	Mobility Service Providers	Offer mobility services	Profit
			vs.
	Municipalities	Offer mobility services	Social welfare

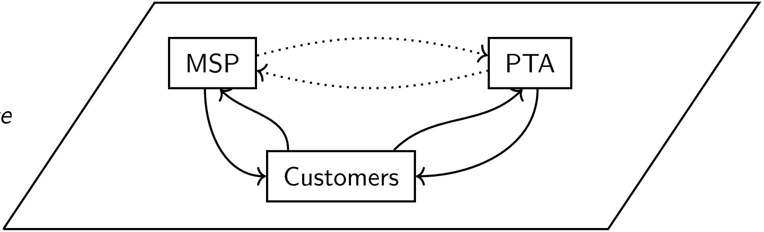
Problem Setting – Who is playing?

	Stakeholder	Role	Goal
	Mobility Service Providers	Offer mobility services	Profit
			vs.
	Municipalities	Offer mobility services	Social welfare
	Customers	Request mobility services	Individual benefit

Problem Setting – A Two-level System

Game Theory

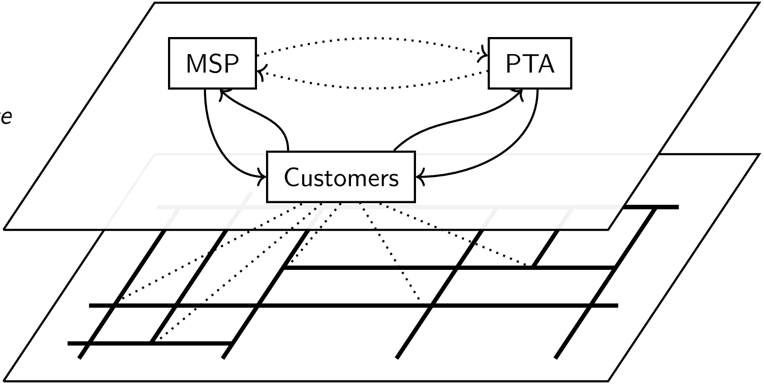
Transportation Market Place



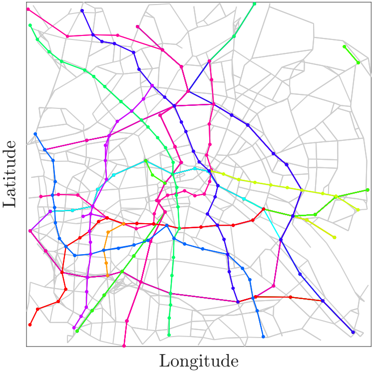
Problem Setting – A Two-level System

Game Theory
Transportation Market Place

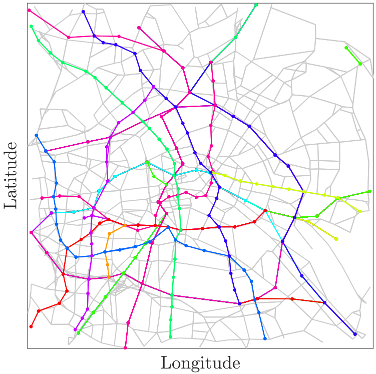
Transportation Research
Transportation Network

