Feedback control

How well can you control a given system or plant?

Limitations arise from:
- Fundamental system/plant properties;
- Limits in knowledge about the plant;
- Variability in the plant.

Can you tell, in advance, whether a plant will be easy or hard to control?
Feedback control

Objectives:
▶ Closed-loop stability
▶ Reference tracking
▶ Disturbance rejection
▶ Noise response

Difficulties:
▶ Model errors
▶ Fundamental limits on controllability of $G(s)$
▶ Actuation constraints
Transfer functions

Loop transfer function
\[ L(s) = G(s)K(s) \]

Sensitivity function
\[ S(s) = \frac{1}{1 + G(s)K(s)} = \frac{1}{1 + L(s)} \]

Complementary sensitivity
\[ T(s) = \frac{G(s)K(s)}{1 + G(s)K(s)} = \frac{L(s)}{1 + L(s)} \]

Output response
\[ y = T(s)r + S(s)G_d(s)d - T(s)n \]

Error response
\[ e = r - y = S(s)r - S(s)G_d(s)d + T(s)n \]

Conflicting objectives

Performance requirements

Reference tracking \[ T(s) \approx 1 \]
Noise rejection \[ T(s) \ll 1 \]
Disturbance rejection \[ S(s)G_d(s) \ll 1 \]
Low closed-loop plant sensitivity \[ S(s) \ll 1 \]

Constraint
\[ S(s) + T(s) = 1 \] for all \( s \)
Sensitivity

Sensitivity to plant gain changes

\[ S(s) = \frac{\text{relative closed-loop response change}}{\text{relative open-loop response change}} = \frac{dT(s)/T(s)}{dG(s)/G(s)} \]

A change of \( \alpha \) percent in the open-loop plant DC gain gives a magnitude change of \(|\alpha S(0)|\) percent in the closed-loop DC gain.

Closed-loop performance

\[ G(s) = \frac{5 e^{-0.1s}}{(s+1)(0.1s+1)} \quad K(s) = \frac{0.5s + 1}{s} \]
Closed-loop performance

\[ L(j\omega) = G(j\omega)K(j\omega) \]

\( \omega_B \): Frequency at which \(|S(j\omega)| = -3\text{dB} = 1/\sqrt{2} \).

\( \omega_c \): Frequency at which \(|L(j\omega)| = 1 \).

If \( \text{PM} < 90^\circ \) then, \( \omega_B < \omega_c < \omega_{BT} \)

\( \text{Maximum control frequency} \): Frequency where \(|K(j\omega)| \) is still significant.

(The \( \omega_c < \omega_{BT} \) statement is true for “reasonable” control designs)
Closed-loop performance

Maximum peak criteria

\[ M_S = \max_\omega |S(j\omega)| \quad \text{and} \quad M_T = \max_\omega |T(j\omega)| \]

\[ = \|S(s)\|_{\mathcal{H}_\infty} = \|T(s)\|_{\mathcal{H}_\infty} \]

Typical specifications:

\[ M_S \leq 2 \quad \text{and} \quad M_T < 1.25 \]

Gain and phase margins:

\[ GM \geq \frac{M_S}{M_S - 1}, \quad PM \geq 2 \arcsin \left( \frac{1}{2M_S} \right) \geq \frac{1}{M_S} \text{ (rad)} \]

Alternative structures

2 degrees-of-freedom structure

\[ u = K(s) \begin{bmatrix} r \\ y_m \end{bmatrix} = \begin{bmatrix} K_r(s) & K_y(s) \end{bmatrix} \begin{bmatrix} r \\ y_m \end{bmatrix} \]

\[ L(s) = -G(s)K_y(s) \]

\[ y = S(s)G_d(s) d + S(s)G(s)K_r(s) r + S(s)G(s)K_y(s) n \]
Alternative structures

Additional loop dynamics

\[ L(s) = G(s)K(s)G_m(s) \]

\[ y = S(s)G_d(s)d + S(s)G(s)K(s)r - T(s)n \]

A general structure

\[ e = \text{performance outputs} \quad w = \text{exogenous inputs} \]

\[ y_m = \text{controller inputs} \quad u = \text{control actuation} \]
Perturbed systems

Nominal case:

\[ G(s) + G_d(s) \]

Perturbed case:

\[ G_\Delta(s) + G_d\Delta(s) \]

\[ G_\Delta(s) = \{ G(s) + \Delta(s) | \Delta(s) \in \text{Set} \} \]

\[ G_d\Delta(s) = \{ G_d(s) + \Delta_d(s) | \Delta_d(s) \in \text{Set} \} \]

Uncertainty

Sources of uncertainty:

- Nonlinear dynamics.
- Operating point variation.
- Neglected dynamics in the model.
- Non-repeatable dynamics.

Which dynamics should we model accurately?
Robustness

Closed-loop configuration:

\[ G \Delta (s) + Gd \Delta (s) + K(s) + dyum - G \Delta (s) = \{ G(s) + \Delta(s) | \Delta(s) \in \text{Set} \} \]
\[ Gd \Delta (s) = \{ Gd(s) + \Delta_d(s) | \Delta_d(s) \in \text{Set} \} \]

What happens for the different \( \Delta \) that may occur in practice?

Bode plot: margins

Magnitude

\[ L(j\omega) \]
\[ G(j\omega) \]
\[ K(j\omega) \]

Phase (deg.)

\[ L(j\omega) \]
\[ G(j\omega) \]
\[ K(j\omega) \]

GM
PM
Nyquist plot: margins

\[ G(s) = \frac{5 e^{-0.1s}}{(s + 1)(0.1s + 1)} \quad K(s) = \frac{0.5s + 1}{s} \]

Nyquist plot: gain perturbation

\[ G(s) = \frac{k e^{-0.1s}}{(s + 1)(0.1s + 1)} \quad k \in [5, 10] \quad K(s) = \frac{0.5s + 1}{s} \]
Nyquist plot: delay perturbation

\[ G(s) = \frac{5 e^{-\tau s}}{(s + 1)(0.1s + 1)}, \quad \tau \in [0.1, 0.2] \]

\[ K(s) = \frac{0.5s + 1}{s} \]

Robustness objectives

**Nominal Stability (NS)**
Closed-loop system stable with no model uncertainty.

**Nominal Performance (NP)**
Closed-loop system satisfies the performance requirements with no model uncertainty.

**Robust Stability (RS)**
Closed-loop system is stable for all models in a prescribed set.

**Robust Performance (RP)**
Closed-loop system satisfies the performance requirements for all models in a prescribed set.
Control design

Main approaches:

Loop shaping

Design $K(s)$ so that the loop, $L(s)$, has the required properties (classical approach).

Signal-based optimal control

Design $K(s)$ to satisfy certain closed-loop system or signal objectives. For example: LQG methods.

Numerical optimisation-based

Use multi-objective optimisation with closed-loop and robustness objectives.

Notes and references

Skogestad & Postlethwaite (2nd Ed.)

Introduction: Sections 1.1 – 1.6
Feedback control: Section 2.2.
Closed-loop performance: Sections 2.3 & 2.4
Controller design: Section 2.5