1 Comparison of Finite Automata

Here are two simple finite automata:

For each, we have a one bit encoding for the states ($x_A$ and $x_B$), one binary output ($y_A$ and $y_B$), and one common binary input ($u$). We want to verify whether or not these two automata are equivalent. This can be done through the following steps:

a) Express the characteristic function of the transition relation for both automaton, $\psi_r(x, x', u)$.

b) Express the joint transition function, $\psi_f$.
   Reminder: $\psi_f(x_A, x'_A, x_B, x'_B) = (\exists u : \psi_A(x_A, x'_A, u) \cdot \psi_B(x_B, x'_B, u))$.

c) Express the characteristic function of the reachable states, $\psi_X(x_A, x_B)$.

d) Express the characteristic function of the reachable output, $\psi_Y(y_A, y_B)$.

e) Are the automata equivalent? Justify with a simple calculus.

2 Temporal Logic

a) We consider the following automaton. The property $a$ is true on states 0 and 3.

For each of the following CTL formula, list all the states for which it holds true.
(i) EF $a$

(ii) EX AX $a$

(iii) EF ( $a$ AND EX NOT($a$) )

b) Given the transition function $\psi_f(x, x')$ and the characteristic function $\psi_Z(x)$ for a set $Z$, write a small pseudo-code which returns the characteristic function of $\psi_{AFZ}(x)$. It can be expressed as symbolic boolean functions, like $x_A x'_A x_B x'_B + x_A x'_A x_B x'_B$.

**Hint:** To do this, simply use the classic boolean operators AND, OR, NOT and $! =$. You can also use an existence selector EXISTS($a$). For a given argument $a$, it returns the set $\{x : \exists x', a(x, x') \text{ is true}\}$.

**Hint:** It can be useful to reformulate AFZ as another CTL formula.