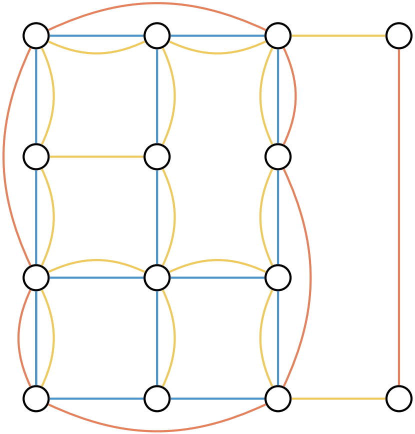


Modeling – Graph Representation



Modeling – Graph Representation



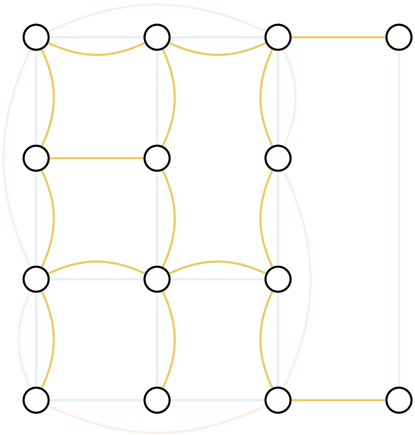


Modeling – Graph Representation

$\mathcal{G}_0$: Free Subgraph

$\mathcal{G}_1$: Subgraph controlled by operator 1

$\mathcal{G}_2$: Subgraph controlled by operator 2

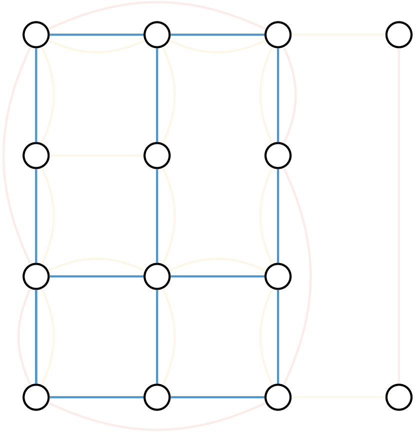


Modeling – Graph Representation

$\mathcal{G}_0$: Free Subgraph

$\mathcal{G}_1$: Subgraph controlled by operator 1

$\mathcal{G}_2$: Subgraph controlled by operator 2

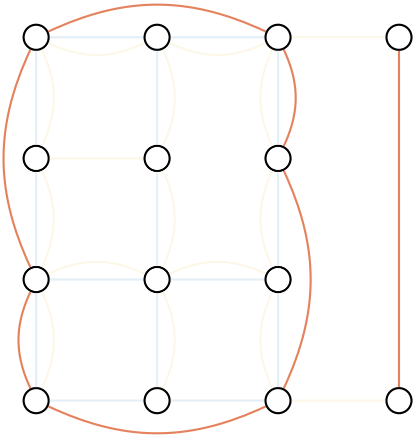


Modeling – Graph Representation

$\mathcal{G}_0$: Free Subgraph

$\mathcal{G}_1$: Subgraph controlled by operator 1

$\mathcal{G}_2$: Subgraph controlled by operator 2

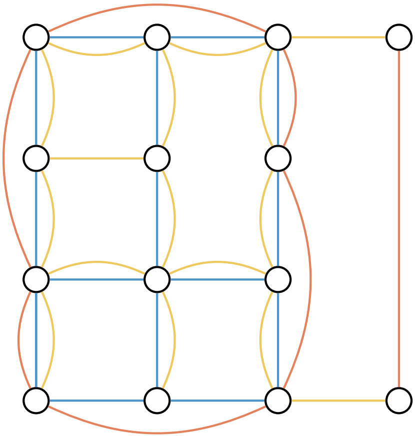


Modeling – Graph Representation

$\mathcal{G}_0$: Free Subgraph

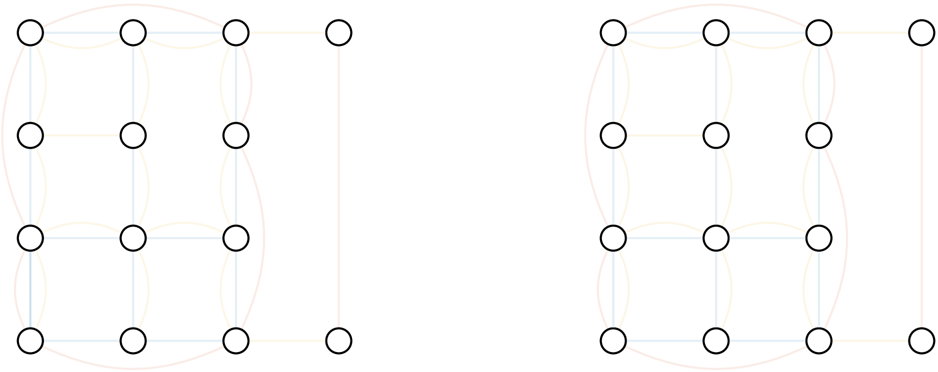
$\mathcal{G}_1$: Subgraph controlled by operator 1

$\mathcal{G}_2$: Subgraph controlled by operator 2



Modeling – Customers

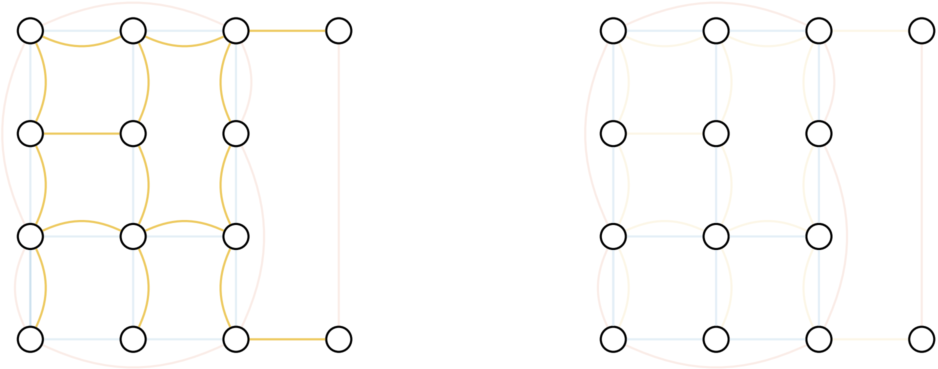
Customers may move:



Modeling – Customers

Customers may move:

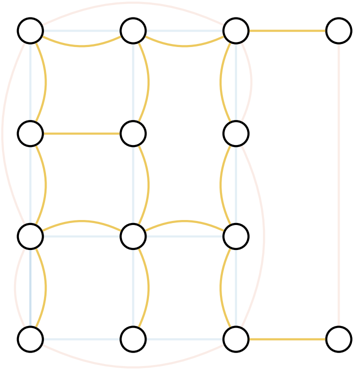
1. on the "free subgraph" $\mathcal{G}_0$, and

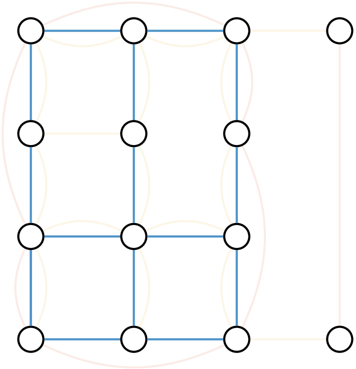


Modeling – Customers

Customers may move:

1. on the "free subgraph" $\mathcal{G}_0$, and
2. on the **fully-connected** operators' subgraphs $\mathcal{G}_1, \ldots, \mathcal{G}_{N_o}$.

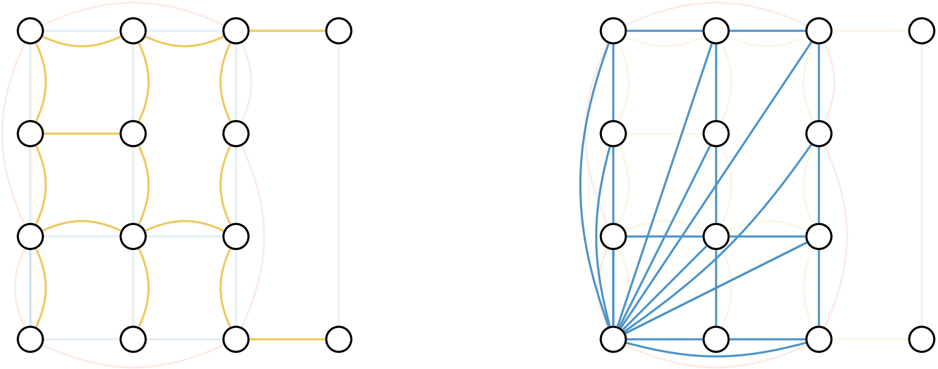




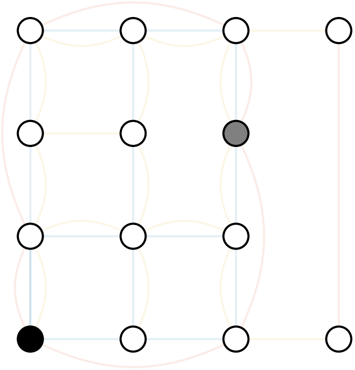
Modeling – Customers

Customers may move:

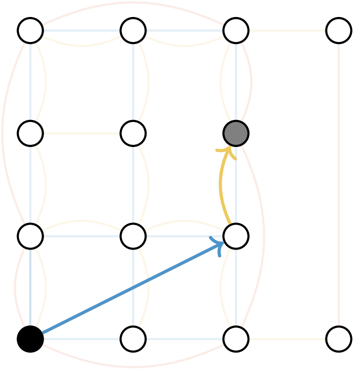
1. on the "free subgraph" $\mathcal{G}_0$, and
2. on the **fully-connected** operators' subgraphs $\mathcal{G}_1, \dots, \mathcal{G}_{N_o}$.



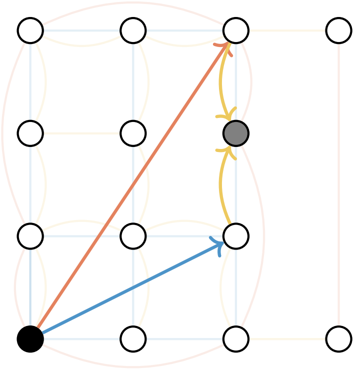
Modeling – Customers



Modeling – Customers



Modeling – Customers



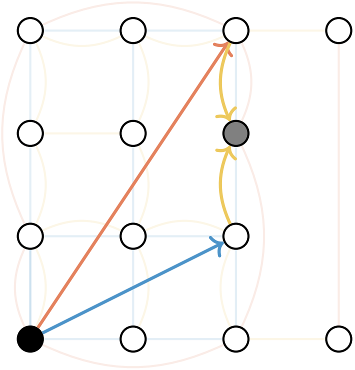
Modeling – Customers

Customers' Route Decision

Select a **reaction curve** ϕ_i:

$$\phi_i(\mathrm{p}) = \alpha \quad \equiv \quad \begin{array}{l}\alpha \text{ customers per unit} \\ \text{time on path p}\end{array}$$

with related cost $J_i(\phi_i, \mathrm{pr}_1, \ldots, \mathrm{pr}_{N_o})$.



Modeling – Customers

Customers' Route Decision

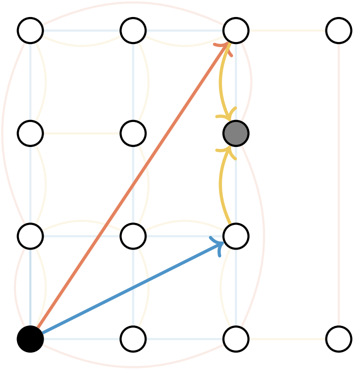
Select a **reaction curve** ϕ_i:

$$\phi_i(\mathrm{p}) = \alpha \quad \equiv \quad \alpha \text{ customers per unit time on path p}$$

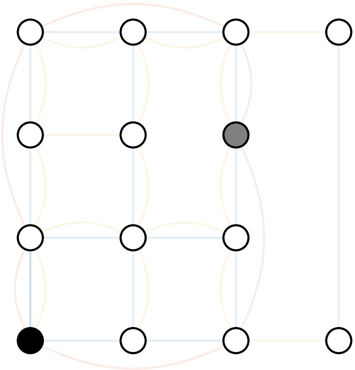
with related cost $J_i(\phi_i, \mathrm{pr}_1, \ldots, \mathrm{pr}_{N_o})$.

Remark (Requirements for ϕ_i)

1. *Demand conservation:* $\phi_i \in \Phi(\mathrm{d}_i)$.
2. *Feasibility:* $\phi_i \in \mathrm{A}_{c,i}$.



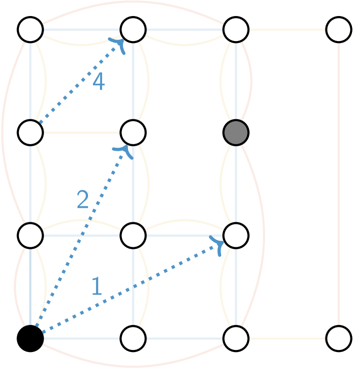
Modeling – Operators



Modeling – Operators

1. Select a pricing strategy pr $\in$ Pr:

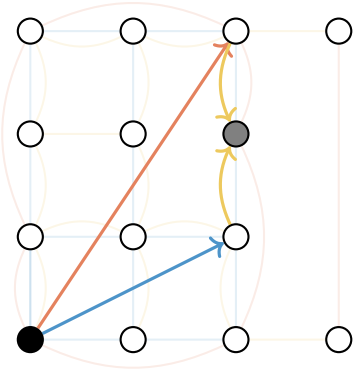
$$\begin{aligned} \text{pr} : \mathcal{V}_j \times \mathcal{V}_j &\to \mathbb{R}_{\geq 0} \cup \{+\infty\} \\ (o, d) &\mapsto \text{price.} \end{aligned}$$



Modeling – Operators

1. Select a pricing strategy $\text{pr} \in \text{Pr}$:

$$\begin{aligned} \text{pr} : \mathcal{V}_j \times \mathcal{V}_j &\to \mathbb{R}_{\geq 0} \cup \{+\infty\} \\ (o, d) &\mapsto \text{price}. \end{aligned}$$

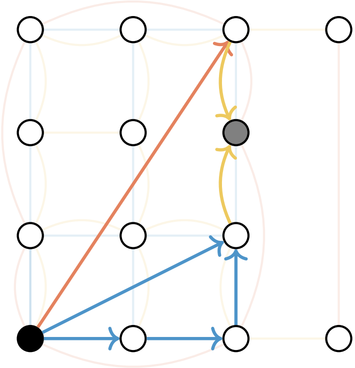


Modeling – Operators

1. Select a pricing strategy $\mathrm{pr} \in \mathrm{Pr}$:

$$\begin{aligned} \mathrm{pr} : \mathcal{V}_j \times \mathcal{V}_j &\to \mathbb{R}_{\geq 0} \cup \{+\infty\} \\ (o, d) &\mapsto \text{price}. \end{aligned}$$

2. Serve each demand i with some flows $\mathrm{F}_i = \{\mathrm{f}_i^1, \ldots, \mathrm{f}_i^{L_i}\}$.

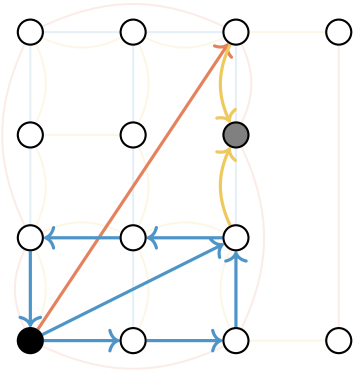


Modeling – Operators

1. Select a pricing strategy $\mathrm{pr} \in \mathrm{Pr}$:

$$\begin{aligned} \mathrm{pr} : \mathcal{V}_j \times \mathcal{V}_j &\to \mathbb{R}_{\geq 0} \cup \{+\infty\} \\ (o, d) &\mapsto \text{price}. \end{aligned}$$

2. Serve each demand i with some flows $\mathrm{F}_i = \{\mathrm{f}_i^1, \ldots, \mathrm{f}_i^{L_i}\}$.
3. Rebalance the system with some flows $\mathrm{F}_0 = \{\mathrm{f}_0^1, \ldots, \mathrm{f}_0^{L_0}\}$.

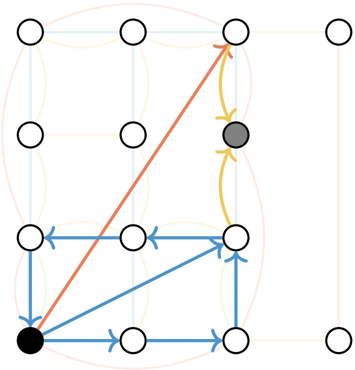


Modeling – Operators

1. Select a pricing strategy pr $\in$ Pr:

$$\begin{aligned} \mathrm{pr} : \mathcal{V}_j \times \mathcal{V}_j &\to \mathbb{R}_{\geq 0} \cup \{+\infty\} \\ (o, d) &\mapsto \text{price}. \end{aligned}$$

2. Serve each demand i with some flows $\mathsf{F}_i = \{\mathsf{f}_i^1, \ldots, \mathsf{f}_i^{L_i}\}$.
3. Rebalance the system with some flows $\mathsf{F}_0 = \{\mathsf{f}_0^1, \ldots, \mathsf{f}_0^{L_0}\}$.



Remark (Requirements for the flows)

1. *Demand satisfaction:* $\mathsf{F}_i \in \mathcal{H}_i(\phi_i)$.
2. *Feasibility:* $(\mathsf{F}_1, \ldots, \mathsf{F}_M, \mathsf{F}_0) \in \mathsf{A}_{o,i}$.

Modeling – Operators

Operators' Profit Maximization

$$\text{Revenue}_j := \sum_{i=1}^{M} \sum_{\mathrm{p}\in\mathcal{S}(\mathrm{d}_i)} \sum_{\substack{a\in\mathrm{p},\\ a\in\bar{\mathcal{A}}_j}} \underbrace{\phi_i(\mathrm{p})}_{\text{Rate}} \cdot \underbrace{\mathrm{pr}_j(\bar{s}_j(a), \bar{t}_j(a))}_{\text{Price}}$$

Modeling – Operators

Operators' Profit Maximization

$$\text{Revenue}_j := \sum_{i=1}^{M} \sum_{\mathrm{p}\in\mathcal{S}(\mathrm{d}_i)} \sum_{\substack{a\in\mathrm{p},\\ a\in\bar{\mathcal{A}}_j}} \underbrace{\phi_i(\mathrm{p})}_{\text{Rate}} \cdot \underbrace{\mathrm{pr}_j(\bar{s}_j(a), \bar{t}_j(a))}_{\text{Price}}$$

$$\text{Cost}_j := \sum_{i=1}^{M} \underbrace{c_j(\mathrm{F}_i)}_{\text{Cost serving demand } i} + \underbrace{c_j(\mathrm{F}_0)}_{\text{Cost rebalancing}}$$

Modeling – Operators

Operators' Profit Maximization

$$\text{Revenue}_j := \sum_{i=1}^{M} \sum_{\mathrm{p} \in \mathcal{S}(\mathrm{d}_i)} \sum_{\substack{a \in \mathrm{p}, \\ a \in \bar{\mathcal{A}}_j}} \overbrace{\phi_i(\mathrm{p})}^{\text{Rate}} \cdot \overbrace{\mathrm{pr}_j(\bar{s}_j(a), \bar{t}_j(a))}^{\text{Price}}$$

$$\text{Cost}_j := \min_{\substack{\mathrm{F}_i \in \mathcal{H}_i(\phi_i), \\ \mathrm{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_j)}, \\ (\{\mathrm{F}_i\}_{i=1}^{M}, \mathrm{F}_0) \in \mathrm{A}_{\mathrm{o},j}}} \sum_{i=1}^{M} \underbrace{c_j(\mathrm{F}_i)}_{\text{Cost serving demand } i} + \overbrace{c_j(\mathrm{F}_0)}^{\text{Cost rebalancing}}$$

Modeling – Operators

Operators' Profit Maximization

$$\text{Revenue}_j := \sum_{i=1}^{M} \sum_{\mathrm{p} \in \mathcal{S}(\mathrm{d}_i)} \sum_{\substack{a \in \mathrm{p},\\ a \in \bar{\mathcal{A}}_j}} \underbrace{\phi_i(\mathrm{p})}_{\text{Rate}} \cdot \underbrace{\mathrm{pr}_j(\bar{s}_j(a), \bar{t}_j(a))}_{\text{Price}}$$

$$\text{Cost}_j := \min_{\substack{\mathrm{F}_i \in \mathcal{H}_i(\phi_i),\\ \mathrm{F}_0 \in 2^{\mathcal{F}(\mathcal{G}_j)},\\ (\{\mathrm{F}_i\}_{i=1}^{M}, \mathrm{F}_0) \in \mathrm{A}_{\mathrm{o},j}}} \sum_{i=1}^{M} \underbrace{c_j(\mathrm{F}_i)}_{\text{Cost serving demand } i} + \underbrace{c_j(\mathrm{F}_0)}_{\text{Cost rebalancing}}$$

Hence

$$U_j(\mathrm{pr}_j, \{\phi_i\}_{i=1}^{M}) := \text{Revenue}_j - \text{Cost}_j$$

Customers Equilibrium

Definition (Customer Equilibrium)

The reaction curve $\phi_i^\star$ is an equilibrium for the demand d_i if

$$J_i(\phi_i^\star, \mathrm{pr}_1, \dots, \mathrm{pr}_{N_o}) \leq J_i(\phi_i, \mathrm{pr}_1, \dots, \mathrm{pr}_{N_o}) \quad \forall\, \phi_i \in \Phi(\mathrm{d}_i) \cap \mathrm{A}_{c,i}$$

The set of equilibria is $\mathcal{E}_i(\mathrm{pr}_1, \dots, \mathrm{pr}_{N_o})$.

Game Equilibrium

Definition (Game equilibrium)

The reaction curves and the pricing strategies $(\{\phi_i^\star\}_{i=1}^M, \{\mathrm{pr}_j^\star\}_{j=1}^{N_o}) \in \prod_{i=1}^M \Phi(\mathrm{d}_i) \cap \mathrm{A}_{c,i} \times \prod_{j=1}^{N_o} \mathrm{Pr}_j$ *are an equilibrium if*

1. *the customers are at equilibrium, and*
2. *no operator can increase her profit by unilaterally deviating from her pricing strategy.*

Game Equilibrium

Definition (Game equilibrium)

The reaction curves and the pricing strategies $(\{\phi_i^\star\}_{i=1}^{M}, \{\mathrm{pr}_j^\star\}_{j=1}^{N_o}) \in \prod_{i=1}^{M} \Phi(\mathrm{d}_i) \cap \mathrm{A}_{c,i} \times \prod_{j=1}^{N_o} \mathrm{Pr}_j$ *are an equilibrium if*

1. *the customers are at equilibrium, and*
2. *no operator can increase her profit by unilaterally deviating from her pricing strategy.*

Game Equilibrium

Definition (Game equilibrium)

The reaction curves and the pricing strategies $(\{\phi_i^\star\}_{i=1}^M, \{\mathrm{pr}_j^\star\}_{j=1}^{N_o}) \in \prod_{i=1}^M \Phi(\mathrm{d}_i) \cap \mathrm{A}_{c,i} \times \prod_{j=1}^{N_o} \mathrm{Pr}_j$ *are an equilibrium if*

1. *the customers are at equilibrium, and*
2. *no operator can increase her profit by unilaterally deviating from her pricing strategy.*

Game Equilibrium

Definition (Game equilibrium)

The reaction curves and the pricing strategies $(\{\phi_i^\star\}_{i=1}^M, \{\mathrm{pr}_j^\star\}_{j=1}^{N_o}) \in \prod_{i=1}^M \Phi(\mathrm{d}_i) \cap \mathrm{A}_{c,i} \times \prod_{j=1}^{N_o} \mathrm{Pr}_j$ *are an equilibrium if*

1. *the customers are at equilibrium, and*
2. *no operator can increase her profit by unilaterally deviating from her pricing strategy.*

Formally, $(\{\phi_i^\star\}_{i=1}^M, \{\mathrm{pr}_j^\star\}_{j=1}^{N_o})$ *is a equilibrium if*

1. *for all* $i \in \{1, \ldots, M\}$
$$\phi_i^\star \in \mathcal{E}_i(\mathrm{pr}_1^\star, \ldots, \mathrm{pr}_{N_o}^\star).$$
2. *for all* $j \in \{1, \ldots, N_o\}$
$$U_j(\mathrm{pr}_j^\star, \{\mathcal{E}_i(\mathrm{pr}_1^\star, \ldots, \mathrm{pr}_{N_o}^\star)\}_{i=1}^M) \geq U_j(\mathrm{pr}_j, \{\mathcal{E}_i(\mathrm{pr}_1^\star, \ldots, \mathrm{pr}_j, \ldots, \mathrm{pr}_{N_o}^\star)\}_{i=1}^M), \quad \forall\, \mathrm{pr}_j \in \mathrm{Pr}_j.$$

AMoD Framework – Settings

Players:

AMoD Framework – Settings

Players:

- M demands

AMoD Framework – Settings

Players:

- M demands
- Two operators:

Name	Graph	Pricing Strategies Set

AMoD Framework – Settings

Players:

- M demands
- Two operators:

	Name	Graph	Pricing Strategies Set
Operator 1	AMoD System	$\mathcal{G}_1$	$\mathrm{Pr}_1 = \bar{\mathbb{R}}_{\geq 0}^{\mathcal{V}_1 \times \mathcal{V}_1} \equiv$ All nonnegative functions

AMoD Framework – Settings

Players:

- M demands
- Two operators:

	Name	Graph	Pricing Strategies Set
Operator 1	AMoD System	$\mathcal{G}_1$	$\mathrm{Pr}_1 = \bar{\mathbb{R}}_{\geq 0}^{\mathcal{V}_1 \times \mathcal{V}_1} \equiv$ All nonnegative functions
Operator 2	PTA/Municipality	$\mathcal{G}_2$	$\mathrm{Pr}_2 = \{\mathrm{pr}_2\}$

AMoD Framework – Settings

Players:

- M demands
- Two operators:

	Name	Graph	Pricing Strategies Set
Operator 1	AMoD System	$\mathcal{G}_1$	$\mathrm{Pr}_1 = \bar{\mathbb{R}}_{\geq 0}^{\mathcal{V}_1 \times \mathcal{V}_1} \equiv$ All nonnegative functions
Operator 2	PTA/Municipality	$\mathcal{G}_2$	$\mathrm{Pr}_2 = \{\mathrm{pr}_2\}$

Assumptions

- Time-invariant setting.
- The time from o to d through path p is known a priori.
- Multimodal route selection.

AMoD Framework – Customers

- Multimodal choice:

AMoD Framework – Customers

- Multimodal choice:

  $p_{\mathrm{AMoD},i}$: AMoD path

AMoD Framework – Customers

- Multimodal choice:

  $p_{\mathrm{AMoD},i}$: AMoD path

  $p_{\mathrm{PT},i}$: public transport and walking path

AMoD Framework – Customers

- Multimodal choice:

  $\mathrm{p}_{\mathrm{AMoD},i}$: AMoD path

  $\mathrm{p}_{\mathrm{PT},i}$: public transport and walking path

$$\Rightarrow A_{c,i} = \{\phi \mid \phi(\mathrm{p}) = 0 \,\forall\, \mathrm{p} \neq \mathrm{p}_{\mathrm{AMoD},i}, \mathrm{p}_{\mathrm{PT},i}\}$$

AMoD Framework – Customers

- Multimodal choice:

  $\mathrm{p}_{\mathrm{AMoD},i}$: AMoD path

  $\mathrm{p}_{\mathrm{PT},i}$: public transport and walking path

$$\Rightarrow \mathrm{A}_{\mathrm{c},i} = \{\phi \mid \phi(\mathrm{p}) = 0 \,\forall\, \mathrm{p} \neq \mathrm{p}_{\mathrm{AMoD},i}, \mathrm{p}_{\mathrm{PT},i}\}$$

- Monetary costs of fares and time:

$$J_i(\phi, \mathrm{pr}_1, \mathrm{pr}_2) = (\mathrm{pr}_1(o, d) + V_\mathrm{T} \cdot t_{\mathrm{AMoD},i}) \cdot \phi(\mathrm{p}_{\mathrm{AMoD},i}) + (\mathrm{pr}_{\mathrm{PT},i} + V_\mathrm{T} \cdot t_{\mathrm{PT},i}) \cdot \phi(\mathrm{p}_{\mathrm{PT},i}).$$

AMoD Framework – Customers

- Multimodal choice:

$\mathrm{p}_{\mathrm{AMoD},i}$: AMoD path

$\mathrm{p}_{\mathrm{PT},i}$: public transport and walking path

$$\Rightarrow \mathrm{A}_{\mathrm{c},i} = \{\phi \mid \phi(\mathrm{p}) = 0 \,\forall\, \mathrm{p} \neq \mathrm{p}_{\mathrm{AMoD},i}, \mathrm{p}_{\mathrm{PT},i}\}$$

- Monetary costs of fares and time:

$$J_i(\phi, \mathrm{pr}_1, \mathrm{pr}_2) = (\mathrm{pr}_1(o,d) + V_{\mathrm{T}} \cdot t_{\mathrm{AMoD},i}) \cdot \phi(\mathrm{p}_{\mathrm{AMoD},i}) + (\mathrm{pr}_{\mathrm{PT},i} + V_{\mathrm{T}} \cdot t_{\mathrm{PT},i}) \cdot \phi(\mathrm{p}_{\mathrm{PT},i}).$$

Equilibrium

$$\phi_i = \operatorname*{arg\,min}_{\phi \in \Phi(\mathrm{d}_i) \cap \mathrm{A}_{\mathrm{c},i}} J_i(\phi, \mathrm{pr}_1, \mathrm{pr}_2)$$

AMoD Framework – Customers

- Multimodal choice:

  $p_{\mathrm{AMoD},i}$: AMoD path

  $p_{\mathrm{PT},i}$: public transport and walking path

  $\Rightarrow A_{c,i} = \{\phi \mid \phi(p) = 0 \, \forall \, p \neq p_{\mathrm{AMoD},i}, p_{\mathrm{PT},i}\}$

- Monetary costs of fares and time:

$$J_i(\phi, \mathrm{pr}_1, \mathrm{pr}_2) = (\mathrm{pr}_1(o, d) + V_\mathrm{T} \cdot t_{\mathrm{AMoD},i}) \cdot \phi(p_{\mathrm{AMoD},i}) + (\mathrm{pr}_{\mathrm{PT},i} + V_\mathrm{T} \cdot t_{\mathrm{PT},i}) \cdot \phi(p_{\mathrm{PT},i}).$$

Equilibrium

$$\phi_i = \mathbb{E}_{V_\mathrm{T}}\left[\operatorname*{arg\,min}_{\phi \in \Phi(d_i) \cap A_{c,i}} J_i(\phi, \mathrm{pr}_1, \mathrm{pr}_2)\right]$$

AMoD Framework – AMoD Operator

- She assigns requests to vehicles (selecting flows).

AMoD Framework – AMoD Operator

- She assigns requests to vehicles (selecting flows).
- Vehicles must be conserved and are limited:

$$\mathsf{A}_{o,1} = \left\{ (\mathsf{F}_1, \ldots, \mathsf{F}_M, \mathsf{F}_0) \,\middle|\, (\mathsf{F}_1, \ldots, \mathsf{F}_M, \mathsf{F}_0) \text{ is balanced} \wedge \text{number of cars} \leq N_{\text{veh}} \right\}.$$

AMoD Framework – AMoD Operator

- She assigns requests to vehicles (selecting flows).
- Vehicles must be conserved and are limited:

$$A_{o,1} = \left\{ (F_1, \dots, F_M, F_0) \,\middle|\, (F_1, \dots, F_M, F_0) \text{ is balanced} \wedge \text{number of cars} \leq N_{\text{veh}} \right\}.$$

- Vehicles flow $F = \{f_1, \dots, f_N\}$ cost:

$$c_{o,1}(F) = \sum_{f \in F} \chi_{\text{rate}}(f) \sum_{a \in \chi_{\text{path}}(f)} c_{d,1}(a)$$

Equilibrium

Theorem (Equilibrium)

If customers have a uniformly distributed value of time, then:

Equilibrium

Theorem (Equilibrium)

If customers have a uniformly distributed value of time, then:

- *The game has a (possibly non-unique) equilibrium.*

Equilibrium

Theorem (Equilibrium)

If customers have a uniformly distributed value of time, then:

- *The game has a (possibly non-unique) equilibrium.*
- *Consider the reaction curves* $\{\phi_i^\star\}_{i=1}^M$ *and the pricing strategies* $\mathrm{pr}_1^\star$ *and* $\mathrm{pr}_2^\star$ *such that*
  1. $\mathrm{pr}_1^\star(o, d) = 0$ *if there is no demand from* o *to* d*;*
  2. $\mathrm{pr}_1^\star(o, d) = p^\star$ *where* $p^\star$ *is the solution of a convex quadratic program;*
  3. $\mathrm{pr}_2^\star(o, d) = \mathrm{pr}_2(o, d)$*;*
  4. $\phi_i^\star \in \mathcal{E}_i(\mathrm{pr}_1^\star, \mathrm{pr}_2^\star)$.

  Then, $(\{\phi_i^\star\}_{i=1}^M, \mathrm{pr}_1^\star, \mathrm{pr}_2^\star)$ *is an equilibrium.*

Equilibrium

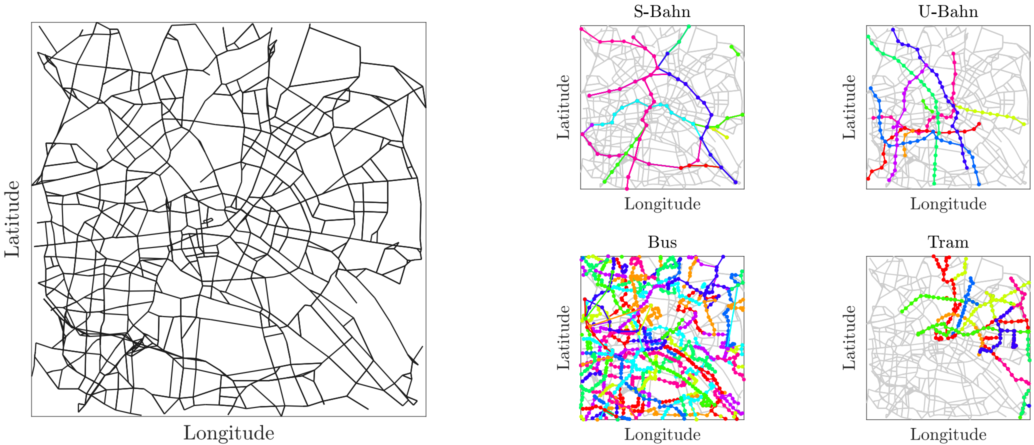
Theorem (Equilibrium)

If customers have a uniformly distributed value of time, then:

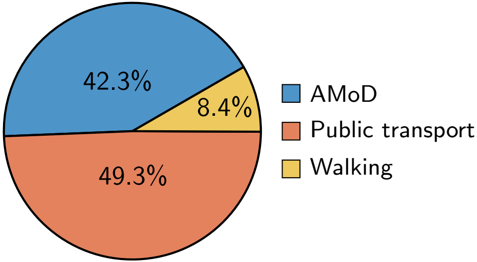
- *The game has a (possibly non-unique) equilibrium.*
- *Consider the reaction curves* $\{\phi_i^\star\}_{i=1}^M$ *and the pricing strategies* $\mathrm{pr}_1^\star$ *and* $\mathrm{pr}_2^\star$ *such that*
  1. $\mathrm{pr}_1^\star(o,d) = 0$ *if there is no demand from* o *to* d*;*
  2. $\mathrm{pr}_1^\star(o,d) = p^\star$ *where* $p^\star$ *is the solution of a convex quadratic program;*
  3. $\mathrm{pr}_2^\star(o,d) = \mathrm{pr}_2(o,d)$*;*
  4. $\phi_i^\star \in \mathcal{E}_i(\mathrm{pr}_1^\star, \mathrm{pr}_2^\star)$.

  Then, $(\{\phi_i^\star\}_{i=1}^M, \mathrm{pr}_1^\star, \mathrm{pr}_2^\star)$ *is an equilibrium.*
- *All equilibria result in the same profit and customers' reaction curves.*

Case Study – Berlin, Germany ($\sim$ 9,000 requests, evening peak)

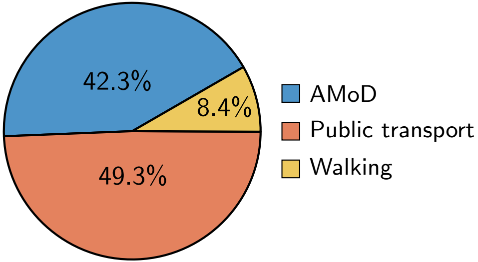


Results – Base Case (fleet of $\sim$ 8,000 vehicles)

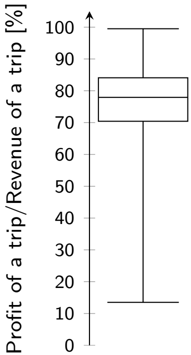


Approx. equal modal split
among AMoD and public transport.

Results – Base Case (fleet of $\sim$ 8,000 vehicles)

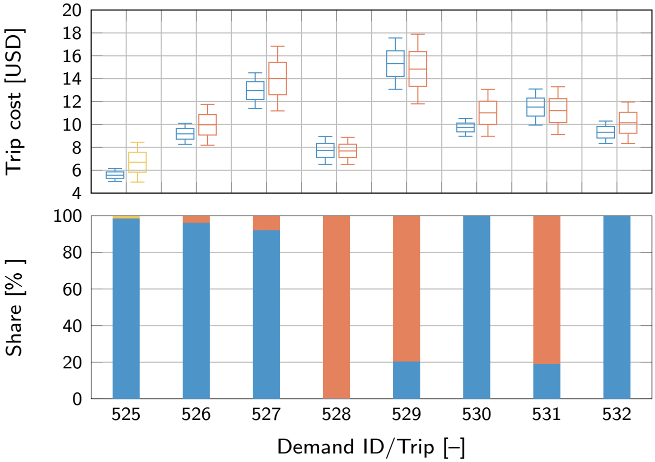


Approx. equal modal split among AMoD and public transport.



Most AMoD trips yield a high profit.

Results – Base Case (fleet of $\sim$ 8,000 vehicles)



At microscopic level, the modal split appears to be less balanced.

Results – Sensitivity of the Equilibrium

AMoD Operator

1. Different vehicles
2. Larger fleet size
3. Heterogenous prices

Municipality

1. Lower public transport prices
2. More efficient public transport infrastructure
3. AMoD service tax

Results – Sensitivity of the Equilibrium

AMoD Operator

1. Different vehicles
2. Larger fleet size
3. Heterogenous prices

Municipality

1. Lower public transport prices
2. More efficient public transport infrastructure
3. AMoD service tax

Results – Sensitivity of the Equilibrium

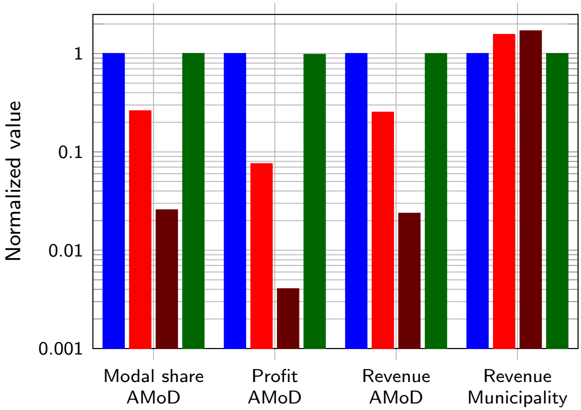
AMoD Operator

1. **Different vehicles**
2. Larger fleet size
3. Heterogenous prices

Municipality

1. **Lower public transport prices**
2. More efficient public transport infrastructure
3. **AMoD service tax**

Results – Vehicles

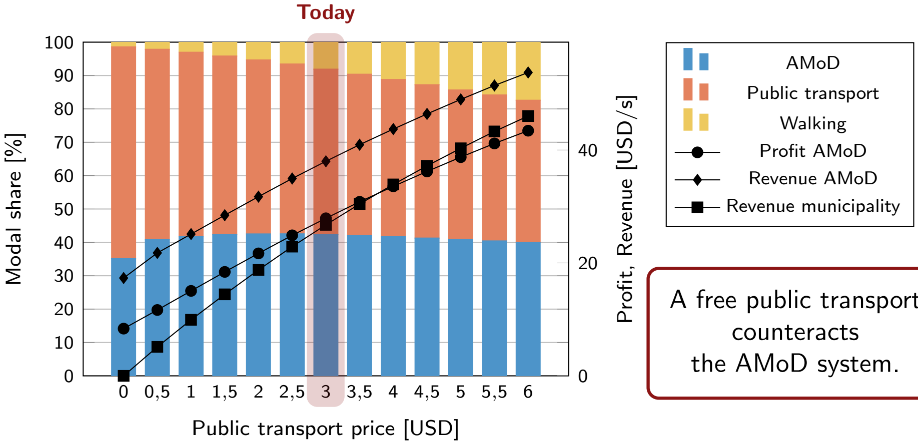


Nominal	0.34 USD/km
Non-autonomous (Low-wage)	1.83 USD/km
Non-autonomous (High-wage)	3.26 USD/km
Non-electrified	0.36 USD/km

Data from:

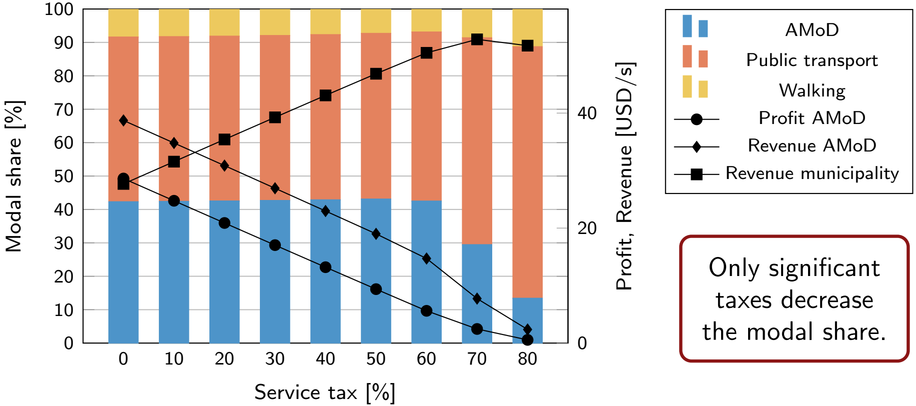
Cost-based analysis of autonomous mobility services [Bösch et al., 2017]

Results – Public Transport Price



A free public transport counteracts the AMoD system.

Results – AMoD Service Tax



Only significant taxes decrease the modal share.

Conclusions

Summary

- General game-theoretical framework for transportation systems.
- Specific framework for an AMoD system competing with the public transport.
- In our case study, the AMoD system attracts 42% of the customers.

Conclusions

Summary

- General game-theoretical framework for transportation systems.
- Specific framework for an AMoD system competing with the public transport.
- In our case study, the AMoD system attracts 42% of the customers.

Managerial Insights

- Vehicles autonomy significantly affects the equilibrium.
- A free public transportation service counteracts the AMoD operator.
- Imposing high taxes on an AMoD system can impact the modal split.

Conclusions

Summary

- General game-theoretical framework for transportation systems.
- Specific framework for an AMoD system competing with the public transport.
- In our case study, the AMoD system attracts 42% of the customers.

Managerial Insights

- Vehicles autonomy significantly affects the equilibrium.
- A free public transportation service counteracts the AMoD operator.
- Imposing high taxes on an AMoD system can impact the modal split.

Outlook

- Competition between multiple AMoD operators.
- Intermodal route selection.

References

- Time in traffic: INRIX.
- Congestion: International Parking Institute (IPI) 2012 Emerging Trends in Parking Study.
- Emissions: Statistical pocketbook 2018.
- Benefits autonomous vehicles: Aptiv, World Economic Forum, and BCG.

Case Study – Data

Road Network: OpenStreetMap.

Public Transit Network: GTFS (topology and travel time).

Origin-destination pairs: MatSim scenario Berlin (scaled with a factor 10).

Considered area: We have:

- 16 km $\times$ 16 km,
- 9052 travel requests (12.8 travel demands per second).

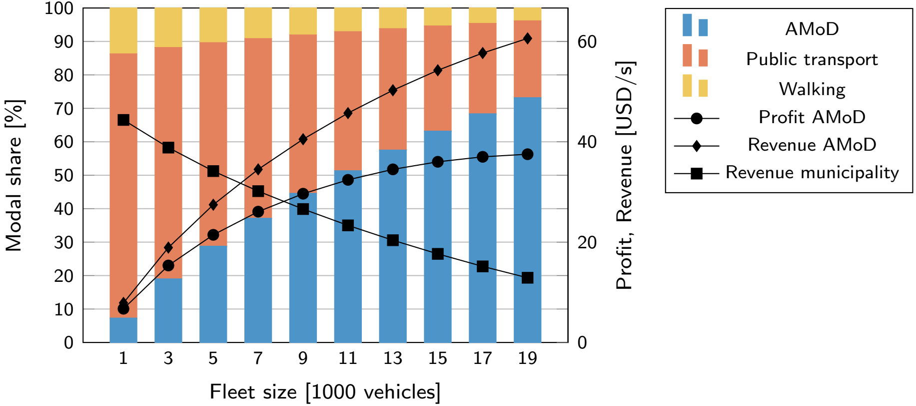
Case Study – Parameters

Parameter	Value
Public transit price	3.12 USD
Value of time minimum	10 USD/h
Value of time maximum	17 USD/h
Operation cost	0.34 USD/km
Walking velocity	1.4 m/s
Average wait S-Bahn/U-Bahn	2.5 min
Average wait tram	3.5 min
Average wait bus	5 min

Case Study – Fleet Size

City	Number of registered cars	Number of taxi licenses	Percentage
Berlin	1,344,000	8,373	0.6%
New York City	3,000,000	13,237	0.4%
San Francisco	494,000	1,800	0.4%

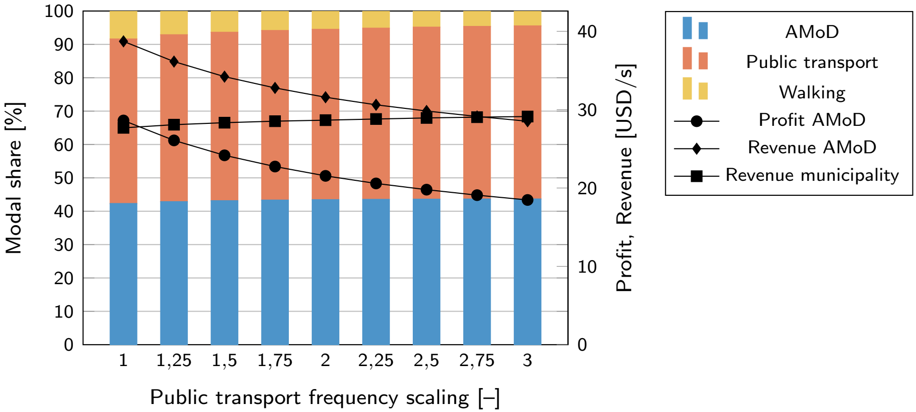
Results – Fleet Size



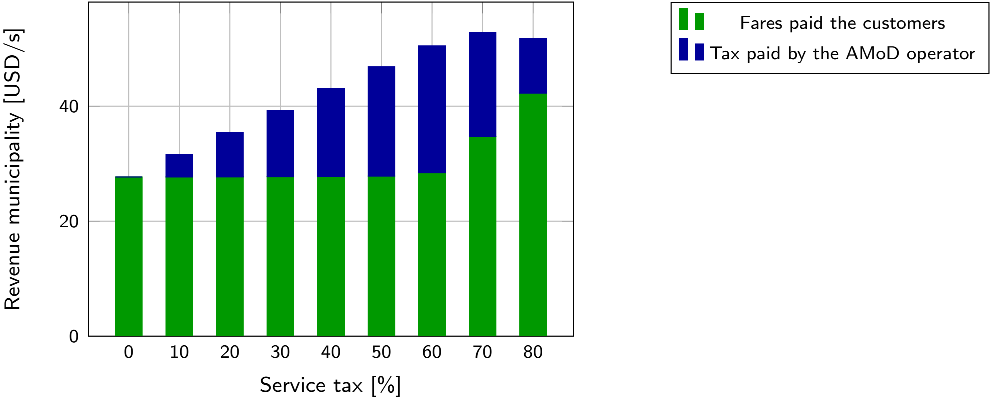
Results – Customers Heterogenity

	Change
Profit AMoD	$+0.3\%$
AMoD modal share	-0.4%
Revenue Municipality	$+0.1\%$

Results – Public Transit Infrastructure



Results – AMoD Service Tax



Results – AMoD Service Tax